Pressure Integral Type Curve Analysis-II: Applications and Field Cases


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ABSTRACT

This paper presents new pressure integral type curves and illustrates their applications with several field examples. The pressure integral plotting functions are applied for the type curve analysis of well test data. Analysis procedures for well tests from homogeneous and vertically fractured reservoirs are outlined with a step-by-step approach, and the analysis techniques for each case are demonstrated with an example. We also show how various pressure ratio functions may be used to fix the time scale, and therefore reduce the uncertainty associated with conventional type curve analyses. Similarly, we show how simultaneous matching of several pressure functions also reduces the problem of uniqueness associated with type curve matching.

INTRODUCTION

The key to using type curves for the analysis of well test data is identification of the correct reservoir type, e.g., homogeneous, dual-porosity, etc. However, the conventional log-log pressure-time plots are relatively insensitive to changes in the pressure data that are characteristic of various reservoir types. As an alternative to pressure-time plots, the pressure derivative has found great utility in the petroleum industry and is often used specifically for identifying reservoir types. Unfortunately, application of pressure derivative plots requires relatively smooth pressure-time data that is free of significant data noise. Reference 8 attempts to obtain smooth derivatives by use of sophisticated splines. Although it is a robust method, Lane et al's method may result in a loss of characteristic shape of the pressure response.

Recently, several authors have developed pressure ratio functions while others have developed pressure integral functions that not only complement the pressure derivative plots, but also provide additional tools for analyzing well test data. The pressure integral functions presented in this paper and in a companion paper are also useful for smoothing noisy data while still maintaining much of the character of the original pressure response. The purpose of this paper is to illustrate applications of these new pressure integral functions, especially for the analysis of noisy field data. In addition, we present several new type curves that are based on the pressure integral functions.

NEW TYPE CURVES

Since the new type curves are developed with several unconventional dimensionless variables, we define these new dimensionless variables in this section. Derivations of most of the dimensionless plotting functions are discussed in Reference 16, and the applications are presented in later sections of this paper.

Definition of Dimensionless Plotting Functions

The dimensionless wellbore pressure, \( p_D \), is defined as

\[
p_D = \frac{kh\Delta p}{141.2 \ qB \mu} \]

where the pressure drop, \( \Delta p \), for drawdown tests is given as

\[
\Delta p = p_i - p_{wf} \]

and for buildup tests,

\[
\Delta p = p_{sw} - p_{wf} \]

The dimensionless time, \( t_D \), based on the wellbore radius is

\[
t_D = \frac{0.0002637 \ kt}{\mu c r_w^2} \]

References and illustrations at end of paper
and the dimensionless time, \( t_{LD} \), based on fracture half-length, \( L_P \), is defined as

\[
\frac{t_{LD}}{t_{LD}} = 0.0002637 \frac{kt}{\phi \mu c L_P^2}
\]

\[\ldots \ldots \ldots \ldots .(5)\]

**Dimensionless Pressure Integral Derivatives**

The dimensionless derivative function, \( P_D' \), is defined in two forms which are identical mathematically. The first form is

\[
P_D' = \frac{dP_D}{d(ln t_D)}
\]

Alternatively,

\[
P_D' = t_D \frac{dP_D}{dt_D}
\]

\[\ldots \ldots \ldots \ldots .(7)\]

We prefer the form in Eq. 7 because it allows us to obtain \( P_D' \) readily from general analytical solutions, which include wellbore storage, bilinear or linear flow regimes for vertically-fractured reservoirs, and radial flow for homogeneous reservoirs.

Using a form similar to Eq. 7, we define the second derivative function, \( P_D'' \), as

\[
P_D'' = \left. \frac{d^2 P_D}{dt_D^2} \right|_{t_D}
\]

\[\ldots \ldots \ldots \ldots .(8)\]

**Dimensionless Pressure Integral Functions**

We also define a new dimensionless pressure integral function, \( P_{Di} \), as

\[
P_{Di} = \frac{1}{t_D} \int_0^{t_D} P_D(\tau) d\tau
\]

\[\ldots \ldots \ldots \ldots .(9)\]

where \( \tau \) is a variable of integration. Further, the first derivative of the pressure integral function is defined as

\[
P_{Di}' = \frac{dP_{Di}}{dt} = \frac{1}{t_D} \frac{dP_{Di}}{dt_D}
\]

\[\ldots \ldots \ldots \ldots .(10)\]

We demonstrated in Reference 16 that Eq. 10 can also be expressed as a dimensionless pressure integral difference

\[
P_{Di}' = P_D - P_{Di}
\]

\[\ldots \ldots \ldots \ldots .(11)\]

That is, the right-hand-sides of Eqs. 10 and 11 are identical mathematically, or

\[
t_D \frac{dP_{Di}}{dt_D} = P_D - P_{Di}
\]

\[\ldots \ldots \ldots \ldots .(12)\]

Other useful pressure integral plotting functions include the second pressure integral, \( P_{Diigma} \)

\[
P_{Diigma} = \frac{1}{t_D} \int_0^{t_D} P_{Di}(\tau) d\tau
\]

\[\ldots \ldots \ldots \ldots .(13)\]

and, the first and second derivatives of the second pressure integral function, \( P_{Diigma} \) and \( P_{Diigma''} \), respectively are

\[
P_{Diigma}' = \frac{dP_{Diigma}}{dt} = t_D \frac{dP_{Diigma}}{dt_D}
\]

\[\ldots \ldots \ldots \ldots .(14)\]

and,

\[
P_{Diigma''} = t_D \frac{d(P_{Diigma})}{dt_D}
\]

\[\ldots \ldots \ldots \ldots .(15)\]

Alternatively, the first derivative of the second pressure integral function may be calculated as a derivative (as given by Eq. 14) or as a pressure integral difference which is given as

\[
P_{Diigma}' = P_{Diigma} - P_{Diigma}
\]

\[\ldots \ldots \ldots \ldots .(16)\]

Similarly, the second derivative of the second pressure integral function may be calculated as a derivative (as given by Eq. 15) or as a pressure integral difference which is given as

\[
P_{Diigma''} = P_{Diigma}' - P_{Diigma'}
\]

\[\ldots \ldots \ldots \ldots .(17)\]

The motivation for expressing the various integral functions (\( P_{Diigma}, P_{Diigma}, \) and \( P_{Diigma''} \)) as differences will allow us to develop an algorithm to compute the dimensionless pressure (or pressure drop) based on these integral difference functions. We have developed a smoothing algorithm that is based on the first and second integral functions, \( P_{Diigma} \) and \( P_{Diigma} \), respectively. This algorithm is given as a task-oriented program in the Appendix.

**Field Variable Forms of the Plotting Functions**

For completeness in this section, we present the derivative and integral relations that were developed in dimensionless form expressed in terms of the appropriate pressure drop, \( \Delta p \), variables. This section is intended to encourage the field analyst to use these new plotting functions by presenting them in their simplest form, that of derivatives and integrals of the pressure drop, \( \Delta p \).

1. The first pressure drop derivative function, \( \Delta p' \), is

\[
\Delta p' = \frac{d\Delta p}{dt}
\]

\[\ldots \ldots \ldots \ldots .(18)\]

2. The second pressure drop derivative function, \( \Delta p'' \), is

\[
\Delta p'' = \left. \frac{d^2 \Delta p}{dt^2} \right|_{t_D}
\]

\[\ldots \ldots \ldots \ldots .(19)\]

3. The first pressure drop integral function, \( \Delta p_i \), is

\[
\Delta p_i = \frac{1}{t} \int_0^{t} \Delta p(\tau) d\tau
\]

\[\ldots \ldots \ldots \ldots .(20)\]

where \( \tau \) is a variable of integration.
4. The first pressure drop integral difference function, $\Delta p_i'$, is

$$\Delta p_i' = t \frac{d \Delta p_i}{dt}$$  \hspace{1cm} (21)

5. The second pressure drop integral function, $\Delta p_{ii}$, is

$$\Delta p_{ii} = \frac{1}{t} \int_0^t \Delta p_i(t) dt$$  \hspace{1cm} (22)

where $t$ is a variable of integration.

6. The second pressure drop integral, first difference function, $\Delta p_{ii}'$, is

$$\Delta p_{ii}' = t \frac{d \Delta p_{ii}}{dt}$$  \hspace{1cm} (23)

7. The second pressure drop integral, second difference function, $\Delta p_{ii}''$, is

$$\Delta p_{ii}'' = t \frac{d \Delta p_{ii}'}{dt}$$  \hspace{1cm} (24)

Dimensionless Ratio Functions

We also illustrate the utility of other plotting functions which are expressed as dimensionless pressure ratios. These functions give unique resolution to flow regimes such as wellbore storage domination (i.e., unit-slope line), bilinear and linear flow for fractured reservoirs, and radial flow for homogeneous reservoirs. The two functions are

$$P_{Dr1} = \frac{P_D''}{P_D} = \frac{\Delta p''}{\Delta p}$$  \hspace{1cm} (25)

and,

$$P_{Dr2} = \frac{P_D}{2P_D} = \frac{\Delta p}{2\Delta p}$$  \hspace{1cm} (26)

where Eq. 26 was proposed by Onur and Reynolds.9

Other useful ratio functions are expressed as various combinations of pressure integrals. The first pressure integral ratio functions are defined as

$$P_{Dir1} = \frac{P_{Di}}{2P_{Di}} = \frac{\Delta p_i}{2\Delta p_i}$$  \hspace{1cm} (27)

$$P_{Dir2} = \frac{P_{Di}'}{P_D} = \frac{\Delta p_i'}{\Delta p}$$  \hspace{1cm} (28)

$$P_{Dir3} = \frac{P_{Di}''}{P_D} = \frac{\Delta p_i''}{\Delta p}$$  \hspace{1cm} (29)

while the second pressure integral ratio functions are defined as

$$P_{Dir1} = \frac{P_{Dii}}{2P_{Dii}} = \frac{\Delta p_{ii}}{2\Delta p_{ii}}$$  \hspace{1cm} (30)

$$P_{Dir2} = \frac{P_{Dii'}}{P_{Di}} = \frac{\Delta p_{ii}'}{\Delta p_i}$$  \hspace{1cm} (31)

$$P_{Dir3} = \frac{P_{Dii''}}{P_{Di}} = \frac{\Delta p_{ii}''}{\Delta p_i}$$  \hspace{1cm} (32)

DEVELOPMENT OF NEW TYPE CURVES

We have developed a comprehensive set of type curves for the application of the derivative and integral functions defined earlier. We considered the homogeneous reservoir case with wellbore storage and skin effects, the case of a well with a finite conductivity vertical fracture in an infinite-acting reservoir, and the case of a well in a naturally fractured (dual porosity) reservoir.

Infinite-acting Homogeneous Reservoir

The infinite-acting homogeneous reservoir solution with wellbore storage and skin effects has been thoroughly developed in terms of dimensionless pressure solutions. The classic work by van Everdingen and Hurst17 introduced this problem to the petroleum literature in 1949. Agarwal, et al.18 later provided tabulated results, type curves, and a methodology for the application of these solutions, using type curves, to the analysis of field data.

Gringarten, et al.19 recognized that the $C_D P_{Di}$ parameter could be used to correlate dimensionless pressure solutions and developed the $P_D$ vs. $t_i/C_D$ format for plotting type curve solutions. However, even using this method for the case of wellbore storage and skin, some difficulty remained in identifying the correct type curve when matching data.

Later, seeking a solution to this problem, Bourdet, et al.1 introduced the pressure derivative type curve. This curve could be applied by itself to field data with the confidence that the derivative of pressure would characterize the formation more accurately than the pressure itself. However, the major utility of the pressure derivative has become matching type curve functions simultaneously using both the pressure and pressure derivative data.

In an attempt to characterize the short-term buildup test, Cinco and Sammeniego20 recently introduced the second derivative of pressure type curve for the case of a well with wellbore storage and skin effects. This type curve shows features similar to the first derivative type curve, but with certain features, such as the region of wellbore storage domination being much more pronounced. This behavior suggests that the second derivative function may be useful for distinguishing subtle changes in data, thereby providing more detail about the formation. Unfortunately, the practical aspects of computing second derivatives of field data has not been considered as a systematic research effort in the petroleum literature. And in fact, the use of this type curve for the analysis of the second derivative of pressure data was not the objective of the Cinco and Sammeniego work.

Fig. 1 illustrates the conventional type curve plotting functions of $P_D$ and $P_D'$. For comparison, we have included the $P_D''$ on this figure. Note that during wellbore storage domination the $P_D''$ curves do not yield a "unit slope" line, but actually a slope of
about 2 is observed. The application of Fig. 1 to analyze well test data is often hampered by errors or noise in the pressure data.

Often, data noise can yield significant qualitative information about data quality, completion problems, or data acquisition problems. However, the effect of data noise on the pressure derivative is rendered completely useless by data noise. This problem of data noise motivated us to develop the original pressure derivative integral in Reference 14. We sought to develop functions which would not be significantly affected by random data noise. This effort led us to Fig. 2.

Fig. 2 is a plot of the pressure derivative function, \( p'_d \), the first pressure integral difference, \( p_{di}' \), and the second pressure integral, first difference function, \( p_{dii}' \). Note the curves appear to be staggered in time. In fact, during wellbore storage the curves are staggered by a factor of 1/2 for the \( p_{di}' \) cases and 1/4 for the \( p_{dii}' \) cases. This is due to the definition of dimensionless pressure during wellbore storage. It is also evident that all of these functions asymptotically approach a value of 1/2 at large times. This behavior is typical of the radial flow regime and in the case of the \( p_d' \) function, when \( p_d' \) approaches 1/2 the formation is experiencing undistorted radial flow.

Some analysts are concerned that while the integral and integral difference functions smooth out the test data, these functions also delay or obscure the observance of characterizing features. For example, if we consider the observation of the 1/2 value asymptotic behavior as being the end of wellbore storage effects, then the \( p_{di}' \) functions experience this behavior about one log cycle later in time than the corresponding \( p_d' \) functions. And about two log cycles later or more when the \( p_{dii}' \) functions are compared to the \( p_d' \) functions.

Although this concern is valid, it is important to note that the \( p_{di}' \) and \( p_{di2}' \) curves exhibit unique characteristics that relate to the properties of the formation. We can use these characteristics for type curve matching of field data with these curves. Also, if we were to match all three data curves simultaneously with Fig. 2, we would find that the \( p_d', p_d', \) and \( p_{di}' \) functions will align themselves to a unique set of type curves. This concept will be demonstrated later by field examples.

The use of pressure ratio functions such as the function proposed by Onur and Reynolds9 reduces the y-axis function to a true dimensionless ratio. By defining ratios of functions of pressure drop divided by functions of pressure drop, we eliminate the formation properties in the dimensionless variables and are left with dimensionless ratios. The utility of these ratios can best be understood by considering the following. For a given correlating parameter, these dimensionless ratios are exactly the same, whether computed from field data or dimensionless solutions, meaning that these functions can only vary in time. This implies that once the field data are plotted on the same scale as the type curve, there is only one degree of freedom; movement along the time axis.

Once the field data ratios are aligned with the type curve, we have uniquely determined the time axis match point. When we know the time axis match point we can use this to force match the pressure functions on other type curves by allowing movement of the data curve only in the vertical dimension. The process is simplified even further by the fact that the correlating parameter (e.g., \( C_p, \rho, \ldots \)) is determined simultaneously with the time axis match point from the ratio function type curves. Knowing the time axis match point and the correlating parameter, it should be relatively simple to determine the pressure match point using the pressure derivative and integral difference function type curves.

Fig. 3 shows the pressure ratio functions \( p_{di2}, p_{dir2}, \) and \( p_{diir2} \). These particular functions were emphasized in the present work because of the following reasons. \( p_{di2} \) is the pressure derivative ratio proposed by Onur and Reynolds9 and this function has gained some familiarity and acceptance. \( p_{dir2} \) was chosen because it has several unique properties. \( p_{dir2} \) has a value of 1/2 during wellbore storage domination, a value of 1 during radial flow, and the maximum value of \( p_{di2} \) for a given value of \( C_p, \rho, \ldots \) roughly corresponds to the time for the start of undistorted radial flow (i.e., the start of the straight line on a semi-log graph).

Since \( p_{dir2} \) is dependent on the accuracy of the pressure derivative function, \( p_d' \), we felt that a variable defined in terms of integral functions, but of the form of \( p_{dir2} \), would be consistent and accurate even in the presence of significant data noise. As shown in Fig. 3, \( p_{diir2} \) is significantly less sensitive than either \( p_{dir2} \) and \( p_{dir2} \). However, \( p_{diir2} \) should always yield a smoother curve than either of the other functions. The use of Figs. 2 and 3 will be illustrated in detail for Examples 1 and 2.

Wells in Vertically Fractured Reservoirs

This section considers the case of a well with a vertical fracture of finite conductivity in an infinite-acting homogeneous reservoir. This problem was recently solved in Laplace space by discretizing the fracture and simultaneously computing the flowrate down the fracture and the pressure at the wellbore. The solution is given in References 21 and 22. This problem and real space solution were introduced by Cinco and Sameniiego23 in 1978.

Both the real space and Laplace space solutions are extremely complicated, but the advantage of the Laplace space solution is its application to problems which are easily worked in Laplace space. These problems include computation of the constant pressure solution, computation of variable-rate solutions, and the computation of pressure responses for dual porosity reservoirs with finite conductivity vertical fractures.

In this section we will consider two cases. First we will consider cases of different fracture conductivities and second we will consider the case of a single fracture conductivity coupled with differing degrees of wellbore storage distortion. We would like to be able to analyze well test data considering both wellbore storage and fracture conductivity, but this will require multiple type curves of different dimensionless fracture conductivities where various cases of wellbore storage are plotted. This is the reason that we present a single dimensionless conductivity case plotted with multiple cases of wellbore storage.

- Cases with no wellbore storage effects

Fig. 4 shows the conventional type curve plotting functions \( p_d \) and \( p_d' \). In addition, we have plotted the second derivative function, \( p_d'' \). Note that \( p_d \) and \( p_d'' \) are very similar and appear to be staggered in time. From this plot, we can suggest that the \( p_d'' \) function will not be of much more use than the \( p_d' \) for fractured wells which are not affected by wellbore storage, although we would certainly use \( p_d'' \) functions if they were available.

In practice, it is usually difficult to apply Fig. 4 for only the \( p_d' \) curve because of uniqueness problems, which arise especially for the low conductivity cases. However, matching multiple curves should improve this problem significantly.

In Fig. 5 we have plotted the \( p_{dir2}, p_{dir2}, \) and \( p_{diir2} \) responses. The most striking feature of this plot is that all of the plotting functions look the same, only staggered in time. At first glance one might discount this behavior and suggest that this plot
is not useful. However, we can use this similarity to our advantage, particularly because we intend to match multiple curves simultaneously. Since all of the curves are similar, we should be able to get good multiple matches, although the low conductivity curves will probably still be a problem due to their similar shapes. One possible solution would be to consider ratio functions and look for behavior which characterizes a particular time period or flow regime.

Fig. 6 illustrates the $p_{DR2}$, $p_{Dir2}$, and $p_{Dii2}$ ratio functions plotted on log-log coordinates. Due to its magnitude, $p_{DR2}$ dominates this plot and multiple curve matching may not always be possible or practical. However, we do see some characteristic behavior. In particular, some solutions transfer from the bilinear flow regime to the linear flow regime (for high conductivity cases, $F_{CD} > 10a$). This behavior is discussed in detail in Reference 14.

Because of the magnitude of $p_{Dii2}$ and $p_{Dii2}$, we plotted these functions in terms of semilog dimensionless time in Fig. 7. Note that we see a similar characteristic behavior of transfer from bilinear to linear flow for the high fracture conductivity cases. This can be very beneficial to our goal of using the ratio functions to identify the correlating parameter $F_{CD}$, and the time axis match point.

To summarize what we have observed so far we can say that finite conductivity vertical fracture solutions for the derivative and integral difference functions may not exhibit distinguishing features, but matching multiple functions may remedy this problem. Also, we have seen that certain ratio functions exhibit unique characteristics that could be used to match field data, and in particular, to determine the correlating parameter $F_{CD}$ and the time axis match point. The use of Figs. 6 and 7 will be illustrated in detail for Example 3.

- Wellbore storage cases ($F_{CD} = 10a$)

Fig. 8 shows the behavior of the $p_{D}$, $p_{D'}$, and $p_{D''}$ functions for a dimensionless fracture conductivity, $F_{CD}$, value of $10a$. We chose this value because it represents a typical value that might be encountered in field analysis. We see that for cases of a very small wellbore storage coefficient, the curves agree with those on Fig. 4 for this conductivity. For larger values of the wellbore storage coefficient, the curves look similar to the curves we saw on Fig. 1 for the homogeneous reservoir case with wellbore storage and skin. Fig. 8 will likely yield good matches if multiple curves are matched simultaneously.

We present the $p_{D'}$, $p_{D''}$, and $p_{Dii2}$ curves for this case in Fig. 9. These curves illustrate behavior similar to the curves shown on Fig. 2 for low values of $C_{D2}$. Although, there are characteristic "humps" for the cases of high values of the wellbore storage coefficient, most of the curves on Fig. 9 would yield uniqueness problems in matching field data. We overcame this difficulty in previous cases by looking at the ratio functions. We shall do the same here.

Fig. 10 shows a log-log plot of the $p_{DR2}$, $p_{Dir2}$, and $p_{Dii2}$ functions. We note immediately that $p_{DR2}$ exhibits characteristic trends which could be used to analyze data. As before with the $p_{Dir2}$ and $p_{Dii2}$ functions for the $F_{CD} = 10a$ case, we find that the range and magnitude of the $p_{Dii2}$ and $p_{Dii2}$ functions is too small to use log-log type curve matching. So again, we will use a semi-log plot to observe the behavior of $p_{Dir2}$ and $p_{Dii2}$ functions.

Fig. 11 shows significant behavior; both sets of curves attempt to rise from 1/2, appear to converge at 2/3 and then approach 1. The high storage cases behave similar to those seen on Fig. 3 where the data rise to some maximum and then approach 1, whereas the low storage cases approach a value of 1 asymptotically from below. The encouraging features of these curves, and others like them, is that the characteristic behaviors representing change from one flow regime to another should yield unique matches with field data. These curves could conceivably be quite useful, but correlation to other values of fracture conductivity will require the use of multiple type curves, which will likely compound the complexity of this type of analysis.

Wells in Naturally Fractured Reservoirs

The solutions for fluid flow in naturally fractured reservoirs have considered two cases of the reservoir matrix behavior. The reservoir matrix can either be assumed to be producing at pseudosteady-state or transient flow conditions. These models may overrealize the physical problem, but these solutions have found wide use in the industry.

The two major problems encountered in the analysis of naturally fractured (dual porosity) reservoir systems is the determination of the appropriate reservoir model and then the interpretation of test data using that model. It would be a significant achievement to develop analysis methods which could be used to systematically determine the reservoir model and the test data.

Bourdert, et al.6 developed a systematic methodology to determine the reservoir model and interpret well test data based on the pressure derivative function, $p_{D'}$. The approach can yield accurate results, but, more often than not, data noise will render this analysis ambiguous at best, uninterpretable at worst. We have seen the smoothing tendency of the integral functions on solutions for unfractured and fractured wells in infinite-acting reservoirs earlier in this work. So it is a natural extension of this work to apply pressure derivative and integral functions to naturally fractured reservoir cases.

The first case that we considered was for the pseudosteady-state matrix flow case as shown in Fig. 12. We used the correlating parameter, $\lambda C_{D2}(1-\omega)$, given by Bourdert, et al. 5 In Fig. 12 we show the $p_{D'}$, $p_{D''}$, $p_{Dii2}$, and $p_{Dii2}$ functions. It is obvious that $p_{D''}$ is the most sensitive function and the functions become less sensitive as the order of the function goes down. However, the $p_{D'}$ and $p_{Dii2}$ functions still mimic the behavior, albeit lower in magnitude and lagged in time, of the $p_{D'}$ and $p_{D''}$ functions. This implies that we could use these functions and develop analysis methods for the pseudosteady-state matrix flow case, although this effort is beyond the scope of the present work.

For the next case we assumed transient flow in the reservoir matrix. Fig. 13 illustrates this example case. Again, the $p_{D'}$, $p_{D''}$, $p_{Dii2}$, and $p_{Dii2}$ functions are used and now the correlating parameter, $\lambda C_{D2}(1-\omega)$, as given by Bourdert, et al. 6 We again find that, as with the pseudosteady-state matrix case, the $p_{D''}$ function is the most sensitive and the other functions are successively less sensitive. We find that, at least for these cases, that only the $p_{D'}$ function exhibits two horizontal lines on the type curve, indicating transition and total system response. $p_{D''}$ dips below the lower limit of the $p_{D'}$ function and $p_{D'}$ and $p_{Dii2}$ never quite reach the lower limit of the $p_{D'}$ function.

The most significant observation that can be made about Figs. 12 and 13 is that the $p_{D'}$ and $p_{Dii2}$ functions both mimic the response of the $p_{D'}$ function. This suggests that $p_{D'}$ and $p_{Dii2}$ can be used to develop a methodology for model identification and data interpretation.

Another improvement would be to use the ratio functions defined earlier for the analysis and interpretation of test data. Fig. 14 shows the response of the pseudosteady-state matrix flow case on the $p_{Dii1}$, $p_{Drr2}$, $p_{Drr3}$, and $p_{Drr2}$ ratio functions. The most dramatic behavior is given by the $p_{Drr1}$ functions and the least
dramatic behavior is given by the $p_{Dir2}$ functions. Fig. 15 illustrates the effect of transient matrix flow on these functions. In this case, there are several very distinct patterns of behavior for each function. It is almost certain that these functions can be used in a systematic manner to determine the reservoir model and interpret test data.

The most important observation that can be made about Figs. 14 and 15 is that the ratio functions do yield unique behavior that can be recognized and correlated using the Bourdet, et al., correlating parameters. This implies that an analysis scheme similar to that developed by Bourdet, et al. can be developed for the integral difference functions.

The Laplace space solutions used to develop the type curves were inverted numerically using the Stehfest algorithm.

APPLICATION OF THE NEW TYPE CURVES

In this section, we outline a procedure for applying these new type curves, including recommendations on how to obtain the pressure drop derivative and integral functions. These procedures are adapted from Ref. 14 and extended to include new functions.

1. Calculate the pressure drop function, $\Delta p$, and the time function, $t$, as follows:
   a. drawdown tests:
      i. $\Delta p = p_i - p_{wf}$
      ii. $t = t$
   b. buildup tests:
      i. $\Delta p = p_{wf} - p_{wf}$
      ii. $t = t = \Delta t = \Delta t/p^2$

2. Calculate the pressure drop derivative function, $\Delta p'$. Many numerical techniques, such as those given in References. 3 and 8, could be used to obtain $\Delta p'$. Piecewise least squares curve fitting of polynomials is also a useful method to compute $\Delta p'$. A good discussion of least squares fitting of polynomials is given by Hornbeck. In general, finite difference formula may not yield accurate derivatives, although the weighted difference algorithm usually gives good results. $\Delta p'$ is defined as

$$\Delta p' = t \frac{d(\Delta p)}{dt}$$

3. Calculate the pressure drop integral function, $\Delta p_1$, using numerical integration techniques such as the trapezoidal rule or the new logarithmic integration technique developed in Appendix C of Ref. 14. The formula for $\Delta p_1$ is

$$\Delta p_1 = \int_0^t \Delta p \ dt$$

4. Calculate the first derivative of the pressure integral function, $\Delta p_1'$, using one of the derivative techniques described in Step 2. The formula for $\Delta p_1'$ is

$$\Delta p_1' = t \frac{d(\Delta p_1)}{dt}$$

5. Calculate the pressure integral ratio functions $p_{Dir1}$ and $p_{Dir2}$ given by Eqs. 25 and 26, $p_{Dir1}$ and $p_{Dir2}$ given by Eqs. 27-29, and $p_{Dir3}$ given by Eqs. 30-32. In general, we have found that the ratio functions $p_{Dir2}$, $p_{Dir2}$, and $p_{Dir2}$ are the most useful forms.

6. Make the following log-log plots using the time function, $t$, as defined in Step 1. Again, we have found the ratio functions $p_{Dir2}$, $p_{Dir2}$, and $p_{Dir2}$ to generally be the most useful.

Pressure Derivative/Integral Difference Matching:

$$\Delta p', \Delta p_1', \text{ and } \Delta p_1'' \text{ vs. } t$$

Pressure Ratio Function Matching:

$$p_{Dir2}, p_{Dir2}, \text{ and, } p_{Dir2} \text{ vs. } t$$

7. From the pressure ratio function plots, obtain a time match point and the type curve correlating parameter (e.g. $C_{pe}$ for wellbore storage and skin solutions or $C_{FD}$ and $F_C$ for vertically fractured solutions).

8. Using the time match point and the type curve correlating parameter from Step 7, obtain the pressure match point from the type curve plots of the pressure derivative/integral difference functions. We may then calculate several well and reservoir parameters as follows:

a. Pressure match point

$$k = \frac{141.2 \frac{qB\mu}{h}}{\Delta p_{M.P.}}$$

where, at the match point,

$$p_D = p_D' = p_D'' = p_{Di}$$

$$\Delta p = \Delta p' = \Delta p_1' = \Delta p_1''$$

b. Time match point

i. Wellbore storage and skin case

$$C_D = \frac{0.0002637 k}{\phi \mu c_f r_p^2} \left( \frac{t}{10 D_C M.P.} \right)$$

ii. Fractured well case

$$L_p^2 = \frac{0.0002637 k}{\phi \mu c_i} \left( \frac{t}{10 D_M M.P.} \right)$$

9. From the type curve correlating parameter (e.g. $C_{pe}$, $C_{FD}$ and $F_C$), we may obtain additional reservoir information. In particular, the skin factor, $S$, for the homogeneous reservoir case is calculated from the $C_{pe}$ parameter, while the fracture conductivity is estimated from the $C_{FD}$ parameter.
a. Wellbore storage and skin case

\[ S = \frac{1}{2} \ln \left( \frac{C_{De}^{2S}}{C_D} \right) \]

b. Fractured well case

\[ k_p = k L_p \frac{F}{CD} \]

FIELD EXAMPLES

Field Example 1. The first example\(^1\) is a pressure buildup test from a well in an infinite-acting, homogeneous reservoir. The data are distorted by wellbore storage but also exhibit a semilog straight line which is indicative of radial flow after about 4 hours. The reservoir data and flow history are given in Table 1.

**TABLE 1**

*Flow History*

<table>
<thead>
<tr>
<th>(t_p, ) hr</th>
<th>15.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q_i, ) STB/D</td>
<td>174</td>
</tr>
</tbody>
</table>

*Well and Reservoir Parameters*

<table>
<thead>
<tr>
<th>(B, ) RB/STB</th>
<th>1.06</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_r, ) psin(^{-1})</td>
<td>4.2 \times 10^{-6}</td>
</tr>
<tr>
<td>(n, ) ft</td>
<td>107</td>
</tr>
<tr>
<td>(\phi, )</td>
<td>0.25</td>
</tr>
<tr>
<td>(\mu, ) cp</td>
<td>2.5</td>
</tr>
<tr>
<td>(r_w, ) ft</td>
<td>0.29</td>
</tr>
</tbody>
</table>

First, we prepare a log-log plot of \(\Delta p / 2 \Delta p', \Delta p'' / \Delta p'\), and \(\Delta p''' / \Delta p''\) vs. \(\Delta t_e\) and match these data with a log-log plot of \(p_D, p_D', p_D''\), and \(p_D''', \) and \(p_D''\) vs. \(t_D/C_D\). Because of the manner in which these variables are defined, we simply align equal values of the pressure ratio functions on the vertical axis and slide the graphs horizontally until a match is obtained, as shown in Fig. 16. Observe that the data have been matched on the curves for \(C_{De}^{2S}\) between \(10^8\) and \(10^{10}\), or approximately \(1 \times 10^9\). Most of the early wellbore storage dominated data are reasonably close to the characteristic value of 0.5, while the later data bend and approach radial flow at about \(t_D/C_D = 75\). From this preliminary analysis, we obtain the following:

**Time Match Point:**

\[ t_D/C_D = 190, \text{ and } \Delta t_e = 10 \text{ hr} \]

**Type Curve Correlating Parameter:**

\[ C_{De}^{2S} = 1 \times 10^9 \] (by interpolation)

Next, we plot log \(\Delta p', \Delta p'', \) and \(\Delta p'''\) vs. log \(\Delta t_e\) and match these data with a log-log plot of \(p_D', p_D'', \) and \(p_D'''\) vs. \(t_D/C_D\). Since we have estimated the time match point from the preliminary analysis using pressure ratio functions, we may now fix this match point and then slide the graphs vertically and simultaneously match all three curves (see Fig. 17). Except for some early and late data points, we were able to obtain an excellent match of most data. After a slight adjustment of the time match point, the final type curve matching results are:

**Pressure Match Point:**

\[ p_D = p_D' = p_D'' = 0.18 \]

\[ \Delta p' = \Delta p'' = \Delta p''' = 10 \text{ psi} \]

**Time Match Point:**

\[ t_D/C_D = 160 \]

\[ \Delta t_e = 10 \text{ hr} \]

**Type Curve Correlating Parameter:**

\[ C_{De}^{2S} = 5 \times 10^9 \] (by interpolation)

The permeability is determined from the pressure match point as follows:

\[ k = \frac{141.2 \ qB\mu \left( \frac{p_D}{\Delta p_{M.P.}} \right) h}{107} \]

\[ k = 10.95 \text{ md} \]

or, \(kh = 1.172 \text{ md-ft}\)

From the time match point, we may solve for the dimensionless wellbore storage coefficient:

\[ C_D = \frac{0.0002637 \ k}{\phi \mu c_r w^2} \left( \frac{t_D}{C_D}_{M.P.} \right) \]

\[ C_D = \frac{(0.0002637)(10.95)}{(0.25)(2.5)(4.2 \times 10^{-6})(0.29)^2} \]

\[ C_D = 817 \]

Using the calculated value of \(C_D\) and the correlating parameter \(C_{De}^{2S}\) taken from the type curves, we may calculate the skin factor as

\[ S = \frac{1}{2} \ln \left( \frac{C_{De}^{2S}}{C_D} \right) \]

\[ S = \frac{1}{2} \ln \left( \frac{5 \times 10^9}{817} \right) \]

\[ S = 7.81 \]

A comparison of the results (Table 2) shows that this analysis compares very well with Bourdet, et al's\(^1\) analysis.
Field Example 2. The second example\(^1\) is a pressure buildup test from an acidized well in a homogeneous reservoir. Although not shown, a log-log plot of the pressure-time data indicates that a unit-slope line, which is characteristic of wellbore storage dominated data, is not present. The flow history and reservoir parameters are summarized in Table 3.

### TABLE 3

**Flow History**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_p ), hr</td>
<td>18.04</td>
</tr>
<tr>
<td>( q ), STB/D</td>
<td>1,500</td>
</tr>
<tr>
<td>( P_{fw} ), psia</td>
<td>2,235.28</td>
</tr>
</tbody>
</table>

**Well and Reservoir Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_r ), RB/STB</td>
<td>1.30</td>
</tr>
<tr>
<td>( c_r ), psin(^{-1})</td>
<td>10.0 x 10(^{-6})</td>
</tr>
<tr>
<td>( h ), ft</td>
<td>73</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.20</td>
</tr>
<tr>
<td>( \mu ), cp</td>
<td>0.50</td>
</tr>
<tr>
<td>( r_w ), ft</td>
<td>0.401</td>
</tr>
</tbody>
</table>

Like the first field example, we prepare a log-log plot of \( \Delta p/2 \Delta t' \), \( \Delta p_i/\Delta t' \), and \( \Delta p_{ii}/\Delta t' \) vs. \( \Delta t_e \) and match these data with a log-log plot of \( p_d/t_D \), \( p_{di}/t_D \), and \( p_{dii}/t_D \) vs. \( t_D/C_D \) (Fig. 18). An excellent match of all data is obtained with the curves for \( C_{de}^{2S} = 3 \). Note that the \( p_{di}/t_D \) and \( p_{dii}/t_D \) ratio functions are approaching a value of one which suggests that the radial flow period has almost been reached. From this preliminary analysis, we obtain the following:

**Time Match Point:**

\( t_D/C_D = 125 \), and \( \Delta t_e = 10 \) hr

**Type Curve Correlating Parameter:**

\( C_{de}^{2S} = 3 \)

Next, we plot log \( \Delta p' \), \( \Delta p'_i \), and \( \Delta p'_{ii} \) vs. log \( \Delta t_e \) and simultaneously match these data with a log-log plot of \( p_{d}'/t'_D \), \( p_{di}'/t'_D \), and \( p_{dii}'/t'_D \) vs. \( t'_D/C_D \) by fixing the time match point from the preliminary pressure ratio function analysis (see Fig. 19). Except for data at the end of the test, we were able to obtain an excellent match of the data. After a slight adjustment of the time match point, the final type curve matching results are:

**Pressure Match Point:**

\( p_{d}' = p_{di}' = p_{dii}' = 0.12 \)

\( \Delta p' = \Delta p'_i = \Delta p'_{ii} = 10 \) psi

**Time Match Point:**

\( t_D/C_D = 110 \)

\( \Delta t_e = 10 \) hr

**Type Curve Correlating Parameter:**

\( C_{de}^{2S} = 3 \)

From the pressure match point, we may calculate the permeability as

\[
k = \frac{141.2}{h} \frac{\mu B_r}{\frac{\Delta p}{M.P.}}
\]

\[
k = \frac{(141.2)(1,500)(1.3)(0.5)}{73} \left( \frac{0.12}{10} \right)
\]

\[k = 22.6 \text{ md}\]

or, \( kh = 1,652 \text{ md-ft}\)

From the time match, we may solve for the dimensionless wellbore storage coefficient,

\[
C_D = \frac{0.0002637}{\phi \mu c_r r_w^2} \left( \frac{t_D}{C_D M.P.} \right)
\]

\[
C_D = \frac{(0.0002637)(22.6)}{(0.20)(0.50)(10 \times 10^{-6})(0.4019)^2} \left( \frac{10}{110} \right)
\]

\[C_D = 3,354\]

Using the calculated value of \( C_D \) and the parameter \( C_{de}^{2S} \) taken from the type curves, we may calculate the skin factor as

\[
S = \frac{1}{2} \ln \left( \frac{C_{de}^{2S}}{C_D} \right)
\]

\[
S = \frac{1}{2} \ln \left( \frac{3}{3,354} \right)
\]

\[S = -3.5\]

A comparison of the results given in Table 4 shows that this analysis compare very well with that obtained by Bourdet, et al\(^1\).
TABLE 4

<table>
<thead>
<tr>
<th>Results From Bourdet, et al1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{Dp}e^{2S}$</td>
</tr>
<tr>
<td>$k$, md</td>
</tr>
<tr>
<td>$S$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results From This Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{Dp}e^{2S}$</td>
</tr>
<tr>
<td>$k$, md</td>
</tr>
<tr>
<td>$S$</td>
</tr>
</tbody>
</table>

Field Example 3. The third example23 is a constant-rate drawdown test from a well intersected by a finite conductivity vertical fracture. The well and reservoir parameters and the flow history are given in Table 5.

TABLE 5

<table>
<thead>
<tr>
<th>Flow History</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_p$, hr</td>
</tr>
<tr>
<td>$q$, STB/D</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Well and Reservoir Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$, RB/STB</td>
</tr>
<tr>
<td>$c_o$, psia^{-1}</td>
</tr>
<tr>
<td>$h$, ft</td>
</tr>
<tr>
<td>$\phi$</td>
</tr>
<tr>
<td>$\mu$, cp</td>
</tr>
<tr>
<td>$r_w$, ft</td>
</tr>
</tbody>
</table>

For this field example, we prepared a semilog plot of $\Delta p_i/\Delta p'$ and $\Delta p_{ii}/\Delta p'$ vs. $t$ and matched these data with a semilog plot of $p_{Di}^{Dir}$ and $p_{Di}^{Dir2}$ vs. $t_{Di}$. These semilog plots help enhance the data. In Fig. 21, we also plotted $p_{Dr}$ vs. $t_{Di}$, Note that the $p_{Di}$ and $p_{Di2}$ ratio functions do not reach a value of one, which suggests the radial flow period has not been reached at the end of the test. From this preliminary analysis, we obtain the following:

Time Match Point:

$ t_{Di} = 0.35$, and $t = 10$ hr

Type Curve Correlating Parameter:

$ F_{CD} = 10\pi$

Next, we plotted log $\Delta p'$, $\Delta p_i'$, and $\Delta p_{ii}'$ vs. log $t$ and simultaneously matched these data with a log-log plot of $p_{Di}'$, $p_{Di}^{Dir'}$, and $p_{Di}^{Dir2'}$ vs. $t_{Di}'$ by fixing the time match point from the preliminary pressure ratio function analysis (see Fig. 22). We were able to obtain an excellent match of the data over the entire test period. After a slight adjustment of the time match point, the final type curve matching results are:

Pressure Match Point:

$ p_{Di}' = p_{Di}^{Dir'} = p_{Di}^{Dir2'} = 0.0395$

$\Delta p' = \Delta p_i' = \Delta p_{ii}' = 10$ psi

Time Match Point:

$ t_{Di}' = 0.30$

$ t = 10$ hr

Type Curve Correlating Parameter:

$ F_{CD} = 10\pi$

The formation permeability is determined from the pressure match point as follows:

$$ k = \frac{141.2 qB\mu}{h} \left( \frac{\Delta p}{\Delta p_{M,P}} \right) $$

$$ k = \frac{(141.2)(195)(1.4)(1.8)}{55} \left( \frac{0.0395}{10} \right) $$

$$ k = 4.98$ md

From the time match, we may solve for the fracture half-length,

$$ L_f^2 = \frac{0.0002637 k}{\phi \mu \tau} \left( \frac{t_{Di}'}{t_{Di}} \right)_{M,P} $$

$$ L_f^2 = \frac{(0.0002637)(4.98)}{(0.18)(1.8)(18 \times 10^{-6})} \left( \frac{10}{0.30} \right) $$

$$ L_f = 86.8$ ft

Using the calculated values of $k$ and $L_f$ and the correlating parameter $F_{CD}$ taken from the type curves, we may calculate the fracture conductivity,

$$ k_{fb} = k L_f F_{CD} $$

$$ k_{fb} = (4.98)(86.8)(10\pi) $$

$$ k_{fb} = 13,580$ md-ft

A comparison of the results given in Table 6 shows that this analysis compare very well with that obtained by Cinco-Ley, et al.23

TABLE 6

<table>
<thead>
<tr>
<th>Results From Cinco-Ley23</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$, md</td>
</tr>
<tr>
<td>$L_f$, ft</td>
</tr>
<tr>
<td>$k_{fb}$, md-ft</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results From This Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$, md</td>
</tr>
<tr>
<td>$L_f$, ft</td>
</tr>
<tr>
<td>$k_{fb}$, md-ft</td>
</tr>
</tbody>
</table>
SUMMARY AND CONCLUSIONS

The major result of this paper is the presentation of systematic procedures to analyze each of the reservoir cases using type curves for that system. These procedures and the new integral type curve families bring a greater degree of consistency to the analysis of pressure test data in that data are verified using additional type curves based on pressure function ratios and type curve analyses which incorporate matching several functions simultaneously. The matching of multiple functions simultaneously also tends to verify the use of a particular reservoir model and reduces the ambiguity of type curve matching since the matching functions must all agree on the same match point and type curve correlating parameter. This represents a significant improvement over matching simply with the pressure and pressure derivative functions.

Although the integral functions do smooth the data and cause the behavior indicative of a particular flow regime to be postponed as much as log cycle (factor of 10), these new functions essentially yield the same or better analysis as the conventional pressure and pressure derivative analysis. The new functions and the smoothing algorithm given in the appendix offer a way around the problems associated with the pressure derivative function evaluated from noisy field data.

Applications:

The types curves and the detailed procedures presented in this work yield a new level of consistency for the analysis of well test data. In particular, matching multiple type curves simultaneously removes the ambiguity that might exist in a highly sensitive function like the pressure derivative. These type curves and analysis methods should be applied to all pressure test data to insure a complete and consistent analysis.

Technical Contributions:

1. A detailed procedure is given for the implementation of pressure integral type curve analysis for homogeneous reservoirs and vertically fractured reservoirs. In addition, methods to analyze naturally fractured reservoirs are suggested.
2. The application of pressure integral type curve analysis for homogeneous reservoirs and vertically fractured reservoirs is demonstrated for each case by use of an illustrative example.

Conclusions:

1. The pressure integral smoothing algorithm can significantly improve data quality prior to analysis. This algorithm systematically integrates and differentiates the test data in order to reconstruct the undistorted pressure behavior. This work suggests that the smoothing procedure may be statistically consistent and could be used as a general smoothing algorithm.
2. The new derivative type curves which utilize the $pD'$, $pDi'$, and $pDii'$ functions should, in general, yield more accurate results than conventional pressure/pressure derivative type curve matching. This improvement is due to the smoothing nature of the $pD'$ and $pDii'$ functions and the matching procedure where all three functions are matched simultaneously to field data.
3. The new ratio type curves reduce the ambiguity in type curve matching by fixing the time match point.
4. The new ratio type curves which utilize the $pDr3$, $pDi2$, and $pDii2$ functions should provide unique characterizations of the formation, such as changes in flow regimes and the presence of heterogeneities. These functions are particularly useful for the analysis of wellbore storage distorted test data and analysis of data from vertically fractured wells.

5. We presented the $pD''$ type curves in the hope that this function would provide a unique characterization of the reservoir. Although this is clearly the most sensitive function, it is unlikely that we will be able to obtain accurate estimates of the second derivative of field pressure data. We could use curve fitting techniques such as splines, but these techniques may significantly bias the data. Application of this function to well testing problems will require further research.

NOMENCLATURE

**Dimensionless Variables**

- $C_D$ = dimensionless wellbore storage coefficient
- $C_Dp$ = dimensionless wellbore storage coefficient based on fracture half-length
- $F_Dr$ = dimensionless fracture conductivity (Eq. 1)
- $pD_i$ = dimensionless pressure (Eqs. 6, 7)
- $pDi'$ = dimensionless pressure derivative function (Eq. 8)
- $pDi''$ = dimensionless pressure derivative function (Eq. 9)
- $pDi''$ = dimensionless pressure integral function (Eqs. 10, 11)
- $pDi$ = dimensionless pressure integral function (Eq. 13)
- $pDi''$ = dimensionless pressure integral function (Eqs. 14, 16)
- $pDi''$ = dimensionless pressure integral function (Eqs. 15, 17)
- $pD1r$ = dimensionless pressure derivative ratio function (Eq. 25)
- $pD2r$ = dimensionless pressure derivative ratio function (Eq. 26)
- $pDi1r$ = dimensionless pressure integral ratio function (Eq. 27)
- $pDi2r$ = dimensionless pressure integral ratio function (Eq. 28)
- $pDi3r$ = dimensionless pressure integral ratio function (Eq. 29)
- $pDi4r$ = dimensionless pressure integral ratio function (Eq. 30)
- $pDi5r$ = dimensionless pressure integral ratio function (Eq. 31)
- $pDi6r$ = dimensionless pressure integral ratio function (Eq. 32)
- $\phi_D$ = dimensionless time based on wellbore radius (Eq. 4)
- $\phi_D$ = dimensionless time based on fracture half-length (Eq. 5)

**Field Variables (Pressure Functions)**

- $\phi_i$ = initial reservoir pressure, psia
- $\phi_{wf}$ = flowing bottomhole pressure, psia
- $\phi_{ws}$ = shut-in bottomhole pressure, psia
- $\phi = $ pressure drop (dd: $\phi = \phi_{wf} - \phi_{ws}$) (Eqs. 2, 3)
- $\phi' = $ pressure drop derivative function, psia (Eq. 18)
- $\phi'' = $ pressure drop derivative function, psia (Eq. 19)
- $\phi_i = $ pressure drop integral function, psia (Eq. 20)
- $\phi_i' = $ pressure drop integral function, psia (Eq. 21)
- $\phi_i'' = $ pressure drop integral function, psia (Eq. 22)
- $\phi_i'' = $ pressure drop integral function, psia (Eq. 23)
- $\phi_i'' = $ pressure drop integral function, psia (Eq. 24)

**Field Variables (Formation and Fluid Properties)**

- $B = $ formation volume factor, RB/STB
- $b = $ fracture width, ft
- $C = $ wellbore storage coefficient, BBL/psia
- $C_t = $ total compressibility, psia$^{-1}$
- $h = $ total formation thickness, ft
- $k = $ permeability, md
- $k_f = $ fracture permeability, md
- $L_f = $ fracture half-length, ft
- $\phi_i = $ initial reservoir pressure, psia
- $\phi_{wf} = $ flowing bottomhole pressure, psia
- $\phi_{ws} = $ shut-in bottomhole pressure, psia
- $q = $ flowrate, STB/D
- $r_w = $ wellbore radius, ft
- $\phi = $ porosity, fraction
- $\lambda = $ interporosity flow coefficient (natural fracture system parameter)
- $\mu = $ viscosity, cp
- $\omega = $ storativity ratio (natural fracture system parameter)
Subscripts
MP = match point on a type curve

ACKNOWLEDGEMENTS

We gratefully acknowledge the assistance of Bobby Poe, Jr. for the use of his algorithm to compute the Cinco and Meng solution for a well with a finite conductivity vertically fractured.

REFERENCES


APPENDIX - Development of the Smoothing Algorithm

In this appendix we present an algorithm for smoothing well test data using the first and second pressure integral functions.

  c Subroutine PLTCAL
  c Computes smoothed pressure function, Ap_cal, and derivative
  c and integral functions.
  c
  c Nomenclature for algorithm:
  c
  c ndata =number of pressure data
  c t =time
  c Ap =input pressure drop data
  c Ap_cal =smoothed pressure drop data
  c Ap' =second derivative of pressure function (Eq. 19)
  c Ap' =first derivative of pressure function (Eq. 18)
  c Ap_i =first integral of pressure function (Eq. 20)
  c Ap_i' =first integral of pressure difference function (Eq. 21)
  c Ap_i'' =second pressure integral function (Eq. 22)
  c Ap_i'' =second integral of pressure difference function (Eq. 23)
  c Ap_i''' =second integral of pressure difference 2nd fcn (Eq. 24)
  c nsnth =number of smoothing cycles
  c mthsm =smoothing methods (1->Ap_i, 2->Ap_i')
  c * =control parameters for derivative and integral calc

  for loop=1, nsnth
  c computes first pressure integral
  call INTGRT(ndata,t,Ap_cal,*,Ap_i)
  c computes second pressure integral
  call INTGRT(ndata,t,Ap_i,*,Ap_i)
  c computes first pressure integral difference function
  call DERCAL(ndata,t,Ap_i,*,Ap_i',dummy)
  c computes second pressure integral difference function
  call DERCAL(ndata,t,Ap_i',*,Ap_i''')
  c computes Ap_cal using first pressure integral
  if (mthsm = 1) then
  c computes Ap_cal using second pressure integral
  elseif (mthsm = 2) then
  endif
  next loop
  c computes pressure derivative functions
  return
end

This algorithm is intentionally devoid of specific computational routines for the derivative and integral functions. We suggest that the reader use algorithms with which he is comfortable. We use the derivative algorithm given in References 3 and 7, as well as piecewise polynomial least squares functions as described by Hornbeck.

For integration we use either the trapezoidal rule or the logarithmic integration scheme introduced in Reference 14. We found that these two relatively simple methods yield accurate and consistent results. We have also used cubic splines (non-smoothing) as discussed in Reference 29, but we have found that...
Fig. 1 - Derivative Type Curve for Homogeneous Reservoir with Wellbore Storage and Skin
Fig. 2 - Integral Type Curve for Homogeneous Reservoir with Wellbore Storage and Skin Functions of Pressure Functions.
Fig. 3 - Ratio Type Curve for Homogeneous Reservoir with Wellbore Storage and Skin
Fig. 4 - Derivative Type Curve for Wells with a Finite Conductivity Vertical Fracture, No Wellbore Storage
Fig. 5 - Integral Type Curve for Wells with a Finite Conductivity Vertical Fracture, No Wellbore Storage
Fig. 7 - Semilog Ratio Type Curve for Wells with a Finite Conductivity Vertical Fracture, No Wellbore Storage
Fig. 8 - Derivative Type Curve for Wells with a Finite Conductivity
Vertical Fracture and Wellbore Storage
Fig. 10 - Ratio Type Curve for Wells with a Finite Conductivity
Vertical Fracture and Wellbore Storage
Fig. 11 - Semilog Ratio Type Curve for Wells with a Finite Conductivity
Vertical Fracture and Wellbore Storage
Fig. 12 - Derivative and Integral Type Curves for Naturally Fractured Reservoirs
(Pseudosteady-state Matrix Flow Case, $\lambda = 1 \times 10^{-2}$, $\omega = 0.01$)
Fig. 13 - Derivative and Integral Type Curves for Naturally Fractured Reservoirs
(Transient Matrix Flow Case, $\lambda = 1 \times 10^{-6}$, $\omega = 0.001$)
Fig. 14 - Ratio Type Curves for Naturally Fractured Reservoirs
(Pseudosteady-state Matrix Flow Case, $\lambda = 1 \times 10^{-2}$, $\omega = 0.01$)
Fig. 15 - Ratio Type Curves for Naturally Fractured Reservoirs
(Transient Matrix Flow Case, $\lambda = 1 \times 10^{-6}$, $\omega = 0.001$)
Fig. 16 - Homogeneous Reservoir Example 1 (from Bourdet, et al.\textsuperscript{1}) using pressure ratio functions.
Fig. 17 - Homogeneous Reservoir Example 1 (from Bourdet, et al.) using pressure derivative and integral difference functions.

Dimensionless Pressure Functions

\[ C_{Dp} = \frac{3}{2} \cdot 10^6 \]

\[ \Delta p_{fr} = \frac{10^6}{10^9} \]
Fig. 18 - Homogeneous Reservoir Example 2 of Bourdet, et al. using pressure derivative and integral difference ratio functions.
Fig. 19 - Homogeneous Reservoir Example 2 (from Bourdet, et al\textsuperscript{1}) using pressure derivative and integral difference functions.
Fig. 20 - Well with a Finite Conductivity Vertical Fracture, Example 3 (from Cinco, et al.\textsuperscript{23}), Ratio Function Analysis (Log-Log Plot)
Fig. 21 - Well with a Finite Conductivity Vertical Fracture, Example 3 (from Cinco, et al. 23), Ratio Function Analysis (Semilog Plot)
Fig. 22 - Well with a Finite Conductivity Vertical Fracture, Example 3 (from Cinco, et al.\textsuperscript{23}), Pressure Derivative and Integral Difference Analysis.