TRANSIENT PRESSURE BEHAVIOR OF
MULTIPLE-FRACTURED GAS WELLS

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TRANSIENT PRESSURE BEHAVIOR OF MULTIPLE-FRACTURED GAS WELLS

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ABSTRACT

Transient Pressure Behavior of Multiple-Fractured Gas Wells

(May 1985)

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Virtually all studies on transient pressure behavior of fractured gas wells assumed a single fracture of equal half-length emanating from each side of the wellbore. However, multiple fractures may result from explosive or massive hydraulic fracturing treatments. This paper presents the use of a cylindrical coordinates simulator to investigate the transient pressure behavior of gas wells with 4-wing, 6-wing, and 8-wing multiple fractures.

Results show that the dimensionless pressure, $P_{WD}$, can be correlated with the dimensionless terms $C_r = \frac{w k_f}{\pi k x_f}$ and $x_{fr12} = \frac{x_{f1}}{x_{f2}}$, where $C_r$ is the dimensionless fracture conductivity and $x_{fr12}$ is the fracture half-length ratio. The symbols, $x_{f1}$ and $x_{f2}$, represent the half-length of the primary fracture and the auxiliary fracture, respectively. Results also show that when $C_r \ll 10$, multiple fracture transient pressure behavior is significantly different from that of a single plane fracture.

A family of type curves was constructed using cross-plots of $P_{WD}$ vs. $tDx_f$. For early times where $tDx_f \ll 0.1$,
and $C_p < 10$, transient pressure data can be analyzed using these type curves to estimate the number of multiple fractures in addition to formation permeability, fracture length, and fracture conductivity. Early time pressure data is necessary to detect the presence of multiple fractures.
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INTRODUCTION

Hydraulic fracturing and massive hydraulic fracturing are effective techniques for improving the productivity of low-permeability gas reservoirs. Much research has been devoted to the study of transient pressure behavior of fractured gas wells to improve the design of hydraulic fractures. Many methods\textsuperscript{1-15} have been proposed to determine formation permeability and fracture characteristics from transient pressure data. Most of these methods are based on either analytical or numerical solutions of the transient flow of fluids toward fractured wells.

Virtually all studies on transient pressure behavior assume that the well is intersected by single vertical fracture of equal half-length on each side of the wellbore. However, under field conditions, multiple fractures may be created either by means of explosives or during massive hydraulic fracturing treatments. No information is available in the petroleum literature regarding the transient pressure behavior of gas wells with multiple fractures.

The first part of this study seeks to review the limitations of existing transient pressure analysis techniques. Next, a two-dimensional, single phase reservoir simulator, MULTIFRAC, in cylindrical "r-theta" coordinates will be developed. This simulator will then be employed to study the transient pressure behavior of low-permeability gas wells with multiple finite-conductivity fractures.

This thesis follows the form and style of the Journal of Petroleum Technology.
Solutions in terms of type curves are presented to determine multiple fracture characteristics and formation permeability. A comparative study is made for investigating the errors in determining formation permeability and fracture characteristics using type curves for single fracture where multiple fractures are present. A discussion regarding the importance of early-time pressure data in determining the presence of multiple fractures is also presented.
LITERATURE REVIEW

The successful economic evaluation of low-permeability gas reservoirs requires reliable estimates of fracture length, fracture conductivity, and formation permeability. Many methods\textsuperscript{1-15} have been developed to determine these parameters from pressure transient data. There are three basic solutions along with analysis methods for vertically fractured gas wells. These solutions are:

(1) infinite-conductivity where fluid entry causes zero pressure drop along the fracture,

(2) uniform-flux where fluid entry into the fracture is the same per unit cross-sectional area near the wellbore and near the tip of fracture,

(3) finite-conductivity where pressure drop along fracture is not uniform.

Conventional pressure transient analysis in fractured wells has relied primarily on numerical and analytic studies based on the concept of infinite-conductivity fractures. The works of Russell and Truitt\textsuperscript{1} pioneered the application of methods based on the assumption of pseudo-radial flow in a reservoir with infinite-conductivity fractures for determining formation permeability and fracture length. Pseudo-radial flow occurs when sufficient time has passed in a buildup or drawdown test during which bottomhole pressure varies linearly with the logarithm of flow time for the drawdown case or the logarithm of the Horner time group for the buildup case. The major limitation of this method is that the time required to
reach the straight line where the slope is related to formation permeability may be impractically long for low-permeability gas reservoirs as shown by Gringarten et al.\textsuperscript{7} and Cinco et al.\textsuperscript{9}

Millheim and Cichowicz\textsuperscript{3} demonstrated that when the linear flow into a fracture dominates at early time, a graph of bottomhole pressure vs. a square root-of-time function will result in a straight line with slope, m. Using the value of this slope, the fracture half-length can be determined provided an independent estimate of formation permeability is available. Limitations of this method which are well-summarized by Lee and Holditch\textsuperscript{4} include:

1. Uniform-flux fracture is present. In practice, the conductivity for the fracture is not large enough to behave as a uniform-flux fracture.

2. An independent estimate of formation permeability is needed to calculate fracture half-length. In practice, a pre-fracture pressure buildup test may be unavailable or difficult to obtain.

3. For the method to be applicable to very early-time data during which linear flow dominates, no wellbore storage distortion can be present. However, wellbore storage distortion can be significant in low-permeability fractured gas wells.

Gringarten et al.\textsuperscript{7} provided analytic solutions for both infinitely-conductive and uniform-flux fractures. Their method allowed the use of early time transient pressure data for the determination of both formation permeability and fracture length. They showed that a graph of dimensionless pressure vs. the logarithm of dimensionless time has a characteristic slope of 1.151 in the absence of boundary
effects. These methods, however, are limited by the assumptions of infinitely-conductive and uniform-flux fracture. In most cases, the fracture conductivity as a result of hydraulic fracturing treatments is not that high.

Agarwal et al.\(^8\) reported that methods which are based on the concept of infinite fracture conductivity are not adequate for analyzing wells with finite-conductivity fractures. They found that such methods yield fracture lengths which are too short for fractures created by massive hydraulic fracturing operations. In addition, fracture conductivity cannot be estimated from conventional methods.

In the past few years, several type curves\(^6,9,10\) have been published which can be used to analyze transient pressure data for low-permeability, fractured gas wells with finite-conductivity fractures. Type curves of Cinco et al.\(^9\) for constant production rate case and those of Agarwal et al.\(^8\) for constant pressure case represent two of the more important type curves. By type-curve matching the transient pressure data, it is possible to determine simultaneously and uniquely reservoir permeability, fracture half-length, and fracture conductivity. However, it is often difficult to obtain the position of best fit. This may lead to different formation and fracture descriptions. As noted by Agarwal et al.\(^8\) the uniqueness problem can be overcome if formation permeability is known independently.

Virtually all studies and solutions of transient pressure behavior assume that the wellbore is intersected by a single vertical fracture of equal half-length emanating from each side of the wellbore. The
fracture is further assumed to occur in a direction perpendicular to the minimum regional stress around the wellbore.

Although such assumptions are necessary for most transient pressure analysis, they may not apply where multiple fractures are present. Abnormal treating pressures during massive hydraulic fracturing treatments have been considered as a possible cause of multiple fractures occuring in gas wells.\textsuperscript{15} Daneshy and Conrad\textsuperscript{16} found that as fracturing pressure increases significantly above the in-situ stress, the potential exists for auxiliary fractures to be created in addition to the primary fracture. These fractures can be found at the wellbore or by opening of natural fissures.

In most cases, multiple fractures are desirable. Much research\textsuperscript{17-26} has been conducted to develop effective techniques for creating multiple fractures to stimulate low-permeability gas reservoirs, especially naturally fractured reservoirs. Eventhough multiple fractures may not increase the drainage area significantly, they can increase productivity by effectively connecting the natural fissures to the wellbore. A significant amount of unconventional gas reserves are present in naturally fractured reservoirs.

Several researchers\textsuperscript{17,18,20,23} at Sandia National Laboratories have studied the use of tailored-pulse-loading as a technique for initiating and extending multiple fractures in a wellbore. This technique involves the shaping of borehole input pressure-time behavior of explosive or a suitable propellant to produce a specific fracture pattern and associated permeability enhancement. They have reported some success in creating multiple fractures during several in-situ
experiments conducted in an ash tuff tunnel complex where mineback facilities permit direct observation of the fractures. A typical multiple fracture pattern obtained is as shown in Figure 1.\textsuperscript{17}

Narendran and Cleary\textsuperscript{27} studied the growth and interaction of multiple hydraulic fractures for maximizing permeable communication of a well to the productive formation using a numerical model. They proposed that multiple fractures can assist in the creation of fracture networks between multiple wellbores for enhanced recovery and solution mining.

In this study, a two-dimensional single phase reservoir simulator using cylindrical "r-theta" coordinates will be developed. This simulator will then be applied to study and present solutions for analysis of transient pressure behavior of gas wells with multiple, finite-conductivity vertical fractures.

![Fracture and Wellbore Diagram](image)

Figure 1: A typical multiple fracture pattern
NUMERICAL MODEL

A two-dimensional single phase reservoir simulator in cylindrical "r-theta" coordinates for studying fractured gas well performance was developed. The model is based on a conventional implicit five-point finite difference equation. The model was programmed in a generalized form so that cartesian coordinates may also be specified. In addition, single phase oil flow can be simulated at unsteady or steady-state conditions. Detailed description of the simulator is given in Appendix A.

The major assumptions used in this study are:

(1) The reservoir is isotropic, homogeneous, horizontal and is bounded by an upper and lower impermeable boundary.

(2) The reservoir has uniform thickness, \( h \), effective permeability, \( k \), and porosity, \( \phi \), which are independent of pressure.

(3) The vertically fractured well is intersected by fully-penetrating, finite-conductivity single and/or multiple fracture(s) of constant half-length, \( x_f \), permeability, \( k_f \) and porosity, \( \phi_f \).

(4) The gas is produced at a constant rate.

(5) Wellbore storage, gravity, turbulence, and skin effects are negligible.

(6) Initially, the reservoir pressure is constant and uniform throughout the entire system.

The physical well-fracture system is illustrated in Figure 2. The drainage pattern is cylindrical with closed boundaries. For the purpose of generating type curves, the drainage boundary was chosen
Figure 2: Multiple finite-conductivity fractures intersecting a wellbore
sufficiently distant from the well-fracture system so that the pressure behavior would not be significantly affected by their presence during the time period of interest. Due to symmetry, it was sufficient to simulate only one-fourth of the well-fracture system. The grid pattern is shown in Figure 3. Fine grid breakup near the wellbore and along the fracture plane is used to improve accuracy of the model during early transient flow period. The fractures are represented by rows of grid cells with narrow widths, and high permeability.

The problem for the single fracture case can be written in dimensionless terms by defining the following variables in oilfield units:

**Dimensionless Pressure**

**Oil well:**

\[
P_{WD} = \frac{k h \Delta P}{141.2 q_o \mu_o B_o}
\]

(1)

where

- \(P_{WD}\) = dimensionless pressure
- \(\Delta P\) = pressure difference, psia
- \(q_o\) = oil flow rate, STB/day
- \(\mu_o\) = oil viscosity, cp
- \(B_o\) = formation volume factor, RBL/STB
- \(k\) = effective formation permeability, md
Figure 3: Grid system for simulation of multiple fractures
\[ h = \text{formation thickness, ft} \]

Gas well:

\[ P_{WD} = \frac{k \, h \, \Delta m(P)}{1422 \, q_g \, T} \quad (2) \]

where \( P_{WD} = \text{dimensionless real gas pseudo-pressure} \)

\( \Delta m(P) = \text{difference in real gas pseudo-pressure, psia}^2/\text{cp} \)

\( q_g = \text{gas flow rate, MSCF/day} \)

\( T = \text{reservoir temperature, degree R} \)

**Dimensionless Time**

\[ t_{DF} = \frac{0.0002637 \, k \, t}{\phi \, \mu_i \, c_t \, x_f^2} \quad (3) \]

where \( t_{DF} = \text{dimensionless time} \)

\( t = \text{time, hours} \)

\( \phi = \text{porosity, fraction} \)

\( c_t = \text{total system compressibility, psia}^{-1} \)

\( x_f = \text{total fracture half-length, ft} \)

\( \mu_i = \text{initial gas viscosity, cp} \)

**Dimensionless Fracture Conductivity**

The dimensionless fracture conductivity, originally proposed by Prats\textsuperscript{5}, and later modified by Cinco et al.\textsuperscript{9}, was adopted in
this study. It is defined as follows:

\[ C_r = \frac{w k_f}{\pi k x_f} \quad (4) \]

where \( C_r \) = dimensionless fracture conductivity  
\( w \) = fracture width, ft  
\( k \) = effective formation permeability, md  
\( k_f \) = fracture permeability, md  
\( x_f \) = total fracture half-length, ft

The solution of the diffusivity equation is obtained using a finite difference program called MULTIFRAC. Time step sizes are automatically controlled within the program so that the pressure difference between any time step does not exceed a specified PMAX value of 400 psia to improve accuracy of the model. A direct solution method using the cyclic-2 ordering scheme is used. This provides for a fast means of solving the system of linear equations describing the reservoir conditions.

Modifications for Multiple Fractures

The orientation and location of auxiliary fractures intersecting the wellbore can also be easily simulated using MULTIFRAC. Pseudo-fracture porosity and permeability was derived in Appendix B to simulate the linear fractures in cylindrical "r-theta" coordinates. This is the major advantage of using cylindrical "r-theta" coordinates. Multiple fractures in any directional orientation can be simulated.
Figure 4 illustrates the schematic drawings of the three different multiple fracture patterns considered in this study. These are:

1. 4-wing star fractures where the primary and auxiliary fractures are at 90 degrees to each other.
2. 6-wing star fractures where the primary and auxiliary fractures are at 45 degrees to each other.
3. 8-wing star fractures where the primary and auxiliary fractures are at 45 and 90 degrees to each other.

The presence of multiple fractures can be simulated by dividing the half-length of a single fracture into n number of half-lengths, where n is the total number of fracture half-lengths. The parameter n is equal to 2, 3, and 4 for the 4-wing, 6-wing, and 8-wing star fracture systems. Additional terms are defined below.

The length of the primary fracture and auxiliary fractures can be determined as a function of half-length ratio:

$$x_{frij} = \frac{x_{fi}}{x_{fj}}$$

for i = 1, 2, ..., n-1, and j = 2, 3, ..., n. The symbols $x_f$ represents the total fracture half-length while $x_{fi}$ represents the half-length of the primary fracture. The rest of the subscripted symbols, $x_{fi}$ and $x_{fj}$ are the fracture half-length of the auxiliary fractures.

In order to simplify the analysis, the fracture width, porosity, and permeability are kept constant. It is further assumed that the total volume occupied by multiple fractures is the same as that of the
Case 1: Single fracture

Case 2: 4-wing star fractures
\( n = 2, \ x_{f1} + x_{f2} = x_f \)

Figure 4: Schematic diagrams of multiple fracture patterns
Case 3: 6-wing star fractures
\(n = 3, \ x_f_1 + 2x_f_2 = x_f\)

Case 4: 8-wing star fractures
\(n = 4, \ x_f_1 + x_f_2 + 2x_f_3 = x_f\)

Figure 4 (continued)
single fracture. Applying these assumptions, we have

\[ \text{wh}x_{f1} + \text{wh}x_{f2} + \ldots + \text{wh}x_{fn} = \text{wh}x_f \]  

(5)

Thus,

\[ x_f = \sum_{i=1}^{n} x_{f_i} \]  

(7)

For a fixed value of \( C_r \) and a desired multiple fracture pattern, we can assign different values of \( x_{frij} \). By solving equations (5) and (7) simultaneously, we can calculate values of individual fracture half-lengths, \( x_{f1}, x_{f2}, \ldots, x_{fn} \). Proper grid breakup will be assigned and the transient pressure behavior of multiple fractures can be simulated.

To illustrate this approach, let us consider a well with a 4-wing star fracture system where \( n = 2 \). The following equations apply:

\[ x_{fr12} = \frac{x_{f1}}{x_{f2}} \]  

(8)

\[ x_f = x_{f1} + x_{f2} \]  

(9)

where \( x_{f1} \) = half-length of primary fracture, ft

\( x_{f2} \) = half-length of auxiliary fracture, ft

For \( x_{fr12} = 50 \) and \( x_f = 500 \) ft, we can calculate \( x_{f1} \) and \( x_{f2} \) to be 490.2 and 9.8 ft, respectively.

For each assumed value of \( x_{fr12} = 5, 10, 20, \) and 50, the corresponding values of \( x_{f1} \) and \( x_{f2} \) are calculated and simulated for \( C_r = 0.2, 1, 2, 10, \) and 100. A set of type curves with the logarithm of \( P_{wD} \) vs. the logarithm of \( t_{Dx_f} \) can be
generated to evaluate the transient pressure behavior of gas wells with multiple fractures.
MODEL VERIFICATION

Most work in fracture simulation was performed using either cartesian or elliptical coordinates.\textsuperscript{5,8,9,12,28} In this study, the use of a two-dimensional single phase model in cylindrical "r-theta" coordinates for simulating single and/or multiple fractures was developed. The model was verified extensively against analytical, semi-analytical, and numerical solutions available in the literature.

A low-permeability, hydraulically fractured gas well with hypothetical properties as presented by Holditch \textit{et al.}\textsuperscript{2} was chosen as an example problem to verify the MULTIFRAC simulator. The data is illustrated in Table 1. The fracture permeabilities were calculated to yield $C_p$ values of 0.2, 1, 2, 10, and 100. Gas PVT properties were calculated in the simulator using published correlations\textsuperscript{6} and are shown in Table 2.

Time step size parameters are shown in Table 3. These allow for an automatic adjustment of the time-step sizes throughout execution of the model. A 26X10 cell grid breakup was used and is shown in Table 4. Fine grids are used near the wellbore and along the fractures to obtain accurate transient pressure results. A radial row of high permeability ($1.0 \times 10^7$ md) was created along the wellbore radius to simulate the physical wellbore.

The model was first checked for material balance. The material balance check is a necessary condition to demonstrate accuracy of any reservoir simulation model. In this study, the material balance errors are expressed in terms of a percentage of the gas in place during every
Table 1

Low-permeability fractured gas well data

<table>
<thead>
<tr>
<th>Reservoir Data</th>
<th>Value</th>
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<tbody>
<tr>
<td>Gas Production Rate, $Q_{sc}$</td>
<td>500.0 MSCF/D</td>
</tr>
<tr>
<td>Initial Pressure, $P_i$</td>
<td>6500.0 psia</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
<td>0.075 fraction</td>
</tr>
<tr>
<td>Permeability, $k$</td>
<td>0.05 md</td>
</tr>
<tr>
<td>Reservoir Thickness, $h$</td>
<td>25.0 ft</td>
</tr>
<tr>
<td>Wellbore Radius, $r_w$</td>
<td>0.25 ft</td>
</tr>
<tr>
<td>Drainage Area</td>
<td>640.0 acres</td>
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<tr>
<td>Formation Temperature, $T$</td>
<td>282 degrees F</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Fracture Data</th>
<th>Value</th>
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<tbody>
<tr>
<td>Fracture Half-Length, $x_f$</td>
<td>500 ft</td>
</tr>
<tr>
<td>Fracture Width, $w$</td>
<td>0.02 ft</td>
</tr>
<tr>
<td>Fracture Porosity, $\phi_f$</td>
<td>0.35 fraction</td>
</tr>
<tr>
<td>Fracture Permeabilities, $k_f$</td>
<td>785.4, 3927, 7854, 39270, 392700 md</td>
</tr>
<tr>
<td>Dimensionless Fracture Conductivities, $C_f$</td>
<td>0.2, 1.0, 2.0, 10.0, 100.0</td>
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Table 2
Gas PVT properties

<table>
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<td>Surface Temperature</td>
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<tr>
<td>Pseudo-Critical Temperature</td>
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<tr>
<td>Pseudo-Critical Pressure</td>
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<tr>
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<td>Mole Percent Carbon Dioxide</td>
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<tr>
<td>Mole Percent Nitrogen</td>
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<table>
<thead>
<tr>
<th>Pressure (psia)</th>
<th>Viscosity (cp)</th>
<th>Compressibility factor</th>
<th>Compressibility (1/psia)</th>
<th>Pseudo-Pressure (psia**2/cp)</th>
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<tr>
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<tr>
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<td>0.9850 E-04</td>
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Table 3

Time step size distribution

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<tr>
<td>Maximum Time Step (DELMAX)</td>
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<td>Time Step Multiplier (DELMLT)</td>
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<td>Total Time Steps (ISTMAX)</td>
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<tr>
<td>Total Cumulative Time (TQCHG)</td>
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Table 4
Grid breakup for gas well with single fracture

Angular Direction - Angles (Degrees)
10 Cells

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<th>Angle</th>
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<th>5.0</th>
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</table>

Radial Direction - Radii (Feet)
26 Cells

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</tr>
<tr>
<td></td>
<td>2979.0</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
time step. It was found that the material balance errors ranged from zero during early transient stage to 0.00001 for late time periods. This shows that the mass of fluid withdrawn matched the change in average pressure.

The model was then checked rigorously against published semi-analytical, analytical, and numerical solutions for transient pressure behavior of a wellbore intersected by finite-conductivity vertical fractures.

For low fracture conductivity ratios of $C_r = 0.2, 1,$ and $2$, excellent agreement was obtained compared with the semi-analytical solution of Cinco et al.\(^9\) Figure 5 shows the plot of the logarithm of dimensionless pressure vs. the logarithm of dimensionless time for $C_r = 0.2, 1,$ and $2$.

For $C_r$ values of 10 and 100, the simulation results were compared with the analytic solutions of Gringarten et al.\(^7\) for an infinite-conductivity fracture. Close agreement was obtained as evidenced in Figure 6. The solution of Gringarten et al. presents a check for an analytic lower bound on dimensionless pressure. Figure 6 shows that the simulation results were slightly lower than the analytic solution of Gringarten et al. For $C_r = 10$, the simulation results matched closely with the analytical solution for an infinite-conductivity fracture. During early transient time, expressed in terms of dimensionless time, the error in $P_{wD}$ for $C_r = 100$ was about 5 to 7%. The error decreased significantly with increasing $t_{DF}$. After a $t_{DF}$ of 1.0, the simulation results agreed very well with the analytic solution.
Figure 5: Comparison of type curves for \( C_r = 0.2, 1.0, \) and 2.0 with semi-analytical solutions of Cinco et al.
Figure 6: Comparison of type curves for Cr = 10 and 100 with analytic solutions of Gringarten et al.
Barker and Ramey have also reported lower dimensionless pressures for high conductivity ratios. They used a finite-element model with a ribbon source connected to a finite fracture width which allowed some minor additional flow into the fracture. Figure 7 illustrates the excellent agreement between the results obtained in this study for $C_r = 100$ and finite element solutions of Barker and Ramey for $C_r = 20,000/r$. Cinco et al. have shown that the well performance for a finite-conductivity vertical fracture with $C_r$ greater than or equal to 100 should be essentially the same as that of an infinite-conductivity fracture.

Several reasons are possible for the discrepancies between the $P_{WD}$ values of this study and the analytic solution of Gringarten et al. for high $C_r$ values. First, numerical round-off errors are inherent in numerical studies, especially in calculating small pressure gradients present during the early transient stage. Next, the model geometries are different. Gringarten et al. used a line source well and an infinitesimal fracture width. In this study, small radial cells with high permeability were used to represent the physical wellbore. Finally, the use of pseudo-porosity and pseudo-permeabilities to represent linear fracture properties in cylindrical "r-theta" coordinates may have contributed some minor errors for high $C_r$ values.

Finally, the simulation results were checked for the characteristic quarter slope and half slope for low and high $C_r$ values respectively on a log-log plot of $P_{WD}$ against $tDX_r$ for early transient flow. Figure 3 shows that the characteristic
Figure 7: Comparison of type curves for $C_r = 100$ with finite element solutions of Barker and Ramey.
quarter slope is present for $C_r = 0.2$ and $1.0$ while the
characteristic half slope is present for $C_r = 100$. Cinco et
al.\textsuperscript{11} have found that the characteristic quarter slope and half
slope are indicative of bilinear and formation linear flow period
respectively. The bilinear flow period consists of a linear
incompressible flow within the fracture, and a linear compressible flow
in the formation. Bilinear flow occurs whenever most of the fluid
entering the wellbore comes from the formation provided that the
fracture tip effects are not felt at the well. Formation linear flow
occurs in a perpendicular direction to the fracture plane. This flow
is only possible at early times for fractures of high conductivity.
TRANSIENT PRESSURE BEHAVIOR OF GAS WELLS

A total of thirty computer runs were made to generate the type curves in this study. Six of them were for the single fracture case while the remainder of them were made for the multiple fractures case. An average run for the single fracture takes about 400 CPU seconds while that for the multiple fractures takes about 1250 CPU seconds using a PRIME 750 minicomputer. Results are presented first for the single fracture case followed by those for the multiple fractures.

Single Fracture Case

Figure 9 shows log-log type curves for gas wells with single-conductivity vertical fractures. The dimensionless fracture conductivity, $C_r$, ranges from 0.2 to 100 while the dimensionless time ranges from $1 \times 10^{-5}$ to 10. Higher values of $C_r$ normally correspond to lower formation permeability and/or shorter fracture length. At lower values of $t_{Dx_f}$, there is a wide separation between the curves of different $C_r$ values. It is observed that this separation diminishes with increasing values of $t_{Dx_f}$. Each type curve corresponding to a $C_r$ value will serve as a reference for comparing type curves of multiple fractures throughout the remainder of this investigation.

The type curves generated in this study as shown in Figure 9 are almost identical to those of Cinco et al. However, the early time range has been extended to $1 \times 10^{-5}$. Agarwal et al. noted
that testing times on low-permeability gas wells will rarely be long enough for the test data to be on the pseudo-radial flow portion of the type curves where conventional semi-log analysis is applicable.

The type curves illustrated in Figure 9 have been generated using pressure drawdown data. However, they may also be extended to analyze pressure buildup data if the producing time, $t_p$, prior to shut-in is sufficiently long compared to shut-in time, $\Delta t$, that is $(t_p + \Delta t)/t_p = 1$. If the producing time is small, the buildup data will resemble one of a lower fracture conductivity unless the values of shut-in time are corrected for producing time using a method proposed by Agarwal. This appears also true for the turbulence effect which is not included in this study.

Hitherto we have considered transient pressure behavior only for gas wells with single finite-conductivity vertical fractures intersecting the wellbore. The effect of multiple vertical fractures on the transient flow of gas wells will now be discussed. All properties for the single fracture system are kept constant except for the grid breakup which is replaced by that of the multiple fracture system as a function of the primary and auxiliary fracture half-lengths.

4-Wing Star Fractures Case

Figures 10 through 14 present type curves for gas wells with 4-wing star fractures intersecting the wellbore. The values of $x_{fr12}$ range from 5 to 50. The grid breakup is shown in Table 5.
Figure 10: Type curves for 4-wing star fractures ($C_p = 0.2$).
Figure 11: Type curves for 4-wing star fractures ($C_r = 1.0$)
Figure 12: Type curves for 4-wing star fractures ($C_r = 2.0$)
Figure 14: Type curves for 4-wing star fractures ($C_r = 100.0$)
### Table 5

Grid Breakup for Gas Well With 4-Wing Star Fractures

#### Angular Direction - Angles (degrees)

<table>
<thead>
<tr>
<th>10 Cells</th>
<th></th>
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<th></th>
</tr>
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</tr>
<tr>
<td>50.0</td>
<td>75.0</td>
<td>85.0</td>
<td>88.0</td>
<td>90.0</td>
<td></td>
</tr>
</tbody>
</table>

#### Radial Direction - Radii (feet)

**Case 1:** \( x_{fr12} = 50 \), \( x_{f1} = 490.20 \) ft, \( x_{f2} = 9.80 \) ft

<table>
<thead>
<tr>
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<th></th>
<th></th>
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<td></td>
</tr>
</tbody>
</table>

**Case 2:** \( x_{fr12} = 20 \), \( x_{f1} = 479.19 \) ft, \( x_{f2} = 23.81 \) ft

<table>
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</tbody>
</table>

**Case 3:** \( x_{fr12} = 10 \), \( x_{f1} = 454.55 \) ft, \( x_{f2} = 45.45 \) ft

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</table>
Table 5 (continued)

Case 4: \( x_{fr12} = 5, x_{r1} = 416.67 \text{ ft}, x_{r2} = 83.33 \text{ ft} \)
30 Cells

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Figure 10 illustrates the low conductivity case where \( C_r = 0.2 \). The solid curve represents the type curve for the single fracture case. The type curves appear to approach the single fracture case asymptotically from below. As the ratio of the primary fracture length, \( x_{f1} \), to the auxiliary fracture length, \( x_{f2} \), decreases, greater deviation from the single fracture type curve is observed at early time. At late times, the type curves appear to converge at \( t_{DX_f} = 10 \).

Next, let us consider the case for \( C_r = 1.0 \). Figure 11 shows that the curves appear to converge at \( t_{DX_f} = 1 \), and remain essentially the same from that point onwards. At early times, there is a wide separation between the type curves. Figure 12 presents similar observations for \( C_r = 2.0 \) but the point of convergence appears to shift to 0.2.

Figure 13 presents the comparison of type curves for \( C_r = 10 \). It is noted that the point of convergence of the type curves occurs at \( t_{DX_f} = 0.02 \). For \( t_{DX_f} \) greater than 0.02, the dimensionless pressure, \( P_{wD} \), for multiple fractures are slightly higher than that of the single fracture case even though the separation between the curves at these times is small compared to the separation at early times. An important observation is that for values of \( x_{f1}/x_{f2} \) of 50 or greater, the type curves match very closely with the curve for single fracture. This is significant because it shows that single fracture at the wellbore exhibits type curve similar to one with 4-wing star fractures for \( C_r = 10 \). Thus, it is extremely difficult to distinguish whether multiple fractures are
present or not for $x_{fr12}$ greater than 50 due to similarity of the type curves.

Figure 14 shows the type curves for $C_r = 100$. Observations similar to those for $C_r = 10$ can be made. The point of convergence decreased to 0.002. Less separation between the type curves is observed at early times. Therefore, it becomes increasingly difficult to distinguish between the transient pressure behavior of gas wells with single fractures or 4-wing star fractures for high $C_r$ values.

In order to evaluate the effect of half-length ratio on the pressure behavior, type curves for $x_{f1}/x_{f2} = 50$, 10, and 5 are re-plotted in Figures 15, 16, and 17 respectively. The fracture conductivity ratios considered are 0.2, 1, and 100 for each $x_{f1}/x_{f2}$ ratio.

It is noted from Figure 15 that for $t_{DX_f}$ greater than 0.01, the type curves for a 4-wing star fracture system closely resemble those for a single fracture system at $x_{f1}/x_{f2}$ greater than 50. For early times of $t_{DX_f}$ equal to or less than 0.01, the separation between the curves increases especially for low $C_r$ cases. This shows that unless early transient pressure data are available, one may not be able to distinguish between the pressure data of a single fracture system and that of a 4-wing star fracture system.

Figure 16 illustrates the type curves for $x_{f1}/x_{f2} = 10$. It can be seen that there is a wider separation between the type curves compared to those of the single fracture system. This is especially true for early times. For a low $C_r$ value of 0.2, the type curves corresponding to a 4-wing star fracture system lie below the curve of a
Figure 15: Effect of half-length ratio on type curves of 4-wing star fractures ($x_{f1}/x_{f2} = 50$)
Figure 16: Effect of half-length ratio on type curves of 4-wing star fractures ($x_{r1}/x_{r2} = 10$)
Figure 17: Effect of half-length ratio on type curves of 4-wing star fractures ($x_{f1}/x_{f2} = 5$)
single fracture system and tend to approach it asymptotically from below. For $C_r = 1.0$, the type curves converge and overlap each other at $t_{Dx_f}$ greater than 0.2. An interesting point can be noted from the case where $C_r = 100$. During the early time period where $t_{Dx_f}$ is less than 0.002, the 4-wing star fracture system exhibit characteristics that corresponds to a higher fracture conductivity system. However, during later time periods where $t_{Dx_f}$ greater than 0.1, it behaves as a fracture system having slightly lower fracture conductivity than the single fracture system.

Observations similar to those for $x_{f1}/x_{f2} = 10$ can be made for a 4-wing star fracture system with a half-length ratio of 5, as evidenced in Figure 17. In practice, however, it is unlikely that one may create any fracture system where the primary fracture length is five times or less than the auxiliary fracture lengths.

Figure 18 shows semi-log plot of type curves for a 4-wing star fracture system with $C_r = 10$. The slope of 1.151 per log cycle reflecting transient pseudo-radial flow is also shown. All these type curves show a slope of much less than 1.151 in the time range for which most well testing will be conducted. This indicates that pseudo-radial flow is not likely to occur. If post-fracture buildup data were plotted on semi-log paper to determine formation permeability, there is a great tendency to determine an optimistic value. This is because the wrong straight line may be used. If formation permeability is obtained from a pre-fracture buildup test, a more accurate determination of fracture length and conductivity may be obtained by type-curve matching techniques.
Figure 18: Semi-log plot of type curves for a 4-wing star fracture system with $C_r = 10$.
The start of the semi-log straight line with the slope of 1.151/cycle appears to decrease as $x_{f1}/x_{f2}$ ratio decreases. This indicates that as the auxiliary fracture grows in length, the pseudo-radial flow regime may be reached earlier.

Comparing Figures 10 through 18, the following summary can be made concerning transient pressure behavior of gas wells with 4-wing star finite-conductivity vertical fractures:

1. There is a greater separation between the type curves as the auxiliary fractures increase in length, i.e. smaller half-length ratio, and as the fracture conductivity decreases.

2. At early times, the type curves indicate that a 4-wing star fracture system exhibits transient pressure behavior representative of higher fracture conductivities than that of a single fracture.

3. At late times and for $C_p$ greater than 10, the type curves show that the 4-wing star fracture system exhibits transient pressure behavior representative of lower fracture conductivities than those of a single fracture system.

4. Unless early time transient pressure data is available, for half-length ratios of greater than 50, it will be extremely difficult to distinguish whether the curves are produced by a 4-wing star fracture system or by a single fracture system. The transient pressure behavior are similar, especially for high fracture conductivity.
6-Wing and 8-Wing Star Fractures Case

Figure 19 presents a comparison of the type curves for a low fracture conductivity case where $C_r = 0.2$. Four curves are shown, one each for single fracture, 4-wing star fractures, 6-wing star fractures, and 8-wing star fractures for $x_{fr12} = x_{fr1}/x_{fr2} = 20$. For the 8-wing star fractures, an additional constraint of $x_{fr23} = x_{fr2}/x_{fr3} = 2$ is used. The grid system for the 6-wing and 8-wing star fracture system is shown in Tables 6 and 7 respectively. It is noted that at early times, there is a wide separation between the type curves. These curves approach the single fracture case asymptotically from below. At $t_{Dx_f} = 0.02$, the curves for the multiple fractures all converge and are not easily distinguishable. This indicates that the transient pressure behavior of wells with multiple fractures could be represented by a simple 4-wing star fracture system for $t_{Dx_f}$ greater than 0.02 for a low fracture conductivity system. Another interesting note is that the type curves for the 6-wing and 8-wing star fracture are almost identical even at the very early transient stage. It appears that the pressure behavior of a multiple fracture system may be sufficiently represented by a 6-wing star fracture system at all times.

Figure 20 shows a comparison of the type curves for $C_r = 100$ and $x_{fr12} = 20$ for single fracture and multiple fracture patterns as mentioned above. The differences between the transient pressure behavior of gas wells with 4-wing or more star fractures may not be significant for high fracture conductivity case.
Figure 19: Comparison of type curves for $C_s = 0.2$ and $x_1/x_{12} = 20$ for single and multiple fractures.
Table 6
Grid Breakup for Gas Well With 6-Wing Star Fractures

\( x_{r12} = 20, \ x_{f1} = 454.55 \text{ ft}, \ x_{f2} = 22.73 \text{ ft} \)

**Angular Direction - Angles (degrees)**
18 Cells

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Table 7

Grid Breakup for Gas Well With 8-Wing Star Fractures

$x_{fr12} = 20$ and $x_{fr23} = 2$ case
$x_f 1 = 454.55$ ft, $x_f 2 = 22.73$ ft, $x_f 3 = 11.36$ ft

**Angular Direction - Angles (degrees)**
18 Cells

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</table>
Figure 20: Comparison of type curves for $C_f = 100$ and $x_{f1}/x_{f2} = 20$ for single and multiple fractures.
The results of several in-situ experiments\textsuperscript{17} and interpretations of fracturing pressures\textsuperscript{29} where multiple fractures are present indicate that the primary fracture half-lengths are usually at least twenty times those of the auxiliary fractures. Therefore, it can be deduced from Figure 19 and 20 that most multiple fracture systems may be represented sufficiently by a simple 4-wing star fracture system for high fracture conductivity.
EXAMPLE OF APPLICATION

To illustrate the application of type curves generated in this study for multiple finite-conductivity vertical fractures, a hypothetical pressure drawdown test for a fractured gas well is considered. The drawdown test data were generated by the reservoir simulator and are presented in Table 8. The data include wellbore pressures which correspond to the early time period.

The objectives are: (1) to demonstrate the need of early time data in order to detect the presence of multiple fractures, (2) to compute the formation permeability, fracture length, and fracture conductivity, and (3) to compare results obtained by type-curve matching the data against the solution for single fracture and the solution for multiple fractures.

The drawdown test data are first matched against the type curves for multiple fractures. Next, the same data are matched against the type curves for single fracture. The steps used in type-curve matching are:

1. \( m(P_i) - m(P_{wf}) \) vs. time data are plotted on tracing paper using the same log-log scale of the type curves.
2. Since the formation permeability is also to be computed, the tracing paper is moved until a match is obtained with a type curve. The \( x\)-axis and \( y\)-axis of the tracing paper need to be parallel to the \( t_{Dx} \)-axis and \( P_{WD} \)-axis, respectively of the type curve.
3. A match point is obtained as shown in Figure 21 for the 4-wing
Table 8
Pressure drawdown test data for a fractured gas well

<table>
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<tr>
<th>( \phi )</th>
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<th>( h )</th>
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<tr>
<td>( c_t )</td>
<td>9.849 E-05 1/psia</td>
<td>( \mu_i )</td>
<td>0.0252 cp</td>
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<tr>
<td>( r_w )</td>
<td>0.25 ft</td>
<td>( T )</td>
<td>282 deg F</td>
</tr>
<tr>
<td>( P_i )</td>
<td>6500 psia</td>
<td>( q_g )</td>
<td>500 MSCF/D</td>
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<tr>
<td>( m(P_i) )</td>
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<table>
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<th>( t ) (hours)</th>
<th>( P_{wf} ) (psia)</th>
<th>( m(P_{wf}) ) (psia**2/cp)</th>
<th>( m(P_i) - m(P_{wf}) ) (psia**2/cp)</th>
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Figure 21: Drawdown test data matched against type curve for 4-wing star fractures with $C_r = 1.0$
star fractures case where

\[ m(P_i) - m(P_{wf}) = 4.2 \times 10^8 \text{ psia}^2/\text{cp}, \quad p_{WD} = 1 \]

\[ t = 35 \text{ hours}, \quad t_{DX_f} = 0.01 \]

\[ C_r = 1.0, \quad x_{f1}/x_{f2} = 50 \]

(4) Formation permeability, \( k \), is calculated using equation (2)

\[ k = \frac{1422(500)(742)(1)/(25)(4.2 \times 10^8)}{0.075(0.02523)(0.9849\times10^{-4})(0.01)} \]

\[ = 0.0502 \text{ md} \]

(5) The fracture half-length is calculated using equation (3)

\[ x_{f}^2 = \frac{0.0002637(0.0502)(35)}{0.075(0.02523)(0.9849\times10^{-4})(0.01)} \]

\[ x_f = 499 \text{ ft} \]

Since \( x_{f1}/x_{f2} = 50 \), we have \( x_{f1} = 489 \text{ ft} \), and \( x_{f2} = 10 \text{ ft} \).

(6) Step (2) was repeated using the same drawdown data to match the type curves of a well with single fracture. The match point obtained is shown in Figure 22 and has the following values:

\[ m(P_i) - m(P_{wf}) = 3.3 \times 10^8 \text{ psia}^2/\text{cp}, \quad p_{WD} = 1 \]

\[ t = 25 \text{ hours}, \quad t_{DX_f} = 0.01 \]

\[ C_r = 1.0 \]

(7) Using these values, and repeating steps (4) and (5), we obtain

\[ \kappa = 0.0639 \text{ md}, \text{ and } x_f = 475 \text{ ft} \]

Table 9 shows a summary of the values and errors of \( \kappa \) and \( x_f \) calculated by matching the drawdown data against the type curves for
Figure 22: Drawdown test data matched against type curve for single fracture with $C_p = 1.0$
<table>
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<th>Input Value</th>
<th>Calculated</th>
<th>% Error</th>
<th>Calculated</th>
<th>% Error</th>
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<td>Single Fracture</td>
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</tbody>
</table>

Table 9
Summary of Type-Curve Matching Results
single and multiple (4-wing star) fractures. The advantage of the type
curves for multiple fractures generated in this study is quite obvious.
If the proper match point is obtained, type-curve matching can
reproduce the input data with high accuracy. In addition, the primary
fracture half-length as well as the auxiliary fracture half-length can
be calculated. It is noted, however, that the data is merely synthetic
and very early pressure data are required to accurately match the
proper type curve. On the other hand, we can see that even though the
pressure data appear to match the $C_r = 1.0$ type curve for a single
fracture system, the calculated values of $k$ and $x_f$ are somewhat
off. In the absence of early time data, it will be almost impossible
to determine if the fracture system is one of single fracture or
multiple fractures. The uniqueness problem of correctly type-curve
matching the data is thus quite evident.
DISCUSSION OF RESULTS

The example calculation shows that formation permeability and fracture characteristics for gas wells with multiple fractures intersecting the wellbore may be estimated using the type curves generated in this study. However, it should be pointed out that type curve analysis for both single and multiple fracture systems does not necessarily provide unique values of fracture length and conductivity. Many factors including wellbore storage distortion, skin damage, turbulence flow, varying fracture conductivities, and reservoir heterogenities are not considered in this study. However, it would be impractically complicated to include all these variables along with multiple finite-conductivity fractures.

If transient pressure are not available for early times, a uniqueness problem arises when type curve matching is used. Extreme care should be used because they are similiar for large values of time. This also demonstrates the need for determining formation permeability from pre-fracture buildup test.

Even though type curves are generated for drawdown tests only, they are also applicable to buildup tests if the producing time before the buildup test is large compared with the maximum shut-in time analyzed in the buildup test. If production and shut-in periods are of comparable magnitude, the values of shut-in time should be corrected for producing time using a method proposed by Agrawal et al.\textsuperscript{14}
CONCLUSIONS

The main purpose of this study is to investigate and provide a solution that can be applied to analyze the transient pressure behavior of gas wells with multiple, finite-conductivity vertical fractures. Following are the conclusions from this study:

(1) A two-dimensional, single phase gas numerical simulator in cylindrical "r-theta" coordinates has been successfully developed and verified.

(2) For the time period of application, dimensionless pressure, $P_{wD}$, can be correlated with dimensionless fracture conductivity ratio, $C_r$, and fracture half-length ratio, $X_{fr12}$.

(3) A plot of log $P_{wD}$ against log $t_{DX}$ show that the presence of multiple fractures greatly affects the transient pressure behavior at early time, especially for low fracture conductivity. They produce type curves which approach those curves for a single fracture from below, and even cross over the single fracture curve for high fracture conductivity.

(4) For $C_r$ values of less than 10, type curves generated in this study may be used to estimate the number of multiple fractures as well as effective formation permeability, fracture length, and fracture conductivity.

(5) Early-time transient pressure data is extremely important to determine the presence of multiple fractures.

(6) The simulator may be applied to history-match field data to
obtain better estimates of effective formation permeability and fracture characteristics where multiple fractures are suspected to be present.
NOMENCLATURE

\( A_2 \) = accumulation coefficient, \( \text{cp/psia}^2 \)

\( b_g \) = reciprocal of gas formation volume factor, \( \text{SCF/RCF} \)

\( C_r \) = dimensionless fracture conductivity 
\[
C_r = \frac{w_k f}{\pi k_x f}
\]

\( c_t \) = total system compressibility, \( 1/\text{psia} \)

\( h \) = formation thickness, \( \text{ft} \)

\( k \) = permeability, \( \text{md} \)

\( m(P) \) = real gas pseudo-pressure function, \( \text{psia}^2/\text{cp} \)

\( n \) = total number of fracture half-lengths

\( P \) = pressure, \( \text{psia} \)

\( PP \) = real gas pseudo-pressure function, \( \text{psia}^2/\text{cp} \)

\( P_{wD} \) = dimensionless real gas pseudo-pressure 
\[
P_{wD} = \frac{k h \Delta m(P)}{1422 q_g T}
\]

\( q_g \) = flow rate, \( \text{MSCF/day} \)

\( r \) = radial space coordinate, \( \text{ft} \)

\( t \) = time, \( \text{hr} \)

\( t_{dx_f} \) = dimensionless time 
\[
t_{dx_f} = 0.0002637kt/\phi u_i c_t x_f^2
\]

\( T \) = temperature, \( ^\circ \text{R} \)

\( T_g \) = coefficient relating gas properties, \( 1/\text{psia} \)

\( TR \) = radial transmissibility coefficient, 
\[
(\text{ft}^3/\text{day})/(\text{psia}^2/\text{cp})
\]

\( TY \) = angular transmissibility coefficient, 
\[
(\text{ft}^3/\text{day})/(\text{psia}^2/\text{cp})
\]
NOMENCLATURE (continued)

\( x_f \) = total fracture half-length, ft
\( x_{f1} \) = half-length of primary fracture, ft
\( x_{f1} \) = half-length of auxiliary fractures, ft
  (for values of \( i \) other than one)
\( x_{frij} \) = fracture half-length ratio
  \( x_{frij} = \frac{x_{fi}}{x_{fj}} \)
\( \text{vol} \) = cell volume, ft\(^3\)
\( w \) = fracture width, ft
\( w_f \) = fracture half-width, ft
\( z \) = vertical space coordinate, ft
\( \lambda \) = transmissibility, md/cp
  \( \lambda = \frac{k_b g}{\mu g} \)

Greek Letters

\( \mu \) = viscosity, cp
\( \phi \) = transformation variable, \( \rho = r^2 \)
\( \theta \) = angle, radians
\( \phi \) = porosity, fraction
NOMENCLATURE (continued)

Subscripts

f = fracture

\( g \) = gas

\( i \) = cell number in radial direction

\( j \) = cell number in angular direction

\( o \) = oil

\( r \) = reservoir condition

\( s \) = standard condition

\( w \) = well
REFERENCES


APPENDIX A

DESCRIPTION OF MULTIFRAC SIMULATOR

MULTIFRAC is a generalized single phase, two-dimensional reservoir simulator developed in this study. Either cylindrical or cartesian coordinates may be specified. It uses a conventional five-point finite difference technique and a point-distributed grid system.

Either single phase oil or single phase gas may be simulated. For gas flow, the real gas pseudo-pressure function is used to linearize the non-linear partial differential equation. A flow diagram illustrating the major components of the simulator is given in Figure 23. Details about the subroutines used in the simulator are given in the Appendix C.

The grid pattern is selected to provide for fine grid breakup near the wellbore and along the fracture to improve accuracy of the model during the early transient flow period. Time-step sizes are also adjusted automatically to improve accuracy of the model.

Fractured gas wells are modeled using one-quarter of the symmetrical drainage pattern. The fractures are represented by narrow grid blocks of one-half fracture width, high permeability, and high porosity. Constant and uniform fracture conductivity is assumed even though change of fracture conductivity with decreasing pressure can be modeled.

A finite difference equation is written for each cell after spatial discretization using the transformation, \( \rho = r^2 \), as suggested by Aziz and Settari.\(^{28}\) These linear equations are then
solved using a direct solution matrix inversion technique, namely, the cyclic-2 ordering matrix banding scheme. Pressure dependent variables such as gas viscosity and compressibility factors are evaluated using chord-slope linear interpolation technique.

**Input Data**

Data input for the MULTIFRAC simulator includes reservoir data, fracture data, and fluid properties data. The reservoir data include the following:

- Formation permeability, $k$
- Formation thickness, $h$
- Formation porosity, $\phi$
- Reservoir temperature, $T$
- Initial pressure, $P_i$
- Dimensions of drainage area, $A$

The fracture data include:

- Fracture half-lengths, $x_f$
- Fracture half-width, $w_f$
- Fracture permeability, $k_f$
- Fracture porosity, $\phi_f$
- Fracture half-length ratio, $x_{fr}$

Fluid properties such as viscosity and formation volume factors are given as a function of pressure. For a gas well, existing correlations are used to generate gas viscosity, compressibility, and compressibility factors as a function of pressure by specifying the
temperature and specific gravity of the gas. The presence of gas 
impurities can also be accounted for in these correlations.

Operating well conditions including constant well pressure or 
constant producing rate or a combination of both drawdown and buildup 
conditions may be specified.

**Flow Equation**

The two-dimensional gas flow in cylindrical coordinates is 
described by the following non-linear partial differential equation:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \lambda R \frac{\partial P}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left( \lambda Y \frac{\partial P}{\partial \theta} \right) = \frac{\partial}{\partial t} (\phi b_g) - q_g \tag{A-1}
\]

where \[ \lambda = \frac{k b_g}{\mu g} \tag{A-2} \]

\[ b_g = \frac{r T_s}{r T_r r} \tag{A-3} \]

Using the transformation \[ \rho = r^2 \], we get

\[
4 \frac{\partial}{\partial \rho} \left( \rho \lambda R \frac{\partial P}{\partial \rho} \right) + \frac{1}{\rho} \frac{\partial}{\partial \phi} \left( \rho \lambda Y \frac{\partial P}{\partial \phi} \right) = \frac{\partial}{\partial t} (\phi b_g) - q_g \tag{A-4}
\]
The discretized equation has several desirable qualities which include the following:

(1) Finite difference solutions will be exact for steady-state Darcy flow in a homogeneous, isotropic medium in the radial and vertical directions.

(2) The total volume of the drainage pattern is independent of discretization.

The real gas pseudo-pressure function is then used to account for change in gas properties with change in pressure. This function also helps to linearize the non-linear partial differential equation. It is defined by Al-Hussainy et al.\textsuperscript{30} as follows:

\[
m(P) = \int_{\rho_b}^{P} \frac{2p}{uZ} \, dp
\]

(A-5)

Combining equations (A-1) through (A-5), and simplifying, we obtain:

\[
4 \frac{\partial}{\partial p} \left( \rho R A k \frac{\partial m(P)}{\partial \rho} \right) + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( YA k \frac{\partial m(P)}{\partial \theta} \right)
\]

\[
= A \frac{\partial (P/Z)}{\partial m(P)} \frac{\partial m(P)}{\partial t} - q_g
\]

(A-6)

where \( A = \frac{Z_s T_r}{P_s T_r} \)

(A-7)
In finite difference form, equation (A-7) becomes:

\[
TR_{i+1/2} \left( PP_{i+1,j} - PP_{i,j} \right) + TR_{i-1/2} \left( PP_{i-1,j} - PP_{i,j} \right) + \\
TY_{j+1/2} \left( PP_{i,j+1} - PP_{i,j} \right) + TY_{j-1/2} \left( PP_{i,j-1} - PP_{i,j} \right) = \\
v_{i,j} A_{i,j} \frac{\Delta(P/P)}{\Delta t} - q_g
\]  
(A-8)

where

\[
TR_{i+1/2} = 2RAk \left( \frac{\rho_{i+1/2}}{\rho_{i+1} - \rho_i} \right) \left( \theta_{j+1/2} - \theta_{j-1/2} \right) 
\]  
(A-9)

\[
TY_{j+1/2} = \frac{1}{2} YAK \left( \frac{\rho_{i+1/2} - \rho_{i-1/2}}{\rho_{i,j} (\theta_{j+1} - \theta_j)} \right) \Delta z
\]  
(A-10)

\[
A_{i,j} = A \frac{\theta(P/Z)}{\theta m(P)}
\]  
(A-11)

\[
v_{i,j} = \frac{1}{2} \left( \theta_{j+1/2} - \theta_{j-1/2} \right) \left( \rho_{i+1/2} - \rho_{i-1/2} \right) \Delta z
\]  
(A-12)

\[
\rho_{i+1/2} = \frac{\rho_{i+1} - \rho_i}{\ln(\rho_{i+1}/\rho_i)}
\]  
(A-13)

\[
\theta_{j+1/2} = \frac{1}{2} \left( \theta_{j+1} + \theta_j \right)
\]  
(A-14)
Note that the terms on the left hand side of the linear equation (A-8) represents gas flow in and out of the system. On the right hand side, the first term represents gas expansion in the system while the last term represents explicit withdrawal out of or injection into the system. The final form of the system of linear equations is given as follows:

\[ A_{PP_{i+1,j}} + C_{PP_{i-1,j}} - P_{i,j} + E_{PP_{i,j-1}} + F_{PP_{i,j+1}} = D_{i,j} \] (A-15)

**Boundary Conditions**

The symmetric cylindrical drainage system is modeled using one-quarter of the whole system. The following boundary conditions are employed in the model:

1. No-flow outer boundary, \( C(MX,J) = 0 \)
2. No-flow inner boundary, \( A(1,J) = 0 \)
3. No-flow at axes of symmetry, \( E(I,1) = F(I,MY) = 0 \)

Gas production is treated by replacing the actual boundary conditions with homogeneous Neumann (no-flow) boundary conditions. This is done by introducing the flow out of the system through sink terms. These sink terms are actually singular (Dirac functions) as they are zero everywhere except at sink points (wells).
APPENDIX B

DERIVATIONS OF PSEUDO-FRACTURE PROPERTIES

In this study, the use of pseudo-porosity and pseudo-permeability in cylindrical coordinates to simulate exact linear fracture porosity and permeability is developed. A major advantage of using these pseudo-fracture properties is that the orientation of multiple fractures can be easily simulated by simply altering the angle "theta" in the model. It also helps to preserve the material balance of the model.

In deriving these pseudo-fracture properties, two major requirements need to be fulfilled. First, the pore volume of the linear fractures has to be preserved. Next, the radial and angular transmissibilities have to equal the transmissibilities of the linear fractures in the x- and y-directions.

The derivations are given as follows:

Pseudo-Porosity

First, we will derive the equation for pseudo-porosity. The pore volume of the linear fractures must be the same as that of the fractures in cylindrical coordinates.

In cylindrical coordinates, we have

\[ ^{pV}R = \frac{1}{2} (\rho_{i+1/2} - \rho_{i-1/2}) (\theta_{j+1/2} - \theta_{j-1/2}) \Delta z \phi_R \]  
(A-16)
In cartesian coordinates, we have

\[ PV_L = \frac{w \Delta z}{\sqrt{\rho_{i+1/2}} - \sqrt{\rho_{i-1/2}}} \phi_L \quad (A-17) \]

Equating (A-16) and (A-17), we obtain

\[ \phi_R = \frac{2w}{(\rho_{i+1/2} - \rho_{i-1/2})} \frac{\sqrt{\rho_{i+1/2}} - \sqrt{\rho_{i-1/2}}}{(\theta_{j+1/2} - \theta_{j-1/2})} \phi_L \quad (A-18) \]

**Pseudo-Permeability in Radial Direction**

In this case, the transmissibility in x-direction must be the same as that in the radial direction in cylindrical coordinates.

In cylindrical coordinates, we have

\[ TR_{i+1/2} = \frac{2c_A A \Delta z \Theta}{\rho_{i+1/2} - \rho_i} k_{r,i+1/2} \]

\[ = \frac{c_A A \Delta z \Theta}{\ln(r_{i+1}/r_i)} \overline{k}_{r,i+1/2} \quad (A-19) \]

where \( \Delta \Theta = \theta_{j+1/2} - \theta_{j-1/2} \)

In cartesian coordinates, we have

\[ TX_{i+1/2} = \frac{c_A w \Delta z}{(x_{i+1} - x_i)} \overline{k}_{x,i+1/2} \quad (A-20) \]

Equating equations (A-19) and (A-20) yields

\[ \overline{k}_{r,i+1/2} = \frac{w \ln(r_{i+1}/r_i)}{\Delta \Theta (x_{i+1} - x_i)} \overline{k}_{x,i+1/2} \quad (A-21) \]
Pseudo-Permeability in Angular Direction

Finally, the transmissibility in the $y$-direction in cartesian coordinates must be the same as that in the angular direction in cylindrical coordinates.

In cylindrical coordinates, we have

$$T_{\theta j+1/2} = \frac{c_1 A\Delta z \Delta \rho}{2\rho_i (\theta_{j+1} - \theta_j)} \overline{k}_{\theta j+1/2}$$ \hspace{1cm} (A-22)

In cartesian coordinates, we have

$$T_{y j+1/2} = \frac{c_1 A\Delta z \Delta x}{w} \overline{k}_{y j+1/2}$$ \hspace{1cm} (A-23)

Equating (A-22) and (A-23) yields

$$\overline{k}_{\theta j+1/2} = \frac{2\rho_i (\theta_{j+1} - \theta_j) \Delta x}{w \Delta \rho} \overline{k}_{y j+1/2}$$ \hspace{1cm} (A-24)

Equations (A-18), (A-21), and (A-24) are the equations used in MULTIFRAC for pseudo-porosity, pseudo-permeability in the radial and angular directions, respectively.
APPENDIX C

DESCRIPTION OF SUBROUTINES USED IN MULTIFRAC

MULTIFRAC was programmed in modular form for ease in programming and debugging. Figure 23 shows the flow diagram of the model and a brief description of each subroutine is given below:

INITL  - Input reservoir data, well data, and fracture properties
        - Initializes parameters

TAB    - Interpolation of gas properties

EXCEPT - Read in data exceptions

AZIZ   - Sets up spatial discretization in cylindrical coordinates
        - Calculate pseudo-porosity and pseudo-permeability for linear fractures

PSEUDO - Sets up gas PVT properties in a working table
        - Calculate pseudo-pressure for real gas flow

ZANDC  - Calculate gas compressibility and compressibility factor

VISCY  - Calculate gas viscosity

XLGR4  - Lagrangian extrapolation to calculate viscosity ratio corresponding to values of pseudo-critical temperature and pseudo-critical pressure

TABLE  - Generate gas PVT properties in tabular form

FLPROP - Updates gas PVT properties

QPCONS - Calculates flow rate for constant wellbore pressure case

INTRUP - Alter new parameters at desired time intervals

EXTRP  - Linear or parabolic extrapolation of pressure dependent properties

FLOCON - Set up flow coefficients

DFLOW  - Calculate flow coefficients under Darcy flow conditions

TFLOW  - Calculate flow coefficients under non-Darcy flow conditions
A3 - Direct solution technique using cyclic-2 ordering scheme

SOLVE - Matrix solver used in conjunction with A3 ordering scheme
Figure 23: Flow diagram of numerical model
VITA

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