WELL TEST ANALYSIS FOR WELLS WITH FINITE CONDUCTIVITY
VERTICAL FRACTURES : APPLICATION TO
THE UPPER CLEARFORK FORMATION

A Thesis
by
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Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE

August 1992

Major Subject: Petroleum Engineering
WELL TEST ANALYSIS FOR WELLS WITH FINITE CONDUCTIVITY
VERTICAL FRACTURES: APPLICATION TO
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August 1992
ABSTRACT

Well Test Analysis for Wells With Finite Conductivity Vertical Fractures:
Application to the Upper Clearfork Formation. (August 1992)
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Chair of Advisory Committee: Dr. Thomas A. Blasingame

For a well in a reservoir that has low permeability, we must use stimulation treatments to increase the production potential of the well. One of the most popular stimulation treatments is the hydraulic fracturing treatment. However, the success of the well stimulation is not guaranteed. We need a method which not only verifies the results of the hydraulic fracturing and subsequent remedial acidizing treatments, but we also need methods to estimate reservoir characteristics and to predict reservoir performance after the treatments.

In this research, we will first consider the effect of an acidizing treatment on a well that has been hydraulically fractured. The fracture is assumed to be vertical and the reservoir is assumed to be infinite-acting and homogeneous. Next, we will develop a semi-analytical model of a well with a finite conductivity vertical fracture in an infinite-acting, homogeneous reservoir. Finally, we will use this fractured well model to develop type curve solutions, which will be used to analyze pressure test data from acid stimulated fractured wells and possibly acid fractured wells. The pressure test data used in this work were obtained from wells in the Upper Clearfork Formation in Ward and Winkler Counties, Texas.

The solution methods developed in this research are new results which can be used to analyze pressure test data from a fractured well that has been acid stimulated. The type curve solutions include wellbore storage effects so that practical conditions can be modeled. In conclusion, these new analysis methods are useful for analyzing fractured wells with wellbore storage effects. We can use these methods to calculate
formation permeability, wellbore storage coefficient, fracture conductivity, and fracture half-length.
DEDICATION

This thesis is dedicated to:

    My parents, Mr. R. and Mrs. V.D. Santivongskul, who always support and motivate me and are patient to see my success;

    Dr. Thomas A. Blasingame, who is comparable as my second father;

    And, my girlfriend, Sirinan Kanchanaprasat, who has stood beside me throughout this work.
ACKNOWLEDGEMENTS

The author would like to thank the following for their contributions to this research:

Dr. Thomas A. Blasingame, for his kindly support of everything to accomplish this investigation and for serving as the chair of the author's advisory committee;

Dr. W.J. Lee, for giving the author valuable knowledge and experience to apply to this investigation and for serving as a member of the author's advisory committee;

Dr. James E. Russell, for his support of this investigation and serving as a member of the author's advisory committee;

Dr. R. Raghavan, for his support of this investigation;

Ms. Elizabeth K. Barboza, for her willing help in preparation of this report and reviewing all documentary, references, and data;

Ms. Jennifer L. Johnston, for her help in preparation of this report and reviewing all documentary, references, and data;

The field data support of Shell Oil Company and the Bureau of Economic Geology of the State of Texas;

The financial support of Energy Research in Applications Program (ERAP) funded through the Coordinating Board of Higher Education for the State of Texas;

Texas A&M University, the most wonderful place to be.
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NOMENCLATURE

\[ B = \text{formation volume factor, bbl/STB} \]
\[ b_f = \text{fracture width, ft} \]
\[ C = \text{wellbore storage constant, bbl/psi} \]
\[ C_i = \frac{k \phi_f c_f}{k_f \phi c_t}, \text{fracture storage factor} \]
\[ C_{Df} = \frac{0.894 C}{\phi c_t h x_f^2}, \text{dimensionless wellbore storage coefficient} \]
\[ c_f = \text{fracture compressibility, psia}^{-1} \]
\[ C_{fD} = \frac{k_f b_f}{k x_f}, \text{dimensionless fracture conductivity} \]
\[ c_t = \text{total compressibility, psia}^{-1} \]
\[ E_1 (x) = \text{exponential integral function} \]
\[ erf(x) = \text{error function} \]
\[ h = \text{formation thickness, ft} \]
\[ h_f = \text{fracture height, ft} \]
\[ k = \text{formation permeability, md} \]
\[ K_0 (x) = \text{modified Bessel function of the second kind of the zero order} \]
\[ k_a = \text{acidized zone permeability, md} \]
\[ k_f = \text{fracture permeability, md} \]
\[ P_{D} = \frac{k h \Delta p}{141.2 q B \mu}, \text{dimensionless pressure} \]
\[ P'_{D} = \text{dimensionless pressure derivative} \]
\[ P_{D,f} = \text{dimensionless pressure from Gringarten, et al}^5 \text{ (Eq.2.1)} \]
\[ P_{D,fin} = \text{dimensionless pressure in Laplace Space for finite conductivity} \]
\[ \text{fracture solution (Eq.2.11)} \]
\[ P_{D,frac} = \text{dimensionless pressure in Laplace Space from Ozkan and Raghavan}^6 \]
\[ \text{ (Eq.2.5)} \]
\[ P_{D,tri} = \text{dimensionless pressure in Laplace Space from Lee and Brockenbrough}^9 \text{ (Eq.2.7)} \]
\[ \Delta p = \text{pressure drop at well, psi} \]
\[ \Delta p' = \text{pressure difference derivative} \]
\( q \) = well production rate, STB/D
\( q_D \) = \( \frac{141.2 \ q \ B \ \mu}{k \ h \ \Delta p} \), dimensionless production rate
\( \bar{q}_{D, \text{fin}} \) = dimensionless production rate in Laplace space for finite conductivity fracture solution (Eq.2.15)
\( q_p \) = well production rate prior to pressure test, STB/D
\( S_f \) = fracture face skin factor
\( t \) = time, hr
\( t_D \) = \( \frac{0.0002637k \ t}{\phi \mu c \ x_f^2} \), dimensionless time
\( t_p \) = production time prior to shut in, hr
\( \Delta t_e \) = \( \frac{\Delta t}{1 + \Delta t / t_p} \), equivalent time function, hr
\( u \) = Laplace transform variable
\( x \) = distance in x - direction, ft
\( x_D \) = \( \frac{x}{x_f} \), dimensionless distance in x - direction
\( x_f \) = fracture half length, ft
\( y \) = distance in y - direction, ft
\( y_D \) = \( \frac{y}{x_f} \), dimensionless distance in y - direction
\( z \) = distance in z - direction, ft
\( \mu \) = fluid viscosity, cp
\( \phi \) = formation porosity, fraction
\( \phi_f \) = fracture porosity, fraction
CHAPTER I
INTRODUCTION

In this chapter, we first introduce the problems associated with the analysis of a vertically fractured well that has been acid stimulated. We then explain the current status of the problem. Next, we review some literature of interest. And finally, we explicitly state the objectives of this research.

1.1. Introduction of Problems

For many decades, petroleum engineers have been developing different methods to increase the efficiency of petroleum production from wells. Hydraulic fracturing and acidizing treatments are two of the most popular methods used to stimulate wells and increase the well productivity. The hydraulic fracturing treatment has been successfully applied in the petroleum industry to increase the productivity of wells in low permeability reservoirs. This treatment stimulates the well by providing a highly permeable path through the reservoir to the well. The acidizing stimulation is a well known treatment in improving the productivity of wells that have been damaged from well completion. This stimulation treatment is used to clean and stimulate the well after the well is completed and is sometimes performed before the well starts producing. More often, the acidizing stimulation is also used as a remedial treatment on old wells.

The Upper Clearfork Formation in Ward and Winkler Counties, Texas has several oil producing regions, often over multiple layers. This reservoir is classified as a "tight reservoir" due to its low permeability. From geological and petrophysical studies, the formation has average porosity of nine percent and permeability less than or equal to one millidarcy. To increase the production potential, hydraulic fracturing
and acidizing treatments were applied to the wells in this formation. To determine whether these treatments significantly improve the production potential of the wells, an appropriate method of analyzing data from hydraulically fractured wells that may have been acid stimulated must be developed.

1.2. Current Status

Many studies have been undertaken to investigate the theoretical performance of hydraulically fractured wells and acidizing wells. These studies show that the characteristics of hydraulically fractured wells can be observed from pressure test data. Consequently, many methods have been proposed for the analysis of well test data. The analysis methods are applied not only to verify the results of hydraulic fracturing and acidizing treatments, but also to estimate reservoir characteristics and predict reservoir performance.

Pressure test data have been collected from the wells located in the Upper Clearfork Formation. Much of these pressure data have been analyzed and the results indicate that most wells behave as vertically fractured wells. Although the test data reveal vertical fracture and transition to radial flow regimes, many well tests have been poorly analyzed or misinterpreted. The major factor that causes misinterpretation of the well test data is the effect of wellbore storage. To analyze these pressure test data correctly, a solution method for interpreting test data from acid stimulated and vertically fractured wells, including the effect of wellbore storage, should be developed and applied.

1.3. Literature Review

Some of the well and reservoir properties, such as permeability and wellbore storage coefficient are important parameters for use in modeling a vertically fractured well. These parameters must be accurately known to correctly predict reservoir performance. For these reasons, emphasis of the development of a solution for
reservoir modeling has been made. Some important reservoir models developed for reservoir study are summarized as following:

1. Single well in homogeneous reservoir

2. Vertically fractured well in homogeneous reservoir
   - Infinite conductivity and uniform flux fractures
   - Finite conductivity fracture
     - Damaged fracture model
     - Trilinear flow model
     - Elliptical flow model

3. Wellbore storage and skin effects

We discuss these reservoir models in more detail below;

1.3.1. Single Well in Homogeneous Reservoir

This model simulates a vertical well in a homogeneous reservoir. Fluid flow in the reservoir is a radial flow regime. Figs. 1.1 and 1.2 show the model of the reservoir and the flow of fluid in the reservoir, respectively. Equations of fluid flow in the reservoir are expressed by Muskat\(^2\), van Everdingen and Hurst\(^3\), and Agarwal, et al.\(^4\)

1.3.2. Vertically Fractured Well in Homogeneous Reservoir

The model of a vertically fractured well in a homogeneous reservoir is used to simulate a well that has been hydraulically fractured in order to study the well and
Figure 1.1 Single Well in Homogeneous Reservoir
Figure 1.2 Fluid Flow in Single Well Reservoir
reservoir behavior. The model is illustrated in Fig. 1.3. There are several concepts used in the model, depending on the properties of the fracture (fracture conductivity).

**Infinite Conductivity and Uniform Flux Fractures**

For a vertically fractured well with infinite fracture conductivity, the fracture conductivity is so high that it is assumed that there is no pressure drop along the fracture during the production period. In the case of a vertically fractured well with a uniform flux fracture, the fracture conductivity is also high. However, there is still some pressure drop along the fracture during the production period. The solutions for a vertically fractured well in homogeneous reservoir with infinite conductivity and uniform flux fracture conditions are presented in Gringarten, et al.\textsuperscript{5} and Ozkan and Raghavan.\textsuperscript{6}

**Finite Conductivity Fracture**

The model of a vertically fractured well in a homogeneous reservoir with a finite conductivity fracture is more applicable for a hydraulically fractured well with low fracture conductivity. In fact, a fracture usually has some damage, such as leakoff, proppant crushing, and incomplete proppant transport, after hydraulic fracturing treatment. This damage causes the fracture to have low fracture conductivity, or reduces flow into the fracture in the case of severe leakoff. Three different models of a vertically fractured well with finite fracture conductivity in a homogeneous reservoir are summarized below:

**Damaged Fracture Model**

This model considers a fractured well with a damaged zone around the fracture. The damaged area obstructs flow of fluid from the reservoir into the fracture and causes some pressure drop during the production period. This model is shown in Fig. 1.4. The solutions for this model are given by Cinco-Ley and Samaniego-V.\textsuperscript{7,8} If a stimulated region is used in place of the damaged region, this concept could be applied to a fractured well that has been stimulated
Figure 1.3 Vertically Fractured Well Model
Figure 1.4 Fractured Well With Damaged Area
by an acidizing treatment.

**Trilinear Flow Model**

The trilinear flow model is an approximate analytical solution for a finite-conductivity vertically fractured well in a homogeneous reservoir. The model is proposed by Lee and Brockenbrough.\(^9\) This model accurately models the performance of a hydraulically fractured well at very early times, but is inaccurate for use at later times. The trilinear flow model is displayed in Fig. 1.5.

**Elliptical Flow Model**

Many studies\(^{10-14}\) use the elliptical flow model to stimulate the flow of fluid from a reservoir to a vertically fractured well. This elliptical flow model is demonstrated in Fig. 1.6. This model yields good prediction of the behavior of the vertically fractured well with finite fracture conductivity. Nevertheless, these solutions require tedious calculations and are not practical for analysis or timely modeling of the reservoir.

1.3.3. **Wellbore Storage and Skin Effects**

Wellbore storage and skin effects are boundary conditions rather than reservoir characteristics. However, these effects can significantly affect the pressure behavior of the well. The wellbore storage is the condition where the oil (or gas in the case of a gas well) that flows from the reservoir accumulates in the well before the well starts producing. Initially, fluid is produced from the wellbore and not the reservoir. The skin factor describes the condition of the region adjacent to the well (or to the fracture for the fractured well). This adjacent area is the path of fluid flow from the reservoir to the well. Wellbore storage and skin effects can have a significant influence on the well pressure performance at very early production times. The wellbore storage and skin effects were investigated by Agarwal, \textit{et al.}\(^4\) and Wattenbarger and Ramey.\(^{15}\)
Figure 1.5 Trilinear Flow Model
Figure 1.6 Elliptical Flow Model
1.4. Objectives

The objectives of this research are:

1. To study the effect of an acidizing stimulation treatment on a vertically fractured well in a homogeneous reservoir.

2. To develop an approximate analytical solution in Laplace space for a finite-conductivity vertically fractured well in an infinite-acting, homogeneous reservoir.

3. To develop type curve solutions for a finite-conductivity vertically fractured well in an infinite-acting, homogeneous reservoir, including wellbore storage effects.

4. To analyze well test data from the Upper Clearfork Formation using the developed type curve solutions.
CHAPTER II

DEVELOPMENT OF THE FRACTURED WELL MODEL

In the previous chapter, we introduced the status of the analysis of well test data from a vertically fractured well that has been acid stimulated. In this chapter, we introduce fractured well models for the solution development. First, we describe the schematic of the reservoir model including reservoir and well characteristics, as well as the important assumptions. Next, we use numerical simulation to study the effect of the acid stimulation treatment on the behavior of a fractured well. Finally, we develop analytical models for both constant production rate and constant production pressure solutions. We also include the effect of the wellbore storage in the constant production rate solution.

2.1. Schematic of Reservoir Model

Before we begin to develop solutions, it is important to identify what appropriate reservoir and well model should be used. In this research, we use the model of a well with vertical fracture in an infinite-acting, homogeneous reservoir. The following statements describe characteristics and assumptions of the reservoir model. The reservoir model requires a rectangular coordinate system (x, y, z). As shown in Fig. 2.1, the fracture is parallel to the x - z plane and the well lies along the z - axis. The origin point (0, 0, 0) is assumed to be at the center of the fracture and of the reservoir so that the model is symmetrical about the z - axis and the x - axis.

The upper and lower boundaries of the formation are impermeable and the reservoir extent is infinite. The formation has constant thickness, \( h \), and the reservoir properties have no variation throughout the entire reservoir. The fracture height, \( h_f \), is also constant and equal to the reservoir thickness. The fracture has a constant width, \( b_f \), and finite conductivity.

Because the model is symmetrical about the z - axis and the x - axis, we can use only one - quarter of the reservoir system to study the behavior of the reservoir.
Figure 2.1 Fractured Well Model With (x, y, z) System
The reservoir model that we used in this study is presented in Fig. 2.2.

2.2. Numerical Modeling

In this section, we use a numerical simulator to model the vertically fractured well with an acidized area around the fracture. We use this model to study the effect of an acidizing treatment on a fractured well.

As previously mentioned, the acidizing treatment can stimulate the well by creating a highly permeable region around the well and/or improving fracture conductivity near the well. In the case of the fractured well, we assume that the acid will make the region around the fracture more permeable. Fig. 2.3 shows the model of a vertically fractured well in a homogeneous reservoir with a zone of altered permeability surrounding the fracture. Although there are some concerns that this model is not physically consistent with acid reaction kinetics, we believe that this model will at least qualitatively describe acid treatments performed on fractured wells.

To study the pressure behavior of the reservoir being modeled, it is necessary to construct an appropriate spatial grid for the numerical reservoir model. In the areas of high pressure gradients such as the areas near the wellbore and fracture tip, we need a small spatial grid to correctly model the pressure change. In the areas that have less pressure gradients, we can use a larger spatial grid. Table 2.1 contains the spatial grid we use in this study. For a better understanding, we give the spatial grid that we use in Fig. 2.4. Note that the spatial grid illustrated in Fig. 2.4 is not proportionally scaled. Moreover, we assume that the well in Fig. 2.4 is rectangular so that we can conveniently construct the spatial grid for the reservoir model.

After we have constructed the reservoir model and the spatial grid (Figs. 2.3 and 2.4, respectively), we use a commercial reservoir simulator, FRACSIM, to simulate the pressure behavior of the reservoir.

First, we have to check whether the pressure data obtained from the simulator is correct. We do this by simulating a constant rate oil production. Once we have run
Figure 2.2 One - Quarter of Reservoir Model
Figure 2.3 Fractured Well With Acidized Area
Table 2.1 Spatial Grid Used in Numerical Study

<table>
<thead>
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<th>x direction, ft</th>
<th>y direction, ft</th>
</tr>
</thead>
<tbody>
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</table>
the simulator, we then compare the result to the dimensionless pressure results from Table 1 of Rodriguez. Fig. 2.5 displays the results of the comparison. In Fig. 2.5, we use the fracture conductivity, $C_f$, equal to $\pi$ and the permeability of the acidized zone, $k_a$, equal to the reservoir formation permeability, $k$. In other words, there is no acidized area around the fracture. The comparison indicates very good agreement between our results and Rodriguez's results.

Next, we investigate the effect of the acidized area around the fracture on the pressure behavior of the well. We do this by assigning the value of the permeability of the acidized zone to be 1, 10, 100, and 1000 times the formation permeability. We also use the following values of dimensionless fracture conductivity: $0.1\pi$, $\pi$, $10\pi$, and $100\pi$.

The results are shown in Fig. 2.6. From Fig. 2.6, we can see that the pressure drop, $\Delta p$, (which is proportional to the dimensionless pressure, $p_D$) at the well decreases when the permeability of the acidized zone increases. This decrease means that the well can produce more fluid with less pressure depletion, or that the acid stimulation treatment is successful in stimulating the fractured well.

Note that the increase in the permeability of the acidized zone does not affect the shape of the pressure curve except at very early times. It is interesting that the increase in the permeability of the acidized zone can yield the same affect on the pressure behavior as the increase in the fracture conductivity (see Fig. 2.6). In conclusion, we assume that we can use a finite conductivity fractured well model without the acidized zone to study the pressure behavior of a vertically fractured well that has had an acid stimulation treatment. Although this assumption may not be valid in terms of acid reaction kinetics, which suggest that most of the acid will be spent near the well, we do assume that field data can be modeled using this concept. We will verify this concept using field data.
Figure 2.5 Comparison of Results From Numerical Simulator and Rodriguez\textsuperscript{16} Results
Figure 2.6: Investigation of Acidized Area Effect in Well Pressure Behavior
2.3. Analytical Modeling

In the previous section, we found that we could use the reservoir model of a vertically fractured well with a finite conductivity fracture in an infinite-acting, homogeneous reservoir to simulate the pressure behavior of a vertically fractured well that has been acidized. In this section, we develop an approximate analytical solution for the finite conductivity fractured well model. The solution developed in this section is valid for both constant production rate and constant production pressure flow conditions. For the constant production rate case, we also consider the effect of the wellbore storage in our solution.

2.3.1. Constant Production Rate Solution

When a vertically fractured well begins production, the fluid in the fracture will flow linearly from the fracture into the well in the x-direction (as shown in Fig. 2.7). Later, fluid in the reservoir near the fracture will flow linearly into the fracture in the y-direction. Then, the fluid in the reservoir that is distant from the well and the fracture will begin flow to the reservoir region near the fracture in the x-direction. This flow behavior is the trilinear flow concept that was proposed by Lee and Brockenbrough.\(^9\) This flow model can explain the behavior exhibited by a finite conductivity vertically fractured well in an infinite-acting, homogeneous reservoir during the early production period. However, after the well produces for a period of time beyond the formation linear flow regime, this concept becomes incorrect because it does not account for radial flow.

For the time beyond the formation linear flow regime, the flow of fluid in the reservoir area near the well and the fracture has two directions. The major portion of the fluid flow is occurring in the y-direction and the minor portion of the flow is in the x-direction. In a similar way, the fluid in the reservoir area that is distant from the well flows in both x- and y-directions. Figs. 2.7 and 2.8 show the flow of fluid in the reservoir to the finite conductivity vertically fractured well at an early time and a later time, respectively. In Fig. 2.8, we can see that the fluid in the reservoir area distant from the well flows similarly in the x-direction as well as the y-direction. For an infinite-acting reservoir, the fluid flow in the reservoir will finally become radial.
Figure 2.7 Early Time Fluid Flow for Finite Conductivity Fractured Well
Figure 2.8 Late Time Fluid Flow for Finite Conductivity Fractured Well
In this research, we develop an approximate analytical solution by combining the trilinear flow model solution of Lee and Brockenbrough and the solutions of an infinite conductivity fractured well of Gringarten, et al. and Ozkan and Raghavan. The trilinear flow model gives good results for the early time pressure behavior of the finite conductivity fractured well. The infinite conductivity fractured well model successfully models the late time behavior where radial flow occurs.

From Gringarten, et al., the solution for a well with an infinite conductivity vertical fracture in a homogeneous reservoir is given as

\[
p_{D,\text{frac}}(t_D, x_D) = \frac{\sqrt{\pi} t_D}{2} \left[ \text{erf} \left( \frac{1 - x_D}{2 \sqrt{t_D}} \right) + \text{erf} \left( \frac{1 + x_D}{2 \sqrt{t_D}} \right) \right] \]

\[+ \left( \frac{1 - x_D}{4} \right) E_1 \left( \frac{(1 - x_D)^2}{4 t_D} \right) + \left( \frac{1 + x_D}{4} \right) E_1 \left( \frac{(1 + x_D)^2}{4 t_D} \right) \]  

(2.1)

where

\[p_D = \frac{k h \Delta p}{141.2 q B \mu} \]  

(2.2)

\[t_D = \frac{0.0002637 k t}{\phi \mu c_s x_f^2} \]  

(2.3)

\[x_D = \frac{x}{x_f} \]  

(2.4)

Eq. 2.1 yields the wellbore pressure for an infinite conductivity vertically fractured well if \(x_D = 0.732\) is used.5

Ozkan and Raghavan give the Laplace space solution for Eq. 2.1 as
\[
\bar{p}_{D, \text{frac}} \left( u, x_D \leq 1, y_D = 0 \right) = \frac{1}{2u \sqrt{u}} \left[ \int_0^{(1+u_D) \sqrt{u}} K_0(z) \, dz + \int_0^{(1-u_D) \sqrt{u}} K_0(z) \, dz \right] \quad (2.5)
\]

where

\[
y_D = \frac{y}{x_f} \quad (2.6)
\]

By using the Stehfest algorithm\textsuperscript{17}, Eq. 2.5 can be inverted to real space to give similar wellbore pressures as Eq. 2.1.

From Lee and Brockenbrough\textsuperscript{9}, the Laplace space solution for the trilinear flow model is given as

\[
\bar{p}_{D, \text{tri}}(u, x_D = y_D = 0, C_1, S_f) = \frac{\pi C_{DP}}{u \psi \tanh(\psi)} \quad (2.7)
\]

where

\[
\psi = \left[ \frac{2(C_{DP})(u + \sqrt{u})^{1/2}}{1 + (u + \sqrt{u})^{1/2} S_f} + C_1 u \right]^{1/2} \quad (2.8)
\]

\[
C_{DP} = \frac{k_f b_f}{k x_f} \quad (2.9)
\]

\[
C_1 = \frac{k \phi_f c_f}{k_f \phi c_i} = 0 \quad (2.10)
\]

Again, we can apply the Stehfest algorithm to invert Eq. 2.7 to obtain the dimensionless wellbore pressure in real time space.

Because the finite conductivity fracture causes a greater pressure drop at the well than the infinite conductivity fracture, a trend of \(p_{D, \text{tri}}\) versus \(t_D\) from the finite
conductivity fractured well is also greater than that calculated from the infinite conductivity fractured well (Fig. 2.9). The difference in pressure drop between these two trends gives us the effect of the finite conductivity fracture. If we add this difference in pressure drop to the trend of pressure drop from a well with an infinite conductivity fracture, we can obtain the pressure drop that includes the finite conductivity effect at an early time and the radial flow response at a later time.

Fig. 2.10 shows the addition of the pressure drop difference from Fig. 2.9 to the pressure drop obtained from Ozkan and Raghavan's infinite conductivity fracture model\(^6\) (evaluated at \(x_D\) equal to 0.732). We can write the new solution in the Laplace space for a well with a finite conductivity vertical fracture in an infinite-acting, homogeneous reservoir as follows:

\[
\overline{p}_{D, \text{fin}}(u, C_{fD}) = \overline{p}_{D, \text{tri}}(u, C_{fD}) - \overline{p}_{D, \text{tri}}(u, C_{fD} = \infty) + \overline{p}_{D, \text{frac}}(u, x_D = f(C_{fD}))
\]

(2.11)

where \(x_D = f(C_{fD})\) means \(x_D\) is a function of \(C_{fD}\). For this work, \(C_{fD} = \infty\) is approximated by \(C_{fD} = \pi \times 10^6\).

The correlation of \(x_D\) and \(C_{fD}\), \(x_D = f(C_{fD})\), is given by

\[
x_D = \frac{a_0 + a_1 \ln (C_{fD}) + a_2 \ln (C_{fD})^2 + a_3 \ln (C_{fD})^3 + a_4 \ln (C_{fD})^4}{1 + b_1 \ln (C_{fD}) + b_2 \ln (C_{fD})^2 + b_3 \ln (C_{fD})^3 + b_4 \ln (C_{fD})^4}
\]

(2.12)

where, values of the coefficients \(a_0 - a_4\) and \(b_1 - b_4\) are given as

\[
\begin{align*}
a_0 &= 0.759919 \\
a_1 &= 0.465301 \\
a_2 &= 0.562754 \\
a_3 &= 0.363093 \\
a_4 &= 0.0298881 \\
b_1 &= 0.994770 \\
b_2 &= 0.896679 \\
b_3 &= 0.430707 \\
b_4 &= 0.0467339
\end{align*}
\]

This functional relationship, \(f(C_{fD})\), must be determined by comparison of Eq. 2.11 and the literature results\(^{16, 18-20}\) for a well with a finite conductivity vertical
Figure 2.9 Different Pressure Drop Between Finite Conductivity and Infinite Conductivity Pressure Curves (Trilinear Flow Model)
Figure 2.10 Finite Conductivity Pressure Curve Obtained by Adding Pressure Drop Difference to Infinite Conductivity Pressure Curve
fracture. Specifically, a value of \( x_D \) was determined (via univariate optimization) for each data trend (that is, for a given value of \( C_{FD} \)). From this optimization procedure, Fig. 2.11 was developed, and Eq. 2.12 was fitted to the data on Fig. 2.11, for the range of \( 0.5 \leq C_{FD} \leq 10,000 \).

For comparison to the literature results\(^{16,18-20}\), we use Eq. 2.11, with the value of \( x_D \) taken from Eq. 2.12, to compute dimensionless pressure values for the constant production rate case, for different values of the dimensionless fracture conductivity. Our results are compared to the results from other literature\(^{16,18-20}\) in Figs. 2.12 - 2.16. In Fig. 2.12 the \( p_D \) values are compared to the results from Cinco-Ley, \textit{et al.}\(^{20}\) for values of \( C_{FD} \) from \( 0.2\pi \) (\( \approx 0.6283 \)) to \( 100\pi \) (\( \approx \) infinite conductivity). The comparison shows excellent agreement for every value of \( C_{FD} \) throughout the entire production period. Fig. 2.13 compares the solution from this research to the results from Rodriguez.\(^{16}\) The comparison is good in the range of \( C_{FD} \) from \( \pi \) (\( \approx 3.1416 \)) to \( 1000\pi \) (\( \approx \) infinite conductivity). Our solution shows some deviation for the value of \( C_{FD} \) equal to \( 0.1\pi \) (\( \approx 0.3142 \)), but this is expected because our correlation is only accurate for the range of \( 0.5 \leq C_{FD} \leq 10,000 \).

A comparison between the solutions from this research and Bennett’s dissertation\(^{19}\) is illustrated in Fig. 2.14. The comparison is good for \( C_{FD} \) values from 0.5 to 500. The comparison shows significant deviation for \( C_{FD} \) equal to 0.1 (which is expected). Figs. 2.15 and 2.16, respectively, compare the values of dimensionless pressure, \( p_D \), and dimensionless pressure derivative, \( p_D' \), calculated from our solution to those values from Cinco and Meng’s solutions.\(^{18}\) The comparisons are good for \( C_{FD} \) from 0.5 to 10,000 (\( \approx \) infinite conductivity). Our solutions also have some deviation for \( C_{FD} \) equal to 0.25 (again, this is expected). In conclusion, the constant production rate solution developed in this research can model the pressure behavior of a finite conductivity vertically fractured well in an infinite-acting, homogeneous reservoir for the range of \( C_{FD} \) from 0.5 to 1,000 (\( \approx \) infinite conductivity).
Figure 2.11 Comparison of $x_D$ Equation to Other Literature\textsuperscript{16, 18-20}
Figure 2.12 Comparison of Results Between Our Solution and Cinco-Ley, et al. Results
Figure 2.13 Comparison of Results Between Our Solution and Rodriguez Results
Figure 2.14 Comparison of Results Between Our Solution and Bennett's Results
2.3.2 Constant Production Rate Solution With Wellbore Storage Effect

From the previous section, we obtained a constant production rate solution for a well with a finite conductivity vertical fracture in an infinite-acting, homogeneous reservoir. The solution given by Eq. 2.11 with the value of \( x_D \) from Eq. 2.12 does not include the effect of the wellbore storage. However, the effect of the wellbore storage is very significant at early times during the production period. To include the wellbore storage effect to the constant production rate solution, we apply Fair's relation\(^{21}\) as follows

\[
\overline{p}_{D, \text{fin}}(u, C_{fD}, C_{Df}) = \frac{1}{\overline{p}_{D, \text{fin}}(u, C_{fD}) + u^2 C_{Df}} \tag{2.13}
\]

where

\[
C_{Df} = 0.894 \frac{C}{\phi c_{\text{f}} h x_f^2} \tag{2.14}
\]

2.3.3 Constant Production Pressure Solution

From Eq. 2.12, we can also develop a constant production pressure solution for a finite conductivity, vertically fractured well in an infinite-acting homogeneous reservoir by using the relation of van Everdingen and Hurst\(^{3}\) that states

\[
\overline{q}_{D, \text{fin}}(u, C_{fD}) = \frac{1}{u^2 \overline{p}_{D, \text{fin}}(u, C_{fD})} \tag{2.15}
\]

where \( \overline{q}_{D, \text{fin}}(u, C_{fD}) \) is the constant production pressure solution in the Laplace space and \( \overline{p}_{D, \text{fin}}(u, C_{fD}) \) is the constant production rate solution from Eq. 2.11.

By applying the Stehfest algorithm\(^{17}\), we can invert \( \overline{q}_{D, \text{fin}}(u, C_{fD}) \) from Eq. 2.15 to obtain the dimensionless rate, \( q_D \), for the finite conductivity, vertically
fractured well in an infinite-acting, homogeneous reservoir. Note that $q_D$ is defined as follows

$$q_D = \frac{141.2qB\mu}{kh\Delta p}$$  \hspace{1cm} (2.16)
CHAPTER III
ANALYSIS OF WELL TEST DATA

In this chapter, we use the approximate analytical solutions that we have developed in the previous chapter to generate both constant production rate and constant production pressure type curves. We then use the constant production rate type curves to analyze the pressure test data from the wells in the Upper Clearfork Formation.

3.1. Type Curve Solutions

In order to study well performance data we must have an appropriate and accurate mathematical model of the reservoir. This model must include all pertinent reservoir and well conditions. Once the mathematical model is established, "type curves" are plotted to illustrate the behavior of the well with time. The type curves which are derived from different well conditions and reservoir characteristics illustrate different performance characteristics. The type curves generated from mathematical model are a convenient method to estimate the reservoir properties. By matching measured well test data to the type curves of the same reservoir model, the reservoir properties in the vicinity of the well can be estimated.

In this section, we use Eqs. 2.13 and 2.15 respectively to generate constant production rate and constant production pressure type curves for a finite conductivity vertically fractured well in an infinite-acting, homogeneous reservoir. Note that we use Eq. 2.13 to include the effect of the wellbore storage for the constant production rate type curves.

3.1.1. Constant Production Pressure Type Curves

We use Eq. 2.15 and the Stehfest algorithm\textsuperscript{17} to calculate dimensionless production rate, \( q_D \), for a constant well pressure. We then plot the reciprocal of the dimensionless production rate, \( 1/q_D \), versus dimensionless time, \( t_D \), for different
values of the fracture conductivity, \( C_f D \). The type curves are illustrated in Fig. 3.1. In Fig. 3.2, we compare our constant production pressure type curves to the numerical results given by Bennett\(^{19}\). The comparison shows an excellent agreement for \( C_f D \) values from 0.5 to 500.

3.1.2. Constant Production Rate Type Curves

For the constant production rate case, we include the effect of the wellbore storage in the calculations. By using Eq. 2.13 and the Stehfest algorithm\(^{17}\), we can compute the dimensionless pressure, \( p_D \), and dimensionless pressure derivative, \( p_D' \), values for the constant production rate case. Then, the \( p_D \) and \( p_D' \) functions are plotted versus dimensionless time divided by dimensionless wellbore storage coefficient, \( t_D/C_{DF} \) to obtain the constant production rate type curves.

To verify our type curves, we generate the constant production rate type curves with \( C_f D \) equal to 1 and vary \( C_{DF} \) from \( 1 \times 10^{-5} \) to 1. These type curves are compared to the type curves generated from the "PAN System" simulator (a commercial simulator). We show this comparison in Fig. 3.3. From Fig. 3.3, we can see that our type curves are in good agreement with the type curves generated from the PAN System. After verifying our type curves, we generated four working type curves with \( C_f D \) equal to 1, 10, 50, and 100, as shown respectively in Figs. 3.4 - 3.7. In the next section, we will use these type curves to analyze pressure test data from the wells in the Upper Clearfork Formation.

3.2. Application of Type Curve Solutions to Field Data

After we have generated the type curves for a well with a finite conductivity vertical fracture, we use these type curves to analyze pressure test data from wells in the Upper Clearfork Formation. Most of the data that we obtained are pressure buildup data although a few of the cases are pressure falloff test data. The pressure buildup data were collected from producing wells after these wells were fractured and acidized while the pressure falloff test data were collected from injection wells which had been previously fractured. For some of the pressure buildup test data, we also
Figure 3.3 Comparison of Constant Production Rate Finite Conductivity Type Curve From Our Research and PAN System With $C_{Df} = 1$ (Including Wellbore Storage Effects)
Figure 3.5 Constant Production Rate Finite Conductivity Type Curve With $C_{dp} = 10$

(including Wellbore Storage Effects)
Figure 3.6: Constant Production Rate Finite Conductivity Type Curve With $C_{bd} = 50$

(Including Wellbore Storage Effects)
Figure 3.7 Constant Production Rate Finite Conductivity Type Curve With $C_m=100$
(including Wellbore Storage Effects)
found some analysis of the pressure test data had been performed.

To analyze the test data, we first plot the data on a log-log scale by plotting pressure difference, \( \Delta p \), and pressure difference derivative, \( \Delta p' \), versus equivalent time difference, \( \Delta t_e \).\(^{22}\) Next, we match the plotted data onto the appropriate type curve and select a match point for data analysis. Details of the analysis calculations are presented in Appendix A. We also summarize the data matching of the pressure test data from twenty-one wells in Appendix B.

We can see in Appendix B that the pressure test data from the actual wells match very well on the type curves we generated although some of the data are severely distorted by the wellbore storage effect (Well No. 35, 140, 151 158, and 159). Some of the test cases also show radial flow behavior at late times (Well No. 35, 66, 67, 72, 135, and 153). More importantly, the pressure test data from the wells No. 140, 151, 159, and 160 behave uniquely and cannot be matched on the conventional fracture type curves.\(^{23-26}\) However, these data can be matched very well on our type curves. Table 3.1 summarizes the results of type curve matching analysis.

Although we matched the pressure test data on the type curves, we could not obtain some important data for computational analysis for all wells. For example, the formation thickness, \( h \), production flow rate prior to the test, \( q_p \), formation volume factor, \( B \), fluid viscosity, \( \mu \), formation porosity, \( \phi \), and total compressibility, \( c_t \), were not known for all cases. As we show in Table 3.2, there are few wells for which we can obtain those important data; for the wells that we have enough information, we can estimate formation properties from the pressure test data. The results of these calculations are provided in Table 3.3. In Table 3.4, we show the results of the type curve analysis using our type curves and the results from the previous analysis as a comparison. We note that the formation permeability values, \( k \), estimated by our method are more consistent and that these results agree with the geological information\(^1\) better than the results from the previous analysis.
Table 3.1 Results From Type Curve Matching

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<th>C_{DF}</th>
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<th>(\Delta t_e/t_D/C_{DF})_{M.P.}</th>
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* Injection well
Table 3.2 Well Data From Upper Clearfork Formation

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<th>( B ), bbl/STB</th>
<th>( \mu ), cp</th>
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* These data are unknown

* Injection well

** Values assumed for calculations
Table 3.3 Calculation Results From Type Curve Analysis

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* nc: Insufficient data, no calculation
* Injection well
Table 3.4 Comparison of Results From This Work and Previous Analysis

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<th>Well Sealy Smith No.</th>
<th>$kh$, md-ft (This Research)</th>
<th>$kh$, md-ft (Previous Analysis)</th>
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nc  Insufficient data, no calculation
nd  No data given
*   Injection well
CHAPTER IV
SUMMARY

In the previous chapter, we presented the development and application of the solution methods used in our research. In this chapter, we summarize our investigation, drawing conclusions from the results of the test data analysis on the wells in the Upper Clearfork Formation. Finally, we summarize possible directions for future research of finite conductivity fractured well analysis.

4.1. Conclusions

For a vertically fractured well that has been acid stimulated, we have assumed that we can study the well behavior by using solution methods based on the model of a well with a finite conductivity vertical fracture in an infinite-acting, homogeneous reservoir. The justification for this assumption is that we also assume that an acid stimulation treatment yields a similar effect on the well behavior as increasing fracture conductivity. Although the validity of these assumptions could be argued, the Clearfork test data reinforce this assumption.

There are several models that have been used to study finite conductivity fractured wells. In our research, we use the trilinear flow model of Lee and Brockenbrough\textsuperscript{9} and the infinite conductivity fractured well model of Gringarten, et al.\textsuperscript{5} and Ozkan and Raghavan.\textsuperscript{6} The trilinear flow model gives excellent performance at early times and the infinite conductivity fractured well model provides good performance at late times for the finite conductivity fractured well behavior.

By combining the trilinear flow model and the infinite conductivity fractured well model, we develop an approximate analytical solution for the analysis of a well with a finite conductivity vertical fracture in an infinite-acting, homogeneous reservoir. The solutions can also include the effect of the wellbore storage, which is very important during the early production period. By using our approximate solutions, we have generated finite conductivity fracture type curves that are
convenient tools for analyzing pressure test data from a fractured well that has been acid stimulated. The type curves that we have generated show good agreement with those from commercial simulators. The type curves also give agreement with pressure test data from wells in the Upper Clearfork Formation.

4.2. Test Data Analysis Conclusions

In this section, we draw conclusions about field data analysis using our new type curve solutions for a well with a finite conductivity vertical fracture.

Although we obtained the pressure test data from twenty-one wells in the Upper Clearfork Formation and successfully matched these data on our type curves, we could not acquire some of the data necessary for analysis calculations. This lack of data prevented us from calculating the fracture properties and reservoir formation characteristics. However, we had enough information to calculate fracture and reservoir properties for a few wells and we have tabulated the results of these calculations in Table 3.3.

The results are consistent with geological information. Some of the test data could only be matched on our type curves and not on any previously developed type curves.

4.3. Directions for Future Research

Upon the completion of this research, we realize that there are several possible directions for further work on the finite conductivity fractured well model. In this section we summarize these directions.

Well With a Finite Conductivity Vertical Fracture in a Finite, Homogeneous Reservoir

In our research, we investigated the case of a well with a finite conductivity vertical fracture in an infinite-acting, homogeneous reservoir. If we work on a
reservoir system of finite extent, we need to consider the effect of the reservoir boundaries. Boundary effects significantly affect the well behavior at late times. As a result, we may or may not have a radial flow period at a late time. This is especially important for the analysis of test data from a small reservoir.

**Multilayered Well**

In an actual field, there may be more than one productive layer in a single well. When we apply a hydraulic fracturing treatment to the well, we may have fractures occurring in more than one layer. These layers may be isolated or may communicate with other layers. This kind of reservoir model is very interesting and is widely applicable to the actual field data.
REFERENCES


APPENDIX A

EXAMPLE OF TYPE CURVE ANALYSIS CALCULATION
APPENDIX A

EXAMPLE OF TYPE CURVE ANALYSIS CALCULATION

Information:

Well: Sealy Smith No.35

\( h = 50 \text{ ft} \)

\( q_p = 47 \text{ STB/d} \)

\( \phi = 0.07 \)

\( \mu = 1 \text{ cp} \)

\( B = 1.4 \text{ bbl/STB} \)

\( c_t = 1.3 \times 10^{-5} \text{ psi}^{-1} \)

Type Curve Match:

Type Curve Matching is shown in Figure A.1.

Matching Results:

\( (\Delta p / p_D)_{\text{M.P.}} = 440 \)

\( (\Delta t_e / (t_D / C_{Df}))_{\text{M.P.}} = 27 \)

\( C_{fD} = 1 \)

\( C_{Df} = 0.1 \)

Calculations:

Determine permeability-thickness product, \( kh \):

From \( p_D = \frac{k h \Delta p}{141.2 \ qB \mu} \)
\[ kh = \frac{141.2 \ q \ B \ \mu}{(\Delta p / p_D)_{M.P.}} \]

\[ kh = \frac{141.2 \ (47) \ (1.4) \ (1)}{440} \]

\[ kh = 21.12 \ \text{md-ft} \]

Determine the formation permeability, \( k \):

\[ k = \frac{21.12}{(50)} \]

\[ k = 0.42 \ \text{md} \]

Determine the fracture half-length, \( x_f \):

From

\[ \frac{t_p}{C_{Df}} = \frac{1}{C_{Df}} \frac{0.0002637 \ k \ \Delta t}{\phi \ \mu \ c_i \ x_f^2} \]

Or,

\[ x_f = \sqrt{\frac{0.0002637 \ k \ (\Delta t / (t_D / C_{Df}))_{M.P.}}{\phi \ \mu \ c_i \ C_{Df}}} \]

\[ x_f = \sqrt{\frac{0.0002637 \ (0.42) \ (27)}{(0.07) \ (1) \ (1.3 \times 10^{-5}) \ (0.1)}} \]

\[ x_f = 181.78 \ \text{ft} \]

Determine the fracture conductivity, \( b_f k_f \):

From

\[ C_{Df} = \frac{b_f k_f}{k \ x_f} \]
Or,

\[ b_f k_f = C_{fD} k x_f \]

\[ b_f k_f = (1) (0.42) (181.78) \]

\[ b_f k_f = 76.77 \text{ md-ft} \]
APPENDIX B

PRESSURE TEST DATA MATCH ON TYPE CURVES
Analysis Results:
\[ \Delta p = 440 \, p_d \]
\[ \Delta t = 27 \, t_d / C_{Drf} \]

Figure B.4 Pressure Data Match: Sealy Smith No.35, Type Curve $C_{FD} = 1$
Figure B.5 Pressure Data Match: Sealy Smith No.39, Type Curve $C_{frD} = 50$
Figure B.6 Pressure Data Match: Sealy Smith No.41, Type Curve C1D = 100

Analysis Results:

\[ Ap = 5000 p \]
\[ Ap = 19 \frac{p}{C_m} \]
Figure B.7 Pressure Data Match: Sealy Smith No.62, Type Curve $C_{lD} = 10$
Analysis Results:
\[ \Delta p = 120 \, p_D \]
\[ \Delta t = 0.81 \, t_D / C_{Df} \]

Figure B.8 Pressure Data Match: Sealy Smith No.66 (Falloff Test), Type Curve \( C_{Df} = 100 \)
Figure B.9  Pressure Data Match: Sealy Smith No.67, Type Curve $C_{fD} = 10$
Figure B.11 Pressure Data Match: Sealy Smith No.114, Type Curve $C_{TD} = 50$
Figure B.12 Pressure Data Match: Scaly Smith No.135, Type Curve C_P = 50

Analysis Results:
$\Delta \rho = 950 \rho_P$
$\Delta \eta = 0.4 \rho_P / \rho_{CR}$
Analysis Results:
\[ \Delta p = 2500 \ P_D \]
\[ \Delta t_s = 12 \ t_D / C_{Df} \]

Figure B.13  Pressure Data Match: Sealy Smith No.140, Type Curve \( C_{fD} = 10 \)
Analysis Results:
\[ \Delta p = 400 \, p_D \]
\[ \Delta t_s = 2.05 \, t_D / C_{DF} \]

Figure B.14 Pressure Data Match: Sealy Smith No.149, Type Curve \( C_{fD} = 50 \)
Analysis Results:
\[ \Delta p = 8800 p_b \]
\[ \Delta t = 59 \frac{t_p}{C_{df}} \]

Figure B.15 Pressure Data Match: Sealy Smith No.151, Type Curve \( C_{FD} = 100 \)
Figure B.16 Pressure Data Match: Sealy Smith No.153, Type Curve $C_{fD} = 50$
Figure B.17 Pressure Data Match: Sealy Smith No.155, Type Curve C, n = 10

Analysis Results:
\[ \Delta p = 165 \frac{\rho_b}{C_{v,fr}} \]
\[ \Delta t_s = 0.121 \frac{t}{C_{v,fr}} \]
Figure B.18 Pressure Data Match: Sealy Smith No.158, Type Curve $C_{ID} = 50$
Figure B.19 Pressure Data Match: Sealy Smith No.159, Type Curve $C_{FD} = 10$
Analysis Results:
$\Delta p = 400 \rho_d$
$\Delta t_c = 12.5 \frac{t_b}{C_{DF}}$

Figure B.21 Pressure Data Match: Sealy Smith No.266, Type Curve $C_{ID} = 50$
VITA

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