PRODUCTION RATE AND CUMULATIVE PRODUCTION MODELS FOR ADVANCED DECLINE CURVE ANALYSIS OF GAS RESERVOIRS

A Dissertation

by

JOSEPH ANSAH

Submitted to the Office of Graduate Studies of Texas A&M University in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 1996

Major Subject: Petroleum Engineering
PRODUCTION RATE AND CUMULATIVE PRODUCTION MODELS FOR ADVANCED DECLINE CURVE ANALYSIS OF GAS RESERVOIRS

A Dissertation

by

JOSEPH ANSAH

Submitted to Texas A&M University in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY

Approved as to style and content by:

[Signatures of Committee Members]

August 1996

Major Subject: Petroleum Engineering
ABSTRACT

Production Rate and Cumulative Production Models for Advanced
Decline Curve Analysis of Gas Reservoirs.
(August 1996)
Joseph Ansah, B.S., Moscow Institute of Oil and Gas
Chair of Advisory Committee: Dr. Thomas A. Blasingame

Production rate and cumulative production data from gas reservoirs are routinely analyzed using dimensionless "type curves", which are plots of numerical, analytical, or sometimes empirical solutions to the gas diffusivity equation. These solutions are theoretically developed by coupling the boundary-dominated flow equation with the gas material balance equation. Due to the mathematical difficulty of this process, past efforts have focused on approximate models based on simplifying assumptions. Two basic limitations of these models are:

- The use of approximate linearization schemes (i.e., the zero-order and first-order polynomial models) for correlating the non-linear viscosity-compressibility term, and
- Assumption of zero or constant bottomhole pressure production.

This work aims to eliminate these restrictive assumptions by proposing semi-analytical solutions developed from the rigorous equations underlying production rate-time analysis models. Pseudopressure and pseudotime functions are also avoided in this work because knowledge of average reservoir pressure is required for computing pseudotime which makes the procedure iterative.

We examine several linearization schemes (zero-order, first-order and general polynomial functions, as well as exponential function) for modeling the non-linear terms. Simulation studies conducted using different gas systems show that the general polynomial function is applicable to all gas reservoirs.
More importantly, this work demonstrates that a third-order polynomial function is adequate for linearizing the gas flow equation during reservoir depletion. Simultaneous solution of the linearized flow and gas material balance equations yields a first-order ordinary differential equation in terms of cumulative production and time. This formulation can be used for variable bottomhole pressure data analysis since bottomhole pressure is isolated explicitly.

The rate and cumulative production models are derived in terms of dimensionless variables to facilitate development of both analytical and graphical solutions (type curves). Closed form predictive equations for cases of constant bottomhole pressure production using the approximate exponential function are presented — these expressions are accurate only for high pressure gas reservoirs. Numerical solutions are also presented for the general polynomial model.

Comparison of the new solutions with Carter type curves shows excellent agreement between the rate responses. Finally, we can estimate reservoir parameters (original-gas-in-place and permeability) by applying these new solutions to field data.
DEDICATION

This dissertation is dedicated:

to my parents Charles K. Ansah and Mary Assabil for your unconditional love and support, and

to Joco and Sam.

You have all been great inspiration to me.
ACKNOWLEDGMENTS

I wish to express my sincere appreciation to the following for their contributions towards the success of this project:

The Chairman of my Graduate advisory committee, Dr. Thomas A. Blasingame, for his support, guidance, and encouragement during my entire course of study at Texas A&M University. Tom, it has been a great pleasure to work with you and to get to know you.

Drs. Robert A. Wattenbarger, W. John Lee and Raytcho D. Lazarov for their advice and suggestions and for agreeing to serve on my advisory committee.

Dr. Enrique Mallén who served as the Graduate Council Representative on my committee.

In addition, I wish to thank the faculty and staff at the Department of Petroleum Engineering, especially Drs. James E. Russell, Ching H. Wu, and Richard A. Startzman, for their support and encouragement during my graduate studies at Texas A&M University. Special thanks to Dr. John P. Spivey for his insightful suggestions at the initial stages of this project. The invaluable assistance of Ms. Gail Krueger is gratefully acknowledged.

I will also like to acknowledge the help of all my colleagues in the Well Testing Research Group.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>DEDICATION</td>
<td>v</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>vi</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>ix</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xv</td>
</tr>
<tr>
<td><strong>CHAPTER I</strong>  INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Background of Decline Curve Analysis</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Objectives of This Research</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Organization of This Dissertation</td>
<td>5</td>
</tr>
<tr>
<td><strong>CHAPTER II</strong> LITERATURE REVIEW</td>
<td>7</td>
</tr>
<tr>
<td><strong>CHAPTER III</strong> DEVELOPMENT OF THE STABILIZED FLOW AND GAS MATERIAL BALANCE EQUATIONS</td>
<td>17</td>
</tr>
<tr>
<td>3.1 Summary</td>
<td>17</td>
</tr>
<tr>
<td>3.2 Description of the Reservoir Model</td>
<td>18</td>
</tr>
<tr>
<td>3.3 Development of the Gas Equations</td>
<td>20</td>
</tr>
<tr>
<td>3.3.1 The Zero-Order Polynomial Linearization Scheme</td>
<td>24</td>
</tr>
<tr>
<td>3.3.2 The First-Order Polynomial Linearization Scheme</td>
<td>28</td>
</tr>
<tr>
<td>3.3.3 The &quot;Exponential&quot; Linearization Scheme</td>
<td>29</td>
</tr>
<tr>
<td>3.3.4 The &quot;General Polynomial&quot; Linearization Scheme</td>
<td>33</td>
</tr>
<tr>
<td><strong>CHAPTER IV</strong> DEVELOPMENT OF THE NEW RATE-TIME SOLUTIONS FOR DRY GAS RESERVOIRS</td>
<td>38</td>
</tr>
<tr>
<td>4.1 Summary</td>
<td>38</td>
</tr>
<tr>
<td>4.2 Analytical Solution of the Zero-Order Polynomial Model</td>
<td>44</td>
</tr>
</tbody>
</table>
4.3 Analytical Solution of the First-Order Polynomial Model ................. 47
4.4 Analytical Solution of the "Exponential" Model ........................................ 57
4.5 Solution of the "General Polynomial" Model ............................................. 66
4.6 Solution Scheme for Variable Bottomhole Pressure Production .......... 72

CHAPTER V VALIDATION OF THE NEW RATE-TIME SOLUTIONS .......... 76
5.1 Summary ........................................................................................................ 76
5.2 Performance Analysis Relations ................................................................. 76
5.3 Production Under Constant Bottomhole Pressure Conditions ............. 79
5.4 Production Under Variable Bottomhole Pressure Conditions .............. 93
   5.4.1 Simulated Case ......................................................................................... 93
   5.4.2 Field Cases .............................................................................................. 102

CHAPTER VI SUMMARY AND CONCLUSIONS .......................................... 114
6.1 Summary ......................................................................................................... 114
6.2 Conclusions .................................................................................................... 115
6.3 Recommendations for Future Work ........................................................... 116

NOMENCLATURE ............................................................................................. 117
REFERENCES ..................................................................................................... 119

APPENDIX A DERIVATION OF RADIAL DIFFUSIVITY EQUATION FOR
REAL GAS FLOW ................................................................................................. 125

APPENDIX B DERIVATIONS OF THE STABILIZED FLOW AND THE GAS
MATERIAL BALANCE EQUATIONS ........................................................................ 130

APPENDIX C PLOTS OF THE GAS VISCOSITY-COMPRESSIBILITY
FUNCTION ............................................................................................................. 137

APPENDIX D LIST OF FORTRAN COMPUTER CODES .................................. 153

VITA ....................................................................................................................... 168
<table>
<thead>
<tr>
<th>FIGURE</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Physical Representation of the Reservoir Model</td>
<td>19</td>
</tr>
<tr>
<td>3.2</td>
<td>Plot of the Viscosity-Compressibility Function (Fluid Sample 1)</td>
<td>26</td>
</tr>
<tr>
<td>3.3</td>
<td>Plot of the Viscosity-Compressibility Function (Fluid Sample 2)</td>
<td>27</td>
</tr>
<tr>
<td>3.4</td>
<td>Exponential Fit: Viscosity-Compressibility Function (Fluid Sample 1)</td>
<td>30</td>
</tr>
<tr>
<td>3.5</td>
<td>Exponential Fit: Viscosity-Compressibility Function (Fluid Sample 2)</td>
<td>31</td>
</tr>
<tr>
<td>3.6</td>
<td>Log-Log Plot of the Viscosity-Compressibility Function (Fluid Sample 1)</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Including the Fourth-Order Polynomial Fit.</td>
<td></td>
</tr>
<tr>
<td>3.7</td>
<td>Log-Log Plot of the Viscosity-Compressibility Function (Fluid Sample 2)</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>Including the Fourth-Order Polynomial Fit.</td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>Fetkovich Type Curves.</td>
<td>40</td>
</tr>
<tr>
<td>4.2</td>
<td>&quot;Zero-Order Polynomial&quot; Solutions of the Real Gas Flow Equation under</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>Boundary-Dominated Flow Conditions.</td>
<td></td>
</tr>
<tr>
<td>4.3</td>
<td>Dimensionless Pressure Solutions from the &quot;Zero-Order Polynomial&quot; Model</td>
<td>49</td>
</tr>
<tr>
<td>4.4</td>
<td>&quot;First-Order Polynomial&quot; Solutions of the Real Gas Flow Equation under</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>Boundary-Dominated Flow Conditions.</td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>Dimensionless Pressure Solutions from the &quot;First-Order Polynomial&quot; Model</td>
<td>56</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>4.6</td>
<td>&quot;Exponential&quot; Solutions of the Real Gas Flow Equation under Boundary-Dominated Flow Conditions. Note the Use of ( p_{wD} ) as the Correlating Parameter........................................ 60</td>
<td></td>
</tr>
<tr>
<td>4.7</td>
<td>Plot of the ( W )-Function.................................................. 61</td>
<td></td>
</tr>
<tr>
<td>4.8</td>
<td>&quot;Exponential&quot; Solutions of the Real Gas Flow Equation under Boundary-Dominated Flow Conditions. Transformed Solutions Using ( \beta ) as the Correlating Parameter (Fluid Sample 1)................................. 63</td>
<td></td>
</tr>
<tr>
<td>4.9</td>
<td>&quot;Exponential&quot; Solutions of the Real Gas Flow Equation under Boundary-Dominated Flow Conditions. Transformed Solutions Using ( \beta ) as the Correlating Parameter (Fluid Sample 2)........................................... 64</td>
<td></td>
</tr>
<tr>
<td>4.10</td>
<td>Dimensionless Pressure Solutions from the &quot;Exponential&quot; Model......... 65</td>
<td></td>
</tr>
<tr>
<td>4.11</td>
<td>&quot;General Polynomial&quot; Solutions of the Real Gas Flow Equation under Boundary-Dominated Flow Conditions (Fluid Sample 1)............................. 68</td>
<td></td>
</tr>
<tr>
<td>4.12</td>
<td>&quot;General Polynomial&quot; Solutions of the Real Gas Flow Equation under Boundary-Dominated Flow Conditions (Fluid Sample 2)............................. 69</td>
<td></td>
</tr>
<tr>
<td>4.13</td>
<td>Dimensionless Pressure Solutions from the &quot;General Polynomial&quot; Model. Low Pressure Gas Reservoir Case (( p_r = 4000 ) psia)....................................... 70</td>
<td></td>
</tr>
<tr>
<td>4.14</td>
<td>Dimensionless Pressure Solutions from the &quot;General Polynomial&quot; Model. High Pressure Gas Reservoir Case (( p_r = 12000 ) psia)................................. 71</td>
<td></td>
</tr>
<tr>
<td>5.1</td>
<td>Comparison of the New Rate-Time Solutions with Carter Type Curves (Numerical Solutions). Low Pressure Gas Reservoir Case (( p_r = 4000 ) psia)................................................................. 81</td>
<td></td>
</tr>
<tr>
<td>5.2</td>
<td>Comparison of the New Rate-Time Solutions with Carter Type Curves (Numerical Solutions). Moderately-Pressured Gas Reservoir Case (( p_r = 8000 ) psia)................................. 82</td>
<td></td>
</tr>
<tr>
<td>FIGURE</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>5.3</td>
<td>Comparison of the New Rate-Time Solutions with Carter Type Curves (Numerical Solutions). High Pressure Gas Reservoir Case $\left(p_l=12000\text{ psia}\right)$.</td>
<td>83</td>
</tr>
<tr>
<td>5.4</td>
<td>Decline Curve Analysis for Case 1 (&quot;West Virginia Gas Well A&quot; — SPE 14238). &quot;Knowles&quot; Type Curve Analysis Approach.</td>
<td>86</td>
</tr>
<tr>
<td>5.5</td>
<td>Decline Curve Analysis for Case 1 (&quot;West Virginia Gas Well A&quot; — SPE 14238). &quot;General Polynomial&quot; Type Curve Analysis Approach.</td>
<td>87</td>
</tr>
<tr>
<td>5.6</td>
<td>&quot;General Polynomial&quot; Type Curve Analysis for Case 2 (Simulated Case). The Analysis Assumes $D_l=0.02039\text{ D}^{-1}$ — Correct Value.</td>
<td>95</td>
</tr>
<tr>
<td>5.7</td>
<td>&quot;General Polynomial&quot; Type Curve Analysis for Case 2 (Simulated Case). The Analysis Assumes $D_l=0.0243\text{ D}^{-1}$ — High Value.</td>
<td>97</td>
</tr>
<tr>
<td>5.8</td>
<td>&quot;General Polynomial&quot; Type Curve Analysis for Case 2 (Simulated Case). The Analysis Assumes $D_l=0.0184\text{ D}^{-1}$ — Low Value.</td>
<td>98</td>
</tr>
<tr>
<td>5.9</td>
<td>Conventional $p/z$ versus $G_p$ Plot for Case 2 (Simulated Variable Bottomhole Pressure Production Case).</td>
<td>101</td>
</tr>
<tr>
<td>5.10</td>
<td>&quot;General Polynomial&quot; Type Curve Analysis of &quot;Gas Well A&quot; (SPE 25909). The Analysis Assumes $D_l=0.006\text{ D}^{-1}$.</td>
<td>104</td>
</tr>
<tr>
<td>5.11</td>
<td>Conventional $p/z$ versus $G_p$ Plot for &quot;Gas Well A&quot; Data (SPE 25909).</td>
<td>105</td>
</tr>
<tr>
<td>5.12</td>
<td>&quot;General Polynomial&quot; Type Curve Analysis of &quot;Gas Well A&quot; (SPE 25909). This Analysis Uses $D_l=0.00582\text{ D}^{-1}$ — Correct Value.</td>
<td>106</td>
</tr>
<tr>
<td>5.13</td>
<td>Plot of Field Production Rate and Flowing Bottomhole Pressure Profiles for &quot;Gas Well B&quot;.</td>
<td>108</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>5.14</td>
<td>&quot;General Polynomial&quot; Type Curve Analysis of &quot;Gas Well B&quot;. This Analysis Assumes $D_i = 0.025$ D$^{-1}$. ........................................ 110</td>
<td></td>
</tr>
<tr>
<td>5.15</td>
<td>Conventional $p/z$ versus $G_p$ Plot for &quot;Gas Well B&quot; Data. .................. 112</td>
<td></td>
</tr>
<tr>
<td>5.16</td>
<td>&quot;General Polynomial&quot; Type Curve Analysis of &quot;Gas Well B&quot;. This Analysis Uses $D_i = 0.02413$ D$^{-1}$. — Correct Value. .................. 113</td>
<td></td>
</tr>
<tr>
<td>A.1</td>
<td>A Unit Cell from a Three-Dimensional Porous Medium. ......................... 125</td>
<td></td>
</tr>
<tr>
<td>C.1</td>
<td>Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.55$ and $T = 150$ °F. ............... 138</td>
<td></td>
</tr>
<tr>
<td>C.2</td>
<td>Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.60$ and $T = 150$ °F. ............... 138</td>
<td></td>
</tr>
<tr>
<td>C.3</td>
<td>Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.70$ and $T = 150$ °F. ............... 139</td>
<td></td>
</tr>
<tr>
<td>C.4</td>
<td>Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.80$ and $T = 150$ °F. ............... 139</td>
<td></td>
</tr>
<tr>
<td>C.5</td>
<td>Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.90$ and $T = 150$ °F. ............... 140</td>
<td></td>
</tr>
<tr>
<td>C.6</td>
<td>Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 1.00$ and $T = 150$ °F. ............... 140</td>
<td></td>
</tr>
<tr>
<td>C.7</td>
<td>Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.55$ and $T = 200$ °F. ............... 141</td>
<td></td>
</tr>
<tr>
<td>C.8</td>
<td>Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.60$ and $T = 200$ °F. ............... 141</td>
<td></td>
</tr>
</tbody>
</table>
FIGURE

C.9  Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.70$ and $T = 200 \, ^\circ\text{F}$ .......... 142

C.10  Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.80$ and $T = 200 \, ^\circ\text{F}$ .......... 142

C.11  Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.90$ and $T = 200 \, ^\circ\text{F}$ .......... 143

C.12  Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 1.00$ and $T = 200 \, ^\circ\text{F}$ .......... 143

C.13  Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.55$ and $T = 250 \, ^\circ\text{F}$ .......... 144

C.14  Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.60$ and $T = 250 \, ^\circ\text{F}$ .......... 144

C.15  Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.70$ and $T = 250 \, ^\circ\text{F}$ .......... 145

C.16  Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.80$ and $T = 250 \, ^\circ\text{F}$ .......... 145

C.17  Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.90$ and $T = 250 \, ^\circ\text{F}$ .......... 146

C.18  Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 1.00$ and $T = 250 \, ^\circ\text{F}$ .......... 146

C.19  Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.55$ and $T = 300 \, ^\circ\text{F}$ .......... 147
FIGURE

C.20 Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.60$ and $T = 300 \, ^\circ\text{F}$............. 147

C.21 Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.70$ and $T = 300 \, ^\circ\text{F}$............. 148

C.22 Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.80$ and $T = 300 \, ^\circ\text{F}$............. 148

C.23 Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.90$ and $T = 300 \, ^\circ\text{F}$............. 149

C.24 Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 1.00$ and $T = 300 \, ^\circ\text{F}$............. 149

C.25 Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.55$ and $T = 350 \, ^\circ\text{F}$............. 150

C.26 Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.60$ and $T = 350 \, ^\circ\text{F}$............. 150

C.27 Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.70$ and $T = 350 \, ^\circ\text{F}$............. 151

C.28 Plot of the Viscosity-Compressibility Ratio versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.80$ and $T = 350 \, ^\circ\text{F}$............. 151

C.29 Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.90$ and $T = 350 \, ^\circ\text{F}$............. 152

C.30 Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 1.00$ and $T = 350 \, ^\circ\text{F}$............. 152
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Summary of Reservoir Fluid Samples</td>
<td>26</td>
</tr>
<tr>
<td>3.2</td>
<td>Coefficients of the &quot;Exponential&quot; Model (Fluid Sample 1)</td>
<td>31</td>
</tr>
<tr>
<td>3.3</td>
<td>Coefficients of the &quot;Exponential&quot; Model (Fluid Sample 2)</td>
<td>32</td>
</tr>
<tr>
<td>3.4</td>
<td>Coefficients of the Fourth-Order Polynomial Model (Fluid Sample 1)</td>
<td>35</td>
</tr>
<tr>
<td>3.5</td>
<td>Coefficients of the Fourth-Order Polynomial Model (Fluid Sample 2)</td>
<td>36</td>
</tr>
<tr>
<td>4.1</td>
<td>Values of the W-Function</td>
<td>62</td>
</tr>
<tr>
<td>5.1</td>
<td>Input Data for Reservoir Simulation</td>
<td>80</td>
</tr>
<tr>
<td>5.2</td>
<td>Reservoir, Fluid, and Production Data for Case 1 (&quot;West Virginia Gas Well A&quot;)</td>
<td>85</td>
</tr>
<tr>
<td>5.3</td>
<td>Analysis Results for Case 1 (&quot;West Virginia Gas Well A&quot;)</td>
<td>92</td>
</tr>
<tr>
<td>5.4</td>
<td>Reservoir, Fluid, and Production Data for Case 2 (Simulated Case)</td>
<td>94</td>
</tr>
<tr>
<td>5.5</td>
<td>Bottomhole Pressure Changes for Case 2 (Simulated Case)</td>
<td>94</td>
</tr>
<tr>
<td>5.6</td>
<td>Analysis Results for Case 2 (Simulated Case)</td>
<td>100</td>
</tr>
<tr>
<td>5.7</td>
<td>Reservoir, Fluid, and Production Data for Case 3 (&quot;Gas Well A&quot; — from SPE 25909)</td>
<td>102</td>
</tr>
<tr>
<td>5.8</td>
<td>Analysis Results for Case 3 (&quot;Gas Well A&quot;) — Variable Bottomhole Pressure Production</td>
<td>107</td>
</tr>
<tr>
<td>5.9</td>
<td>Reservoir, Fluid, and Production Data for Case 4 (&quot;Gas Well B&quot;)</td>
<td>109</td>
</tr>
<tr>
<td>TABLE</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>5.10 Analysis Results for Case 4 (&quot;Gas Well B&quot;) — Variable Bottomhole Pressure Production</td>
<td>111</td>
<td></td>
</tr>
<tr>
<td>D.1 Test1.dat — Example Data File for the Program RKFV (Constant BHP Case)</td>
<td>166</td>
<td></td>
</tr>
<tr>
<td>D.2 WellA.dat — Example Data File for the Program RKFV (Variable BHP Case)</td>
<td>166</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

This chapter introduces the historic concept of production rate-time analysis (popularly described as "Decline Curve Analysis"). The method involves matching of actual production rate and cumulative production data as a function of time against a theoretical model generated analytically or numerically, and presented graphically in the form of "type curves". Values of a match point from this exercise are used to calculate formation parameters such as the original-gas-in-place, effective permeability, skin factor, etc.

1.1 Background of Decline Curve Analysis

As a result of the increased demand on the world crude oil resources, together with the need for environmentally clean-burning fuels for the large fleets of automobiles, planes, ships, power plants, etc. — more and more attention is being directed to the use of natural gas. In the United States alone, the demand for natural gas has been increasing steadily at around 4 percent annually for the past decade. It is projected that the world demand of natural gases will be approximately 3 percent annually well into the early part of the 21st Century.¹ This high level of demand requires the use of efficient methods for the evaluation of well performance and the prediction of future production potential of gas reservoir systems. Development of such evaluation and prediction methodologies is the overall objective of this dissertation.

Natural gas resources are important for the U.S. as well as the global economies in so many ways. Among the obvious reasons are availability and abundance, the relatively low cost of production, as well as the minimum adverse impact of natural gas on the environment. Environmentally, natural gas is a clean burning fuel. Technology will play a key role in the future for attaining the necessary equilibrium between rising demand and the supply of gas resources — from seismic methods (2-D and 3-D) and drilling technologies (horizontal, multi-lateral, etc.) to integrated reservoir characterization, as well as the development and management of discovered reservoirs.

¹ This dissertation follows the style of the Journal of Petroleum Technology.
To provide for the optimal management of gas reservoirs we focus on the development and application of improved methods to estimate reservoir properties, predict reserves, and forecast production performance potential. Specifically, these methods are directed at determining the following parameters:

- Original-gas-in-place (OGIP)
- Formation properties (e.g. permeability or flow capacity, and "skin factor")
- Type and strength of the reservoir drive mechanism
- The future performance potential of individual wells
- The economic limit of the well and reservoir, and
- The expected ultimate recovery (EUR).

Historically, estimates of gas reserves have been obtained from the analysis of reservoir pressure data (using material balance methods) — specifically from the analysis of plots of \( p/z \) versus cumulative gas production, \( G_p \). However, obtaining representative (average) reservoir pressure data is often difficult due to the significant shut-in times required to establish stabilized reservoir conditions, especially in tight (i.e., low permeability) reservoirs. In most cases static reservoir pressures are simply not available.

One important piece of information that is measured accurately and routinely (in fact daily) is the well production rates. Hence, production rate-time analysis represents an important method for obtaining accurate and timely estimates of reservoir parameters. The unique advantage of not requiring average reservoir pressure data directly for such analyses makes decline curve analysis one of the most cost-effective methods in reservoir engineering. The method is based on fundamental flow models, which include the stabilized (boundary-dominated) flow equation and the gas material balance equation.

Decline curve analysis is an integrated type curve approach, which provides the engineer with the means to infer parameters of individual wells, or using the aggregated performance (i.e., the total rate), an entire reservoir can be analyzed. The method typically involves a plot of production rate, \( q_g \), and/or other rate functions (e.g. cumulative gas production, rate derivative, rate integral-derivative, etc.) versus time. This plot is then "matched" against a theoretical model, either explicitly (using regression analysis), or graphically in the form of type curves. As mentioned above, we can estimate OGIP as
well as formation properties from this match. These results are then used to predict the well's economic limit and the expected ultimate recovery for a particular well or an entire reservoir.

Decline curve analysis has historically been performed using theoretical models which assume production at constant bottomhole pressure. 2-5 For a long time, development of these models, which are products of simultaneous solution of the stabilized rate and material balance equations, also use either zero or a constant non-zero bottomhole pressure assumption. This technique has been applied quite successfully over the years, allowing for the development of several semi-analytical expressions for the analysis of production rate and cumulative production data.

However, there are limitations to the current approaches, which are mainly twofold. The first basic limitation is the assumption of zero or constant (non-zero) bottomhole pressure production which ignores actual production practices in the petroleum industry where bottomhole pressures are lowered steadily over the productive life of the well. Changes in flowing bottomhole pressure are very common, and are caused by changes in line pressures at the wellhead, changes in separator working pressures, and occasional well shut-ins due to low market demand and equipment failures, among other reasons. The second limitation is the use of pseudotime functions, as well as other approximate functional models, to linearize the non-linear gas flow equations. The pseudotime approach is computationally intensive because it requires iterative techniques to compute average reservoir pressures needed for the calculation of these pseudotime functions. The use of pseudotime also makes performance predictions unwieldy (some sort of extrapolation must be used).

1.2 Objectives of This Research

The purpose of our current research is to introduce simple linearization models, as well as a general polynomial linearization scheme, for the non-linear gas flow equation by correlating the variation of the viscosity-compressibility product with pressure \( (p/z) \) during reservoir depletion. This approach allows for direct use of pressure and real time in decline curve analysis.

Our focus is on the development of rigorous, but simple, theoretical rate prediction equations, which will provide a means for comparison directly with field production data.
and which can be used to estimate formation properties and gas reserves. Specifically, this study attempts to accomplish the following major objectives:

1. To develop explicit rate prediction equations for gas reservoir systems. We accomplish this by coupling
   - the most accurate stabilized flow equation (in terms of normalized pseudo-pressure), and
   - the gas material balance equation (MBE).

2. To develop a rigorous approach for analyzing rate and flowing bottomhole pressure data directly (without the use of pseudotime) using decline type curve analysis. These analyses will include both
   - production at a constant bottomhole pressure, and more importantly
   - production under variable flowing bottomhole pressure conditions.

The models developed in this work are presented both in terms of dimensionless variables to facilitate the development of analytical relations and graphical solutions (type curves), as well as field variables for performance predictions. Specifically, closed form predictive equations for special cases of zero and constant non-zero flowing bottomhole pressure (BHP) cases are presented to investigate the effect of bottomhole pressure on production rate decline of gas wells. In addition, a closed form predictive model for the general variable-BHP production case is presented. This model is solved by numerical integration and the results are presented in the form of dimensionless type curves. These results are correlated in terms of a new parameter $\beta$.

Another product of this work is the development of explicit equations for estimating average reservoir pressures for both low and high pressure gas reservoirs during boundary-dominated flow. These solutions are also presented in dimensionless format for convenience, as well as in field variables for specific applications.

Our intention is to develop the most accurate rate-time models for the gas well performance analysis, which includes analysis of production rates and bottomhole pressure data. This analysis provides estimates of such reservoir parameters as

- OGIP
• Reservoir properties (effective permeability, \( k_g \), the permeability-thickness product, \( k_g h \), and the skin factor, \( s \)) and

By establishing the pertinent reservoir parameters via history matching of the past production data using our new models, we are able to predict future performance of individual wells as well as the reserves of an entire reservoir.

1.3 Organization of This Dissertation

Chapter II presents a detailed historic perspective of the decline curve analysis concept, including a summary of important developments in the literature on this subject as they relate to gas reservoir systems. The chapter demonstrates the fact that the use of pseudofunctions (\( i.e., \) pseudopressure and pseudotime) for the decline curve analysis of gas wells has been quite extensive in the past. The important reason that these pseudofunctions rigorously linearizes the gas flow equation has been emphasized by various authors. The chapter concludes with a discussion of recent techniques for using simple functional models to linearize the gas flow equation. These models allow for the use of real variables for production rate-time analysis and provide approximate, but direct, solutions for analyzing gas well production data.

The fundamental equations used in the development of our new "polynomial" models are summarized in Chapter III. The first section summarizes these equations followed by a description of the reservoir system used in this study. The main assumptions in the "polynomial" models are also discussed. The last section presents a systematic derivation of the different linearized forms of the gas flow equation using simple functional models to characterize fluid property changes during reservoir depletion. We note that these fundamental equations comprise of the stabilized rate equation for boundary-dominated flow and the gas material balance equation. Time is introduced into this model by using the simple differential expression relating gas production rate and cumulative production.

Due to the mathematical difficulties of using the non-linear stabilized rate equation in terms of pseudopressure, this equation has been linearized using simple functional expressions. Chapter III examines our motivation behind the use of these simple linearization schemes as well as the rationale for developing the "General Polynomial" model. The chapter concludes with the development of the most accurate linearized flow equation using a general polynomial function.
Chapter IV outlines the process for the simultaneous solution of the fundamental equations to yield direct rate prediction equations for decline curve analysis. The objective in this process is to eliminate average reservoir pressure (or pseudopressure) from these fundamental relations thereby yielding a single equation involving cumulative gas production and time. The rate prediction equations are then obtained by differentiating this cumulative production versus time solution. Explicit analytical expression for high pressure gas reservoirs involving an approximate exponential linearization model is also presented in this chapter.

Finally, Chapter IV discusses important computational issues involving the "General Polynomial" model. These issues include the development of a Runge-Kutta-Fehlberg algorithm with adaptive timestep selection option for solving the resulting first-order differential equation and application of this new model to the analysis of variable bottomhole pressure production data.

In Chapter V, we present validation of the new rate-time solutions using simulated rate responses (numerical solutions) as well as practical application of our new solutions to the analysis of actual field data. The example cases include data from gas wells with both constant and variable flowing bottomhole pressure profiles.

Chapter VI discusses the important conclusions as well as our recommendations for applying the results of this research. The fact that a simple semi-analytical rate prediction model has been developed for universal application is highlighted, including the potential extension of this approach to the analysis of production data from solution-gas drive reservoir systems.
CHAPTER II

LITERATURE REVIEW

Attempts to theoretically model production rate decline and cumulative production curves of gas and oil wells date as far back as the early part of this century. In 1921, a detailed summary of the most important findings of the early research activities in this area was documented in the Manual for the Oil and Gas Industry.⁶ This treatise, which is mainly a compilation of the research work of the U.S. Bureau of Mines personnel, first noted the exponential decline model for oil wells, as well as the use of graphical techniques in the form of percentage decline curves (i.e., "hyperbolic" declines) for analysis of production rate data from gas wells.

Several efforts were made during the years immediately thereafter, and probably the most significant contribution towards the development of the modern decline curve analysis concept is the classic paper by Arps⁷ presented in 1944. In this paper, Arps presented a set of exponential and hyperbolic equations for production rate analysis. Although the basis of Arps' development was purely statistical, and therefore empirical in nature, these historic results have found widespread appeal in the oil and gas industry. The continuous use of these so-called "Arps equations" to date is basically due to the explicit nature of the relations and the ease of application of these equations to field data.

Arps also introduced the concepts of a decline exponent, \( b \), and a decline rate constant, \( D_r \), both of which have become the cornerstone of many subsequent research efforts. By estimating these important parameters through history matching, Arps demonstrated the technique of extrapolating rate-time data following exponential and hyperbolic declines using a semilog plot. While the exponential decline is the simplest model to use, especially in decline curve analysis of oil wells, this model also yields the most conservative estimates of in-place fluids and production rates. On the other hand, the harmonic decline model, gives the most optimistic estimates when used for predicting future production rates.
Research following Arps' publication concentrated on improving the forecasting of production data based on a general hyperbolic model. In 1968, Slider\(^8\) presented a new curve matching technique for obtaining a more accurate extrapolation of production rate data following the hyperbolic decline model. This approach was also based on semilog analysis. In addition, the author demonstrated a practical curve-fitting method using preconstructed theoretical decline curves based on Arps' equations. This technique was presented as simple and more effective than other decline curve analysis techniques although significant amount of work was required in data preparation.

In 1972, Gentry\(^9\) published a new set of type curves, again with the same goal of improving the extrapolation of production decline curves. The main feature of these new type curves lies in the re-arrangement of Arps' original rate prediction equations into simpler plotting functions — the ratio of the initial rate to the production rate at other times versus decline rate constant-time product. The author also derived cumulative production plotting functions and presented the results graphically for each of the same decline exponents introduced by Arps. Gentry showed that, by matching the actual production history against these type curves, the basic reservoir parameters needed for any rate extrapolation (decline exponent and the initial decline rate) could be estimated with much confidence.

The next major development in production decline analysis technology occurred in 1980, when Fetkovich\(^2\) presented a unified type curve based on Arps' empirical equations, coupled with the transient rate solutions for bounded reservoir systems. It is worth mentioning that this publication was the final form of an earlier work presented in 1973 on exactly the same subject. Fetkovich showed that transient solutions for a well in a bounded reservoir, producing at a constant bottomhole pressure could be combined with the Arps' solutions to yield a single type curve. In fact, the idea of using type curves for the analysis of rate data had been introduced earlier in the literature by Gentry (Ref. 9), although Gentry's approach was not as robust as the approach presented by Fetkovich.

In addition, Fetkovich introduced the important concept of combining the pseudosteady-state rate equation with the material balance equation in order to generate semi-analytical rate-time solutions for both gas and oil reservoirs. These attempts were aimed at providing a theoretical basis to Arps' empirical equations which were used in developing the new type curves. Due to difficulties with certain variables, Fetkovich defined new
dimensionless variables for rate and time which facilitated the unification of the transient flow solutions with Arps’ boundary-dominated equations. Unfortunately, the analytical solutions derived by Fetkovich were only for a zero flowing bottomhole pressure and, as such, these results have proved of little practical use.

In 1981, Carter\(^3\) presented different type curves exclusively for the analysis of production rate data of gas reservoir systems. The difficulty associated with the analysis of production data from gas reservoirs, as was noted by the author, is accounting for changes in fluid properties during reservoir depletion — in particular, the variation in the viscosity-compressibility product, \(\mu c_g\), which represents the principal source of non-linearity in gas flow models. We note that, the variation in the \(\mu c_g\) product was ignored by Fetkovich.\(^2\) As part of the new methodology, Carter introduced an average correlating parameter, \(\lambda\), to describe fluid property changes for a pressure range traversed during reservoir depletion. It was established that \(\lambda \leq 1.0\).

Carter\(^3\) defined the parameter \(\lambda\) as

\[
\lambda = \frac{\mu c_g i}{\mu c_g} \tag{2.1}
\]

Using a first-order finite-difference formulation Carter showed that \(\lambda\) depends only on reservoir fluid properties, as well as the initial and the constant flowing bottomhole pressures. As an approximation, the author expressed the average viscosity-compressibility product in terms of the difference between initial and flowing pressures and pseudopressures, which led to the following definition for \(\lambda\)

\[
\lambda = \frac{\mu c_i}{2} \left[ \frac{m(p_i) - m(p_{wf})}{z_i(p_i) - z_{wf}(p_{wf})} \right] \tag{2.2}
\]

where the real gas pseudopressure, \(m(p)\),\(^{10}\) is defined as

\[
m(p) = 2 \int_{p_b}^{p} \frac{p'}{\mu(p)z(p)} dp' \tag{2.3}
\]

We again note that the general solutions used in developing Carter’s type curves were obtained with the help of a reservoir simulator. The average correlating \(\lambda\)-parameter was then used to correlate the different rate solutions (obtained for different initial and producing conditions) into a single type curve. The only variable used in this type curve
development was the parameter $\lambda$, which was initially reported to lie in the range 0.5 and 1.0 — with $\lambda=1.0$ representing the limiting case of liquid flow. Simulation of high pressure gas reservoir systems, beyond those given in Carter’s original publication, shows that the $\lambda$-parameter can have values less than 0.5.

Other attempts have been made to correlate variations in the $\mu c_g$ product during depletion of gas reservoirs. Following an earlier publication in 1985, Fraim and Wattenbarger used a normalized pseudotime function to eliminate the non-linear $\mu c_g$ component present in real gas flow models. The motivation for this work, as indicated by the authors, was that simulation results showed that no case of gas flow produced at constant bottomhole pressure matched any of the exponential or hyperbolic depletion stems presented by Fetkovich type curves when ordinary time is used as a plotting function. On the other hand, using the normalized pseudopressure and pseudotime functions linearizes the gas diffusivity and allows for the use of the liquid flow solution (the exponential decline stem) to analyze gas production data.

Unfortunately, the normalized pseudotime approach requires knowledge of average reservoir pressures as a function of time, data which are difficult, if not impossible, to obtain. Therefore, the authors presented an iterative scheme to facilitate average reservoir pressure calculations. In short, Fraim and Wattenbarger showed that, under constant pressure production, the use of normalized pseudotime will result in a match on the exponential stem of the Fetkovich type curves, provided the correct value of OGIP is used in calculating the average reservoir pressure profile.

In 1986, Aminian, et al. also presented a different set of type curves exclusively for gas production rate decline and cumulative production analyses. These type curves were developed based on a semi-analytical model. Although the development of these type curves is purported to eliminate most of the assumptions made by earlier researchers, the results have not received wide acceptance due to the problem of non-uniqueness when applied to field data. These type curves have been observed to be sensitive to formation characteristics and to variations in reservoir pressures. Aminian, et al. used a correlating parameter $X_i=p/p_{wf}$ for these type curves, where $X_i$ can have virtually any number between zero and infinity.
Similar type curves as those presented by Aminian, *et al.*\(^{11}\) were presented by Schmidt, Caudle, and Miller\(^{12}\) in 1986. As with the previous efforts, these authors included in their development a non-Darcy flow term. Schmidt, *et al.* stressed the importance of considering turbulent flow on gas deliverability, especially at high flow rates around the wellbore. However, due to this additional term, Schmidt, *et al.*'s type curves seem especially sensitive to estimates of reservoir permeability and initial reservoir pressure. The authors emphasized the need to incorporate the effect of permeability in gas decline curve analysis. Schmidt, *et al.* adopted a new expression, \((p_{wf}/z_{wf})/(p_{f}/z_{f})\), (which lies in the range of 0.0 and 1.0) as a correlating parameter for their type curves.

In 1986, Fraim, Lee, and Gatens\(^{13}\) presented a new type curve for the decline curve analysis of production data from vertically fractured gas wells. One of the major contributions of this work is a new set of unified type curves that combines transient solutions with the boundary-dominated flow solutions of vertically fractured wells. The authors presented the new fractured well solutions in the form of a modified-Fetkovich type curve for constant bottomhole pressure production. These type curves were developed for dimensionless fracture conductivities, \(F_{CD}\), from 0.2 to 100 and have, in addition, depletion stems for ratios of drainage area length to fracture half-length, \(x_{e}/x_{f}\), less than 10.

To correct for variations in real gas properties, as well as for non-Darcy flow in the hydraulic fractures, Fraim, *et al.* used a normalized pseudotime function in place of real time. The authors observed that the new type curves are exceptionally useful for modeling long-term deliverability from stimulated low permeability formations with long fractures. Fraim, *et al.* used transformed dimensionless rate and time variables similar to those used by Fetkovich (although redefined in terms of fractured well variables), which enabled all the bounded reservoir solutions to overlay each other.

Taking the lead on the significant advances in the use of type curve for the analysis of pressure transient data in 1987, Harrington, Lee, and Taylor\(^{14}\) presented a paper on practical application of type curve techniques for decline curve analysis and forecasting gas deliverabilities. These authors presented a computer-based superposition approach that attempts to eliminate some of the critical limitations of the available type curves, which assume constant pressure or constant rate production. Harrington, *et al.* noted that the
superposition method enables the correct application of type curve matching techniques to actual field cases where both pressure and rate vary throughout the productive life of a well.

As in most previous methods, Harrington et al.'s technique has a major drawback in the sense that it is computationally intensive. Variations in gas properties during reservoir depletion are accounted for through a continuous update of the gas viscosity-compressibility product at the average pressure within the drainage area of the reservoir. The second drawback of this approach is that permeability, skin, and other near wellbore properties must be known. Nevertheless, the authors claim this as a flexible procedure that overcomes the limitations caused by constant pressure or the constant rate assumptions in most type curve methods.

A series of attempts to model the performance of wells with variable flowing bottomhole pressures began in 1986 with the publication by Blasingame and Lee. These authors presented a new theoretical model for predicting reservoir drainage area size and shape from a well producing a single-phase liquid of small and constant compressibility. The key contribution of this work was that it introduced an approximate model for analyzing actual field production data, characterized by random changes in flowing bottomhole pressure and flowrate data. It was emphasized that this model was valid as long as changes in flow rates are not so severe so as to violate the boundary-dominated flow assumptions made in the development of the method.

Blasingame and Lee's method involves a Cartesian plot of pressure drop divided by rate versus a plotting time function. This plot yields a straight line from which the slope and intercept are used to estimate reservoir pore volume and shape factor, respectively. The authors demonstrated the practical application of the new method to a wide range of variable-rate scenarios for wells in bounded reservoirs. We note that this initial approach was focused on the analysis of liquid flow data.

In 1988, Blasingame and Lee adapted their method to the analysis of variable-rate and variable-pressure data from gas wells. Similar to the case of a slightly compressible liquid, the goal of this work was to estimate reservoir size and shape from production data without resorting to average reservoir pressure data. In this case, the gas diffusivity equation was linearized with pseudotime and pseudopressure functions, where the
pseudotime function now included a variable-rate term, similar to the "material balance time" function presented in the liquid case.\textsuperscript{15} We recall that using the non-linear gas flow model requires estimates of average reservoir pressures versus time in order to evaluate the gas properties needed in the pseudotime calculations.

To address this requirement, Blasingame and Lee developed an iterative procedure for the simultaneous estimation of OGIP which was then used in the average reservoir pressure calculation. These authors provided example applications of the new method for both fractured and unfractured gas wells, and stressed the need for boundary-dominated flow conditions for such analysis. However, Blasingame and Lee made an important observation, from simulation studies, that the calculated average pressure profile may also be valid for analyzing transient gas flow data.

In 1990, Ding, Onur, and Reynolds\textsuperscript{17} presented a new method for directly computing pseudotime functions based on average reservoir pressures obtained from stabilized rate or measured bottomhole pressure data. Using pseudotime for the linearization of the gas flow equation the authors validated the fact that the exponential decline stem of the Fetkovich type curve can be used for decline curve analysis of gas well data. This new method was a significant improvement over the available methods for analyzing gas reservoirs because it was non-iterative. However, as was noted in the paper, this method was approximate, and in addition, required an estimate of the time to the start of pseudosteady state flow in the reservoir. Ding, et al. applied the new method to cases of both constant pressure and constant rate production.

Also in 1990, Aminian, et al.\textsuperscript{18} presented a method for the analysis of a multiwell reservoir system. In this paper, the authors introduced new type curves that take into account many important flow phenomena in gas reservoir systems, including changes in viscosity-compressibility product with pressure depletion and non-Darcy flow effects. To fully account for gas property changes these authors developed a robust iterative scheme which was used to generate the type curves.

Aminian, et al. used a shape/skin factor, $s_{CA}$, introduced by Fetkovich and Vienot\textsuperscript{19} to characterize the interference between gas wells in a multiwell reservoir. One of the key conclusions of the work is that interference between gas wells influences only the Darcy
component of the pressure drop because this term is affected by the shape and relative location of the drainage boundaries. Moreover, the non-Darcy flow effects are significant only in the immediate vicinity of the wellbore and are therefore not significantly affected by interference.

In 1991 Blasingame, McCray and Lee\textsuperscript{20} presented a new method for the analysis of production decline data where the flowing bottomhole pressure varied significantly. These authors observed that, for oil wells, the changes in flowing bottomhole pressure during reservoir depletion precludes the use of the exponential model, or a constant rate exponent, $b$, for conventional decline curve analysis. To correct for the changes in production rate Blasingame, et al. introduced a new superposition time function, which was shown to transform the variable pressure drop/variable rate problem into an equivalent constant bottomhole pressure problem. Blasingame et al. also presented the necessary theoretical foundation to justify the use of the superposition time function. In addition, a recursive algorithm was provided to calculate this constant pressure equivalent time function.

In 1992, Spivey, et al.\textsuperscript{21} presented another approach for analyzing constant and variable flowing bottomhole pressure production data, using a rate integral type curve. The prime motivation of this approach, based on experiences from pressure transient analysis where most engineers use both pressure and pressure derivative type curves, was that the simultaneous matching of both rate and cumulative production data improves the confidence of analyzing noisy field data. For the analysis of variable bottomhole pressure data, these authors adopted the concept of "constant pressure analog time" introduced in the petroleum literature by Blasingame, McCray and Lee\textsuperscript{20}. Moreover, Spivey, et al. provided an algorithm for computing these new cumulative production functions. The liquid functions were based on ordinary time while pseudotime was recommended for gas reservoir systems.

Further work on the development of theoretically more rigorous models for the analysis of variable-rate/variable pressure data was presented by Palacio and Blasingame\textsuperscript{22} in 1993, with the introduction of "modified time functions." The authors demonstrated that the use of these modified pseudotime functions rigorously aligns production data to the harmonic stem ($b=1$) of the Fetkovich type curve. These authors provided a detailed theoretical development of the new concept involving the use of "material balance pseudotime
function." Again, since computation of the pseudotime function involved knowledge of average reservoir pressures, or indirectly original-gas-in-place, the authors also provided a method for the explicit computation of original-gas-in-place without the use of an iterative routine.

Palacio and Blasingame also presented a modified "Fetkovich-Carter" type curve, which incorporates the performance of constant rate, constant pressure, and transient gas flow solutions along with the Arps' decline curve stems. We recall that previous efforts to linearize the gas flow model using pseudotime functions resulted in type curves that align themselves onto the liquid solution (i.e., the exponential stem of Arps' family of curves). This new method, however, uses instead the harmonic stem as a guide for type curve matching and future production forecasting. This is a rigorously correct approach, as shown by the detailed derivation provided in the Appendix (Ref. 22).

A new method for directly computing original-gas-in-place (OGIP) was introduced by Keating, Chen and Wattenbarger\textsuperscript{23} in 1994. These authors used the rigorous form of the stabilized flow equation and the gas material balance equation to develop this method. Keating, \textit{et al.} observed that decline curve analysis can be reduced to a two parameter problem involving the prediction of two unknown parameters — the stabilized the flow constant (the well productivity index), \(J_g\), and the original-gas-in-place, \(G\). The authors noted that once either of these parameters is known, the other can be calculated exactly using either the gas flow equation or the gas material balance equation.

This new method is unique in the sense that both upper and lower bounds of the OGIP can be established exactly, from which the actual value of OGIP is then estimated. However, the approach is semi-iterative, since it is based on "reasonable" estimates of the well productivity index, \(J_g\), which includes the formation properties and parameters of the wellbore. We note that the method also assumes constant pressure production. However, it provides an important tool for estimating the OGIP without resorting to reservoir pressure data which, as noted earlier, require lengthy shut-in of gas wells.

In November 1994, Spivey and Frantz\textsuperscript{24} presented a pressure superposition technique for the analysis of variable-rate/variable pressure data from oil and gas wells. In this case the flowing bottomhole pressure was assumed to vary linearly over time in order to mimic field production practices more realistically. These authors concluded that production rates
obtained from using this superposition method follow actual production data more closely than do those calculated using superposition of constant pressure solutions.

In 1996, Knowles⁴ presented a novel approach for linearizing the gas flow equation by correlating variations in fluid properties during reservoir depletion. We observe that this technique follows the approach introduced by Carter³ in 1981. Instead of the constant parameter linearization (zero-order polynomial function) given in Eq. 2.1, Knowles introduced a first-order polynomial function to model the non-linear viscosity-compressibility term (i.e., \( \mu_c \partial \mu_c/\partial p_l \)) during boundary-dominated flow. This linearization scheme, which is valid for low pressure gas reservoirs, resulted in a pressure-squared form of the stabilized rate equation.

The first-order polynomial model provides the means for directly coupling the stabilized flow equation with the gas material balance equation to yield a simple analytical rate-time equation. In addition, this approach gives a relation for the prediction of average reservoir pressure as a function of time.

Recently, Ansah, Knowles, and Blasingame²⁵ have expanded the theoretical developments by Carter³ and Knowles⁴ with new functional models for analyzing production data from high pressure gas reservoirs. In this paper, Ansah, et al. demonstrated that the approaches given by Carter and Knowles using pressure and real time, instead of pseudofunctions, represent special cases of a general polynomial linearization scheme. Closed form predictive equations for the special cases of zero and constant (non-zero) bottomhole flowing pressures were developed using simple functional models for the viscosity-compressibility term.
CHAPTER III

DEVELOPMENT OF THE STABILIZED FLOW AND GAS
MATERIAL BALANCE EQUATIONS

3.1 Summary

In this chapter we develop the fundamental equations used in the rate and cumulative production models. They comprise of the stabilized flow equation (for boundary-dominated flow) and the gas material balance equation. To ensure the most consistent development we begin our work with the most rigorous forms of the stabilized flow equation expressed in terms of normalized pseudopressure and the gas material balance equation expressed in terms of $p/z$.

For the flow of a single-phase gas, the fundamental equations for modeling the boundary-dominated flow in a reservoir are

\[ q_g = J_g [p_p - p_{pwf}] \]  \hspace{1cm} \text{(stabilized gas flow equation)} \hspace{1cm} (3.1.1)

\[ \frac{p}{z} = \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right] \]  \hspace{1cm} \text{(gas material balance equation)} \hspace{1cm} (3.1.2)

with \( q_g = \frac{dG_p}{dt} \) \hspace{1cm} (3.1.3)

Eqs. 3.1.1 and 3.1.2 are derived in detail in Appendix B. The key to successful development of the rate-time model is to combine these three equations by eliminating pressure (or pseudopressure). We couple these equations together in such a fashion as to yield a single ordinary differential equation (ODE) in terms of cumulative gas production and time. Where appropriate, we use the Method of Separation of Variables to solve this resulting ODE analytically. In cases where the result cannot be obtained analytically, for example the "General Polynomial" model, we have solved the ordinary differential equation numerically.

We will, first, present a brief description of the reservoir model used in this study. The next section presents various schemes to linearize the non-linear stabilized flow equation
by correlating the changing gas properties (in particular, the $\mu_{ef}/\mu_{ct}$ profile) during reservoir depletion. These linearized equations provide the necessary functional forms for combining these fundamental equations.

### 3.2 Description of the Reservoir Model

The physical model under consideration is a homogeneous porous medium (reservoir) containing a single-phase dry gas. The reservoir is penetrated fully or partially by a wellbore of radius, $r_w$, as shown in Figure 3.1.

The following assumptions are made regarding the reservoir and the reservoir fluid:

1. Gas flow in the reservoir is assumed to obey Darcy's Law, with no water drive mechanism or liquid production.
2. The volume of the reservoir is finite and the gas flow is dominated by the reservoir boundaries.
3. The porosity, permeability, water saturation, and reservoir thickness are assumed to be constant throughout the reservoir.
4. The initial pressure is also assumed to be uniform throughout the entire reservoir.
5. The flowing bottomhole pressure is either a constant fraction of the initial reservoir pressure or can vary in any arbitrary fashion.
6. Gravitational effects are negligible.

Although the reservoir in Fig. 3.1 is circular, the new models presented in this study are applicable to reservoirs of all shapes. The reservoir geometries are characterized by the reservoir shape factor, $C_A$ (Eq. 3.3.3).
Fig. 3.1 - Physical Representation of the Reservoir Model.
3.3 Development of the Gas Equations

In this section, we introduce the fundamental equations used in the development of the new boundary-dominated gas flow model together with the various approximations. Detailed derivations of the stabilized rate equation (Eq. 3.1.1) and the gas material balance equation (Eq. 3.1.2) are presented in Appendix B. To couple these equations we express the stabilized rate equation in terms of $p/z$ or dimensionless pressure, $p_D$, where $p_D$ is defined by

$$p_D = \frac{p/z}{p_l z_i} \tag{3.3.1}$$

Recalling the stabilized rate equation

$$q_g = J_g [p_p - p_{pwf}] \tag{3.1.1}$$

where, the normalized pseudopressure is defined in Appendix A as

$$p_p = \left(\frac{\mu z}{p}\right)_n \int_{p_b}^p \frac{p'}{\mu z} \, dp' \tag{3.3.2}$$

For a circular reservoir, the well productivity index, $J_g$, is defined as

$$J_g = \frac{0.703kh}{T \left[ \ln \left( \frac{r_e}{r_w} - \frac{3}{4} + s + Dq_g \right) \right]_{n} \left( \frac{2p}{\mu z} \right)_{n}}$$

Ignoring the non-Darcy skin term, $J_g$ can be expressed in general (for a reservoir of arbitrary shape characterized by the shape factor, $C_A$, and with drainage area, $A_r$) as

$$J_g = \frac{0.703kh}{2 \left[ \ln \left( \frac{2.24584}{C_A r_w^2} \right) \right]_{n} \left( \frac{2p}{\mu z} \right)_{n}} \tag{3.3.3}$$

The effective wellbore radius, $r_{wa}$, used in Eq. 3.3.3 is related to the actual wellbore radius by the expression

$$r_{wa} = r_w \exp (-s) \tag{3.3.4}$$

Substituting Eq. 3.3.2 into Eq. 3.1.1 gives
\[ q_g = J_g \left( \frac{\mu z}{P} \right)_n \left[ \int_{P_b}^{P} p' \frac{\mu z}{p' \mu z} \, dp' - \int_{P_b}^{P_{wf}} p' \frac{\mu z}{p' \mu z} \, dp' \right] \]

Using the chain rule, the above expression can be re-written as

\[ q_g = J_g \left( \frac{\mu z}{P} \right)_n \left[ \int_{P_b/z_b}^{P/z} p' \frac{\mu z}{\mu z} \frac{dp'}{d(p'/z)} \, d(p'/z) - \int_{P_b/z_b}^{P_{wf}/z_b} p' \frac{\mu z}{\mu z} \frac{dp'}{d(p'/z)} \, d(p'/z) \right] \]  \hspace{1cm} (3.3.5)

In Eq. 3.3.5 the subscript \( b \) represents some base pressure used for the computation of the pseudopressure function. Usually, this base pressure is taken as atmospheric pressure or zero gauge pressure.

By definition, the coefficient of isothermal compressibility of a gas is

\[ c_g = \frac{1}{\rho_g} \frac{dp_g}{dp} \] \hspace{1cm} (3.3.6)

The density of the reservoir gas, \( \rho_g \), is derived from the Universal Gas Law as

\[ \rho_g = \frac{pM}{zRT} \] \hspace{1cm} (3.3.7)

Substituting Eq. 3.3.7 into Eq. 3.3.6 yields

\[ c_g = \frac{1}{\rho_g} \frac{dp_g}{dp} = \left( \frac{zRT}{pM} \right) \frac{d\left( \frac{pM}{zRT} \right)}{dp} \]

which reduces to

\[ c_g = \frac{1}{p/z} \frac{d(p/z)}{dp} \] \hspace{1cm} (3.3.8)

We substitute Eq. 3.3.8 into Eq. 3.3.5. This gives

\[ q_g = J_g \left( \frac{\mu z}{P} \right)_n \left[ \int_{P_b/z_b}^{P/z} \frac{1}{\mu c_g} \frac{d(p'/z)}{d(p'/z)} - \int_{P_b/z_b}^{P_{wf}/z_b} \frac{1}{\mu c_g} \frac{d(p'/z)}{d(p'/z)} \right] \]

or multiplying through by \( \mu c_g \), we have
\[ q_g = \frac{J_g}{\mu_i c_{gi}} \left( \frac{\mu_i}{P_n} \right) \int_{P_{b/g}}^{P_{b/g}} \left[ \frac{\mu_i c_{gi}}{\mu_i c_{gi}} \right] d(p'/l_z) + \int_{P_{b/g}}^{P_{b/g}} \left[ \frac{\mu_i c_{gi}}{\mu_i c_{gi}} \right] d(p'/l_z) \]

This expression may further be simplified by combining the two integral terms in the bracket, which yields

\[ q_g = J_g' \int_{P_{w/g}}^{P_{w/g}} \left[ \frac{\mu_i c_{gi}}{\mu_i c_{gi}} \right] d(p'/l_z) \] \hspace{1cm} (3.3.9)

where

\[ J_g' = \frac{J_g}{\mu_i c_{gi}} \left( \frac{\mu_i}{P_n} \right) = \frac{2C}{\mu_i c_{gi}} \]

Substituting in the definition of \( J_g \) from Eq. 3.3.3 the last expression becomes

\[ J_g' = \frac{0.703kh}{\mu_i c_{gi} T \ln \left( \frac{2.2458A_C}{A_T^2} \right)} \left( \frac{2P}{\mu_i} \right) \left( \frac{\mu_i}{P_n} \right) = \frac{0.703(2kh)}{\mu_i c_{gi} T \ln \left( \frac{2.2458A_C}{A_T^2} \right)} \] \hspace{1cm} (3.3.10)

and

\[ C = \frac{0.703kh}{\frac{1}{2} T \ln \left( \frac{2.2458A_C}{A_T^2} \right)} \] \hspace{1cm} (3.3.11)

Throughout this study, the normalized conditions in the pseudopressure function will be chosen as the values at the initial pressure, \( p_i \). Therefore, we re-write \( J_g' \) as

\[ J_g' = \frac{J_g}{\mu_i c_{gi}} \left( \frac{\mu_i}{P_n} \right) = \frac{J_g}{\mu_i c_{gi}} \left( \frac{\mu_i c_{gi}}{\mu_i c_{gi}} \right) = \frac{J_g}{\mu_i c_{gi}} \frac{\mu_i c_{gi}}{\mu_i c_{gi}} = J_g \frac{c_{gi}}{p_i} \] \hspace{1cm} (3.3.12)

and \( J_g \) becomes

\[ J_g = \frac{0.703kh}{\frac{1}{2} T \ln \left( \frac{2.2458A_C}{A_T^2} \right)} \left( \frac{2p_i}{\mu_i c_{gi}} \right) \]

or, more compactly (Appendix B), we have

\[ J_g = \frac{2kh}{141.2 \mu_i B_{gi} \ln \left( \frac{2.2458A_C}{A_T^2} \right)} \] \hspace{1cm} (3.3.13)
Substituting Eq. 3.3.12 into Eq. 3.3.9 gives our "fundamental" relation for the stabilized flow rate

\[ q_g = \frac{J_g}{c_g} \left( \frac{z_i}{P_i} \right) \int_{P_{wD}z_{wf}}^{P/z} \left[ \frac{\mu c_{gi}}{\mu c_g} \right] d\left( p'z \right) \] \hspace{1cm} (3.3.14)

Eq. 3.3.14 may be expressed in terms of dimensionless pressure (Eq. 3.3.1) as follows

\[ q_g = \frac{J_g}{c_g} \int_{P_{wD}}^{P_D} \left[ \frac{\mu c_{gi}}{\mu c_g} \right] dP' \] \hspace{1cm} (3.3.15)

Note that in Eq. 3.3.15 we have defined the dimensionless bottomhole pressure, \( p_{wD} \), as

\[ p_{wD} = \frac{P_{wD}z_{wf}}{P_{i}z_i} \] \hspace{1cm} (3.3.16)

When compressibility of the reservoir rock is significant, Eq. 3.3.15 should be written in terms of the total system compressibility, \( c_t \). In that particular case, we write Eq. 3.3.15 as

\[ q_g = \frac{J_g}{c_t} \int_{P_{wD}}^{P_D} \left[ \frac{\mu c_{ti}}{\mu c_{t}} \right] dP' \] \hspace{1cm} (3.3.17)

where the total system compressibility is defined as

\[ c_t = c_g S_g + c_w S_w + c_f \] \hspace{1cm} (3.3.18)

Eq. 3.3.17 is the most rigorous form of the stabilized rate equation for modeling the flow of real gases in porous medium during the boundary-dominated period. Note that up to this point we have introduced no limiting assumptions into this non-linear gas flow equation.

Application of Eq. 3.3.17 requires an appropriate functional model to describe the fluid property changes during reservoir depletion. Specifically, practical application of this integral form of the flow equation requires the use of an accurate model to characterize the viscosity-compressibility function \( \left[ \frac{\mu c_{ti}}{\mu c_{t}} \right] \) versus \( P_D \). We refer to such functional models as
"linearization schemes," and several of these models are discussed in the following sections.

3.3.1 The Zero-Order Polynomial Linearization Scheme

Carter\textsuperscript{3} presented the first model to characterize changes of the viscosity-compressibility ratio as a function of pressure. Carter assumed that within a narrow interval of any two pressures, \( p_i \) and \( p_{wf} \), the viscosity-compressibility function can be approximated by a constant parameter. Hence, he defined a "Zero-order polynomial" approximation for this function which is given as

\[
\frac{\mu_{ig} c_i}{\mu_c} = \lambda
\]  

\text{(2.1)}

or in terms of total system compressibility

\[
\frac{\mu_{it} c_{it}}{\mu_c} = \lambda
\]

Although Carter did not provide the theoretical developments presented in this dissertation, it is obvious that the simple linearization of Eq. 3.3.17 by choosing a constant integrand will be a prime motivation for such assumption. In fact, substitution of the last expression into Eq. 3.3.17 yields

\[
q_g = \frac{J_g}{c_{it}} \int_{P_{wD}}^{P_D} \lambda dp_D' = \frac{J_g}{c_{it}} \int_{P_{wD}}^{P_D} \lambda dp_D
\]

or

\[
q_g = \left. \frac{J_g \lambda}{c_{it}} \frac{P_D}{P_{wD}} \right|_{P_{wD}}^{P_D}
\]

This means

\[
q_g = \frac{J_g \lambda}{c_{it}} [P_D - P_{wD}]
\]  

\text{(3.3.19)}

Substituting the definition of the dimensionless pressure into Eq. 3.3.19 we obtain

\[
q_g = \frac{J_g \lambda}{c_{it} \mathcal{Z}_{ii}(p_d z_i)} \left[ \frac{P}{Z} - \frac{p_{wf}}{z_{wf}} \right]
\]

or

\[
q_g = C_1 \left[ \frac{P}{Z} - \frac{p_{wf}}{z_{wf}} \right]
\]  

\text{(3.3.20)}
where we have defined the constant $C_1$ as

$$C_1 = \frac{J_e}{c_H(p_i/z_i)}$$

or

$$C_1 = \frac{0.703 kh(2\lambda)}{\frac{1}{2} \mu ic_H T \ln \left[ \frac{2.2458A}{C_{a/r}^2} \right]}$$

(3.3.21)

Eq. 3.3.20 shows that using the "Zero-order polynomial" linearization model proposed by Carter results in a $p/z$ form of the stabilized rate equation. We note, by inspection, that this expression is more rigorous than the approach given by Fetkovich, where the $z$-factor was assumed to be 1. As will be shown in Chapter IV, coupling this equation with the gas material balance equation yields an explicit rate-time equation, which may be used to model production rate decline of gas wells. The only variable in this development is the $\lambda$-parameter.

Before proceeding further we will examine the nature of the $[\mu c_i/T]$ vs. $p_D$ function. Figs. C.1 - C.30 in Appendix C present the viscosity-compressibility functions for a variety of natural gases under different reservoir conditions. Typical plots of these functions are shown in Figs. 3.2 and 3.3 below for the two fluid samples described in Table 3.1.

The following basic features about these plots are worth noting:

1. From these figures we observe that there is no interval of the dimensionless pressure profile (or pressure for that matter) in which the viscosity-compressibility function is constant (i.e., a horizontal line).

2. For low pressure reservoir systems we note that the viscosity-compressibility function is fairly linear with respect to $p_D$, and

3. There is more and more deviation from this straight line trend as the initial pressure of the reservoir increases.

It is obvious therefore that the "Zero-order polynomial" model is a gross approximation, to say the least.
Fig. 3.2 - Plot of the Viscosity-Compressibility Function (Fluid Sample 1).

Table 3.1
Summary of Reservoir Fluid Samples

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sample 1</th>
<th>Sample 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir Temperature, °F</td>
<td>150</td>
<td>300</td>
</tr>
<tr>
<td>Gas Specific Gravity of Gas (air=1.0)</td>
<td>0.55</td>
<td>0.70</td>
</tr>
</tbody>
</table>
Fig. 3.3 - Plot of the Viscosity-Compressibility Function (Fluid Sample 2).

We note, however, that the success of the $\lambda$ concept lies in its application as a correlating parameter as defined by Eq. 2.2

$$\lambda = \frac{\mu c_{gi} \left[ m(p_i) - m(p_{win}) \right]}{2 \left( \frac{p_i}{z_i} \right) - \left( \frac{p_{win}}{z_{win}} \right)}$$  \hspace{1cm} (2.2)$$

Carter used Eq. 2.2 to correlate a variety of numerical rate solutions for cases of reservoirs with different initial and producing conditions into a unique type curve. This idea is used in our study for developing the $\beta$-concept.
3.3.2 The First-Order Polynomial Linearization Scheme

A significant improvement in the modeling of fluid property changes during reservoir depletion was presented by Knowles.\textsuperscript{4} The author introduced the $p/z$-squared form of the stabilized rate equation. In Ref. 25 Ansah, et al. demonstrated that the $p/z$-squared form of the stabilized rate equation has an underlying assumption of a first-order polynomial approximation (straight line model) for the viscosity-compressibility term as a function of dimensionless pressure.

The motivation behind this first-order linearization scheme was that coupling this linearized form of the stabilized flow equation with the gas material balance equation enabled Knowles\textsuperscript{4} to develop explicit rate-time solutions. The first-order polynomial function is defined as

$$\frac{\mu_i C_{ti}}{\mu C_t} = P_D$$

(3.3.22)

Eq. 3.3.22 represents an equation of a straight line with a slope of one which passes through the origin of a Cartesian coordinate system. We observe from the Cartesian plots in Figs. 3.2 and 3.3 that such an approximation can only be valid for low pressure gas reservoirs.

Substituting Eq. 3.3.22 into Eq. 3.3.17 yields

$$q_g = \frac{J_G}{c_i} \int_0^{p_D} p_D dp_D = \frac{J_G}{2c_{ti}} \int_0^{p_D} p_D dp_D$$

or

$$q_g = \frac{J_G}{2c_{ti}} \left[ p_D^2 - p_{wD}^2 \right]$$

(3.3.23)

Using the definition of dimensionless pressure Eq. 3.3.23 may be re-written as

$$q_g = \frac{J_G}{2c_i \left(p_j z_i\right)^2} \left[ \left(\frac{p}{z}\right)^2 - \left(\frac{p_{wf}}{z_{wf}}\right)^2 \right]$$

or

$$q_g = C_2 \left[ \left(\frac{p}{z}\right)^2 - \left(\frac{p_{wf}}{z_{wf}}\right)^2 \right]$$

(3.3.24)

In Eq. 3.3.24 the parameter $C_2$ is given by
\[
C_2 = \frac{J_g z_i^2}{2 \mu c_i T} \\
\text{or}
\]
\[
C_2 = \frac{0.703kh}{\frac{1}{2} \mu c_i T \ln \left[ \frac{2.2458A}{C_A r_{wa}^2} \right]} \left( \frac{z_i}{p_i} \right) \tag{3.3.25}
\]

We note that in Ref. 4 the author defines \( C_2 \), for a circular reservoir system, as
\[
C_2 = \frac{0.703kh}{T \left[ \ln (r_d/r_w) - \frac{3}{4} + s + Dq_g \right]} \left( \frac{z}{\mu c_i p_{ref}} \right) \tag{3.3.26}
\]

where, the gas properties in \( C_2 \) are evaluated at some reference pressure. Detailed discussion on this subject is also presented in Ref. 4.

We note from Eq. 3.3.24 that the first-order polynomial linearization model (Eq. 3.3.22) results in the \( p/z \)-squared form of the stabilized rate equation (which was proposed by Knowles\(^4\)). Eq. 3.3.24 can, again, be coupled directly with the gas material balance equation to generate a predictive rate-time equation, with \( p_{wf} \) (or \( p_{wD} \)) as the only correlating parameter.

### 3.3.3 The Exponential Linearization Scheme

In this section, we investigate the use of the exponential function to model fluid property variation during the depletion of gas reservoirs. In particular, our exponential function is defined as
\[
\frac{\mu c_i}{\mu c} = B_0 \exp (B_1 p_D) \tag{3.3.27}
\]

The exponential linearization scheme is appealing because, as will be shown in Chapter IV, it also allows for the development of explicit analytical rate-time relations. Figs. 3.4 and 3.5 show the use of the exponential function to model the viscosity-compressibility curves presented in Figs. 3.2 and 3.3. Note that we have presented only high pressure reservoir cases — \( p_t=8000-12000 \) psia.

Tables 3.2 and 3.3 below present the coefficients of these exponential fits. We note from Figs. 3.4 and 3.5 that although the exponential function appears to model the behavior of
the $\mu_c c_l / \mu c_l$ data well at the initial stages of reservoir depletion, the model deviates quite significantly at the late stages of depletion (i.e., at very low pressures). As will be shown in Chapters IV and V this fact has a minor consequence as far as analysis of field production data are concerned, as we tend to abandon most gas reservoirs before they reach this late stage of depletion.

We, however, emphasize from the results of simulation studies that Eq. 3.3.27 is applicable only to gas reservoirs with high initial pressures.

**Fig. 3.4** - Exponential Fit: Viscosity-Compressibility Function (Fluid Sample 1).
Fig. 3.5 - Exponential Fit: Viscosity-Compressibility Function (Fluid Sample 2).

Table 3.2

Coefficients of the "Exponential" Model (Fluid Sample 1)

<table>
<thead>
<tr>
<th>Pressure (psia)</th>
<th>Coefficient</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B_0$</td>
<td>$B_1$</td>
<td></td>
</tr>
<tr>
<td>8000</td>
<td>0.08694</td>
<td>2.43276</td>
<td></td>
</tr>
<tr>
<td>9000</td>
<td>0.07368</td>
<td>2.59466</td>
<td></td>
</tr>
<tr>
<td>10000</td>
<td>0.06323</td>
<td>2.74661</td>
<td></td>
</tr>
<tr>
<td>11000</td>
<td>0.05512</td>
<td>2.88399</td>
<td></td>
</tr>
<tr>
<td>12000</td>
<td>0.04873</td>
<td>3.00825</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.3  
Coefficients of the "Exponential" Model (Fluid Sample 2)

<table>
<thead>
<tr>
<th>Pressure (psia)</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B_0$</td>
</tr>
<tr>
<td>8000</td>
<td>0.11303</td>
</tr>
<tr>
<td>9000</td>
<td>0.10066</td>
</tr>
<tr>
<td>10000</td>
<td>0.09026</td>
</tr>
<tr>
<td>11000</td>
<td>0.08148</td>
</tr>
<tr>
<td>12000</td>
<td>0.07409</td>
</tr>
</tbody>
</table>

Substituting Eq. 3.3.27 into Eq. 3.3.17, we obtain

$$q_g = \frac{J_g}{c_{li}} \int_{P_{WD}}^{P_D} B_0 \exp(B_1 P_D) \, dP_D = \frac{J_g B_0}{c_{li}} \int_{P_{WD}}^{P_D} \exp(B_1 P_D) \, dP_D$$

or

$$q_g = \frac{J_g B_0}{c_{li} B_1} \exp(B_1 P_D) \bigg|_{P_{WD}}^{P_D} ............................................... (3.3.28)$$

Expanding Eq. 3.3.28 further yields

$$q_g = \frac{J_g B_0}{c_{li} B_1} \left[ \exp(B_1 P_D) - \exp(B_1 P_{WD}) \right]$$

or

$$q_g = C_3 \left[ \exp(B_1 P_D) - \exp(B_1 P_{WD}) \right] ............................................... (3.3.29)$$

In Eq. 3.3.29 the parameter $C_3$ is defined by

$$C_3 = \frac{J_g B_0}{c_{li} B_1}$$

or

$$C_3 = \frac{0.703 \, (2kh)B_0 P_i}{\frac{1}{2} \mu_i \zeta_i c_{li} B_1 T \ln \left[ \frac{2.2458A}{C_A r_w^2} \right]} ............................................... (3.3.30)$$
Eq. 3.3.29 represents the linearized form of the stabilized flow equation for the exponential model, which can now be coupled directly with the gas material balance equation to develop a rate prediction equation.

Finally, it is important to note that the exponential model (Eq. 3.3.27) is also a special case (an approximation) of a general polynomial function. In particular, Eq. 3.3.27 may be expanded as

$$\frac{\mu_i C_{ti}}{\mu C_t} = B_0 \exp(B_1 p_D) = B_0 \left[ 1 + B_1 p_D + \frac{B_2}{2} p_D^2 + \frac{B_3}{6} p_D^3 + \frac{B_4}{24} p_D^4 + \ldots \right]$$

which can be simplified as

$$\frac{\mu_i C_{ti}}{\mu C_t} = B_0 + (B_0 B_1) p_D + \left( \frac{B_0 B_2}{2} \right) p_D^2 + \left( \frac{B_0 B_3}{6} \right) p_D^3 + \left( \frac{B_0 B_4}{24} \right) p_D^4 + \ldots \quad \text{...............}(3.3.31)$$

Eq. 3.3.31 is an $n$-th order polynomial function with special coefficients involving the constants $B_0$ and $B_1$ in the exponential function.

### 3.3.3 The General Polynomial Linearization Scheme

In this work, we also present a general polynomial model for correlating the viscosity-compressibility ratio as a function of dimensionless pressure. We have established from experiments on a wide range of gases that the viscosity-compressibility ratio can be modeled by a third or fourth-order polynomial expression. The viscosity-compressibility term is presented in the form

$$\frac{\mu_i C_{ti}}{\mu C_t} = a_0 + a_1 \left( \frac{p}{z} \right) + a_2 \left( \frac{p}{z} \right)^2 + a_3 \left( \frac{p}{z} \right)^3 + a_4 \left( \frac{p}{z} \right)^4 + \ldots$$

or, in terms of dimensionless pressure, we have

$$\frac{\mu_i C_{ti}}{\mu C_t} = a_0 + a_1 p_D + a_2 p_D^2 + a_3 p_D^3 + a_4 p_D^4 + \ldots \quad \text{...............}(3.3.32)$$

where

$$a_0 = a_0 \quad \text{..............................................}(3.3.33)$$

$$a_1 = a_1 \left[ \frac{p_i}{z_i} \right] \quad \text{..............................................}(3.3.34)$$
\[ a_2 = a_2 \left( \frac{p_i}{z_i} \right)^2 \] \hfill (3.3.35)

\[ a_3 = a_3 \left( \frac{p_i}{z_i} \right)^3 \] \hfill (3.3.36)

\[ a_4 = a_4 \left( \frac{p_i}{z_i} \right)^4 ; \text{ etc} \] \hfill (3.3.37)

The validity of Eq. 3.3.32 is demonstrated in the examples of a fourth-order polynomial fit to the curves presented in Figs 3.1 and 3.2, which are demonstrated in Figs. 3.6 and 3.7 below.

The coefficients of the fourth-order polynomial fit to these curves are given in Tables 3.4 and 3.5. Note that we have presented these plots on a log-log scale to demonstrate the highly accurate fit of the curves with the polynomial function.

It is worth noting also that the "General polynomial" data model should be exact for any reservoir condition. Truncating this function depends on the initial reservoir conditions. For reservoirs with low initial pressures we noted that the first-order approximation to this generalized model appears to be more than adequate for developing the performance relations, whereas more and more terms are required for reservoirs with higher initial pressures.
Fig. 3.6 - Log-log Plot of the Viscosity-Compressibility Function (Fluid Sample 1) Including the Fourth-Order Polynomial Fit.

Table 3.4
Coefficients of the Fourth-Order Polynomial Model (Fluid Sample 1)

<table>
<thead>
<tr>
<th>Pressure (psia)</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_0$</td>
</tr>
<tr>
<td>1000</td>
<td>0.00000</td>
</tr>
<tr>
<td>3000</td>
<td>0.00000</td>
</tr>
<tr>
<td>5000</td>
<td>0.00000</td>
</tr>
<tr>
<td>8000</td>
<td>0.00000</td>
</tr>
<tr>
<td>10000</td>
<td>0.00000</td>
</tr>
<tr>
<td>12000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>
Fig. 3.7 - Log-log Plot of the Viscosity-Compressibility Function (Fluid Sample 2) Including the Fourth-Order Polynomial Fit.

Table 3.5
Coefficients of the Fourth-Order Polynomial Approximation (Fluid Sample 2)

<table>
<thead>
<tr>
<th>Pressure (psia)</th>
<th>(a_0)</th>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(a_3)</th>
<th>(a_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>0.00000</td>
<td>1.05114</td>
<td>0.19430</td>
<td>-0.43537</td>
<td>0.18903</td>
</tr>
<tr>
<td>3000</td>
<td>0.00000</td>
<td>1.18927</td>
<td>-0.22001</td>
<td>-0.20668</td>
<td>0.23789</td>
</tr>
<tr>
<td>5000</td>
<td>0.00000</td>
<td>1.06796</td>
<td>-0.48287</td>
<td>0.22959</td>
<td>0.18609</td>
</tr>
<tr>
<td>8000</td>
<td>0.00000</td>
<td>0.80478</td>
<td>-0.39173</td>
<td>0.22398</td>
<td>0.36289</td>
</tr>
<tr>
<td>10000</td>
<td>0.00000</td>
<td>0.66836</td>
<td>-0.25383</td>
<td>0.04727</td>
<td>0.53781</td>
</tr>
<tr>
<td>12000</td>
<td>0.00000</td>
<td>0.67585</td>
<td>-0.52169</td>
<td>0.38627</td>
<td>0.45840</td>
</tr>
</tbody>
</table>
Now, substituting Eq. 3.3.32 into Eq. 3.3.17 yields the following stabilized flow equation

\[
q_g = \frac{J_g}{c_{hi}} \int_{p_{WD}}^{p_D} \left[ a_0 + a_1 p_D + a_2 p_D^2 + a_3 p_D^3 + a_4 p_D^4 + \ldots \right] \mathrm{d}p_D
\]

or, completing the integration, we have

\[
q_g = \frac{J_g}{c_{hi}} \left[ a_0 p_D + \frac{a_1}{2} p_D^2 + \frac{a_2}{3} p_D^3 + \frac{a_3}{4} p_D^4 + \frac{a_4}{5} p_D^5 + \ldots \right] \bigg|_{p_{WD}}^{p_D}
\]

which can further be simplified as

\[
q_g = \frac{J_g}{c_{hi}} \left[ A_0 + A_1 p_D + A_2 p_D^2 + A_3 p_D^3 + A_4 p_D^4 + A_5 p_D^5 + \ldots \right] \bigg|_{p_{WD}}^{p_D} \tag{3.3.38}
\]

In Eq. 3.3.38, the constants are defined as follows

\[
A_0 = - \left[ A_1 p_{WD} + A_2 p_{WD}^2 + A_3 p_{WD}^3 + A_4 p_{WD}^4 + A_5 p_{WD}^5 + \ldots \right] \tag{3.3.39}
\]

\[
A_1 = a_0 \tag{3.3.40}
\]

\[
A_2 = \frac{a_1}{2} \tag{3.3.41}
\]

\[
A_3 = \frac{a_2}{3} \tag{3.3.42}
\]

\[
A_4 = \frac{a_3}{4} ; \text{etc} \tag{3.3.43}
\]

Eq. 3.3.38 will similarly be coupled with the gas material balance equation to generate the most general rate-time and cumulative production-time solutions for gas decline curve analysis.
CHAPTER IV

DEVELOPMENT OF THE NEW RATE-TIME SOLUTIONS
FOR DRY GAS RESERVOIRS

4.1 Summary

In this chapter we present analytical and semi-analytical rate-time solutions along with pressure and cumulative production-time solutions for gas well performance analysis. The solutions are developed in terms of dimensionless variables and presented graphically in the form of dimensionless "type curves" for easy comparison with other semi-analytical and numerical results available in the petroleum literature.

We begin with a brief introduction of the Arps' equations, which are the most commonly used solutions for conventional decline curve analysis today. These equations establish the general exponential, hyperbolic, and harmonic relations between production rate and time. The hyperbolic equation is given by

\[ q_g = \frac{q_{gi}}{[1 + bD_i t]^{1/b}} \]..........................(4.1.1)

where \( b > 0 \). For the case of \( b = 0 \), Arps observed the following exponential relation

\[ q_g = \frac{q_{gi}}{e^{D_i t}} \]..........................(4.1.2)

In 1980, Fetkovich expressed Arps' empirical equations in terms of dimensionless "decline" variables. The dimensionless production rate was expressed as

\[ q_{Dd} = \frac{1}{[1 + b t_{Dd}]^{1/b}} \]..........................(4.1.3)

and

\[ q_{Dd} = \frac{1}{e^{t_{Dd}}} \]..........................(4.1.4)

where the dimensionless "decline" variables are defined by

\[ q_{Dd} = \frac{q_g}{q_{gi}} \text{ and } t_{Dd} = D_i t \]..........................(4.1.5)
Fetkovich combined these boundary-dominated solutions with transient flow solutions to develop a "unified" type curve. The dimensionless "decline" variables of this composite type curve were defined as

\[ q_{DD} = q_D \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{1}{2} \right] \] .............................................. (4.1.6)

and

\[ t_{DD} = \frac{t_D}{\frac{1}{2} \left( \frac{r_e}{r_{wa}} \right)^2 - 1 \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{1}{2} \right]} \] .............................................. (4.1.7)

In terms of field units the dimensionless rate function becomes

\[ q_{DD} = \frac{141.2 \mu B}{kh} \frac{q_k(t)}{(p_i - p_{wf})} \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{1}{2} \right] \] .............................................. (4.1.8)

and the dimensionless time is given as

\[ t_{DD} = \frac{0.00633kt}{\phi \mu c_r^2 r_{wa}^2} \frac{1}{\frac{1}{2} \left( \frac{r_e}{r_{wa}} \right)^2 - 1 \left[ \ln \left( \frac{r_e}{r_{wa}} \right) - \frac{1}{2} \right]} \] .............................................. (4.1.9)

Fig. 4.1 presents the Fetkovich "composite" type curve which has found widespread use in decline curve analysis of production data from both gas and oil reservoir systems. We note that Eqs. 4.1.3 and 4.1.4 represent explicit analytical relations between production rate and time. The fundamental drawback of these equations is that they were derived specifically for liquid flow, and often yield significant errors when applied to gas reservoir systems.

Theoretically, we derive the production rate-time model from simultaneous solution of the stabilized flow and the gas material balance equations. The rigorous forms of these equations were derived in Chapter III. The results are

\[ q_g = \frac{J_g}{c_{ti}} \int_{P_{wD}}^{P_D} \frac{\mu c_{ti}}{\mu c_t} dP_D \] (stabilized gas flow equation) .......................... (3.3.17)

\[ \frac{p}{z} = \frac{p_i}{\frac{G_p}{G}} \left[ 1 - \frac{G_p}{G} \right] \] (gas material balance equation) .......................... (3.1.2)
Fig. 4.1 - Feltovich Type Curves (after Feltovich).
and the flowrate-cumulative production relation is given by

\[ q_g = \frac{dG_p}{dt} \]  

(3.1.3)

The productivity index of the well, \( J_g \), in Eq. 3.3.17 is defined as

\[ J_g = \frac{2kh}{141.2\mu_i B_g \mu_i \ln \left[ \frac{2.2458A}{C_A R_w^2} \right]} \]  

(3.3.13)

We express the rate equation in dimensionless form by defining the following general relation

\[ q_{Dd} = \frac{q_g}{\left( J_g/c_{ii} \right)^\alpha} = \frac{q_g}{(J_g/c_{ii})^\alpha} \]  

(4.1.10)

where, we have introduced the constant \( \alpha \) defined by

\[ \alpha = \int_{p_{wD}}^{1} \left[ \frac{\mu_i c_{ii}}{\mu_i c_t} \right] dp_D \]  

(4.1.11)

Our motivation for adopting this definition is that the dimensionless rate, \( q_{Dd} \), will always be unity at the start of production when dimensionless time is equal to zero and \( p_D = 1.0 \).

Substituting Eq. 4.1.10 into Eq. 3.3.17, the stabilized flow equation then becomes

\[ q_{Dd} = \frac{1}{\alpha} \int_{p_{wD}}^{p_D} \left[ \frac{\mu_i c_{ii}}{\mu_i c_t} \right] dp_D \]  

(4.1.12)

We also re-write the gas material balance equation in terms of dimensionless variables. The final form is

\[ p_D = 1 - G_{pD} \]  

(4.1.13)

where we have defined the dimensionless cumulative production, \( G_{pD} \), as

\[ G_{pD} = \frac{G_p}{G} \]  

(4.1.14)

Substituting these definitions of \( q_{Dd} \) and \( G_{pD} \) into Eq. 3.1.3 gives

\[ q_{Dd} = \frac{dG_{pD}}{dt_D} \]

or
\[
q_{Dd} = \frac{1}{\alpha} \frac{dG_{pD}}{dt_{Dd}} \tag{4.1.15}
\]

where "ordinary" dimensionless time, \( t_D \), is defined as
\[
t_D = \frac{J_g \alpha}{c_i G} t \tag{4.1.16}
\]

and the dimensionless "decline" time, \( t_{Dd} \), is defined as
\[
t_{Dd} = \frac{J_g}{c_i G} t \tag{4.1.17}
\]

We represent the viscosity-compressibility function in Eq. 4.1.12 by a general expression, \( f(p_D) \). Thus
\[
\frac{\mu_i c_{ii}}{\mu c_i} = f(p_D) \tag{4.1.18}
\]

Substituting Eq. 4.1.18 into Eq. 4.1.12, and summarizing the pertinent equations so far, we have
\[
q_{Dd} = \frac{1}{\alpha} \int_{P_{wD}}^{PD} f(p_D) dp_D' \tag{4.1.19}
\]

\[
p_D = 1 - G_{pD} \tag{4.1.13}
\]

and finally, the dimensionless flowrate-cumulative production identity
\[
q_{Dd} = \frac{1}{\alpha} \frac{dG_{pD}}{dt_{Dd}} \tag{4.1.15}
\]

Eqs. 4.1.13, 4.1.15, and 4.1.19 represent the complete set of dimensionless equations needed to develop our decline curve analysis models. We combine these equations by differentiating Eq. 4.1.13 with respect to dimensionless "decline" time. This gives
\[
\frac{dp_D}{dt_{Dd}} = -\frac{dG_{pD}}{dt_{Dd}}
\]

which, according to Eq. 4.1.15, implies that
\[
q_{Dd} = -\frac{1}{\alpha} \frac{dp_D}{dt_{Dd}} \tag{4.1.20}
\]

Substituting Eq. 4.1.20 into Eq. 4.1.19 yields
\[-\frac{1}{\alpha} \frac{dp_D}{dr_{Dd}} = \frac{1}{\alpha} \int_{P_{wD}}^{P_D} f(p_D) dp_D\]

or canceling the identical terms on both sides of this equation, we obtain

\[\frac{dp_D}{dt_{Dd}} = -\int_{P_{wD}}^{P_D} f(p_D) dp_D\] (4.1.21)

We immediately note that evaluating the right-hand-side of Eq. 4.1.21 by using an appropriate functional form for the function \(f(p_D)\) results in a first-order ordinary differential equation (ODE), which can easily be solved numerically for the dimensionless pressure. The initial condition of this problem is

At \(t = 0\) (or \(t_{Dd} = 0\)); \(P_D = 1.0\) \ ...................................................(4.1.22)

Using the Method of Separation of Variables and the initial condition (Eq. 4.1.22), we integrate Eq. 4.1.21 as follows

\[\int_{1}^{P_D} \frac{dp_D}{f(p_D) dp_D} = -\int_{0}^{t_{Dd}} dt_{Dd}\]

or

\[\int_{1}^{P_D} \frac{dp_D}{f(p_D) dp_D} = -t_{Dd} \] ...................................................(4.1.23)

We will solve Eq. 4.1.23 analytically by specifying the function \(f(p_D)\). Extensive discussion of appropriate linearization functions of \(f(p_D)\) has been presented in Chapter III. We summarize these results as follows
\[ f(p_D) = \frac{\mu_i c_H}{\mu c_t} = \lambda \quad \text{(Zero-Order Polynomial Model)} \] ........................(4.1.24)

\[ f(p_D) = \frac{\mu_i c_H}{\mu c_t} p_D \quad \text{(First-Order Polynomial Model)} \] ........................(4.1.25)

\[ f(p_D) = \frac{\mu_i c_H}{\mu c_t} = B_0 \exp(B_1 p_D) \quad \text{(Exponential Model)} \] ........................(4.1.26)

\[ f(p_D) = \frac{\mu_i c_H}{\mu c_t} = a_0 + a_1 p_D + a_2 p_D^2 + a_3 p_D^3 + a_4 p_D^4 + \ldots \] ........................(4.1.27)

Eq. 4.1.27 represents the most general polynomial model, which by its generality should be applicable to all gas reservoir systems. We note that Eqs. 4.1.24-4.1.26 are approximations of this general model and, in particular, Eqs. 4.1.24 and 4.1.25 are only applicable to low pressure gas reservoirs. By contrast, Eq. 4.1.26 is applicable only to high pressure gas reservoir systems.

Detailed derivations of the rate-time and cumulative production models for each of Eqs. 4.1.24-4.1.27 are provided in the following sections.

4.2 Analytical Solution of the Zero-Order Polynomial Model

Substituting the zero-order polynomial model into Eq. 4.1.23, and integrating yields

\[
\frac{1}{\lambda} \int_1^{p_D} \frac{dp_D}{[p_D - p_{wD}]} = - t_{Dd}
\]

which upon further integration gives

\[
\ln \left[ \frac{p_D - p_{wD}}{1 - p_{wD}} \right] = - \lambda t_{Dd} \quad \text{..........................}(4.2.1)
\]

Exponentiating Eq. 4.2.1, we obtain

\[
\frac{p_D - p_{wD}}{1 - p_{wD}} = \exp(-\lambda t_{Dd})
\]

which may be solved for the dimensionless pressure, \(p_D\), as

\[
p_D = p_{wD} + (1 - p_{wD}) \exp(-\lambda t_{Dd}) \quad \text{..........................}(4.2.2)
\]

It is important to note that Eq. 4.2.2 is actually a time-dependent material balance equation,
but rather an approximation. In terms of real variables (field units) this equation can be expressed as

\[
\frac{p}{z} = \frac{p_w}{z_w} + \left[\frac{p_i}{z_i} - \frac{p_w}{z_w}\right] \exp\left(-\frac{J_g \lambda}{c_i G} t\right)
\]

or

\[
\frac{p}{z} = \frac{p_w}{z_w} + \left[\frac{p_i}{z_i} - \frac{p_w}{z_w}\right] \exp\left(-D_i \lambda t\right)
\]

(4.2.3)

where we have defined the initial decline curve constant, \(D_i\), as

\[
D_i = \frac{J_g}{c_i G}
\]

(4.2.4)

Substituting Eq. 4.2.2 into Eq. 4.1.13 and solving for \(G_{PD}\) gives us the cumulative production result for this case. Completing this operation, we obtain

\[
G_{PD} = 1 - P_D = 1 - p_{WD} - (1 - p_{WD}) \exp(-\lambda t_{DD})
\]

or

\[
G_{PD} = (1 - p_{WD})[1 - \exp(-\lambda t_{DD})]
\]

(4.2.5)

Eq. 4.2.5 may also be expressed in real variables as

\[
G_p = \frac{z_i G}{p_i} \left[\frac{p_i}{z_i} - \frac{p_w}{z_w}\right] \left[1 - \exp\left(-\frac{J_g \lambda}{c_i G} t\right)\right]
\]

or

\[
G_p = \frac{z_i G}{p_i} \left[\frac{p_i}{z_i} - \frac{p_w}{z_w}\right] \left[1 - \exp\left(-D_i \lambda t\right)\right]
\]

(4.2.6)

The dimensionless rate-time model is derived by substituting Eq. 4.2.5 into Eq. 4.1.15. This results in

\[
q_{DD} = \frac{1}{\alpha} \frac{dG_{pD}}{dt_{DD}} = \frac{1}{\alpha} \frac{d}{dt_{DD}} \{(1 - p_{WD})[1 - \exp(-\lambda t_{DD})]\}
\]

or

\[
q_{DD} = \frac{\lambda}{\alpha} (1 - p_{WD}) \exp(-\lambda t_{DD})
\]

(4.2.7)

We note that Eq. 4.2.7 suggests that *all constant pressure gas flow cases will have an exponential rate decline*. This observation is disputed by comparison with numerical simulation solutions, but the result nonetheless provides a limiting behavior for the \(q_{DD}\)
function.

The $\alpha$-parameter for this case is obtained using the definition given by Eq. 4.1.11. Substituting Eq. 4.1.24 into Eq. 4.1.11 yields

$$\alpha = \int_{P_{WD}}^{1} \lambda dp_D' = \lambda (1 - P_{WD})$$ .............................................. (4.2.8)

We now substitute Eq. 4.2.8 into Eq. 4.2.7, which gives

$$q_{Dd} = \frac{\lambda}{\lambda (1 - P_{WD})} (1 - P_{WD}) \exp(-\lambda t_{Dd}) = \exp(-\lambda t_{Dd})$$

or

$$q_{Dd} = \frac{1}{\exp(\lambda t_{Dd})}$$ .............................................. (4.2.9)

In his original publication Carter$^3$ showed via numerical simulation of dry gas reservoirs that the $\lambda$-parameter lies in the range of 0.5 to 1.0. Carter noted that a $\lambda$-value of 0.5 represents the limiting case for the maximum drawdown from a gas reservoir, while a $\lambda$-value of approximately 1.0 represents the exponential depletion model, which is characteristic of slightly compressible liquid systems. In fact, substitution of $\lambda = 1$ into Eq. 4.2.9 above yields the exponential model given by Arps (Eq. 4.1.4). Physically, this result represents a constant value of the viscosity-compressibility function during reservoir depletion. Following Carter's original publication it has been observed that for high pressure gas reservoirs, the $\lambda$-parameter can have values less than 0.5.

In this work, we define a slightly different correlating parameter $\beta$, rather than the $\lambda$-parameter proposed by Carter$^3$. Our $\beta$-parameter is closely related to the $\lambda$-parameter and provides a rigorous linkage with the various solutions proposed in this work. By definition

$$\beta \equiv \frac{1}{(1 - P_{WD})} \int_{P_{WD}}^{1} \left[ \frac{\mu_i c_i}{\mu c_t} \right] dp_D'$$ .............................................. (4.2.10)

Comparison of this definition with Eq. 4.1.11 establishes the following relation between the $\alpha$ and $\beta$-parameters
\[ \beta = \frac{1}{(1 - p_{WD})} \int_{p_{WD}}^{1} \left[ \frac{\mu_c C_t}{\mu_c} \right] dp_D = \frac{\alpha}{(1 - p_{WD})} \]  \hspace{1cm} (4.2.11)

Substituting the expression for \( \alpha \) from Eq. 4.2.8 into this equation yields
\[ \beta \equiv \lambda \]  \hspace{1cm} (4.2.12)

Hence the \( \beta \) and \( \lambda \)-parameters are equivalent for this case. Using our definition of \( \beta \) for this "Zero-Order polynomial" model, the rate equation becomes
\[ q_{Dd} = \frac{1}{\exp(\beta t_{Dd})} \]  \hspace{1cm} (4.2.13)

We can use Eq. 4.2.13 to generate graphical solutions in the form of dimensionless type curves, noting that the only correlating variable in this equation is the \( \beta \)-parameter. Fig. 4.2 shows graphical solutions of this equation for different values of \( \beta \), which are compared here with typical Carter type curves generated using a single-phase gas reservoir simulator. A plot of the dimensionless pressure solutions (Eq. 4.2.2) is presented in Fig. 4.3.

We observe from these plots that the "Zero-order polynomial" model is indeed a gross approximation. Due to the large errors of these solutions compared to the numerical simulation results we will not recommend the "Zero-order polynomial" model for gas well performance analysis.

### 4.3 Analytical Solution of the First-Order Polynomial Model

In this section, we present the rate-time solutions obtained from the pressure-squared form of the stabilized rate equation proposed in Ref. 4. We showed in Chapter III that the use of this equation stems from an implicit assumption of a "First-order polynomial" approximation for the viscosity-compressibility function.

We substitute this "First-order polynomial" model (Eq. 4.1.25) into Eq. 4.1.23, which upon integrating yields
\[ 2 \int_{1}^{P_D} \frac{dp_D}{(p_D^2 - p_{WD}^2)} = -t_{Dd} \]  \hspace{1cm} (4.3.1)
Fig. 4.2 - "Zero-Order Polynomial" Solutions of the Real Gas Flow Equation under Boundary-Dominated Flow Conditions. Solutions Assume a Constant $\mu c_f$ Profile.
Fig. 4.3 - Dimensionless Pressure Solutions from the "Zero-Order Polynomial" Model. Solutions Assume a Constant μC Profile.
The integral at the left-hand-side of Eq. 4.3.1 can be found in p. 323 of Ref. 32. That is

\[ \int \frac{dx}{(x^2 - a^2)} = \frac{1}{2a} \ln \frac{x - a}{x + a} \text{ (for } a \neq 0) \] .................................(4.3.2)

Hence, using Eq. 4.3.2, and completing the integration of Eq. 4.3.1, we obtain

\[ \frac{2}{2p_{wD}} \ln \left( \frac{p_D - p_{wD}}{p_D + p_{wD}} \right) \bigg|_1^{p_D} = -t_{Dd} \text{ (for } p_{wD} \neq 0) \]

or

\[ \ln \left( \frac{p_D - p_{wD}}{p_D + p_{wD}} \right) \left( \frac{1 + p_{wD}}{1 - p_{wD}} \right) = -p_{wD}t_{Dd} \] .................................(4.3.3)

Exponentiating Eq. 4.3.3 yields

\[ \left( \frac{p_D - p_{wD}}{p_D + p_{wD}} \right) \left( \frac{1 + p_{wD}}{1 - p_{wD}} \right) = \exp(-p_{wD}t_{Dd}) \]

or, rearranging, we have

\[ \left( \frac{p_D - p_{wD}}{p_D + p_{wD}} \right) = \left( \frac{1 - p_{wD}}{1 + p_{wD}} \right) \exp(-p_{wD}t_{Dd}) \]

Solving for the dimensionless pressure, \( p_D \), we obtain

\[ p_D = p_{wD} \left[ \frac{1 + \left( \frac{1 - p_{wD}}{1 + p_{wD}} \right) \exp(-p_{wD}t_{Dd})}{1 - \left( \frac{1 - p_{wD}}{1 + p_{wD}} \right) \exp(-p_{wD}t_{Dd})} \right] \]

which may further be simplified to yield

\[ p_D = p_{wD} \left[ \frac{(1 + p_{wD}) + (1 - p_{wD}) \exp(-p_{wD}t_{Dd})}{(1 + p_{wD}) - (1 - p_{wD}) \exp(-p_{wD}t_{Dd})} \right] \] .................................(4.3.4)

We again note that the pressure solution given by Eq. 4.3.4 is an approximate, time-dependent material balance equation. Given the more representative data model in this case (i.e., a straight line linearization data model as compared to the assumption of a constant data function), we would expect Eq. 4.3.4 to perform significantly better than the result from the previous case (Eq. 4.2.2).

In terms of real variables Eq. 4.3.4 is expressed as
\[
P_D = \frac{p_{wf}}{z_{wf}} \frac{[\frac{p_i}{z_i} + \frac{p_{wf}}{z_{wf}}] + \left[\frac{p_i}{z_i} - \frac{p_{wf}}{z_{wf}}\right] \exp\left(-\frac{p_{wf} z_i J_g}{z_{wf} c_i G t}\right)}{\left[\frac{p_i}{z_i} + \frac{p_{wf}}{z_{wf}}\right] - \left[\frac{p_i}{z_i} - \frac{p_{wf}}{z_{wf}}\right] \exp\left(-\frac{p_{wf} z_i J_g}{z_{wf} c_i G t}\right)}
\]


which can be simplified as
\[
P_D = \frac{p_{wf}}{z_{wf}} \frac{[\frac{p_i}{z_i} + \frac{p_{wf}}{z_{wf}}] + \left[\frac{p_i}{z_i} - \frac{p_{wf}}{z_{wf}}\right] \exp\left(-\frac{p_{wf} z_i D_i t}{z_{wf} c_i G t}\right)}{\left[\frac{p_i}{z_i} + \frac{p_{wf}}{z_{wf}}\right] - \left[\frac{p_i}{z_i} - \frac{p_{wf}}{z_{wf}}\right] \exp\left(-\frac{p_{wf} z_i D_i t}{z_{wf} c_i G t}\right)}
\]

Substituting Eq. 4.3.4 into Eq. 4.1.13 and solving for \(G_{pD}\) gives the cumulative production relation for this case. This result is given as
\[
G_{pD} = \frac{(1 - p_{WD}^2) [1 - \exp(-p_{WD} t D_d)]}{[(1 + p_{WD}) - (1 - p_{WD}) \exp(-p_{WD} t D_d)]} \tag{4.3.6}
\]

The rate-time model is now derived by differentiating Eq. 4.3.6 and substituting this result into Eq. 4.1.15. This gives us
\[
q_{Dd} = \frac{1}{\alpha} \frac{d G_{pD}}{d t_{Dd}} = \frac{1}{\alpha} \frac{d}{d t_{Dd}} \left[\frac{(1 - p_{WD}^2) [1 - \exp(-p_{WD} t D_d)]}{[(1 + p_{WD}) - (1 - p_{WD}) \exp(-p_{WD} t D_d)]}\right]
\]

or
\[
q_{Dd} = \frac{(1 - p_{WD}^2)}{\alpha} \left\{ \frac{p_{WD} \exp(-p_{WD} t D_d)}{[(1 + p_{WD}) - (1 - p_{WD}) \exp(-p_{WD} t D_d)]}ight. \\
\left. - \frac{[1 - \exp(-p_{WD} t D_d)](1 - p_{WD}) p_{WD} \exp(-p_{WD} t D_d)}{[(1 + p_{WD}) - (1 - p_{WD}) \exp(-p_{WD} t D_d)]^2} \right\}
\]

This equation can be simplified further to yield
\[
q_{Dd} = \frac{1}{\alpha} \frac{2 p_{WD}^2 (1 - p_{WD}^2) \exp(-p_{WD} t D_d)}{[(1 + p_{WD}) - (1 - p_{WD}) \exp(-p_{WD} t D_d)]^2} \tag{4.3.7}
\]

For this case, the \(\alpha\)-parameter is defined by substituting Eq. 4.1.25 into Eq. 4.1.11. This gives us
\[
\alpha = \int_{p_{WD}}^{1} \left[\frac{\mu c_i}{\mu c_i'}\right] dp' = \int_{p_{WD}}^{1} p_D dp'
\]
or completing the integration, we obtain
\[ \alpha = \frac{1}{2} (1 - p_{wd}^2) \] .................................................................(4.3.8)

Substituting Eq. 4.3.8 into Eq. 4.3.7 gives us
\[ q_{Dd} = \frac{2p_{wd}^2 (1 - p_{wd}^2) \exp(-p_{wd} t_{Dd})}{\left(1 - p_{wd}^2\right)\left[(1 + p_{wd}) - (1 - p_{wd})\exp(-p_{wd} t_{Dd})\right]^2} \]

which upon simplification yields
\[ q_{Dd} = \frac{4p_{wd}^2 \exp(-p_{wd} t_{Dd})}{\left[(1 + p_{wd}) - (1 - p_{wd})\exp(-p_{wd} t_{Dd})\right]^2} \] .........................................................(4.3.9)

From Eq. 4.3.9 we see that the only variable in this explicit rate-time model is \( p_{wd} \), hence graphical solutions in the form of type curves can be generated for different values of \( p_{wd} \). However, we have observed that using \( p_{wd} \) as a correlating parameter results in different rate-time solutions for reservoirs with different initial pressures. Therefore, to generalize this result, we will express Eq. 4.3.9 in terms of our \( \beta \)-parameter presented in Section 4.2.

Note from Eq. 4.2.11 we defined the \( \beta \)-parameter as follows
\[ \beta = \frac{\alpha}{(1 - p_{wd})} \] .................................................................(4.3.10)

Substituting Eq. 4.3.8 into Eq. 4.3.10, we obtain
\[ \beta = \frac{1}{2} (1 + p_{wd}) \text{ or } p_{wd} = 2\beta - 1 \] .................................................................(4.3.11)

Now, substituting Eq. 4.3.11 into Eq. 4.3.9 we obtain the following result in terms of the \( \beta \)-parameter
\[ q_{Dd} = \frac{(2\beta - 1)^2 \exp[-(2\beta - 1) t_{Dd}]}{[\beta - (1 - \beta) \exp[-(2\beta - 1) t_{Dd}]]^2} \] .................................................................(4.3.12)

We note that Eqs. 4.3.9 and 4.3.12 are valid only for a non-zero \( p_{wd} \). We derive similar expressions for the zero bottomhole pressure case. Note that when \( p_{wd}=0 \), Eq. 4.3.1 reduces to
\[
\int_{1}^{p_D} \frac{dp_D}{p_D^2} = -\frac{1}{2} t_{dd} \tag{4.3.13}
\]

Integrating the left-hand-side of Eq. 4.3.13, we get

\[-(p_D^{-1} - 1) = -\frac{1}{2} t_{dd}\]

or

\[\frac{1}{p_D} - 1 = \frac{1}{2} t_{dd}\]

Solving for dimensionless pressure, \(p_D\), gives

\[p_D = \frac{1}{1 + 0.5 t_{dd}} \tag{4.3.14}\]

In terms of real variables, Eq. 4.3.14 becomes

\[
\frac{p}{z} = \frac{p_i}{z_i} \left[ \frac{1}{1 + \frac{J_g}{2c_R G}} \right] \left[ 1 + \frac{1}{1 + 0.5 D_i t} \right]
\]

or

\[\frac{p}{z} = \frac{p_i}{z_i} \left[ 1 + 0.5 D_i t \right] \tag{4.3.15}\]

Substituting Eq. 4.3.14 into Eq. 4.1.13, and solving for \(G_{pD}\), we derive the cumulative production model for the \(p_{wD}=0\) case. The result is given as

\[G_{pD} = \frac{0.5 t_{dd}}{1 + 0.5 t_{dd}} \tag{4.3.16}\]

The rate-time model is then computed by substituting Eq. 4.3.16 into Eq. 4.1.15. This gives

\[q_{dd} = \frac{1}{\alpha} \frac{dG_{pD}}{dt_{dd}} = \frac{1}{\alpha} \frac{d}{dt_{dd}} \left[ 0.5 t_{dd} (1 + 0.5 t_{dd})^{-1} \right] = \frac{0.5}{\alpha} \frac{1}{(1 + 0.5 t_{dd})^2}\]

We observe from Eq. 4.3.8 that when \(p_{wD}=0\), the \(\alpha\)-parameter becomes

\[\alpha = \frac{1}{2} \tag{4.3.17}\]

Substitution Eq. 4.3.17 into the rate equation, we obtain
\[ q_{Dd} = \frac{1}{(1 + 0.5t_{Dd})^2} \] ......................................................(4.3.18)

As \( p_{wD} \) values lie between zero and one, it is obvious that for this "First-order polynomial" model \( 0.5 \leq \beta \leq 1.0 \). In summary, we have the following explicit rate-time equations

\[
q_{Dd} = \begin{cases} 
\frac{1}{(1 + 0.5t_{Dd})^2} ; & \beta = 0.5 \\
\frac{(2\beta - 1)^2 \exp[-(2\beta - 1)t_{Dd}]}{[\beta - (1 - \beta)\exp[-(2\beta - 1)t_{Dd}]]^2} ; & \beta > 0.5 
\end{cases} \] ......................................................(4.3.19)

A plot of various cases of Eq. 4.3.19 is presented in Fig. 4.4. These rate-time solutions are compared with typical Carter type curves as was done previously. We note that these "First-order polynomial" solutions, which we will henceforth call the "Knowles" solutions, compare very well with the numerical solutions — except for high drawdown cases (i.e., low values of \( \beta \)). For these low pressure reservoir cases, we see some deviation of the "Knowles" solutions from the Carter type curves. This discrepancy is due to the fact that the Carter type curves were reproduced using a high pressure gas reservoir (\( p_i = 8000 \) psia, \( \gamma_g = 0.55 \), \( T = 150 \) °F).

We will observe in Chapter V that the rate-time solutions from "Knowles" model compare extremely well with the numerical solutions for low pressure gas reservoir system. This fact reinforces the non-linear nature of the gas flow problem as the solutions are functions of the depletion path. Fig. 4.4 can therefore be used as type curves for the analysis and prediction of gas well performance in low pressure reservoirs (\( p_i \leq 5000 \) psia).

The dimensionless pressure solutions from this "First-order polynomial" model are also developed by substituting Eq. 4.3.11 into Eq. 4.3.4 and 4.3.14. This gives

\[
p_D = \begin{cases} 
\frac{1}{(1 + 0.5t_{Dd})} ; & \beta = 0.5 \\
\frac{(2\beta - 1)[\beta + (1 - \beta)\exp[-(2\beta - 1)t_{Dd}]]}{[\beta - (1 - \beta)\exp[-(2\beta - 1)t_{Dd}]]} ; & \beta > 0.5 
\end{cases} \] ......................................................(4.3.20)

These results are plotted in Fig. 4.5.
Fig. 4.4 - "First-Order Polynomial" Solutions of the Real Gas Flow Equation under Boundary-Dominated Flow Conditions. Solutions Assume a Linear $\mu c_f$ Profile During Reservoir Depletion. (Valid only for Low Pressure Gas Reservoirs).
Fig. 4.5 - Dimensionless Pressure Solutions from the "First-Order Polynomial" Model. Solutions Assume a Linear $\mu c_i$ Profile During Reservoir Depletion.
4.4 Analytical Solution of the "Exponential" Model

In this section, we derive the explicit rate-time solutions from the exponential linearization of the stabilized flow equation. We recall from Chapter III that the "Exponential" model (Eq. 4.1.26) represents a special approximation of the "General polynomial" model for the viscosity-compressibility function \( \mu_c/\mu_c \).

Substitution of Eq. 4.1.26 into the double integral expression in Eq. 4.1.23 yields

\[
\frac{B_1}{B_0} \int_{1}^{p_D} \frac{dP_D}{\exp(B_1 P_D) - \exp(B_1 P_{wD})} = -t_{Dd} \tag{4.4.1}
\]

The integral on the left-hand-side of Eq. 4.4.1 is of the type

\[
\int \frac{dx}{[\exp(ax) + b]} = \frac{1}{ab} (ax - \ln[\exp(ax) + b])
\]

Substituting this result into Eq. 4.4.1 gives

\[
-\frac{1}{B_0 \exp(B_1 P_{wD})} \left( B_1 P_D - \ln[\exp(B_1 P_D) - \exp(B_1 P_{wD})] \right) \bigg|_{1}^{p_D} = -t_{Dd}
\]

and rearranging, we obtain

\[
\left( \ln[\exp(B_1 P_D) - \exp(B_1 P_{wD})] - \ln[\exp(B_1 P_{wD})] \right) \bigg|_{1}^{p_D} = -B_0 \exp(B_1 P_{wD}) t_{Dd} \ldots (4.4.2)
\]

We simplify Eq. 4.4.2 to obtain

\[
\ln\left[ \frac{\exp(B_1 P_D) - \exp(B_1 P_{wD})}{\exp(B_1 P_{wD})} \right] \bigg|_{1}^{p_D} = -B_0 \exp(B_1 P_{wD}) t_{Dd}
\]

which can be expressed as

\[
\ln(1 - \exp[-B_1 (P_D - P_{wD})]) \bigg|_{1}^{p_D} = -B_0 \exp(B_1 P_{wD}) t_{Dd}
\]

Expanding this relation further yields

\[
\ln(1 - \exp[-B_1 (P_D - P_{wD})]) - \ln(1 - \exp[-B_1 (1 - P_{wD})]) = -B_0 \exp(B_1 P_{wD}) t_{Dd}
\]

or using the rules of logarithms, we have
\[
\ln \left( \frac{1 - \exp[-B_1(p_D - p_wD)]}{1 - \exp[-B_1(1-p_wD)]} \right) = -B_0 \exp(B_1 p_wD) t_{Dd} \tag{4.4.3}
\]

We define a simplified variable \( \theta \) as
\[
\theta = B_0 \exp(B_1 p_wD) \tag{4.4.4}
\]

Substituting Eq. 4.4.4 into Eq. 4.4.3, we obtain
\[
\ln \left( \frac{1 - \exp[-B_1(p_D - p_wD)]}{1 - \exp[-B_1(1-p_wD)]} \right) = -\theta t_{Dd}
\]

Exponentiating this result gives us
\[
\frac{1 - \exp[-B_1(p_D - p_wD)]}{1 - \exp[-B_1(1-p_wD)]} = \exp(-\theta t_{Dd})
\]

which can be expanded to yield
\[
\exp[-B_1(p_D - p_wD)] = 1 - (1 - \exp[-B_1(1-p_wD)]) \exp(-\theta t_{Dd}) \tag{4.4.5}
\]

Taking the logarithm of Eq. 4.4.5 gives
\[
-B_1(p_D - p_wD) = \ln[1 - (1 - \exp[-B_1(1-p_wD)]) \exp(-\theta t_{Dd})]
\]

and finally solving for dimensionless pressure, \( p_D \), we obtain
\[
p_D = p_{wD} - \frac{1}{B_1} \ln[1 - (1 - \exp[-B_1(1-p_wD)]) \exp(-\theta t_{Dd})] \tag{4.4.6}
\]

We derive an expression for dimensionless cumulative gas production, \( G_{pD} \), by substituting Eq. 4.4.6 into Eq. 4.1.13. This gives
\[
G_{pD} = (1 - p_{wD}) + \frac{1}{B_1} \ln[1 - (1 - \exp[-B_1(1-p_wD)]) \exp(-\theta t_{Dd})] \tag{4.4.7}
\]

The expression for the dimensionless rate, \( q_{Dd} \), is derived from differentiation of the cumulative production profile using Eq. 4.1.15. This gives
\[
q_{Dd} = \frac{dG_{pD}}{dt_{Dd}} = \frac{1}{\alpha} \frac{d}{dt_{Dd}} \left\{ (1 - p_{wD}) + \frac{1}{B_1} \ln[1 - (1 - \exp[-B_1(1-p_wD)]) \exp(-\theta t_{Dd})] \right\}
\]

or
\[
q_{Dd} = \frac{\theta (1 - \exp[-B_1(1-p_wD)]) \exp(-\theta t_{Dd})}{\alpha B_1 [1 - (1 - \exp[-B_1(1-p_wD)]) \exp(-\theta t_{Dd})]} \tag{4.4.8}
\]
The expression for $\alpha$ for this case is derived by substituting Eq. 4.1.26 into Eq. 4.1.11. That is

$$
\alpha = B_0 \int_{p_{WD}}^{1} [\exp(B_1 p_D')] dp_D = \frac{B_0}{B_1} \exp(B_1 p_{WD}) \bigg|_{p_{WD}}^{1}
$$

or expanding this equation yields

$$
\alpha = \frac{B_0}{B_1} [\exp(B_1) - \exp(B_1 p_{WD})] \tag{4.4.9}
$$

Substituting Eqs. 4.4.4 and Eq. 4.4.9 and into Eq. 4.4.8 gives

$$
q_{Dd} = \frac{B_0 \exp(B_1 p_{WD}) (1 - \exp[-B_1(1-p_{WD})]) \exp(-\theta_{Dd})}{B_0 \exp(B_1) - \exp(B_1 p_{WD})} B_1 \{1 - (1 - \exp[-B_1(1-p_{WD})]) \exp(-\theta_{Dd})\}
$$

which reduces to

$$
q_{Dd} = \frac{(1 - \exp[-B_1(1-p_{WD})]) \exp(-\theta_{Dd})}{(\exp[B_1(1-p_{WD})] - 1)[1 - \exp[-B_1(1-p_{WD})]] \exp(-\theta_{Dd})} \tag{4.4.10}
$$

Plots of Eq. 4.4.10 for two cases (reservoirs with initial pressures of 8000 and 12000 psia) are shown in Fig. 4.6. Values of the coefficients $B_0$ and $B_1$ of the exponential model used in these cases are given in Table 3.2 in Chapter III. These examples show that the rate-time solutions presented in terms of the correlating parameter $p_{WD}$ (Eq. 4.4.10) depend on the initial reservoir pressure.

We will, therefore, express Eq. 4.4.10 in terms of the new $\beta$-parameter by substituting Eq. 4.4.9 into Eq. 4.3.9. This yields

$$
\beta = \frac{\alpha}{(1 - p_{WD})} = \frac{B_0}{B_1(1 - p_{WD})} [\exp(B_1) - \exp(B_1 p_{WD})] \tag{4.4.11}
$$

Solving for $p_{WD}$ in Eq. 4.4.11 gives us

$$
p_{WD} = 1 - \frac{1}{B_1} \ln \left[ - \frac{B_0}{\beta} \exp[B_1 - \frac{B_0}{\beta} \exp(B_1)] \right] - \frac{B_0}{B_1 \beta} \exp(B_1) \tag{4.4.12}
$$
Fig. 4.6 - "Exponential" Solutions of the Real Gas Flow Equation under Boundary-Dominated Flow Conditions (i.e., $(\mu c_l)/(\mu c_0) = B_0 \exp(B_1 p_D)$). Note the Use of $p_{WD}$ as the Correlating Parameter.
Eq. 4.4.12 involves the $W$-function, $W(x),$\textsuperscript{27-28} which is a solution to the transcendental equation

$$W(x) \exp [W(x)] = x \quad \text{..........................................................(4.4.13)}$$

A plot of the $W$-function is presented in Fig. 4.7. The plot shows the two main branches of $W(x)$ — the upper branch, $W_u(x),$ and the lower branch, $W_m(x).$ The values of interest to us in our decline curve analysis model are described by the function $W_u(x),$ in the limited range of $-e^{-1} < x < 0.$

Fig. 4.7 - Plot of the $W$-Function (after Barry et al.\textsuperscript{27}).
Note that values of $W(x)$ can be approximated by the following expression\textsuperscript{27}

$$W(x) = -1.0 + \frac{\sqrt{\eta} (1080 + 384\sqrt{\eta})}{(1080 + 744\sqrt{\eta} + 83\eta)}.$$ \hspace{0.5cm} (4.4.14)

where

$$\eta = 2.0 + 2x\exp(1).$$ \hspace{0.5cm} (4.4.15)

Table 4.1 shows the values from Eqs. 4.4.14 and 4.4.15 compared to the exact values of $W(x)$ for the range of $x$-values of interest in this study.

Substituting Eq. 4.4.12 into Eq. 4.4.10 yields an explicit rate-time equation in terms of the $\beta$-parameter. Figs. 4.8 and 4.9 below present graphical solutions of this equation for both reservoirs with initial pressures of 8000 psia and 12000 psia. Observe that all the solutions do overlay each other quite well and compare excellently with the simulated Carter type curves.

A plot of the dimensionless pressure profile (Eq. 4.4.6) during reservoir depletion is presented in Fig. 4.10. These solutions also compare excellently with the simulated average pressure responses. Once again, we emphasize that the "Exponential" model is only valid for high pressure gas reservoir systems ($p \geq 8000$ psia).

<table>
<thead>
<tr>
<th>$x$</th>
<th>$W(x)$ (Exact)</th>
<th>$W(x)$ (Eq. 4.4.14)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>-0.001229</td>
</tr>
<tr>
<td>-0.05</td>
<td>-0.052706</td>
<td>-0.053652</td>
</tr>
<tr>
<td>-0.10</td>
<td>-0.111833</td>
<td>-0.112525</td>
</tr>
<tr>
<td>-0.15</td>
<td>-0.179491</td>
<td>-0.179962</td>
</tr>
<tr>
<td>-0.20</td>
<td>-0.259171</td>
<td>-0.259457</td>
</tr>
<tr>
<td>-0.25</td>
<td>-0.357403</td>
<td>-0.357545</td>
</tr>
<tr>
<td>-0.30</td>
<td>-0.489402</td>
<td>-0.489448</td>
</tr>
<tr>
<td>-0.35</td>
<td>-0.716639</td>
<td>-0.716641</td>
</tr>
<tr>
<td>-0.367879</td>
<td>-0.998452</td>
<td>-0.998452</td>
</tr>
</tbody>
</table>
Fig. 4.8 - "Exponential" Solutions of the Real Gas Flow Equation under Boundary-Dominated Flow Conditions (i.e., \( (\mu c T (\mu c) = B_0 \exp (B_1 \rho D) \)). Transformed Solutions Using \( \beta \) as the Correlating Parameter (Fluid Sample 1).
Fig. 4.9 - "Exponential" Solutions of the Real Gas Flow Equation under Boundary-Dominated Flow Conditions (i.e., \((\mu c_d)/(\mu c_d) = B_0 \exp(B_1 p_d)\)). Transformed Solutions Using \(\beta\) as the Correlating Parameter (Fluid Sample 2).
Fig. 4.10 - Dimensionless Pressure Solutions from the "Exponential" Model.
4.5 Solution of the "General Polynomial" Model

In this section we present the rate and cumulative production solutions obtained from using the "General polynomial" model for correlating the viscosity-compressibility changes that occur during gas reservoir depletion. As was noted earlier, this equation represents the most general linearization scheme for the non-linear stabilized gas flow equation. In Chapter III we showed that the viscosity-compressibility ratio can be modeled accurately using this general polynomial function.

Substituting the "General polynomial" model (Eq. 4.1.27) into Eq. 4.1.23 and integrating once yields

\[
\int_1^{P_D} \frac{dP_D}{[A_0 + A_1 P_D + A_2 P_D^2 + A_3 P_D^3 + A_4 P_D^4 + A_5 P_D^5 + \ldots]} = -t_{dd} \quad \ldots \quad (4.5.1)
\]

Unfortunately, the integral on the left-hand-side of Eq. 4.5.1 cannot be readily evaluated analytically. Hence, we re-write this integral expression in the form of a simple first-order ordinary differential equation (ODE). Differentiating both sides of Eq. 4.5.1 with respect to dimensionless time, we obtain

\[
\frac{dP_D}{dt_{dd}} = -[A_0 + A_1 P_D + A_2 P_D^2 + A_3 P_D^3 + A_4 P_D^4 + A_5 P_D^5 + \ldots] \quad \ldots \quad (4.5.2)
\]

Note that Eq. 4.5.2 can easily be formulated into an ordinary differential equation involving only dimensionless cumulative production by using Eq. 4.1.13. In fact, substituting Eq. 4.1.13 into Eq. 4.5.2 yields

\[
\frac{dG_{PD}}{dt_{dd}} = [A_0 + A_1(1 - G_{PD}) + A_2(1 - G_{PD})^2 + A_3(1 - G_{PD})^3 + A_4(1 - G_{PD})^4 + \ldots] \quad \ldots \quad (4.5.3)
\]

where \(A_0, A_1, A_2, \) etc. in the last three equations are defined by

\[
A_0 = -[A_1 P_{wD} + A_2 P_{wD}^2 + A_3 P_{wD}^3 + A_4 P_{wD}^4 + A_5 P_{wD}^5 + \ldots] \quad \ldots \quad (4.5.4)
\]

\[
A_1 = a_0 \quad \ldots \quad (4.5.5)
\]

\[
A_2 = \frac{a_1}{2} \quad \ldots \quad (4.5.6)
\]

\[
A_3 = \frac{a_2}{3} \quad \ldots \quad (4.5.7)
\]
Eq. 4.5.3 is evaluated numerically. A Fortran code for the Runge-Kutta-Fehlberg integration routine (presented in Appendix D) with adaptive time step size selection was written for this problem. The initial condition used in this problem is derived from Eq. 4.1.22 as

At \( t = 0 \) (or \( t_{D_d} = 0 \)); \( G_{pD} = 0 \) .........................................................(4.5.8)

After generating the dimensionless cumulative production versus dimensionless time, the dimensionless rate profile is obtained by differentiating this cumulative production profile according to Eq. 4.1.15. For this "General polynomial" model, the \( \alpha \)-parameter is determined by substituting Eq. 4.1.27 into Eq. 4.1.11. This substitution gives

\[
\alpha = \int_{p_{wD}}^{1} \left[ a_0 + a_1 p_D + a_2 p_D^2 + a_3 p_D^3 + a_4 p_D^4 + a_5 p_D^5 + \ldots \right] dp_D
\]

or completing this integration yields

\[
\alpha = \left[ A_0 + A_1 + A_2 + A_3 + A_4 + A_5 + \ldots \right] .........................................................(4.5.9)
\]

The \( \beta \) correlating parameter in this case is defined by

\[
\beta = \frac{\alpha}{1 - p_{wD}} = \frac{1}{1 - p_{wD}} \left[ A_0 + A_1 + A_2 + A_3 + A_4 + A_5 + \ldots \right] .........................................................(4.5.10)
\]

Fig. 4.11 presents solutions of the "General polynomial" model for the cases of a low, moderate and high pressure gas reservoirs (i.e., \( p_r=4000 \), 8000, and 12000 psia, respectively) for Fluid Sample 1. A similar plot of \( q_{D_d} \) versus \( t_{D_d} \) for Fluid Sample 2 is shown in Fig. 4.12.

We note from both of these plots that the "General polynomial" solutions do overlay each other and compare very well with the simulated Carter type curves. The mismatch in the case of the \( \beta=0.5 \) stem for the low pressure case has been explained in Section 4.3 (i.e., the fact that the Carter type curves used in this plot was generated using a high pressure gas reservoir). The accuracy of the "General polynomial" solutions will be verified in Chapter V. Dimensionless pressure solutions for these cases are presented in Figs. 4.13 and 4.14 for the low and high pressure gas reservoir systems, respectively. In these plots, we again find excellent agreement between the new dimensionless pressure solutions and the simulated results.
Fig. 4.11 - "General Polynomial" Solutions of the Real Gas Flow Equation under Boundary-Dominated Flow Conditions (i.e., \(\frac{(\mu c_i)}{(\mu c)} = a_0 + a_1 P_D + a_2 (P_D)^2 + a_3 (P_D)^3 + \ldots\)). (Fluid Sample 1).
Fig. 4.12 - "General Polynomial" Solutions of the Real Gas Flow Equation under Boundary-Dominated Flow Conditions \((\mu c_t) / (\mu c) = a_0 + a_1 p_D + a_2 (p_D)^2 + a_3 (p_D)^3 + ...\). (Fluid Sample 2).
Fig. 4.13 - Dimensionless Pressure Solutions from the "General Polynomial" Model. Low Pressure Gas Reservoir Case ($p_i = 4000$ psia).
Fig. 4.14 - Dimensionless Pressure Solutions from the "General Polynomial" Model. High Pressure Gas Reservoir Case ($p_i = 12000$ psia).
4.6 Solution Scheme for Variable Bottomhole Pressure Production

In this section we use the first-order ordinary differential equation (ODE), developed in Section 4.5, to analyze gas well performance under variable bottomhole pressure conditions. We note that bottomhole pressure profiles of producing gas wells change frequently as a result of many different factors, such as changes in surface line pressure, changes in surface choke size, routine maintenance of the well, proration, etc. These pressure changes affect the daily gas rates, which are manifested as spikes in gas production rate profiles, and must be accounted for in the analysis of gas well performance data.

We recognize that changes in bottomhole flowing pressure during reservoir depletion introduce transient flow behavior. However, in this work, we assume that the pressure changes are relatively small, so as not to violate the assumption of boundary-dominated flow made in the development of these rate-time and cumulative production models.

Recall the dimensionless form of the boundary-dominated flow model

\[
\frac{dG_{PD}}{dF_{PD}} = \left[ A_0 + A_1(1 - G_{PD}) + A_2(1 - G_{PD})^2 + A_3(1 - G_{PD})^3 + A_4(1 - G_{PD})^4 + \ldots \right] \ldots (4.5.3)
\]

where \( A_0, A_1, A_2, \) etc. are given as

\[
A_0 = -\left[ A_1 p_{wD} + A_2 p_{wD}^2 + A_3 p_{wD}^3 + A_4 p_{wD}^4 + A_5 p_{wD}^5 + \ldots \right] \ldots (4.5.4)
\]

\[
A_1 = a_0 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldOTS
\]

\[
A_2 = \frac{a_1}{2}; \quad \text{etc.} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldOTS
\]

The solution of Eq. 4.5.3 depends on the mode of depletion of the reservoir. For constant bottomhole pressure production all the coefficients in this equation are constant. In such a case the integration of the ODE is straightforward when the initial condition is specified. This problem was described in Section 4.5, and the rate-time solutions were presented, graphically, in the form of type curves.

However, under varying flowing bottomhole pressure the coefficient \( A_0 \) is no longer constant, but changes as a function of dimensionless time. When these changes are assumed to be relatively small so that the boundary-dominated model is still applicable, we
can solve the ODE similar to the constant pressure case, using the assumption of constant \( P_{wD} \) terms over discrete intervals of time. The important question remains as to how to establish \( P_{wD} \) from past production history that corresponds exactly to a given dimensionless time, or vice versa. We do this via the initial decline curve constant, \( D_i \).

Note here that the dimensionless "decline" time was defined in Section 4.1 as

\[
t_{Dd} = \frac{J_g}{c_i G} t = D_i t
\]

where

\[
D_i = \frac{J_g}{c_i G}
\]

Under variable bottomhole pressure production, an estimate of \( D_i \) is necessary for transforming the dimensionless time values into real time values. Using these time values we establish exactly the corresponding flowing bottomhole pressures, and hence \( P_{wD} \) values. The need to assume a value of \( D_i \) (because it is unknown) makes the variable bottomhole pressure problem iterative in nature.

It is worth noting that the original problem of boundary-dominated depletion of gas reservoir systems involves two main unknowns — the original-gas-in-place (OGIP) and the well productivity index (i.e., the gas flow constant), \( J_g \). Once either variable is estimated, the other can be calculated using the flow equation or the gas material balance equation (Ref. 23). In that case, production performance can be predicted by solving these two equations separately. Specifically, if the OGIP is known, then we can use the gas material balance equation given by

\[
\frac{p}{z} = \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right]
\]

(3.1.2)

to estimate values of average reservoir pressure as a function of time. We use a table of gas properties to convert the pressures into normalized pseudopressures which are then used in the stabilized flow equation to compute the well productivity index, \( J_g \). We recall the stabilized rate equation

\[
q_g = J_g [p_p - p_{wD}]
\]

(3.1.1)
Eq. 3.1.1 suggests that a plot of \( q_g \) versus \( \Delta p_p = [p_p - p_{pwf}] \) should yield a straight line with a slope of \( J_g \).

On the other hand, if the value of \( J_g \) is known, then we solve this problem of computing OGIP in the reverse order. First, we solve for values of \( p_p \) using Eq. 3.1.1 (\( q_g, p_{pwf}, \) and \( J_g \) are known). Second, we convert these normalized pseudopressure functions into average reservoir pressures (using a table of gas properties), and then use the conventional \( p/z \) versus cumulative gas production plot to estimate OGIP.

Decline curve analysis gives us a graphical means of estimating these two unknowns simultaneously using the entire production history. Note that by coupling the stabilized flow equation and the gas material balance equation into the ordinary differential equation (Eq. 4.5.3), we are solving for the same two unknown parameters, but now using a single equation. The important fact here is that the two-parameter problem has been transformed into a single parameter problem. The two unknown parameters have been grouped into a single unknown parameter, \( D_i \), which physically represents the initial rate decline constant in our rate prediction models.

The result of using \( D_i \) to incorporate the changing bottomhole pressure data into the solution routine is a dimensionless rate profile that exactly reflects all the pertinent features (i.e., sharp rate changes) that occurred during past production. The important issue at this stage is how to obtain a highly precise value of \( D_i \) in order to improve the accuracy of the original-gas-in-place calculation.

In general, there are two approaches. The first - using a minimization algorithm and second - using a graphical approach, which has been described earlier. We consider the latter approach in this work. The \( J_g \)-parameter obtained from the rate match point along the horizontal axis does not require knowledge of the precise value of the constant \( D_i \). In other words, the correct value of the well productivity index \( J_g \) can be estimated using an approximate estimate of \( D_i \).

This fact is very important from an analysis standpoint because the estimate of the original-gas-in-place can be improved by using the conventional \( p/z \) versus cumulative gas production method. In effect, the value of \( J_g \) can be estimated graphically with high level
of precision using an approximate value of $D_i$ to compute the variable rate production response (recall $J_g$ is estimated using the rate match point).

Knowing the value of $J_g$, we can then use the stabilized rate equation to calculate the average reservoir pressure profile. This relation in terms of normalized pseudopressure is derived from Eq. 3.1.1 as

$$p_p = \frac{q_g}{J_g} + p_{pwf} \tag{4.6.2}$$

Using this calculated pseudopressure profile, we compute the corresponding average reservoir pressure profile using fluid property correlations and a table look-up routine. A conventional $p/z$ versus cumulative gas production plot can be generated and the original-gas-in-place extrapolated from the straight line portion of the data. The final result of this graphical technique is to generate a production rate-time profile that reproduces the characteristic features of the production history. This technique will be demonstrated with practical examples in Chapter V.
CHAPTER V

VALIDATION OF THE NEW RATE-TIME SOLUTIONS

5.1 Summary
In this chapter we present verification of the new rate-time equations using simulated gas production data. More importantly, this chapter presents practical applications of the new solutions for the analysis of actual field data. The simulated production rate data were generated using a single-phase gas reservoir simulation package. Examples of both constant and variable flowing bottomhole pressure production are presented.

The basic data requirements for the analysis and prediction of gas well performance are the past production data, the initial reservoir pressure, formation temperature and specific gravity of the reservoir fluid. The production data should include gas flow rate and flowing bottomhole pressure as a function of time. Additional data include porosity, formation thickness, water saturation, and the rock compressibility, as well as wellbore radius.

We note that flowing bottomhole pressures are seldom recorded in the field, however, for single-phase gas reservoirs, bottomhole pressure data can be computed from wellhead pressure measurements. For the case of a gas well producing under a constant flowing bottomhole pressure, the production rate data are plotted on a log-log scale and matched against the new type curves developed in Chapter IV, from which we estimate reservoir parameters.

5.2 Performance Analysis Relations
The fundamental goal of decline curve analysis is to determine the two main parameters of a particular well — the original-gas-in-place and the well productivity index, $J_g$. In particular, we are interested in estimating the following parameters:

- Original-gas-in-place, $G$,
- Formation permeability, $k$ (or $kh$), and
Reservoir Drainage area, $A$.

These reservoir parameters are estimated using the following relations, derived from the definitions of the dimensionless variables ($q_{Dd}$ and $t_{Dd}$) together with the flowrate and time values obtained from a match point.

**Original-Gas-In-Place, $G$**

From the definition of $t_{Dd}$ (Eq. 4.1.17), we have

$$ t_{Dd} = \frac{J_g}{c_i t} $$

we compute the original-gas-in-place from the time match point using

$$ G = \frac{J_g}{c_i t} \left[ \frac{L}{t_{Dd, MP}} \right] ...................................................(5.2.1) $$

The well productivity index, $J_g$, is estimated from the rate match point as

$$ J_g = \frac{c_i}{\alpha \left[ \frac{q_g}{q_{Dd, MP}} \right]} ...................................................(5.2.2) $$

In Eq. 5.2.2 the $\alpha$-parameter is determined from the flowing bottomhole pressure, $p_{wf}$, at the start of gas production together with the coefficients of the different data functions (Eqs. 4.1.24-4.1.27) used to model the viscosity-compressibility profile during gas reservoir depletion. Dimensionless flowing bottomhole pressure is calculated using the following definition

$$ p_{WD} = \frac{(p_{wf} z_{wf})}{(P_w z_i)} ...................................................(5.2.3) $$

Substituting $p_{WD}$ into the individual expressions derived in Chapter IV for the different functional models, we have

**The Zero-Order Polynomial Model:**

$$ \alpha = \lambda (1 - p_{WD}) = \beta (1 - p_{WD}) ...................................................(5.2.4) $$

**Knowles First-Order Polynomial Model:**

$$ \alpha = \frac{1}{2} (1 - p_{WD}^2) ...................................................(5.2.5) $$
The Exponential Model:

\[ \alpha = \frac{B_0}{B_1} [\exp(B_1) - \exp(B_1 p_{wd})] \] .................................................... (5.2.6)

The General Polynomial Model:

\[ \alpha = [A_0 + A_1 + A_2 + A_3 + A_4 + A_5 + \ldots] \] .................................................... (5.2.7)

We note that in Eq. 5.2.7 \( p_{wd} \) is incorporated in the parameter \( A_0 \). Specifically, the constants in Eq. 5.2.7 are given by

\[ A_0 = -[A_1 p_{wd} + A_2 p_{wd}^2 + A_3 p_{wd}^3 + A_4 p_{wd}^4 + A_5 p_{wd}^5 + \ldots] \] .................................................... (5.2.8)

\[ A_1 = a_0 \] .................................................... (5.2.9)

\[ A_2 = \frac{a_1}{2} \] .................................................... (5.2.10)

\[ A_3 = \frac{a_3}{3} \] .................................................... (5.2.11)

\[ A_4 = \frac{a_3}{4} \); etc .................................................... (5.2.12)

Reservoir drainage area, \( A \)

From volumetric relation, the reservoir drainage area is given by

\[ A = 5.615 \frac{B_{gi} G}{\phi h (1 - S_{wi})} \] .................................................... (5.2.13)

Formation permeability, \( k \)

The effective permeability is estimated from the definition of \( J_g \), where \( J_g \) is defined by

\[ J_g = \frac{2kh}{141.2 \mu_i B_{gi} \ln \left[ \frac{2.2458A}{C_A r_{wa}^2} \right]} \]

Rearranging this expression gives

\[ k = \frac{141.2 \mu_i B_{gi} J_g}{2h} \frac{2.2458A}{\ln \left[ \frac{2.2458A}{C_A r_{wa}^2} \right]} \] .................................................... (5.2.14)

And, the permeability-thickness product, \( kh \), is given by
\[ kh = \frac{141.2 \mu_l B_{gi} \varepsilon_g}{2} \ln \left[ \frac{2.2458A}{C_A r_w^2} \right] \] \hspace{1cm} (5.2.15)

### 5.3 Production Under Constant Bottomhole Pressure Conditions

This section presents application of the new rate-time equations for the decline curve analysis of production data from gas wells flowing at fairly constant bottomhole pressure. We consider both simulated as well as actual field examples. Our goal is to provide a comprehensive verification to illustrate the validity of individual rate solutions, as well as the applicability of the solutions as decline curve analysis tools.

The simulated gas well performance cases are considered by matching the calculated rate solutions using our rate-time models against typical Carter type curves generated from a low, moderate, and high pressure gas reservoirs \((p_i=4000, 8000, \text{and} 12000 \text{ psia}, \) respectively). Table 5.1 presents a summary of the input parameters used in the simulation of these cases. For the numerical simulation cases the gas flow rate versus time data were converted into dimensionless variables using the following relations\(^{22}\)

\[ q_{Dd} = \frac{141.2 \mu_l B_i}{kh} \frac{q_g(t)}{(p_{pi} - p_{pwf})} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} \right] \] \hspace{1cm} (5.3.1)

and

\[ t_{Dd} = \frac{0.00633k t}{\phi \mu_i c_{ii} r_w^2} \frac{1}{2} \left[ \left( \frac{r_e}{r_w} \right)^2 - 1 \right] \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} \right] \] \hspace{1cm} (5.3.2)

Rate responses were generated using the four linearization models presented in this study. Due to the poor performance of the zero-order polynomial model, this model was not included in our verification process. The fluid parameters given in Table 5.1 (specifically, gas gravity and temperature) were used to generate the gas properties using correlations given in Refs. 47-49. These gas properties were then used to determine the coefficients of the functional models by plotting the viscosity-compressibility ratio versus dimensionless pressure and fitting these trends with the appropriate data model.

Fig. 5.1 shows a plot of \(q_{Dd}\) versus \(t_{Dd}\) from the "First-order polynomial" and the "General polynomial" models compared to the numerical simulation solutions (the "Carter" type curves) generated for the low pressure gas reservoir \((p_i=4000 \text{ psia})\). As is the case throughout this work, our focus in this verification process is only on the boundary-
dominated flow region, where our new rate-time models are applicable. Transient rate solutions have been attached to these type curves for completeness, as was also done by Fetkovich. These transient solutions were obtained from analytical solutions for a bounded circular reservoir system.

We note from Fig. 5.1 that both the "Knowles" and the "General polynomial" models perform excellently for this low pressure reservoir case. In contrast to the plots presented in Chapter IV, these solutions match the simulated Carter type curves very well. Note also that, in this figure, we did not include the "Exponential" rate solutions, because, as was stated earlier, the "Exponential" model is not valid for low pressure gas reservoirs.

**Table 5.1**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Reservoir Pressure, $p_i$</td>
<td>4000, 8000, and 12000 psia</td>
</tr>
<tr>
<td>Reservoir Temperature, $T$</td>
<td>150 and 300 °F</td>
</tr>
<tr>
<td>Gas Specific Gravity, $\gamma_g$</td>
<td>0.55 and 0.7 (air=1)</td>
</tr>
<tr>
<td>Irreducible Water Saturation, $S_{wc}$</td>
<td>0.0 (fraction)</td>
</tr>
<tr>
<td>Wellbore Radius, $r_w$</td>
<td>0.3 ft</td>
</tr>
<tr>
<td>Reservoir Outer Boundary, $r_e$</td>
<td>300 ft</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
<td>0.1 (fraction)</td>
</tr>
<tr>
<td>Reservoir Thickness, $h$</td>
<td>10 ft</td>
</tr>
<tr>
<td>Permeability, $k$</td>
<td>1.0 md</td>
</tr>
</tbody>
</table>

A similar plot for a moderately-pressured gas reservoir case ($p_i$=8000 psia) is presented in Fig. 5.2. In this case, we present only the "Exponential" and "General Polynomial" models as the "Knowles" solutions are not valid for pressures greater than 5000 psia. Again, we find excellent agreement between the simulated results and the new rate-time solutions. The production performance is matched extremely well for all of the cases (note that solutions for both Fluid Samples 1 and 2 are presented in this figure).
Fig. 5.2 - Comparison of the New Rate-Time Solutions with Carter Type Curves (Numerical Solutions). Moderately-Pressured Gas Reservoir Case ($p_i = 8000$ psia).
Fig. 5.3 - Comparison of the New Rate-Time Solutions with Carter Type Curves (Numerical Solutions). High Pressure Gas Reservoir Case \((p_i = 12000 \text{ psia})\).
Finally, Fig. 5.3 presents a similar comparison for the case of a high pressure gas reservoir system \( p_r = 12000 \) psia. Once again, we observe excellent agreement between the new rate prediction models and the simulated type curves — and therefore, any of the new models can be used to generate type curves for the analysis and prediction of high pressure gas well performance.

At this point, it is important to observe from the shapes of these simulated and computed rate-time curves (Figs. 5.1-5.3) that these solutions do not perfectly overlay one another for the different pressure systems. This fact is true for both the numerically simulated rate responses as well as the new analytical solutions, and confirms the highly non-linear nature of the gas flow equation. In short, the semi-analytical and numerical solutions agree on a case-by-case basis, but not necessarily for cases of different pressure levels — hence, illustrating the non-linear behavior of the gas flow equations.

We therefore recommend that, as much as possible, type curves corresponding to the initial pressure of the reservoir be used for performance analysis. Simple analytical expressions have been provided in this work (Eqs. 4.3.19 and 4.4.10) for generating rate-time solutions without having to resort to a reservoir simulator. We consider, in the next sections, example cases of gas performance analysis using our new solutions.

**Case 1 (West Virginia Gas Well A) — Field Case**

In this section, we present an analysis of real field data using the new rate-time equations. This example shows production data from a well flowing gas at a fairly constant bottomhole pressure from a low pressure reservoir. We chose this case from the petroleum literature because these rate data have been analyzed extensively in the past, and provide a good basis for establishing the accuracy of our new equations.

West Virginia Gas Well A is a vertical well which has been hydraulically fractured in a reservoir undergoing depletion. The original production data were given by Fetkovich, *et al.*\(^{34}\) and were later analyzed in Refs. 5 and 20. Summary of the reservoir and fluid properties is given in Table 5.2.
Table 5.2
Reservoir, Fluid, and Production Data for Case 1 ("West Virginia Gas Well A")

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reservoir Properties:</strong></td>
<td></td>
</tr>
<tr>
<td>Wellbore Radius, ( r_w )</td>
<td>0.354 ft</td>
</tr>
<tr>
<td>Net Pay Thickness, ( h )</td>
<td>70 ft</td>
</tr>
<tr>
<td>Average Porosity, ( \phi )</td>
<td>0.06 (fraction)</td>
</tr>
<tr>
<td>Average Reservoir Temperature, ( T )</td>
<td>160.0 °F</td>
</tr>
<tr>
<td>Skin Factor, ( s )</td>
<td>-5.17</td>
</tr>
<tr>
<td>Permeability (Original Estimate), ( k )</td>
<td>0.07 md</td>
</tr>
<tr>
<td><strong>Fluid Properties:</strong></td>
<td></td>
</tr>
<tr>
<td>Gas Specific Gravity, ( y_g )</td>
<td>0.57 (air=1)</td>
</tr>
<tr>
<td>Gas FVF at Initial Conditions, ( B_{gi} )</td>
<td>7.101x10^{-4} RB/Scf</td>
</tr>
<tr>
<td>Viscosity at Initial Conditions, ( \mu_{gi} )</td>
<td>0.0255 cp</td>
</tr>
<tr>
<td>Total Compressibility, ( c_H )</td>
<td>1.824x10^{-4} psia^{-1}</td>
</tr>
<tr>
<td><strong>Production Parameters:</strong></td>
<td></td>
</tr>
<tr>
<td>Initial Reservoir , ( p_i )</td>
<td>4175 psia</td>
</tr>
<tr>
<td>Average BHP, ( p_{wf} )</td>
<td>710 psia</td>
</tr>
<tr>
<td>Dimensionless BHP, ( p_{WD} )</td>
<td>0.1702</td>
</tr>
</tbody>
</table>

Figs. 5.4 and 5.5 present analyses of these rate-time data using both the "Knowles" and "General Polynomial" type curves, respectively. Detailed calculations of the reservoir parameters using the match point values and the performance analysis relations developed in Section 5.2 are as follows:

**Type Curve Match:** Knowles Type Curve--Pressure-Squared Flow Model (Fig. 5.4)

Matching Parameters:

\[ [q_{Dd}]_{MP} = 1.0 \quad [q_g]_{MP} = 1.98 \times 10^6 \text{ Scf/D} \]
\[ [t_{Dd}]_{MP} = 1.0 \quad [t]_{MP} = 695 \text{ Days} \]
Fig. 5.4 - Decline Curve Analysis for Case 1 ("West Virginia Gas Well A" — SPE 14238). "Knowles" Type Curve Analysis Approach.
Fig. 5.5 - Decline Curve Analysis for Case 1 ("West Virginia Gas Well A" — SPE 14238). "General Polynomial" Type Curve Analysis Approach.
Analysis Results:

\[ J_g = 743.758 \text{ Scf/D/psia} \]

\[ G = 2.8344 \times 10^9 \text{ Scf} \]

\[ A = 4.1398 \times 10^6 \text{ ft}^2 \text{ or } 95.04 \text{ acres} \]

\[ k = 0.0518 \text{ md} \]

Calculations:

Well Productivity Index:

The well productivity index, \( J_g \), is computed from the production rate match point. Using Eq. 5.2.2 we obtain

\[ J_g = \frac{c_{nn} \left[ \frac{q_g}{q_{Df}} \right]}{\alpha_{[f_{Df,M}]} \alpha} = \frac{(1.8237 \times 10^{-4} \text{ psia}^{-1})}{(0.4855)} \left[ \frac{(1.98 \times 10^6 \text{ Scf/D})}{(1.0)} \right] \]

or

\[ J_g = 743.758 \text{ Scf/D/psia} \]

In these calculations, we have used the fact that for this "Knowles" model, the \( \alpha \)-parameter is given by Eq. 5.2.5

\[ \alpha = \frac{1}{2}(1 - p_w^2) = \frac{1}{2} \left[ 1 - (0.1702)^2 \right] \]

or

\[ \alpha = 0.4855 \]

Original-Gas-In-Place:

The original-gas-in-place is obtained from the time match point using Eq. 5.2.1 and the computed \( J_g \). This result is given by

\[ G = \frac{J_g}{c_{nn} \left[ \frac{t}{f_{Df,M}} \right]} = \frac{(743.758 \text{ Scf/D/psia}) \left[ \frac{(695 \text{ Days})}{(1.0)} \right]}{(1.8237 \times 10^{-4} \text{ psia}^{-1})} \]

or

\[ G = 2.8344 \times 10^9 \text{ Scf} \]

Reservoir Drainage Area:

Using the computed value of original-gas-in-place, we estimate the reservoir drainage area, \( A \), from Eq. 5.2.13
\[ A = 5.615 \frac{B_{gi}G}{\phi h (1-S_{wi})} = 5.615 \frac{(7.1012 \times 10^{-4} \text{RB/Scf})(2.8344 \times 10^9 \text{Scf})}{(0.06)(70 \text{ ft})(1-0.35)} \]

or

\[ A = 4.1398 \times 10^6 \text{ ft}^2 \text{ or } 95.04 \text{ acres} \]

**Permeability:**

The formation permeability, \( k \), is calculated from Eq. 5.2.14 as

\[ k = \frac{141.2 \mu_i B_{gi}J_g}{2h} \ln \left[ \frac{2.2458A}{CAr_w^2} \right] \]

\[ = 141.2 \frac{(0.0225 \text{cp})(7.1012 \times 10^{-4} \text{RB/Scf})(743.76 \text{Scf/psi})}{2(70 \text{ ft})} \]

\[ \times \ln \left[ \frac{2.2458 \left(4.1398 \times 10^6 \text{ ft}^2\right)}{(31.62)(62.27 \text{ ft})^2} \right] \]

or

\[ k = 0.0518 \text{ md} \]

Note that in this calculation we have used the apparent wellbore radius, \( r_{wa} \), defined by Eq. 3.3.4

\[ r_{wa} = r_w \exp(-s) = 0.354 \text{ ft} \exp(+5.17) = 62.274 \text{ ft} \]

And, for a radial reservoir system, the Dietz shape factor is \( C_A = 31.62 \).

**Type Curve Match:** Polynomial Type Curve--"General Polynomial" Flow Model (Fig. 5.5)

Similar analysis was conducted for Field Case 1 using the "General Polynomial" type curve. Results of this analysis are as follows

**Matching Parameters:**

\[ [Q_{Dd}]_{MP} = 1.0 \quad [Q_{d}]_{MP} = 1.98 \times 10^6 \text{ Scf/D} \]

\[ [t_{Dd}]_{MP} = 1.0 \quad [t]_{MP} = 680 \text{ Days} \]
Analysis Results:

\[ \dot{J}_g = 768.119 \text{ Scf/D/psia} \]
\[ G = 2.864 \times 10^9 \text{ Scf} \]
\[ A = 4.183 \times 10^6 \text{ ft}^2 \text{ or 96.03 acres} \]
\[ k = 0.0537 \text{ md} \]

Calculations:

Well Productivity index:

Solving the production rate match point relation for \( \dot{J}_g \) (Eq. 5.2.2), we obtain

\[ \dot{J}_g = \frac{c_{ti}}{\alpha} \left[ \frac{q_g}{q_{ld,imp}} \right] = \left( \frac{1.8237 \times 10^4 \text{ psia}^{-1}}{0.4701} \right) \left( \frac{1.98 \times 10^6 \text{ Scf/psia}}{1.0} \right) = 768.119 \text{ Scf/psia} \]

where in this case the parameter \( \alpha \) is computed from the coefficients of the 4-th order polynomial function used for this analysis

\[ a_0 = 0.0000 \]
\[ a_1 = 1.3970 \]
\[ a_2 = -1.4215 \]
\[ a_3 = 1.1737 \]
\[ a_4 = -0.1494 \]

Therefore, using Eqs. 5.2.8-5.2.13 we compute the intermediate coefficients

\[ A_1 = a_0 = 0.0000 \]
\[ A_2 = \frac{a_1}{2} = 0.6985 \]
\[ A_3 = \frac{a_2}{3} = -0.4738 \]
\[ A_4 = \frac{a_3}{4} = 0.2934 \]
\[ A_5 = \frac{a_4}{5} = -0.0299 \]

and

\[ A_0 = -[A_1 p_w D + A_2 p_w^2 + A_3 p_w^3 + A_4 p_w^4 + A_5 p_w^5 + \ldots] \]

\[ = -[0.0 + 0.6985(0.1702)^2 - 0.4738(0.1702)^3 + 0.2934(0.1702)^4] \]
- 0.0299(0.1702)^5] 

or 

\[ A_0 = -0.01815 \]

Finally, the \( \alpha \)-parameter is estimated using Eq. 5.2.7

\[ \alpha = [A_0 + A_1 + A_2 + A_3 + A_4 + A_5 + \ldots] \]

\[ = [-0.01815 + 0.0 + 0.6985 - 0.4738 + 0.2934 - 0.0299] \]

or 

\[ \alpha = 0.4701 \]

**Original-Gas-In-Place:**

Estimate of the original-gas-in-place is obtained from the time match point using Eq. 5.2.1 and the computed value of \( J_g \)

\[ G = \frac{J_g}{c_{ti}} \left[ \frac{t}{t_{Dd,MP}} \right] = \left( \frac{768.119 \text{ Scf/D/psia}}{1.8237 \times 10^{-4} \text{ psia}^{-1}} \right) \left[ \frac{(680 \text{ Days})}{(1.0)} \right] \]

or 

\[ G = 2.8640 \times 10^9 \text{ Scf} \]

**Reservoir Drainage Area:**

Using the computed value of gas-in-place we estimate the reservoir drainage area as

\[ A = 5.615 \frac{B_{gi}G}{\phi h (1-S_{wi})} \]

\[ = 5.615 \frac{(7.1012 \times 10^{-4} \text{ Scf}) (2.8640 \times 10^9 \text{ RB/Scf})}{(0.06) (70 \text{ ft}) (1-0.35)} \]

or 

\[ A = 4.18304 \times 10^6 \text{ ft}^2 \text{ or } 96.03 \text{ acres} \]

**Permeability:**

The formation permeability, \( k \), is estimated from Eq. 5.2.14 as

\[ k = \frac{141.2 \mu_i B_{gi} J_g}{2h} \ln \left[ \frac{2.2458A}{C_{i,i_{wa,}}^2} \right] \]
\[ = 141.2 \frac{(0.0225 \text{cp})(7.1012 \times 10^{-4} \text{RB/Scf})(768.119 \text{ Scf/D/psi})}{2(70 \text{ ft})} \times \ln \left[ \frac{2.2458 (4.18304 \times 10^6 \text{ ft}^2)}{(31.62)(62.27)^2} \right] \]

or

\[ k = 0.0537 \text{ md} \]

**Summary for Case 1 (Field Case):**

For this field case we find consistent analysis results from both the "Knowles" and the "General polynomial" type curves. This is true for our estimates of the well productivity index as well as the original-gas-in-place. We conclude that the original-gas-in-place in the drainage volume associated with this well is about 2.864 Bscf.

Table 5.3 below presents a comparison of our results with other analyses reported in the literature.

### Table 5.3

<table>
<thead>
<tr>
<th>Reservoir Parameter</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SPE 14238</td>
</tr>
<tr>
<td>OGIP, ( G ) (Bscf)</td>
<td>3.034</td>
</tr>
<tr>
<td>Well Productivity Index, ( J_g ) (Scf/D/psi)</td>
<td>1086.859</td>
</tr>
<tr>
<td>Drainage Area, ( A ) (acres)</td>
<td>101.73</td>
</tr>
<tr>
<td>Permeability-Thickness Product, ( k_h ) (md-ft)</td>
<td>5.39</td>
</tr>
<tr>
<td>Formation Permeability, ( k ) (md)</td>
<td>0.077</td>
</tr>
</tbody>
</table>
5.4 Production Under Variable Bottomhole Pressure Conditions

In this section, we demonstrate the technique for applying the new "General polynomial" model to the analysis of gas well production data where the flowing bottomhole pressures are not constant. We have noted that bottomhole flowing pressures of a well can vary due to several factors during reservoir depletion. The most obvious case is a shut-in caused by mechanical failure of a wellbore equipment. In addition, it is a common field practice to artificially induce changes in flowing bottomhole pressure by an operator to increase or decrease production by adjusting the choke setting at the wellhead due to market demands (seasonal demands).

We consider one simulated case and two field example cases. For the field cases, we have chosen one low pressure and one relatively high pressure reservoir examples to demonstrate applicability the "General polynomial" model.

5.4.1 Simulated Case

In this section, we present analysis of a simulated production rate data generated from a high pressure gas reservoir \((p_i = 10000\ \text{psia})\).

Case 2 — Simulated Example

Table 5.4 presents the pertinent input data used to simulate this rate profile. The production schedule is summarized in Table 5.5.

Fig. 5.6 shows a plot of the gas production rate data matched against the computed response. In this case, we have used the correct value of the decline constant, \(D_i\), of 0.02039 \(D^{-1}\) to generate this dimensionless rate profile. We note an excellent match of the computed rate profile with the production history. The basic characteristics of the rate history (fluctuations or "jumps") caused by changes in flowing bottomhole pressure are well reflected in our computed solution. Recall that this solution (i.e., the "General polynomial" model) assumes only boundary-dominated flow behavior. A match point yields the following parameters

\[
\begin{align*}
[q_{Dd}]_{MP} &= 1.0 & [q_g]_{MP} &= 4.75 \times 10^6 \ \text{Scf/D} \\
[t_{Dd}]_{MP} &= 1.0 & [t]_{MP} &= 49.5 \ \text{Days}
\end{align*}
\]
### Table 5.4
Reservoir, Fluid, and Production Data for Case 2 (Simulated Case)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reservoir Properties:</strong></td>
<td></td>
</tr>
<tr>
<td>Wellbore Radius, $r_w$</td>
<td>0.3 ft</td>
</tr>
<tr>
<td>Net Pay Thickness, $h$</td>
<td>20 ft</td>
</tr>
<tr>
<td>Average Porosity, $\phi$</td>
<td>0.10 (fraction)</td>
</tr>
<tr>
<td>Average Reservoir Temperature, $T$</td>
<td>250.0 °F</td>
</tr>
<tr>
<td>Drainage Area, $A$</td>
<td>40.0 acres</td>
</tr>
<tr>
<td>Permeability, $k$</td>
<td>1.0 md</td>
</tr>
<tr>
<td><strong>Fluid Properties:</strong></td>
<td></td>
</tr>
<tr>
<td>Gas Specific Gravity, $y_g$</td>
<td>0.60 (air=1)</td>
</tr>
<tr>
<td>Gas FVF at Initial Conditions, $B_{gi}$</td>
<td>$4.9204 \times 10^{-4}$ RB/Scf</td>
</tr>
<tr>
<td>Viscosity at Initial Conditions, $\mu_{gi}$</td>
<td>0.0343 cp</td>
</tr>
<tr>
<td>Total Compressibility, $c_{ti}$</td>
<td>$4.6105 \times 10^{-5}$ psia$^{-1}$</td>
</tr>
<tr>
<td><strong>Production Parameters:</strong></td>
<td></td>
</tr>
<tr>
<td>Initial Reservoir Pressure, $p_i$</td>
<td>10000 psia</td>
</tr>
<tr>
<td>Initial BHP, $p_{wf}$</td>
<td>6000 psia</td>
</tr>
<tr>
<td>Decline Parameter, $\alpha$ (Dimensionless)</td>
<td>0.1864</td>
</tr>
</tbody>
</table>

### Table 5.5
Bottomhole Pressure Changes for Case 2 (Simulated Case)

<table>
<thead>
<tr>
<th>Time (Days)</th>
<th>Bottomhole Pressure, $p_{wf}$ (psia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 — 50</td>
<td>6000</td>
</tr>
<tr>
<td>50 — 100</td>
<td>4000</td>
</tr>
<tr>
<td>100 — 200</td>
<td>5000</td>
</tr>
<tr>
<td>200 — 300</td>
<td>6000</td>
</tr>
</tbody>
</table>
Fig. 5.6 - "General Polynomial" Type Curve Analysis for Case 2 (Simulated Case).
The Analysis Assumes $D_i = 0.02039 \text{ D}^{-1}$ — Correct Value.

**Legend:** Variable Flowing Bottomhole Pressure Production Case. Simulated Example.

- **Data Points**
- **Computed Rate Profile**

**Boundary-Dominated Flow Region**

**Data for Case 2 (Simulated Example):**

- Variable BHP Production
  - $p = 10000 \text{ psia} \ (z = 1.3765)$
  - $\gamma_g = 0.60 \ (air = 1)$
  - $T = 250 ^\circ \text{F}$
  - $r_w = 0.3 \text{ ft}$
  - $h = 20 \text{ ft}$
  - $\alpha = 0.1864 \ (\text{dimensionless})$

**Analysis Results:**

- $G = 1.2617 \text{ Bscf}$
  - (Note: Actual $= 1.2613 \text{ Bscf}$)
- $J_g = 1175.19 \text{ Scf/D/psi}$
- $k = 0.9907 \text{ md}$

**Results Comparison:** Variable $p_{wf}$ Analysis

<table>
<thead>
<tr>
<th></th>
<th>Input Data</th>
<th>Analysis Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$ (Bscf)</td>
<td>1.2613</td>
<td>1.2617</td>
</tr>
<tr>
<td>$A$ (Acres)</td>
<td>40.0</td>
<td>40.0</td>
</tr>
<tr>
<td>$k$ (md)</td>
<td>1.000</td>
<td>0.9907</td>
</tr>
</tbody>
</table>
Going through the calculation procedures we obtain the following results

**Analysis Results:**

\[ J_g = 1175.1937 \text{ Scf/D/psia} \]
\[ G = 1.2617 \times 10^9 \text{ Scf} \]
\[ A = 1.7429 \times 10^6 \text{ ft}^2 \text{ or } 40.01 \text{ acres} \]
\[ k = 0.9907 \text{ md} \]

We note that the actual original-gas-in-place for this run was 1.26133 Bscf, so our results from this match are very consistent with the input data. To investigate the sensitivity of the productivity index estimate on the assumed value of \( D_t \) we chose to generate the same rate profile assuming \( \pm 10\% \) error in the assumed \( D_t \). Figs. 5.7 and 5.8 show the generated rate profiles assuming \( D_t = 0.02433 \text{ D}^{-1} \) (High) and \( D_t = 0.0184 \text{ D}^{-1} \) (Low), respectively.

In these examples the horizontal match points give the same value of \( q_g = 4.75 \times 10^6 \text{ Scf/D} \) for a dimensionless rate \( qD_t = 1.0 \), although the values of \( D_t \) are incorrect. The effects of the incorrect \( D_t \)-values are obvious from the shapes of the computed dimensionless rate profiles which do not exactly match the correct production history, especially, at very late time.

It is worth mentioning that, for erratic field data, such errors in the estimate of \( D_t \) would be common and it would be difficult to distinguish the correct trend. The important observation is that regardless of the error associated with the assumption of \( D_t \), we can estimate \( J_g \) quite accurately.
Fig. 5.7 - "General Polynomial" Type Curve Analysis for Case 2 (Simulated Case).
The Analysis Assumes $D_i = 0.0243 \text{ D}^{-1} — \text{High Value.}$
Fig. 5.8 - "General Polynomial" Type Curve Analysis for Case 2 (Simulated Case).
The Analysis Assumes $D_i = 0.0184 \text{ D}^{-1}$ — Low Value.
In this particular case, the parameter estimates from Figs. 5.7 and 5.8 are as follows

**Match Parameters: (Fig. 5.7 — High Value of \( D_i \))**

\[
\begin{align*}
[q_{DD}]_{MP} &= 1.0 & [q_g]_{MP} &= 4.75 \times 10^6 \text{ Scf/D} \\
[u_{DD}]_{MP} &= 1.0 & [t]_{MP} &= 45.0 \text{ Days}
\end{align*}
\]

**Analysis Results:**

\[
\begin{align*}
J_g &= 1175.1937 \text{ Scf/D/psia} \\
G &= 1.1470 \times 10^9 \text{ Scf} \\
A &= 1.5845 \times 10^6 \text{ ft}^2 \text{ or } 36.38 \text{ acres} \\
k &= 0.9840 \text{ md}
\end{align*}
\]

**Match Parameters: (Fig. 5.8 — Low Value of \( D_i \))**

\[
\begin{align*}
[q_{DD}]_{MP} &= 1.0 & [q_g]_{MP} &= 4.75 \times 10^6 \text{ Scf/D} \\
[u_{DD}]_{MP} &= 1.0 & [t]_{MP} &= 54.0 \text{ Days}
\end{align*}
\]

**Analysis Results:**

\[
\begin{align*}
J_g &= 1175.1937 \text{ Scf/D/psia} \\
G &= 1.3764 \times 10^9 \text{ Scf} \\
A &= 1.9014 \times 10^6 \text{ ft}^2 \text{ or } 43.65 \text{ acres} \\
k &= 0.9968 \text{ md}
\end{align*}
\]

In Chapter IV we established that if we can solve for \( J_g \) exactly (or at least quite accurately), then we can use the stabilized flow equation to estimate values of average reservoir pressure during depletion. The appropriate computational relation, which is in theory exact for boundary-dominated flow data, is given by Eq. 4.6.2

\[
p_p = \frac{q_g}{J_g} + p_{pwf} .................................................................(4.6.2)
\]

In order to apply Eq. 4.6.2 we must use fluid properties table to convert the flowing bottomhole pressure data to \( p_{pwf} \) and then calculate the \( p_p \) values. We then use the same
fluid properties table to convert the $p_p$ values to values of average pressure. Knowing the average reservoir pressures we can then use the conventional $p/z$ versus $G_p$ plot to verify the actual value of the original-gas-in-place.

Fig. 5.9 shows the plot constructed for this simulated example. Extrapolating the straight line trend of the data we estimate the value of the original-gas-in-place to be 1.261 Bscf which agrees quite well with our input value of 1.2613 Bscf.

**Summary for Simulated Example Case 2:**

We have demonstrated through this simulated example that the "General Polynomial" model is indeed an accurate analysis tool. We find very consistent analysis results (Table 5.6) compared with the input parameters.

| Table 5.6 |
| Analysis Results for Case 2 (Simulated Example) |

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulator Input</th>
<th>Analysis Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original-Gas-In-Place, $G$ (Bscf)</td>
<td>1.2613</td>
<td>1.261</td>
</tr>
<tr>
<td>Well Productivity Index, $J_g$ (Scf/D/psia)</td>
<td>1186.244</td>
<td>1175.194</td>
</tr>
<tr>
<td>Drainage Area, $A$ (acres)</td>
<td>40.0</td>
<td>40.01</td>
</tr>
<tr>
<td>Formation Permeability, $k$ (md)</td>
<td>1.0</td>
<td>0.9907</td>
</tr>
</tbody>
</table>
Fig. 5.9 - Conventional $p/z$ versus $G_p$ Plot for Case 2 (Simulated Variable Bottomhole Pressure Production Case).
5.4.2 Field Cases

In this section we present analyses of two field cases. Case 3 is an example of a gas well producing from a low pressure reservoir while Case 4 produces from a high pressure gas reservoir. The latter case represents a classic example of a normal field production scenario, where the flowing bottomhole pressures are reduced at any time the well shows a significant decline in gas production.

Case 3 — Gas Well A

"Gas Well A" is a gas well producing from a tight (low permeability) formation. This case was first reported in the literature by Palacio and Blasingame. Due to apparent liquid loading problems there are large fluctuations in the flowing bottomhole pressures.

A summary of the reservoir and fluid properties, as well as production information is given in Table 5.7.

| Table 5.7 |
| Reservoir, Fluid, and Production Data for Case 3 ("Gas Well A" — from SPE 25909) |

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reservoir Properties:</strong></td>
<td></td>
</tr>
<tr>
<td>Wellbore Radius, $r_w$</td>
<td>0.3 ft</td>
</tr>
<tr>
<td>Net Pay Thickness, $h$</td>
<td>48 ft</td>
</tr>
<tr>
<td>Average Porosity, $\phi$</td>
<td>0.09 (fraction)</td>
</tr>
<tr>
<td>Reservoir Temperature, $T$</td>
<td>275.0 °F</td>
</tr>
<tr>
<td><strong>Fluid Properties:</strong></td>
<td></td>
</tr>
<tr>
<td>Gas Specific Gravity, $\gamma_g$</td>
<td>0.72 (air=1)</td>
</tr>
<tr>
<td>Gas FVF at Initial Conditions, $B_{gi}$</td>
<td>$1.3061 \times 10^{-3}$ RB/Scf</td>
</tr>
<tr>
<td>Gas Viscosity at Initial Conditions, $\mu_{gi}$</td>
<td>0.0193 cp</td>
</tr>
<tr>
<td>Total Compressibility, $c_{ti}$</td>
<td>$3.6417 \times 10^{-4}$ psia$^{-1}$</td>
</tr>
<tr>
<td><strong>Production Parameters:</strong></td>
<td></td>
</tr>
<tr>
<td>Initial Reservoir Pressure, $p_i$</td>
<td>2625 psia</td>
</tr>
<tr>
<td>Initial BHP, $p_{wf}$</td>
<td>2000 psia</td>
</tr>
<tr>
<td>Decline Parameter, $\alpha$ (Dimensionless)</td>
<td>0.2110</td>
</tr>
</tbody>
</table>
Fig. 5.10 shows our analysis plot of the production rate-time data, using an assumption of $D_t=0.006$ D$^{-1}$. The analysis results are as follows:

**Matching Parameters:**

\[
[q_{Dd}]_{MP} = 1.0 \quad [q_g]_{MP} = 3.5 \times 10^6 \text{ Scf/D}
\]
\[
[t_{Dd}]_{MP} = 1.0 \quad [t]_{MP} = 167.0 \text{ Days}
\]

**Analysis Results:**

\[
J_g = 604.019 \text{ Scf/D/psia}
\]
\[
G = 0.2764 \times 10^9 \text{ Scf}
\]
\[
A = 4.6929 \times 10^5 \text{ ft}^2 \text{ or } 10.773 \text{ acres}
\]
\[
k = 0.2875 \text{ md}
\]

Using this estimate of $J_g=604.019$ Scf/D/psia, we compute the normalized pseudopressure values according to Eq. 4.6.2 and then the average reservoir pressure profile as a function of time. These values are plotted in Fig. 5.11. Extrapolating the apparent straight line trend in Fig. 5.11 yields an estimate of the original-gas-in-place, $G=0.263$ Bscf.

We use this value of the original-gas-in-place to adjust the $D_t$ estimate. Using the definition of $D_t$ in Eq. 4.6.1, we obtain

\[
D_t = \frac{J_g}{c_n G} = \frac{(604.019 \text{ Scf/D/psia})}{(3.6417 \times 10^{-4} \text{ psia}^{-1})(0.263 \times 10^9 \text{Scf})}
\]

or

\[
D_t = 0.00631 \text{ D}^{-1}
\]

Fig. 5.12 shows a plot of the computed rate profile using this corrected value of $D_t$ matched against the original field data. We find the match of this field data to be quite good. Note that most of the major features in the data are well reflected in this figure.
Fig. 5.10 - "General Polynomial" Type Curve Analysis of "Gas Well A" (SPE 25909). The Analysis Assumes $D_i = 0.006 D^{-1}$. 


Data for "Gas Well A":
(Variable BHP Production)
- $p_f = 2625$ psia ($z = 0.9285$)
- $\gamma_g = 0.72$ (air = 1)
- $T = 275^\circ F$
- $r_w = 0.3$ ft
- $h = 48$ ft
- $\alpha = 0.2110$ (dimensionless)

Analysis Results:
- $G = 0.2764$ Bscf
(Note: Estimate using $D_i = 0.006$)
- $J_g = 604.019$ Scf/D/psia
- $k = 0.2875$ md

Results Comparison: Variable $p_{wf}$ Analysis

<table>
<thead>
<tr>
<th></th>
<th>SPE 25909</th>
<th>This Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$ (Bscf)</td>
<td>0.2450</td>
<td>0.2764</td>
</tr>
<tr>
<td>$k$ (md)</td>
<td>0.3060</td>
<td>0.2875</td>
</tr>
</tbody>
</table>

Boundary-Dominated Flow Region
Fig. 5.11 - Conventional $p/z$ versus $G_p$ Plot for "Gas Well A" Data (SPE 25909).
**Legend:** "Gas Well A" Data — from SPE 25909. Variable Bottomhole Pressure Analysis Using the "General Polynomial" Approach.

**Boundary-Dominated Flow Region**

**Data for "Gas Well A":**
(Variable BHP Production)

- $p_f=2625$ psia ($z_f=0.9265$)
- $\gamma_g=0.72$ (air=1)
- $T=275$ °F
- $r_w=0.3$ ft
- $h=48$ ft
- $\alpha=0.2110$ (dimensionless)

**Analysis Results:**

- $G=0.263$ Bscf
- $J_g=604.019$ Scf/D/psia
- $k=0.2864$ md

**Results Comparison: Variable $p_{wf}$ Analysis**

<table>
<thead>
<tr>
<th></th>
<th>SPE 25909</th>
<th>This Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$ (Bscf)</td>
<td>0.2450</td>
<td>0.2630</td>
</tr>
<tr>
<td>$k$ (md)</td>
<td>0.3060</td>
<td>0.2864</td>
</tr>
</tbody>
</table>

**Fig. 5.12 - "General Polynomial" Type Curve Analysis of "Gas Well A" (SPE 25909).**

This Analysis Uses $D_i = 0.00582$ D$^{-1}$ — Correct Value.
Table 5.8 below presents a comparison of our results with previous analysis reported in the literature (SPE 25909).

<table>
<thead>
<tr>
<th>Reservoir Parameter</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SPE 25909</td>
</tr>
<tr>
<td>Original-Gas-In-Place, ( G ) (Bscf)</td>
<td>0.245</td>
</tr>
<tr>
<td>Well Productivity Index, ( J_g ) (Scf/D/psia)</td>
<td>661.934</td>
</tr>
<tr>
<td>Drainage Area, ( A ) (acres)</td>
<td>6.6</td>
</tr>
<tr>
<td>Permeability-Thickness Product, ( kh ) (md-ft)</td>
<td>14.544</td>
</tr>
<tr>
<td>Formation Permeability, ( k ) (md)</td>
<td>0.303</td>
</tr>
</tbody>
</table>

**Table 5.8**

Analysis Results for Case 3 ("Gas Well A") — Variable Bottomhole Pressure Production

**Case 4 — Gas Well B**

"Gas Well B" is a well also producing from a low permeability formation that has had a very large fracture stimulation treatment. The initial flowrate of this well is 16.04 MMScf/D. This is a very interesting well because although the reservoir is undergoing depletion the rate-time plot shows that the well maintains relatively higher flowrates close to the original values until quite late in the life of the well.

It is important to note that production levels are being maintained as a result of gradual lowering of the flowing bottomhole pressure as shown in Fig. 5.13. The changing bottomhole pressure makes conventional decline curve analysis of this well unlikely as the approach assumes constant bottomhole pressure production.

Table 5.9 presents a summary of the reservoir and fluid properties, as well as production information. We note there are limited reservoir data about this well and so we will focus only on the determination of the two fundamental parameters of the well — the OGIP and the well productivity index.
### Table 5.9
Reservoir, Fluid, and Production Data for Case 4 ("Gas Well B")

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reservoir Properties:</strong></td>
<td></td>
</tr>
<tr>
<td>Wellbore Radius, $r_w$</td>
<td>Unknown</td>
</tr>
<tr>
<td>Net Pay Thickness, $h$</td>
<td>Unknown</td>
</tr>
<tr>
<td>Average Porosity, $\phi$</td>
<td>Unknown</td>
</tr>
<tr>
<td>Reservoir Temperature, $T$</td>
<td>310.0 °F</td>
</tr>
<tr>
<td><strong>Fluid Properties:</strong></td>
<td></td>
</tr>
<tr>
<td>Gas Specific Gravity, $\gamma_g$</td>
<td>0.683</td>
</tr>
<tr>
<td>Gas FVF at Initial Conditions, $B_{gi}$</td>
<td>5.8532 x10^{-4}</td>
</tr>
<tr>
<td>Gas Viscosity at Initial Conditions, $\mu_{gi}$</td>
<td>0.0331</td>
</tr>
<tr>
<td>Total Compressibility, $c_{ti}$</td>
<td>6.2818 x10^{-5}</td>
</tr>
<tr>
<td><strong>Production Parameters:</strong></td>
<td></td>
</tr>
<tr>
<td>Initial Reservoir Pressure, $p_i$</td>
<td>8400</td>
</tr>
<tr>
<td>Initial BHP, $p_{wf}$</td>
<td>7436</td>
</tr>
<tr>
<td>Decline Parameter, $\alpha$ (Dimensionless)</td>
<td>0.0609</td>
</tr>
</tbody>
</table>

Fig. 5.14 shows the analysis plot for the rate versus time data in dimensionless format. We find that using a value of $D_i=0.025$ D^{-1} yields a computed dimensionless rate profile that reproduces the important features of the past production history. Our match of the field data against the computed rate response yields the following results:

**Match Parameters:** (Gas Well B):

\[
[q_{Dd}]_{MP} = 1.0 \quad [q_g]_{MP} = 1.75 \times 10^7 \text{ Scf/D}
\]

\[
[t_{Dd}]_{MP} = 1.0 \quad [t]_{MP} = 40.0 \text{ Days}
\]

**Analysis Results:**

\[ J_g = 18037.542 \text{ Scf/D/psia} \]

\[ G = 11.4856 \times 10^9 \text{ Scf} \]
Fig. 5.14 - "General Polynomial" Type Curve Analysis of "Gas Well B". This Analysis Assumes $D=0.025$ D$^{-1}$.
As in the previous case, we use this estimate of $J_g$ in the stabilized rate equation to generate the average pressures in the reservoir during depletion. The conventional $p/z$ versus $G_p$ plot for this production data is provided in Fig. 5.15. We estimate the original-gas-in-place from this plot to be approximately 11.90 Bscf. This value is used to correct the $D_i$ estimate. According to Eq. 4.6.1 we have

$$D_i = \frac{J_g}{c_t G} = \frac{18037.5421 \text{ Scf/D/psia}}{6.28182 \times 10^{-5} \text{ psia}^{-1} 11.90 \times 10^9 \text{Scf}} = 0.02413 \text{ D}^{-1}$$

Fig. 5.16 presents our plot of the new rate profile generated using $D_i = 0.02413 \text{ D}^{-1}$. We note from this plot that most of the basic features of the rate profile are well reflected in the computed response. We present a summary of our final results of this analysis in Table 5.10 below.

### Table 5.10
Analysis Results for Case 4 ("Gas Well B") — Variable Bottomhole Pressure Production

<table>
<thead>
<tr>
<th>Reservoir Parameter</th>
<th>Previous Analysis (Unpublished)</th>
<th>This Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>OGIP, $G$ (Bscf)</td>
<td>10.80</td>
<td>11.90</td>
</tr>
<tr>
<td>Well Productivity Index, $J_g$ (Scf/D/psia)</td>
<td>17000</td>
<td>18037.5</td>
</tr>
</tbody>
</table>
Fig. 5.15 - Conventional $p/z$ versus $G_p$ Plot for "Gas Well B" Data.
Fig. 5.16 - "General Polynomial" Type Curve Analysis of "Gas Well B". This Analysis Uses $D_i = 0.02413$ D$^{-1}$ — Correct Value.
CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1 Summary

A theory for the development of semi-analytical flow models for decline curve analysis has been presented and verified in this work. The basis of this theory lies in the use of linearized forms of the stabilized (boundary-dominated) flow equation, rigorously presented in terms of pseudopressure, and the direct coupling of this relation with the gas material balance equation.

We began this study by outlining the relevance of using theoretical models for matching past production performance and for the prediction of future production rates of gas wells. Theoretically, these dimensionless decline curve models are developed from the stabilized gas flow equation and the gas material balance equation. Due to the non-linear nature of the boundary-dominated flow equation, we have introduced a series of functional models for correlating the behavior of the viscosity-compressibility product as a function of $p/z$ (or dimensionless pressure, $p_D$).

These data models allow us to develop explicit semi-analytical solutions for the gas flow problem, with the exception of the "General polynomial" model — which requires a numerical solution.

Our prime focus in the development of the rate-time and cumulative production solutions has been to eliminate pressure ($p/z$) or dimensionless pressure, $p_D$. We have achieved this for each of the functional models, enabling us to derive explicit rate-time and cumulative production-time equations necessary for the analysis and prediction of gas reservoir performance.

More importantly, this theory has enabled us to use real variables (field variables) for decline curve analysis of gas well data, rather than using pseudopressure and pseudotime functions generally required to perform such analyses.
6.2 Conclusions

Based on the theoretical developments and results from this work we identify the following conclusions:

- We have presented and verified a novel approach for linearizing the stabilized (pseudosteady-state) flow equation. We achieved this by correlating the changes in the viscosity-compressibility product of gases which represent the primary source of non-linearity in the gas flow equation.

- Our approach uses functional relationships of the fluid property changes during reservoir depletion. We note that linearization of the stabilized flow equation using approximations of the proposed "General polynomial" model has been applied in the past rather inadvertently (e.g., the Carter\textsuperscript{3} and Knowles\textsuperscript{4} approaches).

- The linearized form of the stabilized flow equation has been coupled with the gas material balance equation to yield rigorous, but simple, performance relations for the analysis of gas well production data under both constant and variable flowing bottomhole pressure conditions. These performance relations include explicit equations for predicting $q_g(t)$, $G_p(t)$, and $\bar{p}(t)$.

- The rate-time equations are simple for engineering applications, as demonstrated by the examples presented in this work. In addition, these results offer the potential for developing new analysis techniques via algebraic manipulation of our new relations.

- The "General polynomial" model introduced in this work is well suited for analyzing variable pressure production data because the flowing bottomhole pressure terms have been isolated explicitly, making it possible to incorporate these data directly into the solution routine. However, in this case, knowledge of the initial decline constant, $D_i$, is required for the analysis. This fact makes the variable pressure production analysis iterative.

- We have demonstrated with both simulated and field examples that the new rate-time models presented in this work are accurate and efficient for the analysis and prediction of gas well performance.
6.3 Recommendations for Future Work

We make the following recommendations regarding our new rate and cumulative production models for future research efforts:

- The "General polynomial" model should be adapted for analyzing solution-gas drive reservoirs by, similarly, linearizing the non-linear boundary-dominated flow equation rigorously written in terms of "multi-phase" pseudopressure.\textsuperscript{31,41}

- The production rate and cumulative production relations developed in this work can also be adapted for use in an inflow-performance-relation (IPR) format for the correlation of well performance on field-wide basis (analogous to the work presented in Refs. 11, 12, and 40).

- Additional efforts at analytically solving the general polynomial formulation should be made, by focusing especially on the cubic function. Other formulations (\textit{i.e.}, functional models) should also be considered. In particular, the second-order polynomial function (\textit{i.e.}, quadratic equation) should yield an explicit rate-time solutions. This model will have the form

\[
\frac{\mu_i c_t}{\mu c_t} = a_0 + a_1 p_D + a_2 p_D^2 \tag{6.3.1}
\]

where \(a_0, a_1, \text{ and } a_2\) are real numbers, but not necessarily integers.

- In addition, with the behavior of the viscosity-compressibility function being constrained to values between zero and one, the following model may also be investigated

\[
\frac{\mu_i c_t}{\mu c_t} \approx a_1 p_D^{a_2} + (1-a_1)p_D^{a_3} \tag{6.3.2}
\]

where \(a_1, a_2, \text{ and } a_3\) are constants.

- Finally, formulations which include transient flow behavior should be developed. Ref. 14 provides some insight, but the tedium of repeated superposition calculations should be avoided, as should also be approaches which require "continuous updating" of the pressure dependent properties.
NOMENCLATURE

$A$ = reservoir drainage area, ft$^2$ [m$^2$]

$b$ = decline exponent (after Arps$^7$ and Fetkovich$^2$)

$B_g$ = gas formation volume factor, RB/Scf

$B_o$ = oil formation volume factor, RB/Scf

$C$ = coefficient of the flow equation (Eq. 3.3.11), Scf/D-psia$^2$/cp [m$^3$/s·kPa$^2$/Pa.s]

$C_A$ = Dietz's shape factor

$c_f$ = formation (rock) compressibility, psia$^{-1}$ [kPa$^{-1}$]

$c_g$ = gas compressibility, psia$^{-1}$ [kPa$^{-1}$]

$c_t$ = total system compressibility, psia$^{-1}$ [kPa$^{-1}$]

$c_w$ = water compressibility, psia$^{-1}$ [kPa$^{-1}$]

$D$ = non-Darcy flow coefficient, 1/Scf/D [1/m$^3$/s]

$D_i$ = initial decline rate constant, D$^{-1}$ [s$^{-1}$]

$E$ = fluid expansion, RB/Stb

$F$ = underground fluid withdrawal, RB [m$^3$]

$F_{CD}$ = Dimensionless fracture conductivity

$G$ = original-gas-in-place(OGIP), Scf [m$^3$]

$G_p$ = cumulative gas production, Scf [m$^3$]

$h$ = reservoir thickness, ft [m]

$J_g$ = well productivity index, Scf/D/psia [m$^3$/s·kPa]

$k$ = average permeability, md [m$^2$]

$m(p)$ = real gas pseudopressure, psia$^2$/cp [kPa$^2$/Pa.s]

$N$ = original-oil-in-place(OOIP), stb [m$^3$]

$p$ = average reservoir pressure, psia [kPa]

$p_D$ = initial reservoir pressure, psia [kPa]

$p_i$ = initial reservoir pressure, psia [kPa]

$p_p$ = normalized pseudopressure, psia [kPa]

$p_{pwf}$ = normalized pseudopressure corresponding to $p_{wf}$, psia [kPa]

$p_{wf}$ = flowing bottomhole pressure, psia [kPa]

$q_{Dd}$ = dimensionless "decline" rate
$q_g$ = gas flow rate, Scf/D [m$^3$/s]
$r$ = radial distance, ft [m]
$r_e$ = external radius of a reservoir, ft [m]
$R_{p_s}$ = cumulative produced gas-oil ratio, Scf/stb
$R_s$ = solution gas-oil ratio, Scf/Scf
$R_y$ = volatile oil-gas ratio, stb/Scf
$r_w$ = wellbore radius, ft [m]
$r_{wa}$ = effective wellbore radius, ft [m]
$s$ = skin factor
$S_g$ = gas saturation, fraction
$S_{wc}$ = irreducible (connate) water saturation, fraction
$t$ = time, D [s]
$T$ = reservoir temperature, °R [K]
$t_{Dd}$ = dimensionless "decline" time
$z$ = gas compressibility factor

Subscript

$b$ = base (reference) conditions
$D$ = dimensionless
$g$ = gas
$i$ = initial conditions
$wf$ = wellbore conditions

Greek Symbols

$\alpha$ = constant defined by Eq. 4.1.11
$\beta$ = correlating parameter defined by Eq. 4.2.10
$\Delta$ = change in a variable
$\eta$ = constant defined by Eq. 4.4.15
$\phi$ = porosity, fraction
$\gamma$ = specific gravity (air = 1)
$\lambda$ = Carter's correlating parameter (Ref. 3)
$\theta$ = variable defined by Eq. 4.4.4
$\mu$ = viscosity, cp [Pa.s]
REFERENCES


APPENDIX A

DERIVATION OF THE RADIAL DIFFUSIVITY EQUATION
FOR REAL GAS FLOW

Derivation of the Mass Continuity Equation

In order to proceed with our derivation of the gas diffusivity equation, we must first consider the general continuity (i.e., the mass balance) equation that governs fluid flow through a control volume. We define our control volume as a unit cell in a three-dimensional Cartesian space with dimensions $\Delta x$, $\Delta y$, and $\Delta z$, as shown in Fig. A.1.

Fig. A.1 - A Unit Cell from a Three-Dimensional Porous Medium.
The Law of Conservation of Mass within this unit cell states that

\[
\begin{pmatrix}
\text{Accumulation of Total Mass within the unit cell during a time period } \Delta t \\
\text{Input of Total Mass into the unit cell during the time period}
\end{pmatrix}
= \begin{pmatrix}
\text{Output of Total Mass from the unit cell during the time period}
\end{pmatrix}
\tag{A-1}
\]

In other words, mass accumulation in the unit cell over a short interval of time \(\Delta t\) will be determined by the total mass in the unit cell at any time, \(t\), compared with the total mass at the time, \(t+\Delta t\). The total mass of fluid can be determined from the product of the volume of the cell and the fluid density. That is

\[
\text{Total Mass} = \phi \Delta x \Delta y \Delta z \rho \tag{A-2}
\]

Along any of the axes in our Cartesian coordinate system the rate of mass input and output are described by the flux across the surface area perpendicular to that axis. Along the x-axis, for instance,

\[
\text{Input of Total Mass} = (\rho v_x)_x \Delta y \Delta z \Delta t \tag{A-3}
\]

Similarly,

\[
\text{Output of Total Mass} = (\rho v_x)_{x+\Delta x} \Delta y \Delta z \Delta t \tag{A-4}
\]

Writing similar expressions along the other remaining two coordinate axes, we can rewrite Eq. A-1 as

\[
(\phi \rho)_{t+\Delta t} \Delta x \Delta y \Delta z - (\phi \rho)_t \Delta x \Delta y \Delta z = (\rho v_x)_x \Delta y \Delta z \Delta t - (\rho v_x)_{x+\Delta x} \Delta y \Delta z \Delta t
\]

\[
+ (\rho v_y)_y \Delta x \Delta z \Delta t - (\rho v_y)_{y+\Delta y} \Delta x \Delta z \Delta t
\]

\[
+ (\rho v_z)_z \Delta x \Delta y \Delta t - (\rho v_z)_{z+\Delta z} \Delta x \Delta y \Delta t \tag{A-5}
\]

Dividing through Eq. A-5 by \(\Delta x \Delta y \Delta z \Delta t\), yields

\[
\frac{(\phi \rho)_{t+\Delta t} - (\phi \rho)_t}{\Delta t} = \frac{(\rho v_x)_x - (\rho v_x)_{x+\Delta x}}{\Delta x} + \frac{(\rho v_y)_y - (\rho v_y)_{y+\Delta y}}{\Delta y} + \frac{(\rho v_z)_z - (\rho v_z)_{z+\Delta z}}{\Delta z}
\tag{A-6}
\]

Now taking the limit as \(\Delta x\), \(\Delta y\), \(\Delta z\), and \(\Delta t\) approach zero, we can express Eq. A-6 in terms of partial derivatives as
\[
\left[ \frac{\partial (\rho \phi)}{\partial t} \right]_{x,y,z} = - \left[ \frac{\partial (\rho v_x)}{\partial x} \right]_{y,z,t} - \left[ \frac{\partial (\rho v_y)}{\partial y} \right]_{x,z,t} - \left[ \frac{\partial (\rho v_z)}{\partial z} \right]_{x,y,t} \quad \text{.................................. (A-7)}
\]

We adopt the vector notation\(^{50}\) by re-writing the terms on the right-hand-side of Eq. A-7 using the divergence symbol, \(\nabla\). That is

\[
\nabla (\rho \vec{v}) = \left[ \frac{\partial (\rho v_x)}{\partial x} \right]_{y,z,t} + \left[ \frac{\partial (\rho v_y)}{\partial y} \right]_{x,z,t} + \left[ \frac{\partial (\rho v_z)}{\partial z} \right]_{x,y,t} \quad \text{.................................. (A-8)}
\]

Substituting Eq. A-8 into Eq. A-7 we obtain

\[
\nabla (\rho \vec{v}) = - \left[ \frac{\partial (\rho \phi)}{\partial t} \right] \quad \text{.................................. (A-9)}
\]

Eq. A-9 is the general continuity equation governing the flow of any fluid of density, \(\rho\), through a three-dimensional (3-D) space. Note that we have used a 3-D Cartesian coordinate system, which can readily be transformed into radial, spherical, or any system of coordinates by re-writing the left-hand-side of Eq. A-9 accordingly.\(^{50}\) This will be done in the next section for real gas flow in a radial coordinate system.

**Derivation of the Gas Diffusivity Equation**

We now consider the flow of a single-phase dry gas in a porous medium. We develop the radial diffusivity equation by combining the continuity equation, Eq. A-9, with Darcy’s Law (the momentum balance for flow in porous media).

Darcy’s Law is given as

\[
\vec{v} = - \frac{k}{\mu} (\nabla p + \rho g) \quad \text{.................................. (A-10)}
\]

and the equation of state (EOS) for real gases is

\[
\rho_g = \frac{pM}{zRT} \quad \text{.................................. (A-11)}
\]

In this development we neglect the effect of the earth’s gravitational field. Substituting Eqs. A-10 and A-11 into Eq. A-9 gives
\[-\nabla \left( \frac{pM}{zRT \mu} \nabla p \right) = -\frac{\partial}{\partial t} \left[ \frac{\phi}{z} \frac{pM}{zRT} \right] \]

Assuming isothermal flow in the reservoir and factoring constant terms, this equation reduces to

\[\nabla \left[ k \frac{P}{\mu z} \nabla p \right] = \frac{\partial}{\partial t} \left[ \phi \frac{P}{z} \right] \] \hspace{1cm} (A-12)

Defining a normalized pseudopressure function \(^{22}\) we have

\[p_p = \left( \frac{\mu z}{P} \right) \int_{P_b}^{P} \frac{P'}{\mu z} \, dp' \] \hspace{1cm} (A-13)

This definition gives us

\[\nabla p_p = \left( \frac{\mu z}{P} \right) \frac{P}{\mu z} \nabla p \] \hspace{1cm} (A-14)

and

\[\frac{\partial p_p}{\partial t} = \left( \frac{\mu z}{P} \right) \frac{P}{\mu z} \frac{\partial p}{\partial t} \] \hspace{1cm} (A-15)

Meanwhile, expanding the right-hand-side of Eq. A-12 using the product rule yields

\[\frac{\partial}{\partial t} \left[ \phi \frac{P}{z} \right] = \phi \frac{\partial}{\partial t} \left[ \frac{P}{z} \right] + \frac{P}{z} \frac{\partial \phi}{\partial t} = \phi \frac{P}{z} \left[ \frac{1}{p/z} \frac{d(p/z)}{dp} \right] + \frac{1}{\phi} \frac{d\phi}{dp} \frac{\partial p}{\partial t} \] \hspace{1cm} (A-16)

We note that the definition of gas compressibility \(^{51}\) is given by

\[c_g = \frac{1}{\rho_g} \frac{dp_g}{dp} \]

Substituting Eq. A-11 into this definition yields

\[c_g = \left( \frac{zRT}{pM} \right) \frac{d\left( \frac{pM}{zRT} \right)}{dp} = \frac{1}{p/z} \frac{d(p/z)}{dp} \] \hspace{1cm} (A-17)

Similarly, the formation compressibility is defined as

\[c_f = \frac{1}{\phi} \frac{d\phi}{dp} \] \hspace{1cm} (A-18)
Therefore, Eq. A-16 may be re-written as

\[ \frac{\partial}{\partial t} \left[ \phi \frac{p}{z} \right] = \phi \left[ \frac{p}{z} \right] \left[ c_g + c_l \right] \frac{\partial p}{\partial t} = \phi \mu c_l \left( \frac{p}{\mu z} \right) \frac{\partial p}{\partial t} \] \hspace{1cm} \text{(A-19)}


\[ \nabla \left[ k \left( \frac{p}{\mu z} \right) \nabla p \right] = \phi \mu c_l \left( \frac{p}{\mu z} \right) \frac{\partial p}{\partial t} \]

and for a uniform (i.e., constant) permeability system

\[ \nabla [\nabla p] = \nabla^2 p = \frac{\phi \mu c_l}{k} \frac{\partial p}{\partial t} \hspace{1cm} \text{(A-20)} \]

For three-dimensional flow in a radial coordinate system the vector expression at the left-hand-side of Eq. A-20 is given by

\[ \nabla^2 p = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial p}{\partial r} \right] + \frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} \] \hspace{1cm} \text{(A-21)}

For the cases we consider in this work, we will ignore gas flow in both the angular direction (\( \theta \)-direction) as well as the vertical direction (\( z \)-direction). Eq. A-21 then simplifies to

\[ \nabla^2 p = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial p}{\partial r} \right] \hspace{1cm} \text{(A-22)} \]

Substituting Eq. A-22 into Eq. A-20 yields

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial p}{\partial r} \right] = \frac{\phi \mu c_l}{k} \frac{\partial p}{\partial t} \]

or in terms of field units

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial p}{\partial r} \right] = \frac{\phi \mu c_l}{0.00633 k} \frac{\partial p}{\partial t} \hspace{1cm} \text{(A-23)} \]

where time in Eq. A-23 is in days.
APPENDIX B

DERIVATIONS OF THE STABILIZED FLOW AND THE GAS MATERIAL BALANCE EQUATIONS

Derivation of the Stabilized Gas Flow Equation

In Appendix A we derived the general diffusivity equation which governs the flow of real gases in porous medium. This result was expressed in terms of the normalized pseudopressure and is given as

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial p_p}{\partial r} \right] = \frac{\phi \mu c_l}{0.00633 k} \frac{\partial p_p}{\partial t} \quad (A-23)$$

Under boundary-dominated flow conditions changes in reservoir pore volume are dictated by the rate of fluid withdrawal. That is

$$q_{res} = \frac{dV_p}{dt} \quad \text{or} \quad q_g B_g = \frac{dV_p}{dt} \quad (B-1)$$

From the definition of the isothermal gas compressibility we have

$$c_g = -\frac{1}{V_p} \left[ \frac{dV_p}{dp} \right] \quad \text{or} \quad \frac{dV_p}{dt} = -c_g V_p \frac{dp}{dt}$$

Rearranging this equation gives

$$\frac{dp}{dt} = -\frac{1}{c_g V_p} \frac{dV_p}{dt} \quad (B-2)$$

Substituting Eq. B-1 into Eq. B-2 gives us

$$\frac{dp}{dt} = -\frac{q_g B_g}{c_g V_p} \quad (B-3)$$

Using the definition of the normalized pseudopressure function (Eq. A-13), we can write

$$\frac{\partial p}{\partial t} = \frac{dp}{dp_p} \frac{\partial p_p}{\partial t} = \left( \frac{p}{\mu z n p} \right) \frac{\partial p_p}{\partial t}$$
which gives

\[
\frac{\partial p_p}{\partial t} = \frac{p}{\mu_z} \frac{\partial p}{\partial t} \left( \frac{\mu_z}{\mu_z} \right) \frac{\partial p}{\partial t} \tag{B-4}
\]

Substituting Eq. B-3 into Eq. B-4 yields

\[
\frac{\partial p_p}{\partial t} = -\frac{p}{\mu_z} \frac{\partial p}{\partial t} \left( \frac{\mu_z}{\mu_z} \right) \frac{q_g B_g}{\rho_{n} \sigma_{g} V_p} \tag{B-5}
\]

Combining Eqs. A-23 and B-5, assuming boundary-dominated flow, we obtain the following expression

\[
\frac{1}{r} \frac{d}{dr} \left[ r \frac{\partial p_p}{\partial r} \right] = -\frac{\phi \mu_c}{0.00633 k} \frac{\partial p}{\partial r} \left( \frac{\mu_z}{\mu_z} \right) \frac{p}{\rho_{n} \sigma_{g} V_p} \frac{q_g B_g}{\rho_{n} \sigma_{g} V_p}
\]

or

\[
\frac{1}{r} \frac{d}{dr} \left[ r \frac{\partial p_p}{\partial r} \right] = -\frac{\phi}{0.00633 k} \frac{\partial p}{\partial r} \left( \frac{\mu_z}{\mu_z} \right) \frac{p}{\rho_{n} \sigma_{g} V_p} \frac{q_g B_g}{\rho_{n} \sigma_{g} V_p} \frac{1}{\pi r^2 h \phi} = -C \tag{B-6}
\]

where the constant \( C \) is defined by

\[
C = \left[ \frac{p}{0.00633 k \mu_z} \right] \frac{q_g B_g}{\rho_{n} \sigma_{g} V_p} = \left[ \frac{p}{0.00633 k \mu_z} \right] \frac{q_g p_s c z T}{\rho_{n} \sigma_{g} V_p \pi r^2 h z_{sc} T_{sc} p}
\]

Simplifying this expression further gives

\[
C = \frac{1}{0.00633} \frac{q_g}{\pi r^2 k h z_{sc} T_{sc} \sigma_{g} V_p} \frac{p}{\mu_z} \tag{B-7}
\]

Eq. B-6 is a second-order ordinary differential equation which can twice be integrated to yield the boundary-dominated flow equation. The boundary conditions for this problem are

**Inner Boundary Condition (IBC): Constant Bottomhole Pressure Inner Boundary**

At the wellbore \((r = r_w); p = p_{wf}\),

which implies that

At \(r = r_w\); \(p = p_{wf}\) \tag{B-8}
Outer Boundary Condition (OBC): No-Flow Outer Boundary

At \( r = r_e \): \( \frac{dp}{dr} = 0 \).

This outer boundary condition implies that

\[
\left[ \frac{r \frac{dp}{dr}}{r_e} \right] = \frac{r \frac{dp}{dr}}{r_e} = 0
\]

Thus, at \( r = r_e \):

\[
\left[ r \frac{dp}{dr} \right]_{r_e} = 0
\] 

\[ \text{(B-9)} \]

The first integration of Eq. B-6 yields

\[
r \frac{dp}{dr} = \frac{1}{2} Cr^2 + C_1
\]

where \( C_1 \) is an integration constant. Using the outer boundary condition we solve this equation for \( C_1 \), which gives

\[
C_1 = \frac{1}{2} Cr_e^2
\]

Hence, the final form of the first integration is

\[
r \frac{dp}{dr} = \frac{1}{2} Cr^2 + \frac{1}{2} Cr_e^2
\]

or

\[
r \frac{dp}{dr} = -\frac{1}{2} C \left[ r - \frac{r_e^2}{r} \right]
\] 

\[ \text{(B-10)} \]

Integration of Eq. B-10 yields

\[
p_p = -\frac{1}{2} C \left[ \frac{1}{2} r^2 - r_e^2 \ln r \right] + C_2
\]

Using the inner boundary condition, we determine \( C_2 \) to be

\[
C_2 = p_{pwf} + \frac{1}{2} C \left[ \frac{1}{2} r_w^2 - r_e^2 \ln r_w \right]
\]

Substitution of \( C_2 \) into the previous result and rearranging gives us

\[
p_p - p_{pwf} = \frac{1}{2} C \left[ r_e^2 \ln \frac{r}{r_w} - \frac{1}{2} r^2 + \frac{1}{2} r_w^2 \right]
\] 

\[ \text{(B-11)} \]
We now use the definition of the $C$-parameter (Eq. B-7) in this equation. Eq. B-11 then becomes

$$p_p - p_{pwf} = \frac{q_g p_{sc} T}{0.00633(2\pi)k h T_{sc}} \left( \frac{\mu_g}{p} \right) n \left[ \ln \frac{r}{r_w} - \frac{1}{2} \frac{r^2}{r_w^2} \right]$$

(B-12)

where, we have ignored the term $r_0^2/r_w^2$ as insignificantly small compared to $r^2/r_w^2$.

To express Eq. B-12 in terms of average reservoir pressure, we will use a pore volume average pressure, $\bar{p}$, which is defined as

$$\bar{p} = \frac{\int_{r_w}^{r_e} p \, dV_p}{\int_{r_w}^{r_e} dV_p}$$

(B-13)

Substituting the appropriate variables for the reservoir pore volume, Eq. B-13 becomes

$$\bar{p} = \frac{\int_{r_w}^{r_e} p \, d(\pi r^2 h \phi)}{\int_{r_w}^{r_e} d(\pi r^2 h \phi)}$$

Reducing the terms, we obtain

$$\bar{p} = \frac{2}{(r_e^2 - r_w^2)} \int_{r_w}^{r_e} p \, r \, dr$$

(B-14)

Again, using the definition of the normalized pseudopressure (Eq. A-13) and considering Eq. B-14 we can write

$$\bar{p}_p = \frac{2}{(r_e^2 - r_w^2)} \int_{r_w}^{r_e} p \, r \, dr$$

(B-15)

Substituting Eq. B-12 into Eq. B-15 gives
\[ \bar{p}_p = \frac{2}{(r_e^2 - r_w^2)} \frac{q_g P_{sc}}{0.00633(2\pi)khT_{sc}} \left( \frac{\mu_z}{\mu} \right) \ln \int_{r_w}^{r_e} \left[ \ln \frac{r_e}{r_w} - \frac{1}{2} \frac{r^2}{r_e^2} \right] r \, dr + \frac{2}{(r_e^2 - r_w^2)} \int_{r_w}^{r_e} p_{pwf} \, r \, dr \]

Evaluating the last term in this expression, and moving this result to the left-hand-side, gives

\[ \bar{p}_p - p_{pwf} = \frac{2}{r_e^2 - r_w^2} \frac{q_g P_{sc}}{0.00633(2\pi)khT_{sc}} \left( \frac{\mu_z}{\mu} \right) \ln \int_{r_w}^{r_e} r \left[ \ln \frac{r_e}{r_w} - \frac{1}{2} \frac{r^2}{r_e^2} \right] \, dr \]

or

\[ \bar{p}_p - p_{pwf} = \frac{2}{r_e^2} \frac{q_g P_{sc}}{0.00633(2\pi)khT_{sc}} \left( \frac{\mu_z}{\mu} \right) \ln \int_{r_w}^{r_e} r \left[ \ln \frac{r_e}{r_w} - \frac{1}{2} \frac{r^2}{r_e^2} \right] \, dr \] .......................... (B-16)

where we have again ignored the \( r_w^2 \) term as very small compared to \( r_e^2 \).

Integrating the right-hand-side of Eq. B-16 by parts, we obtain

\[ \bar{p}_p - p_{pwf} = \frac{2}{r_e^2} \frac{q_g P_{sc}}{0.00633(2\pi)khT_{sc}} \left( \frac{\mu_z}{\mu} \right) \left[ \ln r_e - \ln r_w - \frac{r_e^2}{4} + r_w^2 - \frac{r^2}{4} + \frac{r^4}{8} \right] \]

which upon simplification gives us

\[ \bar{p}_p - p_{pwf} = \frac{2}{0.00633(2\pi)khT_{sc}} \left( \frac{\mu_z}{\mu} \right) \left[ \ln \frac{r_e}{r_w} - \frac{3}{4} \right] \] .......................... (B-17)

We note that this development does not include near-wellbore phenomena such as formation damage and non-Darcy effects, associated with high flowrates. By including these effects, as reported extensively in the petroleum literature\(^{11-12}\), we can re-write Eq. B-17 in its final form as

\[ q_g = 0.703kh \left( \frac{2p}{\mu} \right) \left( \frac{\mu_z}{\mu} \right) \ln \left[ \frac{\bar{p}_p - p_{pwf}}{\ln \frac{r_e}{r_w} - \frac{3}{4} + s + Dq_g} \right] \]

or

\[ q_g = J_g \left[ \bar{p}_p - p_{pwf} \right] \] .......................... (B-18)

where
\[ J_g = \frac{0.703kh}{T \ln \left( \frac{r_e}{r_w} \right) + \frac{s + Dq_g}{2}} \left( \frac{2p}{\mu z} \right)_n \]  

(B-19)

When we ignore the non-Darcy skin term, \( J_g \) can be expressed in general (for a reservoir of arbitrary shape characterized by the shape factor, \( C_A \), and with drainage area, \( A_r \)) as

\[ J_g = \frac{0.703kh}{\frac{1}{2} T \ln \left( \frac{2.2458A}{C_A r_w^2} \right) \left( \frac{2p}{\mu z} \right)_n} \]  

(B-20)

where we have used the effective wellbore radius, \( r_{wa} \), defined by

\[ r_{wa} = r_w \exp (-s) \]  

(B-21)

Throughout this study, the normalized conditions will be chosen as the values at the initial pressure, \( p_i \). Therefore, Eq. B-20 can be expressed as

\[ J_g = \frac{0.703kh}{\frac{1}{2} T \ln \left( \frac{2.2458A}{C_A r_w^2} \right) \left( \frac{2p_i}{\mu z_i} \right)} \]  

(B-22)

Recall that the definition of the gas formation volume factor (FVF)\(^51\)

\[ B_g = \frac{1}{5.615} \frac{z T p_s c}{z_s c_s T_p c} = \frac{0.00502 z T}{p} \]  

(B-23)

Substituting Eq. B-23 into Eq. B-22 yields

\[ J_g = \frac{2kh}{141.2 \mu_i B_g^1 \ln \left( \frac{2.2458A}{C_A r_w^2} \right) \left( \frac{2p_i}{\mu z_i} \right)} \]  

(B-24)

Henceforth, we will write Eq. B-18 simply as

\[ q_g = J_g [p_p - p_{pwf}] \]  

(B-25)

**Derivation of the gas Material Balance Equation**

Walsh, *et al.*\(^36\) recently presented the generalized material balance equation (MBE) for volumetric reservoir systems, which involves the underground withdrawal, \( F \), and fluid expansion, \( E \). The general material balance equation is given as
\[ F = NE_o + GE_g + \Delta W \] .................................(B-26)

where

\[ F = N_p \left[ \frac{B_o (1 - R_o R_{ps}) + (R_{ps} - R_s) B_g}{(1 - R_v R_s)} \right] \] .................................(B-27)

\[ E_o = \frac{B_o (1 - R_v R_{si}) + (R_{si} - R_s) B_g}{(1 - R_v R_s)} - B_o \] .................................(B-28)

and

\[ E_g = \frac{B_g (1 - R_v R_s) + (R_{vi} - R_s) B_o}{(1 - R_v R_s)} - B_g \] .................................(B-29)

For a strictly dry gas reservoir the original-oil-in-place (OOIP), \( N \), is zero — and the reservoir gas phase contains no volatile-oil (i.e., \( R_v = 0 \)). Ignoring the expansion of connate water and reservoir rock (\( \Delta W \)) Eqs. B-26 to B-29 reduce to

\[ F = GE_g \] .................................(B-30)

where

\[ F = G_p B_g \]

and

\[ E_g = B_g - B_{gi} \]

We can therefore re-write Eq. B-30 as

\[ G_p B_g = G (B_g - B_{gi}) \] .................................(B-31)

Substituting the definition of the gas formation volume factor (Eq. B-23) into Eq. B-31 and canceling out the constants, we obtain

\[ G_p \frac{zT}{P} = G \left( \frac{zT}{P} - \frac{zT}{P_i} \right) \]

Rearranging the terms yields the final form of the gas material balance equation

\[ \frac{P}{z} = \frac{P_i}{z_i} \left[ 1 - \frac{G_p}{G} \right] \] .................................(B-32)
APPENDIX C

PLOTS OF THE GAS VISCOSITY-COMPRESSIBILITY FUNCTION

In this Appendix we present plots of the viscosity-compressibility ratio as a function of dimensionless time for a very broad range of gas compositions, system temperatures and system pressures. It is important to note that non-linearities in the gas flow equation are caused by fluid property changes during reservoir depletion. In particular, this non-linear behavior is caused primarily by changes in gas viscosity and gas compressibility. We can therefore linearize the gas flow model by correlating the gas viscosity-compressibility product versus pressure (and eventually, over time).

Our objective in this section is to investigate the behavior of the viscosity-compressibility function for different gas reservoir systems. We have used industry-standard correlations\textsuperscript{47-49} for gas properties to generate these plots. The primary data requirements in this exercise are specific gravity of the reservoir gas, formation temperature, and initial pressure of the reservoir system.

Several reservoir fluids have been selected ranging from methane ($\gamma_g = 0.55$) to real gases containing heavier hydrocarbon components (maximum, $\gamma_g = 1.00$). The range of temperatures has been selected to reflect practical reservoir conditions encountered in most fields (150 - 350 °F).

We note that most of these plots exhibit similar basic features. In particular, all the trends are fairly straight for gas reservoir systems with low initial pressures ($p_i$ less than approximately 5000 psia). Secondly, none of these curves show any region where the viscosity-compressibility function is constant (i.e., a horizontal trend). Finally, all the trends deviate away from the straight line trend as the initial pressure of the system increases.
Fig. C.1 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.55$ and $T = 150 \, ^\circ F$.

Fig. C.2 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.60$ and $T = 150 \, ^\circ F$. 
Fig. C.3 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.70$ and $T = 150 ^{\circ}\text{F}$.

Fig. C.4 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.80$ and $T = 150 ^{\circ}\text{F}$. 
**Fig. C.5** - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.90$ and $T = 150 \, ^\circ F$.

**Fig. C.6** - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 1.00$ and $T = 150 \, ^\circ F$. 
Fig. C.7 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.55$ and $T = 200 \, ^\circ\text{F}$.

Fig. C.8 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.60$ and $T = 200 \, ^\circ\text{F}$.
Fig. C.9 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.70$ and $T = 200 \, ^\circ F$.

Fig. C.10 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.80$ and $T = 200 \, ^\circ F$. 
Fig. C.11 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.90$ and $T = 200 \, ^\circ\text{F}$.

\[ p_D = \frac{(p/z)}{(p_i/z_i)} \]

Fig. C.12 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 1.00$ and $T = 200 \, ^\circ\text{F}$.

\[ p_D = \frac{(p/z)}{(p_i/z_i)} \]
Fig. C.13 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.55$ and $T = 250 \, ^{\circ}F$.

Fig. C.14 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.60$ and $T = 250 \, ^{\circ}F$. 

[preamble]

$P_D = (p/z)/(p_i/z_i)$
Fig. C.15 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.70$ and $T = 250 \, ^\circ F$.

Fig. C.16 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.80$ and $T = 250 \, ^\circ F$. 
Fig. C.17 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with \( \gamma_g = 0.90 \) and \( T = 250 \, ^\circ F \).

Fig. C.18 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with \( \gamma_g = 1.00 \) and \( T = 250 \, ^\circ F \).
**Fig. C.19** - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.55$ and $T = 300^\circ F$.

**Fig. C.20** - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.60$ and $T = 300^\circ F$. 
Fig. C.21 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with \( \gamma_g = 0.70 \) and \( T = 300 \, ^\circ\text{F} \).

\[
p_D = \frac{p}{z} / \left( \frac{p_1}{z_1} \right)
\]

Fig. C.22 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with \( \gamma_g = 0.80 \) and \( T = 300 \, ^\circ\text{F} \).
Fig. C.23 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.90$ and $T = 300 \, ^\circ\text{F}$.

Fig. C.24 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 1.00$ and $T = 300 \, ^\circ\text{F}$.
Fig. C.25 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.55$ and $T = 350 \, ^\circ\text{F}$.

Fig. C.26 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.60$ and $T = 350 \, ^\circ\text{F}$.
Fig. C.27 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.70$ and $T = 350$ °F.

Fig. C.28 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.80$ and $T = 350$ °F.
Fig. C.29 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 0.90$ and $T = 350 \, ^{\circ}\text{F}$.

Fig. C.30 - Plot of the Viscosity-Compressibility Function versus Dimensionless Pressure. Data for Gas System with $\gamma_g = 1.00$ and $T = 350 \, ^{\circ}\text{F}$.
APPENDIX D

LIST OF FORTRAN COMPUTER CODES

In this appendix we present a description of the Runge-Kutta-Fehlberg solution algorithm which is used to solve the "General Polynomial" model. Two data files are provided, at the end, as a guide to the use of this program.

The computer program RKFV consists of two main blocks:

RKFC45 — which is used to compute dimensionless rate, dimensionless pressure, and cumulative production solutions for the case of constant bottomhole pressure production, and

RKFV45 — which is used to compute dimensionless rate, dimensionless pressure, and cumulative production solutions in the case of variable bottomhole pressure production.

In addition, the program includes the following subroutines and functions:

RKF — Runge-Kutta-Fehlberg subroutine for solving a first-order ordinary differential equation (ODE).

F(X,Y) — the first-order ODE function.

DER — subroutine for calculating the first derivative of a function.

GRID — subroutine for calculating a logarithm time grid.

PROPS — driver used for computing the gas properties (compressibility factor, viscosity, isothermal gas compressibility, and formation volume factor).

ZFACT — subroutine for calculating gas compressibility factor and isothermal gas compressibility.

VISCQ — subroutine for calculating gas viscosity.

BGAS — subroutine for calculating gas formation volume factor.

TABLEQ — a table look-up subroutine.

POLFIT — subroutine for calculating the coefficients of a polynomial function.

GJD — subroutine for solving a matrix.
PROGRAM RKFV

C RKFV45 - FIFTH-ORDER RUNGE-KUTTA-FEHLMAN PROGRAM FOR THE
C DECLINE CURVE ANALYSIS OF GAS WELL PRODUCTION DATA
C
C LAST EDITED: MAY 31, 1996
C
IMPLICIT REAL*8 (A-H, O-Z)
IMPLICIT INTEGER*4 (I-N)
DIMENSION POZ(301), RMCT(301), VGTAB(301)
CHARACTER*12 DAT, OUT
COMMON /DATA1/ PWD, AA(10)
COMMON /DATA2/ PWF(12)
COMMON /DATA3/ PFWF(1001), TIM(1001), PTAB(301), ZTAB(301)

WRITE(*,*) ' ENTER THE INPUT FILE NAME
READ(*,10) DAT

10 FORMAT(A12)
WRITE(*,*) ' ENTER THE OUTPUT FILE NAME
READ(*,10) OUT
OPEN(UNIT=7,FILE=DAT,STATUS='OLD')
OPEN(UNIT=8,FILE=OUT)
OPEN(UNIT=9,FILE='RKFV.SCR')
READ(7,*) GRAV
READ(7,*) TEMP
READ(7,*) CROCK
READ(7,*) PINIT
READ(7,*) NT
READ(7,*) NPIT
READ(7,*) YINT, TSTART, TEND, DTIN
READ(7,*) DTMX, ITMX, OMEGA
READ(7,*) IFLAG
IF(IFLAG .EQ. 0) THEN
  READ(7,*) N
  READ(7,*) (PWF(I), I=1,N)
  GO TO 30
ELSE
  READ(7,*) DI
  READ(7,*) NP
  DO 20 J=1, NP
    READ(7,*) TIM(J), PFWF(J)
  20 CONTINUE
ENDIF

30 CONTINUE
WRITE(8,40) YINT, TSTART, TEND, DTIN
40 FORMAT(1X, 'INITIAL Y VALUE =',F12.5,/, 
& 1X, 'INITIAL T VALUE =',F12.5,/, 
& 1X, 'FINAL T VALUE =',F12.5,/, 
& 1X, 'INITIAL TIMESTEP SIZE =',F12.5,/)
CALL POLFIT(NFIT, NT, POZ, RMCT, AA)
WRITE(8,50)
50 FORMAT('/',' OUTPUT DATA DECK :','/)
WRITE(8,60)
60 FORMAT(' COEFFICIENTS OF THE POLYNOMIAL FUNCTION ')
DO 70 J = 1, Nev
    WRITE(8,*) AA(J)
    AA(J) = AA(J)/(REAL(J) + 1.D0)
70 CONTINUE
CII = CGINIT + CROCK
IF(IFLAG .EQ. 0) THEN
    CALL RKFC45(YINT, TSTART, TEND, DTIN, DELT, DTMX, ITMX, OMEGA, N)
ELSE
    CALL RKFV45(YINT, TSTART, TEND, DTIN, DELT, DTMX, ITMX, OMEGA, NT,
                NP, DI, PINIT, ZINIT)
ENDIF
STOP
END

SUBROUTINE RKFC45(YINT, TSTART, TEND, DTIN, DELT, DTMX, ITMX, OMEGA, N)
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C RKFC45 - FIFTH-ORDER RUNGE-KUTTA-FEHLBERG PROGRAM FOR THE
C ANALYSIS OF CONSTANT BHP PRODUCTION DATA
C
C LAST EDITED: MAY 31, 1996
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
IMPLICIT REAL*8 (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)
DIMENSION TDD(501),RATE(501,12),RS(11001),TS(11001)
COMMON /DATA1/ PWD, AA(10)
COMMON /DATA2/ PWF(12)
C
NPAN = 120
TDS = 0.0001D0
CALL GRID(NPAN, TDD, TDS, TEND)
DO 10 J=1,N
    PWD = PWF(J)
    AA0 = -(AA(1)*(PWD**2.) + AA(2)*(PWD**3.) + AA(3)*(PWD**4.)
          + AA(4)*(PWD**5.) )
    ALPHA = AA0 + AA(1) + AA(2) + AA(3) + AA(4)
    BETA = ALPHA/(1.D0 - PWD)
    TVAL = TSTART
    TOLD = TSTART
    YOLD = YINT
    DELT = DTIN
    QCAL = 1.D0
    NN = 1
    TS(NN) = TSTART
    RS(NN) = QCAL
    WRITE(9,30) DELT, TVAL, QCAL
    TN = 0.D0
    YN = 0.D0
    DO WHILE(TVAL .LE. TEND)
        NN = NN + 1
10 CONTINUE
CALL RKF(TOLD, YOLD, DELT, RK4, RK5)
ERR = (DABS(RK4-RK5)/(RK5+1.D-12))*100.D0
NCUT = 0
DO WHILE (ERR .GT. 1.E-8)
DELT = DELT/OMEGA
NCUT = NCUT + 1
CALL RKF(TOLD, YOLD, DELT, RK4, RK5)
ERR = (DABS(RK4-RK5)/(RK5+1.D-12))*100.D0
IF (NCUT .GE. ITMX) THEN
WRITE(*,*) 'CONVERGENCE CRITERIA NOT MET IN ', ITMX, ', ITER.'
WRITE(8,*) 'CONVERGENCE CRITERIA NOT MET IN ', ITMX, ', ITER.'
STOP
ENDIF
ENDDO
YCAL = RK5
TVAL = TVAL + DELT
CALL DER(NN, YN, TN, YOLD, TOLD, YCAL, TVAL, QCAL)
QCAL = QCAL/ALPHA
WRITE(9, 30) DELT, TVAL, QCAL
TS(NN) = TVAL
RS(NN) = QCAL
DELT = DELT*OMEGA
IF (DELT .GT. DTMX) DELT = DTMX
YOLD = YCAL
TOLD = TVAL
IF (QCAL .LT. 1.E-5) GO TO 40
ENDDO
30 FORMAT (1X, F15.7, 1X, F15.7, 1X, F15.7)
40 CONTINUE
DO 60 L=1, NPAN
   CALL TABSEQ(TS, RS, NN, TDD(L), RATE(L, J))
60 CONTINUE
10 CONTINUE
C .........................................................WRITING OUTPUT
WRITE(8, 70)
70 FORMAT(3X, ' TD ', ' 5X', ' QD0 ', ' 3X', ' QD1 ', ' 3X', ' QD2 ', ' 3X', ' QD3 ', &
       ' 4X', ' QD4 ', ' 3X', ' QD5 ', ' 3X', ' QD6 ', ' 3X', ' QG7 ', &
       ' 3X', ' QD8 ', ' 3X', ' QD9 ', ' 3X', ' QD10 ')
DO 80 I=1, NPAN
   WRITE(8, 90) TDD(I), (RATE(I, J), J=1, N)
80 CONTINUE
90 FORMAT(1X, F10.6, 1X, (11F9.6))
RETURN
END

SUBROUTINE RKFV45(YINT, TSTART, TEND, DTIN, DELT, DTMX, ITMX, OMEGA, NT, & NP, DI, PINIT, ZINIT)
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
IMPLICIT INTEGER*4(I-N)
COMMON /DATA1/ PWD, AA(10)
COMMON /DATA3/ PWFV(1001), TIM(1001), PTAB(301), ZTAB(301)

CALL TABSEQ(PTAB, ZTAB, NT+1, PWFV(1), ZWF1)
PWF = PWFV(1)*ZINIT/(PINIT*ZWF1)
AA0 = -(AA(1)*(PWD**2.) + AA(2)*(PWD**3.) + AA(3)*(PWD**4.)
   + AA(4)*(PWD**5.) )
ALPHA = AA0 + AA(1) + AA(2) + AA(3) + AA(4)
BETA = ALPHA/(1.D0 - PWD)
WRITE(8,*) 'PWD1 = ', PWD, ' BETA= ', BETA
WRITE(8,*) ' ALPHA = ', ALPHA
TVAL = TSTART
TOLD = TSTART
YOLD = YINT
DELT = DTIN
QCAL = 1.D0
NN = 1
WRITE(8,20) TVAL, QCAL
WRITE(9,30) DELT, TVAL, QCAL
TN = 0.D0
YN = 0.D0
DOWHILE(TVAL .LE. TEND)
   TIMG = (TVAL+DELT)/DI
   IF(TIMG .GT. TIM(NP)) GO TO 40
   NN = NN + 1
   CALL TABSEQ(TIM, PWFV, NP, TIMG, PWF)
   CALL TABSEQ(PTAB, ZTAB, NT+1, PWF, ZWF)
   PWD = PWF*ZINIT/(PINIT*ZWF)
   CALL RKF(TOLD, YOLD, DELT, RK4, RK5)
   ERR = (DABS(RK4-RK5)/(RK5+1.D-20)) * 100.D0
   NCUT = 0
   DOWHILE(ERR .GT. 1.E-8)
      DELT = DELT/OMEGA
      TIMG = (TVAL+DELT)/DI
      CALL TABSEQ(TIM, PWFV, NP, TIMG, PWF)
      CALL TABSEQ(PTAB, ZTAB, NT+1, PWF, ZWF)
      PWD = PWF*ZINIT/(PINIT*ZWF)
      NCUT = NCUT + 1
      CALL RKF(TOLD, YOLD, DELT, RK4, RK5)
      ERR = (DABS(RK4-RK5)/(RK5+1.D-20)) * 100.D0
      IF(NCUT .GE. ITMX) THEN
         WRITE(*,*) 'CONVERGENCE CRITERIA NOT MET IN ',ITMX,' ITER.'
         WRITE(8,*) 'CONVERGENCE CRITERIA NOT MET IN ',ITMX,' ITER.'
         STOP
      ENDIF
   ENDDO
   CONTINUE
   YCAL = RK5
   TVAL = TVAL + DELT
   CALL DER(NN, YN, TN, YOLD, TOLD, YCAL, TVAL, QCAL)
   QCAL = QCAL/ALPHA
   WRITE(8,20) TVAL, QCAL
   WRITE(9,30) DELT, TVAL, QCAL
   DELT = DELT/OMEGA
   IF(DELT .GT. DTMX) DELT = DTMX
   YOLD = YCAL
TOLD = TVAL
ENDDO
20 FORMAT(1X,F15.7,1X,F15.7)
30 FORMAT(1X,F15.7,1X,F15.7,1X,F15.7)
40 CONTINUE
RETURN
END

SUBROUTINE RKF(X1,Y1,DELX,R4,R5)
C
C
C LAST EDITED: MARCH 15, 1996
C
C X1 - X-VARIABLE IN THE ODE
C Y1 - Y-VARIABLE IN THE ODE
C DELX - STEP-SIZE
C R4 - FOURTH-ORDER RUNGE-KUTTA SOLUTION
C R5 - FIFTH-ORDER RUNGE-KUTTA SOLUTION
C
C
C
C
C
C
C
C
C
C
C IMPLICIT REAL*8 (A-H,O-Z)
C IMPLICIT INTEGER*4 (I-N)
C
C TK1=DELX*F(X1,Y1)
C TK2=DELX*F((X1+(0.25D0*DELX)),(Y1+(0.25D0*TK1)))
C TK3=DELX*F((X1+(0.375D0*DELX)),(Y1+((3.D0/32.D0)*TK1) +
C & ((9.D0/32.D0)*TK2)))
C TK4=DELX*F((X1+((12.D0/13.D0)*DELX)),(Y1+((1932.D0/2197.D0)*TK1) -
C & ((7296.D0/2197.D0)*TK2)+((7296.D0/2197.D0)*TK3))
C TK5=DELX*F((X1+DELX),(Y1+((439.D0/216.D0)*TK1) - (8.D0*TK2) +
C & ((3680.D0/513.D0)*TK3)-((845.D0/4104.D0)*TK4))
C TK6=DELX*F((X1+(0.5D0*DELX)),(Y1-(8.D0/27.D0)*TK1+(2.D0*TK2) -
C & ((3544.D0/2565.D0)*TK3)+((1859.D0/4104.D0)*TK4)-
C & ((11.D0/40.D0)*TK5))
C
C R4 =Y1 + ((25.D0/216.D0)*TK1) + ((1408.D0/2565.D0)*TK3) +
C & ((2197.D0/4104.D0)*TK4) - (0.2D0*TK5)
C R5 =Y1 + ((16.D0/135.D0)*TK1) + ((6656.D0/12825.D0)*TK3) +
C & ((28561.D0/56430.D0)*TK4) - (9.D0/50.D0)*TK5 + (2.D0/55.D0)*TK6
C
RETURN
END

REAL*8 FUNCTION F(X,Y)
C
C
C FUNCTION F - FIRST-ORDER ODE FUNCTION
C
C X - X-VARIABLE IN THE ODE
C Y - Y-VARIABLE IN THE ODE
C
C
C
C IMPLICIT REAL*8 (A-H,O-Z)
C IMPLICIT INTEGER*4 (I-N)
COMMON/DATA1/ FWD, AA(10)

C
AA0 = -( AA(1)*(FWD**2.) + AA(2)*(FWD**3.) + AA(3)*(FWD**4.)
& + AA(4)*(FWD**5.) )
F = -( AA0 + AA(1)*(Y**2.) + AA(2)*(Y**3.) + AA(3)*(Y**4.)
& + AA(4)*(Y**5.) )
RETURN
END

SUBROUTINE DER(K, YN, TN, YOLD, TOLD, YCAL, TT, DYDT)

C
C    DER - FINITE-DIFFERENCE DERIVATIVE ROUTINE
C
C    K - TIMESTEP COUNTER
C    YN - Y-VARIABLE AT N-2 TIME LEVEL
C    TN - X-VARIABLE AT N-2 TIME LEVEL
C    YOLD - Y-VARIABLE AT N-1 TIME LEVEL
C    TOLD - X-VARIABLE AT N-1 TIME LEVEL
C    YCAL - Y-VARIABLE
C    TT - X-VARIABLE
C    DYDT - DERIVATIVE OUTPUT
C
C
C
C
IMPLICIT REAL*8(A-H,O-Z)
IMPLICIT INTEGER*4(I-N)

C

Y = YN
YN = 1.D0 - YOLD
YY = 1.D0 - YCAL
TN2 = TN
TN = TOLD
IF(K .EQ. 2) THEN
    DYDT = (YY - YN)/(TT - TN)
ELSE
    DYDT = YN2*((2.D0*TT - TN - TT)/((TN - TN2)*(TN2-TT)))
& + YY*((2.D0*TT - TN2 - TT)/((TN - TN2)*(TN-TT)))
& + YY*((2.D0*TT - TN2 - TN)/((TT - TN2)*(TT-TN)))
ENDIF
RETURN
END

SUBROUTINE GRID(NPAN, XW, TSTART, TEND)

C
C
C
C
C
IMPLICIT REAL*8(A-H,O-Z)
IMPLICIT INTEGER*4(I-N)
DIMENSION XW(500)

C
TSTRT = DLOG10(TSTART)
TENDD= DLOG10(TEND)
XW(1) = TSTRT
DX = (TENDD - TSTRT)/(NPAN - 1)
DO 10 J=2,NPAN
   XW(J) = DX + XW(J-1)
10 CONTINUE
DO 20 I=1,NPAN
   XW(I) = 10.D0**(XW(I))
20 CONTINUE
RETURN
END

SUBROUTINE PROPS(GRAV,T,PINIT,CROCK,NT,ZI,POZTAB,"
& RMCT,PTAB,ZTAB,VGTAB,VGINIT,CGINIT,BGINIT)
IMPLICIT REAL*8(A-H,O-Z)
IMPLICIT INTEGER*4(I-N)
DIMENSION PTAB(301),ZTAB(301),VGTAB(301),CGTAB(301),"
& POZTAB(301),RMCT(301),BGTAB(301)
C C
PTAB(1) = 0.
PSC = 14.7
TSC = 520.0D0
PINC = (PINIT+100.0)/(NT-1)
DO 10 I = 2,NT+1
   PTAB(I) = PTAB(I-1) + PINC
   CALL ZFACT(PTAB(I),T,GRAV,ZTAB(I),CGTAB(I))
   CALL VISCG(PTAB(I),T,GRAV,ZTAB(I),VGTAB(I))
   CALL BGAS (PTAB(I),T,ZTAB(I),PSC,TSC,BGTAB(I))
10 CONTINUE
C CALL TABSEQ(PTAB,VGTAB,NT+1,PINIT,VGINIT)
CALL TABSEQ(PTAB,CGTAB,NT+1,PINIT,CGINIT)
CALL TABSEQ(PTAB,BGTAB,NT+1,PINIT,BGINIT)
CALL TABSEQ(PTAB,ZTAB,NT+1,PINIT,ZI)
WRITE(9,40)
POZTAB(1) = 0.0
RMCT(1) = 0.0
WRITE(9,50)POZTAB(1),RMCT(1)
DO 30 I=2,NT
   POZTAB(I) = PTAB(I)*ZI/(ZTAB(I)*PINIT)
   RMCT(I) = (VGINIT*(CGINIT+CROCK))/(VGTAB(I)*(CGTAB(I)+CROCK))
WRITE(9,50)POZTAB(I),RMCT(I)
30 CONTINUE
FORMAT(/,6X,'POZ ',7X,'MCTi/MCT')
RETURN
END

SUBROUTINE ZFACT(PTAB,T,GRAV,ZTAB,CGTAB)
C
C ZFACT - SUBROUTINE FOR COMPUTING GAS COMPRESSIBILITY
C
C PTAB - PRESSURE VALUES
C T - RESERVOIR TEMPERATURE
C GRAV - GAS GRAVITY
C ZTAB - GAS COMPRESSIBILITY FACTOR
C CGTAB - ISOTHERMAL GAS COMPRESSIBILITY
C
C IMPLICIT REAL*8(A-H,O-Z)
C
C ...... pseudo-critical temp. & press. (Sutton)
PC = 756.8D0 - 131.D0*GRAV - 3.6D0*GRAV*GRAV
TC = 169.2D0 + 349.5D0*GRAV - 74.D0*GRAV*GRAV
C
C ...... pseudo-reduced temp. & press.
TR = T/TC
PR = PTAB/PC
C
C ...... Dranchuk & Abou-Kassem Eq.
A1 = 0.3265
A2 =-1.07
A3 =-0.5339
A4 = 0.01569
A5 =-0.05165
A6 = 0.5475
A7 =-0.7361
A8 = 0.1844
A9 = 0.1056
A10 = 0.6134
A11 = 0.7210
C1 = A1+A2/TR+A3/TR**3+A4/TR**4+A5/TR**5
C2 = A6+A7/TR+A8/TR**2
C3 = A9*(A7/TR+A8/TR**2)
C
C
Z = 1.0
DO 10 ITER = 1,100
   DR = 0.27*PR/(Z*TR)
C
   WRITE(*,*)'HERE 4 ZFACT Z DR',Z,DR
   C4 = A10*(1+A11*DR**2)*(DR**2/TR**3)*EXP(-A11*DR**2)
   DC4DR=(2.d0*A10*DR/(TR**3))
   & (1.D0 + A11*(DR**2) - (A11*(DR**2))**2)
   & DEXP(-A11*(DR**2))
   DZDR = C1 + 2.D0*C2*DR
   & - 5.D0*C3*(DR**4) + DC4DR
C
   ...... function statement for DAK Eq.
   FUN = Z - (1.D0 + C1*DR + C2*(DR**2)
   & - C3*(DR**5) + C4)
   DFUN = 1.D0 + C1*DR/Z + 2.D0*C2*(DR**2)/Z
   & - 5.D0*C3*(DR**5)/Z + DC4DR*DR/Z
   DEL = -(FUN/DFUN)
Z = Z + DEL
IF(DABS(DEL) .LT. 1.E-8) GO TO 20
10 CONTINUE
WRITE(8,*) '****WARNING**** ZFACT DID NOT CONVERGE', PTAB
20 CONTINUE
ZTAB = Z
C-----------------------------------------------------------------------
C Computes Gas Compressibility, cg
C-----------------------------------------------------------------------
TERM1 = DZDR/(1.D0 +(DR/Z)*DZDR)
CR = (1.D0/PR) - (0.27D0/((Z**2)*TR))*TERM1
CTAB = PR*CR/PTAB
RETURN
END

SUBROUTINE VISCG(PTAB, T, GRAV, ZTAB, VGTAB)
C
C VISCG - SUBROUTINE FOR GAS VISCOSITY
C
C PTAB - PRESSURE VALUES
C T - RESERVOIR TEMPERATURE
C GRAV - GAS GRAVITY
C ZTAB - GAS COMPRESSION FACTOR
C VGTAB - COMPUTED GAS VISCOSITY
C
C IMPLICIT REAL*8 (A-H, O-Z)
C
C ... gas viscosity (Lee, Gonzales & Eakin)
C
WIMOL= GRAV*28.97D0
D = 1.4935E-3*PTAB*WIMOL/(ZTAB*T)
AK = (9.379D0+0.01607D0*WIMOL)*(T**1.5)/(209.2+19.26*WIMOL+T)
X = 3.448+(986.4/T)+0.0109D0*WIMOL
Y = 2.447-0.2224*X
VGTAB= (1.E-4)*AK*DEXP(X*(D**Y))
RETURN
END

SUBROUTINE BGAS(PTAB, T, ZTAB, PSC, TSC, BGTAB)
C
C ZFACT - SUBROUTINE FOR COMPUTING GAS COMPRESSIBILITY
C
C PTAB - PRESSURE VALUES
C T - RESERVOIR TEMPERATURE
C ZTAB - GAS COMPRESSION FACTOR
C VGTAB - COMPUTED GAS FORMATION VOLUME FACTOR
C
C IMPLICIT REAL*8 (A-H, O-Z)
C
BGTAB = ((PSC*T*ZTAB)/(TSC*PTAB))
RETURN
END
SUBROUTINE TABSEQ(X, Y, N, XX, YY)

C
C TABSEQ - TABLE LOOK-UP ROUTINE USING SEQUENTIAL SEARCH
C - (LINEAR INTERPOLATION BETWEEN TABLES USED)
C X - VECTOR OF INDEPENDENT VARIABLES (ARGUMENTS)
C Y - VECTOR OF DEPENDENT VARIABLES (FUNCTION VALUES)
C N - NUMBER OF TABLE ENTRIES
C XX - ARGUMENT
C YY - INTERPOLATED FUNCTION
C

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION X(*), Y(*)
IF(XX.LT.X(1))GO TO 20
I = 1
10 CONTINUE
   I = I+1
   IF(I.GT.N)GO TO 30
   IF(XX.GT.X(I))GO TO 10
   YY = Y(I-1) + (Y(I) - Y(I-1))*(XX - X(I-1))/(X(I)-X(I-1))
   RETURN
20 CONTINUE
   YY = Y(I)
   WRITE(8,40)XX
   WRITE(*,40)XX
   RETURN
30 CONTINUE
   YY = Y(N)
   WRITE(8,40)XX
   WRITE(*,40)XX
   C WRITE(*,*),'OUT OF TABSEQ - RKFV'
   RETURN
40 FORMAT(' WARNING - ARGUMENT OUT OF TABLE, XX = ',',F12.5)
END

SUBROUTINE POLFIT(NMAX, NPTS, X, Y, COF)

C
C POLFIT - REGRESSION ROUTINE USING A GENERAL POLYNOMIAL FUNCTION
C
C NMAX - NUMBER OF DATA ENTRIES
C NPTS - ORDER OF THE POLYNOMIAL FUNCTION
C X - VECTOR OF INDEPENDENT VARIABLES (ARGUMENTS)
C Y - VECTOR OF DEPENDENT VARIABLES (FUNCTION VALUES)
C COF - COMPUTED COEFFICIENTS OF THE POLYNOMIAL FUNCTION
C

IMPLICIT REAL*8(A-H,O-Z)
IMPLICIT INTEGER*4(I-N)
DIMENSION X(*), Y(*), COF(10), B(10), A(10,10), DUM(20)

C
DO 10 I = 1, NMAX
   B(I) = 0.
   COF(I) = 0.
10 CONTINUE
DO 20 J = 1,NMAX
DO 20 I=1,NPTS
   RJ = REAL(J)
   B(J) = B(J) + Y(I)*(X(I)**RJ)
20 CONTINUE

DOM(1) = NPTS
NT = 2*NMAX
DO 40 K = 1,NT
   DOM(K) = 0.0
   DO 30 J = 1,NMAX
      RK = REAL(K) + 1.0
      DOM(K) = DOM(K) + X(J)**RK
   CONTINUE
30 CONTINUE
40 CONTINUE

DO 50 J = 1,NMAX
DO 50 I = 1,NMAX
   K = I + J
   A(J,I) = DOM(K-1)
50 CONTINUE

CALL GJD(A,B,COF,NMAX)
RETURN
END

SUBROUTINE GJD(A, RHS, X, N)

C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C C
C C N          - ORDER OF MATRIX
C C RHS         - RIGHT HAND SIDE VECTOR
C C A           - LINEAR SYSTEM MATRIX
C C X           - SOLUTION VECTOR
C C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

IMPLICIT REAL*8(A-H,P-Z)
IMPLICIT INTEGER*4(I-N)
DIMENSION X(*), RHS(*), A(10, 10)

C DO 10 I = 1,N
   X(I) = 0.0
10 CONTINUE

DO 20 K = 1,N
   TEMP = 1.0/D(A(K,K))
   J = K
   DO 30 CONTINUE
      A(K,J) = A(K,J)*TEMP
      IF(J.LT. N) THEN
         J = J + 1
         GO TO 30
      ENDIF
   CONTINUE
   RHS(K) = RHS(K)*TEMP
20 CONTINUE

IF(K.EQ. J) GO TO 40
   TEMP = A(J,K)
   L = K
40 CONTINUE
   A(J,L) = A(J,L) - A(K,L)*TEMP
   IF(L.LT. N) THEN
L = L + 1  
GO TO 50  
ENDIF  
RHS(J) = RHS(J) - RHS(K) * TEMP  
40 CONTINUE  
20 CONTINUE  
DO 60 I = 1, N  
   X(I) = RHS(I)  
60 CONTINUE  
RETURN  
END
### Table D.1

Test1.dat — Example Data File for the Program RKFV (Constant BHP Case)

<table>
<thead>
<tr>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.55</td>
</tr>
<tr>
<td>610</td>
</tr>
<tr>
<td>1.e-8</td>
</tr>
<tr>
<td>10000</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>1.0 0.0 100. 0.0001</td>
</tr>
<tr>
<td>0.05 10 1.03</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 0.998</td>
</tr>
</tbody>
</table>

### Table D.2

Wella.dat — Example Data File for the Program RKFV (Variable BHP Case)

<table>
<thead>
<tr>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.72</td>
</tr>
<tr>
<td>735</td>
</tr>
<tr>
<td>1.e-8</td>
</tr>
<tr>
<td>2625</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>1.0 0.0 100. 0.0001</td>
</tr>
<tr>
<td>0.05 10 1.03</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>0.008</td>
</tr>
<tr>
<td>38</td>
</tr>
<tr>
<td>0.0 2000.0</td>
</tr>
<tr>
<td>10.0 329.42</td>
</tr>
<tr>
<td>20.0 1017.95</td>
</tr>
<tr>
<td>30.0 511.77</td>
</tr>
<tr>
<td>40.0 326.75</td>
</tr>
</tbody>
</table>
Table D.2. Continued.

<table>
<thead>
<tr>
<th>Value</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.0</td>
<td>500.54</td>
</tr>
<tr>
<td>60.0</td>
<td>401.08</td>
</tr>
<tr>
<td>70.0</td>
<td>614.64</td>
</tr>
<tr>
<td>80.0</td>
<td>287.90</td>
</tr>
<tr>
<td>90.0</td>
<td>264.93</td>
</tr>
<tr>
<td>100.0</td>
<td>264.79</td>
</tr>
<tr>
<td>110.0</td>
<td>368.60</td>
</tr>
<tr>
<td>120.0</td>
<td>246.59</td>
</tr>
<tr>
<td>130.0</td>
<td>703.62</td>
</tr>
<tr>
<td>140.0</td>
<td>268.91</td>
</tr>
<tr>
<td>150.0</td>
<td>339.48</td>
</tr>
<tr>
<td>160.0</td>
<td>304.59</td>
</tr>
<tr>
<td>170.0</td>
<td>350.58</td>
</tr>
<tr>
<td>180.0</td>
<td>467.19</td>
</tr>
<tr>
<td>190.0</td>
<td>479.00</td>
</tr>
<tr>
<td>200.0</td>
<td>1156.23</td>
</tr>
<tr>
<td>210.0</td>
<td>1360.44</td>
</tr>
<tr>
<td>220.0</td>
<td>1312.34</td>
</tr>
<tr>
<td>230.0</td>
<td>528.01</td>
</tr>
<tr>
<td>240.0</td>
<td>356.87</td>
</tr>
<tr>
<td>250.0</td>
<td>339.06</td>
</tr>
<tr>
<td>260.0</td>
<td>306.48</td>
</tr>
<tr>
<td>270.0</td>
<td>280.82</td>
</tr>
<tr>
<td>280.0</td>
<td>327.63</td>
</tr>
<tr>
<td>290.0</td>
<td>1048.54</td>
</tr>
<tr>
<td>300.0</td>
<td>312.87</td>
</tr>
<tr>
<td>310.0</td>
<td>306.25</td>
</tr>
<tr>
<td>320.0</td>
<td>280.25</td>
</tr>
<tr>
<td>330.0</td>
<td>280.28</td>
</tr>
<tr>
<td>340.0</td>
<td>233.61</td>
</tr>
<tr>
<td>350.0</td>
<td>597.07</td>
</tr>
<tr>
<td>360.0</td>
<td>596.54</td>
</tr>
<tr>
<td>370.0</td>
<td>596.58</td>
</tr>
</tbody>
</table>
VITA

JOSEPH ANSAH

Personal Information:

Date and Place of Birth: June 14, 1960
Accra, Ghana.

Parents: Charles K. Ansaah and Mary Assabil

Permanent Address: 19322 Winding Branch
Katy, Texas 77449

Education:

Ph.D. Texas A&M University
College Station, Texas
Ph.D., Petroleum Engineering
(August 1996)

B.S. Moscow Institute of Oil and Gas
Currently, State Academy of Oil and Gas
Moscow, Russia
B.S., Petroleum Engineering
(June 1989)

Affiliations:

Society of Petroleum Engineers
Pi Epsilon Tau

Professional Interests:

Reservoir and Production Engineering
Pressure Transient and Production Decline Analyses
Enhanced Oil Recovery
Upstream Project Management