

# Two-Phase Compressibility Factors for Retrograde Gases

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**Summary.** The two-phase compressibility factor should be used in material-balance calculations for rich-gas-condensate systems. This paper presents a correlation for estimating this factor from field data. The correlation, which is based on 67 fluid-depletion studies with  $C_{7+}$  concentrations  $\geq 4$  mol% also applies to rich gases with large amounts of  $CO_2$ ,  $H_2S$ , and  $N_2$ . The correlation gives an average error of 3.66% but may result in larger errors for some rich gases.

## Introduction

The Standing and Katz<sup>1</sup> correlation for the gas compressibility factor is valid only for dry-gas systems. Retrograde gas-condensate reservoirs experience liquid fallout during depletion below the dewpoint. The two-phase compressibility factor accounts for the formation of a liquid phase.

Compressibility factors are used in material-balance equations to estimate initial gas in place and reserves. Gas compressibility factors normally are used when a reservoir fluid-depletion study is not available. This practice is acceptable for retrograde gases if the gas condensate is lean; however, if the gas is rich, the reserves may be seriously underestimated if the two-phase compressibility factor is not used.<sup>2</sup>

Fig. 1 shows the relationship of the compressibility factors of a rich gas condensate as a function of pressure.<sup>3</sup> In this system, the two-phase compressibility factor is uniformly less than the gas compressibility factor, and the two compressibility factors diverge as the pressure depletes. Systems that exist at higher pressures and temperatures may display a different behavior near the dewpoint. High-pressure regions have been observed in which the two-phase compressibility factor is greater than the gas compressibility factor. At low pressures, however, the two-phase compressibility factor is less than the gas compressibility factor and both diverge as the pressure decreases. For the case in Fig. 1, failure to use the two-phase compressibility factor will cause  $p/z$  to be too low, underestimating the initial gas in place and the reserves.

This paper presents correlations that aid in determining when a two-phase compressibility factor should be used and how to calculate this factor from field data. The correlation for the two-phase compressibility factor is based on the pseudoreduced properties of the produced gas that can be calculated from the well-stream gas composition or the primary-separator gas specific gravity corrected to well stream. The pseudoreduced pressure and pseudoreduced temperature were calculated with the methods presented by Sutton.<sup>4</sup> These methods include correlations for calculating the pseudocritical properties of the heptane-plus fraction,<sup>5</sup> adjustment of the pseudocritical properties for impurities,<sup>6</sup> and use of the modified Stewart-Burkhardt-Voo<sup>7</sup> mixing rules. Sutton also gives a correlation for calculating pseudoreduced pressure and temperature from the well-stream gas specific gravity. The correlation for determining when to use the two-phase compressibility factor is based on the initial well-stream gas specific gravity that can be calculated from surface separation data<sup>8-11</sup> if well-stream-composition data are not available. The correlations were verified with simulated production data.

## Theory

The material-balance equation for volumetric performance of a retrograde gas reservoir is derived easily from the real gas law. Initially the reservoir contains  $n_i$  moles of gas:

$$n_i = p_i V_i / z_i RT \quad (1)$$

Assuming that only gas is produced, a liquid phase condenses in the reservoir as the pressure depletes below the dewpoint, leaving  $n_r$  moles of gas and liquid remaining in the reservoir:

$$n_r = n_i - n_p = p V_i / z_{2p} RT \quad (2)$$

Because the compressibility factor in Eq. 2 now applies to both gas and liquid, it is a two-phase compressibility factor.<sup>2,12</sup>

Dividing Eq. 2 by Eq. 1 gives, after substitution of  $G_p/G$  for  $n_p/n_i$  and rearrangement,

$$\frac{p}{z_{2p}} = \frac{p_i/z_i}{1 - (G_p/G)} \quad (3)$$

a form of the material-balance equation similar to that given by Duke.<sup>13</sup>

Rearrangement and expansion of Eq. 2 to account for the volumes occupied by the gas and liquid remaining in the reservoir give a relationship between the two-phase compressibility factor and the compressibility factors of the equilibrium gas and liquid,

$$z_{2p} = \frac{n_g}{n_r} \left( \frac{pV_g}{n_g RT} \right) + \frac{n_L}{n_r} \left( \frac{pV_L}{n_L RT} \right) = f_g z_g + f_L z_L \quad (4)$$

Eq. 4, recently given by Vo *et al.*,<sup>14</sup> indicates that the two-phase compressibility factor is a weighted average of the liquid and gas compressibility factors and a function of pressure, temperature, composition, and the amount of liquid remaining in the reservoir.

## Development of Correlations

Our correlations were developed with data from 131 constant-volume depletion studies performed on retrograde gas samples collected worldwide. Table 1 shows the range of the data at dewpoint conditions. The data was partitioned on the bases of  $C_{7+}$  concentrations and impurities (Table 2). A sensitivity study was performed with the final correlations that supported these partitions.

The two-phase compressibility factor was correlated with pseudoreduced pressure and pseudoreduced temperature. The compositions of the produced gas from each step of the constant-volume depletion studies were used to calculate both the pseudoreduced pressure and temperature with the methods presented by Sutton. Fig. 2 shows the laboratory-determined two-phase compressibility factor for the entire data set as a function of pseudoreduced pressure. Although the data spreads at low pseudoreduced pressures, a general linear trend is apparent.

Figs. 3 through 6 show similar plots for Data Sets 1 through 4. These figures indicate that the data have two different trends as a result of  $C_{7+}$  concentration. The two-phase compressibility factor for the rich gases (Figs. 3 and 5) shows a general linear trend with pseudoreduced pressure, while the two-phase compressibility factor for the lean gases (Figs. 4 and 6) shows a curvature similar to that of a single-phase gas. While not indicated in Figs. 2 through 6, the two-phase compressibility factor was found to be a linear function of the inverse of pseudoreduced temperature. In fact, the variation in pseudoreduced temperature partially accounts for the spreading of data noted at low values of pseudoreduced pressure.

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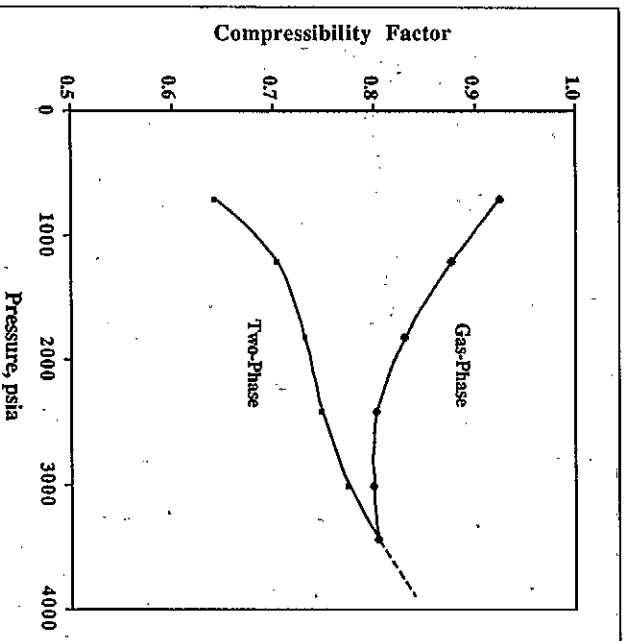


Fig. 1—Compressibility factors of a rich gas condensate as functions of pressure.

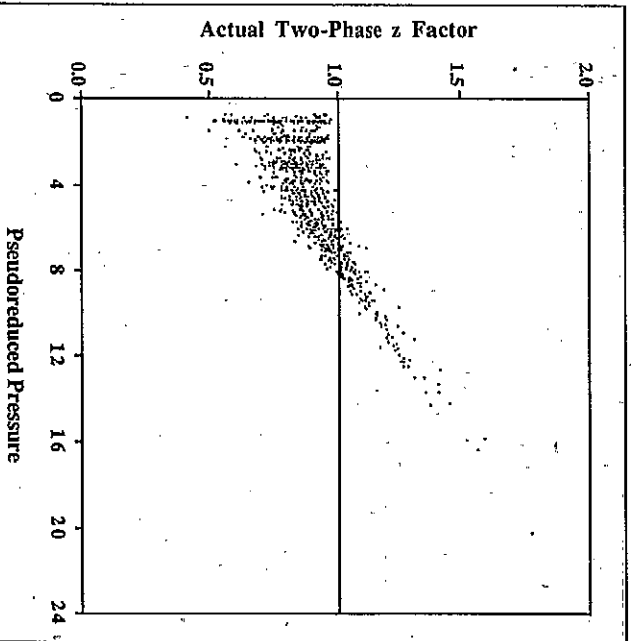


Fig. 2—Two-phase compressibility factor as a function of pseudoreduced pressure for all available data.

TABLE 1—RANGE OF DATA AT DEWPOINT CONDITIONS

Variable	Mean	Minimum	Maximum
H <sub>2</sub> S	0.99	0.00	28.16
CO <sub>2</sub>	2.54	0.00	63.52
N <sub>2</sub>	1.84	0.01	12.74
C <sub>1</sub>	73.23	19.37	94.20
C <sub>2</sub>	7.58	1.95	16.66
C <sub>3</sub>	3.94	0.62	12.27
IC <sub>4</sub>	0.85	0.18	2.53
n-C <sub>4</sub>	1.48	0.12	5.02
IC <sub>5</sub>	0.61	0.08	1.62
n-C <sub>5</sub>	0.63	0.08	2.09
C <sub>6</sub>	0.85	0.14	2.04
C <sub>7+</sub>	5.40	0.85	12.68
Molecular weight of C <sub>7+</sub>	154.0	108.0	253.0
Specific gravity of C <sub>7+</sub>	0.794	0.736	0.850
γ <sub>d</sub> (air = 1)	1.018	0.644	1.557
Z <sub>d</sub>	1.048	0.704	1.775
P <sub>d</sub> , psia	5,240.0	1,930.0	11,844.0
T <sub>d</sub> , °F	239.0	94.0	325.0

These observations were used to fit Data Set 1 with a linear model. The independent variables selected were pseudoreduced pressure and the inverse of pseudoreduced temperature of the produced gas. These variables are calculated easily from data normally available in the field. Both quadratic and cubic models and other variables, such as initial pressure, initial compressibility factor, gas specific gravity, and the molecular weight and specific gravity of the C<sub>7+</sub> fraction, were tried. However, a full quadratic model with two-phase compressibility proportional to pressure and inversely proportional to temperature, as suggested by Eq. 4, produced the best results. The SAS15 software package was used for the analysis.

The final form of the correlation to estimate the two-phase compressibility factor is

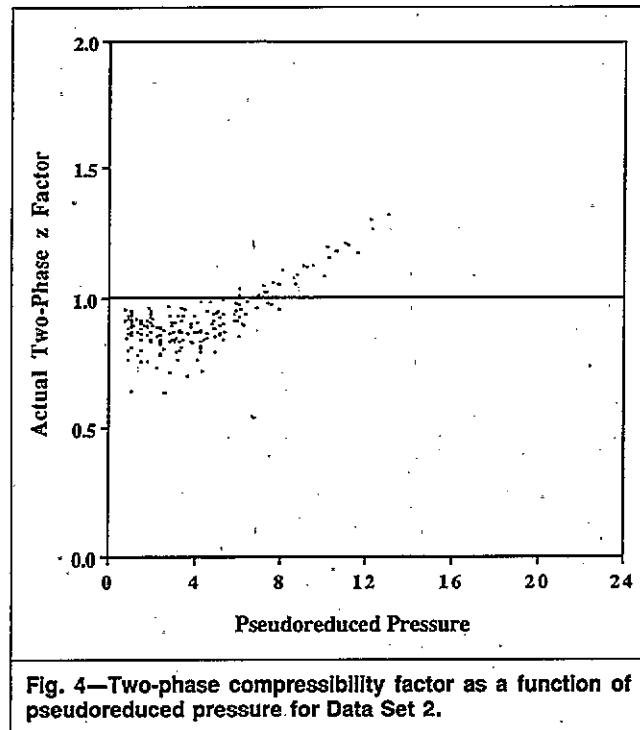
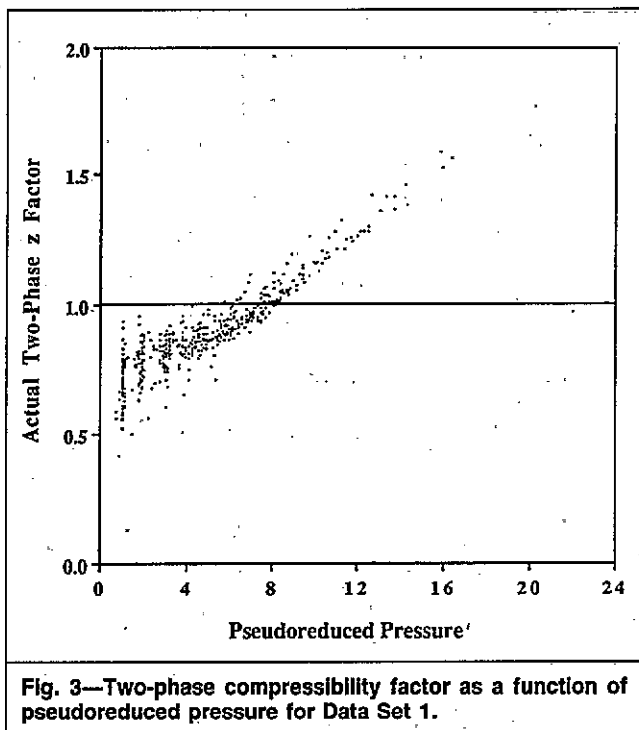
$$z_{2P} = A_0 + A_1(P_r) + A_2\left(\frac{1}{T_r}\right) + A_3(P_r)^2 + A_4\left(\frac{1}{T_r}\right)^2 + A_5\left(\frac{P_r}{T_r}\right), \quad (5)$$

for  $0.7 \leq P_r \leq 20.0$  and  $1.1 \leq T_r \leq 2.1$ , where  $A_0 = 2.24353$ ,  $A_1 = -0.0375281$ ,  $A_2 = -3.56539$ ,  $A_3 = 0.000829231$ ,  $A_4 = 1.53428$ , and  $A_5 = 0.131987$ .

Although the coefficients have been shown with six digits, the data obtained from the fluid reports are accurate to only three significant figures. The sum of the coefficients in Eq. 5 is 0.308 and can be interpreted as the critical compressibility factor. As Table 3 shows, the coefficient of determination ( $R^2$ ) for Eq. 5 is 0.948 and the average absolute error for Data Set 1 is 3.38%. Also note in Table 3 that the use of Eq. 5 may result in large errors for some rich gas condensates. The large maximum errors can be attributed to two factors: samples containing high concentrations of C<sub>2</sub> and C<sub>3</sub> and samples depleted to pressures < 700 psig; both had relatively high errors.

TABLE 2—CLASSIFICATION OF FLUID SAMPLES IN DATA BASE

Data Set	Description	Initial C <sub>7+</sub> (%)	Initial Impurities (%)	Fluid Samples	Depletion Steps
1	Relatively pure rich gases	≥4.0	<5.0	67	478
2	Relatively pure lean gases	<4.0	<5.0	29	191
3	Relatively impure rich gases	≥4.0	≥5.0	14	97
4	Relatively impure lean gases	<4.0	≥5.0	21	141
	All gases	—	—	131	907



While the final correlation is based on the 67 depletion studies of Data Set 1, approximate two-phase compressibility factors also can be obtained for Data Set 3. Fig. 7 shows the accuracy of the correlation with Data Sets 1 and 3 compared with the actual two-phase compressibility factor. These are the rich gases, including those with large concentrations of impurities. As Table 1 shows, these data sets include a sample with more than 63% CO<sub>2</sub> and a sample with more than 28% H<sub>2</sub>S.

Fig. 8 shows the accuracy of the correlation with Data Sets 2 and 4 compared with the actual two-phase compressibility factor. Fig. 8 illustrates that the two-phase compressibility factor of a lean gas cannot be predicted by Eq. 5. The average absolute error for Data Sets 2 and 4 combined is 7.35%. Later, we show that the single-phase gas compressibility factor<sup>1</sup> should be used in the material-balance equations for lean gases.

Eq. 5 is recommended for use with gases having C<sub>7+</sub> concentrations of ≥4.0 mol%. Consequently, a method is needed to estimate the concentration of the C<sub>7+</sub> fraction when the well-stream gas-composition data are not available. We used the entire data set to develop the following correlation to calculate the concentration of the C<sub>7+</sub> fraction from the well-stream gas specific gravity:

$$n_{C_{7+}} = -0.0885119 + 0.141013(\gamma_g) \dots \dots \dots (6)$$

Fig. 9 shows the accuracy of this correlation with the actual C<sub>7+</sub> fraction. Thus, if the well-stream gas specific gravity is <0.911, the correlation should not be used. The single-phase gas compressibility correlation should be used for these lean gases. We summarize the recommended procedure for determining the two-phase compressibility factor below.

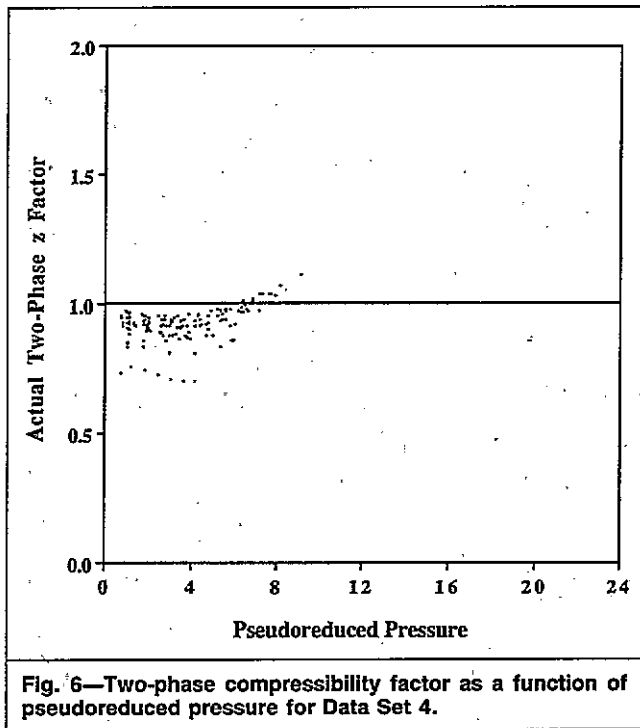
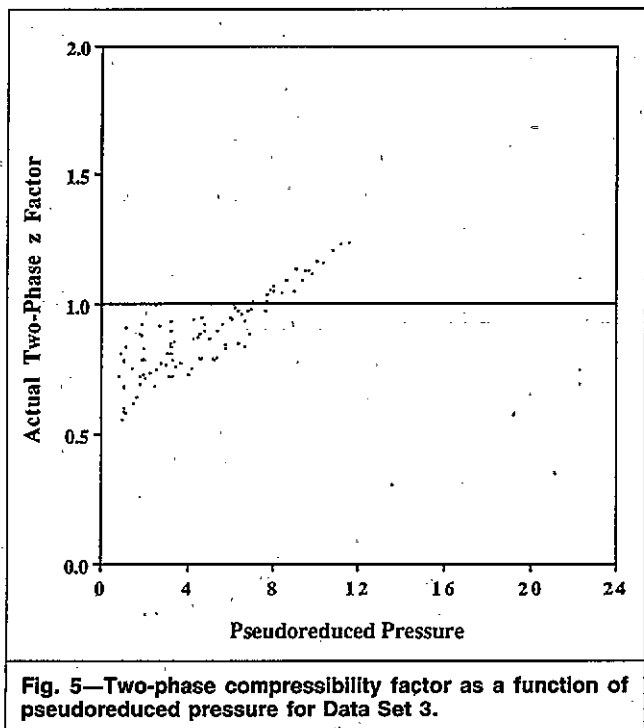


TABLE 3—PERCENT ERROR OF SELECTED DATA SETS

Data Set	Percent Error		$R^2$
	Average	Maximum	
1	3.38	34.95	0.948
2	7.95	34.51	NA
3	4.81	27.58	NA
4	6.55	25.39	NA
All	4.98	34.95	NA

NA = not applicable.

1. Calculate pseudoreduced properties of the produced gas from well-stream gas composition or well-stream gas specific gravity.
2. If the concentration of  $C_{7+}$  is  $\geq 4.0$  mol% (or if the well-stream gas specific gravity is  $>0.911$ ), use Eq. 5 to calculate the two-phase compressibility factor.
3. If the well-stream gas composition and the well-stream gas specific gravity are both unknown, as will be the case when predictions are made of reserves at an abandonment pressure, use the last known gas composition or specific gravity.

**Verification of Correlation**

The correlation was tested by simulating natural reservoir depletion processes with the VIP-COMPTM module of the VIP-EXECUTIVETM simulator.<sup>16</sup> The reservoir simulator was set up as an  $r$ - $Z$  model with one producing well in the center of the reservoir. The simulated reservoir was 150 ft thick with a 440-ft diameter. The porosity was 0.13, and the permeability was 400 md in the radial direction and 40 md in the vertical direction. The grid for the simulation model had nine concentric rings around the producer, with external radii at 10, 20, 30, 40, 70, 100, 130, 170, and 220 ft and had nine layers, each of equal thickness, porosity, and permeability. The constant flow rate was 4,000 Mscf/D. All other parameters, including saturation data, rock and fluid properties, and separator temperatures and pressures, were taken from Ref. 3. Constant-volume depletion data from a rich<sup>3</sup> and a lean<sup>17</sup> gas condensate were matched with the Peng-Robinson<sup>18</sup> equation of state (EOS) and the simulator's EOS-PAKTM. Table 4 shows the fluid analyses.

The simulated data were analyzed two different ways. Both analysis methods used the pseudoreduced properties of the produced

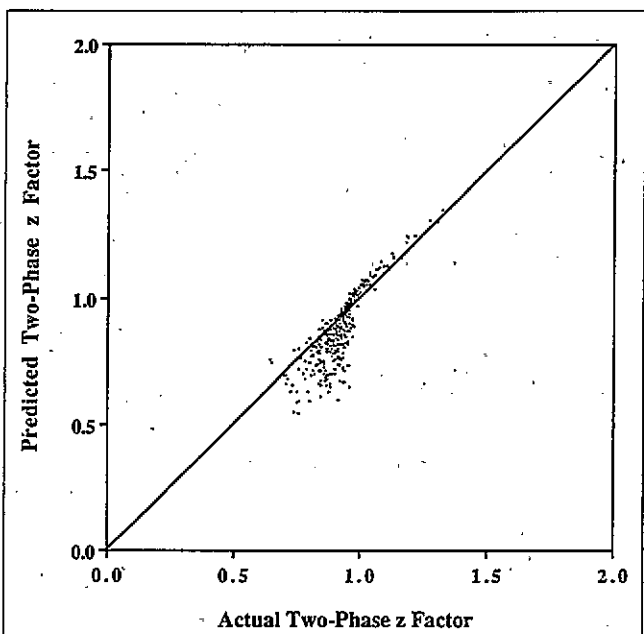


Fig. 8—Accuracy for two-phase compressibility correlation for Data Sets 2 and 4.

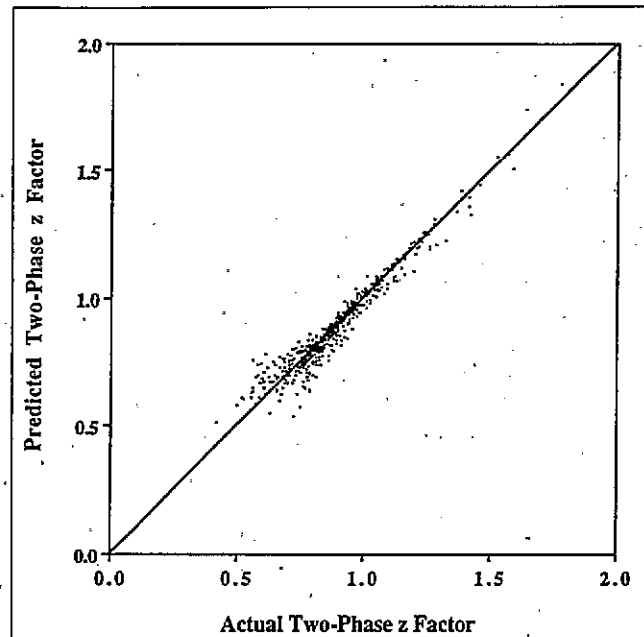


Fig. 7—Accuracy of two-phase compressibility correlation for Data Sets 1 and 3.

gas at several pressures to calculate the two-phase compressibility factor. The major difference in the two methods is that, in the first method, the well-stream gas compositions were used to calculate the pseudoreduced properties, and in the second method, the well-stream gas specific gravities were used. The Gold *et al.*<sup>9</sup> correlation was used for the latter calculations.

Fig. 10 shows the results of these calculations in the form of a material-balance plot for the rich gas condensate. For comparison, Fig. 10 also shows the actual data from the constant-volume depletion study. As expected, the actual  $p/z_{2P}$  extrapolates to 1.0, and the actual  $p/z$  extrapolates to  $\approx 0.9$ , indicating that initial gas in place and reserves are underestimated by 10%. The calculated  $p/z_{2P}$  estimates initial gas in place and reserves with an error of  $\pm 3.0\%$ .

Fig. 11 shows similar calculations for the lean-gas-condensate system. For this system, the actual  $p/z$  extrapolates to  $\approx 0.98$ , in-

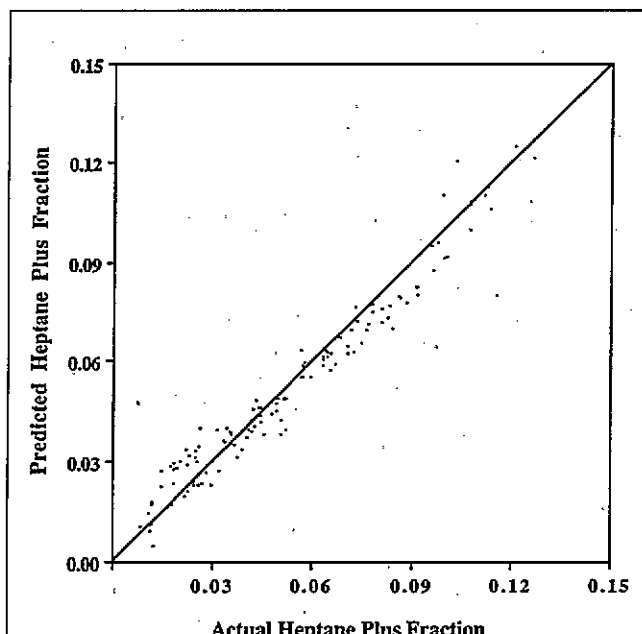


Fig. 9—Accuracy of  $C_{7+}$  correlation for initial compositions.

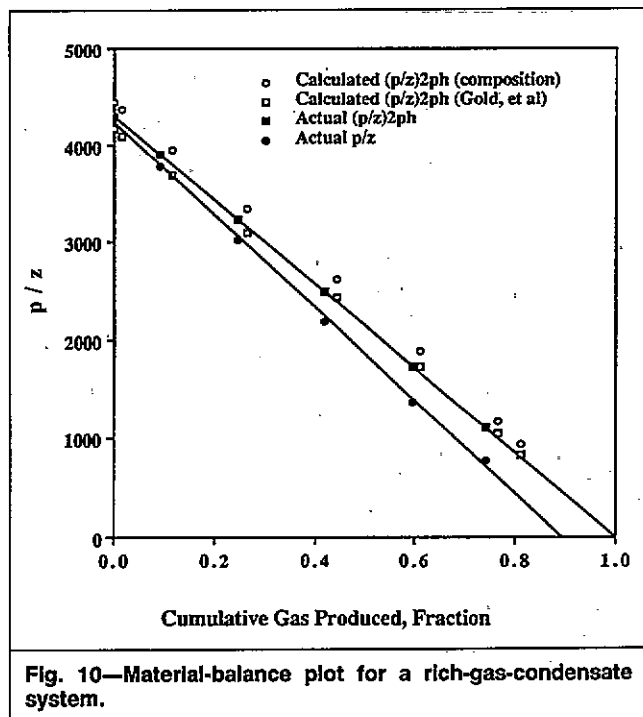


Fig. 10—Material-balance plot for a rich-gas-condensate system.

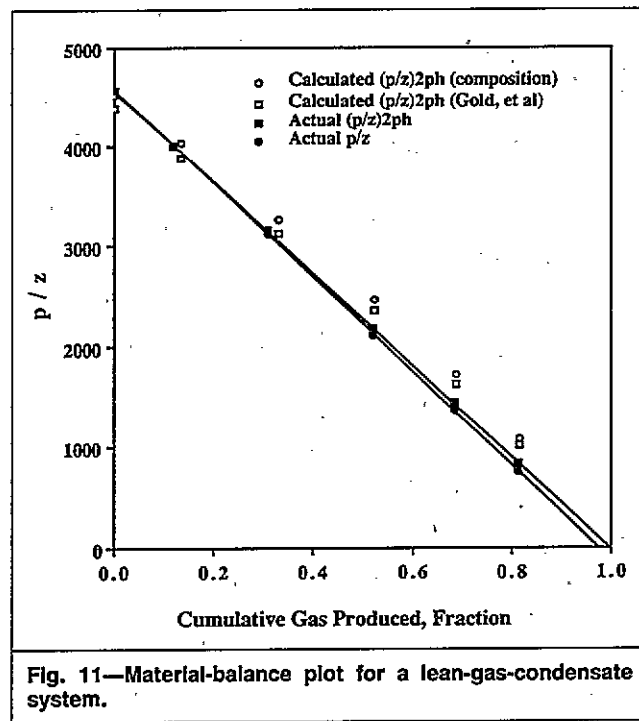


Fig. 11—Material-balance plot for a lean-gas-condensate system.

dicating that an acceptable estimate of initial gas in place and reserves can be obtained with the gas compressibility factor.

### Conclusions

1. An estimate of the two-phase compressibility factor can be made with the correlation presented if the  $C_{7+}$  concentration of the initial gas is  $\geq 4.0$  mol% or the well-stream gas specific gravity is  $> 0.911$ .
2. When the initial  $C_{7+}$  concentration is  $< 4.0$  mol% or the well-stream gas specific gravity is  $< 0.911$ , the single-phase gas compressibility factor should be used in the  $p/z$  material-balance plot.
3. The correlation for the two-phase compressibility factor can be used for rich gases with large amounts of  $CO_2$ ,  $H_2S$ , and  $N_2$ .

### Nomenclature

- $A$  = coefficient for Eq. 5  
 $f_g$  = mole fraction of gas  
 $f_L$  = mole fraction of liquid  
 $G$  = initial gas in place, scf  
 $G_p$  = cumulative gas produced, scf  
 $n_{C_{7+}}$  = heptane-plus fraction  
 $n_g$  = total moles of gas, lbm-mol  
 $n_L$  = total moles of liquid, lbm-mol  
 $n_p$  = total moles produced, lbm-mol  
 $n_r$  = total moles remaining in reservoir ( $p < p_d$ ), lbm-mol  
 $n_i$  = total moles, lbm-mol  
 $p$  = pressure, psia  
 $p_d$  = dewpoint pressure, psia  
 $p_i$  = initial pressure, psia  
 $p_r$  = pseudoreduced pressure  
 $r$  = radial direction, ft  
 $R$  = universal gas constant,  $(10.73 \text{ psia-ft}^3)/[(\text{lbm-mol})-\text{°R}]$   
 $R^2$  = statistical parameter  
 $T$  = temperature, °R  
 $T_r$  = pseudoreduced temperature  
 $V_g$  = volume of gas, ft<sup>3</sup>  
 $V_L$  = volume of liquid, ft<sup>3</sup>  
 $V_t$  = total volume, ft<sup>3</sup>  
 $z$  = gas compressibility factor  
 $z_d$  = dewpoint gas compressibility factor

TABLE 4—HYDROCARBON ANALYSIS OF DEWPOINT FLUID SAMPLES

Component	Rich Gas Condensate	Lean Gas Condensate
$CO_2$	1.21	2.44
$N_2$	1.94	0.08
$C_1$	65.99	82.10
$C_2$	8.69	5.78
$C_3$	5.91	2.87
$iC_4$	2.39	0.56
$n-C_4$	2.78	1.23
$iC_5$	1.57	0.52
$n-C_5$	1.12	0.60
$C_6$	1.81	0.72
$C_{7+}$	6.59	3.10
Molecular weight of $C_{7+}$	140.0	132.0
Specific gravity of $C_{7+}$	0.774	0.774
$\gamma_g$ (air = 1)	1.126	0.823

- $z_g$  = gas compressibility factor  
 $z_i$  = initial gas compressibility factor  
 $z_L$  = liquid compressibility factor  
 $z_{2p}$  = two-phase compressibility factor  
 $Z$  = vertical direction; ft  
 $\gamma_g$  = gas gravity

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### SI Metric Conversion Factors

ft	× 3.048*	E-01	= m
ft <sup>3</sup>	× 2.831 685	E-02	= m <sup>3</sup>
°F	(°F-32)/1.8		= °C
md	× 9.869 233	E-04	= μm <sup>2</sup>
psi	× 6.894 757	E+00	= kPa

\*Conversion factor is exact.

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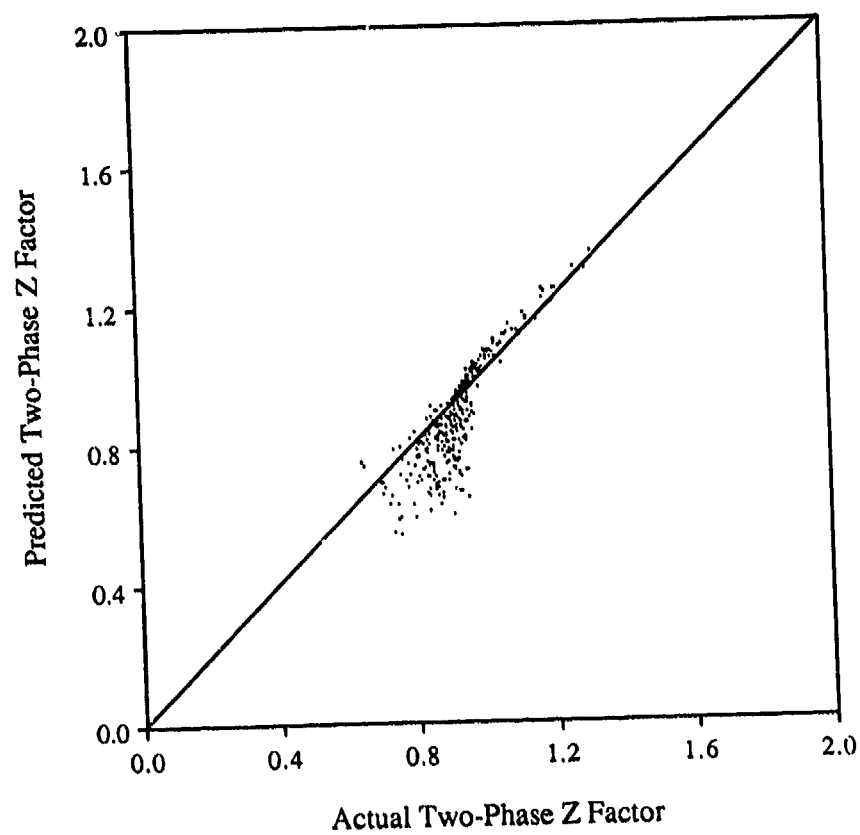


Fig. 8 - Accuracy of Two-Phase Compressibility Correlation for Data Sets 2 and 4.

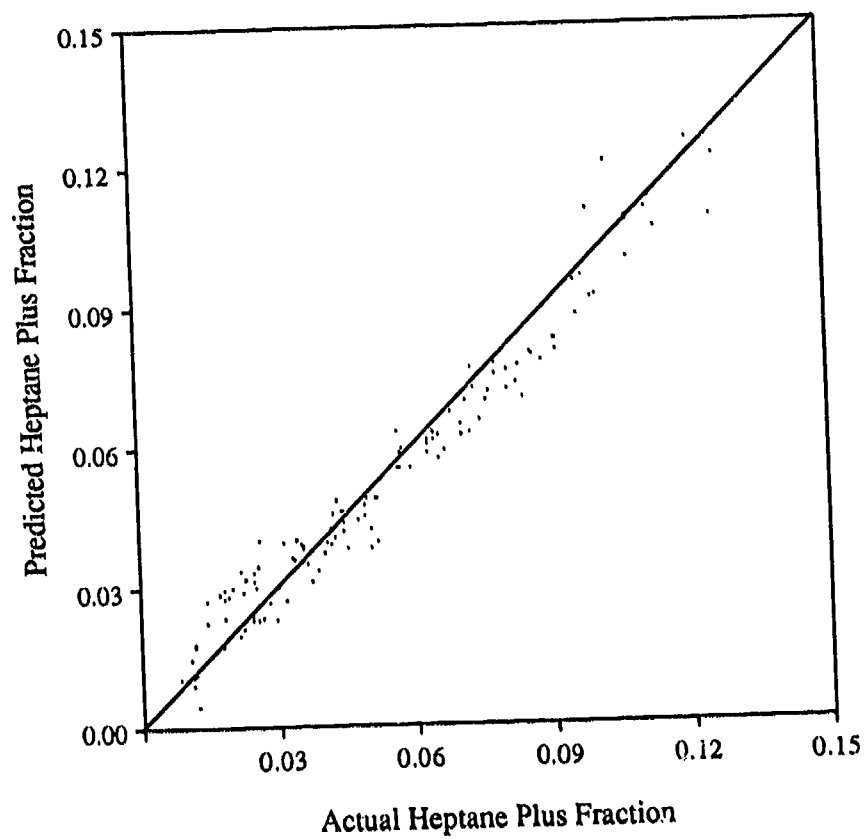


Fig. 9 - Accuracy of Heptane Plus Correlation for Initial Compositions.

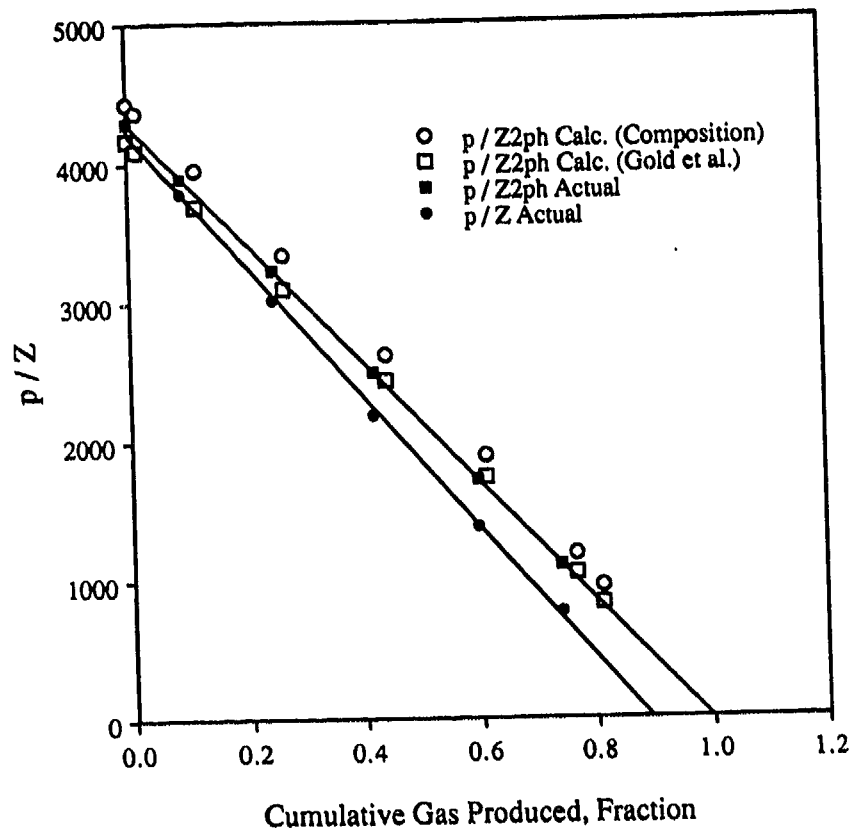


Fig. 10 - Material Balance Plot for a Rich Gas-Condensate System.

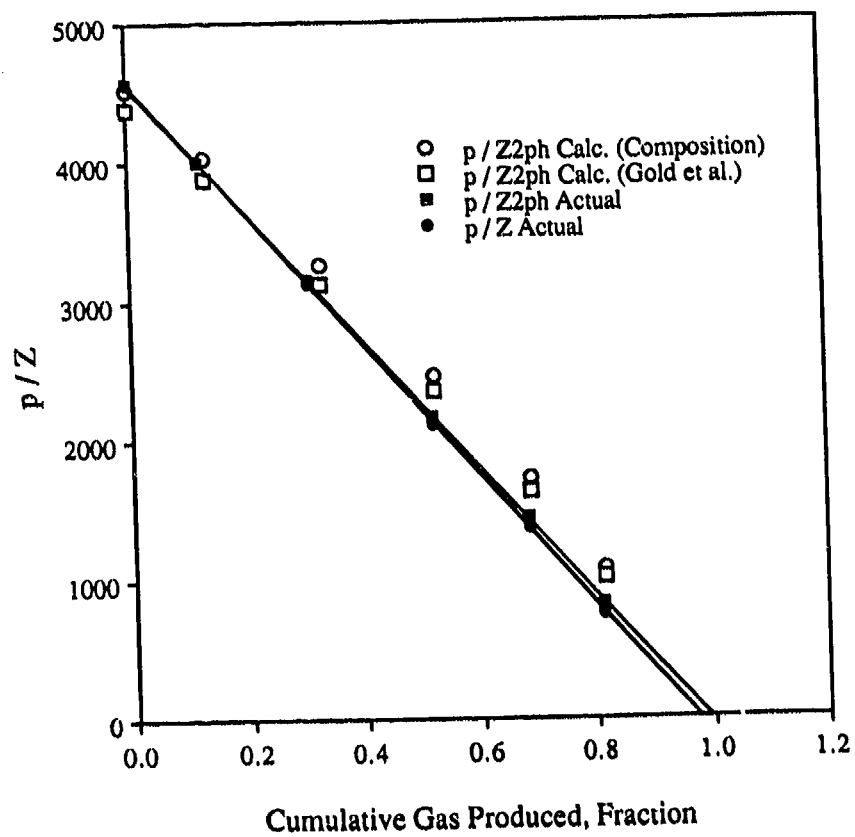


Fig. 11 - Material Balance Plot for a Lean Gas-Condensate System.