RESERVOIR BEHAVIOR DURING
BOUNDARY-DOMINATED TWO-PHASE FLOW

A Dissertation

by

MOHAMMED ABDULRAHEEM AL-SADDIQUE

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

December 1994

Major Subject: Petroleum Engineering
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December 1994

Major Subject: Petroleum Engineering
ABSTRACT

Reservoir Behavior During Boundary-Dominated Two-Phase Flow. (December 1994)
Mohammed AbdulRaheem Al-Saddique, B.S. King Saud University;
M.S. King Saud University
Co-Chairs of Advisory Committee: Dr. Thomas A. Blasingame
Dr. Maria A. Barrufet

In this work we determined the overall in situ composition path during depletion of
different fluid types in the reservoir including gas condensates, volatile oil, and black oil
reservoirs. A "tie-line relationship" concept was introduced to describe the overall
compositional changes in the reservoir. The tie-line relationship can be used to determine
the accuracy with which the fluid phase properties can be estimated from constant
composition expansion experiments.

The applicability of laboratory experiments like the constant composition expansion
and constant volume depletion to model reservoir performance was investigated. We
developed alternative procedures that are more applicable to determine the average
pressure-saturation relation for the entire depletion history of the reservoir. A single-cell
simulator was developed to model the average reservoir performance and was tested with
the results from a full compositional simulator. The results obtained from the single-cell
simulator gave accurate prediction of the average reservoir performance where other
laboratory procedures failed.

The pressure and saturation profiles in the reservoir were determined by developing
the necessary equations and using the pressure-saturation relation from the single-cell
simulator. The application of this method lead to the confirmation of the non-uniqueness
of pressure-saturation relation throughout the reservoir and the saturation dependence on
location.
DEDICATION

This dissertation is dedicated to my family.
ACKNOWLEDGEMENTS

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King Saud University for their financial support throughout this work.

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NOMENCLATURE

\( B_o \) = oil formation volume factor, \( BBL/STB \)

\( B_g \) = gas formation volume factor, \( ft^3/SCF \)

\( C_1 \) = conversion constant, \( \frac{5.615}{141.2 \cdot 2\pi} \cdot \frac{ft^3}{bbl} \)

\( h \) = reservoir thickness, \( ft \)

\( k_r \) = relative permeability

\( L \) = liquid mole fractions

\( M_i \) = original total mass in place, \( lb-mole \)

\( M_p \) = cumulative total mass produced, \( lb-mole \)

\( m_{pD} \) = dimensionless pseudopressure integral

\( p \) = pressure, psia

\( \bar{p} \) = average reservoir pressure evaluated at \( \bar{r} \), psia

\( q_o \) = oil production rate, \( STB \)

\( q_t \) = total molar production rate, \( lb-mole/day \)

\( R \) = universal gas constant, \( 10.73 \, psi \, ft^3/lb\-mole \, ^\circ R \)

\( r \) = radius, \( ft \)

\( \bar{r} \) = average reservoir radius, \( 0.54928 \, r_e, ft \)

\( r_D \) = dimensionless radius, \( r/r_w \)

\( R_s \) = solution gas-oil ratio, \( SCF/STB \)

\( S \) = saturation

\( t \) = time, days

\( T \) = temperature, \( ^\circ R \)

\( T_c \) = critical temperature, \( ^\circ R \)

\( t_{DA} \) = dimensionless time based on drainage area

\( V \) = vapor mole fractions
\( v_m \) = specific volume of phase \( m \), \( ft^3/lb\text{-mole} \)
\( V_p \) = pore volume, \( ft^3 \)
\( x \) = mole fractions in the liquid phase
\( y \) = mole fractions in the vapor phase
\( z_g \) = compressibility factor of gas
\( z_i \) = overall mole fraction of component \( i \)
\( z_o \) = compressibility factor of oil

Greek
\( \lambda_t \) = total fluid mobility
\( \mu \) = viscosity, cp
\( \phi \) = porosity
\( \rho_m \) = molar density of phase \( m \), \( lb\text{-mole}/ft^3 \)

Subscripts
\( 2\phi \) = two phase
\( c \) = critical
\( e \) = external boundary
\( g \) = gas
\( i \) = initial conditions; component number
\( o \) = oil
\( t \) = total
\( w \) = wellbore
CHAPTER I

INTRODUCTION

The main objective of this work was to establish a simplified procedure for the
analysis and interpretation of well test and production data in solution-gas-drive
reservoirs. In addition, we will build our study systematically from the estimation of
pressure and composition dependent fluid properties to the "material balance" modeling
of the pressure and saturation performance, to the determination of the pressure and
saturation profiles in the reservoir, and finally, to the analysis of well test data. Many of
the objectives for our study are designed to complement as well as validate previous for
the modeling of multiphase flow in porous media.\textsuperscript{1-8}

In the following section we describe the problems we must address and the directions
we will take towards resolving them.

1.1 Estimation of Vapor and Liquid Phase Properties

Fluid properties are usually determined from laboratory procedures which simulate
the reservoir depletion conditions. However, Jacoby and Yarborough\textsuperscript{9} and Fussell\textsuperscript{10}
reported that $K$-values (equilibrium constants) are insensitive to overall composition
during depletion. This observation suggests that a simple constant composition expansion
laboratory procedure can be used to estimate the fluid phase properties in the reservoir
regardless of the overall compositional changes that take place. On the other hand, Coats\textsuperscript{11}
observed the agreement of compositional properties from simulation and laboratory
constant volume depletion experiments for very rich and near-critical gas condensate
fluids. We here investigate these observations and establish the guideline necessary for its
use.

\footnote{This dissertation follows the style of the \textit{Journal of Petroleum Technology}.}
1.1.1 Constant Composition Expansion (CCE)

constant composition expansion (CCE) or flash process, occurs when the fluid composition and temperature remain constant in a laboratory cell, where the process usually starts at the saturation pressure and pressure is then reduced in decrements to simulate reservoir depletion. In this process the overall composition does not change with pressure since no fluid is withdrawn from the cell from either phase. This process represents the reservoir performance early in its life before the evolving phase (gas or oil) becomes significant.

1.1.2 Constant Volume Depletion (CVD) and Its Variations

In the constant volume depletion (CVD) process, the volume of the laboratory cell is maintained constant during pressure depletion similar to the situation in the reservoir. The constant volume is typically set at the saturation pressure and reservoir temperature. Later the pressure is reduced in equal decrements holding the temperature constant. At each pressure decrement, the increase in the cell volume is recorded and an equivalent volume of gas is produced to restore the cell to the original volume. The CVD process can be used to model reservoir performance after the gas phase is established beyond the critical gas saturation. The CVD process is better suited for gas condensate reservoirs where the critical oil saturation is high and most of the production is from the gas phase.

We chose to modify the CVD process in an attempt to develop a better description for modelling the reservoir depletion mechanism. To account for the production of the liquid phase from the cell, we tested the applicability of using fractional flow curves from production data to establish volume of fluid produced from each phase. These results from these simulation are reported in Chapter III.

1.2 Composition Path During Depletion

Production of both phases from the reservoir can cause the overall in-situ fluid composition to change dramatically. We studied the path the overall composition takes in the reservoir during depletion on ternary diagrams and developed a simplified approach of
how it changes. We established a “tie-line relationship” concept that establishes guidelines for the applicability of CCE derived phase compositions to model reservoir phase properties. We have established this concept for the three and five component fluids that are used here.

1.3 Zero-Dimensional Compositional Simulator

We developed zero dimensional compositional reservoir simulator to predict the performance of reservoirs and to establish the applicability of the material balance concept to reservoirs exhibiting large compositional changes. The use of such a model can help us determine the applicability of Muskat’s simplified material balance equation based on black-oil fluid properties to compute the reservoir average pressure-saturation relation when the compositional changes are taken into consideration. The success of using the zero-dimensional compositional simulator to model reservoir performance will confirm that the reservoir performance does not depend on the mode of production (constant pressure or constant rate), and that even for the compositional formulation we will be able to use a tank type model to characterize reservoir performance at average reservoir pressure.

1.4 Estimation of Original Mass in Place

The computation of the original mass in place in a single phase reservoir system is quite simple since all the data necessary to estimate the pressure and compressibility factor are usually available. For a two-phase reservoir, however, this is not the case since it is necessary to compute a two-phase compressibility factor based on the properties and the relative amounts of the two phases at average reservoir conditions. The results of the zero-dimensional compositional simulator helped us determine the proper pressure-saturation relation for the average reservoir. We tested the accuracy of our method and further studied the magnitude of error that results from using a single-phase compressibility factor in material balance computations for a two-phase system. We also investigated the use of simple procedures like CCE and CVD to determine the liquid
volume fractions (equivalent to saturation) and hence the two-phase compressibility factor and to determine the magnitude of error in the material-balance computation that results.

1.5 Pressure and Saturation Relations

The pressure-saturation relations we investigate here are those applicable to boundary-dominated flow. Unlike the many attempts to model the pressure-saturation relation during transient flow like that of Boe, et al., very few methods are available to compute the saturation for the average reservoir. Material-balance based formulation like Muskat’s were developed for black-oil reservoirs using black-oil fluid properties and variables, but they have not been tested for accuracy when compositional changes take place throughout the reservoir. The gas-oil ratio method developed by Evinger and Muskat were based on the steady-state assumption and black-oil properties which do not include the influence of compositional changes in the reservoir. The formulation of the steady-state theory in terms of compositional variable by Jones can also be used to approximate the pressure-saturation at the average level, but its reliance on steady-state concept renders it an approximation to the average saturation at best. We developed and tested a compositional equation based on the mass in place, phase compressibility factors, pressure, and production rate. The results of the full compositional simulator and the zero-dimensional compositional simulator are given in Chapter IV. We found this equation equivalent to both full compositional simulation (on the average level) and zero-dimensional compositional simulation.

1.6 Pressure and Saturation Profile

The only method available for computing the pressure and saturation profiles in the reservoir is the one developed 23 years ago by Levine and Prats that is based on two assumptions. The first is that the rate of decline of stock-tank oil in place is constant everywhere, and the second is that pressure-saturation relation could be represented by the gas-oil ratio at the outer boundary. Furthermore, they assumed that the value of the gas-oil ratio to be constant throughout the reservoir based on their observation from simulation that the gas-oil ratio changed only about 10% throughout the reservoir (at fixed times).
There are two major concerns about this approach. First is that the method require estimates of the pressure and the gas-oil ratio at the outer boundary; the second is that the gas-oil ratio does not always remain constant, and we need to develop an understanding of the parameters affecting its change.

Raghavan\textsuperscript{8} later chose the producing gas-oil ratio as a constant for the computation of the pressure-saturation relation in the reservoir and also assumed that it would remain constant to within 10\% throughout the reservoir.

In this work we determined the gas-oil ratio from compositional simulation and studied its variation in the reservoir to determine parameters that affect its value. We used the zero-dimensional compositional simulator results to determine the average pressure-saturation relation in the reservoir. We also determined the pressure-time data from the compositional material-balance equation when the original mass in place was known. We have developed the necessary equations similar to those of Levine and Prats with compositional variables to compute the pressure-saturation profile in the reservoir. Our equations use the average pressure-time and average pressure-saturation data to determine the saturation profile in the reservoir. We incorporated the average reservoir pressure-saturation relation at the average reservoir radius. We based this on both the study we conducted on the variation with distance of the gas-oil ratio and changes in saturation caused by the compositional changes that take place in the reservoir.

1.7 Saturation Dependence on Location

The computed pressure and saturation profiles in the reservoir suggested that when compositional changes are considered in the reservoir, the oil saturation does not become a function of pressure alone. We have found that the saturation has strong dependence on location in the reservoir especially for heavier fluids.
1.8 Computation of Pseudopressure

We used the computed pressure and saturation profiles to compute the pseudopressure integral and compared it to the pseudopressure integral derived from based on the actual pressure-saturation profiles from the compositional simulation. The results were satisfactory.

1.9 Characteristics of the Simulation Model

In this work we used the compositional simulator developed by Jones\textsuperscript{2} which was used in testing the reservoir integral and sandface integral during the transient two-phase flow period. These pseudopressure integrals form the analogue to the liquid solution during transient flow. The simulator was also used to test the compositional form of the steady-state theory Jones developed.

The model is a fully implicit, finite-difference compositional simulator similar to the “Minimum Variable Newton-Raphson” model presented by Coats.\textsuperscript{13} At every timestep and at every grid in the reservoir, the method is used to compute the pressure, saturation, and phase compositions in an implicit manner. The reservoir-fluid model has the following properties:

- Homogeneous reservoir of uniform thickness and isotropic reservoir with constant porosity and permeability.

- Block-centered grids. The grid blocks expand logarithmically in a radial fashion towards the outer boundary as shown in Fig. 1.9.1. This approach increases the accuracy of the computed pressure and saturation profiles close to the wellbore where pressure gradients are high.

- Phase equilibria calculated using the Redlich-Kwong\textsuperscript{14} equation-of-state as modified by Zudkevitch and Joffe.\textsuperscript{15} Mass transfer between the phases is accounted for by assuming that phase equilibrium exists at each time step.
Fig. 1.9.1
Block-Centered Radial Grid Distribution Used in the Simulator.
• Instantaneous equilibrium between the gas and oil phases in any grid block.

• The skin effect modeled by introducing a zone of altered permeability around the wellbore. The skin factor can then be computed using the Hawkins formula.\textsuperscript{16}

• The effects of gravity, capillary pressure, non-Darcy flow, and wellbore storage are not considered in this study.

• When the simulations start at saturation pressure, the saturation of the evolving phase is computed from phase equilibria at the corresponding iteration pressure. When the pressure drops below saturation pressure, a flash process is performed on the initial overall composition to the corresponding iteration pressure. This approach yields estimates of the liquid and vapor phase compositions and fractions of liquid and vapor moles in each phase. The phase compositions are used in the equation-of-state to compute the phase molar densities. The densities and the molar fractions are then used to compute the saturation using the following equation:

\[
S_o = \frac{L/\rho_o}{L/\rho_o + V/\rho_g}
\]  \hspace{1cm} (1.9.1)

Equation (1.9.1) is only used when the liquid first starts to develop in a gas condensate reservoir or when the gas first develops as the pressure drops beyond the saturation pressure in an oil reservoir. Afterwards, the saturation is computed as dictated by the flow equations and phase equilibria at each timestep in the simulator.

The simulator was mainly developed to study early time responses\textsuperscript{2}. In our study we were interested in observing the late time behavior during boundary-dominated flow. We were also interested in depleting the reservoir to fully test the methods we are developing; this includes producing the reservoir to an abandonment pressure of approximately 50 psia. We tested the simulator to obtain such results, and we recognized some difficulties in converging to a solution when the pressure at the outer boundary dropped below the
saturation pressure. We found that the simulator oscillated around the solution during the iterations. In order to alleviate this problem, we applied a damping factor to the change in pressure per iteration as suggested by Coats\textsuperscript{13}. After applying a damping factor of 0.3 to the pressure, we obtained convergence of the solution at every location in the reservoir. When the damping factor was applied, the number of iterations per time step increased from 4 to 9. To reduce the number of iterations and ensure convergence, the damping factor was applied until the pressure at the outer boundary dropped to a level where the two phases were well established. We started decreasing the effect of the damping factor by increasing its value towards unity, and then we kept the damping factor constant at that value (0.95) for the remainder of the simulation.

The simulator was also modified to work with five components to further investigate the fluid composition path during transient and boundary dominated flow and to test our ability to determine phase properties from flash phase compositions for more than 3-component fluids.

The simulator was developed to work in a single mode of production (either constant pressure or constant rate, or build-up). When the simulator is running in constant rate mode, the pressure at the wellbore becomes very low at late stages of production and can even be negative. To alleviate this problem we have modified the simulator to switch to constant wellbore pressure mode when the pressure at the wellbore drops to a certain value that is predetermined from the simulator’s input data. The switch to constant wellbore pressure mode enables us to deplete the reservoir to a very low average pressure which we need for our study.

1.10 Properties of the Equation of State

The Redlich-Kwong equation of state is well established in the industry and is found to model gas properties better than other cubic equations of state. The Redlich-Kwong equation of state as modified by Zudkevitch and Joffe\textsuperscript{15} is used here to model the phase behavior for both the compositional simulator and the single-cell simulator.
The Redlich-Kwong EOS for a phase $m$ which is a mixture is given by

$$p = \frac{RT}{v_m - b_m} - \frac{a_m}{T^{0.5} v_m (v_m + b_m)} \quad (1.10.1)$$

where $v_m$ is the specific volume, $ft^3/lb$-mole

and $a_m$ and $b_m$ are parameters given by

$$a_m = \sum_{i=1}^{N_c} \sum_{j=1}^{N_c} x_{i,m} x_{j,m} a_{i,j} \quad (1.10.2)$$

and

$$b_m = \sum_{i=1}^{N_c} x_{i,m} b_{i,j} \quad (1.10.3)$$

where,

$x_i$ is the mole fraction of component $i$ in phase $m$.

$a_{i,j}$ and $b_i$ are determined from

$$a_{i,j} = (1 - k_{i,j}) (a_i a_j)^{0.5} \quad (1.10.4)$$

where, $k_{i,j}$ are the binary interaction coefficients. The attraction term ($a_i$) in the equation of state is obtained from the critical constraints for each pure component as

$$a_i = \Omega_{ai} R^2 T_{ci}^{2.5} / p_{ci} \quad (1.10.5)$$

Likewise, the repulsion term is

$$b_i = \Omega_{bi} R^2 T_{ci} / p_{ci} \quad (1.10.6)$$

Zubkevitch and Joffe\textsuperscript{15} introduced temperature dependence in computing the parameters $a_i$ and $b_i$ of the EOS. This was done by adjusting the parameters $\Omega_{ai}$ and $\Omega_{bi}$ which are dependent on the properties of component $i$ along its vapor pressure curve and are temperature dependent. The binary interaction coefficients $k_{i,j}$ are zero except for
$k_{1,2} = 0.024$, and $k_{2,3} = 0.025$ which were obtained from Zudkevitch and Joffe for methane, butane, and decane respectively.

The engineering equation of state given by

$$p v_m = z_m RT \quad (1.10.7)$$

can be used in the modified Redlich-Kwong equation of state (1.10.1) to obtain the cubic form of the equation of state that provides us with the compressibility factor for each phase, $z_m$,

$$z_m^3 - z_m^2 + (A_m - B_m^2 - B_m) z_m - A_m B_m = 0 \quad (1.10.8)$$

where

$$A_m = a_m p / R^2 T^{2.5} \quad (1.10.9)$$

and

$$B_m = b_m p / RT \quad (1.10.10)$$

Solving the cubic equation for each phase can provide the vapor and liquid compressibility factor, which then enables us to compute each phase's physical properties.

1.11 Fluid Compositions and Physical Properties

The fluid molar density in lb-mole/ft$^3$ is computed from the engineering equation of state where the phase compressibility factor is obtained from the solution of the cubic equation defined in equation 1.10.8,

$$\rho_m = p / z_m RT \quad (1.11.1)$$

where $m$ refers to liquid or vapor phase.

Of the three roots obtained from the solution of the cubic equation, the smallest root is chosen for the vapor phase and the largest root is chosen for the liquid phase.
The phase viscosities are computed using the procedure reported by Lohrenz, Bray, and Clark.\textsuperscript{17} We apply the process outlined by Lohrenz et. al. for the liquid phase to calculate both liquid and vapor phase viscosities. This is possible because we do not use the $c_7+$ fractions. The method is composed of two parts: first, computing the phase viscosity at low pressure using the correlation of Stiel and Thodos\textsuperscript{18} for the pure components, and then the mixture viscosity at low pressure is computed using the Herning and Zipperer\textsuperscript{19} equation. Second, the correction for increased pressure is applied on the basis of the Josse, Stiel, and Thodos\textsuperscript{20} correlation between residual viscosity and reduced density. The details of the method are discussed in Jones.\textsuperscript{2}

1.11.1 Fluid Compositions

The fluid compositions used here cover the different fluid types we are interested in, namely gas condensates, volatile oils, and black oils. These compositions were used in earlier studies by Jones\textsuperscript{2} and Vo.\textsuperscript{3} We are using the same compositions here for the purpose of continuity since we intend to expand on their results.

We use two gas condensate mixtures used by Jones,\textsuperscript{2} and volatile oil and black oil mixtures used by Vo.\textsuperscript{3} We will refer to these as fluids 1, 2, 3, and 4 respectively. These data were selected by Jones and Vo from the experimental data presented by Reamer, Sage, and Lacey.\textsuperscript{21} Fluids 1 and 2 have a maximum liquid drop-out as obtained from flash of 10.5\% and 12.4\% in mole fractions, corresponding to 9.69\% and 11.29\% in volume fractions (saturation). The maximum liquid drop-out for each fluid is obtained at the corresponding reservoir temperature for that fluid listed in Table 1.11.1. The primary difference between these two compositions is the richness of the condensate mixture given by the maximum liquid dropout obtained from flash. The isothermal behavior of these two fluids is shown in Fig. 1.11.1 and Fig. 1.11.2. Fluid 3 is a volatile oil which has a saturation pressure of 2,173 psia, and fluid 4 is a black oil that has a saturation pressure of 2,328.3 psia. Figs. 1.11.3 and 1.11.4 show the isothermal behavior of these two fluids.
The fluids compositions and $\Omega_{ai}$ and $\Omega_{bi}$ are given in Table 1.11.1 for all the 3-component mixtures.

Fig. 1.11.1  Maximum Liquid Drop-Out From Constant Composition Expansion for Gas Condensate (Fluid 1)
Fig. 1.11.2  Maximum Liquid Drop-Out From Constant Composition Expansion for Gas Condensate (Fluid 2)

Fig. 1.11.3  Constant Composition Expansion Data for the Volatile Oil (Fluid 3)
Fig. 1.11.4  Constant Composition Expansion Data for the Black Oil (Fluid 4)

Fig. 1.11.5  Maximum Liquid Drop-Out From Constant Composition Expansion for the 5-Component Gas Condensate (Fluid 5)
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Vapor Composition</th>
<th>Liquid Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_1$</td>
<td>$n-C_4$</td>
</tr>
<tr>
<td><strong>Fluid 1 (Condensate)</strong></td>
<td>Composition</td>
<td>0.89064</td>
</tr>
<tr>
<td>$T = 620 ^\circ R$</td>
<td>$\Omega_a$</td>
<td>0.42510</td>
</tr>
<tr>
<td>$P_{sat} = 3976.0$ psia</td>
<td>$\Omega_b$</td>
<td>0.08590</td>
</tr>
<tr>
<td>$L_{max} = 10.5 %$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Fluid 2 (Condensate)</strong></td>
<td>Composition</td>
<td>0.896283</td>
</tr>
<tr>
<td>$T = 740 ^\circ R$</td>
<td>$\Omega_a$</td>
<td>0.42510</td>
</tr>
<tr>
<td>$P_{sat} = 4189.7$ psia</td>
<td>$\Omega_b$</td>
<td>0.08590</td>
</tr>
<tr>
<td>$L_{max} = 12.4 %$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Fluid 3 (Volatile Oil)</strong></td>
<td>Composition</td>
<td>0.91529</td>
</tr>
<tr>
<td>$T = 620 ^\circ R$</td>
<td>$\Omega_a$</td>
<td>0.42510</td>
</tr>
<tr>
<td>$P_{sat} = 3173$ psia</td>
<td>$\Omega_b$</td>
<td>0.08590</td>
</tr>
<tr>
<td><strong>Fluid 4 (Black Oil)</strong></td>
<td>Composition</td>
<td>0.97262</td>
</tr>
<tr>
<td>$T = 620 ^\circ R$</td>
<td>$\Omega_a$</td>
<td>0.42510</td>
</tr>
<tr>
<td>$P_{sat} = 2328.3$ psia</td>
<td>$\Omega_b$</td>
<td>0.08590</td>
</tr>
</tbody>
</table>

* Shaded compositions correspond to initial reservoir composition and state of fluid at $p > p_{sat}$.
TABLE 1.11.2 Fluid Compositions at Saturation Pressure for the 5-Component Fluid Used in This Study

Reservoir temperature: 620 °R.  
Saturation Pressure: 3655.76 psia
$L_{max} = 7.09\%$

<table>
<thead>
<tr>
<th>Fluid 5</th>
<th>$C_1$</th>
<th>$C_3$</th>
<th>$n-C_4$</th>
<th>$n-C_7$</th>
<th>$n-C_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vapor</td>
<td>0.89000</td>
<td>0.03000</td>
<td>0.05000</td>
<td>0.00500</td>
<td>0.02500</td>
</tr>
<tr>
<td>Liquid</td>
<td>0.67196</td>
<td>0.04653</td>
<td>0.09015</td>
<td>0.02074</td>
<td>0.17061</td>
</tr>
<tr>
<td>$\Omega_a$</td>
<td>0.42510</td>
<td>0.41200</td>
<td>0.41540</td>
<td>0.45050</td>
<td>0.46512</td>
</tr>
<tr>
<td>$\Omega_b$</td>
<td>0.08590</td>
<td>0.07420</td>
<td>0.07590</td>
<td>0.07300</td>
<td>0.07259</td>
</tr>
</tbody>
</table>

A 5-component hypothetical mixture that resembles gas condensate properties was created to test the results pertaining to fluid properties estimation from flash (constant composition expansion) obtained here and those reported by Vo.\textsuperscript{3} This is mainly done to verify that the results obtained are not particular to 3-component mixtures. The composition data and the $\Omega_{ai}$ and $\Omega_{bi}$ for the 5-component mixture are given in Table 1.11.2, and its constant composition expansion behavior is shown in Fig. 1.11.5.

1.12 Relative Permeability Data

We used two relative permeability data sets. One of them is an imbibition data set used in simulating gas condensate reservoir performance where gas displaces oil, and the other is for a drainage system used to model the volatile and black oil fluids where oil displaces the gas phase. We refer to these data sets as Set 1 and Set 2.

Relative Permeability Data Set 1 is an experimental data set that was originally supplied by Fetkovich and used in the Jones\textsuperscript{2} study. Set 1 has zero critical gas saturation and a relatively high critical oil saturation ($S_{oc}=0.38$) which is a characteristic of gas condensate reservoirs. Relative Permeability Data Set 2 was used by Raghavan\textsuperscript{8} in his study and has a low critical gas saturation ($S_{gc}=0.07$) and a critical oil saturation $S_{oc}=0.234$. These two relative permeability data sets appear in Figs. 1.12.1 and 1.12.2.
Fig. 1.12.1  Relative Permeability Set 1 (Imbibition)

Fig. 1.12.2  Relative Permeability Set 2 (Drainage)
CHAPTER II

COMPOSITION PATH DURING DEPLETION

An important part in the analysis of the multiphase fluid flow problem lies in the determination of fluid properties as a function of pressure for both the liquid and vapor phases. The description of fluid properties is typically complicated by the fact that the overall in-situ composition in the reservoir changes during the depletion process. Moreover, the different fluid types (gas condensate, volatile oil, and black oil) exhibit different compositional changes during depletion. The magnitude of compositional changes in heavy fluids like black oils is usually minor and can therefore be approximated as a two-component system. However, in lighter fluids the composition may change dramatically during the depletion process, which makes us question the validity of using laboratory (PVT) data to determine the vapor and liquid properties from a cell experiment without completely understanding what happens in the reservoir.

The validity of the cell process is questioned here because the cell experiments allow changes in the fluid composition only by withdrawing fluid from the cell, where as in the reservoir the compositional changes are caused due to two factors. The first is equivalent to the cell process in which fluid from either phase or both phases is allowed to move out of the reservoir according to its mobility, thereby causing the overall fluid composition to change. The second is that in the reservoir the overall composition can be altered by the moving fluid since the moving fluid has a flowing composition that is different from the in-situ fluid in the reservoir over which it is passing on the way to the wellbore. The flowing fluid moves from the higher pressure region of the reservoir to the lower pressure zone when the pressure drops, the heavier components of the gas phase in the flowing composition condense on the way to the wellbore thus altering the overall in-situ composition in the reservoir. The question we have here is whether this change is large enough to affect the accuracy with which we can determine the phase fluid properties from the cell experiments.
To clarify the compositional changes in the reservoir and how these changes affect the phase fluid properties, we will use the compositional simulator discussed in Chapter I to evaluate the reservoir performance of three fluid prototypes:

a. Black Oil  
b. Volatile Oil  
c. Gas Condensate

In our study, we tracked the compositional changes in the reservoir using ternary diagrams to develop results for determining individual phase properties at reservoir conditions.

Our goal was to resolve the following questions:

- Is it possible for us to accurately determine the fluid phase properties at reservoir conditions during depletion using a constant composition expansion (CCE) process to model performance throughout the life of the reservoir?
- If the constant composition expansion (CCE) process is a successful model, why is this process able to model compositional changes for individual phases during depletion, even though the overall composition could change dramatically in the reservoir.

In the previous works regarding the insensitivity of K-values to composition, Coats, Jacoby and Yarborough, Fussell, and Vo's work regarding the success of modelling phase compositions in the reservoir using a constant composition expansion (CCE) process at least for a short pressure range. We note that these studies could help us to determine the basis for estimating phase compositions solely by a CCE process.

We constructed Fig. 2.1 and 2.2 for two hypothetical cases where the overall composition changes in a completely different manner for each case. In Fig. 2.1 we show a hypothetical composition case $z (z_1, z_4, z_{10})$ along with its phase envelope (two-phase region) at pressure $p_1$. The line connecting the points $y (y_1, y_4, y_{10})$ and $x (x_1, x_4, x_{10})$ is the tie-line connecting the vapor and liquid phase compositions $y$ and $x$ respectively. The
Fig. 2.1 Compositional Changes During Depletion for a 3-Component System
phase compositions at $\hat{y}$ and $\hat{x}$ can be obtained at this stage by a simple flash (CCE) process for any overall composition along the tie-line $(\hat{y} - \hat{x})$ to the pressure $p_1$. When the reservoir fluid is produced at isothermal conditions, two parameters change: first, the pressure drop causes the phase envelope to expand, $(p_2)$, and the gas phase and liquid phase compositions change accordingly. Second, the overall in-situ composition changes because of the following factors:

a. The mobility of the vapor phase is usually higher than that of the liquid phase, and therefore the two phases are not produced at the same percentage.

b. The fluid moving towards the wellbore will condense its heavier components along the low pressure path in the reservoir, thereby creating a region of heavier compositions towards the wellbore.

These two factors are compounded to produce a net change in the overall in-situ composition that makes the composition heavier, and hence the successive changes of this composition follow a path towards the heavier components on the ternary diagram. Considering these net compositional changes and considering that a pressure drop took place from $p_1$ to $p_2$, we can consider the possibility of the overall in-situ composition to be at point $\hat{z}^*$. Figs. 2.1 and 2.2 show two different positions that would be the next location for the overall in-situ composition. Although the two compositions look similar, there is a distinct difference between these compositions as shown in Fig. 2.1; we note that the overall in-situ composition appears to follow a path that is neither related to the initial overall composition nor to its phase compositions. The location of the overall composition here poses two problems. First, the location of this composition does not explain why it is possible to model phase compositions with a CCE process since there seems to be no relation between this overall in-situ composition and the initial overall composition. Second, we have no way of determining the value of the new overall compositions for subsequent pressures without resorting to fully compositional simulation. In Fig. 2.2 the overall in-situ composition is located in a position that alleviates the two problems posed in Fig. 2.1. This is true since the composition $\hat{z}^*$ has a
Fig. 2.2 A Schematic Diagram Showing the Tie-Line Relationship Between the Initial Overall Composition and its Subsequent Values During Depletion
tie-line that passes through the initial overall composition \( \hat{z} \). When either overall composition \( \hat{z} \) or \( \hat{z}^* \) is flashed to pressure \( p_2 \) we will obtain identical vapor and liquid phase compositions given by the ends of the tie-line \( \hat{y} \) and \( \hat{x} \) respectively.

To find out if the overall in-situ composition follows the scenarios presented in Fig. 2.1 or Fig. 2.2, we determine the vapor and liquid compositions (tie-line) for each pressure and overall in-situ composition from simulation and compare the phase composition with the value we obtain by performing a flash (CCE) process on the initial overall composition to the simulation pressure. If all the phase compositions obtained from the CCE process and from the overall in-situ composition determined from the compositional simulator for all the pressures during depletion are equal, then this will confirm that the overall in-situ composition always follows a path that has its tie-line pass through the initial overall composition. We will call this the “tie-line relationship”.

In this study we investigated the extent of the applicability of the tie-line relationship for the different types of fluids we used. We depleted the reservoir to an average reservoir pressure of approximately 50 psia and compared the phase compositions obtained from simulation with the compositions obtained by flashing the initial overall composition to the same pressure.

In the following section and throughout the remainder of this dissertation, we refer to average reservoir pressure and properties at average reservoir pressure. The average pressure we consider is equivalent to the volumetric average pressure of the entire reservoir and is obtained from the radial distribution of pressures at the point

\[
\hat{r} = 0.54928 \times r_e
\]  \( (2.1) \)

Where Eq. (2.1) was derived for single-phase flow conditions and was reported by Camacho and Raghavan\(^5\) for solution-gas drive reservoirs and was confirmed by Vo\(^3\) later for gas condensate systems. We have checked the accuracy of the pressure computed at this radius with the volumetric average pressure in every simulation case we performed and found that the values estimated using Eq. (2.1) always compared to within 1 psia.
We must stress here that even if this hypothesis is correct and in fact we confirm the tie-line relationship for all the fluids considered in this study, the only information we will obtain is the fluid phase compositions. We do not obtain the overall in-situ composition nor do we obtain the relative amounts of vapor and liquid phases or the spatial location of the overall composition on the ternary diagram. Determining the location of the overall composition on the ternary diagram is equivalent to determining a pressure-saturation relation since the relative amounts of vapor and liquid are directly related to the phase saturations at the given pressure. Chapter III will consider this problem in an expanded way and will ultimately determine the pressure-saturation relation for the average reservoir radius \( \bar{r} \).

2.1 Composition Path During Depletion for 3-Component Fluids and Phase

Compositions Estimation by Constant Composition Expansion Process

In this section we will investigate the applicability of the tie-line relationship as discussed earlier. When this is confirmed we will be able to compute the phase compositions and hence the phase properties for all fluids by simply performing a constant composition expansion process on their initial overall composition.

The fluids used for this study cover the different fluid behavior that might be encountered in the oilfield. The simulation data used to generate these results are given in Table 2.1.1.

We have used the compositional simulator to generate the results shown in Fig. 2.1.1 through Fig. 2.2.4. However, to obtain the constant composition data shown in the figures, we have extracted the equation-of-state code from the simulator and used it to predict the constant composition expansion (flash) behavior.

Fig. 2.1.1 was generated for fluid 1 (gas condensate) by running the simulator in a constant total molar rate of 5000 lb-moles/day until the pressure at the wellbore reaches a low value (in this case 46 psia) and then the simulator was switched to constant wellbore pressure mode (pwf=46 psia) for the rest of the simulation until the reservoir was depleted
TABLE 2.1.1 Simulation Data Used in the Study of Composition Path

\[ r_e = 5000 \text{ ft}, \quad \text{Porosity} = 0.2 \]

\[ r_w = 0.25 \text{ ft}, \quad \text{Thickness} = 40 \text{ ft} \]

<table>
<thead>
<tr>
<th>Fluid Type</th>
<th>Permeability ( md )</th>
<th>Initial pressure psia</th>
<th>Production Rate ( \text{Lb-Mole/Day} )</th>
<th>Wellbore Pressure Held Constant at</th>
<th>Average Pressure During Switch</th>
<th>Average Pressure at End of Run psia</th>
<th>Percentage Mass Recovery</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas Condensate (1)</td>
<td>50</td>
<td>3979</td>
<td>5000</td>
<td>46</td>
<td>300</td>
<td>51</td>
<td>95.5</td>
</tr>
<tr>
<td>Gas Condensate (2)</td>
<td>5</td>
<td>4300</td>
<td>5000</td>
<td>55</td>
<td>1110</td>
<td>88</td>
<td>94.4</td>
</tr>
<tr>
<td>Volatile Oil</td>
<td>10</td>
<td>3175</td>
<td>1000</td>
<td>49</td>
<td>508</td>
<td>170</td>
<td>70.0</td>
</tr>
<tr>
<td>Black Oil</td>
<td>5</td>
<td>2386</td>
<td>2000</td>
<td>66</td>
<td>2347</td>
<td>70</td>
<td>61.5</td>
</tr>
<tr>
<td>5-Component Gas Condensate</td>
<td>5</td>
<td>3700</td>
<td>5000</td>
<td>44</td>
<td>958</td>
<td>530</td>
<td>84.4</td>
</tr>
</tbody>
</table>
(\(p_{\text{avg}}=51 \text{ psia}\)). This graph mainly shows the magnitude of change that the overall in-situ composition goes through during the entire reservoir life. The compositional changes appear to be monotonic for the lighter component \(C_1\) so that it continuously decreases during the simulation. This is mainly due to the continuous condensation of the heavier components once the saturation pressure is reached. Fig. 2.1.2 was generated for the richer gas condensate; The results obtained are similar.

The volatile oil (Fluid 3) and black oil (Fluid 4) are shown in Fig. 2.1.3 and Fig. 2.1.4 respectively. Although the fluid behavior is different, the overall composition of \(C_1\) decreases monotonically throughout the reservoir life. In this case, this is due to the lighter component transfer from the liquid phase to the vapor phase when the saturation pressure is reached, and also due to the higher mobility of the gas phase that causes the gas phase (lighter components) to be withdrawn from the reservoir at a higher rate than the liquid (heavier components) phase.

The mole fractions of the heavier components considered here (namely \(C_4\) and \(C_{10}\)) start to increase as the reservoir is depleted. For gas condensates this is due to condensation into oil and the entrapment of the oil phase caused by the naturally high critical oil saturation in gas condensate reservoirs. For the volatile oil or black oil case, the evolving gas phase is trapped until the critical gas saturation is reached, after which the gas starts to move out of the reservoir, with a higher mobility than the liquid. This causes the overall composition to be richer in heavier components.

When the pressure at the wellbore is held constant, the pressure becomes lower in the reservoir and actually low enough for \(C_4\) to start evaporating from the liquid phase and move out of the reservoir causing the \(C_4\) fractions to drop sharply at very low pressures. This behavior can be seen in both gas condensates in Fig. 2.1.1 and Fig. 2.1.2 and also in the black oil case shown in Fig. 2.1.4. The volatile oil case shown in Fig. 2.1.3 did not exhibit the same behavior since the pressure was not low enough for it to evaporate (\(p_{\text{avg}}=170 \text{ psia at end of simulation}\)).

The results shown so far confirm the magnitude of change in the overall in-situ fluid composition in all the cases discussed and shows the importance of studying this change
Overall in situ Composition for C_{10}

\[ T = 620^\circ R \quad p_{\text{sat}} = 3976 \text{ psia} \]

\[ \text{AC}_4 \text{ Max} \]

\[ \text{AC}_{10} \text{ Max} \]

\[ \text{AC}_4 \text{ Max} \]

Overall in situ Composition for \( C_4 \) and \( C_{10} \)

Fig. 2.1.1  Overall in situ Compositional Changes During Depletion for Gas Condensate Fluid 1
Fig. 2.1.2 Overall in situ Compositional Changes During Depletion for Gas Condensate Fluid 2
Fig. 2.1.3  Overall in-situ Compositional Changes During Depletion for Volatile Oil Fluid 3
Fig. 2.1.4 Overall in situ Compositional Changes During Depletion for Black Oil Fluid.
and its impact on phase compositions, including whether it is possible to obtain the same phase compositions by the simple CCE process using the initial overall composition.

What remains to be shown is the comparison between the fluid phase compositions obtained from simulation and those obtained by flashing the initial overall composition to the pressure in the reservoir.

Fig. 2.1.5 and Fig. 2.1.6 show comparison between simulation and flash phase compositions for the vapor and liquid phases respectively for gas condensate Fluid 1. The simulation and flash vapor compositions agree completely from the saturation pressure of 3,976 psia until the pressure reaches a value of 1,200 psia, when the two compositions start to depart. The liquid phase compositions from simulation and flash in Fig. 2.1.6 also agree for the same pressure range. The basic conclusion from these two figures is that the vapor and liquid compositions obtained from simulation and from CCE are almost identical for a fairly long pressure range before they start deviating.

The agreement between simulation and CCE obtained phase compositions confirmed that the tie-line relationship will exist for this case from the start of depletion until the pressure reaches the value of 1,200 psia, and then it will start to fail. This could also be interpreted to mean that both phase compositions and phase fluid properties will be obtainable simply by flashing the initial overall composition to the target pressure using an equation of state or PVT data from a flash experiment until the pressure reaches the value of 1,200 psia. After this pressure, the properties obtained from flash will deviate more as the pressure drops.

The departure of flash phase compositions between simulation and flash phase compositions up to a low average pressure of 300 psia is within 3%, which will cause a minor error in the computation of fluid phase properties represented mainly in the vapor and liquid compressibility factors. Fig. 2.1.7 shows the magnitude of departure that is caused by the difference in phase compositions from simulation and flash. The maximum error results at the end of simulation ($p_{avg}$=51 psia) in the vapor phase compressibility factor and is less than 4.6%. This result encourages us to use the constant composition
Fig. 2.1.5  Comparison of Simulation and Flash Vapor Phase Compositions for Gas Condensate Fluid 1
Fig. 2.1.6 Comparison of Simulation and Flash Liquid Phase Compositions for Gas Condensate Fluid 1

- Liquid Phase Compositions
- Pressure, psia

$T = 620^\circ R$
$P_{sat} = 3976$ psia
Fig. 2.1.7  Comparison of Simulation and Flash Compressibility Factors for Gas Condensate Fluid 1

\[ T = 620^\circ R \]
\[ p_{\text{sat}} = 3976 \text{ psia} \]
expansion process to obtain phase fluid properties since the benefits over weigh the error in properties estimation.

Fig. 2.1.8 is a ternary diagram showing the composition path of the overall in-situ composition, vapor phase composition, and liquid phase composition during depletion of the reservoir at the average reservoir radius \( r \). The crosses indicate the overall in-situ composition change, while the hollow circles and the solid circles indicate the vapor and liquid compositions respectively. The numbers shown next to each point are the pressures at which this composition exist. This graph shows the dramatic changes that the overall in-situ composition undergoes during depletion. At the beginning of simulation before the saturation pressure is reached, the overall in-situ composition, the initial composition, and the vapor phase compositions are identical and exist on the ternary diagram as one point at 3,975 psia. As the reservoir is depleted, the overall composition changes (becomes heavier as discussed earlier) and moves towards the heavy component in the ternary diagram. As was indicated earlier from the phase composition data (Fig. 2.1.5 and Fig. 2.1.6), the tie-line relationship should be valid from the beginning of simulation until the average pressure reaches 1,200 psia. In Fig. 2.1.9 we have reproduced Fig. 2.1.8 and drawn the line A-B passing through the vapor and liquid phase compositions at 1,203 psia. The initial overall composition and the overall in-situ composition at 1,203 psia by definition of the tie-line relationship should lie on this line. By examining the position of the initial overall composition, we find that it lies on the line A-B, thereby confirming the tie-line relationship. This case is similar to the schematic diagram shown in Fig. 2.2 where it is possible to predict the phase compositions by a CCE process using the initial overall composition. We have drawn the line C-D for a later overall in-situ composition at a pressure of 149 psia passing through its vapor and liquid phase compositions. We observe the position of the initial overall composition (at pressure 3,975 psia) and find that it does not lie on the this line, and therefore the tie-line relationship is not honored. This is confirmed by Fig. 2.1.5 and Fig. 2.1.6 where the phase compositions from simulation and CCE deviate. The failure of the tie-line relationship indicates that the fluid phase compositions can not be predicted reliably at this or later pressures by using a CCE process.
The behavior of the other fluid compositions was similar to Fluid 1, where the tie-line relationship was honored for a limited pressure range, after which it failed and error resulted from using the CCE process to estimate phase compositions.

It appears also that the heavier mixtures do follow the tie-line relationship more closely for lower pressures. This can be seen in Fluid 2 which is a gas condensate of larger liquid drop-out from flash than Fluid 1 (at reservoir temperature). Fig. 2.1.10 shows the smaller departure between the two vapor compositions obtained from simulation and CCE. The liquid phase compositions from flash compare better with simulation than the vapor compositions for the same pressure range, as can be seen in Fig. 2.1.11.

The vapor and liquid compressibility factors based on vapor and liquid compositions obtained from simulation and CCE for Fluid 2 are shown in Fig. 2.1.12. Excellent agreement between the two values can be seen throughout the pressure range.

The ternary diagram in Fig. 2.1.13 shows close agreement until the average pressure reaches 400 psia. The graphs for the other two fluids (Fluid 3 and Fluid 4) performed even better in terms of obtaining phase compositions by constant composition expansion and in terms of the tie-line relationship. The results for Fluids 3 and 4 are shown in Fig. 2.1.14 through Fig. 2.1.21.
Vapor Composition of C₄ and C₁₀

$P_{sat} = 4189.7$ psia

$T = 740° R$

$Y_1$ (Simul.)

$Y_4$ (Flash)

$Y_{10}$ (Simul.)

$Y_{10}$ (Flash)

Pressure, psia

Vapor Composition of C₁₀
Fig. 2.1.11 Comparison of Simulation and Flash Liquid Phase Compositions for Gas Condensate Fluid 2
Fig 2.1.12 Comparison of Simulation and Flash Compressibility Factors for Gas Condensate Fluid 2
Fig. 2.1.13 Composition Path During Depletion for Gas Condensate Fluid 2
Fig. 2.1.14 Comparison of Simulation and Flash Vapor Phase Compositions for Volatile Oil Fluid 3
Fig. 2.1.15 Comparison of Simulation and Flash Liquid Phase Compositions for Volatile Oil Fluid 3

$T = 620^\circ R$

$p_{sat} = 3173$ psia
Fig 2.1.16 Comparison of Simulation and Flash Compressibility Factors for Volatile Oil Fluid 3
Fig. 2.1.18 Comparison of Simulation and Flash Vapor Phase Compositions for Black Oil Fluid 4

$T = 620^\circ R$

$p_{\text{sat}} = 2328.3$ psia
Vapor Compressibility Factors

Fig. 2.1.20 Comparison of Simulation and Flash Compressibility Factors for Black Oil Fluid 4
Fig. 2.1.21  Composition Path During Depletion for Black Oil Fluid 4
2.2 5-Component Mixture Composition Path

We have two objectives here, first is to investigate the applicability of flash driven phase compositions to represent the reservoir values based on the reservoir overall in-situ composition, and the second to test the tie-line relationship for this 5-component system. Although we cannot visualize the tie-line relationship for the 5-component system, the applicability of the principle can be tested by comparing the vapor and liquid mole fractions from both simulation and constant composition expansion as we did earlier.

In following the same line of thought, the tie-line relationship exists when the overall composition in the reservoir will always follow a tie-line passing through the initial overall composition. For this to happen, both phase compositions (vapor and liquid) obtained from flashing the initial overall composition and from the reservoir composition should be equal.

The results we show here confirm the existence of the tie-line relationship for the 5-component mixture, and therefore this phenomenon is not specific to 3-component mixtures.

Fig. 2.2.1 shows the overall in-situ composition change in the reservoir during depletion. As is the case with the 3-component mixtures, the overall composition did become heavier by producing most of \( C_1 \) and condensing the heavier components. Fig. 2.2.2 compares the vapor composition values obtained from simulation to those obtained from constant composition expansion. The values from simulation and flash seem to agree quite well until the pressure reaches 1,200 psia, where the two values start to depart. The maximum departure of both values occurs at the end of simulation and constitutes approximately 1\% of the vapor composition. Fig. 2.2.3 is similar to Fig. 2.2.2 but is generated for the liquid phase. The compared compositions agree within some tolerance, as is the case with the 3-component mixtures.

The phase compressibility factors obtained from the vapor and liquid phase compositions from both simulation and flash are shown in Fig. 2.2.4. The very close agreement between the compressibility values computed in simulation and from flash confirms the accuracy of the properties obtained from constant composition expansion.
Overall In-Situ Composition of C₃, C₄, C₇, C₁₀

$T = 620^\circ R$

$P_{\text{sat}} = 3655.76$ psia

Pressure, psia

Overall In-Situ Compositional Changes During Depletion for Gas Condensate Fluid 5

Fig. 2.2.1
Vapor Composition of $C_3$, $C_4$, $C_7$, and $C_{10}$

Fig. 2.2.2 Comparison of Simulation and Flash Vapor Phase Compositions for Gas Condensate Fluid 5

1 Vapor Compositions of $C^1$
Liquid Composition of C₃, C₄, and C₇

$T = 620^\circ R$
$P_{sat} = 3655.76$ psia

Fig. 2.2.3 Comparison of Simulation and Flash Liquid Phase Compositions for Gas Condensate Fluid 5
Vapor Compressibility Factors

$P_{\text{sat}} = 365.76 \text{ psia}$
$T = 620^\circ \text{R}$

Fig. 2.2.4 Comparison of Simulation and Flash Compressibility Factors for Gas Condensate Fluid 5
2.3 Concluding Remarks

We have seen from the results shown for 3-component and 5-component mixtures the following

- Vapor and liquid compositions can be obtained with good accuracy from a constant composition expansion process even though the overall in-situ composition in the reservoir changes dramatically during depletion.

- Since we are able to predict the vapor and liquid compositions accurately during depletion, this means that the tie-line relationship holds.

- When the tie-line relationship holds, the two end-points of the tie line are fixed at each pressure. This means that the change in the overall composition could be attributed solely to the overall in-situ composition movement along the tie-line far from the initial overall composition.

- The distance the overall in-situ composition moves away from the initial overall composition is directly related to the liquid mole fractions, and therefore it should be sufficient for us to predict the correct liquid mole fraction in order to determine the overall in-situ composition in the reservoir. This mainly forms the basis for our study in Chapter III.

- Since the phase compositions and properties from simulation and flash are virtually the same, obtaining a value for the liquid mole fractions is equivalent to obtaining the oil saturation since both values are related to the molar density.
CHAPTER III

AVERAGE SATURATION ESTIMATION

Our goal is to be able to predict the oil saturation at the average reservoir level as a function of average reservoir pressure for the different fluid types in the reservoir during the entire depletion history of the reservoir. We investigated the applicability of different laboratory procedures such as Constant Composition Expansion (CCE) and Constant Volume Depletion (CVD) to model the oil saturation relation with pressure.

We found that the reservoir depletion mechanism depends on the fluid type in the reservoir and the stage of depletion. For example, a gas condensate reservoir flows at or above saturation pressure in single-phase. When the pressure drops below saturation pressure, the oil phase starts to develop. Because of the high critical oil saturation in gas condensate reservoirs, the oil phase becomes trapped in the reservoir and only the gas phase is allowed to flow out of the reservoir. This behavior is better suited for a CVD process in which only the gas phase is allowed to be produced.

An oil reservoir, on the other hand, exhibits a different behavior for different stages of depletion. When the pressure drops below the saturation pressure in an oil reservoir, the gas phase starts to come out of solution. Depending on the critical gas saturation value, the gas saturation starts to build-up and no gas is allowed to move out of the reservoir until the critical gas saturation is reached. This results in a change to the overall in-situ composition and makes it lighter, but considering that the critical gas saturation is usually low in an oil reservoir and that the overall compositional changes from gas accumulation are minor, the overall in situ composition may be considered constant and could be represented by a CCE process. When the gas saturation reaches the critical gas saturation it starts to move out of the reservoir, causing significant compositional changes that should be accounted for; therefore, the CVD is better suited for this part of depletion. When the pressure becomes very low in the reservoir and gas phase flow becomes dominant because of its high mobility, the differential vaporization process may best suit the situation in which the evolving gas phase will immediately leave the reservoir, leaving mainly the liquid phase.
behind. We should keep in mind that the saturation behavior may be different in areas close to the wellbore for either type of fluid, but our main concern is with the average reservoir which constitutes the main reservoir performance.

Although the laboratory tests are attractive to use mainly because of their simplicity to perform and the fact that they require no knowledge of either the relative permeabilities or the critical saturations, we found from the previous discussion that none of these methods was able to predict the entire reservoir behavior for oil reservoirs. The path we followed to accurately predict the average reservoir parameters is to describe the reservoir behavior initially by following the laboratory procedures outlined above (CCE and CVD) in a laboratory cell, and on the basis of the applicability of the results we modified the procedure to account for all the controlling factors that influence the pressure-saturation relation in the reservoir.

Initially, the oil reservoirs follow a path close to the CCE in which the gas phase becomes trapped in the reservoir below its critical saturation. This is contrary to the CVD procedure. Next the gas phase starts to move out of the reservoir simultaneously with the liquid phase, but in different proportions which are determined by the mobility of each phase. We expect this part of the depletion process to deviate from the initial part that could be modeled by the CCE process since the overall composition changes during the gas phase withdrawal. The CVD process is partly applicable here since the gas phase production is accounted for, but it cannot account for the liquid production. To investigate the magnitude of compositional changes that occur as the liquid phase flows out of the reservoir, we have developed a process in which we model the initial part of the depletion process (the entrapment of the gas phase up to its critical saturation) by following a CCE procedure followed by a CVD process to allow the gas phase to move out of the reservoir. This process is a simple cloning of the reservoir process in which we have made two simplifications:

- The effect of oil phase production on the overall composition is not considered.
- Only the gas phase is allowed to be produced from the reservoir.
These two simplifications were designed to eliminate the need for additional information to perform the procedure and to keep the procedure as simple as possible. The results obtained did improve our estimate in the early part of depletion, but after the gas phase became mobile the method failed.

This result made it necessary to account for the flow of the oil phase and its impact on the overall composition in the cell. To allow the liquid phase to flow, we needed a criterion by which we could selectively determine the fraction to be produced from each phase. In the reservoir the flow of each phase is controlled by its mobility. To use a similar criterion and available data we selected fractional flow curves based on the volumetric rates of each phase at the sandface as the criterion to control the gas and oil flow out of the cell. The fractional flow curves have the advantage that each phase mobility has already controlled its flow and therefore should be representative of the reservoir mobility. Although the oil saturation may be different at the sandface from that at the average radius because of a large compositional changes that occur close to the wellbore compared to those at the average radius, we have used these values as an approximation to the process, this should yield better results than completely ignoring the oil phase flow and its impact on composition.

The results we obtained from applying the fractional flow curves constituted an improvement over the previous process in which we ignored oil flow, but the results were not accurate or consistent for the different fluids. For example, the volatile oil fluid performance followed the same trend as that of the reservoir, but there was a difference between the two values. This difference is smaller than using the standard CVD process, however, and does not require the knowledge of the critical gas saturation. The results of the black oil fluid were not as encouraging as those of the volatile oil fluid since it appears that the compositional changes close to the wellbore for the heavier fluid are greater than those of the volatile oil, causing the applied mobilities to differ from those in the average radius at the same pressure.

Application of the fractional flow curves to control the gas-and-oil phase flow from the reservoir did not yield an accurate pressure-saturation profile as we had anticipated.
Consistent with our goal of obtaining an accurate description of the reservoir behavior for the entire depletion history of the reservoir, we found it necessary to develop a compositional simulator that is composed of one cell to form a link between the laboratory procedures and the full compositional simulation and to explicitly determine if a laboratory procedure that is done in a laboratory cell is adequate to describe the reservoir behavior. The “single-cell” or the “zero-dimensional” compositional simulator will have access to the relative permeability data used in the full compositional simulator and will use fluid properties (viscosities) that are based on the composition at each pressure. The success of such an effort will indicate for us the possibility of using material balance equations to express the pressure-saturation relations in the reservoir regardless of the magnitude of compositional changes that occur, and will extend the tank model concept to the compositional reservoirs.

### 3.1 Depletion Mechanisms

In the reservoir, fluid flow is governed by the relative permeability of each phase and its mobility. The occurrence of multi-phase is dictated by the initial fluid composition and the reservoir temperature and pressure. The preferential movement of a certain phase out of the reservoir based on its mobility causes the overall in-situ composition in the reservoir to change. This change causes the variations in fluid phase compositions and hence the phase properties as well as the oil saturation will change.

The general equation to compute the overall in-situ molar composition during depletion in a laboratory cell is

\[
\bar{z}_i = \frac{\bar{L} - l}{(\bar{L} + \bar{V}) - (l + v)} x_i + \frac{\bar{V} - v}{(\bar{L} + \bar{V}) - (l + v)} y_i
\]  

Where we start with a cell of arbitrary volume containing \( \bar{F} \) number of moles of initial composition \( \bar{z}_i \),

- \( \bar{L} \) and \( \bar{V} \) are the number of liquid and vapor moles respectively obtained by
flashing the overall composition in the cell to the current pressure and are constrained by $\bar{L} + \bar{V} = \bar{F}$.

- $l$ and $v$ are the produced number of moles from the liquid and vapor phases respectively. The number of moles produced may be governed by a functional form that includes the relative permeability and the fluid phase properties as necessary for the composition in the cell to match that of the reservoir.

- $x_i$ and $y_i$ are the phase compositions obtained by flashing the overall composition $z_i$ to the current pressure.

The important parameters to consider here are $l$ and $v$ since they are the only variables we can change during depletion to match what happens in the reservoir. We will keep this equation in mind when trying to model the reservoir performance with simple functions to calculate $l$ and $v$.

### 3.1.1 Constant Composition Expansion (CCE)

The overall composition in the reservoir is considered to be constant and no fluid is produced from either phase. This is equivalent to assigning a value of zero to both $l$ and $v$ in Eq. (3.1.1) which becomes

$$z_i = \frac{\bar{L}}{\bar{L} + \bar{V}} x_i + \frac{\bar{V}}{\bar{L} + \bar{V}} y_i$$

(3.1.2)

or in terms of mole fractions,

$$z_i = Lx_i + Vy_i$$

(3.1.3)

The constant composition expansion or flash vaporization is performed as follows:

1. Start with the initial overall composition at saturation pressure for a given reservoir temperature.

2. Reduce the pressure in decrements that correspond to the simulation pressure drop per time step at the average radius of the reservoir (obtained from simulation output file).
3. Flash the initial overall composition using the equation of state to the pressure established in Step 2 to obtain the vapor and liquid compositions for each phase and the corresponding liquid mole fractions.

4. Compute the oil saturation from the definition

\[ S_o = \frac{Lz_o}{Lz_o + Vz_g} \]  \hspace{1cm} (3.1.4)

where \( z_o \) and \( z_g \) are the oil and gas compressibility factors.

5. At the subsequent pressures, we flash again the initial overall composition to the desired pressure to obtain the phase compositions \((x_i, y_i)\) and liquid mole fractions \((L)\).

### 3.1.2 Constant Volume Depletion

In the CVD process we produce only the vapor phase; therefore, the value of \( l \) in Eq. (3.1.1) is set to zero, and the resulting equation for calculating the overall composition is

\[ z_i = \frac{\bar{L}}{(\bar{L} + \bar{V}) - \nu} x_i + \frac{\bar{V} - \nu}{(\bar{L} + \bar{V}) - \nu} y_i \]  \hspace{1cm} (3.1.5)

We should note here that the only restriction on the production of the vapor phase is to remove the volume of the fluid (gas and liquid) that exceeds the volume of the fluid at saturation pressure. This is in contrast to the reservoir, where the vapor phase has to remain in the reservoir until the critical saturation is reached, and then the vapor phase is produced depending on the vapor phase mobility that stems the gas phase relative permeability and gas viscosity.

The constant volume depletion process is performed here as follows:

1. Start the process at the initial reservoir pressure \((p_i)\) and the initial overall in-situ composition of the reservoir fluid \((z_i)\).

2. Reduce the pressure until the saturation pressure of the fluid is reached. The molar liquid volume \((V_m, ft^3/lb\cdotmole)\) and the cell volume \((V_{cell}, ft^3)\) should be recorded at
this point. We mark the volume at this pressure as the reference volume that we need to keep constant. The \textit{lb-moles} available at this pressure ($M_{cell}$) are calculated from this volume:

\[ M_{cell} = \frac{V_{cell}}{V_m}, \text{ lb-moles} \]  \hspace{1cm} (3.1.6)

3. At each pressure decrement from there on, do a constant composition expansion (CCE) process on the overall composition in the cell to obtain the liquid and vapor phase compositions ($x_p, y_i$) and the liquid mole fraction ($L$).

4. Compute the vapor and liquid molar phase densities at the current pressure from the engineering equation of state. These densities are part of the results obtained from flash and are based on the computed compressibility factors:

\[ \rho_o = \frac{p}{z_o RT}, \text{ lb-moles/ft}^3, \hspace{2cm} (3.1.7) \]

\[ \rho_g = \frac{p}{z_g RT}, \text{ lb-moles/ft}^3 \hspace{1cm} (3.1.8) \]

5. Calculate the \textit{lb-moles} available in each phase from the phase partition determined by the phase equilibria calculation (liquid and vapor mole fractions):

\[ M_V = M_{cell}, \text{ lb-moles} \hspace{2cm} (3.1.9) \]

\[ M_L = L M_{cell}, \text{ lb-moles} \hspace{1cm} (3.1.10) \]

6. Use the densities and mass computed for each phase to determine the volume occupied by the two phases, and hence the total volume of the cell at this pressure:

Vapor Volume \hspace{1cm} \[ V_V = M_V / \rho_g, \text{ ft}^3 \]  \hspace{1cm} (3.1.11)

Liquid Volume \hspace{1cm} \[ V_L = M_L / \rho_o, \text{ ft}^3 \]  \hspace{1cm} (3.1.12)

Total Volume \hspace{1cm} \[ V_{total} = V_V + V_L, \text{ ft}^3 \]  \hspace{1cm} (3.1.13)

7. Determine the excess volume by comparing the current cell volume with the volume marked at saturation pressure. The excess volume based on the pressure reduction needs to be produced:

\[ V_{excess} = V_{total} - V_{cell}, \text{ ft}^3 \]  \hspace{1cm} (3.1.14)
8. Calculate the equivalent vapor mass corresponding to the excess volume. This calculation is based on the vapor density and the excess volume and corresponds to the excess cell volume that needs to be produced:

\[ M_{\text{vap.prod.}} = \rho_v \times V_{\text{excess}}, \text{ lb-moles} \]  

(3.1.15)

9. Remove the calculated gas mass and recalculate the liquid and vapor mole fractions on the basis of the remaining mass of each phase:

\[ M_V = M_V - M_{\text{vap.prod.}} \]  

(3.1.16)

\[ M_{\text{cell}} = M_V + M_L \]  

(3.1.17)

\[ V = M_V / M_{\text{cell}} \]  

(3.1.18)

\[ L = 1 - V \]  

(3.1.19)

10. Use Eq. (3.1.3) to combine the phase compositions obtained at step 3 and the newly calculated liquid mole fractions in step 9 into the overall composition of the cell \( z_i \).

This composition is then carried to the next pressure stage.

11. Report the liquid and vapor phase compositions \((x_i, y_i)\) and the newly calculated liquid mole fraction \(L\) after producing the excess volume for the current pressure.

12. Proceed to the second pressure decrement and repeat the same procedure.

We will show the results obtained from using the CCE and CVD procedure discussed above and compare it with full compositional simulation results for the different fluids involved in this study.

Table 3.1.1 lists the input data for the simulation conducted here for the different fluid types used. The case number is used to identify each case shown. The compositions for the different fluids used in this study are given in Table 1.11.1 and Table 3.1.1.

Fig. 3.1.1 shows the liquid mole fractions from the full compositional simulator (hollow circles), from flash, and from CVD vs. average reservoir pressure. These results were obtained for Fluid 1 (gas condensate). We notice that all the three methods yield similar results until the pressure becomes 3,650 psia where the flash-computed liquid mole fractions start to deviate. The reason for this departure is that the overall composition
in the flash process remains constant because there is no removal of gas from the system, while for the compositional reservoir simulator and CVD the overall composition becomes heavier because of released gas. This can be seen more clearly in Fig. 3.1.2 where the oil saturation builds up more in the simulation and the CVD case than for the flash. The oil saturation values obtained from the flash start to depart rapidly after the pressure reaches 3,100 psia, which is the pressure at which the generated liquid starts to re-evaporate. The CVD process follows the reservoir behavior throughout the simulation. This is attributed to the liquid entrapment in the cell caused by producing only the gas phase and the continuous change in the overall composition. In the simulation, the oil generated is trapped in the reservoir rock because of the critical oil saturation that is usually high for gas condensates. We note here that the critical oil saturation is 38% and the maximum liquid developed in the simulation (at the average radius) is 11.6%, which is much lower than the critical oil saturation. This confirms that no liquid will be produced from the average radius throughout the simulation, and that the only possibility for it to be produced is for the pressure to drop much lower and reach a value at which the liquid phase starts to re-evaporate. We notice from Fig. 3.1.2 for the reservoir simulation case and the CVD case that we cannot recover most of the liquid phase once it is generated in the reservoir, even when the pressure becomes very low.

The results observed for Fluid 2 (gas condensate) which has a higher liquid drop-out (maximum liquid drop-out from flash is 12.4%) follows the same results and reasoning for gas condensate Fluid 1 (maximum liquid drop-out from flash is 10.5%). Fig. 3.1.3 shows that although the fluid has a higher richness in the liquid drop-out, the amount of fluid developed from flash cannot match that of simulation and constant volume depletion. The results from constant volume depletion follow the same results of simulation basically for the same reason discussed earlier. The oil saturation vs. average reservoir pressure for Fluid 2 is shown in Fig. 3.1.4. This figure shows that the maximum oil saturation obtained from simulation or from CVD is much lower than the critical oil saturation of 38%. This high critical oil saturation ensures that the oil will be trapped in the reservoir, and therefore, the reservoir behavior can be approximated by a CVD process.
TABLE 3.1.1 Simulation Cases Considered in the Study of Average Pressure and Average Saturation Relation

\[ r_e = 5000 \text{ ft, Porosity} \]
\[ r_w = 0.25 \text{ ft, Thickness} \]

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Fluid Type</th>
<th>Permeability (md)</th>
<th>Initial Pressure psia</th>
<th>Production Rate Lb-Mole/Day</th>
<th>Wellbore Pressure Held Constant at</th>
<th>Mass Recovery at Mode Switch, %</th>
<th>Average Pressure at Mode Switch</th>
<th>Average Pressure at End of Run psia</th>
<th>Percentage Mass Recovery</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Condensate Fluid 1</td>
<td>50</td>
<td>3979</td>
<td>5000</td>
<td>46</td>
<td>89.5</td>
<td>302</td>
<td>51</td>
<td>95.5</td>
</tr>
<tr>
<td>2</td>
<td>Condensate Fluid 2</td>
<td>5</td>
<td>4300</td>
<td>5000</td>
<td>55</td>
<td>70.0</td>
<td>1110</td>
<td>88</td>
<td>94.4</td>
</tr>
<tr>
<td>3</td>
<td>Volatile Oil</td>
<td>10</td>
<td>3175</td>
<td>1000</td>
<td>49</td>
<td>61.3</td>
<td>508</td>
<td>170</td>
<td>70.0</td>
</tr>
<tr>
<td>4</td>
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<td>5</td>
<td>2386</td>
<td>2000</td>
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<td>2347</td>
<td>70</td>
<td>61.5</td>
</tr>
<tr>
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<td>3700</td>
<td>5000</td>
<td>44</td>
<td>73.1</td>
<td>958</td>
<td>531</td>
<td>84.4</td>
</tr>
<tr>
<td>6</td>
<td>Volatile Oil</td>
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<td>3175</td>
<td>5000</td>
<td>49</td>
<td>61.3</td>
<td>510</td>
<td>164</td>
<td>70.2</td>
</tr>
<tr>
<td>7</td>
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<td>5000</td>
<td>42</td>
<td>41.4</td>
<td>780</td>
<td>338</td>
<td>53.5</td>
</tr>
<tr>
<td>8</td>
<td>Volatile Oil</td>
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<td>3175</td>
<td>10000</td>
<td>43</td>
<td>54.7</td>
<td>775</td>
<td>475</td>
<td>62.1</td>
</tr>
<tr>
<td>9</td>
<td>Volatile Oil</td>
<td>50</td>
<td>3175</td>
<td>-----</td>
<td>100</td>
<td>0.0</td>
<td>3175</td>
<td>103</td>
<td>71.8</td>
</tr>
<tr>
<td>10</td>
<td>Volatile Oil</td>
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<td>3175</td>
<td>200</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>2685</td>
<td>9.9</td>
</tr>
<tr>
<td>11</td>
<td>Black Oil</td>
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<td>2386</td>
<td>500</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>1150</td>
<td>31.6</td>
</tr>
<tr>
<td>12</td>
<td>Black Oil</td>
<td>5</td>
<td>2386</td>
<td>10000</td>
<td>35</td>
<td>0.0</td>
<td>2386</td>
<td>754</td>
<td>13.9</td>
</tr>
<tr>
<td>13</td>
<td>Black Oil</td>
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<td>-----</td>
<td>100</td>
<td>0.0</td>
<td>2386</td>
<td>211</td>
<td>57.1</td>
</tr>
<tr>
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<td>3175</td>
<td>5000</td>
<td>41</td>
<td>2.7</td>
<td>3173</td>
<td>573</td>
<td>59.6</td>
</tr>
<tr>
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<td>3490</td>
<td>5000</td>
<td>24</td>
<td>0.59</td>
<td>3336</td>
<td>1271</td>
<td>43.2</td>
</tr>
<tr>
<td>16</td>
<td>Black Oil</td>
<td>10</td>
<td>2330</td>
<td>1000</td>
<td>42</td>
<td>41.6</td>
<td>777</td>
<td>350</td>
<td>53.1</td>
</tr>
</tbody>
</table>
Fig. 3.1.1 Comparison of Liquid Mole Fractions Computed From Simulation, Flash, and Constant Volume Depletion for Gas Condensate Fluid 1 (Case 1)

$T=620^\circ R$

$p_{\text{sat}}=3976$ psia
It is important to note that the constant volume depletion process did model the simulation value only because the liquid accumulated in the reservoir did not move out (at average radius) because the critical saturation of oil is high (38%). It is worth mentioning that the oil critical saturation in gas condensate reservoirs is usually high, and we believe that even with richer gas condensates the amount of liquid build-up in the reservoir (average radius) will not exceed the critical oil saturation. Because of this analysis, we conclude that the constant volume depletion process will model the average reservoir saturation as long as the oil volume fractions obtained from constant volume depletion experiment do not exceed the critical oil saturation.

The results obtained for the volatile oil case (Fluid 3) are shown in Fig. 3.1.5. We find that the liquid mole fractions obtained from flash tend to match the simulation results early in the depletion but at lower pressures deviate considerably, while those from constant volume depletion depart from the break out of two-phase flow to the end of simulation but follow the trend of simulation data. The constant composition expansion (flash) process matched simulation results early in the simulation, while a pressure value was not too far from the saturation pressure. By examining the oil saturation values shown in Fig. 3.1.6, we notice that the simulation and flash data continue to match until we reach an oil saturation of 80%; then they start to depart (at $p/p_{sat}=0.85$). The reason for this early match is that all the gas generated (evolved from the liquid phase) during the pressure drop was trapped in the reservoir while its saturation is below the critical gas saturation (0.07). Although the gas phase is trapped in the reservoir causing the overall composition in the reservoir to change (become somewhat lighter), the magnitude of change is small because the small critical gas saturation and therefore will not constitute a visible change in the oil saturation. When the gas saturation increases beyond 20%, the gas relative permeability starts to increase sharply (as seen in the relative permeability curves in Fig. 1.12.2), thus allowing the gas phase to move out of the reservoir (average radius) and at the same time reducing the oil relative permeability significantly. The sharp increase in gas mobility and the decrease in the oil mobility cause more of the liquid phase to be trapped in the reservoir, thus reducing the amount of liquid produced and limiting the reduction in the oil saturation in the reservoir (Fig. 3.1.6). In the flash process, since the
Fig. 3.1.3 Comparison of Liquid Mole Fractions Computed From Simulation, Flash, and Constant Volume Depletion for Gas Condensate Fluid 2 (Case 2)
Fig. 3.1.4 Comparison of Oil Saturation Computed From Simulation, Flash, and Constant Volume Depletion for Gas Condensate Fluid 2 (Case 2)
Fig. 3.1.5 Comparison of Liquid Mole Fractions Obtained From Simulation, Flash, and Constant Volume Depletion for Volatile Oil Fluid 3 (Case 3)
composition remains constant throughout the process, the liquid formed is lighter than that in the reservoir and the amount of liquid formed also drops more as the pressure is reduced.

The constant volume depletion case disagrees completely with the simulation data from the start. This is mainly caused by the process itself which produces the gas phase as soon as it is developed. At the same time, the simulated overall composition is affected by the gas saturation that fills 7% of the reservoir rock by gas (critical gas saturation) before it starts to move. As the process goes on and the simulation data starts to be affected by the gas phase movement out of the reservoir, the two saturation values from constant volume depletion and from simulation become dephased. This behavior was used as a basis for the hybrid procedure we proposed to model the average oil saturation for volatile oils in the reservoir.

The results for the black oil fluid are similar to those of the volatile oil case. Fig. 3.1.7 shows the liquid mole fraction obtained from simulation, flash, and constant volume depletion. The results from flash follow the simulation initially but start to deviate with the compositional changes discussed earlier. The constant volume depletion results also depart early but retain the same pattern as those of simulation because of their similarity in producing the gas phase. One last observation for both the volatile oil case and the black oil case is that the two liquid mole fractions obtained from simulation and constant volume depletion agree at the end of reservoir depletion and that most of the mass remaining in the reservoir is in the liquid phase. This is true since the mass of the gas phase is negligibly small at the very low pressure at end of simulation.

As was the case with the phase compositions and phase fluid properties discussed in Chapter II, the 5-component simulation, flash, and constant volume depletion results did not differ from those of the 3-component gas condensates. The results of the comparison are shown in Fig. 3.1.8. The simulation and constant volume depletion values match completely to the end of simulation. We note also here that the magnitude of oil saturation resulting from simulation (at average radius) is well below the critical oil saturation, and therefore no oil will be produced from the reservoir (at average radius).
Fig. 3.1.7  Comparison of Liquid Mole Fractions Obtained From Simulation, Flash, and Constant Volume Depletion for Black Oil Fluid 4 (Case 4)
Fig. 3.1.8  Comparison of Liquid Mole Fractions Obtained From Simulation, Flash, and Constant Volume Depletion for Gas Condensate Fluid 5 (5-Components, Case 5)
3.1.3 Hybrid Process

The hybrid process was prompted by the agreement in the shape of the liquid mole fractions obtained from simulation and constant volume depletion for the volatile oil and black oil cases discussed in Fig. 3.1.5 and Fig. 3.1.7. The reason for the disagreement between the two procedures was that the gas phase was produced in the constant volume depletion case as soon as it evolved, while in simulation the gas phase was retained until the critical gas saturation was reached and then it was produced based on the basis of the relative permeability curves. On the other hand, the agreement between the early flash data and simulation encouraged us to try this hybrid procedure that is a combination of the flash and constant volume expansion procedure. If this procedure were successful in modeling the average reservoir saturation, the only additional information needed would be a value of the critical gas saturation.

The approach we take here is to start the depletion process as a constant composition expansion (flash) process until the gas saturation builds up to the level of the critical gas saturation dictated by the relative permeability data (in this case \( S_{eg} = 0.07 \)). When that critical gas saturation is reached, the process is switched to constant volume depletion. The constant volume to be used for this process is the volume at the pressure at which the critical saturation is reached. From here on we proceed with the procedure described for the constant volume depletion discussed in Section 3.1.2.

The results we obtained for the volatile oil (Fluid 3) are shown in Fig. 3.1.9. The input data to the simulator are given in Table 3.1.1. The solid line in Fig. 3.1.9 represent the liquid mole fractions obtained from simulation, the solid circles represents the flash response, and the normal void triangles represent the original constant volume depletion procedure response. All the other curves are derived from the application of the hybrid process with different critical gas saturations.

The curve represented by solid squares shows the hybrid response when the critical gas saturation is considered 10%. This means that we followed the constant composition expansion procedure until the fraction of gas volume reached 10% and then we switched the process to constant volume depletion. We can see here that there is an improvement
Fig. 3.1.9  Hybrid Process Applied to Volatile Oil Fluid 3 (Case 6)
Fig. 3.1.10 Hybrid Process Applied to Black Oil Fluid 4 (Case 4)
over the standard constant volume depletion procedure since we match the simulation response for a longer pressure range. By repeating the process with a larger critical gas saturation of 23.4% and then to 30%, we seem to get closer to the simulation response and still preserve the same shape of the simulation curve.

Fig. 3.1.10 shows similar results to those shown in Fig. 3.1.9 for the black oil mixture (Fluid 4) for a shorter pressure range. The results for zero critical gas saturation (standard constant volume depletion) and 10% show the liquid accumulation in the cell is more than that in simulation but follows approximately the shape of simulation. As we increase the critical gas saturation beyond the saturation at which the gas starts to effectively move out of the reservoir as dictated by the relative permeability (20% as shown in Fig. 1.12.2), the liquid fractions decrease beyond those in simulation as seen in the case for the critical gas saturations of 23.4% and above in Fig. 3.1.9. As soon as the gas is allowed to move out (without relative permeability restriction), the amount of gas that moves out exceeds that in simulation, causing the gas mole fractions to decrease and the liquid mole fractions to increase past the simulation values. This fast drop in the gas mole fractions caused the liquid mole fractions to cross the simulation values and therefore depart from the value we are trying to model.

The problem we face with this approach is that it appears that the critical gas saturation is not the only factor influencing the liquid mole fractions in the reservoir but rather the combined effect of the critical gas saturation and the shape of the relative permeability data. This can be explained in a similar fashion to our reasoning of the departure between flash data and simulation data for the volatile oil case in that the actual departure between the simulation value and the constant volume depletion should be attributed to both the critical gas saturation and the magnitude of gas relative permeability from the relative permeability curves (ignoring the effects of gas viscosity).

With the results obtained so far, we can conclude that the critical gas saturation is not the only parameter that needs to be introduced to the CVD process to match the reservoir behavior. If the introduction of the critical gas saturation is sufficient to model reservoir performance then the results for the case of 10% critical gas saturation should match
closely those of simulation in Fig. 3.1.10 since the actual critical gas saturation in the reservoir is 7%.

This conclusion is rather disappointing since we were hoping to be able to model the reservoir performance with minimal data requirements and by using fluid properties alone. What we have found is that neither the constant composition expansion process nor the constant volume depletion process can model the full behavior of this fluid system, nor can a hybrid process between the two procedures model it. The alternative would require more data to help us predict the correct average oil saturation.

3.2 Constant Volume Depletion Using Fractional Flow Curves

By now we have established that we need to define some criteria by which we can produce both the liquid and vapor phases from the lab cell to obtain compositional changes that are similar to those taking place in the reservoir.

To avoid the need to use the relative permeability data for controlling the fluid production from each phase, we have tried here to use the production rate history data to develop fractional flow curves based on the vapor and liquid rates as a function of pressure. The motivation for using the fractional flow curves comes from the fact that the rate responses that we get at the sandface have phase mobilities already influencing them, and therefore might give us better production criteria than the ones we used so far. The question we need to answer before we use the fractional flow curves as the production criteria is whether the pressure alone is enough to correlate the changes that happen at the sandface to those at the average radius.

By using the fractional flow curves at the average radius found only in the pressure correspondence between the two locations we are in fact assuming that the pressure and saturation in the reservoir have a unique relationship regardless of position. As we will see later in Chapter IV, this is not entirely true since the compositional changes that happen at the sandface are much greater than those occurring at the average radius simply because the heavier components from the flowing stream condensate close to the wellbore where
the pressure is low. Keeping this in mind, we have computed the fractional flow curves from the phase rate data and also computed the actual fractional flow curves at the average radius from the simulator using the mobilities in the simulation at that location.

The equation we use to calculate the fractional flow curves is based on the volumetric flow rates at the surface and on the phase relative permeability and viscosity at the average radius,

\[
f_l = \frac{q_l}{q_l + q_v} = \frac{1}{1 + \frac{k_{rg}}{k_{ro}} \frac{\mu_o}{\mu_g}}
\]  

(3.2.1)

and

\[
f_v = 1 - f_l
\]  

(3.2.2)

where \((q_l \text{ and } q_v)\) are the volumetric flow rates at reservoir conditions and as such do not require the usual conversion from standard conditions to reservoir conditions.

Fig. 3.2.1 shows the computed fractional flow curves from rates (sandface) and from simulation (average radius). The two values show a difference when plotted at the same pressure, which indicates that using the pressure alone to correlate the two fractional flow values at the two locations is not sufficient because of the compositional changes that take place close to the wellbore. These compositional changes influence both the saturations and the viscosities of each phase, causing the mobility for each phase to be different at the two different locations at the same pressure.

The difference in the fractional flow curves shown in Fig. 3.2.1 could be explained as follows. initially there is all liquid flow in the reservoir until the saturation pressure is reached and the gas phase breaks out of liquid but remains in the pores. The vapor phase evolves first at the sandface where the pressure is lowest and then progresses inward to the reservoir. The composition of the liquid phase flowing from deep in the reservoir is mainly equal to the initial overall composition. As this flowing liquid composition passes over the sandface (lower pressure), more gas evolves from it, thus increasing the gas saturation at the sandface at a faster rate than that at the average radius for the same pressure. This explains the rapid gas build-up at the sandface compared to that at the
Fig. 3.2.1 Fractional Flow Curves Obtained at Sandface and at the Average Radius for Volatile Oil Fluid 3 (Case 6)
average radius (from saturation pressure to \( p=2930 \) psia in Fig. 3.2.1). This phase continues until the critical gas saturation is exceeded, after which the mechanism is reversed. Since the gas is allowed to move out of the reservoir, the composition of liquid at the sandface becomes heavier because of the low pressure and the evaporation of its lighter components. The flowing composition also contributes to these compositional changes when it passes over the low pressure zone close to the wellbore, the gas phase breaks out and is allowed to move out of the reservoir because of its mobility; the heavier liquid mixes with the liquid phase close to the sandface, making the liquid composition heavier and contributing to a faster oil saturation buildup at the sandface than that at the average radius.

The procedure for applying the fractional flow curves as criteria for preferential phase withdrawal from the constant volume depletion cell is as follows:

- Start at the initial pressure and use a simple flash process until the saturation pressure is reached.
- Mark volume at the saturation pressure is marked and is considered to be the constant volume to be preserved throughout the entire process. We calculate the total mass available in the cell for this volume (similar to CVD).
- Drop the pressure below saturation pressure and perform a constant composition expansion (CCE) process on the overall composition in the cell to obtain the liquid and vapor phase compositions and the liquid mole fractions.
- Using the obtained phase compositions, evaluate the vapor and liquid phase molar densities at the current pressure and temperature.
- Knowing the molar density of each phase and the mass in each phase, determine the volume occupied by each phase. Adding the two phase volumes will determine the total cell volume which exceeds the cell volume at saturation pressure. Determine this excess volume.
- In the standard CVD computation, calculate the equivalent gas phase mass to be produced from the cell. In this case we produce both the vapor and liquid phases.
The fractions of vapor and liquid production is controlled by the value of vapor and liquid fractional flow data for the current pressure obtained from the rates.

- Apply the obtained fractions (fractional flow) at the current pressure to the excess volume to determine the volume to be produced from each phase. We then determine the corresponding mass to be produced and remove that mass from each phase from the cell. By producing this mass, the cell volume will return to the volume at saturation pressure.

- Calculate the liquid and vapor mole fractions after production based on the remaining mass of each phase.

- Use Eq. (3.1.3) to combine the phase compositions obtained from flash at the current pressure and the newly calculated liquid mole fractions into the overall composition of the cell. This composition is then carried to the next pressure stage.

\[ z_i = Lx_i + Vy_i \]  \hspace{1cm} (3.1.3)

- Repeat the process for the consecutive pressures.

The results obtained from this procedure for the volatile oil case are shown in Fig. 3.2.2 along with simulation, flash and standard constant volume depletion results. The simulation input data are given in Table 3.1.1

As expected, this method performed better than both flash and standard constant volume depletion results and also eliminated the problems encountered with the hybrid procedure in that we do not need to specify a critical gas saturation, nor do we need to know the shape of the relative permeability. The results shown here followed the simulation values as did the flash at early pressure, and also departed from the flash data to follow the simulation data shape but lagged in its transition.

The difference between the liquid mole fractions obtained from this procedure and from simulation is caused by the difference between the fractional flow curves obtained from the rates of data at sandface and those at the average radius. At the pressure of 2,950 psia the vapor phase movement started to increase faster at the average radius than that at
Fig. 3.2.2 Results of Applying the Fractional Flow Curves as a Criterion for Phase Volatile Oil Fluid 3 (Case 6).
the sandface, as seen in Fig. 3.2.1 and explained earlier. This caused the liquid movement at the average radius to lag behind that at the sandface for the same pressure. When the liquid fractional flow curve at the sandface is applied at this pressure, it causes more liquid to be produced from the cell than is actually produced from the reservoir at the average radius. This started to show in Fig. 3.2.2 at 2720 psia when the computed values from this procedure started to deviate from those of simulation. The net result is that we have liquid mole fractions lower than the actual.

At first it may seem that the use of this procedure could enhance our ability to predict the average pressure-saturation relation using data that is already available in the field, but by investigating further we find that the gap between simulation and computed values from this procedure increases as the initial fluid composition becomes heavier. This can be seen in Fig. 3.3.2 where the difference between the two values is attributed to the difference in the fractional flow curves at the sandface and at the average radius as shown in Fig. 3.3.1.

3.3 Development of the Single-Cell Simulation Module

Exploring ways to predict the average pressure-saturation relation for the volatile oil and black oil without requiring the relative permeability data did not provide a significant improvement in the results. We found it necessary to investigate the use of the relative permeability data and its derived mobility to control the fluid withdrawal from the constant volume depletion cell during depletion.

In this procedure we assumed the availability of relative permeability data. Our objective was to use the relative permeability data as a mechanism to preferentially produce the liquid and vapor phases during the CVD process in order to history match the reservoir performance.

The outcome of the simulator will be used to investigate the following

- Test if it is possible for us to determine the average pressure-saturation relation in the reservoir from a single cell, and if the compositional changes at the average radius could inhibit this model from working.
Fig. 3.3.1  Fractional Flow Curves Obtained at Sandface and at the Average Radius for Black Oil Fluid 4 (Case 7)
Fig. 3.3.2 Results of Applying the Fractional Flow Curves as a Criterion for Phase Production as Compared With Simulation, Flash, and Standard CVD Results, Black Oil Fluid 4 (Case 7)
• If the first objective is possible, then investigate the influence of the operating schedule of the reservoir on the total recovery from the reservoir. This includes testing the reservoir performance with different production rates and constant pressure production.

• Test the success of the single-cell simulator and its accuracy in modeling the reservoir performance at the average level, and can conclude whether a tank model could be used to model the reservoir performance regardless of the composition, production rate, permeability or initial pressure level.

The parts required for this process to work are the following:

a. Field data consisting of average reservoir pressure and total mass produced.

b. Relative permeability data (as a function of saturation) obtained from laboratory experiment.

c. CVD lab experiment (simulated using the equation of state)

The data needed for the CVD experiment are as follows:

• \( z_i \) : Initial overall composition of the in-place reservoir fluid.

• \( p_i \) : Initial pressure of the reservoir.

• \( T \) : Reservoir Temperature.

• \( k_{rm} \) : Relative permeability of the vapor and liquid phases \((m= g, o)\).

**Single-Cell Simulator Algorithm**

1. Start at the initial pressure and use a simple flash process at each pressure until the saturation pressure is reached.

2. Mark a unit volume at the saturation pressure and consider it the constant volume to be preserved throughout the entire process \((V_{sat}, ft^3)\). Calculate the total mass available in the cell for this volume and from the molar volume \((V_m, \text{lb-mole/ft}^3)\) computed from the equation of state:
\[ M_{\text{cell}} = V_{\text{sat}}/V_m, \text{ lb-mole} \] (3.3.1)

3. Drop the pressure below saturation pressure and perform a flash process on the overall composition in the cell to obtain the liquid and vapor phase compositions and the liquid mole fractions.

4. Using the obtained phase compositions, evaluate the vapor and liquid phase molar densities at the current pressure (lb-mole/ft³).

5. Using the liquid and vapor mole fractions \( (L \text{ and } V) \), the total mass in the cell, and the phase densities, determine the mass and volume occupied by each phase:

\begin{align*}
\text{Vapor Mass } M_V &= V M_{\text{cell}}, \text{ lb-moles} \tag{3.3.2} \\
\text{Liquid Mass } M_L &= L M_{\text{cell}}, \text{ lb-moles} \tag{3.3.3} \\

\text{Vapor Volume } V_V &= M_V/\rho_g, \text{ ft}^3, \text{ and} \\
\text{Liquid Volume } V_L &= M_L/\rho_o, \text{ ft}^3 \tag{3.3.4} \\
\end{align*}

6. Calculate the combined volume of the vapor and liquid phases and compare it with the control volume at saturation pressure. The excess volume \( V_{\text{excess}} \) should be produced in order to maintain a constant volume process:

\[ V_{\text{excess}} = V_{\text{sat}} - (V_V + V_L), \text{ ft}^3 \tag{3.3.6} \]

7. To define the criteria for producing vapor and liquid phases we determine the liquid volume fractions (saturation) from the computed volumes in 5 and use them to enter the relative permeability table and determine each phase relative permeability at this saturation:

\[ S_g = \frac{V_V}{V_V + V_L}, \text{ and } S_o = \frac{V_L}{V_V + V_L} \tag{3.3.7} \]

8. Use the phase-molar densities \( (\rho_o, \rho_g) \) to compute the phase viscosities \( (\mu_o, \mu_g) \) using the Lohrenz, Bray, and Clark¹⁷ correlation as discussed in Section 1.11.

9. Calculate the selective production criteria (fractional flow curves) for each phase at the current pressure:
For the vapor phase = \( f_v = \frac{1}{1 + \frac{k_{ro}}{k_{rg}} \frac{\mu_g}{\mu_o}} \) \hspace{1cm} (3.3.8)

For the liquid phase = \( f_l = \frac{1}{1 + \frac{k_{rg}}{k_{ro}} \frac{\mu_o}{\mu_g}} = 1 - f_v \) \hspace{1cm} (3.3.9)

The drainage relative permeability (Set 2) is used for the volatile and black oils.

10. Apply the obtained fractional flow fractions (\( f_v, f_l \)) at the current pressure to the excess volume \( V_{\text{excess}} \) to determine the volumes to be produced from each phase:

The volume to be produced from the vapor phase is

\[ V_{\text{vap. prod.}} = V_{\text{excess}} \times f_v \times ft^3 \] \hspace{1cm} (3.3.10)

The volume to be produced from the liquid phase is

\[ V_{\text{liq. prod.}} = V_{\text{excess}} \times f_l \times ft^3 \] \hspace{1cm} (3.3.11)

11. Compute the mass equivalent to the excess volume (in lb-moles) to be produced from each phase using the volumes computed in 10 and the molar densities computed in 4. This is the excess mass to be produced.

\[ M_{\text{vap. prod.}} = V_{\text{vap. prod.}} \times \rho_g \times \text{lb-moles} \] \hspace{1cm} (3.3.12)

\[ M_{\text{liq. prod.}} = V_{\text{liq. prod.}} \times \rho_o \times \text{lb-moles} \] \hspace{1cm} (3.3.13)

12. Remove the excess mass from each phase and calculate the mass remaining in each phase after production. When the excess mass is removed the cell will return to its marked volume at saturation pressure.

Mass remaining in each phase:

\[ M_V = V M_{\text{cell}} - M_{\text{vap. prod.}} \], and

\[ M_L = L M_{\text{cell}} - M_{\text{liq. prod.}} \] \hspace{1cm} (3.3.14)

\( M_{\text{cell}} \) is the mass in the cell at any pressure.
13. Calculate mass left in the cell after fluid withdrawal:

\[ M_{cell} = M_L + M_V \] 

(3.3.15)

14. Calculate the new vapor and liquid volume fractions based on the volumes of each phase in the cell:

\[ V_V = V_{vap} - V_{vap,prod} \]
\[ V_L = V_{liq} - V_{liq,prod} \] 

(3.3.16)

15. Calculate each phase saturation,

\[ S_g = \frac{V_V}{V_V + V_L} , \quad S_o = \frac{V_L}{V_V + V_L} \] 

(3.3.7)

16. Since the fluid is withdrawn from each phase at different proportions, the saturation will change during the fluid withdrawal, so the relative permeability and each phase mobility should be adjusted accordingly. The phase viscosities are considered constant at each stage because we are withdrawing fluid at a fixed pressure and because each phase composition remains constant. Therefore the only parameter we need to adjust is the saturation and its derived relative permeability. The way we handle the changing saturation problem during fluid withdrawal is as follows (this is an iterative procedure):

a. Record the total mass available at the current pressure, \( M_{cell} \).

b. Assume that the first saturation computed (based on flash of the cell overall composition in Eq. 3.3.7) is correct and determine its corresponding relative permeability and fractional flow values from Eq. 3.3.8 and 3.3.9.

c. Withdraw the fluid from each phase according to the fractional flow value.

d. After the fluid withdrawal compute the volume fractions (saturations) of the remaining phases based on their volumes Step 15.

e. Compare the resulting saturation at the end of the process to the saturation used for the production criteria. If the two saturations agree within tolerance we report this saturation for the current pressure and move to the next pressure decrement. Otherwise, the newly computed saturation is assumed to be correct.
(as an iterate value) and we return the cell to the total mass that was available
at this pressure as recorded in Step \( a \).

f. Using the new saturation (iterate value) and the phase densities, we can
determine the adjusted liquid and vapor mole fractions:

\[
L = \frac{S_o}{S_o + S_g (\rho_g / \rho_o)} \tag{3.3.17}
\]

\[
V = 1 - L \tag{3.3.18}
\]

g. Using the original mass in the cell and the computed vapor and liquid mole
fractions, determine the mass in each phase.

h. Compute the volume occupied by each phase based on the mass of each phase
and its molar density. Calculate their combined volume, and determine the
excess volume to be produced.

i. Using the new saturation, Compute a new fractional flow value and use it as a
criterion for preferentially producing the equivalent mass of the excess volume
in each phase.

j. Compute the new vapor and liquid mole fractions based on the remaining mass
in each phase.

k. Compute the resulting saturation using Eq. 3.3.7 Go to Step \( e \).

17. When a saturation value is converged, record the total mass remaining in the cell
to be used as an input for the next pressure decrement. The resulting vapor and
liquid mole fractions are computed as follows:

\[
L = \frac{M_L}{M_{cell}}, \quad V = 1 - L = \frac{M_V}{M_{cell}} \tag{3.3.19}
\]

18. Combine the vapor and liquid mole fractions and the phase compositions to update
the overall composition in the cell:

\[
z_{cell} = L x_i + V y_i \tag{3.3.20}
\]

19. Continue simulating the reservoir process by repeating the procedure from Step 3
to 18.
From this process we developed a table of pressure and saturation values that are applicable at the average radius of the reservoir and are equivalent to material balance computations.

A sample result of the single-cell simulator shown in Fig. 3.3.3 compares it with the different methods discussed before. The results from the single cell simulator (hollow squares) compare very well with the simulation values at the average radius (hollow circles). The values actually follow those from simulation throughout the entire depletion history considering that the simulation values were obtained initially at constant total molar rate and then switched to constant wellbore pressure production mode. Fig. 3.3.4 compares the single cell simulation results for the black oil mixture (Fluid 4) with simulation data. Although matching the simulation data with fractional flow curves derived from rates was unsatisfactory for the black oil case, the results here come naturally and predict the reservoir values very closely.

We found that the single cell simulator can provide us with accurate information about the reservoir behavior (depletion mechanism) without the processing time the full simulation requires. A typical run of the program to describe the whole reservoir depletion performance can be done in approximately 10 minutes on a 486-PC, which encourages the use of such method as an analysis tool.

The effect of the pressure decrement used in the single cell simulator was tested with the results shown in Fig. 3.3.5 for the volatile oil case and in Fig. 3.3.6 for the black oil case. We have tested the simulator with three pressure decrements: 0.1, 1.0, and 10 psia. The reason for selecting larger pressure drops is to increase the speed of the simulation process. The results obtained indicate that there is a slight loss of accuracy when the pressure drop is large (above 5 psia). A high pressure decrement can cause the end-value of the saturation obtained from the iterative process not to be representative of the whole pressure range but to act as an average value that causes a slight inaccuracy in the obtained fractional flow values. Hence it can deviate the process from that of the full simulation. Such results can be seen in Fig. 3.3.5 for the volatile oil case when the pressure decrement selected is 10 psia. For the black oil case (Fig. 3.3.6) the deviation from simulation at
Fig. 3.3.4 Single Cell Simulator Results Compared to Full Simulation and Flash Results, Black Oil Fluid 4 (Case 4)
Fig. 3.3.5   Effect of the Pressure Decrement on the Single-Cell Simulator Results, Volatile Oil Fluid 3 (Case 6)
10 psia pressure drop is not large because the changes in saturation with pressure in this case are not as large as those for the volatile oil. Therefore, for heavier fluids we can use larger pressure decrements without loss of accuracy.

We have tested the simulation responses for three different production rates for the volatile oil composition as shown in Fig. 3.3.7 where the rates chosen cover a wide range of conditions (200, 5000, and 10000 \( \text{lb-mole/day} \)) and a single constant wellbore pressure simulation \( (p_{wf}=100 \ \text{psia}) \). We have also tested the responses for the black oil fluid as shown in Fig. 3.3.8 with three different production rates (500, 2000, and 10000 \( \text{lb-mole/day} \)) and a single constant wellbore pressure simulation \( (p_{wf}=100 \ \text{psia}) \). All the results obtained show that there is virtually no change in the liquid mole fractions as functions of pressure at the average radius, regardless of the different compositional changes that take place there and regardless of the way we produce the reservoir. This indicates also that the average saturation in the reservoir will always be the same regardless of the production history since we can model the whole reservoir performance based on a tank model.

Since all the simulations conducted in Fig. 3.3.7 were based on the same absolute permeability of 50 \( \text{md} \), and those in Fig. 3.3.8 were based on the absolute permeability of 5 \( \text{md} \), we conducted additional simulations to investigate the effect of the absolute permeability and the pressure difference between the initial pressure and saturation pressure. The combination of a low absolute permeability and a larger difference between initial pressure and saturation pressure can increase the compositional changes that take
Fig. 3.3.7 Comparison of the Different Production Rates Effects on the Liquid Mole Fractions and the Applicability of the Single-Cell Simulator Results to Model Full Compositional Simulation Results, Volatile Oil Fluid 3 (Cases 3, 6, 9, 10)
Fig. 3.3.8 Comparison of Different Production Rate Effects on the Liquid Mole Fractions and the Applicability of the Single-Cell Simulator Results to Model Full Compositional Simulation Results, Black Oil Fluid 4 (Cases 4, 11, 12, 13)
place close to the wellbore and can show if those compositional changes can affect the pressure-saturation at the average reservoir. Fig. 3.3.9 was generated with a fixed total molar production rate of 5000 lb-mole/day and two absolute permeabilities of 5 md and 50 md. Two initial pressure values were chosen. The first is 3,175 psia, which is close to the saturation pressure of 3,173 psia, and the other is much higher than the saturation pressure at 3,490 psia. The results from the single cell simulator are also shown on the graph. The results obtained indicate that there is no significant difference among all the three cases considered here, and that the single cell simulator can model all the considered cases so far with very good accuracy.

We finally show the pressure-saturation relation computed from the single-cell simulator and at the average radius from the full compositional simulator in Fig. 3.3.10 and Fig. 3.3.11 for the volatile and black oil respectively. Clearly we can see the very good prediction that is obtained from the single-cell simulator.

We finally conclude that the results obtainable from the single-cell simulator can model the reservoir performance on the average level accurately and that knowing the fluid's initial composition and relative permeability is enough for us to predict the reservoir performance.
Fig. 3.3.9  Effect of Permeability and Initial Pressure Level on Liquid Mole Fractions for Volatile Oil Fluid 3 (Cases 6, 14, and 15)
Fig. 3.3.10  Effect of Permeability and Initial Pressure Level on Oil Saturation for Volatile Oil Fluid 3 (Cases 6, 14, and 15)
Fig. 3.3.11  Comparison of Oil Saturation for Different Production Rates and Constant Wellbore Pressure for Black Oil Fluid 4 (Cases 11, 4, 12, and 13)
3.4 Pressure-Time Relation from the Single-Cell Simulator

The presence of the average reservoir pressure-saturation relation allows us to determine the average reservoir pressure as function of time when the following information is available:

a. Original mass in place ($M_i$).

b. Initial reservoir pressure ($p_i$).

c. Total molar rate of production (constant rate production).

The initial reservoir pressure is used to compute the initial fluid density ($\rho_{oi}$) which is needed in the equation to compute time:

$$t = \frac{M_i}{Q_t} \cdot \frac{\rho_{oi} - \rho_{2\phi}}{\rho_{oi}}$$  \hspace{1cm} (3.4.1)

where,

$$\rho_{2\phi} = S_o \bar{\rho}_o + S_g \bar{\rho}_g$$ \hspace{1cm} (3.4.2)

To compute the relation between the pressure and time, we introduce Eq. 3.4.1 in the single cell-simulator. At each pressure decrement, we evaluate Eq. 3.4.2 based on the phase densities and the oil and gas saturations.

Note that when we have single-phase flow the equation still holds and the two-phase density will fall back to the single-phase density of the initial fluid.
Fig. 3.4.1 Fractional Mass Recovery History From Full Compositional Simulation Compared with Results From the Single-Cell Simulator as a Function of Pressure for Volatile Oil Fluid 3 (Case 6)
Fig. 3.4.2 Fractional Mass Recovery From Full Compositional Simulation Compared with Results From the Single-Cell Simulator as a Function of Pressure for Black Oil Fluid 4 (Case 16)
The computation of the average reservoir pressure with time was possible because the mass recovery from the single-cell simulator and the full compositional simulator were identical throughout the production history as seen in Fig. 3.4.1 for the volatile oil and Fig. 3.4.2 for the black oil. The input data to simulation are shown in Table 3.1.1 for all the cases in this section. The time computation in Eq. 3.4.1 assumes constant total molar production rate, and the time obtained from its use will be identical to that in simulation as long as the reservoir is producing in a constant total molar rate mode. When the full compositional simulator switches to constant wellbore pressure mode, the results from the single-cell simulator and the full compositional simulator will differ.

The average pressure computed from Eq. 3.4.1 using the single-cell simulator pressure-saturation relation is compared with the full compositional simulation in Fig. 3.4.3 for the volatile oil and Fig. 3.4.4 for the black oil. Both values follow the full simulator response until the simulator switches to constant wellbore pressure mode, and then they start to deviate. The oil saturation history performed similarly for both fluids as seen in Fig. 3.4.5 for the volatile oil and Fig. 3.4.6 for the black oil. The fractional recovery history from the single-cell simulator also followed that from full simulation until the full simulation mode was switched to constant wellbore pressure mode. The time of the fractional recovery for the single-cell simulator was obtained from the pressure-time relation computed from Eq. 3.4.1, and the result for the volatile oil is shown in Fig. 3.4.7.
Fig. 3.4.3 Pressure History From Full Compositional Simulation Compared With Results From the Single-Cell Simulator for Volatile Oil Fluid 3 (Case 6)
Fig. 3.4.4 Pressure History From Full Compositional Simulation Compared With Results From the Single-Cell Simulator for Black Oil Fluid 4 (Case 16)
Fig. 3.4.5 Oil Saturation History From Full Compositional Simulation Compared With Results From the Single-Cell Simulator for Volatile Oil Fluid 3 (Case 6)
Fig. 3.4.6  Oil Saturation History From Full Compositional Simulation Compared With Results From the Single-Cell Simulator for Black Oil Fluid 4 (Case 16)
Fig. 3.4.7  Fractional Mass Recovery History From Full Compositional Simulation Compared With Results From the Single-Cell Simulator for Volatile Oil Fluid 3 (Case 6)
3.5 Generalized Material Balance Computation

The general material balance equation used to predict the original oil and gas in place in volumetric reservoirs uses the production data, the compressibility factor, and the average reservoir pressure to generate a straight line relationship between $p/z$ and the mass produced. The vapor compressibility factor is used for gas reservoirs and the liquid compressibility factor is used for oil reservoirs. However, when a liquid phase develops in gas condensate reservoirs or when the gas phase starts to evolve in oil reservoirs, the use of either phase compressibility factor can introduce an error into the computed original oil or gas in place. We will look first at the original form of the equation and look at how the two-phase influence could be incorporated in the compressibility factor to obtain the correct original oil or mass in-place.

The general material balance equation for predicting the initial oil or gas in place is:

$$\left( \frac{\bar{p}}{z_i} \right) = \frac{p_i}{z_i} \left( 1 - \frac{M_p}{M_i} \right)$$  \hspace{1cm} (3.5.1)$$

where $M_p$ and $M_i$ are total number of moles produced from both phases in the reservoir and total moles initially in place respectively, and $p_i$ and $z_i$ are the initial reservoir pressure and the corresponding compressibility factor.

We will use the derivation by Vo\textsuperscript{3} to show first the relationship between the average compressibility factor used in Eq. 3.5.1 and the individual phases that exist, and second the dependence of the resulting compressibility factor on the reservoir depletion mechanism represented in the average oil saturation (or liquid mole fractions).

Vo started by defining the average reservoir molar density by weighing the average vapor and liquid densities by their saturations:

$$\bar{\rho} = \bar{\rho}_o S_o + \bar{\rho}_g S_g$$  \hspace{1cm} (3.5.2)$$

Using the engineering equation of state, this expression can be written in terms of the compressibility factor of each phase:
\[ \frac{1}{\bar{z}} = \frac{\bar{S}_o}{\bar{z}_o} + \frac{\bar{S}_g}{\bar{z}_g} \]  

(3.5.3)

The oil and gas saturations can be expressed in terms of the vapor and liquid mole fractions and the corresponding molar densities:

\[ \bar{S}_o = \frac{L/\bar{\rho}_o}{L/\bar{\rho}_o + V/\bar{\rho}_g}, \quad \bar{S}_g = \frac{V/\bar{\rho}_g}{L/\bar{\rho}_o + V/\bar{\rho}_g} \]  

(3.5.4)

which is equivalent to

\[ \bar{S}_o = \frac{L\bar{z}_o}{L\bar{z}_o + V\bar{z}_g}, \quad \bar{S}_g = \frac{V\bar{z}_g}{L\bar{z}_o + V\bar{z}_g} \]  

(3.5.5)

Using Eq. 3.5.5 in Eq. 3.5.4, we get

\[ \frac{1}{\bar{z}} = \frac{L + V}{L\bar{z}_o + V\bar{z}_g} = \frac{1}{L\bar{z}_o + V\bar{z}_g} \]  

(3.5.6)

which is

\[ \bar{z} = L \bar{z}_o + V \bar{z}_g \]  

(3.5.7)

where \( L \) and \( V \) used here are the liquid and vapor mole fractions obtained at the average reservoir conditions.

The average compressibility factor given by Eq. 3.5.7 is usually termed the two-phase compressibility factor \( \bar{z}_{2p} \) and it indicates its dependence on two parameters:

- Phase compressibility factors defined at average reservoir pressure and evaluated from the phase compositions at the average reservoir conditions.
- Vapor and liquid mole fractions obtained at the average reservoir conditions and based on the overall composition at the average radius, and as such dependent on the reservoir depletion mechanism as described throughout the previous sections of this chapter.
Eq. 3.5.7 tells us that we need to use the two-phase compressibility factor to actually predict the correct mass in place during depletion, although it involves determining the average pressure-saturation relation.

We will show here the application of the material balance equation for the gas condensate fluid when using the gas compressibility factor, and then we will show the results of the volatile and black oil fluid when using the liquid compressibility factor. These results will show us the magnitude of error involved in ignoring the evolving phase and ignoring the reservoir depletion mechanism. The vapor and liquid compressibility factors are obtained as discussed in Chapter II by flashing the initial overall in-situ composition to the average reservoir pressure and using the resulting phase compositions to compute each phase-compressibility factor.

Fig. 3.5.1 and Fig. 3.5.2 show the results of using the two-phase compressibility factor and the vapor phase compressibility factor for the gas condensate Fluids 1 and 2. The plots using the two-phase compressibility factor extrapolate to 100% total mass in-place while those using the vapor compressibility factors extrapolate to 95.5% of the original mass in-place for Fluid 1 and 94.7% for Fluid 2. The difference between the extrapolated value between the two fluids could be attributed to the richness of Fluid 2 compared to Fluid 1 which means that the deviation will be larger for richer compositions.

To determine the benefit of using the constant composition expansion data over the single-phase vapor compressibility factor as suggested by Vo,\(^3\) we have regenerated Fig. 3.5.1 and Fig. 3.5.2 by adding the results obtained by computing the two-phase compressibility factor from constant volume depletion and constant composition expansion (flash) along with the simulation two-phase compressibility factor. The results are shown in Figures 3.5.3, 3.5.4, and 3.5.5 for Fluids 1, 2, and 5 respectively. The results from constant volume depletion were identical to those in simulation, as we would expect from the combined liquid mole fractions and vapor and liquid compressibility factors (see Chapter II and previous sections in this chapter). The results based on flash data when extrapolated to zero pressure yielded original mass in place data close to those obtained using the vapor compressibility factor but with the prediction improved by flash data.
Fig. 3.5.1  $p/z$ Plot Comparing the Results of Using the Two-Phase Compressibility Factor and the Vapor Phase Compressibility Factor for Gas Condensate Fluid 1 (Case 1)
Fig. 3.5.2 $p/z$ Plot Comparing the Results of Using the Two-Phase Compressibility Factor and the Vapor Phase Compressibility Factor for Gas Condensate Fluid 2 (Case 2)
Fig. 3.5.3  Comparison of p/z2phase Values Obtained From Simulation, Flash, and Constant Volume Depletion for Gas Condensate Fluid 1 (Case 1)
Fig. 3.5.4  Comparison of p/z2phase Values Obtained From Simulation, Flash, and Constant Volume Depletion for Gas Condensate Fluid 2 (Case 2)
Fig. 3.5.5  Comparison of p/z2phase Values Obtained From Simulation, Flash, and Constant Volume Depletion for Gas Condensate Fluid 5 (Case 5)
We have performed similar computations for the volatile and black oil compositions by using the liquid compressibility factor and comparing the results with those obtained from simulation values of the two-phase compressibility factor. The results are shown in Fig. 3.5.6 and Fig. 3.5.7. These two figures clearly show that the liquid compressibility factor should not be used in predicting the original mass in-place even as an approximation.

We know from the previous sections of this chapter that the liquid mole fractions obtained from constant composition expansion and constant volume depletion cannot model the reservoir behavior when volatile oil or black oil is in the reservoir. Nevertheless, we have investigated the possibility of using these two procedures to compute the two-phase compressibility factor as seen in Fig. 3.5.8 and Fig. 3.5.9. The values obtained from the constant volume depletion procedure seem to follow those of simulation more closely and therefore yield better approximation than the flash values. Fig. 3.5.10 and Fig. 3.5.11 show the results obtained for the volatile oil and black oil respectively. By extrapolating the flash and constant volume depletion data to zero pressure, we obtained the results shown in Table 3.5.1. Using constant volume depletion, the predicted original mass in place is the closest to the actual (109.41% of the actual for volatile oil and 122.82% for black oil). The values obtained from flash and from the liquid compressibility factor were far off the correct value.

Because of the results shown so far, we concluded that the prediction of the original mass in place based on any of the methods discussed so far will be inappropriate for the volatile oil and black oil compositions, and there is no way to determine the magnitude of error that might result. We therefore suggest here the use of the results obtained from the single-cell simulator to compute the two-phase compressibility factor to obtain a reliable estimate of the original mass in place.

In Fig. 3.5.12 we show the two-phase compressibility factor computed from the single-cell simulator and compared to reservoir simulations using different flow rates and constant wellbore pressure. The emphasis here is on the unique values
Fig. 3.5.6 p/z Plot Comparing the Results of Using the Two-Phase Compressibility Factor and the Liquid Phase Compressibility Factor for Volatile Oil Fluid 3 (Case 3)
Fig. 3.5.7  $p/z$ Plot Comparing Results of Using the Two-Phase Compressibility Factor and the Liquid Phase Compressibility Factor for Black Oil Fluid 4 (Case 4)
Fig. 3.5.8  Comparison of Simulation, Flash, and CVD Two-Phase Compressibility Factor for Volatile Oil Fluid 3 (Case 3)
Fig. 3.5.9  Comparison of Simulation, Flash, and CVD Two-Phase Compressibility Factor for Black Oil Fluid 4 (Case 4)
Fig. 3.5.10  p/z2phase Plot vs. Mass Fraction Recovered for Simulation, Flash, and CVD for Volatile Oil Fluid 3 (Case 3)
Fig. 3.5.11 p/z2phase Plot vs. Mass Fraction Recovered for Simulation, Flash, and CVD for Black Oil Fluid 4 (Case 4)
TABLE 3.5.1  Comparison of the Different Procedures to Predict Original Mass in Place

<table>
<thead>
<tr>
<th>Fluid</th>
<th>(\frac{(M_i)_{\text{pred}}}{M_i}) (Flash), %</th>
<th>(\frac{(M_i)_{\text{pred}}}{M_i}) ((z_v)), %</th>
<th>(\frac{(M_i)_{\text{pred}}}{M_i}) ((z_l)), %</th>
<th>(\frac{(M_i)_{\text{pred}}}{M_i}) (CVD), %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas Condensate 1</td>
<td>96.49</td>
<td>95.5</td>
<td>-----</td>
<td>100.00</td>
</tr>
<tr>
<td>Gas Condensate 2</td>
<td>95.88</td>
<td>94.7</td>
<td>-----</td>
<td>100.00</td>
</tr>
<tr>
<td>Gas Condensate 5</td>
<td>97.28</td>
<td>-----</td>
<td>-----</td>
<td>100.00</td>
</tr>
<tr>
<td>Volatile Oil</td>
<td>77.25</td>
<td>-----</td>
<td>173.60</td>
<td>109.41</td>
</tr>
<tr>
<td>Black Oil</td>
<td>65.17</td>
<td>-----</td>
<td>176.10</td>
<td>122.82</td>
</tr>
</tbody>
</table>

of the obtained two-phase compressibility factors from all the cases which are based on their respective pressure-saturation profiles as function of pressure. This also confirms the applicability of a tank model to predict the reservoir performance for volatile and black oil fluids. Fig. 3.5.13 shows the resulting \(p/z\) plot using the computed two-phase compressibility factor. All the results obtained extrapolate to 100% original mass in place.
Fig. 3.5.12 Two-Phase Compressibility Factor for Different Production Rates and Constant Wellbore Pressure Compared with the Single-Cell Simulator for Black Oil Fluid 4 (Cases 11, 4, 12, and 13 Respectively)
Fig. 3.5.13  \( p/z2phase \) for Different Production Rates and Constant Wellbore Pressure Compared With the Single-Cell Simulator for Black Oil Fluid 4 (Cases 11, 4, 12, and 13 Respectively)
CHAPTER IV
PRESSURE AND SATURATION CONSIDERATIONS
DURING BOUNDARY DOMINATED FLOW

Our ability to analyze pressure data from a multiphase flowing reservoir depends directly on our ability to determine both a pressure-saturation relation and pressure and saturation profiles in the reservoir. This is important on two counts: First, we need to be able to determine the relative permeability at any pressure which is a function of saturation. Second, we need to identify the changes in the reservoir since the differential equation is a function of both the spatial and temporal coordinates.

In this chapter we focus on the developments in both areas which we hope will to help us advance the understanding of the problem. First we look at the different formulations for the computation of pressure and saturation profiles in the reservoir. This is mainly the black oil and compositional formulations. Second we look at the methods available to compute the pressure-saturation relations during boundary dominated flow based on both black oil and compositional formulations. We will also show that the use of black oil methods could contribute to our lack of understanding of the multiphase problem.

We investigate the constant gas-oil ratio assumption used to compute saturation profiles in the reservoir and show the region of its applicability.

Finally we use the results obtained in our earlier developments of pressure-saturation relations from the single-cell simulator to compute the pressure and saturation profiles and point to the source of inaccuracies that might result.

4.1 Pressure and Saturation Relations

Many pressure-saturation relations have been developed before. We will concentrate here on those relations applicable during the boundary-dominated flow period. The available methods, their applicability and limitations will be discussed here.
4.1.1 Material Balance

Applying the material balance principle to an element of unit volume and accounting for the mass entering and leaving the element yields the following equation for saturation that is directly related to pressure and compressibility factors of both phases and of the relative mass remaining in the system:

\[ S_o = \frac{z_l}{z_v - z_l} \left[ \frac{p_f}{z_l} \left( 1 - \frac{M_p}{M_l} \right) - 1 \right] \quad (4.1.1) \]

This equation is equivalent to the pressure-saturation relation obtained from the single-cell simulator and offers the advantage that it does not require the relative permeability data for the computation. It requires, however, the initial mass in place.

4.1.2 Gas-Oil Ratio

The producing gas-oil ratio was used to provide a pressure-saturation relation as early as 1941 when Evinger\(^1\) integrated the gas and oil flow equation under steady-state conditions and divided the gas flow rate by the oil flow rate to obtain the expression for the producing gas-oil ratio relating fluid properties to the relative permeability data:

\[ R_p = R_s + \frac{k_{rg}}{k_{ro}} \frac{\mu_o}{\mu_g} \frac{B_o}{B_g} \quad (4.1.2) \]

4.1.3 Steady-State Theory

The steady state saturation change with pressure can be computed from the steady state theory as shown by Jones\(^2\):

\[ \frac{k_{rg}}{k_{ro}} = \frac{V \rho_o \mu_g}{L \rho_g \mu_o} \quad (4.1.3) \]

where the parameters at the right-hand side are computed from phase compositions which are obtained by performing a flash process on the initial overall composition. In this method the relative permeability data is needed to compute the pressure-saturation relation.
The gas-oil ratio method and the compositional formulation of the steady-state theory have similarities in that they are both based on the steady-state flow assumption, and that the relative permeability ratio is obtained using fluid properties alone. This can be seen when we rewrite equation (4.1.2) and (4.1.3) as follows,

\[
\frac{k_{rg}}{k_{ro}} = \frac{V}{\frac{\rho_g \mu_o}{\rho_o \mu_g}}, \text{ and}
\]

\[
\frac{k_{rg}}{k_{ro}} = \frac{R_p - R_s}{\frac{B_o \mu_2}{B_g \mu_g}}
\]

Both the gas-oil ratio and the compositional formulation of the steady-state theory are based on steady-state assumptions and therefore represent an approximation of the process in the reservoir during the boundary-dominated flow period. Furthermore, they apply mainly at the region close to the wellbore where steady-state flow may prevail. The two methods also have a common disadvantage that limits their use and can lead to inaccuracies in estimating the saturation in the reservoir beyond the steady-state acting zone close to the wellbore. This disadvantage is inherent in the use of the relative permeability ratio to back-calculate the corresponding saturation from the relative permeability curves. Using this approach, we cannot compute the gas saturation before the critical gas saturation is reached. That is, the minimum gas saturation we can compute is the critical gas saturation.

4.2 Prediction of Saturation and Pressure Profiles in the Reservoir During Boundary-Dominated Flow Period

In 1961 Levine and Prats suggested a method to compute the pressure and saturation profiles in the reservoir for black oil during the boundary-dominated flow period using the following equation:
\[
\int_{P_D}^{B_o} \left( \frac{k_{ro}(s_o)}{\mu_o B_o} \right) dp = \frac{70.6 q_o}{kh p_b} \left[ \frac{r_D^2 - r_{D_e}^2}{r_{D_e}^2} - 2 \ln \left( \frac{r_D}{r_{D_e}} \right) \right]
\]  
(4.2.1)

where \( P_D \) is the dimensionless pressure based on the bubble point pressure \( (p_b) \),

\[
P_D = \frac{p}{p_b}, \quad r_D = \frac{r}{r_w}
\]  
(4.2.2)

The equation was developed on the assumption that the rate of decline of stock tank oil in place is constant throughout the reservoir (at the same instant of time); \( i.e., \)

\[
\frac{\partial (s_o)}{\partial t} \bigg|_{average} = \text{constant} = \frac{-q_o}{\pi \phi h (r_e^2 - r_w^2)}
\]  
(4.2.3)

To compute the pressure-saturation relation Levine and Prats used the gas-oil ratio values obtained from simulation at the outer boundary. They also assumed that the total gas oil ratio will remain constant throughout the reservoir at each instant of time.

The previous two assumptions were necessary to compute the pressure-saturation profile in the reservoir, but they have two disadvantages. First, we have no way of evaluating the pressure at the outer boundary or its corresponding total gas-oil ratio (at the outer boundary) as they suggested. Second, is that the producing gas-oil ratio that can be evaluated at the wellbore does not always remain constant throughout the reservoir.

Raghavan\(^8\) later suggested using the producing gas-oil ratio rather than the gas-oil ratio at the outer boundary and referred to the observation made by Levine and Prats\(^7\) that the gas-oil ratio remains constant to within 10%.

This method mainly works for black-oil fluids because there are no compositional changes that take place in the reservoir, and therefore, saturation remains a strong function of pressure.

We have computed the gas-oil ratio throughout the reservoir from the compositional simulation results by using the equation

\[
R_p = R_s + \frac{k_{rg}}{k_{ro}} \frac{\mu_o}{\mu_g} \frac{B_g}{B_o}
\]  
(4.1.2)
and

$$R_s = \frac{V^{sc}/\rho_g^{sc}}{L^{sc}/\rho_o^{sc}} \times 5.615 \text{, SCF/STB}$$  \hspace{1cm} (4.2.4)$$

where $L^{sc}$ and $V^{sc}$ and the phase densities obtained by flashing the liquid composition from reservoir pressure and temperature to separator conditions (14.7 psia and 60° R). The phase densities are in $Lb$/Mole/ft$^3$, and $L^{sc}$ and $V^{sc}$ are the liquid and vapor mole fractions. The oil formation volume factor is computed from

$$B_o = \frac{\rho_o^{sc}}{\rho_o}.\frac{1}{L^{sc}}, \text{ bbl/STB}$$  \hspace{1cm} (4.2.5)$$

The gas formation volume factor is computed from

$$B_g = \frac{\rho_{g, \text{separator}}^{sc}}{\rho_g^{rc}} \times \frac{1}{5.615}, \text{ bbl/SCF}$$  \hspace{1cm} (4.2.6)$$

A sample of the results obtained here is shown in Fig. 4.2.1 and Fig. 4.2.4 for the volatile oil fluid. The simulation input parameters are given in Table 4.2.1. All results shown here are obtained for the boundary-dominated flow period, as can be seen from the dimensionless time based on the drainage area of the reservoir.

Fig. 4.2.1 initially shows a constant producing gas-oil ratio in the zone close to the wellbore at $t_{DA}=0.15$ until it reaches 28 ft deep in the reservoir and then it starts to increase. To understand why the gas-oil ratio increases beyond the 28 ft in the reservoir we need to examine both Fig. 4.2.2 and Fig. 4.2.3, and Eq. 4.1.5. First, Fig. 4.2.2 shows that the saturation profile extends smoothly from the wellbore until it reaches the radius of 28 ft where it jumps suddenly because of the relative permeability effects. The jumping point 28 ft is the point at which the gas saturation reached the critical gas saturation. The gas is trapped there causing the rate of gas saturation build up to increase rapidly. The same effect is seen at a later time $t_{DA}=4.95$. If we consider now Fig. 4.2.3 and compare the solution gas-oil ratio to the producing gas-oil ratio of Fig. 4.2.1 at their respective times, we find that the value of the producing gas-oil ratio beyond the 28 ft zone for $t_{DA}=0.15$ and beyond 245 ft at $t_{DA}=4.95$ equals to the solution gas-oil ratio.
TABLE 4.2.1 Simulation Cases Considered in the Study of Pressure and Saturation Profiles

<table>
<thead>
<tr>
<th>Case No</th>
<th>Fluid Type</th>
<th>Permeability ( \mu ) md</th>
<th>Initial Pressure ( P_{\text{init}} ) psia</th>
<th>Production Rate ( q ) Ml-Mole/Day</th>
<th>Wellbore Pressure at Mode Switch ( P_{\text{sw}} ) psia</th>
<th>Average Pressure at End of Run ( P_{\text{end}} ) psia</th>
<th>Percentage Mass Recovery</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Volatile Oil</td>
<td>50</td>
<td>3175</td>
<td>5000</td>
<td>49</td>
<td>164</td>
<td>70.2</td>
</tr>
<tr>
<td>2</td>
<td>Black Oil</td>
<td>10</td>
<td>2330</td>
<td>2000</td>
<td>-----</td>
<td>1367</td>
<td>25.6</td>
</tr>
</tbody>
</table>
Fig. 4.2.1  Gas-Oil Ratio Profiles at Different Times for Volatile Oil Fluid 3 (Case 1)
Fig. 4.2.2 Oil Saturation Profiles at Different Times for Volatile Oil Fluid 3 (Case 1)
Fig. 4.2.3  Solution Gas-Oil Ratio Profiles at Different Times for Volatile Oil Fluid 3 (Case 1)
value. This can also be seen from the producing gas-oil ratio Eq. 4.1.2 where the value of the producing gas-oil ratio will equals the solution gas-oil ratio when the gas relative permeability is zero. This confirms that the producing gas-oil ratio can not be constant throughout the reservoir as long as both phases are not mobile.

Considering the other two curves shown in the previous Figures $t_{DA}=11.03$, and $t_{DA}=20.16$, we notice that the producing gas-oil ratio for these times do not remain constant, nor do they change smoothly. If we compare the response at these times with the oil saturation responses shown in Fig. 4.2.2, we notice that the oil saturation changes are smoother than the responses at earlier times. The drop in the gas-oil ratio can be attributed here to the drop in the solution gas-oil ratio which is shown in Fig. 4.2.3. Although the second term in Eq. 4.1.5 is contributing to the gas-oil ratio (because the gas phase started to move), the drop in solution gas-oil ratio is greater, causing the gas-oil ratio to decrease.

As the pressure drops further both factors in Eq. 4.1.5 contribute to the gas-oil ratio; that is first the solution gas-oil ratio continues to decrease, and second, the higher gas saturation increases the gas mobility and thus increases the flowing gas-oil ratio. When the two parameters become equal, that is, the reduction in the solution gas-oil ratio is equal to the increase in the flowing gas-oil ratio, we will have a constant gas-oil ratio. In other words, if all the gas evolving from solution flows out of the reservoir, then we have a true constant gas-oil ratio. We have to consider also that although this depends largely on the gas mobility, the fluid composition in the reservoir is not constant; and even if the evolving gas is equal to the flowing gas, the ratios between their two amounts could be different at different locations in the reservoir.

We finally conclude that the producing gas-oil ratio should not be set at an arbitrary set constant throughout the reservoir without considering the parameters discussed. From what we have seen so far, we consider that the gas saturation has the most effect on the constant gas-oil ratio concept followed by the gas mobility, and finally the overall composition distribution in the reservoir.
Fig. 4.2.4: Gas-Oil Ratio Profiles at Late Stages of Depletion for Volatile Oil Fluid 3 (Case I)
Fig. 4.2.4 shows the gas-oil ratio at a late stage of the reservoir life where the gas mobility is considerably higher than the earlier times throughout the reservoir. This resulted in the gas-oil ratio being constant except for a small zone at the outer boundary. The oil saturation corresponding to the times shown in Fig. 4.2.4 are somewhat constant and are shown in Fig. 4.2.5.

Later in the life of the reservoir when the gas starts to build-up at the low pressure zone around the wellbore the gas-oil ratio starts to build up sharply in that zone. The deeper zone in the reservoir remains unaffected and the gas-oil ratio remains constant. Fig. 4.2.6 and Fig. 4.2.7 were generated for the black oil fluid using the simulation data given in Table 4.2.1. The oil saturation dropped from 70% to 54% over the 10 ft. zone around the wellbore causing the producing gas-oil ratio to increase from 4560 (SCF/STB) to 5000 SCF/STB over the same distance.

All the gas-oil ratio profiles we looked at have followed the same properties summarized here:

- The gas-oil ratio profile in the reservoir begins as constant around the wellbore up to the region where the gas saturation is below the critical saturation where the value of the gas-oil ratio follows that of the solution gas-oil ratio.
- When the critical gas saturation is reached in the reservoir but the gas mobility is still very low and can not compensate for the reduction of solution gas-oil ratio, the gas-oil ratio starts to drop
- When the rate of gas flow is large enough and can compensate for the decrease in the solution gas-oil ratio (with pressure) the gas-oil ratio becomes constant throughout the reservoir.
- At the late stage of production when the pressure drop becomes high around the wellbore and the gas saturation starts to build up in the wellbore region, the producing gas-oil ratio increases rapidly around the wellbore but remains constant in the deeper region of the reservoir. At this point it will be misleading to use the constant gas-oil ratio assumption to compute the saturation profile in the reservoir since there will be large difference as time progresses.
Fig. 4.2.6 Gas-Oil Ratio Profiles at Different Times for Black Oil Fluid 4 (Case 2)
Fig. 4.2.7 Oil Saturation Profiles at Different Times for Black Oil Fluid 4 (Case 2)
4.2.1 Compositional Formulation

The General flow equation for a homogeneous reservoir is

$$\frac{\partial}{\partial r} \left[ r \left( k_{ro} \frac{\rho_o}{\mu_o} + k_{rg} \frac{\rho_g}{\mu_g} \right) \frac{\partial p}{\partial r} \right] = -\frac{\phi r}{k C_1} \frac{\partial}{\partial t} \left( S_o \rho_o + S_g \rho_g \right)$$

(4.2.7)

In deriving a workable form for the calculation of the pressure and saturation profile in the compositional case we can not follow exactly the procedure of Levine and Prats\(^7\) in assuming that the rate of change of \((S_o/B_o)\) is constant everywhere in the reservoir since \(\frac{\partial p}{\partial t}/r\) can be as much as ten times different from \(\frac{\partial p}{\partial t}/r\) as reported by Aanonsen\(^2\) and Jones.\(^2\) However, if the rate of decline of average mass in the reservoir is constant

\[ \frac{\partial}{\partial t} (\bar{S}_o \rho_o + \bar{S}_g \rho_g) = \text{constant} \]

for each time, we can follow the same development as the liquid solution.

For a constant production rate, the number of moles produced is equal to the rate of change of the average molar density of reservoir fluid per unit reservoir volume. The average fluid density is computed based on the phase density and phase volume fractions (saturations):

$$\bar{\rho} = \bar{S}_o \rho_o + \bar{S}_g \rho_g$$

(4.2.8)

where \(\bar{\rho}_o, \bar{\rho}_g\) are the average phase densities, and \(\bar{S}_o, \bar{S}_g\) are the average phase saturations. Using this expression we obtain

$$q_i = -V_p \frac{\partial}{\partial t} (\bar{S}_o \rho_o + \bar{S}_g \rho_g)$$

(4.2.9)

where \(V_p\) is the pore volume of the reservoir and is defined as:

$$V_p = \pi (r_c^2 - r_w^2) \phi h$$

(4.2.10)

Invoking the constant rate of mass decline \[\frac{\partial}{\partial t} (\bar{S}_o \rho_o + \bar{S}_g \rho_g) = \text{constant}\] condition, the right hand side could then be represented by
\[ \frac{\partial}{\partial t} \left( S_o \rho_o + S_g \rho_g \right) = - \frac{q_t}{\pi (r_e^2 - r_w^2) \phi h} \] (4.2.11)

Substituting equation (4.2.11) into equation (4.2.7), we get

\[ \frac{\partial}{\partial r} \left[ r \left( k_{ro} \frac{\rho_o}{\mu_o} + k_{rg} \frac{\rho_g}{\mu_g} \frac{dp}{dr} \right) \right] = \frac{r q_t}{\pi k h C_1 (r_e^2 - r_w^2)} \] (4.2.12)

By moving the spatial derivative to the right-hand-side we can integrate equation (4.2.12) over the entire reservoir, but by performing this process we are concluding that the relative permeabilities will be a sole function of pressure and therefore will ignore all the changes of saturations with respect to distance.

Integrating over the entire reservoir and invoking the no flow outer boundary condition, we obtain an intermediate form (4.2.13) that can be further integrated to the final form (4.2.14).

\[ r \left( k_{ro} \frac{\rho_o}{\mu_o} + k_{rg} \frac{\rho_g}{\mu_g} \right) \frac{dp}{dr} = \frac{-q_t}{\pi k h C_1 (r_e^2 - r_w^2)} \left( \frac{r_e^2 - r_w^2}{2} \right) \] (4.2.13)

Which can be integrated again;

\[ \int p \left( k_{ro} \frac{\rho_o}{\mu_o} + k_{rg} \frac{\rho_g}{\mu_g} \right) dp = \frac{q_t}{2 \pi k h C_1} \left[ \frac{r_e^2}{(r_e^2 - r_w^2)} \ln \left( \frac{r_e}{r} \right) - \frac{r_e^2 - r_w^2}{2 (r_e^2 - r_w^2)} \right] \] (4.2.14)

Equation (4.2.14) is the equation we use to compute the pressure and saturation distributions in the reservoir for a fixed time.

To compute the pressure and saturation distribution in the reservoir we need the saturation as function of pressure. we use here the saturation-pressure relation obtained from the single-cell simulator discussed in Chapter III. we assume that the pressure-saturation relation applies equally to all locations in the reservoir.

Computation Procedure

1. Use the results of the single cell simulator to provide the following information for all the pressure range necessary:
   - Fluid properties, i.e. densities and viscosities for each phase.
• Phase saturations at each pressure.
• Average reservoir pressure for the given time.

2. Start at the average reservoir pressure which corresponds to \( \bar{p} = 0.54924 \times r_c \) and current time.

3. Select a pressure decrement value \( \Delta p \) to reduce the pressure by. Use the results of the single-cell simulator and the relative permeability data to compute the integral in the left-hand side of equation (4.2.14).

4. With the left-hand side defined in equation (4.2.14) we can solve for the radius in the right-hand side (which corresponds to the lower limit of the integral). This radius defines the pressure and corresponding saturation at this location in the reservoir.

5. Repeat steps 3 and 4 until the pressure at the lower limit of the integral reaches the pressure at the wellbore. This gives the pressure and saturation profile for all the reservoir.

6. To obtain the pressure and saturation profile for a different time use the average pressure and wellbore pressure corresponding to that time. The same calculation procedure applies.

Fig. 4.2.8 shows the pressure profile in the reservoir from simulation (solid lines) and the computed pressure profile (dashed lines) based on equation (4.2.14) and the procedure outlined above for the volatile oil. Since the computation starts with known average pressure and average saturation at the average radius and progresses towards the wellbore, the computed pressure profile starts very close to the simulation value until it reaches close to the wellbore where the two values start to depart slightly. We should note that the computed profile departs from that of simulation only close to the wellbore at an approximate distance of (10 ft) only. The accompanying saturation profile that corresponds to the pressure profile in Fig. 4.2.8 are shown in Fig. 4.2.9. The computed profiles (dashed lines) match the simulation (solid lines) values closely before the critical gas saturation is reached where the gas becomes mobile. This is in contrast to the gas-oil ratio method and the steady-state computed saturations which can be predicted
Fig. 4.2.8  Pressure Profiles at Different Times Obtained From Simulation as Compared to Computed Profiles for Volatile Oil Fluid 3 (Case 1)
Fig. 4.2.9  Oil Saturation Profiles Obtained From Simulation at Different Times as Compared to Computed Profiles for Volatile Oil Fluid 3 (Case 1)
Fig. 4.2.10  Pressure Profiles From Simulation at Late Stages of Depletion as Compared to Computed Profiles for Volatile Oil Fluid 3 (Case 1)
Fig. 4.2.11  Oil Saturation Profiles Obtained From Simulation at Late Stages of Depletion as Compared to Computed Profiles for Volatile Oil Fluid 3 (Case 1)
only after the gas phase becomes mobile. We note here that the maximum departure between the simulation saturation profile and the computed profile occurs at the wellbore and is less than the 7% critical gas saturation. Fig. 4.2.10 and Fig. 4.2.11 correspond to the pressure and saturation profiles at the late times for the same simulation.

Fig. 4.2.12 through Fig. 4.2.14 show the results for case 2 in Table 4.2.1 which is for the black oil fluid. The computed pressure profiles shown in Fig. 4.2.12 and Fig. 4.2.14 are again close to the simulation values up to the region close to the wellbore where it starts to deviate. The computed saturation profiles in Fig. 4.2.13 and Fig. 4.2.15 seem to depart from the simulation values (to a maximum of 10% saturation units).

To help explain the departure between oil saturation profiles computed in this case as well as in the volatile oil case discussed before we need to look back at our assumption to represent the pressure-saturation relation in the entire reservoir by the relation from a single location (at the average radius). Generally using this approach when conducting black oil simulations will not create much deviation as seen here. Although in this study we use a black oil composition, by using a compositional simulator we are in fact allowing the composition to change in the entire reservoir similar to what happens in nature. The drawback of applying such a method is that we are not yet able to quantify the magnitude of compositional changes that happen in the reservoir at different locations due to fluid movement and evaporation of lighter components from the liquid phase and condensation of heavier components from the vapor phase and its impact on saturation.

To examine the extent of the saturation changes in the reservoir as an impact of the compositional changes that occur at different locations in the reservoir we will determine from simulation at each location and pressure the magnitude of the oil saturation. In other words we want to determine the saturation value in the entire reservoir for a fixed pressure. If no compositional changes take place in the reservoir or if the compositional changes have no significant effect on the oil saturation then we should obtain a constant oil saturation profile for the entire reservoir at any given pressure and therefore we will be able to represent the saturation as a sole function of pressure at any location in the reservoir. Fig. 4.2.16 and Fig. 4.2.17 show the results we have obtained for simulation
Fig. 4.2.12  Pressure Profiles at Different Times Obtained From Simulation as Compared to Computed Profiles for Black Oil Fluid 4 (Case 2)
Fig. 4.2.13  Oil Saturation Profiles Obtained From Simulation at Different Times as Compared to Computed Profiles for Black Oil Fluid 4 (Case 2)
Fig. 4.2.14  Pressure Profiles From Simulation at Late Stages of Depletion as Compared to Computed Profiles for Black Oil Fluid 4 (Case 2)
Fig. 4.2.15  Oil Saturation Profiles From Simulation at Late Stages of Depletion As Compared to Computed Profiles for Black Oil Fluid 4 (Case 2)
case 2 in TABLE 4.2.1. To generate these Figures we first set a pressure value below the saturation pressure, we determine the oil saturation at the wellbore corresponding to that pressure. We advance to the next location in the reservoir away from the wellbore, the pressure at this location is not equal to the pressure we are obtaining the saturation for, therefore we remain at this position and look at the data at the next timestep and check the pressure value (which should be lower than the value at the previous timestep). We continue advancing time at the same location until the pressure we have set is reached or the pressure is lower in which case we interpolate between the pressure values to determine the oil saturation corresponding to the fixed pressure at this location. By now we should have two saturation values corresponding to the same pressure at two locations.

To continue to determine the saturation values as function of location alone (at fixed pressure) we advance to the next location deeper in the reservoir and repeat the process again until we obtain the oil saturation for the entire reservoir for the fixed pressure we have set. To determine the saturation values at a lower pressure we repeat the entire search for all the reservoir.

Fig. 4.2.16 shows the saturation profile in the reservoir at fixed pressure for the range of pressure from the saturation pressure to 75% of the saturation pressure, and Fig. 4.2.17 shows the range of pressure down to 55% of the saturation pressure. Initially at a pressure of (2326 psia, saturation pressure = 2328.3 psia) the oil saturation is independent of location since there are no compositional changes that took place yet in the entire reservoir. As the pressure drops to (2210 psia) we can see two regions, one at a constant oil saturation of 94% close to the wellbore and the second at 95%. The difference between the two regions could be attributed to the evaporation of the lighter components which continue to be trapped in the same location ($S_g < S_{gc}$) and the continual feed to the trapped gas phase from the flowing oil (as pressure drops the lighter components from the flowing liquid phase evolve and becomes trapped). Although deeper in the reservoir the gas evolving from solution is still trapped in that location but the gas saturation does not build-up as fast as the region close to the wellbore since the feed to the gas phase from the flowing liquid is minimal. As the pressure is dropped further we notice that the trend is reversed and the same trend continues to the end of simulation as seen in Fig. 4.2.17. The
Fig. 4.2.16  Oil Saturation Changes Caused by Compositional Changes in Different Locations in the Reservoir at Fixed Pressures for Black Oil Fluid 4 (Case 2)
Fig. 4.2.17  Oil Saturation Changes Caused by Compositional Changes in Different Locations in the Reservoir at Fixed Pressures for Black Oil Fluid 4 (Case 2), Data Shown for Lower Pressures
saturation profile trend reversal is due to the accumulation of the liquid phase close to the wellbore after the gas phase becomes mobile. When the gas phase becomes mobile the flowing (liquid and vapor phases) start to contribute to the compositional changes in the reservoir as the pressure drops close to the wellbore in two ways, first the heavier components in the gas phase start to condensate as the fluid moves towards the wellbore, second, when the gas saturation increases, the liquid mobility is decreased and the gas mobility is increased thus helping in accumulating more liquid in the region close to the wellbore.

In the region deep in the reservoir the contribution to the liquid phase from the flowing vapor is minimal, and therefore no major changes happen to the oil saturation due to compositional changes induced by the flowing phase. This means that the oil saturation obtained at the deepest location in the reservoir will not change with location and should follow what we obtain from the single-cell simulator at the corresponding pressure. Indeed, by comparing the single-cell simulator results shown in Fig. 3.3.11 the black oil composition at a pressure of (1744 psia) with the deepest saturation point in Fig. 4.2.16 at the same pressure yield the same oil saturation of 79.5%.

If we go back now to the computed saturation profiles shown in Fig. 4.2.15 and try to explain the difference between the simulation values and the computed values at a location 4 ft. away from the wellbore at the time corresponding to \( T_{DA} = 11.04 \). Fig. 4.2.12 tells us that the pressure at that location and the given time will be approximately (1278 psia). If we look at Fig. 4.2.17 which tells us the saturation that we should have used in our computation of the pressure-saturation profile in equation (4.2.14) at this particular pressure and location. The value we obtained here for the oil saturation is 83.4% while the value used from the single cell simulator is only 72.6%. This leaves a difference in the input to the method of 11.2% saturation units. Comparing this value to the value obtained from our calculation of the pressure-saturation profile at this location and pressure we find the computed saturation of 73.1% only 10.4% far from the actual simulation saturation. This indicates that the procedure for computing the pressure and saturation profiles used here will give us very good approximation if we can find a way to model the saturation change with location.
4.3 Pseudopressure Integral Considerations

We can express the result obtained in equation (4.2.14) by means of a dimensionless pseudopressure integral of the form

\[
m_{PD}(r) = \frac{2\pi kh c_i}{q_i} \int_{p}^{\bar{p}} \lambda_i [p(r)] \, dp
\]  

(4.3.1)

where,

\[
\lambda_i = k_{ro} \frac{\rho_o}{\mu_o} + k_{rg} \frac{\rho_g}{\mu_g}
\]  

(4.3.2)

Based on our assumption of boundary dominated flow, the pseudopressure integral becomes

\[
m_{PD}(r) = \left[ \frac{r_e^2}{(r_e^2 - r_w^2)} \ln \left( \frac{r}{\bar{r}} \right) - \frac{r_e^2 - \bar{r}^2}{2 (r_e^2 - r_w^2)} \right]
\]  

(4.3.3)

Note that the right-hand-side of equation (4.3.3) depends only on the location and is constant for all times while the left-hand-side is a function of the average pressure for a specific time under consideration, and therefore, the profile is a function of time.

A sample calculation of the pseudopressure integral using pressure and saturation profiles from simulation is shown in Fig. 4.3.1 for different production times using the volatile oil fluid. The computed pseudopressure integral reveals a semilog straight line for all the times used in the calculation (except at the onset of boundary dominated flow period, i.e. before the boundary dominated flow is fully developed). A sample calculation using the pressure and saturation values obtained from the single-cell simulator for the time \(T_{DA}=0.15\) is shown here. We note that although the pressure-saturation relation obtained from the single-cell simulation do not apply to all locations in the reservoir, we still get a semi-log straight line as shown in Fig. 4.3.1. Another case is shown in Fig. 4.3.2 for the black oil fluid. Both results using the pressure-saturation relation based on the single cell simulator yield identical results to the integral based on the actual pressure and saturation profiles from the simulation.
Fig. 4.3.1  Pseudopressure Integral Based on Pressure and Saturation Profiles From Simulation Compared to that Based on the Pressure-Saturation Relation From the Single-Cell Simulator for Volatile Oil Fluid 3 (Case 1)
Fig. 4.3.2  Pseudopressure Integral Based on Pressure and Saturation Profiles From Simulation Compared to that Computed Based on the Pressure-Saturation Relation From the Single-Cell Simulator for Black Oil Fluid 4 (Case 2)
If we take the logarithmic derivative of the pseudopressure integral with respect to \( r \) using the chain rule we can get the slope of the semi-log straight line

\[
\frac{\partial m_{PD}}{\partial \ln r} = \frac{\partial m_{PD}}{\partial p} \frac{\partial p}{\partial r} \frac{\partial r}{\partial \ln r} \tag{4.3.4}
\]

where,

\[
\frac{\partial m_{PD}}{\partial p} = \frac{2\pi khC_1}{q_t} \cdot \lambda_t[p(r)] \tag{4.3.5}
\]

therefore,

\[
\frac{\partial m_{PD}}{\partial \ln r} = \frac{2\pi khC_1}{q_t} \cdot \lambda_t[p(r)] \cdot r \frac{\partial p}{\partial r} = \frac{r_e^2 - r_w^2}{r_e^2 - r_w^2} \tag{4.3.6}
\]

The total molar production rate for any location in the reservoir is defined as,

\[
q_t(r) = -2\pi khC_1 \cdot \left( k_{ro} \frac{p_o}{\mu_o} + k_{rg} \frac{p_g}{\mu_g} \right) \cdot r \frac{\partial p}{\partial r} \tag{4.3.7}
\]

and the total molar production rate at the sandface is given by,

\[
q_t(r_w) = -2\pi khC_1 \cdot \left( k_{ro} \frac{p_o}{\mu_o} + k_{rg} \frac{p_g}{\mu_g} \right) \cdot r_w \frac{\partial p}{\partial r} \tag{4.3.8}
\]

Rearranging equation (4.3.6),

\[
q_t(r_w) = -2\pi khC_1 \cdot \left( k_{ro} \frac{p_o}{\mu_o} + k_{rg} \frac{p_g}{\mu_g} \right) \cdot r \frac{\partial p}{\partial r} \cdot \frac{r_e^2 - r_w^2}{r_e^2 - r_w^2} \tag{4.3.9}
\]

\[
= q_t(r) \cdot \frac{r_e^2 - r_w^2}{r_e^2 - r_w^2}
\]

Equation (4.3.9) allows us to compute first the flow rate at any location in the reservoir based on the flow rate at the sandface. Second, it allows us to compute the pressure derivative at the radius of the average pressure in the reservoir by applying equation (4.3.7) at the average radius \( \bar{r} \) and substituting it in equation (4.3.9). The resulting expression is
\[
\left( \frac{\partial p}{\partial r} \right) = \frac{q_t(r_w)}{\lambda_t(\bar{r})} \cdot \frac{r_e^2}{r_w^2} \cdot \frac{\bar{r} - \bar{r}}{\lambda_t(\bar{r})} \cdot \frac{-1}{2 \pi \rho \rho C_f}
\] (4.3.10)

We can apply this expression at the average radius since we can evaluate the average total mobility \( \lambda_t(\bar{r}) \) from the single-cell simulator and the total molar rate is held constant.

Application of equation (4.3.10) to a black oil fluid is shown in Fig. 4.3.3. The simulation derivative is computed by a simple central difference formula based on the simulation values at the average radius (\( \bar{r} \)), and the computed derivative is based total fluid mobility at the average radius as determined from the single cell simulator and was used in equation (4.3.10) to compute the corresponding pressure derivative.

The result in equation (4.3.9) could also be used to determine the total mobility at the sandface if a method is available to determine the spatial pressure derivative at the wellbore. The equation then becomes to be applied is,

\[
\lambda_t(r_w) = \lambda_t(\bar{r}) \left( \frac{\partial p/ \partial r}{\partial p/ \partial r}_{r_w} \right) \cdot \frac{r_e^2}{r_w^2} \cdot \frac{r_w - r_w}{r_e^2/\bar{r} - \bar{r}}
\] (4.3.11)

Note that all the data necessary to evaluate the total fluid mobility at the sandface is available once the spatial pressure derivative at the sandface is determined.
Fig. 4.3.3 Spatial Pressure Derivative from Simulation Compared to Derivative Computation Based on the Single-Cell Simulator and Equation (4.3.10) for Black Oil Fluid 4 (Case 2)
CHAPTER V

SUMMARY

5.1 Conclusions
1. The overall composition in the reservoir follows a path characterized by the ‘tie-line relationship’ for most of the reservoir life.
2. When the tie-line relationship is valid the vapor and liquid phase compositions in the reservoir for all the fluid types discussed here (gas condensate, volatile oil, and black oil) can be estimated accurately by performing a simple constant composition expansion (CCE) process on the overall composition of the initial reservoir fluid. This accuracy remains even when the overall in-situ composition in the reservoir changes greatly.
3. When the composition path in the reservoir deviates from the tie-line relationship the CCE process yields phase compositions that are not accurate.
4. The tie-line relationship is valid for 3-component fluids as well as for higher number of components.
5. The tie-line relationship when applicable constitutes a straight line on the ternary diagram that passes through all the vapor and liquid compositions during depletion.
6. The vapor and liquid compositions at the point where the tie-line relationship starts to fail can be used to simulate the reservoir performance in a black oil simulator to predict the reservoir performance from the start of production up to the point where the tie-line relationship fails.
7. The tie-line relationship fails when the gas phase composition becomes heavier.
8. The applicability of the constant composition expansion (CCE) or the constant volume depletion (CVD) laboratory experiments to model reservoir performance depends on both the type of fluid in the reservoir and the stage of depletion.
9. The (CVD) experiment can predict the average pressure-saturation relation accurately for gas condensate reservoirs for the entire depletion history when the critical oil saturation is higher than the liquid drop-out from the fluid in the reservoir.

10. Although the CCE process can model the gas condensate depletion behavior early in the reservoir life, this method does deviate from the reservoir performance at later times due to the compositional changes in the reservoir. The use of this method without taking this into consideration can lead to errors in estimating the original mass in-place.

11. The Standard CVD process can not model the performance of volatile oil or black oil reservoirs at any point of their life. While the CCE process can model the initial part of depletion of these reservoirs, it deviates greatly later and can not be reliably used for predicting the pressure-saturation relation for the average reservoir.

12. A hybrid procedure proposed here that combines the CVD and CCE effects for volatile and black oil fluids showed that the depletion behavior in the reservoir does not only depend on the critical saturation but also depends on the shape of the relative permeability curves.

13. The use of fractional flow curves based on the production rate at the sandface as a production criteria for a modified CVD procedure (to produce both phases) gave better match to reservoir performance than either CCE or CVD and has the advantage of not requiring the critical gas saturation values.

14. The difference between the reservoir performance and the performance based on the fractional flow curves is attributed to the difference in the pressure-saturation relation at the wellbore from that at the average radius.

15. A single-cell simulator (zero-dimensional material balance model) developed here and uses the relative permeability and its derived mobility to control the flow of both phases from the laboratory cell can provide accurate reservoir performance prediction for the entire reservoir life.

16. The ability of the single-cell simulator to model the reservoir performance throughout the reservoir life proves the applicability of a tank model to predict reservoir performance even when large compositional changes occur in the reservoir.
17. The single-cell simulator not only predicts the average pressure-saturation relation but also accurately predicts the reservoir recovery as function of pressure for both volatile oil and black oil reservoirs.

18. When the original mass, in-place is known, the average pressure and average saturation of the reservoir can be predicted throughout the reservoir life as function of time using the results obtained from the single-cell simulator.

19. The reservoir performance (pressure-saturation relation) is not affected by the mode of production (constant pressure or constant total molar rate) for any rate of production.

20. Reservoir permeability or initial pressure level with respect to saturation pressure have no effect on the reservoir performance (pressure-saturation relation).

21. The two-phase compressibility factor is the correct compressibility factor that should be used in estimating the original mass in-place (OMIP). The use of a two-phase compressibility factor based on CCE improved the estimate of the (OMIP) by only 1.2% for gas condensate reservoirs compared to the estimate based on the vapor phase compressibility factor alone.

22. The use of a two-phase compressibility factor based on CCE in an oil reservoir can results of underestimation of OMIP by up to 34%, while the use of the liquid phase compressibility factor can result in overestimation of the OMIP by up to 76%. The use of a two-phase compressibility factor based on CVD is closer than the other two methods and results in overestimation of the OMIP by 10 to 20%.

23. The use of a compressibility factor based on the single-simulator results can predict the OMIP accurately.

24. The producing gas-oil ratio does not become constant throughout the reservoir until both phases are mobile.

25. At late stages of depletion the gas-oil ratio increases sharply close to the wellbore compared to deeper regions in the reservoir.

26. Pressure and saturation profiles in the reservoir can be predicted by using the pressure-saturation relation obtained from the single-cell simulator.
27. The overall in-situ compositional changes during depletion in the reservoir can affect the uniqueness and applicability of the pressure-saturation relation obtained at the average radius to other locations in the reservoir.

28. The accuracy of the predicted pressure and saturation profiles in the reservoir depends on the applicability of the uniqueness of the pressure-saturation relation throughout the reservoir.

29. The pressure-saturation relation obtained from the single-cell simulator can provide a good estimate to the pseudopressure integral based on the simulation pressure and saturation profiles.

5.2 Directions for Further Research

1. Tie-Line Relationship:
   
   a. Investigate the effects that cause the tie-line relationship to fail. It always failed late in the life of the reservoir when the pressure becomes low. It mainly fails when the gas composition starts to get heavier indicating the vaporization of the heavier components. More study is needed to establish the limits of the tie-line relationship.

   b. Test the tie-line relationship for pseudo-components on a pseudo-ternary diagram. In this case we can have more than three components and still can visualize the relationship.

2. Develop an analytic expression for the saturation changes with location. Using such relationship will allow us to use the single-cell simulator results to compute the saturation distribution for each location and as a function of pressure.
REFERENCES


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