Analysis of Slug Test Data From Hydraulically Fractured Coalbed Methane Wells

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ABSTRACT

This paper presents new type curves for analyzing slug tests in hydraulically fractured coal seams. The type curves were developed using a finite-conductivity, vertical fracture model and are presented in terms of three parameters -- dimensionless wellbore storage coefficient, dimensionless fracture conductivity, and fracture-face skin. With these new curves, we may estimate the hydraulic fracture half-length, the formation permeability, and the fracture conductivity. We also present a procedure for using the new curves and illustrate the procedure with an example.

INTRODUCTION

Slug testing has been proven to be an effective method for characterizing the production potential of coal seams. A slug test involves the imposition of an instantaneous change in pressure (or fluid head) in a well and the measurement of the resulting change in pressure as a function of time. This change in pressure is created by either injecting into or withdrawing from the well a specific volume of fluid (i.e., a slug). From this measured pressure response, we may estimate the permeability and near-wellbore conditions.

Initially, slug testing methods and analysis techniques were developed for estimating the transmissivity of shallow, underpressured aquifers, but also have found applications in the petroleum industry, especially for analyzing the flow period during drill stem tests. Recently, slug testing has been extended to the evaluation of the production potential of coal seams. Since most coal seams are saturated initially with water, slug testing provides a simple but effective method for estimating flow properties early in the productive life before the initiation of gas production. Reference 7 provides an overview of conventional slug testing in coal seams.

Conventional slug test analysis techniques are based on radial flow models. However, many wells completed in coal seams require hydraulic fracturing in order to become economically viable producers. Therefore, these conventional slug test analysis techniques cannot be used to either assess the success of the fracture treatment or to evaluate the post-fracture potential of these stimulated coal seams. Karsaki, et al. studied the pressure response of slug tests in infinite-conductivity vertical fractures, but they did not investigate the behavior of finite-conductivity fractures. The purposes of this paper are to develop a model for slug testing in coal seams with finite-conductivity vertical fractures and to illustrate application of this model to the analysis of slug tests.

MATHEMATICAL MODEL

We developed our slug test model using Cinco-Ley, et al.'s model which considers a well intersected by a fully-penetrating, finite-conductivity, vertical fracture. The reservoir is assumed to be an isotropic, homogeneous, infinite medium having a uniform thickness, h, permeability, k, and porosity, \( \phi \). In addition, the reservoir contains a slightly compressible fluid of viscosity, \( \mu \), and compressibility, c, that are independent of pressure.

Cinco-Ley, et al.'s model assumes the fracture to be a homogeneous, uniform slab with height, h, width, \( b_f \), and half length, \( L_f \). Because the fracture width is much smaller than fracture length and height, the model assumes the flow in the fracture is linear and that fluid influx at the fracture tips is negligible. In addition, the model assumes that fluid production from the reservoir to the wellbore occurs only through the fracture. Further, since the fracture volume is small, the model neglects the fracture compressibility and assumes flow within the fracture is relatively incompressible. Additional details concerning the model formulation and problem solution may be obtained in Ref. 9 and Appendix A.

Under these conditions, Cinco-Ley, et al.'9 derived an expression for the dimensionless pressure drop at the wellbore (i.e., \( x_D = 0 \)) during constant rate production from a well intersected by a finite-conductivity fracture as...
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\[ \frac{\bar{p}_w(s)}{1 + 2\bar{C}_{\text{FD}} \bar{p}_D(s)} = \frac{\bar{p}_D(s)}{1 + \bar{p}_D(s)} \]

where the wellbore storage coefficient based on fracture half-length is

\[ C_{\text{FD}} = \frac{0.894C}{\phi \mu L_f^2} \]

and \( C \) is the wellbore storage coefficient for a changing liquid level.

The conventional pressure plotting function for production slug tests is defined as

\[ \bar{p}_D(s) = \frac{\bar{p}_f(s) - \bar{p}_i(s)}{\bar{p}_f(s) - \bar{p}_o(s)} \]

and, for an injection slug test,

\[ \bar{p}_D(s) = \frac{\bar{p}_i(s) - \bar{p}_o(s)}{\bar{p}_f(s) - \bar{p}_o(s)} \]

where \( \bar{p}_f \) is the original reservoir pressure, \( \bar{p}_o \) is the wellbore pressure prior to the beginning of the slug test, and \( p(t) \) is the pressure measured during the test. According to Ramey and Agarwal [13], Eqs. 8 and 9 also define the annulus unloading rate,

\[ \bar{p}_D(s) = \frac{\bar{p}_f(s) - \bar{p}_i(s)}{\bar{p}_f(s) - \bar{p}_o(s)} \]

which may be computed in Laplace space as

\[ \bar{p}_D(s) = \bar{C}_{\text{FD}} \bar{p}_D(s) \]

Following Ramey and Agarwal [13], we also define a dimensionless sandface rate as

\[ \bar{q}_s(s) = 1 - \frac{\bar{p}_D(s)}{\bar{p}_f(s)} \]

or,

\[ \bar{q}_s(s) = 1 - \frac{\bar{p}_D(s)}{\bar{p}_f(s)} \]

where the damage to the fracture face is quantified by a positive skin factor, \( \bar{S}_f \).

Recently, Cinco-Ley and Meng [10] and Meehan, et al. [11] presented a method for solving Eqs. 4 and 5. The fracture half-length is divided into \( n \) discrete elements, and each element is modeled as a uniform flux fracture. Since the flux is unknown, we write an equation describing the pressure and flux distribution in each element, and from this system of equations, we solve for the wellbore pressure and flux in Laplace space. We then compute the real space solutions using the Stehfest inversion algorithm [12]. Details of this solution technique are presented in Refs. 10 and 11 and Appendix B.

Before inverting the system of equations and solving for the dimensionless pressure at the wellbore, we include wellbore storage as follows:

\[ \bar{p}_{\text{FD}}(s) = \frac{\bar{p}_D(s)}{1 + \bar{C}_{\text{FD}} \bar{p}_D(s)} \]

\[ \bar{p}_D(s) = \frac{\bar{p}_f(s) - \bar{p}_i(s)}{\bar{p}_f(s) - \bar{p}_o(s)} \]

where the wellbore storage coefficient based on fracture half-length is

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or,

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The dimensionless sandface rate defined by Eqs. 12 and 13 is useful as a slug test plotting function to provide better resolution of very early data.

**SLUG TEST PRESSURE RESPONSE**

In this section, we investigate the effects of various fracture and reservoir parameters on the slug test pressure response. Cinco-Ley, et al. [9] have described four distinct flow periods that may occur in hydraulically fractured wells -- fracture linear, bilinear, formation linear, and pseudoradial flow. These periods and their associated flow patterns are illustrated in Fig. 1.

Fracture linear flow occurs at very small values of dimensionless time. During this period, most of the fluid flow comes from expansion of the fluid contained in the fracture, and the flow pattern is essentially linear. Bilinear flow occurs when fluid flows linearly from the formation into the fracture. During this flow period, the fracture-tip effects have not yet influenced the pressure response. For most practical values of dimensionless time, bilinear flow appears only in fractures with \( F_{\text{CD}} < 100\pi \). Following a transition period, the next flow pattern is formation...
linear flow, which occurs only in fractures with \( F_{CD} \geq 100\pi \) (i.e., essentially infinite-conductivity fractures). The final flow period, pseudoradial flow, occurs with fractures of all conductivities. However, the higher the conductivity, the later the flow pattern can be characterized as essentially radial.

**Effects of Wellbore Storage Coefficient and Fracture Conductivity.**

For hydraulically fractured wells in which skin effects are negligible, the parameters controlling the pressure response are the dimensionless wellbore storage coefficient and fracture conductivity. The dimensionless wellbore storage coefficient, defined by Eq. 7, is based on fracture half-length. In this study, we investigate values of \( C_{LP} \) ranging from 10^4 to 1. Small values of \( C_{LP} \) represent small volumes of fluid in the wellbore and/or long hydraulic fractures. Conversely, large values of \( C_{LP} \) are typical of large volumes of fluid in the wellbore and/or short fracture half-lengths. We also investigate the effects of dimensionless fracture conductivities ranging from 0.1 \( \pi \) to 100\( \pi \). As defined by Eq. 3, small values of \( F_{CD} \) represent long, low-conductivity fractures and/or very permeable formations, while values of \( F_{CD} \geq 100\pi \) are indicative of high or even infinite conductivity fractures.

Similar to the type curves presented by Ramey, et al., we developed four type curves using \( t_{LP}/C_{LP} \) as the time plotting function. Figure 2 is a semilog plot of \( p_{Dslug} \) vs. \( t_{LP}/C_{LP} \) for \( F_{CD} = 0.1\pi \) and several values of \( C_{LP} \). All of the curves have the characteristic S-shape exhibited by radial flow models. In fact, the curves do not display any distinct characteristics which would allow us to distinguish them readily from curves generated with radial flow models. Similarly, semilog plots showing the pressure responses for values of \( F_{CD} = \pi, 10\pi, \) and 100\( \pi \) are shown in Figs. 3-5, respectively. Note that as \( F_{CD} \) increases, the curve shapes become more distinct and are more sensitive to the value of \( C_{LP} \). In addition, we observe that for all values of dimensionless fracture conductivity, the level of essentially constant or static pressure response is maintained for longer periods for larger values of \( C_{LP} \), which suggests larger radii of investigation are achieved.

The late-time pressure responses shown in Figs. 6-9, which are log-log plots of \( p_{Dslug} \) vs. \( t_{LP}/C_{LP} \) as a function of \( C_{LP} \), illustrate that many of the curves form a unit-slope line at large values of \( t_{LP}/C_{LP} \). From a comparison with the behavior of slug tests with radial flow, the appearance of the late-time unit-slope line suggests that a stabilized pseudoradial flow period is reached. As we would expect, this pseudoradial flow period appears sooner for small values of \( F_{CD} \). In addition, we observe that regardless of the value of \( F_{CD} \), the curves for \( C_{LP} \geq 0.1 \) converge to form a unit-slope line by \( t_{LP}/C_{LP} = 100 \). Unfortunately, these same curves have similar shapes for the range of fracture conductivities studied, which indicates that unique type curve matches may be difficult to obtain for large values of \( C_{LP} \) (i.e., for short fracture half-lengths and/or large wellbore storage coefficients). Conversely, the curves for \( C_{LP} < 0.1 \) exhibit unique shapes for different values of \( F_{CD} \).

The early-time pressure responses are illustrated in Figs. 10-13, which are log-log plots of \( q_{Dslug} \) vs. \( t_{LP}/C_{LP} \). For \( F_{CD} = 0.1\pi \) and \( C_{LP} \leq 0.01 \), we observe a unit-slope line at very early times (i.e., \( t_{LP}/C_{LP} \geq 10^{-3} \)). For radial models Sageev16 has shown that in the absence of wellbore skin, the early-time response also has a unit-slope prior to the final pressure buildup. Note that when we use log-log plots of the early pressure response, the curves for values of \( C_{LP} \geq 0.1 \) exhibit slightly more character than with semilog plots, especially at large values of \( F_{CD} \). Therefore, if the early data are available, these log-log plots may help reduce the ambiguity in type-curve matching. Unfortunately, the small values of \( p_{Dslug} \) and \( t_{LP}/C_{LP} \) shown in Figs. 10-13 correspond to very small pressures changes and times that may be impractical to obtain from field tests.

To estimate the time at which the slug test pressure response reflects bilinear flow, we compared the semi-analytical solution given by Eq. 4 to the approximate bilinear flow solution given by Cinco-Ley, et al.14,

\[
\bar{p}_{wb}(s) = \frac{1}{s} \left[ \frac{\pi}{4} F_{CD} + \pi^2 C_{LP} \right] \tag{14}
\]

or, the slug test pressure response in Laplace space is

\[
\bar{p}_{Dslug}(s) = \frac{1}{s} \left[ \frac{\pi C_{LP}}{4 F_{CD} + \pi^2 C_{LP}} \right] \tag{15}
\]

We then compute \( p_{Dslug} \) by inverting Eq. 15 with the Stehfest12 algorithm. For the range of dimensionless times studied, we observed bilinear flow patterns for all values of \( F_{CD} \) and \( C_{LP} \), but especially at the lower values of \( C_{LP} \). In general, for \( C_{LP} \geq 0.1 \), bilinear flow occurs at \( t_{LP}/C_{LP} < 0.01 \); however, for larger dimensionless fracture conductivities. At smaller values of \( C_{LP} \), a significant portion of the pressure response represents bilinear flow. For example, for \( F_{CD} = 0.1\pi \) and \( C_{LP} = 10^{-4} \), the end of bilinear flow occurs at approximately \( t_{LP}/C_{LP} = 1 \), and as shown by Fig. 6, the pressure appears to be approaching a unit-slope line within three or four log cycles after this time.

When \( F_{CD} = 100\pi \) and \( C_{LP} = 10^{-4} \), bilinear flow ends at approximately \( t_{LP}/C_{LP} < 0.01 \); however, as shown by Fig. 11, the subsequent pressure response does form a unit-slope line for the range of \( t_{LP}/C_{LP} \) investigated. We suggest that much of the pressure behavior after \( t_{LP}/C_{LP} = 0.01 \) reflects formation linear flow. In general, we observe that the pressure responses for small values of \( C_{LP} \) and large values of \( F_{CD} \) are more sensitive to the various flow patterns.

**Effects of Fracture-Face Skin.** Cinco-Ley and Samaniego17 have suggested that two types of fracture damage may occur during the hydraulic fracturing process — within the fracture adjacent to the wellbore and in the formation around the fracture face. The first type of damage, often described as a choked fracture, is thought to be caused by proppant crushing and embossing in the fracture face, which is not included in this study. In general, the second type of damage, as a fracture-face skin, is probably caused by fluid losses into the formation. In this study, we address only the effects of fracture-face damage on the slug test pressure response.

Appendices A and B describe the manner in which damage to the fracture face is modeled as an infinitesimal skin with no storage.

In a slug test, a damaged zone around the wellbore reduces the rate of fluid flow into the wellbore and, therefore, tends to reduce the rate at which the pressure changes. We would expect a similar reduction in fluid flow from the reservoir to the fracture when fracture-face skin is present. However, when compared to the curve shape for no skin, Figs. 14 and 15 show that the presence of fracture-face skin has little effect on the pressure response in low-conductivity fractures, thus greatly reducing the character of the curve shapes. Specifically, fracture-face skin factors less than 0.1 are indistinguishable from the zero skin case. In addition, note that these skin effects are less pronounced for small values of \( C_{LP} \) and \( F_{CD} \) (Fig. 15), which suggests that we cannot estimate the fracture-face skin accurately in short, low-conductivity fractures. The early-time effects shown in Figs. 16 and 17 for \( F_{CD} = 0.1\pi \) and \( C_{LP} = 10^{-4} \) and 1.0, respectively, help distinguish the pressure response at early times, but the pressure differences may be too small to be measured accurately with conventional pressure gauges.
For highly-conductive fractures with small values of $C_{LFD}$ (Fig. 18), we see that fracture-face skin affects the pressure behavior significantly. We note that larger values of skin tend to maintain the static pressure response longer (i.e., skin reduces the rate at which the pressure changes). The early-time pressure responses for these same parameters are plotted in Fig. 19. As the skin increases, we observe the formation of the unit-slope line earlier than when skin is not present. For high-conductivity fractures with large values of $C_{LFD}$ (Fig. 20), we see that fracture-face skin, especially values less than 0.1, has little effect on the pressure response. Only the early-time pressure behavior is affected by the presence of skin, as illustrated by Fig. 21. From these plots, we may conclude again that fracture-face skin effects are difficult to discern from slug tests in wells with short fractures (i.e., large values of $C_{LFD}$). As we would expect, fracture face skin affects the pressure response most when formation linear flow behavior predominates (i.e., large values of $FCD$ and small values of $C_{LFD}$).

**SLUG TEST ANALYSIS PROCEDURE**

In this section, we present a procedure for analyzing slug tests in hydraulically fractured wells, and we illustrate the procedure with a simulated injection slug test.

**Analysis Procedure**

1. Prepare semilog and log-log plots of the field data. We suggest plotting both $PD_{slug}$ and $PD_{slag}$ vs. time using the same scales as the type curves. For production slug tests, $PD_{slug}$ is defined by Eq. 8, while for injection type tests, $PD_{slug}$ is defined by Eq. 9. In addition, the early-time plotting variable, $q_{Dslug}$, is defined by Eq. 13. As we have discussed previously, semilog and log-log plots of $PD_{slug}$ are best suited for analyzing intermediate- and late-time data, while semilog and log-log plots of $q_{Dslug}$ provide resolution of early-time data, and under some conditions, help to reduce the ambiguity of type curve matching.

2. Next, we select the type curves for analysis of the slug test. As we discussed previously, the slug test response depends on three parameters -- $C_{LFD}$, $FCD$, and $S_f$. Therefore, we should use all available information to estimate these parameters and reduce the uncertainty in our analysis. For example, if previous fracture treatments in the particular coal seam have not resulted in significant fluid losses into the formation, we may select type curves for no fracture-face skin.

3. Next, we find a match between the field data and type curve plots. Because of the manner in which $PD_{slug}$ and $PD_{slag}$ have been defined, we simply align equal values of the functions on the vertical axes and slide the graphs horizontally until we obtain a match.

4. From the type curves, read values of the correlating parameters, $C_{LFD}$, $FCD$, and $S_f$. In addition, obtain a time match point, i.e., $t_{MP}$ and the corresponding value of $t$ from the plot of the field data.

5. Using the definition of dimensionless wellbore storage coefficient, estimate the hydraulic fracture half-length. From Eq. 7,

$$C_{LFD} = \frac{0.894C}{\phi c_b h_f^2}$$

or, the fracture half-length $L_f$ is

$$L_f = \sqrt{\frac{0.894C}{\phi c_b h_f^2}}$$

Note that we must have estimates of $\phi$ and $c_b$. The wellbore storage coefficient (C, bbl/psi) for a changing liquid level in the wellbore is defined as

$$C = \frac{A_{wb}}{\rho_f}$$

where $A_{wb}$ is the wellbore area in bbl/ft and $\rho_f$ is the density of the fluid (psi/ft) used for the slug test.

6. Using the time match point obtained in Step 4, calculate the formation permeability as follows:

$$k = \frac{3.390 \mu_c C_P}{h} \left( \frac{t_{MP}}{q_{Dslug} C_{LFD}} \right)$$

If pre-fracture estimates of permeability are available, we may estimate the porosity-compressibility product using the time match point and the correlating parameter $C_{LFD}$ from the type curve match, or

$$\phi c_b = \frac{0.0002637 k (\frac{t_{MP}}{q_{Dslug} C_{LFD}})}{\rho_f}$$

7. From the dimensionless fracture conductivity obtained in Step 4, we may estimate fracture conductivity as

$$k_p f = (k_L f) FCD$$

**Example Problem**

We illustrate the application of our type curves with a simulated injection slug test in a hydraulically fractured coal seam. The test data were generated with the coal properties summarized in Table 1. We have assumed the slug test was conducted in a well completed with 7-inch casing (I.D. = 6.094 in.) and using fresh water. The wellbore pressure at the beginning of the test was $P_b = 600$ psia. In addition, we note that previous fracture treatments have not suffered significant fluid losses, so we assume fracture-face skin is negligible.

**TABLE 1**

Well and Coal Seam Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>1.0</td>
</tr>
<tr>
<td>$c_p$</td>
<td>3 x 10^{-3}</td>
</tr>
<tr>
<td>$h$</td>
<td>10</td>
</tr>
<tr>
<td>$\phi$</td>
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</tr>
<tr>
<td>$\mu$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\rho_f$</td>
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</tr>
<tr>
<td>$\rho_p$</td>
<td>500</td>
</tr>
<tr>
<td>$L_f$</td>
<td>100</td>
</tr>
<tr>
<td>$k$</td>
<td>5</td>
</tr>
<tr>
<td>$FCD$</td>
<td>$\pi$</td>
</tr>
</tbody>
</table>

The first step is to calculate the slug test plotting functions, $PD_{slug}$ and $PD_{slag}$, and prepare semilog and log-log plots of both. Next, we attempt to match the log-log plot of the field data with the slug test type curves for zero skin. Note that we may obtain reasonable matches of $PD_{slug}$ for several values of $FCD$ and $C_{LFD}$; however, the best match of both plotting functions was obtained...
The time match point is
\[ t = 1 \text{ hr and } t_{LD}/C_{LD} = 0.19 \]
Using this same match point, we also obtain a good match of the semilog plot of the data, as shown by Fig. 23.
We estimate the wellbore area \( (A_{wb}, \text{ bbl/ft}) \) to be
\[ A_{wb} = \frac{m(0.25)^2}{5.615} = 0.033 \text{ bbl/ft} \]
and the wellbore storage coefficient is
\[ C = \frac{A_{wb}}{N} = 0.033 \text{ bbl/ft} = 0.081 \text{ bbl/psi} \]
The hydraulic fracture half-length is estimated to be
\[ L_f = \sqrt{\frac{0.894 C}{f_0 h C_{LD}}} \]
which agrees with the value used to simulate the slug test. The permeability of the coal seam is estimated to be
\[ k = \frac{3.390 \mu C}{h} \left( \frac{t_{LD}/C_{LD}}{t} \right) \]
where \( f_0 \) is 0.19, and
\[ k = 5.2 \text{ md} \]
which also agrees with the value given in Table 1. The fracture conductivity is estimated to be
\[ k_f y_f = (K_L)F_{CD} \]
\[ k_f y_f = 5.2 (98.3) (\pi) \]
\[ k_f y_f = 1,606 \text{ md-ft} \]

**SUMMARY AND CONCLUSIONS**

We have developed new type curves for analyzing slug tests in hydraulically fractured reservoirs and have presented a procedure, which is illustrated with an example, for applying these new type curves. With these type curves, we may estimate the hydraulic fracture half-length, formation permeability, and fracture conductivity. We have also investigated the effects of fracture face skin and found that these effects are less pronounced at large values of \( C_{LD} \) and \( F_{CD} \), which suggests that we cannot estimate the fracture face skin accurately from slug tests in short, low-conductivity fractures.

As we have discussed, our slug test model was developed using Cinco-Ley, et al.'s finite-conductivity, vertical fracture model and is limited to the assumptions listed earlier in the paper. In addition, our model assumes a constant wellbore storage coefficient during the test. Further, our model is valid only for single-phase flow conditions. Finally, we should note that we assume the fracture fully penetrates a single zone and that this single zone is the only zone in pressure communication with the wellbore.

**NOMENCLATURE**

- \( A_{wb} \) = area of wellbore, bbl/ft
- \( B \) = formation volume factor, RB/STB
- \( b_f \) = fracture width, ft
- \( C \) = wellbore storage coefficient based on a changing liquid level, bbl/psi
- \( C_{LD} \) = dimensionless wellbore storage coefficient based on fracture half-length
- \( c \) = fluid compressibility, psi\(^{-1}\)
- \( c_i \) = total system compressibility, psi\(^{-1}\)
- \( F_{CD} \) = dimensionless fracture conductivity
- \( h \) = formation thickness, ft
- \( K_0 \) = modified Bessel function of the second kind, zero order
- \( k \) = formation permeability, md
- \( k_f \) = fracture permeability, md
- \( L_f \) = hydraulic fracture half-length, ft
- \( p \) = measured pressure during slug test, psia
- \( P_D \) = dimensionless pressure
- \( P_{Dslag} \) = slug test pressure plotting function
- \( P_{Dlag} \) = Laplace transform of slug test pressure plotting function
- \( P_f \) = pressure in fracture, psia
- \( P_{DF} \) = dimensionless fracture pressure
- \( P_{DP} \) = Laplace transform of dimensionless fracture pressure
- \( p_{ipi} \) = initial pressure, psia
- \( p_o \) = measured pressure at beginning of slug test, psia
- \( p_{WD} \) = dimensionless pressure at the wellbore
- \( P_{WD} \) = Laplace transform of dimensionless wellbore pressure
- \( p_{w} \) = bottomhole flowing pressure, psia
- \( q \) = flow rate, bbl/day
- \( q_f \) = dimensionless flow rate
- \( q_{DF} \) = dimensionless fracture flow rate
- \( q_{Dslag} \) = slug test rate plotting function
\[ S_f \] = fracture face skin factor
\[ s \] = Laplace transform variable
\[ t_{pD} \] = dimensionless time based on fracture half-length
\[ x \] = distance along fracture, ft
\[ x_{pD} \] = dimensionless distance along the fracture
\[ y \] = distance perpendicular to fracture
\[ y_{pD} \] = dimensionless distance perpendicular to fracture
\[ \phi \] = porosity, fraction
\[ \mu \] = viscosity, cp.
\[ \rho_f \] = density of liquid in wellbore, psi/ft

REFERENCES


Appendix A - Derivation of Finite Conductivity Vertical Fracture Model

Following the model formulation and problem solution presented by Cinco-Ley, et al.\(^9\), the partial differential equation in dimensionless variables describing the transient flow in the fracture is

\[
\frac{\partial^2 p_{D}}{\partial x_{pD}^2} + 2\frac{\partial p_{D}}{F_{CD} \partial y_{pD}} = 0
\]  

where the dimensionless variables are defined as follows:

\[
p_{D}(x_{pD}, t_{pD}) = \frac{k h (p_{i} - p(x,t))}{141.2 q p 0 \mu f}
\]  

\[
t_{pD} = 0.0002637 \frac{kr}{\mu f L_{f}}
\]  

\[
F_{CD} = \frac{k_{h} r_{f}}{\mu f L_{f}}
\]  

\[
x_{pD} = \frac{x}{L_{f}}
\]  

\[
y_{pD} = \frac{y}{L_{f}}
\]

The fracture is at initial reservoir pressure \(p_i\), or

\[
p_{D}(x_{pD}, t_{pD} = 0) = 0, \text{ for } 0 \leq x \leq x_{pD} \quad \text{(A-7)}
\]

while the boundary condition describing constant rate production at the wellbore (i.e., \(x_{pD} = 0\)) is

\[
\frac{\partial p_{D}}{\partial x_{pD}} = -\frac{Q}{F_{CD}} \quad \text{(A-8)}
\]

The boundary condition describing the no flux condition at the fracture tips (i.e., \(x_{pD} = \pm 1\)) is

\[
\frac{\partial p_{D}}{\partial x_{pD}} = 0 \quad \text{(A-9)}
\]

After integrating Eq. A-1 twice with respect to \(x_{pD}\) and applying the initial and boundary conditions, Cinco-Ley, et al.\(^9\) derived the pressure drop between the wellbore (i.e., \(x_{pD} = 0\)) and any point \(x_{pD}^*\) through the fracture as
where the dimensionless pressure at the wellbore for constant rate conditions is

\[ p_D(x_D=0,t_D) = \frac{k h(t - p_D(x=0,t))}{141.2 q \beta \mu} \]  

and \( q_D \) is defined as the dimensionless flow rate per unit of fracture length going from the reservoir to the fracture,

\[ q_D(x,t) = \frac{q(x,t)}{q_D(x',t)} \]  

Modeling the fracture as a plane source with dimensionless flux density \( g_D(x',t_D) \), the dimensionless pressure drop at any point in the reservoir is

\[ p_D(x_D,y_D,t_D) = \frac{1}{4} \int_0^{l_D} \int_0^{l_D} q_D(x',t) \exp \left[ \frac{(x_D-x')^2}{4k(t_D-x'^2)} \right] dx' dt \]  

or, solving for the pressure drop at the face of the fracture (i.e., \( y_D = 0 \)),

\[ p_D(x_D,y_D=0,t_D) = \frac{1}{4} \int_0^{l_D} \int_0^{l_D} q_D(x',t) \exp \left[ \frac{(x_D-x')^2}{4k(t_D-x'^2)} \right] dx' dt \]  

If we equate Eqs. A-10 and A-14, we may solve for the pressure in the fracture at the wellbore,

\[ p_D(x_D=0,t_D) = \frac{1}{4} \int_0^{l_D} \int_0^{l_D} q_D(x',t) \exp \left[ \frac{(x_D-x')^2}{4k(t_D-x'^2)} \right] dx' dt \]  

or, solving for the pressure drop at the face of the fracture (i.e., \( y_D = 0 \)),

\[ p_D(x_D,y_D=0,t_D) = \frac{1}{4} \int_0^{l_D} \int_0^{l_D} q_D(x',t) \exp \left[ \frac{(x_D-x')^2}{4k(t_D-x'^2)} \right] dx' dt \]  

If we include the effects of fracture face skin (\( S_f \)), Eq. A-15 becomes

\[ \bar{p}_D(s) = \frac{1}{2} \int_0^{l_D} \bar{q}_D(x',s) \left[ K_D(x_D - x') + K_0(x_D + x') \right] dx' \]  

The Laplace transform of Eq. A-15 is

\[ \bar{p}_D(s) = \frac{1}{2} \int_0^{l_D} \bar{q}_D(x',s) \left( K_D(x_D - x') + K_0(x_D + x') \right) dx' \]  

Applying Eqs. B-2 and B-3 into B-1, we have
In addition, we utilize the condition that the summation of the flow entering each fracture element is equal to the flow at the wellbore, or, in Laplace space,

\[ \Delta x \sum_{i=1}^{n} \bar{q}_{ij}(s) = \frac{1}{3} \] ..........................(B-5)

We may write an equation for each fracture element and with Eq. B-5 develop a system of \(n+1\) equations with \(n+1\) unknowns, i.e., \(\bar{q}_{ij}(s), i = 1, \ldots, n\) and \(\bar{P}(s)\). The unknowns are obtained by solving the system of equations. Once we have the dimensionless flux and pressure in Laplace space, we may then use the Stehfest\(^{12}\) algorithm to obtain these variables in real space. A similar methodology is used to solve Eq. A-18 when fracture face skin is included. Additional details of the solution procedure are provided in Refs. 10 and 11.
(a) Fracture Linear Flow
(b) Bilinear Flow
(c) Formation Linear Flow
(d) Pseudoradial Flow

Fig. 1: Flow Periods for Hydraulically Fractured Wells

Fig. 2: Slug Test Pressure Response for $FCD = 0.1 \pi$ and $S_t = 0$

Fig. 4: Slug Test Pressure Response for $FCD = 10 \pi$ and $S_t = 0$

Fig. 5: Slug Test Pressure Response for $FCD = \pi$ and $S_t = 0$
Fig. 5 - Slug Test Pressure Response for $F_{CD} = 10\pi$ and $S_f = 0$

Fig. 6 - Intermediate- and Late-Time Slug Test Response for $F_{CD} = 0.1\pi$ and $S_f = 0$

Fig. 7 - Intermediate- and Late-Time Slug Test Response for $F_{CD} = \pi$ and $S_f = 0$

Fig. 8 - Intermediate- and Late-Time Slug Test Response for $F_{CD} = 10\pi$ and $S_f = 0$
Fig. 9 - Intermediate- and Late-Time Slug Test Response for $F_{CD} = 100\pi$ and $S_f = 0$

Fig. 10 - Early-Time Slug Test Response for $F_{CD} = 0.1\pi$ and $S_f = 0$

Fig. 11 - Early-Time Slug Test Response for $F_{CD} = \pi$ and $S_f = 0$

Fig. 12 - Early-Time Slug Test Response for $F_{CD} = 10\pi$ and $S_f = 0$
Fig. 21 - Early-Time Slug Test Response for $F_{CD} = 100$ m and $C_{L_{d}} = 1$

Fig. 22 - Log-log Match of the Example Slug Test

Fig. 23 - Semilog Match of the Example Slug Test