Correlation of Black Oil Properties at Pressures Below Bubblepoint Pressure—A New Approach

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Abstract

Virtually all of the published correlations for solution gas-oil ratios for pressures at and below bubblepoint pressures suffer from two major deficiencies. First, calculated values of solution gas-oil ratios do not match the concave up, point of inflection, concave down shapes as pressure declines below the bubblepoint that are evident in experimental data, especially at high initial solution gas-oil ratios. Second, bubblepoint pressure must be estimated using the same correlations; there is no convenient way to impose a known bubblepoint pressure—such as one derived from reservoir pressure data. We propose a correlation where the equations are formulated to solve both of these problems.

Published correlations for oil formation volume factors for pressures at and below bubblepoint pressures suffer the same two deficiencies plus a third problem: the material balance connection of oil formation volume factors, solution gas-oil ratios, and reservoir oil densities is not honored. We propose an oil formation volume factor calculation which is based on the solution gas-oil ratio correlation described above so that the first two deficiencies are alleviated. Further, the proposed correlation connects these fluid properties in material balance format.

In addition to solving the deficiencies discussed above, the proposed correlations fit the data set of 2097 laboratory measured values much more closely than other published correlations.

Also, we propose a modification of an existing bubblepoint pressure correlation which can be used in the event that a field-data derived bubblepoint pressure is not available.

This correlation fits the data set of 728 laboratory measured bubblepoint pressures more closely than other published correlations.

All three of the proposed correlations require the usual field data; solution gas-oil ratio at the bubblepoint, reservoir temperature, separator gas specific gravity, and stock tank oil gravity.

Introduction

Many correlations for estimating crude oil PVT properties have been published in the past 50 years. Most of these correlations yield reasonably accurate results when applied at the bubblepoint pressure. However, for pressures below the bubblepoint, the computed PVT properties may have considerable error.

The results of several correlations [Standing(1), Vasquez and Beggs(2), Petrosky and Farshad(3), and Kartoatmodjo and Schmidt(4)] were compared with laboratory data. To illustrate the problem, two sets of laboratory data were selected: one with relatively high initial solution gas-oil ratio and one with low initial solution gas-oil ratio. Figures 1 and 2 show the typical concave up, point of inflection, concave down shapes of both solution gas-oil ratios and oil formation factors as pressure declines below the bubblepoint for a reservoir oil with high initial gas-in-solution. Figures 3 and 4 show data which illustrate that the concave up
portion of the curves are more nearly linear for oils with lower initial gas-in-solution.

Note that all the correlations result in curves which do not have the correct shapes. This is important because many applications of these fluid properties require differences as pressure declines, i.e., the slopes of the lines are critical. The incorrect shapes produced by these correlations persist throughout the entire data set.

This paper introduces new correlations to model the performance of the solution gas-oil-ratio, \( R_s \), and the oil formation volume factor, \( B_o \), for black oils for pressures at and below the bubblepoint. The solution gas-oil ratio correlation is based on a “reduced” variable approach which permits use of bubblepoint pressure from any source.

Oil formation volume factors are calculated with a material balance equation using solution gas-oil ratios and oil densities.

The paper also proposes a bubblepoint pressure correlation developed using reservoir temperature, gas specific gravity, oil gravity, and the solution gas-oil ratio (at the bubblepoint pressure) which fits our data set more closely than the other correlations.

Data Description

Data from a total of 195 PVT laboratory reports were used to develop the solution gas-oil-ratio correlation and to verify the oil formation volume factor calculations. The “measured” data were derived from differential vaporization and separator tests.

The solution gas-oil ratios in the database are total gas-oil ratios, obtained by adding the solution gas-oil ratios obtained at the separator and the stock tank. A correlation\(^6\) for obtaining stock-tank gas-oil ratios is available for use if field data are not available for use in these correlations.

The gas specific gravities used in the correlation process are separator gas gravities. We used these estimates of the gas specific gravity instead of the more rigorous weighted average (separator and stock tank) because stock tank gas specific gravities are rarely measured in field operations.

The ranges of data that were used to develop the \( R_s \) correlation (195 lab reports/2,097 data sets) are given in Table 1.

| \( p_b \) (psig) | \( T \) (°F) | \( B_{ob} \) (resbl/STB) | \( R_{sb} \) (scf/STB) | \( g_{API} \) | \( g_{gs} \) (air = 1)
---|---|---|---|---|---|
106 | 70 | 1.040 | 102 | 11.6 | 0.561

An additional database of PVT properties at the bubblepoint pressure (728 sample points) was assembled; these data were used specifically to develop a bubblepoint pressure correlation.

Both of these data sets cover the usual range of properties encountered in the field.

Correlation for Solution Gas-oil-ratio

In contrast to many approaches presented in the past\(^1, 3, 4\), our correlation of solution gas-oil ratio, \( R_s \), is not derived from rearranging a bubblepoint pressure correlation. Our solution uses a reduced variable approach.

Definition of Reduced Functions

Two sets of dimensionless functions were calculated with data from each PVT report. These functions, originally defined by Cronquist\(^6\) are “reduced pressure,” and “reduced gas-oil-ratio.” The reduced pressure variable, \( p_r \), is defined as the pressure divided by the bubblepoint pressure [Equation (1)], and the reduced gas-oil-ratio variable, \( R_{sr} \), is defined as the solution gas-oil ratio divided by the solution gas-oil ratio at the bubblepoint [Equation (2)].

\[
p_r = \frac{p}{p_b} \quad (1)
\]

\[
R_{sr} = \frac{R_s}{R_{sb}} \quad (2)
\]

Since, by definition, solution gas-oil ratio is zero at atmospheric pressure (0 psig), the pressures used in Equation (1) are in units of psig.

Figure 5 is a plot of these “reduced” variables for several of the PVT reports in the data set. Notice that the endpoints of reduced functions, \( R_{sr} \) and \( p_r \), are (0, 0) and (1, 1).

General Model

The “reduced” variables were correlated with the three-coefficient, two-term power series model given as Equation (3).

\[
R_{sr} = a_1 p_r^{a_2} + (1 - a_1) p_r^{a_3} \quad (3)
\]

The performance of the power series model was superior to any other model we tested. The power series model was tested on indi-
individual PVT reports and as a correlating equation for the entire data base: the results were excellent.

**Final Form of Solution Gas-Oil Ratio Correlation**

Coefficients $a_1$, $a_2$, and $a_3$, for Equation (3) were derived for each of the 195 PVT reports. These coefficients were then correlated with commonly available field data which in this case were obtained from the laboratory separator test data.

We used non-linear regression methods on several different functional forms.

The best regression analysis results were obtained using the following functions [Equations (4), (5), and (6)].

\[
a_1 = A_0 \gamma_s^A \gamma_{API}^A T^A B_0^A p_b^{A_0} \tag{4}
\]

\[
a_2 = B_0 \gamma_s^B \gamma_{API}^B T^B B_0^B p_b^{B_0} \tag{5}
\]

\[
a_3 = B_0 \gamma_s^C \gamma_{API}^C T^C B_0^C p_b^{B_0} \tag{6}
\]

The regression coefficients are shown in Table 2.

The bubblepoint pressures, $p_b$, in Equations (4) to (6) are in units of psig. An attempt was made to develop these equations without including bubblepoint pressure, but its introduction into the equations substantially improved the prediction of gas-oil ratios throughout the entire pressure range. Bubblepoint pressures for correlation purposes were taken from the PVT reports. In application of the solution gas-oil ratio correlation, bubblepoint pressures can be taken from reservoir pressure measurements, if available, or from a bubblepoint pressure correlation of the user’s choice.

Table 3 gives measures of the accuracy of the proposed correlation. A comparison of solution gas-oil ratios calculated with Equations (1) to (6) with the data used in their development is given in Figure 6.

### Table 2: Regression coefficients for the new solution gas-oil ratio correlation.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Equation (4)</th>
<th>Coefficients</th>
<th>Equation (5)</th>
<th>Coefficients</th>
<th>Equation (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$</td>
<td>$9.73 \times 10^{-7}$</td>
<td>$B_1$</td>
<td>0.022339</td>
<td>$C_1$</td>
<td>0.725167</td>
</tr>
<tr>
<td>$A_1$</td>
<td>1.672608</td>
<td>$B_1$</td>
<td>-1.004750</td>
<td>$C_1$</td>
<td>-1.485480</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.929870</td>
<td>$B_2$</td>
<td>0.337711</td>
<td>$C_2$</td>
<td>-0.164741</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.247235</td>
<td>$B_3$</td>
<td>0.132795</td>
<td>$C_3$</td>
<td>0.091330</td>
</tr>
<tr>
<td>$A_4$</td>
<td>1.056052</td>
<td>$B_4$</td>
<td>0.302065</td>
<td>$C_4$</td>
<td>0.047094</td>
</tr>
</tbody>
</table>

### Table 3: Statistical accuracy of proposed solution gas-oil ratio correlation (using laboratory measured values of bubblepoint pressure).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of Squared Residuals, $(\text{scf}/\text{STB})^2$</td>
<td>737.333</td>
</tr>
<tr>
<td>Standard Deviation, $(\text{scf}/\text{STB})$</td>
<td>18.2</td>
</tr>
<tr>
<td>Variance, $(\text{scf}/\text{STB})^2$</td>
<td>333.9</td>
</tr>
<tr>
<td>Average Absolute Error, per cent</td>
<td>4.73</td>
</tr>
</tbody>
</table>

**Calculation of Oil Formation Volume Factor**

Oil formation volume factors, solution gas-oil ratios, and reservoir oil densities are connected by material balance. Thus, oil formation volume factors can be calculated using this material balance formulation.

### Reservoir Oil—Surface Fluid Material Balance

The connection between surface fluids and reservoir oil is given as Equation (7).

\[
R_o = \rho_{STO} + 0.01357 R_s \gamma_g 
\]

This is not a correlation but is a material balance of the surface gas and surface liquid with the reservoir liquid. The use of Equation (7) to calculate oil formation volume factor honors the material balance relationship among $B_o$, solution gas-oil ratios and reservoir oil densities. Further, since reservoir oil densities monotonically increase as pressure decreases, the correct shape of oil formation volume factor as pressure declines is maintained and the linear relationship between oil formation volume factor and solution gas-oil ratio is preserved. These important empirical relationships would probably not be honored if an independent correlation for oil formation volume factor was used.

### Solution Gas-oil Ratios for Use in Equation (7)

Solution gas-oil ratio, $R_s$, in Equation (7) may be obtained as a function of pressure with the correlation presented in the previous section.

### Reservoir Oil Densities for Use in Equation (7)

The density of the reservoir oil, $\rho_{oR}$, in Equation (7) is also a function of pressure and may be estimated using the correlations given as Equations (8) to (12).
The term \( \rho_{po} \) is the density of a pseudoliquid composed of the surface liquid plus the surface gas treated as if it were a liquid at standard conditions. Equation (8) requires the use of the apparent liquid density of the gas, \( \rho_g \), which represents the density of the gas if the gas could be liquefied at standard conditions. Equation (9) is a correlation\(^8\) for apparent liquid density.

\[
\rho_{po} = -49.8930 + 85.0149 \gamma_{gs} - 3.7037 \gamma_{gs} \rho_{po} + 0.047982 \gamma_{gs} \rho_{po}^2 + 2.98914 \rho_{po} - 0.035689 \rho_{po}^2
\]  

\[
\rho_g = -0.01 \rho_{po} + 0.167 + 16.181(10^{-0.042\rho_{po}}) \left( \frac{p}{1000} \right)
\]

\[
-0.01 \left[ 0.299 + 263(10^{-0.0063\rho_{po}}) \right] \left( \frac{p}{1000} \right)^2
\]

Equation (11)\(^{9, 10}\) is used to further adjust density to reservoir temperature, resulting in the density of reservoir oil at reservoir conditions, \( \rho_{or} \).

\[
\rho_{or} = \rho_{bs} - \left( 0.00302 + 1.505 \rho_{bs}^{0.951} \right) (T - 60)^{0.938}
\]

\[
+ \left[ 0.0216 - 0.0233 (10^{-0.0161\rho_{bs}}) \right] (T - 60)^{0.475}
\]

Equation (9) was derived using separator gas specific gravity, rather than the more rigorous total gas specific gravity (weighted average of separator and stock-tank gas specific gravities), since stock-tank gas specific gravity is seldom measured in the field.

Values of oil formation volume factors as functions of pressure were calculated using Equations (7) to (12) and compared with values from the data base described in Table 1. Figure 7 gives a comparison of calculated and laboratory measured oil formation volume factors for the 2,097 data points. Most of the outliers in Figure 7 result from three of the PVT reports. Table 4 shows several statistical measures of the oil formation volume factor calculations.

### Bubblepoint Pressure Correlation

An estimate of bubblepoint pressure is required to use the gas-oil ratio correlation presented in this paper. A second data base of 728 data sets at bubblepoint pressures was used to develop a correlation. The ranges of data in this data set are given in Table 5.

Non-linear regression methods were used to adjust the coefficients of all of the bubblepoint pressure correlations listed in Table 11. The Petrosky-Farshad modification\(^3\) of Standing’s equation\(^4\) was found to be the most appropriate model for describing bubblepoint pressure as a function of commonly measured field data. However the equation presented in this study introduces one additional coefficient to the model in order to increase the accuracy of the correlation. This correlation with coefficients modified to best fit the bubblepoint pressures in our data base is given as Equations (13) and (14).

\[
\rho_b = 1091.47 \left[ R_{sp}^{0.08465} \gamma_{gs}^{-0.161488} \right] \left( 10^2 - 0.740152 \right)^{3.534991}
\]  

| TABLE 4: Statistical accuracy of oil formation volume factor calculations. |
|-----------------------------|-----------------------------|
| Parameter | Value |
| Sum of Squared Residuals, (resbbl/STB)\(^2\) | 3.31 |
| Standard Deviation, (resbbl/STB) | 0.038 |
| Variance, (resbbl/STB)\(^2\) | 0.0014 |
| Average Absolute Error, per cent | 1.74 |

| TABLE 5: Ranges of data in data set used to develop bubblepoint pressure correlation. |
|-----------------------------|-----------------------------|
| 70 < \( R_b \) < 6700 psia |
| 74 < T < 327 °F |
| 10 < \( R_{sb} \) < 1870 scf/STB |
| 12 < \( \gamma_{API} \) < 55 °API |
| 0.556 < \( \gamma_{gs} \) < 1.367 (air = 1) |

| TABLE 6: Statistical accuracy of bubblepoint pressure correlation. |
|-----------------------------|-----------------------------|
| Parameter | Value |
| Sum of Squared Residuals, psia\(^2\) | 50,261,226 |
| Standard Deviation, psia | 263 |
| Variance, psia\(^2\) | 69,517 |
| Average Absolute Error, per cent | 11.5 |

| TABLE 7: Statistical accuracy of solution gas-oil ratio using \( \rho_b \) from new correlation. |
|-----------------------------|-----------------------------|
| Parameter | Value |
| Sum of Square Residuals, (scf/STB)\(^2\) | 4,614,251 |
| Standard Deviation, scf/STB | 45.7 |
| Variance, (scf/STB)\(^2\) | 2089 |
| Average Absolute Error, per cent | 10.5 |
Several statistical measures of the accuracy of bubblepoint pressure calculated with Equations (13) and (14) may be found in Table 6. Table 7 gives a similar evaluation of the proposed solution gas-oil ratio correlation using values of bubblepoint pressure from Equations (13) and (14) to calculate $p_r$.

Figure 8 shows a comparison of values of bubblepoint pressure calculated with Equations (13) and (14) with the corresponding laboratory measured values.

**Validation of the Correlations**

The solution gas-oil ratio and oil formation volume factor curves resulting from the use of the proposed methods have the correct shapes. Figures 9 and 10 show the results of the solution gas-oil ratio correlation [Equations (1) to (6)] compared with the same laboratory data of Figures 1 and 3. Two curves are shown: one calculated using the laboratory measured value of bubblepoint pressure, the other calculated with a value of bubblepoint pressure from Equations (13) and (14). The agreement in both the shapes and the values is excellent.

Figures 11 and 12 show the results of the formation volume factor calculation [Equation (7) using values of reservoir oil density from Equations (8) to (12)] compared with the laboratory data of Figures 2 and 4. The two lines result from different options of obtaining bubblepoint pressures for the solution gas-oil ratios as discussed above. The agreement in both the shapes and the values is excellent.

An independent data set, a subset of the data used by Kartoatmodjo and Schmidt (4), was obtained after this research was completed. The data consist of 541 PVT studies with a total of 541 bubblepoint observations and 4,103 solution gas-oil ratio and oil formation volume factor observations. Table 8 gives the range of data in this data set.

This data set was used to test the accuracies of the correlations presented here and several correlations from the literature. Tables 9, 10 and 11 show the results of comparison of correlation calculations with these data.

**Conclusions**

1. We have developed statistical as well as graphical evidence
that the currently available black oil correlations do not accurately model solution gas-oil-ratios and oil formation volume factors for pressures below the bubblepoint.

2. We have developed and verified a new set of correlations for black oils that can be used to describe the behaviour of the solution gas-oil-ratios and oil formation volume factors for pressures at and below the bubblepoint.

3. The calculation of solution gas-oil ratios using the methodology proposed here is flexible since it allows the use of bubblepoint pressure from either our \( p_b \) correlation, another \( p_b \) correlation, or field pressure measurements.

4. A three-coefficient, two-term power law series model is proposed to represent the character of the reduced solution gas-oil-ratio (\( R_s / R_{sb} \)) versus reduced pressure (\( p/p_b \)) trends.

5. We recommend a material balance method of calculating oil formation volume factors which honors the connection among oil formation volume factors, solution gas-oil ratios, and reservoir oil densities as functions of pressure(7).

6. We have developed a new bubblepoint pressure correlation using the relation proposed by Petrosky and Farshad(3) [which is a modification of the original form proposed by Standing(1)].

7. The correlations proposed in this study have been tested with an independent data set (the quantity of data in the independent data set is approximately double the quantity of data used to develop the proposed correlations), and the results validate our findings.

NOMENCLATURE

\[
\begin{align*}
B_o &= \text{oil formation volume factor, resbbl/STB} \\
B_{ob} &= \text{oil formation volume factor at the bubblepoint, resbbl/STB} \\
p &= \text{pressure, psi [p is in psig for Equation (1)]}
\end{align*}
\]

Greek Letter Variables

\[
\begin{align*}
\gamma_{API} &= \text{stock tank oil gravity, } ^\circ\text{API} \\
\gamma' &= \text{gas specific gravity, } \text{air} = 1 \\
\gamma_{sp} &= \text{gas specific gravity measured at the separator, } \text{air} = 1 \\
\gamma_{STO} &= \text{stock tank oil specific gravity, 141.5}/(\gamma_{API} + 131.5) \\
\rho_s &= \text{apparent density of surface gas if it were a liquid, lbm/cu ft.} \\
\rho_{ns} &= \text{pseudoliquid density at reservoir pressure and standard temperature, } 60^\circ \text{F, lbm/cu ft.} \\
\rho_{AR} &= \text{reservoir oil density at reservoir conditions, lbm/cu ft.} \\
\rho_{po} &= \text{pseudoliquid density at standard conditions, lbm/cu ft.} \\
\rho_{STO} &= \text{stock-tank oil density at standard conditions, lbm/cu ft.}
\end{align*}
\]

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9. STANDING, M.B., Volumetric and Phase Behaviour of Oil Field Hydrocarbon Systems; SPE, Dallas, TX, 1951.


Authors' Biographies

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