RESERVOIR ANALYSIS USING PRODUCTION DECLINE DATA
AND ADJUSTED TIME

A Thesis
by
THOMAS LEE MCCRAY

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE

December 1990

Major Subject: Petroleum Engineering
RESERVOIR ANALYSIS USING PRODUCTION DECLINE DATA
AND ADJUSTED TIME

A Thesis
by
THOMAS LEE MCCRAY

Approved as to style and content by:

W. J. Lee
(Chairman of Committee)

R. Raghavan
(Member)

W. D. Von Gonten
(Head of Department)

R. R. Berg
(Member)

December 1990
ABSTRACT

Reservoir Analysis Using Production Decline Data and Adjusted Time. (December 1990)
Thomas Lee McCray, B.S., Texas A&M University
Chair of Advisory Committee: Dr. W. John Lee

Dimensionless type curves have been used extensively in the petroleum industry to analyze and interpret production decline data. The accuracy of decline curve analysis with current type curves relies on the assumption of production at constant bottomhole pressure. Also, current analysis procedures become increasingly difficult as data scatter increases (common in production data).

Several methods have been used, with partial success, to account for discrete flowing bottomhole pressure changes. This work proposes a new method to convert the variable-pressure rate data to the constant bottomhole pressure production case by normalizing rate and using an adjusted time. The new rate correction method was applied successfully to simulated data from a homogeneous, undersaturated liquid (black oil), closed circular reservoir.

Excellent results were obtained considering simulated wells with discrete changes in flowing bottomhole pressure, wells which were shut-in during their productive life, and wells which were produced at constant rate. Results also verify the use of the new method with a combination of these flow scenarios. The method proposed has advantages over current methods in the literature. These advantages include stability, speed, and simplicity.

New type curves can more accurately estimate reservoir parameters such as skin, permeability, radial extent, and reserves, given scattered production data. All new dimensionless type curves were verified by error analysis. Random error of up to 25% was added to the production rates to test the application of the type curves. Advantages over the dimensionless rate type curve include data "smoothing," distinct shapes to distinguish between curves, and separation of the transient solutions for varying dimensionless radial extent. A ratio function is also proposed with the advantage of a fixed vertical match.

In summary, this work presents a new procedure for decline curve analysis with data affected by bottomhole flowing pressure changes and random error. Simulated data were analyzed successfully using the new procedure.
DEDICATION

This thesis is dedicated to:

My parents, Tom and Norma McCray, who are both models of the person I hope to become, who have supported me with their boundless love and wisdom throughout my life, and who have rejoiced with me in the good times and encouraged me to move forward through the bad;

My grandparents, Bill and Doris Snavely and Breeta Critchfield, who have helped mold me into the person that I am;

My committee chairman and friend, Dr. W.J. Lee, who has shown nothing less than perfect patience and who has kept me on the right path to complete this work;

My friend, Dr. Thomas Blasingame, who has held my hand throughout my college endeavors and allowed me to stumble only when the lesson learned far exceeded the inconvenience;

My teachers and mentors, especially Dr. W.J. Lee, Dr. R. Raghavan, Dr. R.R. Berg, Dr. L.D. Piper, and Dr. S.A. Holditch, who, possibly without realizing it, have taught me that learning is more than just a grade;

My good friends, Mark and Michelle Zama, Paul and Julie Figel, Dan and Nancy Rayes, Scott Meyers, and Mike Ramsay, for their patience and good humor with me throughout this work;

The band;

My friends and co-students in the well testing group;

And to God for the sacrifice of His Son and the gift of life.

"When your stomach's bad, a few crackers can do wonders. One-hundred thousand pregnant women can't be wrong."

-Charles Everett Decker

(fictional Stephen King character)
ACKNOWLEDGEMENTS

I would like to thank the following for their contributions to this work:

Dr. W.J. Lee, for his guidance and support of this investigation;
Dr. Thomas A. Blasingame, for his guidance and support of this investigation;
Dr. R. Raghavan, for his guidance and support of this investigation;
Dr. R.R. Berg, for his support and review of this investigation;
Dr. S.W. Poston, for his review of this investigation;
The Well Testing Research Group, for financial support;
Ashley Barnett, for his help with computer graphics; and
Drs. R. Raghavan, R.R. Berg, and W.D. Von Gonten, for serving as members of my advisory committee.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>DEDICATION</td>
<td>iv</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>v</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>viii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>ix</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td></td>
</tr>
<tr>
<td>Decline Curve Analysis</td>
<td>1</td>
</tr>
<tr>
<td>Variable-Pressure Rate Analysis</td>
<td>2</td>
</tr>
<tr>
<td>Variable-Rate Pressure Analysis</td>
<td>3</td>
</tr>
<tr>
<td>Objectives of This Research</td>
<td>3</td>
</tr>
<tr>
<td>DEVELOPMENT AND VERIFICATION OF THE VARIABLE-PRESSURE FLOW MODEL</td>
<td>4</td>
</tr>
<tr>
<td>Variable-Pressure Flow Model</td>
<td>4</td>
</tr>
<tr>
<td>Application of the Variable-Rate Approximation to Variable-Pressure</td>
<td>4</td>
</tr>
<tr>
<td>Analysis</td>
<td>6</td>
</tr>
<tr>
<td>Verification of Variable-Pressure Approximation with Simulated Data</td>
<td>7</td>
</tr>
<tr>
<td>Discussion of Results</td>
<td>16</td>
</tr>
<tr>
<td>Section Summary</td>
<td>25</td>
</tr>
<tr>
<td>DEVELOPMENT AND VERIFICATION OF ADJUSTED TIME</td>
<td>26</td>
</tr>
<tr>
<td>Application of the Variable-Pressure Flow Model to Type Curve</td>
<td>26</td>
</tr>
<tr>
<td>Matching</td>
<td>26</td>
</tr>
<tr>
<td>Discussion of Results</td>
<td>28</td>
</tr>
<tr>
<td>Forecasting Production Schedules Using the New Time</td>
<td>35</td>
</tr>
<tr>
<td>Superposition</td>
<td>35</td>
</tr>
<tr>
<td>Section Summary</td>
<td>40</td>
</tr>
<tr>
<td>DEVELOPMENT OF NEW DIMENSIONLESS TYPE CURVES</td>
<td>41</td>
</tr>
<tr>
<td>Assumptions</td>
<td>41</td>
</tr>
<tr>
<td>Approach to New Type Curve Development</td>
<td>42</td>
</tr>
<tr>
<td>Development of Time Averaged Rate</td>
<td>48</td>
</tr>
<tr>
<td>Development of Dimensionless Integral Derivative</td>
<td>50</td>
</tr>
<tr>
<td>Development of Dimensionless Integral Ratio</td>
<td>52</td>
</tr>
<tr>
<td>Estimating Skin and Permeability</td>
<td>53</td>
</tr>
<tr>
<td>Application of Pressure Normalized Functions to Type Curve Matching</td>
<td>55</td>
</tr>
</tbody>
</table>
ERROR ANALYSIS................................................................. 58
Discussion of Results.......................................................... 59

A NEW ALGORITHM TO ANALYZE AND INTERPRET PRODUCTION DATA WITH AN EXAMPLE CASE............................................ 67
  New Algorithm to Analyze Production Data ................................ 67
  Example Case........................................................................ 70

SUMMARY AND CONCLUSIONS.................................................... 92

NOMENCLATURE ................................................................. 93

REFERENCES ........................................................................ 96

SUPPLEMENTARY SOURCES CONSULTED.................................... 97

APPENDIX A - DERIVATION OF EMPIRICAL DIMENSIONLESS RATE AND EMPIRICAL DIMENSIONLESS CUMULATIVE PRODUCTION FOR POST-TRANSIENT FLOW ............. 98

APPENDIX B - DERIVATION OF NEW EMPIRICAL DIMENSIONLESS FUNCTIONS FOR POST-TRANSIENT FLOW ......................... 101

APPENDIX C - LAPLACE SOLUTIONS FOR CONSTANT TERMINAL PRESSURE PRODUCTION FROM A BOUNDED, CIRCULAR RESERVOIR .............................................. 103

APPENDIX D - CONSIDERATIONS FOR ANALYZING SINGLE-PHASE GAS AND MULTI-PHASE FLOW ........................................... 105

VITA....................................................................................... 109
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SIMULATOR INPUT DATA FOR VERIFICATION OF VARIABLE-PRESSURE APPROXIMATION</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>PRODUCTION SCHEDULES USED TO VERIFY VARIABLE-PRESSURE APPROXIMATION</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>RESULTS FROM INTERPRETED BEST FIT STRAIGHT LINE</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>COMPUTED RESULTS FROM STRAIGHT LINE APPROXIMATION</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>ERROR ANALYSIS SUMMARY OF NEW PLOTTING FUNCTIONS COMPARED WITH DIMENSIONLESS RATE</td>
<td>66</td>
</tr>
<tr>
<td>6</td>
<td>ROCK AND FLUID PROPERTIES FOR EXAMPLE CASE</td>
<td>72</td>
</tr>
<tr>
<td>7</td>
<td>FLOWING WELL DATA FOR EXAMPLE CASE</td>
<td>73</td>
</tr>
<tr>
<td>8</td>
<td>CALCULATED VALUES FOR STRAIGHT LINE APPROXIMATION FOR EXAMPLE CASE</td>
<td>74</td>
</tr>
<tr>
<td>9</td>
<td>CALCULATED ADJUSTED TIME VALUES FOR EXAMPLE CASE</td>
<td>78</td>
</tr>
<tr>
<td>10</td>
<td>CALCULATED VALUES OF PLOTTING FUNCTIONS FOR EXAMPLE CASE</td>
<td>79</td>
</tr>
<tr>
<td>11</td>
<td>SUMMARY OF RESULTS FOR EXAMPLE CASE</td>
<td>90</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simulated rate schedule for a well centered in a bounded, circular drainage area, being produced at constant pressure (Case 1).</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>Simulated rate schedule for a well centered in a bounded, circular drainage area, being produced at constant rate (Case 2).</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>Simulated rate schedule for a well centered in a bounded, circular drainage area, being produced with a step-pressure profile (Case 3).</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>Simulated rate schedule for a well centered in a bounded, circular drainage area, being produced at constant pressure, interrupted by 21-day shut-ins (Case 4).</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>Simulated rate schedule for a well centered in a bounded, circular drainage area, being produced with a step-pressure profile, interrupted by 21-day shut-ins (Case 5).</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>Simulated rate schedule for a well centered in a bounded, circular drainage area, being produced at constant rate, followed by constant pressure production (Case 6).</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>Variable pressure approximation for a well centered in a circular drainage area, being produced at constant pressure (Case 1).</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>Variable pressure approximation for a well centered in a circular drainage area, being produced at constant rate (Case 2).</td>
<td>18</td>
</tr>
<tr>
<td>9</td>
<td>Variable pressure approximation for a well centered in a circular drainage area, being produced with a step-pressure profile (Case 3).</td>
<td>19</td>
</tr>
<tr>
<td>10</td>
<td>Variable pressure approximation for a well centered in a circular drainage area, being produced at constant pressure, interrupted by 21-day shut-ins (Case 4).</td>
<td>20</td>
</tr>
<tr>
<td>11</td>
<td>Variable pressure approximation for a well centered in a circular drainage area, being produced with a step-pressure profile, interrupted by 21-day shut-ins (Case 5).</td>
<td>21</td>
</tr>
<tr>
<td>12</td>
<td>Variable pressure approximation for a well centered in a circular drainage area, being produced at constant rate, followed by constant pressure production (Case 6).</td>
<td>22</td>
</tr>
<tr>
<td>13</td>
<td>Adjusted time plot for a well centered in a circular drainage area, being produced at constant pressure (Case 1).</td>
<td>29</td>
</tr>
<tr>
<td>14</td>
<td>Adjusted time plot for a well centered in a circular drainage area, being produced at constant rate (Case 2).</td>
<td>30</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Adjusted time plot for a well centered in a circular drainage area, being produced with a step-pressure profile (Case 3)</td>
<td>31</td>
</tr>
<tr>
<td>16</td>
<td>Adjusted time plot for a well centered in a circular drainage area, being produced at constant pressure, interrupted by 21-day shut-ins (Case 4)</td>
<td>32</td>
</tr>
<tr>
<td>17</td>
<td>Adjusted time plot for a well centered in a circular drainage area, being produced with a step-pressure profile, interrupted by 21-day shut-ins (Case 5)</td>
<td>33</td>
</tr>
<tr>
<td>18</td>
<td>Adjusted time plot for a well centered in a circular drainage area, being produced at constant rate, followed by constant pressure production (Case 6)</td>
<td>34</td>
</tr>
<tr>
<td>19</td>
<td>Variable pressure approximation for a well centered in a circular drainage area, being produced with a step-pressure profile (Case 3) with extrapolated values</td>
<td>36</td>
</tr>
<tr>
<td>20</td>
<td>Variable pressure approximation for a well centered in a circular drainage area, being produced with a step-pressure profile, interrupted by 21-day shut-ins (Case 5) with extrapolated values</td>
<td>37</td>
</tr>
<tr>
<td>21</td>
<td>Adjusted time plot for a well centered in a circular drainage area, being produced with a step-pressure profile (Case 3) using extrapolated values</td>
<td>38</td>
</tr>
<tr>
<td>22</td>
<td>Adjusted time plot for a well centered in a circular drainage area, being produced with a step-pressure profile, interrupted by 21-day shut-ins (Case 5) using extrapolated values</td>
<td>39</td>
</tr>
<tr>
<td>23</td>
<td>Composite dimensionless rate type curve for a well centered in a circular drainage area, producing at constant pressure (after Fetkovich)</td>
<td>43</td>
</tr>
<tr>
<td>24</td>
<td>Composite dimensionless cumulative type curve for a well centered in a circular drainage area, producing at constant pressure (after Fram)</td>
<td>46</td>
</tr>
<tr>
<td>25</td>
<td>Composite dimensionless time average rate type curve for a well centered in a circular drainage area, producing at constant pressure</td>
<td>49</td>
</tr>
<tr>
<td>26</td>
<td>Composite dimensionless integral derivative type curve for a well centered in a circular drainage area, producing at constant pressure</td>
<td>51</td>
</tr>
<tr>
<td>27</td>
<td>Composite dimensionless integral ratio type curve for a well centered in a circular drainage area, producing at constant pressure</td>
<td>54</td>
</tr>
<tr>
<td>28</td>
<td>Type curve match of new type curve functions calculated with 5% maximum error added to rate (r_{eD}=10, b=0)</td>
<td>60</td>
</tr>
<tr>
<td>29</td>
<td>Error analysis of new dimensionless type curve functions with 5% maximum error added to dimensionless rate (r_{eD}=10, b=0)</td>
<td>61</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>30</td>
<td>Type curve match of new type curve functions calculated with 15% maximum error added to rate ($r_{eD}=10$, $b=0$)</td>
<td>62</td>
</tr>
<tr>
<td>31</td>
<td>Error analysis of new dimensionless type curve functions with 15% maximum error added to dimensionless rate ($r_{eD}=10$, $b=0$)</td>
<td>63</td>
</tr>
<tr>
<td>32</td>
<td>Type curve match of new type curve functions calculated with 25% maximum error added to rate ($r_{eD}=10$, $b=0$)</td>
<td>64</td>
</tr>
<tr>
<td>33</td>
<td>Error analysis of new dimensionless type curve functions with 25% maximum error added to dimensionless rate ($r_{eD}=10$, $b=0$)</td>
<td>65</td>
</tr>
<tr>
<td>34</td>
<td>Rate schedule for example well flowing at constant rate, followed by constant pressure production.</td>
<td>71</td>
</tr>
<tr>
<td>35</td>
<td>Straight line approximation for example case.</td>
<td>76</td>
</tr>
<tr>
<td>36</td>
<td>Adjusted time plot of pressure-normalized rate for example case.</td>
<td>80</td>
</tr>
<tr>
<td>37</td>
<td>Adjusted time plot of time average rate function for example case.</td>
<td>81</td>
</tr>
<tr>
<td>38</td>
<td>Adjusted time plot of rate derivative function for example case.</td>
<td>82</td>
</tr>
<tr>
<td>39</td>
<td>Adjusted time plot of ratio function for example case.</td>
<td>83</td>
</tr>
<tr>
<td>40</td>
<td>Type curve match of the ratio plotting function vs. adjusted time on the $q_{Ddir}$ type curve (example case)</td>
<td>84</td>
</tr>
<tr>
<td>41</td>
<td>Type curve match of the derivative plotting function vs. adjusted time on the $q_{Ddid}$ type curve (example case)</td>
<td>85</td>
</tr>
<tr>
<td>42</td>
<td>Type curve match of the time average rate plotting function vs. adjusted time on the $q_{Dd}$ type curve (example case)</td>
<td>87</td>
</tr>
<tr>
<td>43</td>
<td>Type curve match of the pressure-normalized rate function vs. adjusted time on the $q_{Dd}$ type curve (example case)</td>
<td>88</td>
</tr>
</tbody>
</table>
INTRODUCTION

In this section we will introduce problems associated with the analysis of production data and some of the methods currently used to analyze these data. We will also state the formal objectives of this research.

DECLINE CURVE ANALYSIS

Since the early 1900's, decline curves have been studied as tools to estimate future performance of wells and to determine important reservoir characteristics. Predictions based on decline curves are made possible by the assumption that the flow rate of an oil well declines in a consistent way when the well is produced at constant flowing bottomhole pressure.

Arps$^{1,2}$ reviewed almost 40 years of work on decline curve analysis and summarized his findings into a mathematical relationship now commonly known as the Arps' loss ratio. Using this relationship he showed how to extrapolate rate-time data, following either an exponential or a hyperbolic decline, using semi-log coordinates. This technique is still popular within the petroleum industry for estimating time to economic limit and future reserves for oil and gas wells. The advantage of this method is its simple application to the common exponential decline. However, semi-log analysis becomes more complicated for hyperbolic decline. Also, transient data are of little value in production forecasting.

Fetkovich$^3$ developed a type curve which combined the transient and boundary-dominated rate solutions. This was accomplished by defining dimensionless rate and dimensionless time as functions of initial rate and initial decline, respectively. Fetkovich$^4$ also showed how the type curve could be matched to estimate reserves, pore volume, skin, permeability, radial extent, and the exponent 'b' in Arps' loss ratio for boundary dominated flow. Advantages of the dimensionless rate type curve include the ability to estimate reservoir parameters by matching transient data and the ability to estimate reserves for hyperbolic decline as easily as for exponential decline data. Accurate matches can be difficult, especially at early times, because of production data scatter and lack of distinct slopes of the solutions plotted on the type curve.

This thesis follows the style of the Journal of Petroleum Technology.
Fraim developed a composite type curve based on cumulative production. Fraim's curve can be used to estimate the same parameters as the rate type curve. The advantage is that cumulative production (integral of rate with respect to time) is a smooth function. Cumulative production provides a more accurate type curve match than rate data does. However, nondistinct slopes of the cumulative production solutions can make this type curve difficult to apply at early times.

Analogous to cumulative production is the pressure integral function developed by Blasingame et al. They also derived the time-averaged pressure to provide a smooth, differentiable function. The derivative of time-averaged pressure provides a means to match noisy pressure data on the pressure derivative type curve. The pressure derivative type curve has been used to estimate reservoir parameters, such as skin and permeability more accurately. Blasingame et al. also derived a pressure integral ratio which is useful because of a fixed vertical match with the dimensionless pressure integral ratio type curve.

VARIABLE-PRESSURE RATE ANALYSIS

One apparent problem, common to all decline curve techniques, is the assumption of production at constant bottomhole pressure (BHP). Flowing pressures change in most oil wells. This can be caused by changes in line pressure, changes in separator pressure, short periods of shut-ins for well testing, and lengthy down time. Also, flowing pressure change is continuous in wells produced at constant rate.

Several methods have been proposed in the literature to convolve rate data obtained for wells with variable BHP to the constant-BHP solution. The adjusted data can then be analyzed using constant-BHP decline curve techniques.

Fetkovich showed that new transients introduced into a well, when reinitialized in time, and corrected for changes in BHP, retrace the initial transient decline of the well. This effect is consistent with superposition theory. Although the reinitialized transient data do not extend the rate curve, the analysis can be simplified because the amount of data available for type curve matching is increased. Fetkovich also used superposition theory developed by Hurst to account for step BHP changes at the wellbore.

Economides and Ramey discuss the theory of step pressure testing. They verified that superposition proposed by Hurst becomes unstable as time from the pressure step approaches flowing time up to the pressure step. They also discuss superposition theory of a well converted from constant rate to constant-BHP production. Theory shows that, after some time following the conversion from constant-rate to constant-pressure production, the effect of the constant-rate production "dies out" and the
analytical techniques to analyze constant-BHP production become valid.

Also, as pressure changes become numerous, the mathematical statement of superposition becomes tedious. Interpolated values of rate must also be used with sparse data. Superposition methods also require downhole rate measurements or calculations when applied to periods of shut-in. When the effects of scattered data are also considered, current superposition methods can produce misleading or non-interpretable results.

VARIABLE-RATE PRESSURE ANALYSIS

Analogous to the variable-pressure rate response is the variable-rate pressure response. Blasingame and Lee\textsuperscript{9} developed a method to analyze pressure data under conditions of variable-rate. By neglecting transient flow, they developed a straight line relationship between $\Delta p/q$ and $Q/q$. The slope and intercept of this straight line provide a means to calculate reservoir area and shape factor. This analysis was verified for square-wave, sinusoidal, and random rate profiles.

OBJECTIVES OF THIS RESEARCH

The formal objectives of this research are four fold:

1. To develop a stable superposition method which corrects the variable-BHP production rate profile to the rate profile which would have existed for constant-BHP production.

2. To verify this superposition method using simulated data for common production profiles, including constant-rate production, step-BHP production, production with intermittent shut-ins, and combinations of these.

3. Develop new production decline type curves based on integral plotting functions which provide less ambiguity in matching than the conventional rate type curve.

4. Develop a systematic procedure to analyze noisy, variable-BHP data using the new superposition method and type curves.
DEVELOPMENT AND VERIFICATION
OF THE VARIABLE-PRESSURE FLOW MODEL

In this section we develop a new method to compute equivalent constant-pressure rate production from variable-pressure rate data. The motivation of this investigation arose from the need for a stable superposition method to incorporate into history matching. The advantage of this technique is the estimation of reservoir parameters and reserves without relying on more tedious material balance methods.

The method applies strictly to pseudosteady-state flow. Step changes in flowing bottomhole pressure impose transients which keep the well from reaching true pseudosteady-state flow. However, this does not keep the well from exhibiting a pseudosteady-state-like flow which will be subsequently referred to as stabilized flow.

The method presented in this section will be verified using simulated data. We will also advance this technique to produce normalized rate as a function of adjusted time to allow type curve matching with variable-BHP production data which is discussed later.

VARIABLE-PRESSURE FLOW MODEL

In this section we will consider the variable-rate pressure analysis considered by Blasingame and Lee\textsuperscript{9}. We will verify that this method is also applicable for variable-pressure rate and can be applied to decline curve analysis.

The pressure change, $\Delta p$, in a variable rate well is

$$
\Delta p = p_i - p_{wf} = 141.2 \frac{B\mu}{kh} \left\{ q(t) \left[ \ln \frac{r_e}{r_w} - \frac{3}{4} + \frac{r_w^2}{2r_e^2} + s \right] + 2\pi(0.0002637) \frac{k}{\phi \mu c_t \Lambda} Q(t) \right. 
$$

$$
- \sum_{j=1}^{m} (q_j - q_{j-1}) \sum_{n=1}^{\infty} \frac{J_0(x_n r_w)}{x_n^2 J_1(x_n r_e)} \left[ -x_n^2 \pi(0.0002637) \frac{k}{\phi \mu c_t \Lambda} (t - t_{j-1}) \right] \right\} 
$$

(1)

The derivation of this solution requires the following assumptions:

1. radial flow into the well over the net pay thickness,
2. homogeneous and isotropic porous medium,
3. pressure independent porosity and permeability,
4. small and constant fluid compressibility,
5. constant fluid viscosity,
6. negligible gravity forces,
7. small pressure gradients, and
8. any rate schedule.

At some time following a step-rate change, the infinite series which dominates during transient flow dies out and stabilized flow will return. By assuming stabilized flow, and generalizing the drainage area shape using the Dietz shape factor, $C_A$, the variable-rate approximation for stabilized flow is given by Blasingame and Lee as

$$\Delta p = 70.6 \frac{q(t)B\mu}{kh} \ln \frac{4A}{\epsilon^2 C_A r_w^2} + 0.2339 \frac{B}{\phi h c_i A} Q(t)$$

(2)

or

$$\frac{\Delta p}{q(t)} = 70.6 \frac{B\mu}{kh} \ln \frac{4A}{\epsilon^2 C_A r_w^2} + 0.2339 \frac{B}{\phi h c_i A} \bar{t}$$

(3)

where

$$\bar{t} = \frac{\int_0^t q(t)dt}{q(t)}$$

(4)

Eq. 3 implies that a plot of $\Delta p/q$ vs. $\bar{t}$ will result in a straight line with a slope of

$$m_{vr} = 0.2339 \frac{B}{\phi h c_i A}$$

(5)

and intercept

$$b_{vr} = 70.6 \frac{B\mu}{kh} \ln \frac{4A}{\epsilon^2 C_A r_w^2}$$

(6)

Blasingame and Lee showed that results are the same for step-rate changes, sinusoidal rate changes, and a random rate profile. With large rate changes, however, transient flow (described by the infinite series in Eq. 1) dominates and is not modeled by Eq. 3. This results in a "spike" on the graph of $\Delta p/q$ vs. $\bar{t}$ which is disregarded in determining slope and intercept.
A special case to which we will apply Eq. 3 is the constant-BHP production rate profile. This will allow us to convert variable-BHP rate data to the equivalent constant-BHP rate solution. We can then apply decline curve analysis to estimate reserves and other reservoir parameters.

APPLICATION OF THE VARIABLE-RATE APPROXIMATION TO VARIABLE-PRESSURE ANALYSIS

Re-writing Eq. 3 to solve for \( q/\Delta p \)

\[
\frac{q}{\Delta p} = \frac{0.01416}{B \mu} \frac{k h}{\ln \frac{4A}{\phi \mu c_t A \ln \frac{4A}{e \gamma C_A R_w^\phi} \Delta p}} - 0.07953 \frac{k}{\phi \mu c_t A \ln \frac{4A}{e \gamma C_A R_w^\phi} \Delta p} \frac{Q}{\Delta p}
\]

Equation 7 suggests that a plot of \( q/\Delta p \) vs. \( Q/\Delta p \) will result in a straight line of slope

\[
m_{vp} = -0.07953 \frac{k}{\phi \mu c_t A \ln \frac{4A}{e \gamma C_A R_w^\phi}}
\]

and intercept

\[
b_{vp} = 0.01416 \frac{k h}{B \mu \ln \frac{4A}{e \gamma C_A R_w^\phi}}
\]

Dividing Eq. 9 by Eq. 8

\[
\frac{b_{vp}}{m_{vp}} = \frac{-1}{5.615} \frac{\phi h A c_t}{B}
\]

and recognizing that

\[
N = \frac{1}{5.615} \frac{\phi h A (1 - S_{wi})}{B}
\]

Eq. 10 becomes

\[
N = -\frac{b_{vp}(1 - S_{wi})}{m_{vp} c_t}
\]
We also note that we can determine shape factor if we have values for $k$ and $h$.

Rearranging Eq. 9

$$C_A = \frac{4A}{e^r \sqrt{r_w} \text{Exp}\left\{0.01416 \frac{kh}{B \mu_\sigma v_p}\right\}}$$

(13)

We replace $r_w$ with $r_{wa}$ (apparent wellbore radius), in all equations to consider the skin effect, where

$$r_{wa} = r_w e^{-s}$$

(14)

We have developed a variable-pressure approximation which provides a straight-line relationship and allows us to determine shape factor and reserves. In the following section we verify this relationship with simulated data.

VERIFICATION OF VARIABLE-PRESSURE APPROXIMATION WITH SIMULATED DATA

We must consider all pressure profiles, typically encountered in the field, to effectively test our variable-pressure approximation. Common production scenarios include the following:

1. constant-BHP production,
2. constant-rate production,
3. step-BHP production,
4. constant-BHP production interrupted by shut-ins,
5. step-BHP production interrupted by shut-ins, and
6. constant rate followed by constant-BHP production.

The simulator used to investigate the accuracy and applicability of Eq. 7 was a fully implicit finite-difference reservoir simulator. The simulator modeled radial, single-phase flow of a liquid with small and constant compressibility. The simulator was verified by comparison with the analytical solutions for transient and pseudosteady-state flow in a bounded circular reservoir at a constant producing rate.9

All reservoir parameters are held constant for all simulation cases to effectively compare results. Table 1 describes all input parameters for the liquid production cases. Production descriptions for all cases considered are summarized in Table 2.

Figures 1 through 6 are plots of each simulated rate profile. These are plotted on log-log coordinates to emphasize the need for a new superposition method. Obviously,
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_0 )</td>
<td>0.4</td>
<td>cp</td>
</tr>
<tr>
<td>( B_0 )</td>
<td>1.0</td>
<td>RB/STB</td>
</tr>
<tr>
<td>( r_e )</td>
<td>745.0</td>
<td>ft</td>
</tr>
<tr>
<td>( k )</td>
<td>1.0</td>
<td>md</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.2</td>
<td>(fraction)</td>
</tr>
<tr>
<td>( S_{wi} )</td>
<td>0.0</td>
<td>(fraction)</td>
</tr>
<tr>
<td>( h )</td>
<td>30.0</td>
<td>ft</td>
</tr>
<tr>
<td>( s )</td>
<td>+2.0</td>
<td></td>
</tr>
<tr>
<td>( c_o )</td>
<td>15 \times 10^{-6}</td>
<td>psi^{-1}</td>
</tr>
<tr>
<td>( c_f )</td>
<td>0.0</td>
<td>psi^{-1}</td>
</tr>
<tr>
<td>( C_D )</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>( p_i )</td>
<td>4800.0</td>
<td>psi</td>
</tr>
<tr>
<td>Case Number</td>
<td>Description</td>
<td>Time span, days</td>
</tr>
<tr>
<td>-------------</td>
<td>-------------</td>
<td>----------------</td>
</tr>
<tr>
<td>1</td>
<td>$P_{wf} = 4000$ psi</td>
<td>0 - 2000</td>
</tr>
<tr>
<td>2</td>
<td>$q_0 = 50$ STB/D</td>
<td>0 - 2000</td>
</tr>
<tr>
<td>3</td>
<td>$P_{wf} = 4000$ psi</td>
<td>0 - 100</td>
</tr>
<tr>
<td></td>
<td>$P_{wf} = 3000$ psi</td>
<td>100 - 200</td>
</tr>
<tr>
<td></td>
<td>$P_{wf} = 3500$ psi</td>
<td>200 - 400</td>
</tr>
<tr>
<td></td>
<td>$P_{wf} = 3000$ psi</td>
<td>400 - 600</td>
</tr>
<tr>
<td></td>
<td>$P_{wf} = 3050$ psi</td>
<td>600 - 2000</td>
</tr>
<tr>
<td>4</td>
<td>$P_{wf} = 4000$ psi</td>
<td>0 - 100</td>
</tr>
<tr>
<td></td>
<td>shut-in</td>
<td>100 - 121</td>
</tr>
<tr>
<td></td>
<td>$P_{wf} = 4000$ psi</td>
<td>121 - 621</td>
</tr>
<tr>
<td></td>
<td>shut-in</td>
<td>621 - 642</td>
</tr>
<tr>
<td></td>
<td>$P_{wf} = 4000$</td>
<td>642 - 2000</td>
</tr>
<tr>
<td>5</td>
<td>$P_{wf} = 4000$ psi</td>
<td>0 - 100</td>
</tr>
<tr>
<td></td>
<td>shut-in</td>
<td>100 - 121</td>
</tr>
<tr>
<td></td>
<td>$P_{wf} = 3000$ psi</td>
<td>121 - 621</td>
</tr>
<tr>
<td></td>
<td>shut-in</td>
<td>621 - 642</td>
</tr>
<tr>
<td></td>
<td>$P_{wf} = 3500$ psi</td>
<td>642 - 2000</td>
</tr>
<tr>
<td>6</td>
<td>$q_0 = 50$ STB/D</td>
<td>0 - 1000</td>
</tr>
<tr>
<td></td>
<td>$P_{wf} = 2000$ psi</td>
<td>1000 - 2000</td>
</tr>
</tbody>
</table>
Figure 1 - Simulated rate schedule for a well centered in a bounded, circular drainage area, being produced at constant pressure (Case 1).
Figure 2 - Simulated rate schedule for a well centered in a bounded, circular drainage area, being produced at constant rate (Case 2).
Figure 3 - Simulated rate schedule for a well centered in a bounded, circular drainage area, being produced with a step-pressure profile (Case 3).
Figure 4 - Simulated rate schedule for a well centered in a bounded, circular drainage area, being produced at constant pressure, interrupted by 21-day shut-ins (Case 4).
Figure 5 - Simulated rate schedule for a well centered in a bounded, circular drainage area, being produced with a step-pressure profile, interrupted by 21-day shut-ins (Case 5).
Figure 6 - Simulated rate schedule for a well centered in a bounded, circular drainage area, being produced at constant rate, followed by constant-pressure production (Case 6).
the profiles affected by pressure steps, shut-ins, and constant rate production would be difficult to type curve match without a superposition approach.

We have developed a method to estimate reserves without type curve matching for exponential decline. We cannot abandon type curve matching because it is useful in estimating skin and permeability and in forecasting a production schedule for both exponential and hyperbolic decline. These topics will be investigated later in this report.

DISCUSSION OF RESULTS

Figures 7 through 12 show the results of the variable-pressure approximation approach. These figures show that \( q/\Delta p \) vs. \( Q/\Delta p \) data, for the cases considered, do represent a straight line suggested by Eq. 7. However, we see that these data are shifted up or down on the straight line depending on the pressure change. On each plot we show the true straight line with a slope and intercept calculated from Eqs. 8 and 9 with the simulator input values from Table 1. This verifies that the true straight line is represented by the variable-BHP data.

The results of these analyses are shown in Tables 3 and 4. In Table 3 we compare the slope and intercept of the straight line shown in Figs. 7 through 12 with our interpretation of the best fit straight line. Table 4 compares calculated reserves and \( C_A \) with the true input values. Reserves and \( C_A \) were calculated using the values of \( m_{vp} \) and \( b_{vp} \) in Table 3 and Eqs. 12 and 13.

These results show excellent agreement between calculated and true reserves with a maximum absolute error less than 3%. Results for \( C_A \) are less impressive with up to 35% error. This error is caused by several factors. First, we must consider that the calculated shape factor is based on the exponential of the intercept. Therefore, small errors in intercept result in large errors in shape factor. We also note that, for cases with discreet pressure steps, (Figs. 9, 11, and 12) the variable-pressure approximation curve tends to be displaced from the true constant-BHP straight line approximation. We weighted all stabilized flow data equally in determining the straight line approximations, since theory (Eq. 7) does not predict this behavior.

We might enhance our method by constructing the straight line through the initial stabilized flow region prior to pressure steps or shut-ins (since this data is not shifted from the true straight line). However, to do so would be assuming that these data are always available. In conclusion, if initial flowing conditions persist to stabilized flow, this data should be used to construct the straight line approximation. This will result in more accurate estimates of \( C_A \). If shut-ins or pressure steps occur before the initial
Figure 8 - Variable pressure approximation for a well centered in a circular drainage area, being produced at constant rate (Case 2).
Figure 9 - Variable pressure approximation for a well centered in a circular drainage area, being produced with a step-pressure profile (Case 3).
Figure 10: Variable pressure approximation for a well centered in a circular drainage area, being produced at constant pressure, interrupted by 21-day shut-ins (Case 4).
Figure 11 - Variable pressure approximation for a well centered in a circular drainage area, being produced with a step-pressure profile, interrupted by 21-day shut-ins (Case 5).
Figure 12 - Variable pressure approximation for a well centered in a circular drainage area, being produced at constant rate, followed by constant-pressure production (Case 6).
TABLE 3
RESULTS FROM INTERPRETED BEST FIT STRAIGHT LINE

<table>
<thead>
<tr>
<th>Case</th>
<th>$b_{vp}$</th>
<th>$b_{vp}$ % Error</th>
<th>$m_{vp}$</th>
<th>$m_{vp}$ % Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.056224</td>
<td>+0.30</td>
<td>-0.001991</td>
<td>+0.76</td>
</tr>
<tr>
<td>2</td>
<td>0.056430</td>
<td>+0.67</td>
<td>-0.001992</td>
<td>+0.71</td>
</tr>
<tr>
<td>3</td>
<td>0.056894</td>
<td>+1.50</td>
<td>-0.001988</td>
<td>+0.91</td>
</tr>
<tr>
<td>4</td>
<td>0.056218</td>
<td>+0.29</td>
<td>-0.001978</td>
<td>+1.41</td>
</tr>
<tr>
<td>5</td>
<td>0.056315</td>
<td>+0.46</td>
<td>-0.001961</td>
<td>+2.25</td>
</tr>
<tr>
<td>6</td>
<td>0.056771</td>
<td>+1.28</td>
<td>-0.001991</td>
<td>+0.76</td>
</tr>
</tbody>
</table>

$m_{vp} = -0.002006$

$b_{vp} = 0.056056$
TABLE 4
COMPUTED RESULTS FROM STRAIGHT LINE APPROXIMATION

<table>
<thead>
<tr>
<th>Case</th>
<th>N, RB</th>
<th>N % Error</th>
<th>CA</th>
<th>CA % Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,882,709</td>
<td>-0.87</td>
<td>33.83</td>
<td>+6.98</td>
</tr>
<tr>
<td>2</td>
<td>1,889,605</td>
<td>+1.24</td>
<td>36.37</td>
<td>+15.03</td>
</tr>
<tr>
<td>3</td>
<td>1,907,676</td>
<td>+2.20</td>
<td>42.81</td>
<td>+35.40</td>
</tr>
<tr>
<td>4</td>
<td>1,838,383</td>
<td>-1.51</td>
<td>32.96</td>
<td>+4.25</td>
</tr>
<tr>
<td>5</td>
<td>1,818,427</td>
<td>-2.58</td>
<td>33.68</td>
<td>+6.51</td>
</tr>
<tr>
<td>6</td>
<td>1,900,993</td>
<td>+1.85</td>
<td>40.97</td>
<td>+29.57</td>
</tr>
</tbody>
</table>

\[ N_{\text{true}} = 1,866,543 \text{ RB} \]
\[ C_{A \text{ true}} = 31.62 \]
stabilized flow regime, all stabilized data should be considered. However, this may result in large errors in the calculations of $C_A$. We must also consider that small errors in $k$, $h$, $\mu$, and $B$ are also exponentiated, resulting in increased error in the calculated $C_A$. Therefore, the estimated shape factor should be verified by other means, such as geological data, well spacing, and well patterns.

SECTION SUMMARY

We have developed a method to determine accurate estimates of initial oil in place for wells following exponential decline with common production schedules. The method was verified using simulation studies for a well centered in a bounded circular reservoir. However, the theory developed is generalized with the use of Dietz shape factor and applies to any shape reservoir with a no-flow outer boundary. Calculated errors in shape factor can be large, but can be minimized considering only stabilized flow data prior to discreet pressure changes (provided these data exist).

In the next section we introduce a stable superposition technique to allow type curve matching with data not following constant-pressure production. This will be useful for estimating skin and permeability and forecasting future production.
DEVELOPMENT AND VERIFICATION OF ADJUSTED TIME

In the previous section we verified that variable-BHP data can be analyzed to yield original oil in place (OOIP) and $C_A$ for liquid production following an exponential decline. We now concentrate on analyzing the variable-BHP data to yield estimations of a skin factor, permeability, radial extent, and a future production schedule for undersaturated liquid reservoirs. These objectives can be easily obtained from type curve matching of constant-BHP data. However, current superposition methods can become unstable for variable-BHP production. In this section we develop a new time superposition method which is stable and simple to apply to undersaturated liquid reservoirs.

APPLICATION OF THE VARIABLE-PRESSURE FLOW MODEL TO TYPE CURVE MATCHING

We begin by stating the definition of cumulative production, which relates $t$, $q$, and $Q$ as

$$Q = \int_0^t q\,d\tau$$

(15)

And, for constant-BHP production

$$\frac{Q}{\Delta p} = \frac{1}{\Delta p} \int_0^t q\,d\tau$$

$$= \int_0^t \frac{q}{\Delta p} d\tau$$

(16)

If we consider the trapezoidal approximation to Eq 15

$$Q = \sum_{j=1}^{n} \frac{q_j + q_{j-1}}{2} (t_j - t_{j-1})$$

(17)

or, from Eq. 16
\[
\frac{Q}{\Delta p} = \sum_{j=1}^{n} \frac{q_j + q_{j-1}}{2} \frac{\Delta p}{\Delta p} (t_j - t_{j-1})
\]  

(18)

Considering only a single partition

\[
Q_j - Q_{j-1} = \frac{q_j + q_{j-1}}{2} (t_j - t_{j-1})
\]  

(19)

or, from Eq. 18

\[
\frac{Q_j}{\Delta p} - \frac{Q_{j-1}}{\Delta p} = \frac{q_j + q_{j-1}}{2} \frac{\Delta p}{\Delta p} (t_j - t_{j-1})
\]  

(20)

Equation 19 is valid for both variable-BHP and constant-BHP cases. Therefore, by rearranging Eq. 19, such that

\[
t_j = \frac{2(Q_j - Q_{j-1})}{q_j + q_{j-1}} + t_{j-1}
\]  

(21)

we can "reconstruct" the real time plots shown in Figs. 1 through 6.

Equation 20 is only valid for constant-BHP production. Rewriting Eq. 20

\[
t_j = \frac{2}{q_j + q_{j-1}} \left( \frac{Q_j}{\Delta p} - \frac{Q_{j-1}}{\Delta p} \right) + t_{j-1}
\]  

(22)

However, our objective is to develop a method to allow variable-BHP data to be matched on the constant-BHP rate type curve. This is accomplished by noting that, since each stabilized variable-BHP data point corresponds to an equivalent constant-BHP data point (i.e. the variable-BHP data lie on the same straight line as the constant-BHP data), Eq. 22 will produce the same results for \(t_j\) using either the constant-BHP or the variable-BHP data. When applying Eq. 22 to variable-BHP data, we must remember that we are not calculating producing time, but a time adjusted to the constant-BHP case \((t_{adj})\).
Also, noting that the pressure may change from \( j-1 \) to \( j \), Eq. 22 becomes

\[
t_{adj} = \frac{Q_j - Q_{j-1}}{\frac{Q_j}{\Delta p_j} + \frac{Q_{j-1}}{\Delta p_{j-1}}} + t_{adj \ j-1}
\]

We note that Eq. 23 reduces to Eq. 22 for constant-BHP production.

Since the plot of \( q \) vs. \( Q \) for variable-BHP data does not follow the \( q \) vs. \( Q \) plot for constant-BHP data, we cannot produce the same results simply using data that has not been pressure normalized. This means that the variable-BHP plot of \( q \) vs. \( t_{adj} \) will not follow the constant-BHP plot of \( q \) vs. \( t \). We must plot \( q/\Delta p \) vs. \( t_{adj} \) and compare this with the constant-BHP plot of \( q/\Delta p \) vs. \( t \).

DISCUSSION OF RESULTS

Figures 13 through 18 show the results of pressure normalized rate on the adjusted time scale. The plots were constructed by evaluating Eq. 23 along the \( q/\Delta p \) vs. \( Q/\Delta p \) plots for the cases considered previously. The first value for adjusted time was determined as

\[
t_{adj \ 1} = t_1
\]

We have included the constant-pressure \( q/\Delta p \) data on all cases. With the exception of Figs. 15 and 17, these figures show that Eq. 23 is an accurate time superposition to yield the equivalent constant-BHP normalized rate from the variable-BHP data. We also see that, although the nonstabilized flow data is adjusted in time, it is not corrected to the constant-BHP production curve. This results as spikes on the graph of \( q/\Delta p \) vs. \( t_{adj} \) (Figs 15 through 18). The initial transients in cases 1 through 6 follow the constant-BHP production curve. These transients therefore follow the true transient on the \( q/\Delta p \) vs. \( t_{adj} \) plot even though they do not lie on the straight-line approximation on the graph of \( q/\Delta p \) vs. \( Q/\Delta p \).

Figures 15 and 17 show the inadequacy of this method to reproduce the accurate \( t_{adj} \) for data which produce large gaps in the variable-pressure approximation curve (Figs. 9 and 11). We can accommodate for this by extrapolating the
Figure 13 - Adjusted time plot for a well centered in a circular drainage area, being produced at constant pressure (Case 1).
Figure 14 - Adjusted time plot for a well centered in a circular drainage area, being produced at constant rate (Case 2).
Figure 15 - Adjusted time plot for a well centered in a circular drainage area, being produced with a step-pressure profile (Case 3).
Figure 16 - Adjusted time plot for a well centered in a circular drainage area, being produced at constant pressure, interrupted by 21-day shut-ins (Case 4).
Figure 17 - Adjusted time plot for a well centered in a circular drainage area, being produced with a step-pressure profile, interrupted by 21-day shut-ins (Case 5).
Figure 18 - Adjusted time plot for a well centered in a circular drainage area, being produced at constant rate, followed by constant pressure production (Case 6).
variable-pressure approximation to "fill" these gaps. The extrapolations are shown in Figs. 19 and 20. Figures 21 and 22 show the resulting constant-BHP profiles determined by calculating \( t_{adj} \) considering the extrapolated data.

The constant-pressure production rate type curve can now be used to estimate skin, permeability, and initial reserves using the method proposed by Fetkovich\(^3\,4\) for constant-BHP cases.

**FORECASTING PRODUCTION SCHEDULES USING THE NEW TIME SUPERPOSITION**

We must verify that future production can be predicted using time superposition. This can be accomplished by rearranging Eq. 23 to solve for \( \Delta t_{adj} \) (from \( j \) to \( j+1 \))

\[
\Delta t_{adj} = \frac{2 \left( \frac{Q_{j+1} - Q_j}{\Delta p_{j+1} - \Delta p_j} \right)}{\frac{q_{j+1} - q_j}{\Delta p_{j+1} - \Delta p_j}}
\]

and rearranging Eq. 21 to solve for \( \Delta t \) (from \( j \) to \( j+1 \))

\[
\Delta t = \frac{2(Q_{j+1} - Q_j)}{q_{j+1} + q_j}
\]

We will assume that the well will continue to produce at the current flowing bottomhole pressure (\( \Delta p_{j+1} = \Delta p_j = \Delta p \)). Under this condition we can rewrite Eq. 25

\[
\Delta t_{adj} = \frac{2 \frac{1}{\Delta p} (Q_{j+1} - Q_j)}{\frac{1}{\Delta p} (q_{j+1} + q_j)} = \frac{2(Q_{j+1} - Q_j)}{q_{j+1} + q_j}
\]

Comparing Eqs. 26 and 27, for time steps which \( \Delta p \) remains constant

\[
\Delta t = \Delta t_{adj}
\]

Equation 28 allows us to extrapolate \( q/\Delta p \) to \( \Delta t_{adj} \) and determine \( q \) at a time increment \( \Delta t \) where

\[
q_{\text{extrapolated}} = \left( \frac{q}{\Delta p} \right)_{\text{extrapolated}} \Delta p
\]
Figure 19 - Variable pressure approximation for a well centered in a circular drainage area, being produced with a step-pressure profile (Case 3) with extrapolated values.
Figure 20 - Variable pressure approximation for a well centered in a circular drainage area, being produced with a step-pressure profile, interrupted by 21-day shut-ins (Case 5) with extrapolated values.
Figure 21 - Adjusted time plot for a well centered in a circular drainage area, being produced with a step-pressure profile (Case 3) using extrapolated values.
Figure 22 - Adjusted time plot for a well centered in a circular drainage area, being produced with a step-pressure profile, interrupted by 21-day shut-ins (Case 5) using extrapolated values.
We must emphasize caution at this point. Since we are superimposing on time, care must be taken to extrapolate from the final $t_{adj}$. This is not necessarily the largest $t_{adj}$.

Equation 27 also reveals an important fact in calculating adjusted time. Unless there is a pressure change between $t_j$ and $t_{j-1}$, adjusted time can be determined directly using the relationship of Eq. 28, without having to apply Eq. 20. This is valuable in saving computation time.

SECTION SUMMARY

We have developed a stable time superposition to convert variable-BHP production data to the equivalent constant-BHP normalized rate profile. We have developed equations which show that the new method can be used to forecast a production schedule. We have stated, without proof, that this data can be matched using the method developed by Fetkovich to allow estimates of skin, permeability, and initial reserves. This will be verified in the following section. We will also develop new type curves which will allow us to more accurately type curve match rough production data. Next, we will summarize all work into an algorithm which will allow us to estimate these reservoir parameters given rough, variable-pressure production data. We will then use the dimensionless type curves to estimate skin, permeability, and reserves for undersaturated oil reservoirs.
DEVELOPMENT OF NEW DIMENSIONLESS TYPE CURVES

In this section we introduce our development of new dimensionless functions by first stating necessary assumptions and background. We then give the general methodology of our solution development of the new type curves for constant-BHP production. To develop a basic understanding of the new type curves, we will limit the discussion to general mathematical concepts. For more rigorous development of the equations, the reader is encouraged to refer to the appendices.

We emphasize that the purpose of the new type curves introduced here is to obtain distinct matches with scattered rate data. Scattered data will be defined as data which contain some random error. This does not include shifts in flow rate resulting from either shut-ins or changes in flowing bottomhole pressure. These effects should be "corrected", using the approach outlined in the previous section, prior to type curve matching.

ASSUMPTIONS

All analytical solutions discussed in this thesis are based on several important limiting assumptions. Not only must we consider the assumptions necessary to solve the radial diffusivity equation, but we must also include the assumptions made in developing the equation.

Derivation Assumptions
Certaint assumptions required to derive the radial diffusivity equation for liquid flow are:

1. homogeneous and isotropic porous reservoir;
2. medium of uniform thickness;
3. pressure independent rock and fluid properties;
4. small, continuous pressure gradients;
5. radial, laminar flow;
6. negligible gravity forces;
7. isothermal reservoir; and
8. small and constant fluid compressibility.

Solution Assumptions

The assumptions necessary to solve the radial diffusivity equation vary with the reservoir model. The assumptions necessary for the solution discussed in this work
(bounded, circular reservoir) follow:

1. constant-pressure production into wellbore,
2. well is centered in circular drainage area,
3. no flow across outer boundary, and
4. reservoir pressure is uniform prior to initial production.

Few, if any, reservoirs strictly adhere to these assumptions. However, decline curves developed under these assumptions have been applied to many wells and fields with accuracy. Although we have developed a method to adjust data for step-pressure changes, the assumption of small, continuous pressure gradients is still required to develop the dimensionless solutions addressed in this chapter.

**APPROACH TO NEW TYPE CURVE DEVELOPMENT**

Much progress has been made in the analysis of pressure build-up and drawdown analysis. In this section we will attempt to incorporate this progress in the analysis of production data.

Since the development of the dimensionless pressure type curve, one of the most useful type curves has been the pressure derivative type curve. The pressure derivative type curve, because of unique matching characteristics, has provided the engineer with a useful tool for analyzing smooth pressure data. Problems arise when attempting to differentiate rough data. Taking derivative of this rough data increases data scatter.

Blasingame et al.\(^6\) developed matching techniques for rough pressure data. New type curves were developed based on pressure integral data. In the following section we will advance this approach and develop dimensionless type curves for noisy, constant-BHP production data affected by data scatter.

**Background - Dimensionless Rate**

The new methods of decline curve analysis are developed based on the dimensionless rate type curve proposed by Fetkovich. The reader should therefore have an understanding of the dimensionless variables and the solution technique developed by Fetkovich. In this section we will provide a general understanding of the solution. The reader is referred to Appendices A and B for more detailed review.

Fetkovich developed a dimensionless rate type curve which combines the analytical transient and the empirical boundary dominated solutions (Fig. 23). This was accomplished by defining dimensionless rate as
Figure 23: Composite dimensionless rate curve for a well centered in a circular drainage area, producing at constant pressure (after Fetkovich).
\[ q_{D_i} = \frac{q(t)}{q_i} \]  

(30)

The initial rate, \( q_i \), is given by

\[ q_i = \frac{kh(p_i - p_{wr})}{141.2 \mu B \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{3}{4} \right]} \]  

(31)

where

\[ r_{wa} = r_{we} e^{-s} \]  

(32)

Replacing \( q_i \) in Eq. 30 with its equivalent expression given in Eq. 31

\[ q_{D_i} = \frac{q(t)}{kh(p_i - p_{wr})} \]  

\[ \frac{141.2 \mu B \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{3}{4} \right]} \]  

(33)

or

\[ q_{D_i} = q_D \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{3}{4} \right] \]  

(34)

Dimensionless time \( t_{D_i} \) is defined as

\[ t_{D_i} = \frac{0.00634kt}{\phi \mu c r_{wa}^2} \]  

\[ \frac{1}{2} \left[ \frac{r_e^2}{r_{wa}} - 1 \right] \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{3}{4} \right] \]  

(35)

Fetkovich discovered that this analytical dimensionless rate solution converges with the dimensionless empirical exponential rate solution for boundary dominated flow

\[ q_{D_i} = e^{-D_t} \]  

(36)

by defining a dimensionless time scale

\[ t_{D_i} = D_t t \]  

(37)

Combining Eqs 35 and 37
\[
D_i = \frac{0.00634k}{\phi \mu c_i r_{wa}^2} \left[ \frac{1}{2} \left( \frac{r_e}{r_{wa}} \right)^2 - 1 \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{3}{4} \right] \right]
\]  

(38)

This provides us with an analytical approach to decline curve analysis.

Initial reserves in place can be determined by type curve matching post-transient flow data (which will match the empirical solutions). Fetkovich shows that reserves can be calculated by

\[
Q_t = \frac{Q_i}{D_i}
\]  

(39)

And by combining Eqs. 30, 37, and 39

\[
Q_t = \left( \frac{q(t)}{q_{Dd/\text{mp}t_{Dd/\text{mp}}} \right) (1 - t)
\]  

(40)

With this type curve we can obtain estimates of radial extent, initial decline, and reserves. Difficulty in obtaining an accurate match increases as data scatter (inherent in many production reports) increases and at the onset of production before sufficient data has been collected. Type curve matching can be made more accurately if we define functions which reduce data scatter and provide more distinct type curve shapes, especially at early times where little data has been acquired.

**Background - Dimensionless Cumulative Production**

Framn^{5} developed a type curve for cumulative production (Fig. 24) from a well produced at constant pressure from a bounded, circular reservoir.

Cumulative production is given by

\[
Q = \int_0^t q d\tau
\]  

(41)

The dimensionless form of this solution is

\[
Q_{Dd} = \int_0^{t_{Dd}} q_{Dd} d\tau
\]  

(42)
Figure 24 - Composite dimensionless cumulative type curve for a well centered in a circular drainage area, producing at constant pressure (after Frahm).
The empirical calculations of this integral are discussed in detail in Appendix A. The dimensional equivalence for \(Q_{Dd}\) can be determined by noting that

\[
\frac{0.00634k}{\phi \mu \sigma r_{wa}^2} \int \frac{1}{\frac{1}{2} \left( \frac{r_e}{r_{wa}} \right)^2 - 1} \left( \ln \left( \frac{r_e}{r_{wa}} \right) \right)^{\frac{3}{4}} dt
\]  

and replacing Eqs. 33 and 43 into Eq. 42

\[
Q_{Dd} = \frac{1.792B}{\phi c_b (r_e^2 - r_{wa}^2)(p_i - p_{wt})} \int_0^t q(t) dt
\]

\[
= \frac{1.792BQ}{\phi c_b (r_e^2 - r_{wa}^2)(p_i - p_{wt})}
\]

From Eqs. 31 and 38, Eq. 44 is equivalent to

\[
Q_{Dd} = \frac{D_i Q}{q_i}
\]

From Eqs. 39 and 45, we can determine initial producible reserves from our type curve match as

\[
Q_t = \frac{Q(t)}{Q_{Dd_{imp}}}
\]

Integration is mathematically a smoothing function. Therefore, cumulative production data (either calculated from rate or directly obtained) is much smoother than rate data.

Referring to Fig. 24, the difficulty in matching data still remains at early times because of the nondistinct shape of the type curve. In the following sections we will discuss the development of new type curves with distinct characteristics for more accurate matches.
DEVELOPMENT OF TIME AVERAGED RATE

We need a smooth, differentiable function to develop a derivative type curve, which has distinct matching characteristics, to be used with noisy rate data. Obviously, differentiating cumulative production results in the original rate. We therefore define time averaged rate

\[
\bar{q} = \frac{Q}{t}
\]

(47)

to give us a smooth function which can be successfully differentiated.

The dimensionless form of Eq. 47 is

\[
\bar{q}_{\text{Dd}} = \frac{Q_{\text{Dd}}}{t_{\text{Dd}}}
\]

(48)

Figure 25 shows the dimensionless time average rate type curve.

Replacing the definitions of \(Q_{\text{Dd}}\) and \(t_{\text{Dd}}\)

\[
\bar{q}_{\text{Dd}} = \frac{Q}{t} \frac{141.2B\mu \left[ \ln \left( \frac{r_e}{r_{wa}} \right) \right] - \frac{3}{4}}{kh(p_i - p_{wft})} = \frac{\bar{q}(t)}{kh(p_i - p_{wft})} \frac{141.2B\mu \left[ \ln \left( \frac{r_e}{r_{wa}} \right) \right] - \frac{3}{4}}{141.2B\mu \left[ \ln \left( \frac{r_e}{r_{wa}} \right) \right] - \frac{3}{4}} = \frac{\bar{q}(t)}{q_i}
\]

(49)

Producible reserves can be determined by type curve matching by combining Eqs. 37, 39, and 49 to show

\[
Q_t = \left( \frac{\bar{q}(t)}{\bar{q}_{\text{Dd}} \text{mp}} \right) \left( \frac{t}{t_{\text{Dd}} \text{mp}} \right)
\]

(50)

Referring to Figs. 24 and 25, the \(\bar{q}_{\text{Dd}}\) and \(Q_{\text{Dd}}\) type curves do not provide distinct matching characteristics. The advantage over the dimensionless rate type curve is the smoothness of the \(\bar{q}_{\text{Dd}}\) function. This will be verified in the following section.
Figure 25 - Composite dimensionless time average rate curve for a well centered in a circular drainage area, producing at constant pressure.

\[ \frac{p_{d1}}{p_{d0}} = \frac{p_{dB}}{b} \]
DEVELOPMENT OF DIMENSIONLESS INTEGRAL DERIVATIVE

Differentiating the smooth $\bar{q}_{Dd}$ function

$$\frac{d\bar{q}_{Dd}}{dt_{Dd}} = \frac{d}{dt_{Dd}} \left( \frac{Q_{Dd}}{t_{Dd}} \right)$$

$$= \frac{d}{dt_{Dd}} \left( \frac{1}{t_{Dd}} \int_{0}^{t_{Dd}} q_{Dd} dt \right)$$

(51)

Combining Leibnitz's rule and the product rule

$$\frac{d\bar{q}_{Dd}}{dt_{Dd}} = \frac{q_{Dd}}{t_{Dd}} - \frac{1}{t_{Dd}^2} \int_{0}^{t_{Dd}} q_{Dd} dt$$

$$= \frac{1}{t_{Dd}} (q_{Dd} - \bar{q}_{Dd})$$

(52)

Multiplying this derivative by $-t_{Dd}$ we produce the dimensionless integral derivative function

$$q_{Ddid} = -t_{Dd} \frac{d\bar{q}_{Dd}}{dt_{Dd}}$$

$$= (\bar{q}_{Dd} - q_{Dd})$$

(53)

The $q_{Ddid}$ type curve is shown in Fig. 26. We determine the dimensional equivalence by replacing $t_{Dd}$ with the analytical definition (Eq. 35) and noting that

$$\frac{d\bar{q}_{Dd}}{dt_{Dd}} = \frac{d\bar{q}_{Dd}}{dt} \times \frac{dt}{dt_{Dd}}$$

$$= \frac{d(\bar{q})}{dt} \times \frac{dt}{dt_{Dd}}$$

$$= \frac{1}{q_{i}} \frac{dq}{dt} \times \frac{dt}{dt_{Dd}}$$

(54)

Equation 54 yields
Figure 26 - Composite dimensionless integral derivative type curve for a well centered in a circular drainage area, producing at constant pressure.
\[ q_{\text{Ddid}} = \frac{q_i}{k \cdot h \cdot (p_i - p_{\text{wrf}})} \]

\[ = \frac{q_i}{141.2 \mu B \cdot \ln \left( \frac{r_s}{r_{\text{wa}}} \cdot \frac{3}{4} \right)} \]

\[ = \frac{q_i}{q_i} \]

where

\[ q_i = \frac{d \bar{q}}{dt} \]

Initial reserves can be determined from

\[ Q_i = \left( \frac{q_i}{q_{\text{Ddid}}} \right)_{\text{mp}} = \left( \frac{1}{q_{\text{Ddid}}} \right)_{\text{mp}} \]

(57)

The \( q_{\text{Ddid}} \) function provides a dimensionless type curve for production decline analysis with distinct characteristics to enable more accurate estimates of reservoir properties. Because \( q_{\text{Ddid}} \) is a derivative function, it will not be as smooth as \( \bar{q}_{\text{Dd}} \). Comparing Eqs. 30, 49, and 55, we note that

\[ q_i = \left( \frac{q_i}{q_{\text{Dd}}} \right)_{\text{mp}} = \left( \frac{\bar{q}_{\text{Dd}}}{q_{\text{Ddid}}} \right)_{\text{mp}} = \left( \frac{q_{\text{Ddid}}}{q_{\text{Ddid}}} \right)_{\text{mp}} \]

(58)

Since \( q_i \) is a constant, Eq. 58 implies that the match points must be identical on the \( q_{\text{Dd}}, \bar{q}_{\text{Dd}}, \) and \( q_{\text{Ddid}} \) type curves. This will allow us to match the curves simultaneously. This will be discussed in further detail.

**DEVELOPMENT OF DIMENSIONLESS INTEGRAL RATIO**

We define the integral ratio as

\[ q_{\text{lr}} = \frac{\bar{q}}{q_{\text{Did}}} \]

(59)

or, in dimensionless terms as

\[ q_{\text{Ddir}} = \frac{\bar{q}_{\text{Dd}}}{q_{\text{Ddid}}} \]

(60)

We can replace the dimensional definitions of \( \bar{q}_{\text{Dd}} \) and \( q_{\text{Ddid}} \) (Eqs. 49 and 55) to
show that

\[ q_{D_{dir}} = \frac{q}{q_{id}} \]
\[ = q_{ir} \]  \hspace{1cm} (61)

Equation 61 shows that the match on the \( q_{D_{dir}} \) type curve (Fig. 27) is fixed vertically. Only a shift in time is required to overlay the \( q_{D_{dir}} \) type curve with production data. Since there is no means of determining \( q_i \) from this type curve, initial reserves cannot be determined using this plotting function. We use this type curve to more accurately determine the best time match on the other type curves. Since \( t_{D_d} \) is defined the same for all functions, the time match must be identical on the \( q_{D_d}, \overline{q}_{D_d}, q_{Ddid}, \) and \( q_{D_{dir}} \) type curves.

**ESTIMATING SKIN AND PERMEABILITY**

Rearranging the dimensional solutions for \( q_{D_d}, \overline{q}_{D_d}, \) and \( q_{Ddid} \) (Eqs. 33, 49, and 55) and generalizing the match points

\[ k = \frac{141.2\mu B}{h(p_i - p_{wf})} \left[ \ln\left(\frac{r_e}{r_{wa}}\right) - \frac{3}{4} \frac{f(t)}{f_{D_d/\text{mp}}} \right] \]  \hspace{1cm} (62)

where

\[ \frac{f(t)}{f_{D_d/\text{mp}}} = \text{match point of curve being used} \]  \hspace{1cm} (63)

Rearranging the definition of \( t_{D_d} \) (Eq 35) and assuming \( (r_e/r_{wa})^2 >> 1 \)

\[ t_{D_d} = \frac{0.01268k}{\phi \mu c r_{D_d}^2} \left[ \ln\left(\frac{r_e}{r_{wa}}\right) - \frac{3}{4} \right] \]  \hspace{1cm} (64)

or

\[ r_e = \sqrt{\frac{0.01268k}{\phi \mu c} \left[ \ln\left(\frac{r_e}{r_{wa}}\right) - \frac{3}{4} \right] t_{D_d/\text{mp}}} \]  \hspace{1cm} (65)
Figure 27 - Composite dimensionless integral ratio type curve for a well centered in a circular drainage area, producing at constant pressure.
We obtain values for \( r_e/r_{wa} \) and \((t/t_{Dd})_{mp} \) from the type curve match. A skin factor can now be determined as

\[
s = \ln(r_w) - \ln(r_{wa})
\] (66)

Fetkovich has shown that the values of permeability and skin are insensitive to the \( r_e/r_{wa} \) stem chosen for the transient data.

**APPLICATION OF PRESSURE NORMALIZED FUNCTIONS TO TYPE CURVE MATCHING**

We earlier introduced adjusted time to allow type curve matching of variable-BHP data. This requires a plot of pressure normalized rate on the adjusted time scale. Since all dimensionless plotting functions are based on constant-BHP production, we must modify the plotting procedure for the new functions to allow the new type curves to be matched with variable-BHP data. This can be accomplished by replacing \( t \) and \( q \) in Eq. 41 with \( t_{adj} \) and \( q/\Delta p \), respectively.

We define the integral

\[
\left( \frac{Q}{\Delta p} \right)^* = \int_0^{t_{adj}} \left( \frac{q}{\Delta p} \right) d\tau
\] (67)

Since \( q/\Delta p \) vs. \( t_{adj} \) lies on the constant-BHP solution of \( q/\Delta p \) vs. \( t \), \( (Q/\Delta p)^* \) vs. \( t_{adj} \) must lie on the constant-BHP solution of \( Q/\Delta p \) vs. \( t \). This provides us with a means to match the \( Q_{Dd} \) type curve with variable-BHP data. Proceeding with the new functions

\[
\left( \frac{\bar{q}}{\Delta p} \right)^* = \left( \frac{Q}{\Delta p} \right)^*/t_{adj}
\] (68)

\[
\left( \frac{q_{id}}{\Delta p} \right)^* = -t_{adj}\frac{d}{dt_{adj}} \left( \frac{\bar{q}}{\Delta p} \right)^*
\] (69)
\[
\left( \frac{q_{ir}}{\Delta p} \right)^* = \left( \frac{\bar{q}}{\Delta p} \right)^* \frac{\Delta p}{q_{id}^*} \Delta p
\]

(70)

Comparing Fig. 1 with Fig. 13, we note that pressure normalization has the effect of vertically shifting the rate data. The variable-BHP plotting functions defined in Eqs. 67 through 70 will also be displaced vertically. We are required to also normalize the type curve matching equations to account for this effect.

Combining Eqs. 31 and 33 we can modify the dimensional solution for \( q_{Dd} \) (Eq. 32) to account for pressure normalization

\[
q_{Dd} = \left( \frac{q}{\Delta p} \right) \frac{q_i}{q} \Delta p
\]

(71)

Modifying the calculation of reserves using the pressure normalized rate functions (\( q_{Dd}, q_{Dd}^* \) and \( q_{Dd1}^* \)) and adjusted time match points,

\[
Q_t = \left( \frac{\bar{q}}{\Delta p} \right) f_{Dd} \left( \frac{\bar{q}_d}{\Delta p} \right) \frac{f_{adj}}{mp} \frac{f_{Dd1}^*}{mp} \Delta P_{abandonment}
\]

(72)

where

\[
\Delta P_{abandonment} = P_i - P_{wf, abandonment}
\]

(73)

All type curve calculations of reserves must be multiplied by this \( \Delta p \) when matching pressure-normalized functions.

Modifying Eq. 62 to account for pressure normalization when calculating permeability

\[
k = \left[ \frac{141.2 \mu B}{h(P_i - P_{wf})} \right] \ln \left( \frac{r_e}{r_{wa}} \right) \left( \frac{f}{r_{Dd}} \right) \frac{\Delta p}{f_{Dd}} \Delta p
\]

\[
= \left[ \frac{141.2 \mu B}{h} \right] \ln \left( \frac{r_e}{r_{wa}} \right) \left( \frac{f}{r_{Dd}} \right) \frac{\Delta p}{f_{Dd}} \Delta p
\]

(74)
And radial extent can be determined as

\[
    r_c = \sqrt{\frac{0.01268 \ k \ \text{ladj}}{\phi \ \mu \ c \ \ln\left(\frac{r_c}{r_{wa}}\right) - \frac{2}{4} \ \text{TrX} / \ \text{mp}}}
\]  

(75)

We will consider the accuracy of estimations of the reservoir parameters considered in this section for rough, variable-BHP production data using type curve matching with an example case in a later section.
ERROR ANALYSIS

An error analysis was completed on all new type curve plotting functions. The analysis was performed by adding up to 25% maximum random error to the analytical dimensionless rate solution ($r_eD=10$) and calculating the other dimensionless analytical functions developed earlier. In this section we will discuss the results of the error analysis performed on the dimensionless functions.

Computation of the new functions requires cumulative production, $Q$. We calculate $Q$ by integrating rate, over time. A piecewise log linear function was used. This function is given as

$$Q_j = Q_{j-1} + \frac{(q_jt_j - q_{j-1}t_{j-1})}{(1 + m)}$$

where $m$ is the slope of the line between $q_j$ and $q_{j-1}$.

Time averaged rate, $\bar{q}$ was then computed at each point by

$$\bar{q}_j = \frac{Q_j}{t_j}$$

We can use Eq. 77 directly provided we have values of cumulative production given.

The derivative function $q_{id}$ was calculated by

$$q_{idj} = t_j \frac{d\bar{q}_j}{dt_j}$$

The derivative in Eq. 78 was determined by first fitting a second order polynomial through 9 points about point $j$. The polynomial used was

$$\bar{q}_j = a_0 + a_1 \ln(t_j) + a_2 \ln(t_j)^2$$

We then used finite difference with a 5-point span about $t_j$ through the polynomial fit. Other derivative methods were attempted, however, we recommend the use of the polynomial fit prior to differentiating because other methods resulted in extreme scatter in the derivative function.
The ratio function was calculated as

$$q_{ir} = \frac{\bar{q}}{q_{id}}$$

(80)

DISCUSSION OF RESULTS

Figure 28 shows the noisy rate data with 5% maximum error added. Also shown are the computed $\bar{q}$, $q_{id}$ and $q_{ir}$ curves. This graph shows that all new functions appear to provide good agreement with the analytical solutions. Figure 29 shows the results of the error analysis. Figures 30 through 33 show the comparison of the match of the computed functions with the analytical function, and the error for each function with 15% and 25% maximum error included in $q$.

We see that $\bar{q}$ is a much smoother function than $q$. The $\bar{q}$ function appears to contain the largest error at early times. This is due to start-up error in our piecewise log-linear approximation for $Q$. This small error is expanded in the calculation of $q_{id}$ and $q_{ir}$. In general, these results show that $\bar{q}$ contains much less error than $q$, and the $q_{id}$ and $q_{ir}$ functions contain approximately the same error. Table 5 shows the maximum error in all functions for each case.
Figure 28 - Type curve match of new type curve functions calculated with 5% maximum error added to rate ($r_{cD} = 10$, $b = 0$).
Figure 29 - Error analysis of new dimensionless type curve functions with 5% maximum error added to dimensionless rate ($r_{dD} = 10$, $b=0$).
Figure 30 - Type curve match of new type curve functions calculated with 15% maximum error added to rate ($r_{elj}=10$, $b=0$).
Figure 31 - Error analysis of new dimensionless type curve functions with 15% maximum error added to dimensionless rate ($r_{ed}=10, b=0$).
Figure 32 - Type curve match of new type curve functions calculated with 25% maximum error added to rate ($r_{eD}=10$, $b=0$).
Figure 33 - Error analysis of new dimensionless type curve functions with 25% maximum error added to dimensionless rate \(r_{eD}=10, \ b=0\).
<table>
<thead>
<tr>
<th>Maximum $q_{Dd}$ Error (%)</th>
<th>Maximum $q_{Dd}$ Error (%)</th>
<th>Maximum $q_{Ddd}$ Error (%)</th>
<th>Maximum $q_{Ddir}$ Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.99</td>
<td>3.30</td>
<td>10.71</td>
<td>13.06</td>
</tr>
<tr>
<td>14.65</td>
<td>9.80</td>
<td>22.15</td>
<td>13.08</td>
</tr>
<tr>
<td>25.00</td>
<td>14.10</td>
<td>35.48</td>
<td>30.25</td>
</tr>
</tbody>
</table>
A NEW ALGORITHM TO ANALYZE AND INTERPRET PRODUCTION DATA WITH AN EXAMPLE CASE

In this section we provide a new algorithm to analyze production data to yield estimates of reserves, radial extent, skin, shape factor, and permeability. This algorithm can be used for both constant-BHP and variable-BHP production from an undersaturated oil reservoir. We also provide a simulated example case which follows the algorithm in a step-by-step manner. We will then compare our results to the true input values.

NEW ALGORITHM TO ANALYZE PRODUCTION DATA

The new algorithm is a 6-step procedure. These steps can be concisely stated:

1. data collection,
2. straight-line approximation,
3. calculation of adjusted time,
4. plotting function calculations,
5. type curve matching, and
6. parameter calculations from type curve matches.

In the remainder of this section, we describe each of these steps in detail.

Data collection

We must first collect all data needed for a complete and accurate analysis. This is probably the most important step, since our results will reflect errors in data gathering. The data required are listed below:

1. results from a fluid analysis, including \( \mu_o, B_o, \) and \( c_o; \)
2. verification of an undersaturated oil reservoir from the fluid analysis;
3. accurate estimations of reservoir properties including \( \phi, c_f, S_{wiv}, \) and \( h; \)
4. initial reservoir pressure, \( p_i; \) and
5. accurate flow data including rate, \( q, \) and bottomhole-flowing pressure, \( p_{wf}, \)
   as functions of time.

Straight line approximation

We must evaluate the flow data to verify that the plot of \( q/\Delta p \) vs. \( Q/\Delta p \) does follow a straight line. This is accomplished in the following manner:

1. integrating rate using a piecewise log-linear calculation (Eq. 76) to yield \( Q \) at each time step;
2. calculate \( q/\Delta p \) and \( Q/\Delta p \) at each time step;
3. plot \( q/\Delta p \) vs. \( Q/\Delta p \) on Cartesian coordinates;
4. locate the stabilized data on the graph (i.e. disregard the initial transient, and all other transient data which produce a "spike" on this graph when locating the true straight line relationship);
5. construct a straight line through the stabilized flow data;
6. calculate slope, \( m_{vp} \), and \( y \)-intercept, \( b_{vp} \), from the straight line; and
7. Calculate OOIP (N) using Eq. 12.

**Calculation of adjusted time**

Adjusted time, \( t_{adj} \), must be determined for each \( q/\Delta p \) data point to allow matching on the constant-BHP production type curves. We recommend the following:
1. make a table showing \( t \), \( q/\Delta p \), and \( Q/\Delta p \);
2. calculate adjusted time for the first row as, \( t_{adj1} = t_1 \); and
3. calculate adjusted time for each subsequent row from eq. 23.

**Calculate and plot all functions**

This is an 8-step procedure:
1. calculate cumulative function as

\[
\left( \frac{Q}{\Delta p} \right)^* = \int_0^{t_{adj}} \left( \frac{q}{\Delta p} \right) dt_{adj}
\]

\[ (81) \]

2. calculate the time average rate function as

\[
\left( \frac{\bar{q}}{\Delta p} \right)^* = \frac{\left( \frac{Q}{\Delta p} \right)^*}{t_{adj}}
\]

\[ (82) \]

3. calculate the derivative function as

\[
\left( \frac{q_{id}^*}{\Delta p} \right) = \frac{q \left( \frac{\bar{q}}{\Delta p} \right)^*}{t_{adj}} dt_{adj}
\]

\[ (83) \]
4. calculate the ratio function as

\[
\frac{q_{ir}}{(\Delta p)^*} = \frac{q}{(\Delta p)^*} \frac{q_{dL}}{(\Delta p)^*}
\]  \hspace{1cm} (84)

5. plot rate function, \( q/\Delta p \) vs. \( t_{adj} \);

6. plot \( (q/\Delta p)^* \) vs. \( t_{adj} \);

7. plot \( (q_{dL}/\Delta p)^* \) vs. \( t_{adj} \) and

8. plot \( (q_{ir}/\Delta p)^* \) vs. \( t_{adj} \).

**Match type curves**

All new type curves developed can be matched simultaneously. Since the ratio function can only be shifted along the \( t_{pd} \) scale, this curve should be matched first to determine the time match point. The derivative function should be matched next due to its distinct matching characteristics. We do this by matching the time scale on the same time match points as the ratio function and then shifting vertically until the best match is obtained. The rate and time average rate functions should be matched using the same match points obtained from the match of the derivative function. If the rate or derivative functions appear to provide a better vertical match than the time average rate function, all type curves should be matched using the new match points. This procedure allows us to find the best match to use in our calculations of reservoir parameters.

**Parameter calculations using type curve matches**

The required calculations are listed below:

1. Calculate \( k \) using Eq. 74;

2. Calculate \( r_e \) using Eq. 75;

3. Calculate apparent wellbore radius as

\[
r_{wa} = \frac{r_e}{r_{eD}}
\]  \hspace{1cm} (85)

4. Calculate \( s \) using Eq. 66;

5. Calculate \( C_A \) using Eq. 13;

6. Calculate reserves, \( Q_b \) using Eq. 72; and
7. Calculate recovery factor as

\[ RF = \frac{Q_t}{N} \]  \hspace{1cm} (86)

**EXAMPLE CASE**

The example case considered is a well flowing at constant rate (≈50 STB/D) until the flowing pressure is too low to "buck" the line pressure \( p_{wf} = 2150 \) psi at 1000 days. The well is then produced at constant-BHP. Figure 34 illustrates the rate profile for this well. In this section, we will follow the algorithm proposed to obtain estimates of N, k, r_e, s, C_A, and RF.

**Data collection - example case**

We will assume that accurate data has been collected from reservoir fluids analysis, core analysis, and log analysis. The rock and fluid properties are shown in Table 6. The first 10 flow rates collected are shown in Table 7.

**Straight line approximation - example case**

We need values of \( q/\Delta p \) and \( Q/\Delta p \) to determine our straight line approximation. These values, for the first 10 time steps, are shown in Table 8. \( Q \) was determined using Eq. 76. The slope, \( m \), for the first point was calculated as

\[ m_1 = +0.01167 \]  \hspace{1cm} (87)

using a least squares fit through the first 5 rate points on the log-log graph of \( q \) vs. \( t \).

\[ Q_1 = \frac{q_1 t_1}{1 + m} \]

\[ = \frac{(49.67)(1.03)}{1 + 0.01167} \]

\[ = 50.41 \text{ STB} \]  \hspace{1cm} (88)

The remaining values of \( Q \) were determined using Eq. 76 with

\[ m_{j>1} = \frac{\log q_{j-1} - q_j}{\log t_{j-1} - t_j} \]  \hspace{1cm} (89)
Figure 34 - Rate schedule for example well flowing at constant rate, followed by constant pressure production.
TABLE 6
ROCK AND FLUID PROPERTIES FOR EXAMPLE CASE

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_o$</td>
<td>0.4 cp</td>
</tr>
<tr>
<td>$B_o$</td>
<td>1.0 RB/STB</td>
</tr>
<tr>
<td>$k$</td>
<td>1.0 md</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.2 (fraction)</td>
</tr>
<tr>
<td>$S_{wi}$</td>
<td>0.0 (fraction)</td>
</tr>
<tr>
<td>$h$</td>
<td>30.0 ft</td>
</tr>
<tr>
<td>$c_o$</td>
<td>$15 \times 10^{-6}$ psi$^{-1}$</td>
</tr>
<tr>
<td>$c_f$</td>
<td>0.0 psi$^{-1}$</td>
</tr>
<tr>
<td>$p_i$</td>
<td>4800.0 psi</td>
</tr>
</tbody>
</table>
### TABLE 7
FLOWING WELL DATA FOR EXAMPLE CASE

<table>
<thead>
<tr>
<th>t, days</th>
<th>( q_0 ), STB/D</th>
<th>( p_{wf} ), psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.03</td>
<td>49.67</td>
<td>4024.46</td>
</tr>
<tr>
<td>1.81</td>
<td>49.39</td>
<td>3995.82</td>
</tr>
<tr>
<td>2.17</td>
<td>47.75</td>
<td>3987.23</td>
</tr>
<tr>
<td>2.53</td>
<td>48.42</td>
<td>3980.11</td>
</tr>
<tr>
<td>3.04</td>
<td>51.60</td>
<td>3971.60</td>
</tr>
<tr>
<td>3.54</td>
<td>50.35</td>
<td>3964.54</td>
</tr>
<tr>
<td>3.99</td>
<td>48.56</td>
<td>3958.90</td>
</tr>
<tr>
<td>4.51</td>
<td>47.56</td>
<td>3953.28</td>
</tr>
<tr>
<td>5.09</td>
<td>52.28</td>
<td>3947.66</td>
</tr>
<tr>
<td>5.57</td>
<td>49.84</td>
<td>3943.46</td>
</tr>
<tr>
<td>t, days</td>
<td>q, STB/D</td>
<td>Q, STB</td>
</tr>
<tr>
<td>---------</td>
<td>-----------</td>
<td>--------</td>
</tr>
<tr>
<td>1.03</td>
<td>49.67</td>
<td>50.41</td>
</tr>
<tr>
<td>1.81</td>
<td>49.39</td>
<td>89.03</td>
</tr>
<tr>
<td>2.17</td>
<td>47.75</td>
<td>106.80</td>
</tr>
<tr>
<td>2.53</td>
<td>48.42</td>
<td>124.08</td>
</tr>
<tr>
<td>3.04</td>
<td>51.60</td>
<td>149.51</td>
</tr>
<tr>
<td>3.54</td>
<td>50.35</td>
<td>174.91</td>
</tr>
<tr>
<td>3.99</td>
<td>48.56</td>
<td>197.45</td>
</tr>
<tr>
<td>4.51</td>
<td>47.56</td>
<td>222.12</td>
</tr>
<tr>
<td>5.09</td>
<td>52.28</td>
<td>251.01</td>
</tr>
<tr>
<td>5.57</td>
<td>49.84</td>
<td>275.58</td>
</tr>
</tbody>
</table>
Therefore

\[ m_2 = \frac{\log(49.67)}{\log(1.03)} \]

\[ = -0.01003 \] \hspace{1cm} (90)

and

\[ Q_2 = 50.41 + \frac{(49.39)(1.81) - ((49.67)(1.03))}{1 + (-0.01003)} \]

\[ = 89.03 \text{ STB} \] \hspace{1cm} (91)

We now plot \( q/\Delta p \) vs. \( Q/\Delta p \) to make the straight line approximation (Fig. 35)

From the straight line through the data

\[ m_{vp} = -0.001963 \] \hspace{1cm} (92)

and

\[ b_{vp} = 0.05615 \] \hspace{1cm} (93)

From Eq. 12

\[ N = -\frac{b_{vp}(1 - S_w i)}{m_{vp}c_t} \]

\[ = -\frac{(0.05615)(1 - 0)}{(- 0.001963)(15 \times 10^{-6})} \]

\[ = 1,909,945 \text{ STB} \] \hspace{1cm} (94)

**Calculation of adjusted time - example case**

We define the first value of adjusted time as

\[ t_{adj1} = t_1 \]

\[ = 1.03 \text{ days} \] \hspace{1cm} (95)
Figure 35 - Straight line approximation for example case.
Subsequent values of adjusted time are calculated from Eq. 23. The second value was determined as

\[
t_{adj2} = \frac{2(0.11071 - 0.06500)}{(0.06142 + 0.06384)} + 1.03
\]

\[= 1.76\]  \hspace{1cm} (96)

Table 9 shows the first 10 calculated values of \(t_{adj}\).

**Calculate and plot all functions - example case**

To determine the new plotting functions, we use Eqs. 67 through 70. Table 10 shows the first 10 values of each plotting function. The values of \((Q/\Delta p)^*\) were determined by integrating \(q/\Delta p\) over \(t_{adj}\) using the log-linear integration scheme described by Eq. 76. The derivative of \((\bar{q}/\Delta p)^*\) was determined using a polynomial fit (Eq. 79) through 9 points about \((\bar{q}/\Delta p)^*\), and using 5-point finite difference at evenly spaced points along the polynomial fit.

Each function is plotted on the adjusted time scale on the same size graph paper as the dimensionless type curves. These plots are shown in Figs. 36 through 39.

**Match type curves - example case**

We now overlay \((q_{ir}/\Delta p)^*\) on the \(q_{D dir}\) type curve and obtain the best match by shifting only horizontally along the \(t_{Dd}\) scale. This match is shown in Fig. 40. Using the origin of \((q_{ir}/\Delta p)^*\) vs. \(t_{adj}\) as a match point, we see that

\[
\left(\frac{t_{adj}}{t_{Dd/im}}\right) = \frac{1}{0.002}
\]

\[(97)\]

We next overlay \((q_{id}/\Delta p)^*\) vs. \(t_{adj}\) on the \(q_{D did}\) type curve at the time match obtained from the \(q_{D dir}\) type curve. The \((q_{id}/\Delta p)^*\) vs. \(t_{adj}\) plot is then shifted vertically to obtain the best match. This match is shown in Fig. 41. Using the origin of \((q_{id}/\Delta p)^*\) vs. \(t_{adj}\) as a match point, we see that

\[
\left[\frac{(q_{id})^*}{\Delta p}\right]_{q_{D did/im}} = \frac{0.001}{0.018}
\]

\[(98)\]
TABLE 9  
CALCULATED ADJUSTED TIME VALUES FOR EXAMPLE CASE

<table>
<thead>
<tr>
<th>t, days</th>
<th>q/Δp, STB/D/psi</th>
<th>Q/Δp, STB/psi</th>
<th>t_adj, days</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.03</td>
<td>0.06384</td>
<td>0.06500</td>
<td>1.03</td>
</tr>
<tr>
<td>1.81</td>
<td>0.06142</td>
<td>0.11071</td>
<td>1.76</td>
</tr>
<tr>
<td>2.17</td>
<td>0.05875</td>
<td>0.13141</td>
<td>2.10</td>
</tr>
<tr>
<td>2.53</td>
<td>0.05905</td>
<td>0.15134</td>
<td>2.44</td>
</tr>
<tr>
<td>3.04</td>
<td>0.06228</td>
<td>0.18048</td>
<td>2.92</td>
</tr>
<tr>
<td>3.54</td>
<td>0.06026</td>
<td>0.20936</td>
<td>3.39</td>
</tr>
<tr>
<td>3.99</td>
<td>0.05773</td>
<td>0.23475</td>
<td>3.82</td>
</tr>
<tr>
<td>4.51</td>
<td>0.05617</td>
<td>0.26233</td>
<td>4.31</td>
</tr>
<tr>
<td>5.09</td>
<td>0.06134</td>
<td>0.29450</td>
<td>4.86</td>
</tr>
<tr>
<td>5.57</td>
<td>0.05819</td>
<td>0.32173</td>
<td>5.31</td>
</tr>
<tr>
<td>$t_{adj}$, days</td>
<td>$q/\Delta p$, STB/psi</td>
<td>$(Q/\Delta p)^*$, STB/psi</td>
<td>$(\bar{q}/\Delta p)^*$, STB/psi</td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------------</td>
<td>-------------------------</td>
<td>-------------------------</td>
</tr>
<tr>
<td>1.03</td>
<td>0.06384</td>
<td>0.06915</td>
<td>0.06714</td>
</tr>
<tr>
<td>1.76</td>
<td>0.06142</td>
<td>0.11805</td>
<td>0.06522</td>
</tr>
<tr>
<td>2.10</td>
<td>0.05875</td>
<td>0.13968</td>
<td>0.06437</td>
</tr>
<tr>
<td>2.44</td>
<td>0.05705</td>
<td>0.16932</td>
<td>0.06361</td>
</tr>
<tr>
<td>2.92</td>
<td>0.06028</td>
<td>0.19195</td>
<td>0.06314</td>
</tr>
<tr>
<td>3.39</td>
<td>0.06026</td>
<td>0.22256</td>
<td>0.06287</td>
</tr>
<tr>
<td>3.82</td>
<td>0.05773</td>
<td>0.24910</td>
<td>0.06243</td>
</tr>
<tr>
<td>4.31</td>
<td>0.06134</td>
<td>0.27876</td>
<td>0.06181</td>
</tr>
<tr>
<td>4.86</td>
<td>0.06134</td>
<td>0.31288</td>
<td>0.06147</td>
</tr>
<tr>
<td>5.31</td>
<td>0.05819</td>
<td>0.34155</td>
<td>0.06132</td>
</tr>
</tbody>
</table>
Figure 36 - Adjusted time plot of pressure-normalized rate for example case.
Figure 37 - Adjusted time plot of time average rate function for example case.
Figure 38 - Adjusted time plot of rate derivative function for example case.
Figure 39 - Adjusted time plot of ratio function for example case.
Figure 40 - Type curve match of the ratio plotting function vs. adjusted time on the $q_{bar}$ type curve (example case).
Figure 41 - Type curve match of the derivative plotting function vs. adjusted time on the $q_{d^4/dt}$ type curve (example case).
We now use these match points to match \((\bar{q}/\Delta p)^*\) vs. \(t_{adj}\) on the \(q_{Dd}\) type curve, and \((q/\Delta p)\) vs. \(t_{adj}\) on the \(q_{Dd}\) type curve. These matches are shown in Figs. 42 and 43. The \((\bar{q}/\Delta p)^*\) and \(q/\Delta p\) matches verify that we have selected the best match points. If any match does not agree, we must shift all curves to obtain the best match.

**Parameter calculations using type curve matches - example case**

From our type curve matches on the \(q_{Ddir}\) and \(q_{Ddid}\) type curves, we see that \(r_e/r_{wa}\) lies between \(1 \times 10^4\) and \(1 \times 10^5\). Since the matches are inconclusive of the exact value, we select

\[
\frac{r_e}{r_{wa}} = 50,000
\]  

(99)

We can now determine permeability from Eq. 74 as

\[
k = \left[ \frac{141.2\mu B}{h} \right] \ln \left( \frac{r_e}{r_{wa}} \right) \frac{3}{4} \left( \frac{\Delta p}{f_{Dd,mp}} \right)^*
\]

\[
= \left( \frac{141.2}{30.0} \right) \left[ \ln(50,000) - \frac{3}{4} \right] \left( \frac{0.001}{0.018} \right)
\]

\[= 1.05 \text{ md}
\]  

(100)

From Eq. 75

\[
r_e = \sqrt{\frac{0.01268 k}{\phi \mu c_{L}} \left( \frac{t_{adj}}{\ln \left( \frac{r_e}{r_{wa}} \right) - \frac{3}{4} \left( \frac{t_{Dd,mp}}{t_{adj}} \right) } \right)}
\]

\[
= \sqrt{\frac{(0.01268)(1.05)}{(0.2)(0.4)(15 \times 10^{-6}) \left[ \ln(50,000) - \frac{3}{4} \right] (0.002)}}
\]

\[= 742.2 \text{ ft}
\]  

(101)

The apparent wellbore radius, \(r_{wa}\), is determined as

\[
r_{wa} = \frac{r_e}{\left( \frac{r_e}{r_{wa}} \right)}
\]

\[= \frac{742.2}{50,000}
\]
Figure 42 - Type curve match of the time average rate plotting function vs. adjusted time on the \( \bar{q}_{Dd} \) type curve (example case).
Figure 43 - Type curve match of pressure normalized rate function vs. adjusted time on the $q_{DI}$ type curve (example case).
= 0.01484 ft \hfill (102)

Skin factor, \( s \), is determined using Eq. 66 as
\[
s = \ln(r_w) - \ln(r_{wa})
= \ln(0.2) - \ln(0.01484)
= +2.6 \hfill (103)
\]

Shape factor is estimated from Eq. 13 as
\[
C_A = \frac{4A}{e^{r_{wa}} \exp \left( \frac{0.01416 - kh}{B \mu \lambda_p} \right)}
= \frac{4\pi(742.2)^2}{e^{0.5772(0.01484)^2} \exp \left( \frac{0.01416}{(1.0)(0.4)(0.05615)} \right)}
= 41.87 \hfill (104)
\]

Initial reserves are determined using Eq. 72 as
\[
Q_t = \left( \frac{f}{f_{Df}} \right)^* \left( \frac{\text{adj}}{\text{mp}} \right) \Delta P_{\text{abandonment}}
= \left( \frac{0.001}{0.018} \right) \left( \frac{1}{0.002} \right) (4800 - 2150)
= 73,611 \text{ STB} \hfill (105)
\]

Primary recovery can now be determined from Eq. 86
\[
RF = \frac{Q_t}{N}
= \frac{73,611 \text{ STB}}{1,909,945 \text{ STB}}
= 0.0385 \hfill (106)
\]

Discussion of results

Table 11 is a summary table of results. We show the calculated values, input values, and the error resulting from our calculations. This table shows that the algorithm proposed to analyze noisy, variable-BHP production data is accurate. Comparing Figs.
### TABLE 11
SUMMARY OF RESULTS FOR EXAMPLE CASE

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calculated values</th>
<th>Input values</th>
<th>Error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>N, STB</td>
<td>1,909,945.0</td>
<td>1,866,543.0</td>
<td>+2.27</td>
</tr>
<tr>
<td>k, md</td>
<td>1.05</td>
<td>1.0</td>
<td>+5.00</td>
</tr>
<tr>
<td>r_e, ft</td>
<td>742.2</td>
<td>745.0</td>
<td>-0.38</td>
</tr>
<tr>
<td>s</td>
<td>+2.6</td>
<td>+2.0</td>
<td>+30.00</td>
</tr>
<tr>
<td>C_A</td>
<td>41.87</td>
<td>31.62</td>
<td>+32.00</td>
</tr>
<tr>
<td>Q_t</td>
<td>73,611.0</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>RF</td>
<td>0.0385</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
40 through 43, we see that the new type curves provide a more accurate estimate of the true $r_0/r_{wa}$ stem for the data. This would be very difficult using only the dimensionless rate type curve. By using multiple type curve matches, we have also chosen more accurate match points than by using only the dimensionless rate type curve. These results show that the systematic algorithm proposed in this section is accurate in determining reservoir parameters from production data.
SUMMARY AND CONCLUSIONS

In this work we have developed theory which can be used to analyzed noisy, variable-BHP flow data. This work supports the following conclusions:

1. The superposition method proposed, using adjusted time, corrects the variable-BHP production rate profile to the rate profile which would have existed for constant-BHP production for undersaturated oil reservoirs.

2. This superposition method was verified using simulated data for common production profiles.

3. New production decline type curves based on integral plotting functions allow simultaneous matching and provide more unique matching characteristics than the conventional dimensionless rate type curve.

4. A systematic procedure was developed to analyzed noisy, variable-BHP production data, using the new superposition method and the new type curves.
NOMENCLATURE

**Dimensionless Variables**

\[ C_D = \frac{0.894}{\phi c_l h^2_w}, \text{ dimensionless wellbore storage coefficient} \]

\[ q_D = \frac{141.2q(t)\mu B}{\ln \left( \frac{r_e}{r_W^a} \right) - \frac{2}{4}}, \text{ dimensionless rate} \]

\[ q_{Di}(t) = q_i, \text{ dimensionless rate defined by Fetkovich} \]

\[ Q_{DD} = \int_0^{t_{DD}} q_{DD} d\tau, \text{ dimensionless cumulative production} \]

\[ \bar{q}_{DD} = \frac{Q_{DD}}{t_{DD}}, \text{ dimensionless time average rate} \]

\[ q_{Dd} = \frac{d}{dt} \frac{Q_{DD}}{t_{DD}}, \text{ dimensionless integral derivative} \]

\[ q_{Ddir} = \frac{\bar{q}_{DD}}{q_{Dd}}, \text{ dimensionless integral ratio} \]

\[ r_{eD} = \frac{r_e}{r_W^a}, \text{ dimensionless drainage radius} \]

\[ t_{Dd} = D_t, \text{ dimensionless time defined by Fetkovich} \]

**Field Variables**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Drainage area, ft²</td>
</tr>
<tr>
<td>b</td>
<td>Constant in Arp's loss ratio</td>
</tr>
<tr>
<td>B</td>
<td>Formation volume factor, RB/STB</td>
</tr>
<tr>
<td>bvp</td>
<td>Y-intercept of straight line approximation, STB/D/psi</td>
</tr>
<tr>
<td>C_A</td>
<td>Dietz Shape Factor</td>
</tr>
<tr>
<td>c_t</td>
<td>Total compressibility, psia⁻¹</td>
</tr>
<tr>
<td>D_i</td>
<td>Initial decline, D⁻¹</td>
</tr>
<tr>
<td>h</td>
<td>Reservoir net pay thickness, ft</td>
</tr>
<tr>
<td>k</td>
<td>Permeability, md</td>
</tr>
<tr>
<td>mvp</td>
<td>Slope of straight line approximation, D⁻¹</td>
</tr>
<tr>
<td>N</td>
<td>Initial oil in place, STB</td>
</tr>
</tbody>
</table>
\[
\frac{\mu_i}{\rho_i} \int_{0}^{p} \frac{d\rho}{\mu} = \text{pseudopressure for gas calculations, psi}
\]

\( p_a \) = Pressure, psi
\( p \) = Average volumetric reservoir pressure, psi
\( \Delta p \) = Wellbore pressure drop, psi
\( q \) = Flow rate, STB/D
\( Q_c \) = Cumulative production, STB
\( Q_{a} \) = Cumulative production determined using pseudotime, STB
\( Q_t \) = Producible reserves for primary recovery, STB
\( q_i \) = Initial flow rate, STB/D
\( \frac{dq}{dt} \), integral derivative, STB/D
\( \bar{q} \) = Time average rate, STB/D
\( N \) = \( Q_t / q \), recovery factor
\( r_e \) = Drainage radius of reservoir, ft
\( r_w \) = Wellbore radius, ft
\( r_{wa} \) = \( r_w e^{-s} \), apparent wellbore radius, ft
\( s \) = Skin factor
\( S_{wi} \) = Initial water saturation, fraction of pore volume
\( t \) = Time, D

\[ \mu_i \sigma_i \int_{0}^{t} \frac{1}{\mu(p)e_d(p)} dt \]

\( t_a \) = Pseudotime for gas calculations, D
\( t_{adj} \) = Adjusted time, D
\( I_0 \) = Modified Bessel function of the first kind of order zero
\( I_1 \) = Modified Bessel function of the first kind of order one
\( J_0 \) = Bessel function of the first kind of order zero
\( K_{0} \) = Modified Bessel function of the second kind of order zero
\( K_{1} \) = Modified Bessel function of the second kind of order one
\( u \) = Laplace transform parameter
\( x_n \) = Root of the first order Bessel function of the first kind (i.e., \( J_1(x_n) = 0 \))

**Greek**

\( \alpha \) = Constant in multiphase flow calculations
\( \beta \) = Constant in multiphase flow calculations
\( \gamma \) = 0.5772 Euler’s constant
\( \psi \) = Normalized pseudopressure for multiphase flow calculations
\( k \)
\( \lambda \) = \( \mu \), mobility
\( \mu \) = Viscosity, cp
\[ \begin{align*} 
\phi & = \text{Porosity, fraction} \\
\tau & = \text{Dummy variable of integration} 
\end{align*} \]

Subscripts

\begin{align*} 
b & = \text{Base condition} \\
g & = \text{Gas parameter} \\
i & = \text{Initial conditions} \\
j & = \text{Step number} \\
mp & = \text{Match point on dimensionless type curve} \\
o & = \text{Oil parameter} \\
r & = \text{Relative condition} \\
vp & = \text{Variable pressure conditions} \\
w & = \text{Water parameter} \\
wf & = \text{Flowing well conditions} 
\end{align*} 

Superscripts

\begin{align*} 
* & = \text{Plotting function based on adjusted time} 
\end{align*}
REFERENCES


SUPPLEMENTARY SOURCES CONSULTED


APPENDIX A

DERIVATION OF EMPIRICAL DIMENSIONLESS RATE
AND EMPIRICAL DIMENSIONLESS CUMULATIVE PRODUCTION
FOR POST-TRANSIENT FLOW

In this appendix, the dimensionless rate and cumulative production solutions for a well in a bounded, circular, homogeneous reservoir are derived. We will consider only the constant bottomhole pressure case in the post-transient flow regime. These equations are all developed from the generalized hyperbolic rate equation used by Fetkovich in the composite dimensionless rate type curve.

DIMENSIONLESS TIME AVERAGED RATE

The generalized hyperbolic rate equation developed by Arps is

\[ q(t) = \frac{q_i}{[1 + bD_i t]^{1/b}} \]  

(A-1)

where: \(0 \leq b \leq 1\)

We will retain the dimensionless variables defined by Fetkovich. Dimensionless rate is defined as

\[ q_{Dd} = \frac{q(t)}{q_i} \]  

(A-2)

and dimensionless time is defined as

\[ t_{Dd} = D_i t \]  

(A-3)

Combining Eqs. A-1 through A-3, the generalized hyperbolic equation for dimensionless rate as a function of dimensionless time is

\[ q_{Dd} = \frac{1}{[1 + bD_i t_{Dd}]^{1/b}} \]  

(A-4)

For the special case of exponential decline (b=0)

\[ q_{Dd} = e^{-t_{Dd}} \]  

(A-5)

And for the special case of harmonic decline (b=1)
\[ q_{Dd} = \frac{1}{[1 + t_{Dd}]} \]  

(A-6)

**DIMENSIONLESS CUMULATIVE PRODUCTION**

A Dimensionless cumulative production type curve was developed by Fraim. Since cumulative production is an integral (smoothing) function, it will be important to the development of the new type curve plotting functions. For completeness, the derivation of the dimensionless cumulative production relationships will be discussed.

Cumulative production is simply defined as

\[ Q = \int_0^t q(\tau) d\tau \]  

(A-7)

Substituting Eq. A-1 into Eq. A-7

\[ Q = \int_0^t \frac{q_i}{[1 + bD_i t]^{1/b}} d\tau \]  

(A-8)

or

\[ Q = \frac{q_i}{(1 - b)D_i} \left[ 1 - (1 + bD_i t)^{(1 - 1/b)} \right] \]  

(A-9)

We now define dimensionless cumulative production as

\[ Q_{Dd} = \frac{QD_i}{q_i} \]  

(A-10)

Substituting Eq. A-8 into Eq. A-9

\[ Q_{Dd} = \frac{1}{(1 - b)} \left[ 1 - (1 + bD_i t)^{(1 - 1/b)} \right] \]  

(A-11)

Combining Eqs. A-1 and A-2 and rearranging

\[ [1 + bD_i t] = q_{Dd}^{-b} \]  

(A-12)

or

\[ [1 + bD_i t]^{(1 - 1/b)} = (q_{Dd})(q_{Dd}^{-b}) \]  

(A-13)
Substituting Eq. A-13 into Eq. A-11

\[ Q_{\text{Dd}} = \frac{1}{(1 - b)} \left[ 1 - q_{\text{Dd}} \right] \cdot b \]  

(A-14)

For the exponential case \((b=0)\), equation Eq. A-14 becomes

\[ Q_{\text{Dd}} = \left[ 1 - q_{\text{Dd}} \right] \]  

(A-15)

For the harmonic case \((b=1)\), we integrate Eq. A-6

\[ Q_{\text{Dd}} = \int_0^{t_{\text{Dd}}} \frac{1}{\left[ 1 + \tau \right]} \, d\tau \]  

(A-16)

or

\[ Q_{\text{Dd}} = \ln(1 + t_{\text{Dd}}) \]  

(A-17)

From Eq. A-6 (for \(b=1\))

\[ \frac{1}{q_{\text{Dd}}} = 1 + t_{\text{Dd}} \]  

(A-18)

Substituting Eq. A-18 into Eq. A-17

\[ Q_{\text{Dd}} = \ln \left( \frac{1}{q_{\text{Dd}}} \right) \]  

(for \(b=1\))  

(A-19)
APPENDIX B

DERIVATION OF NEW EMPIRICAL DIMENSIONLESS FUNCTIONS
FOR POST-TRANSIENT FLOW

In this appendix we develop new dimensionless type curve functions for constant pressure production, post-transient flow. Cumulative production is a smooth function, however we need a plotting function which provides distinct matching characteristics to provide more accurate estimates of reservoir parameters from type curve analysis. The logical procedure is to develop a smooth differentiable function based on cumulative production.

DIMENSIONLESS TIME AVERAGED RATE

We proceed by defining dimensionless time averaged rate as

$$\bar{q}_{Dd} = \frac{Q_{Dd}}{t_{Dd}}$$  \hspace{1cm} (B-1)

For 0≤b≤1, we combine eqs. B-1 and A-14 to yield the general hyperbolic expression for $\bar{q}_{Dd}$

$$\bar{q}_{Dd} = \frac{1}{(1 - b)t_{Dd}}[1 - q_{Dd}(1 - b)]$$  \hspace{1cm} (B-2)

For the exponential decline (b=0), eq. B-2 reduces to

$$\bar{q}_{Dd} = \frac{1}{t_{Dd}}[1 - q_{Dd}]$$  \hspace{1cm} (B-3)

Combining eqs. B-1 and A-19 yields (for b=1)

$$\bar{q}_{Dd} = \frac{1}{t_{Dd}}\ln\left(\frac{1}{q_{Dd}}\right)$$  \hspace{1cm} (B-4)

DIMENSIONLESS INTEGRAL DERIVATIVE

To produce a type curve with distinct matching characteristics, we define the dimensionless integral derivative as

$$q_{Dd id} = -t_{Dd} \frac{d\bar{q}_{Dd}}{dt_{Dd}}$$  \hspace{1cm} (B-5)
Substituting the definition of $\bar{q}_{Dd}$ into Eq. B-5

$$q_{Dd\text{d}} = -t_{Dd} \left[ \frac{d}{dt_{Dd}} \left[ \frac{Q_{Dd}}{t_{Dd}} \right] \right]$$  \hspace{1cm} (B-6)

or

$$q_{Dd\text{d}} = -t_{Dd} \left[ \frac{d}{dt_{Dd}} \left[ \int_{0}^{t_{Dd}} q_{Dd\text{d}} \, \tau \right] \right]$$  \hspace{1cm} (B-7)

Applying the quotient rule to carry out the differentiation

$$q_{Dd\text{d}} = -t_{Dd} \left[ \frac{t_{Dd}Q_{Dd} - \int_{0}^{t_{Dd}} q_{Dd\text{d}} \, \tau}{t_{Dd}^2} \right]$$  \hspace{1cm} (B-8)

Simplifying

$$q_{Dd\text{d}} = \frac{Q_{Dd}}{t_{Dd}} - q_{Dd}$$  \hspace{1cm} (B-9)

or

$$q_{Dd\text{d}} = \bar{q}_{Dd} - q_{Dd}$$  \hspace{1cm} (B-10)

This solution is valid for all cases and can be verified by substituting for $\bar{q}_{Dd}$ in Eq. B-5 and differentiating.
APPENDIX C

LAPLACE SOLUTIONS FOR
CONSTANT TERMINAL PRESSURE PRODUCTION
FROM A BOUNDED, CIRCULAR RESERVOIR

The constant terminal rate solution, in Laplace space, for flow from a bounded, circular reservoir, (of small and constant compressibility fluid) is given as

\[ \bar{P}_D(u) = \frac{K_1(r_d r_D u)[K_0(r_D u)] + I_1(r_d r_D u)K_0(r_D u)}{u^{3/2}[I_1(r_d r_D u)K_1(r_D u) - K_1(r_d r_D u)I_1(r_D u)]} \]  

(C-1)

van Everdingen and Hurst\textsuperscript{10} give the relationship

\[ \bar{P}_D(u) \bar{q}_D(u) = \frac{1}{u^2} \]  

(C-2)

or

\[ \bar{q}_D(u) = \frac{1}{u^2 \bar{P}_D(u)} \]  

(C-3)

Combining Eqs C-1 and C-3

\[ \bar{q}_D(u) = \frac{[I_1(r_d r_D u)K_1(r_D u) - K_1(r_d r_D u)I_1(r_D u)]}{\sqrt{u} \left[ K_1(r_d r_D u)I_0(r_D u) + I_1(r_d r_D u)K_0(r_D u) \right]} \]  

(C-4)

Inverting Eq. C-4 to real time, using the algorithm proposed by Stehfest, we solve for \(q_D(t)\). Defining \(q_{Dd}\) after Fetkovich

\[ q_{Dd} = qD \left[ \ln(r_d D) - \frac{3}{4} \right] \]  

(C-5)

Integrating in Laplace space

\[ \bar{Q}_D(u) = \int_0^t \bar{q}_D(u) e^{-u \tau} d\tau \]

\[ = \frac{\bar{q}_D(u)}{u} \]  

(C-6)
Inverting Eq. C-6 with the Stehfest algorithm, we solve for \( Q_D(t) \). Defining \( Q_{Dd} \) after Fraim

\[
Q_{Dd} = \frac{Q_D(t)}{\sqrt{r_D^2 - 1}}
\]  

(C-7)

All other analytical type curve solutions were developed directly from \( Q_{Dd} \) and \( q_{Dd} \).
APPENDIX D

CONSIDERATIONS FOR ANALYZING SINGLE-PHASE GAS AND MULTI-PHASE FLOW

We have developed and verified a superposition method in this work to produce a constant-pressure profile given variable-pressure rate data. The methods presented are strictly applicable to the flow of undersaturated liquids. In this appendix we will discuss possible superposition methods for gas and multi-phase flow. Although these methods have not been verified, past work indicates that these ideas are strong candidates for solutions to common problems with gas and multi-phase flow. The methods presented should be considered for future research.

A POSSIBLE SUPERPOSITION METHOD FOR VARIABLE-PRESSURE GAS FLOW

Blasingame and Lee\textsuperscript{11} have developed a method to estimate the size and shape of gas reservoirs. This method is similar to the variable-rate reservoir limit testing of undersaturated oil. The post-transient flow equation for gas is given as

\[
\frac{\Delta p}{q} = \frac{p_{ai} - p_{awf}}{q} \approx 70.6ln\left(\frac{4a}{c_0C_ar_w}\right) + \frac{t_a}{24c_0G} \tag{D-1}
\]

where

\[
\tilde{t}_a = \int_0^{t_a} \frac{q(\tau) d\tau}{q(t_a)} \tag{D-2}
\]

The variables \( p_a \) and \( t_a \) are defined as

\[
p_a = \frac{\mu_i}{\rho_i} \int_0^p \frac{p}{\mu} dp \tag{D-3}
\]

and

\[
t_a = \mu_i c_i \int_0^t \frac{1}{\mu(p)c_i(p)} dp \tag{D-4}
\]
The straight line relationship between $\Delta p/q$ and $t_a$ was verified. This strongly implies that we can calculate an adjusted time similar to liquid flow, which will allow use to use type curve matching to estimate reservoir parameters. The proposed solution is

\[
 t_{adj} = \frac{2 \left( Q_{sj} - Q_{sj-1} \right)}{\Delta p_a j - \Delta p_a j-1} + t_{adj-1}
\]

where

\[
 Q_a^* = \frac{\int_0^{t_a} q(\tau) \, d\tau}{\Delta p_a}
\]

This method should convert the variable-pressure gas rate to an equivalent constant-pressure liquid rate with the same advantages as the methods proposed earlier. One apparent disadvantage would be the inability to predict future production as easily as the liquid case, since the adjusted time suggested (Eq. D-5) would be a function of the time function defined by Eq. D-4.

A POSSIBLE SUPERPOSITION METHOD FOR MULTIPHASE FLOW

Poston and Chen\textsuperscript{12} have developed a method to convert oil flow rate in a multiphase system to the equivalent single-phase oil rate. This is accomplished by defining normalized pseudopressure and normalized pseudotime as

\[
 \psi_p = \int_{p_b}^{p} \frac{k_{ro}(S_o)}{\mu_o(p')B_o(p')} \, dp'
\]

and

\[
 (D-7)
\]
\[
\begin{align*}
    t_p &= \int_{t_b}^{t} \frac{\lambda_{ir}(S_0, p)}{c(S_0, p)} \, dt \\
    &= \frac{\lambda_{ir}(S_0, p)}{c(S_0, p)} \left. \frac{d\tau}{\lambda_{ir}(S_0, p)} \right| _{t_b} ^{t} \tag{D-8}
\end{align*}
\]

where

\[
\lambda_{ir} = \left( \frac{k_{ro}}{\mu_0} \right) + \left( \frac{k_{rg}}{\mu_g} \right) + \left( \frac{k_{rw}}{\mu_w} \right) \tag{D-9}
\]

The dimensionless rate terms defined by Fetkovich become

\[
q_{dpD} = \exp(-t_{dpD}) \tag{D-10}
\]

and

\[
\begin{align*}
    t_{dpD} &= \left( \frac{0.00633}{\alpha} \right) \frac{k}{c} \frac{\lambda_{ir}}{\mu} \frac{1}{\phi} t_p \tag{D-11}
\end{align*}
\]

where

\[
\alpha = \frac{(r_{eD}^2 - 1)}{2} \tag{D-12}
\]

and

\[
\beta = \frac{[4r_{eD}^4\ln(r_{eD}) - 3r_{eD}^4 + 4r_{eD}^2 - 1]}{4(r_{eD}^2 - 1)^2} \tag{D-13}
\]

The constant terminal oil rate solution is given by Chen and Poston as

\[
q_0 = \frac{kh(w_p - w_p^*)}{141.2\beta} \left[ \frac{k_{ro}}{\mu_o B_o} \right] \tag{D-14}
\]

This transformation converts the multiphase oil flow rate to the single-phase solution. From this we propose

\[
\begin{align*}
    t_{adj} &= \frac{2 \left( \frac{Q_{s1} - Q_{s1-1}}{\Delta \psi_{pa j} - \Delta \psi_{pa j-1}} \right)}{q_{o1} \cdot \Delta \psi_{pa j} + q_{o1-1} \cdot \Delta \psi_{pa j-1}} + t_{adj-1} \tag{D-15}
\end{align*}
\]
where

\[ Q_a = \frac{\int_0^{t_p} q_0 \, dt}{\Delta \psi_p} \]  \hspace{1cm} (D-16)

The solutions proposed for gas and multiphase flow should be verified before application to field problems.
VITAE

Name: Thomas Lee McCray

Born: March 2, 1965
Kansas City, Missouri

Parents: Thomas E. and Norma L. McCray

Permanent Address: 9323 Fairdale
Houston, Texas 77063

Education
Texas A&M University
College Station, Texas
B.S., Petroleum Engineering
(December 1988)

Texas A&M University
College Station, Texas
M.S., Petroleum Engineering
(December 1990)