Advances in the Use of Convolution Methods in Well Test Analysis

This is a preprint -- subject to correction.

ABSTRACT

This paper introduces explicit (discrete data) methods to compute the effects of wellbore storage and wellbore phase redistribution distortion. The advantage of these methods over calculation techniques in Laplace space requiring numerical inversion is that these methods can be used to predict analytically, in real space, the early and late time behavior of the wellbore storage and phase redistribution distortion of constant rate pressure data. These methods can be applied to discrete data from dimensionless solutions (or field data, if formation properties are known). Detailed derivations of the wellbore storage and wellbore phase redistribution cases are given.

The major result of this paper is the development of explicit, real space analytical solutions to compute wellbore storage and phase redistribution effects in pressure test data. In general, the computations are quite accurate compared to results obtained using Laplace transform inversion of the Laplace space solutions. As a practical consideration, we have found that generally it is better to compute the derivative functions numerically from the pressure function rather than by analytical differentiation methods. In some cases the analytical derivatives are too tedious for hand calculations. The computed derivative functions also are shown to be accurate when compared to the Laplace inversion derivative functions.

INTRODUCTION

The computation of pressure responses in the presence of wellbore storage and/or wellbore phase redistribution effects requires inversion of the Laplace space solutions for these cases. In general, the Laplace space solutions cannot be inverted analytically and typically are inverted numerically using an algorithm such as the one proposed by Stehfest.1 Often, the numerical inversions are computationally intensive. The purpose of this paper is to provide approximations in Laplace space that, upon analytical inversion, yield simple and accurate closed form approximations of the wellbore storage and wellbore phase redistribution solutions.

These approximations are significant to well test analysis since the approximations can be used to model the behavior of test data and may provide new methods of interpretation. The original development of the computation of wellbore storage solutions is given in Refs. 2-6. van Everdingen7 and, later, Ramsey8 provide relations to compute the wellbore pressure response in the presence of wellbore storage effects using an exponential sandface rate model. Ramsey provided some limited analysis methods for wellbore storage distorted pressure data based on the exponential sandface rate model.

Although we do not develop methods to interpret well test data in this paper, we do expect the development of several analysis techniques for wellbore storage distorted pressure data to arise from the computational formulas derived in this work. In particular, the relations derived in this work should be useful in interpreting the "unit slope" line on a type curve plot, and some relations may be useful for convolution and deconvolution analyses.

We have verified the relations developed in this work for the computation of wellbore storage and skin effects for fractured and unfractured wells in homogenous reservoirs and for wells in naturally fractured reservoirs. The Laplace space solutions were obtained as follows: from Ref. 3 for unfractured wells in homogeneous reservoirs, from Ref. 9 for fractured wells (uniform flux or infinite conductivity) in homogeneous reservoirs, and from Refs. 10 and 11 for unfractured wells in naturally fractured reservoirs.

It is worth noting that Ref. 12 presents a novel concept to compute sandface flow rates and pressure responses in the presence of wellbore storage distortion effects using exponential models for flow rates and a power law model for pressure drop. Although the work in Ref. 12 has not found utility in well testing technology, Ref. 12 does provide insight into the exponential model for sandface flow rate.

Finally, we develop explicit techniques to compute the pressure response in the presence of wellbore phase redistribution effects.9 To our knowledge, this result has not been presented in the literature previously. This calculation uses only the wellbore
storage and wellbore phase redistribution dimensionless pressures to compute the total wellbore dimensionless pressure. The utility of this result is twofold. First, we can compute the effects of wellbore phase redistribution without using an numerical inversion algorithm to invert the Laplace space solution. Second, this result may provide insight into the analysis of pressure test data which exhibit the effects of wellbore phase redistribution.

DEFINITIONS OF DIMENSIONLESS VARIABLES

**Dimensionless Pressure Functions**

The dimensionless wellbore pressure, \( p_D \), for a constant rate flow system is defined as

\[
p_D = \frac{k b h p}{141.2 Q B H} \tag{1}
\]

where the pressure drop, \( dP \), for drawdown tests is

\[
dP = P_i - P_wd \tag{2}
\]

and for buildup tests,

\[
dP = P_w + P_wd \tag{3}
\]

The dimensionless wellbore pressure, \( p_{WD} \), for a constant rate system with skin effects is defined as

\[
p_{WD} = p_D + S \tag{4}
\]

The dimensionless wellbore pressure, \( p_{wCD} \), for a system with wellbore storage and skin effects is defined as

\[
p_{wCD}(tD) = \int_0^{tD} \frac{d}{dt} \left[ f_{wCD}(\tau) \right] p_{WD}(tD - \tau) d\tau \tag{5}
\]

Similarly, the dimensionless wellbore pressure, \( p_{wCD} \), for a system with wellbore phase redistribution and skin effects is defined as

\[
p_{wCD}(t) = \int_0^{t} \frac{d}{dt} \left[ f_{wCD}(\tau) \right] p_{CD}(t - \tau) d\tau \tag{6}
\]

**Dimensionless Time Functions**

The dimensionless time, \( t_{WD} \), based on the wellbore radius, \( r_w \), is

\[
t_{WD} = \frac{0.0002637 k r_w}{\Phi p_w^2} \tag{7}
\]

and the dimensionless time, \( t_{wCD} \), based on fracture half-length, \( L_h \), is

\[
t_{wCD} = \frac{0.0002637 k}{\Phi u c L_h^2} \tag{8}
\]

**Dimensionless Pressure Derivative Functions**

The dimensionless derivative function, \( p_D^* \), which is used in type curve analysis, is defined in two forms which are mathematically identical. The first form is

\[
p_D^* = \frac{d p_D}{d(\ln t_D)} \tag{9}
\]

and the second form is

\[
p_D^* = \frac{d p_D}{d t_D} \tag{10}
\]

**Dimensionless Sandface Flow Rate Functions**

For the constant wellbore storage case, the dimensionless sandface flow rate, \( q_{wCD} \), is defined as

\[
q_{wCD} = 1 - C_D \frac{d p_D}{d t_D} \tag{11}
\]

For the wellbore phase redistribution case, the dimensionless sandface flow rate, \( q_{wCD} \), is defined as

\[
q_{wCD} = 1 - C_D \left( \frac{d p_D}{d t_D} \right)^2 \tag{12}
\]

**Dimensionless Wellbore Storage Coefficients**

The dimensionless wellbore storage coefficient, \( C_D \), based on the wellbore radius, \( r_w \), is

\[
C_D = 0.894 \frac{C}{r_w^2} \tag{13}
\]

while the dimensionless wellbore storage coefficient, \( C_{wCD} \), based on fracture half-length, \( L_h \), is defined as

\[
C_{wCD} = 0.894 \frac{C}{r_w^2} \tag{14}
\]

**APPROXIMATIONS FOR \( p_{wCD}(tD) \): ANALYTICAL RESULTS**

In this section, we discuss the analytical developments of the approximations we have created for the dimensionless pressure function, \( p_{wCD}(tD) \), which includes the effects of wellbore storage and skin effects. In Appendix A, we derive the Laplace transform identities that are required to develop the \( p_{wCD}(tD) \) approximations. These identities are developed rigorously from the convolution integral for the case of a well with wellbore storage and skin effects. In Appendices B, C, and D, we develop approximations for \( p_{wCD}(tD) \) based on an assumed behavior of the constant rate dimensionless pressure, \( p_{CD}(tD) \).

Case 1: \( p_{wCD}(tD) \) approximation based on constant \( p_{CD}(tD) \)

Appendix B develops the case where we assume that the \( p_{CD}(tD) \) function is constant near a particular time of interest. For this case, the \( p_{wCD}(tD) \) approximation is

\[
p_{wCD}(tD) = p_{CD}(tD) \left[ 1 - \frac{tD}{p_{CD}(tD) C_D} \right] \tag{15}
\]

Equation 15 states that the \( p_{wCD}(tD) \) function is an exponentially increasing function of the \( p_{CD}(tD) \) relation. This is a somewhat intuitive result since we know that the \( p_{CD}(tD) \) function increases monotonically over time until it is identical to the \( p_{CD}(tD) \) function. The accuracy of Eq. 15 as a general model for wellbore storage will be illustrated in the following section, where all of the approximate solutions are verified against the numerical inversion solutions of chosen reservoir systems.

Case 2: \( p_{wCD}(tD) \) approximation based on linear \( p_{CD}(tD) \)

Appendix C develops the case where we assume that the \( p_{CD}(tD) \) function is linear near a particular time of interest. For this case, the \( p_{wCD}(tD) \) approximation is

\[
p_{wCD}(tD) = p_{CD}(tD) \left[ 1 - \frac{tD}{\alpha} \right] \left[ 1 - \frac{tD}{\beta} \right] \left[ 1 - \frac{tD}{\gamma} \right] \tag{16}
\]

where the generalized coefficients for this case, \( \alpha, \beta, \text{ and } \gamma \), are defined as

\[
\alpha = 1 + C_D b \frac{C}{C_D a} \tag{16}
\]
\[
\beta = \frac{a}{C_D} \\
\theta = \frac{b}{C_D}
\]

The coefficients \( a \) and \( b \) appear in the \( p_{LD}(t) \) model which is given as:

\[
p_{LD}(t) = a + bt_D
\]  \hspace{1cm} (17)

To use Eq. 16 to compute \( p_{wCD}(t) \), requires that we know values for the coefficients \( a \) and \( b \). We can solve for \( b \) by differentiating Eq. 17 with respect to \( t_D \). This gives

\[
b = \frac{\frac{da}{dr_D}}{p_{LD}(t)}
\]  \hspace{1cm} (18)

The coefficient \( a \) can be determined directly from Eq. 17 and using the value of \( b \) from Eq. 18. This gives

\[
a = p_{LD}(t) - t_D \left( \frac{da}{dr_D} \right)
\]  \hspace{1cm} (19)

Again, the validity of the Eq. 16 will be determined by comparison with numerical inversion solutions.

**Case 3: \( p_{wCD}(t) \) approximation based on quadratic \( p_{LD}(t) \)**

Appendix D develops the case where we assume that the \( p_{wCD}(t) \) function behaves as a quadratic near a particular time of interest. For this case, the \( p_{wCD}(t) \) approximation and its associated coefficients are summarized Appendix D. The validity of this approximation will be determined in the next section.

**APPROXIMATIONS FOR \( p_{wCD}(t) \): VERIFICATION**

In this section, we verify each of the three approximations for \( p_{wCD}(t) \) summarized in the previous section. All of the verification cases use the infinite-acting (transient) flow solutions for the given reservoir system.

**Unfractured Wells in an Infinite-Acting Homogeneous Reservoir**

The infinite-acting homogeneous reservoir solution with wellbore storage and skin effects is discussed in detail in Refs. 2-6. All of these solutions, except those presented in Ref. 5 (where a finite-difference reservoir simulator was used), require that the Laplace transform of the function be numerically inverted to yield the real time solution. One purpose of this paper is to develop closed form approximations which model the pressure response distorted by wellbore storage and skin effects. Therefore, we compare our new approximations to the numerical inversion solutions.

**Case 1: \( p_{wCD}(t) \) approximation based on constant \( p_{LD}(t) \)**

The premise of case 1 is that the \( p_{LD}(t) \) function can be considered constant near a particular time of interest. Although this premise clearly violates the theory of Laplace transforms, we make this assumption in an attempt to obtain an accurate approximation to the correct (numerical inversion) solution.

The behavior of \( p_{CD}(t) \) for \( 10^{-4} < C_D < 10^4 \) is shown in Fig. 1 for 15 values of \( C_D \) ranging from \( 10^1 \) to \( 10^4 \). These parameters will be used for all of the unfractured well cases for the \( p_{wCD}(t) \), \( P_{wCD}(t) \), and \( q_{wCD}(t) \) solutions. We note from Fig. 1 that, except for a slight disagreement for \( C_D = 10^4 \) and during the time period \( 1 \leq t_D \leq 5 \times 10^5 \), the case 1 approximation (Eq. 15) accurately predicts the \( p_{wCD}(t) \) solution.

In Fig. 2, we see the behavior of the \( p_{wCD}(t) \) function. As we would expect, the approximate derivative solution deviates over the same parameter range as the \( p_{wCD}(t) \) function. In fact, the deviation is slightly more pronounced, because this is a derivative function. In the case of the approximate derivative functions, all derivatives were determined numerically from the computed \( p_{wCD}(t) \) function, except for the derivatives for case 1 which were computed using Eq. B-5. Again, the results in Fig. 2 suggest that Eq. 15 and its derivatives should provide reasonably accurate results compared to the numerical inversion solutions.

Fig. 3 shows the behavior of the \( q_{wCD}(t) \) function computed using the \( p_{wCD}(t) \) function and Eq. 11. We again note the disagreement in the solutions for \( C_D = 10^4 \) during \( 1 \leq t_D \leq 1 \times 10^5 \). Although the approximate \( q_{wCD}(t) \) function deviates significantly for those parameter ranges, this should not rule out the use of \( p_{wCD}(t) \) and \( q_{wCD}(t) \) functions based on Eq. 15. It is important to note that the \( q_{wCD}(t) \) function contains errors in the \( p_{wCD}(t) \) function by use of the derivative function and by the subtraction required in Eq. 11. In other words, the \( q_{wCD}(t) \) function will magnify any errors in the \( p_{wCD}(t) \) function. Therefore, the accuracy of the \( q_{wCD}(t) \) function should be considered the most sensitive test of the approximate solutions.

**Case 2: \( p_{wCD}(t) \) approximation based on linear \( p_{LD}(t) \)**

The pressure behavior of \( p_{wCD}(t) \) for case 2 is shown in Fig. 4. Recall that this case assumes the \( p_{wCD}(t) \) function varies linearly with time near a particular time of interest. We note that the \( p_{wCD}(t) \) functions compare extremely well with the numerical inversion solutions. The only variance of any significance occurs for \( C_D = 10^4 \) during the time period \( 1 \leq t_D \leq 1 \times 10^5 \). This agreement suggests that Eq. 16 is an excellent approximation to the \( p_{wCD}(t) \) function.

Fig. 5 illustrates the behavior of the \( p_{wCD}(t) \) function. In this case, we again observe excellent agreement with the numerical inversion solutions. The relatively small deviation affects only solutions for \( C_D = 10^4 \) and during the time period \( 1 \leq t_D \leq 5 \times 10^5 \). The deviation observed in Fig. 5 is slightly greater than the deviation we saw in Fig. 4 for the approximate \( p_{wCD}(t) \) function.

It is important to note that, because of the extremely tedious nature of the closed form derivative for this case, the derivative functions were computed numerically. We do not believe that the numerical differentiation contributes any significant deviation to the approximate \( p_{wCD}(t) \) function. Because of the excellent agreement between the approximate and numerical inversion solutions, Eq. 16 and its derivatives should provide accurate estimates of the \( p_{wCD}(t) \) and \( q_{wCD}(t) \) functions.

Fig. 6 illustrates the behavior of the \( q_{wCD}(t) \) function for case 2, which was computed using the numerical derivative of the \( p_{wCD}(t) \) function and Eq. 11. As we noted earlier, the \( q_{wCD}(t) \) function is the most sensitive quantity that we can compute. In this case, there is some disagreement between approximate and numerically inverted solutions. This deviation only affects solutions for \( C_D = 10^4 \) during \( 1 \leq t_D \leq 1 \times 10^5 \). Because of the relatively small magnitude of the error, we believe that Eq. 16 can be used for the computation of the \( p_{wCD}(t) \), \( P_{wCD}(t) \), and \( q_{wCD}(t) \) solutions. The errors associated with each function should be within engineering accuracy for most applications.

**Case 3: \( p_{wCD}(t) \) approximation based on quadratic \( p_{LD}(t) \)**

Case 3 assumes that the \( p_{wCD}(t) \) function varies as a quadratic function of time near a particular time of interest. The behavior of \( p_{wCD}(t) \) for case 3 is shown in Fig. 7. The approximate \( p_{wCD}(t) \) solutions were computed using Eq. D-5 and the computational procedure given in Appendix D. Although the approximate solution is slightly high for \( C_D = 10^4 \) during all times, the agreement between the approximate and numerical inversion solutions for \( p_{wCD}(t) \) is good.

The \( P_{wCD}(t) \) functions are shown in Fig. 8. We immediately
note the excellent agreement of approximate and numerically inverted \( p_{wCD}(t_p) \) functions. There is only a very slight deviation for \( C_D=10^4 \) during the time period \( 1 \leq t_p \leq C_D 10^4 \). In addition, significant deviation is observed at later times \( (t_p>C_D 10^4) \) for \( 10^5 < C_D < 2 \times 10^6 \). This later deviation probably can be attributed to some property of Eq. D-5 that causes the \( p_{wCD}(t_p) \) function to decay at a slower rate, as this deviation is essentially a time lag. We believe Eq. D-3 is an accurate model for the \( p_{wCD}(t_p) \) and \( p_{CD}(t_p) \) functions, but doubt its worth as a hand method since the relation is quite tedious.

The \( q_{wCD}(t_p) \) function for case 3 is shown in Fig. 9. The disagreement in this case is virtually identical to that for case 2 (Fig. 6). The deviation of the \( p_{wCD}(t_p) \) functions that was noted for case 3 in Fig. 8 is not evident in Fig. 9. This is due to the effect of the \( p_{CD}(t_p) \) function and the subtraction in Eq. 11. Although Eq. D-5 has been shown to accurately represent the \( q_{wCD}(t_p) \) function, we believe that similar, if not more consistent, results will be obtained using Eq. 16 (case 2 result). Therefore, we recommend the use of Eq. 15 over Eq. D-5 due primarily to the observation that these relations should yield the same results. Also, Eq. 16 is much less tedious to use than Eq. D-5.

**Fractured Wells in an Infinite-Acting Homogeneous Reservoir**

This section considers the case of a well with a vertical fracture of infinite conductivity in an infinite-acting homogeneous reservoir. This problem was recently solved in Laplace space by Chow and Ragab. In the previous section, case 2 (Fig. 6) was found to be the most accurate and consistent model of the \( p_{wCD}(t_p) \) function for an unfractured well in an infinite-acting homogeneous reservoir. The purpose of this section is to determine the most accurate and consistent model of the \( p_{wCD}(t_p) \) function for a fractured well in an infinite-acting homogeneous reservoir.

**Case 1: \( p_{wCD}(t_p) \) approximation based on constant \( p_{CD}(t) \)**

The behavior of \( p_{wCD}(t_p) \) is shown in Fig. 10 for the time period \( 10^5 < t_p < C_D 10^5 \) and for 6 values of \( C_D \) ranging from \( 3 \times 10^3 \) to 1. These parameters will be used for all of the fractured well cases for the \( p_{wCD}(t_p) \), \( p_{CD}(t_p) \), and \( q_{wCD}(t_p) \) functions. In the case 1 approximation (Eq. 15) does not accurately predict the \( p_{wCD}(t_p) \) solution for the time period \( t_p > C_D 10^5 \) and for all \( C_D \). This is likely due to this period being the linear flow region for the wellbore storage case, and the assumption of a constant \( p_{CD}(t) \) function is insufficient for this period. This disagreement suggests that Eq. 15 is not a good approximation of the \( p_{wCD}(t_p) \) function.

In Fig. 11, we see the behavior of the \( p_{wCD}(t_p) \) function and, as we would expect, the approximate function given by Eq. B-9 yields poor results for \( t_p > C_D 10^5 \) and for all \( C_D \). Recall that the \( p_{CD}(t_p) \) function also experienced significant deviation during this same time period. Similarly, the behavior of the \( q_{wCD}(t_p) \) function, as shown in Fig. 12, cannot be modeled using the approximate function for the same parameter range mentioned above.

Because of the nature of Eq. 11, the deviation for the \( q_{wCD}(t) \) function is the greatest of all the functions. As we noted above, disagreement between the approximate and numerical inversion solutions suggests that Eq. 15 should not be used to compute the \( p_{wCD}(t_p) \), \( p_{CD}(t_p) \), and \( q_{wCD}(t_p) \) solutions.

**Case 2: \( p_{wCD}(t_p) \) approximation based on linear \( p_{CD}(t_p) \)**

Fig. 13 shows the behavior of the \( p_{CD}(t_p) \) function. This plot suggests that the assumption of a linear \( p_{CD}(t_p) \) function yields relatively good results compared to the assumption of \( p_{CD}(t_p) \) being constant. The largest deviation occurs for the time period \( 1 \leq t_p < C_D 10^5 \), for \( C_D = 1 \). However, this deviation is acceptable when compared to the results for case 1 (constant

**PwCD(t_p)).**

Fig. 14 shows that the \( PwCD(t_p) \) function is accurately estimated using numerical differentiation of Eq. 16. The most significant deviations occur for \( 10^5 < t_p < C_D 10^5 \), for \( C_D = 0.3 \) and \( 0.1 \). Again, this behavior is considered acceptable when compared to case 1. Fig. 15 shows that the approximate \( q_{wCD}(t_p) \) function is quite accurate at low values of \( C_D \) and successively more inaccurate for higher values of \( C_D \). This behavior is expected, given the previous observations of the \( PwCD(t_p) \) and \( PCD(t_p) \) functions. These comparisons suggest that use of Eq. 16 should yield accurate approximations of the \( p_{wCD}(t_p) \) and \( PCD(t_p) \) functions, and acceptable estimates of the \( q_{wCD}(t_p) \) function.

**Case 3: \( PCD(t_p) \) approximation based on quadratic \( P_{CD}(t_p) \)**

Fig. 16 shows that the \( P_{CD}(t_p) \) function may not be accurately represented by the assumption of a quadratic relation for \( P_{CD}(t_p) \). In fact, Fig. 16 illustrates that the approximate solutions are slightly greater than the numerical inversion solutions, except at very early times, \( t_p < C_D 1 \). This exaggerated behavior of the approximate \( P_{CD}(t_p) \) function suggests that the solution for this case, Eq. D-5, may not be generally suitable for fractured wells, although we recommend further investigation.

From Fig. 16, we suspect that the approximate solutions for \( P_{wCD}(t_p) \) and \( Q_{CD}(t_p) \) will not be accurate. Fig. 17 illustrates that the approximate derivative function, \( P_{CD}(t_p) \), always exhibits values greater than the numerical inversion solution. This observation is identical to the one in Fig. 16 for the \( P_{wCD}(t_p) \) function. Similarly, Fig. 18 shows that the approximate \( Q_{CD}(t_p) \) function is in significant error except for low values of \( C_D \).

In summary, the results for the \( PwCD(t_p) \), \( P_{CD}(t_p) \) and \( Q_{CD}(t_p) \) approximate functions suggest that Eq. D-5 should not be used for fractured wells.

**Wells in an Infinite-Acting Naturally Fractured Reservoir**

This section considers the application of the explicit relations for wellbore storage and skin effects to a naturally fractured reservoir system. This study considers the pseudosteady-state and transient interporosity flow models. We have chosen to investigate the application of all three \( p_{CD}(t) \) relations to a single case. First we will consider the transient interporosity flow case and later, the interporosity flow case.

**Naturally Fractured Reservoir: Transient Interporosity Flow Case (C_D=10^5, S=10, L=10^-4, and \( \omega=10^{-3} \))**

Fig. 19 shows the comparison of the three \( \mu_{CD}(t) \) relations to the numerically inverted Laplace space solution. Each of the relations yields an accurate approximation of the \( \mu_{CD}(t) \) function for this case. The constant rate solutions, \( P_{CD}(t) \) and \( P_{CD}(t) \), are presented in Fig. 19 to illustrate the agreement of the \( P_{CD}(t) \) and \( P_{CD}(t) \), and \( P_{CD}(t) \) solutions once wellbore storage effects have diminished.

Fig. 19 also shows good agreement with the inversion solution for \( \mu_{CD}(t_p) \), for cases 1 and 3. However, the case 2 approximation for \( \mu_{CD}(t_p) \) agrees almost exactly with the numerical inversion solution. This again suggests that the case 2 method (Eq. 16) should be the general explicit model for wellbore storage and skin effects.

**Naturally Fractured Reservoir: Pseudosteady-State Interporosity Flow Case (C_D=10^4, S=10, L=10^{-4}, and \( \omega=10^{-3} \))**

Fig. 20 shows the comparison of the three \( \mu_{CD}(t) \) relations to the numerically inverted Laplace space solution for the pseudosteady-state interporosity flow case. The approximate
functions agree well with the numerical inversion solution. Next, we consider the \( p_{WCD}(t,\rho) \) functions for this case. Fig. 20 shows that the results of cases 1 and 2 agree well with the numerical inversion solution, but case 3 shows oscillatory deviation and should be deemed a failure for this particular case.

It is important to note that modeling the dip in the derivative function, \( p_{WCD}(t,\rho) \), is no small accomplishment. We feel that since cases 1 and 2 agree very well with the \( p_{WCD}(t,\rho) \) solution that these methods should find utility in these types of reservoir cases. The cause of the failure of the case 3 method (Eq. (D-5)) is not clear, but further investigation is warranted before discarding this method. In contrast, it is very clear that Eq. (16) (case 2 solution) should provide accurate estimates of the \( p_{WCD}(t,\rho) \) and \( p_{WCD}(t,\rho) \) functions for any of the reservoir systems considered.

**EXPLICIT CALCULATION OF THE WELLBORE PHASE REDISTRIBUTION DIMENSIONLESS PRESSURE, \( p_{\Phi\Phi}(t) \)**

In this section, we verify two formulas which can be used to explicitly compute the total dimensionless pressure for a system with wellbore storage, skin, and wellbore phase redistribution effects. These relations are developed in Appendix E.

These formulas were constructed in such a way that the total dimensionless pressure, \( p_{\Phi\Phi}(t) \), is given as

\[
p_{\Phi\Phi}(t) = p_{WCD}(t) + p_{\Phi\Phi}(t)
\]

(20)

where \( p_{WCD}(t) \) is the dimensionless wellbore storage pressure and the dimensionless wellbore phase redistribution pressure, \( p_{\Phi\Phi}(t) \), is given as

\[
p_{\Phi\Phi}(t) = \sum_{l=1}^{n} \left( \frac{d}{dt} \left[ p_{\Phi\Phi}(t, l) \right] - \frac{d}{dt} \left[ p_{\Phi\Phi}(t, l-1) \right] \right) p_{WCD}(t, l-1)
\]

(21)

or, alternatively as

\[
p_{\Phi\Phi}(t) = -\sum_{l=1}^{n} \frac{d}{dt} \left[ p_{\Phi\Phi}(t, l-1) \right] p_{WCD}(t, l-1)
\]

(22)

Also, the dimensionless phase redistribution pressure, \( p_{\Phi\Phi}(t) \), is given by \( \Phi^{12} \) as

\[
p_{\Phi\Phi}(t) = C_{\Phi\Phi} \left[ 1 - \exp \left( -\frac{t}{\tau_{\Phi\Phi}} \right) \right]
\]

(23)

Our experience suggests that, when using the model of \( p_{\Phi\Phi}(t) \) given by Eq. 23, Eq. 22 will yield the most accurate results for \( p_{\Phi\Phi}(t) \) relative to the numerical inversion of Eq. E-10.

We present Fig. 21 as a verification of Eq. 22. Fig. 21 assumes an unfractured well in an infinite-acting homogeneous reservoir. The properties used to generate Fig. 21 are: \( C_{D} = 10^{5}, C_{D} = 20, D_{P} = 10^{5}, S = 0 \). Note that the numerical inversion solution and the explicit computation (Eq. 22) yield identical results for the \( p_{\Phi\Phi}(t) \) function. This is a very substantial result which shows that the total dimensionless pressure, \( p_{\Phi\Phi}(t) \), can be computed without inversion of Laplace space relations (for \( p_{\Phi\Phi}(t) \) given in explicit form by Eq. 23). The advantages of using the explicit calculation also include the prospect for de-coupling of the wellbore storage and wellbore phase redistribution solutions, which may lead to a rigorous analysis method for pressure data distorted by wellbore phase redistribution.

Fig. 21 also shows that the explicit and numerical inversion calculations of the derivative function, \( p_{\Phi\Phi}(t) \), agree almost exactly. This agreement of derivatives suggests that for any case of wellbore phase redistribution, the numerical inversion and explicit calculations should agree very well.

**SUMMARY AND CONCLUSIONS**

In summary, we have developed accurate, real time approximations for the dimensionless pressure function which includes effects of wellbore and skin effects, \( p_{WCD}(t) \). Of the three relations we present, the case which assumes that the constant rate \( p_{WCD}(t) \) function is linear in time appears to give the most accurate and consistent results.

We have also successfully de-coupled the wellbore storage and wellbore phase redistribution problems in such a fashion that explicit relations can be written for the total wellbore dimensionless pressure, \( p_{WCD}(t) \), which includes the effects of wellbore storage, skin, and wellbore phase redistribution effects. These explicit relations for \( p_{WCD}(t) \) yield very good agreement with the numerical inversion solutions and should be considered to have the same order of accuracy as the numerical inversion solutions.

**Conclusions:**

1. Three explicit relations have been developed for the computation of the dimensionless wellbore pressure with wellbore storage and skin effects, \( p_{WCD}(t) \). These relations have been verified for fractured and unfractured wells in homogeneous reservoirs and for unfractured wells in naturally fractured (dual porosity) reservoirs.

2. Of the three relations we present for \( p_{WCD}(t) \), the case which assumes that the constant rate \( p_{WCD}(t) \) function is linear in time (case 2) exhibits the most accurate and consistent behavior of all of the approximations.

3. The explicit relations developed for the computation of the dimensionless wellbore pressure which includes wellbore storage, skin effects, and the effect of wellbore phase redistribution (\( p_{WCD}(t) \)) have been verified to be on the same order of accuracy as the Laplace transform numerical inversion solutions for this problem.

**NOMENCLATURE**

**Dimensionless Variables**

- \( C_{D} \) = dimensionless "apparent" wellbore storage coefficient for wellbore phase redistribution model
- \( C_{L} \) = dimensionless wellbore storage coefficient
- \( C_{L}_{D} \) = dimensionless wellbore storage coefficient based on fracture half-length
- \( C_{W} \) = dimensionless coefficient for wellbore phase redistribution model
- \( D_{P} \) = dimensionless pressure with skin effects
- \( D_{P} \) = dimensionless pressure with wellbore storage, skin, and wellbore phase redistribution effects
- \( D_{W} \) = dimensionless pressure with wellbore storage and skin effects
- \( D_{WCD} \) = dimensionless pressure with wellbore storage, skin, and wellbore phase redistribution effects
- \( D_{WCD} \) = dimensionless pressure with wellbore storage, skin, and wellbore phase redistribution effects
- \( D_{WCD} \) = dimensionless pressure with wellbore storage, skin, and wellbore phase redistribution effects
- \( D_{WCD} \) = dimensionless pressure with wellbore storage, skin, and wellbore phase redistribution effects
- \( D_{WCD} \) = dimensionless pressure with wellbore storage, skin, and wellbore phase redistribution effects
- \( D_{WCD} \) = dimensionless pressure with wellbore storage, skin, and wellbore phase redistribution effects
- \( D_{WCD} \) = dimensionless pressure with wellbore storage, skin, and wellbore phase redistribution effects
- \( S \) = dimensionless skin factor
- \( \tau_{P} \) = dimensionless time based on wellbore radius

**Eqs.**

- Eq. (1)
- Eq. (2)
- Eq. (3)
- Eq. (4)
- Eq. (5)
- Eq. (6)
- Eq. (7)
- Eq. (8)
- Eq. (9)
- Eq. (10)
- Eq. (11)
- Eq. (12)
- Eq. (13)
- Eq. (14)
- Eq. (15)
- Eq. (16)
- Eq. (17)
- Eq. (18)

Appendix A: Derivation of Laplace Transform Identities for Wellbore Storage Distortion

The convolution integral for wellbore storage is given as

$$P_{wCD}(t) = \int_{0}^{t} \frac{d}{ds} \left[ Q_{wCD}(s) \right] P_{ID} (t-s) \; ds$$

(A-1)

where

$$Q_{wCD}(t) = 1 - C_D \frac{d}{dt} [P_{wCD}(t)]$$

(A-2)

and

$$P_{ID}(t) = P_D(t) + S$$

(A-3)

Taking the Laplace transform of Eq. A-1 gives

$$\tilde{P}_{wCD}(s) = u \tilde{Q}_{wCD}(s) \tilde{P}_D(s)$$

(A-4)

Taking the Laplace transform of Eq. A-2 gives

$$\tilde{Q}_{wCD}(s) = \frac{1}{u} - C_D \tilde{P}_{wCD}(s)$$

(A-5)

Combining Eqs. A-4 and A-5 and solving for $\tilde{P}_{wCD}(s)$ gives

$$\tilde{P}_{wCD}(s) = \frac{1}{\tilde{P}_D(s) + C_D s^2}$$

(A-6)

or alternatively

$$\tilde{P}_{wCD}(s) = \frac{\tilde{P}_D(s)}{1 + C_D s^2 \tilde{P}_D(s)}$$

(A-7)

Eqs. A-6 and A-7 are mathematically equivalent, but these relations also provide convenient simplifications when used with certain relations of $P_{ID}(t)$. These relations will be discussed as they arise in the following derivations.

Appendix B: Derivation of Approximations for $P_{wCD}(t)$

We do not assume that $P_D(t)$ is constant for all $t_0$, only that $P_D(t_0)$ can be approximated as being constant near a particular time of interest. Proceeding along this theme...
\[ P_{CD}(u) = \alpha \] \hspace{1cm} (B-1)

Taking the Laplace transform of Eq. B-1 gives
\[ \tilde{P}_{CD}(s) = \frac{\alpha}{u} \] \hspace{1cm} (B-2)

Substituting Eq. B-2 into Eq. A-6 gives
\[ \tilde{P}_{wCD}(u) = \frac{1}{\frac{u}{2} + C_D u^2} \] \hspace{1cm} (B-3)

Factoring the denominator of Eq. B-3 and rearranging gives
\[ \tilde{P}_{wCD}(u) = \frac{1}{C_D u} \left[ u + \frac{1}{\delta C_D} \right] \]

or
\[ \tilde{P}_{wCD}(u) = \frac{\alpha}{u} \left[ u + \frac{1}{\delta C_D} \right] \] \hspace{1cm} (B-4)

where
\[ \alpha = \frac{1}{C_D} \]

and
\[ \beta = \frac{1}{\delta C_D} \]

The inverse Laplace transform of Eq. B-4 is
\[ p_{wCD}(t) = \frac{\alpha}{\beta} \left[ 1 - \exp\left(-\frac{t}{\beta C_D}\right) \right] \]

Using the definitions of \( \alpha \) and \( \beta \) for this problem, we obtain
\[ p_{wCD}(t) = \frac{a}{u} \left[ 1 - \exp\left(-\frac{t}{\beta C_D}\right) \right] \] \hspace{1cm} (B-5)

From Eq. B-1 we find that
\[ a = p_{LD}(t) \] \hspace{1cm} (B-6)

Combining Eqs. B-5 and B-6 gives
\[ p_{wCD}(t) = p_{LD}(t) \left[ 1 - \exp\left(-\frac{t}{\beta C_D}\right) \right] \] \hspace{1cm} (B-7)

At this point we would like to differentiate Eq. B-7 with respect to \( t \). We have two options. First we will "blindly" differentiate Eq. B-7 assuming that \( p_{LD}(t) \) constant. This gives
\[ \frac{d}{dt} [p_{wCD}(t)] = \frac{1}{C_D} \exp\left(-\frac{t}{\beta C_D}\right) \] \hspace{1cm} (B-8)

Second, we will differentiate Eq. B-7 assuming \( p_{LD}(t) = f(t) \). This gives
\[ \frac{d}{dt} [p_{wCD}(t)] = \left[ 1 - \exp\left(-\frac{t}{\beta C_D}\right) \right] \frac{d}{dt} [p_{LD}(t)] \]

\[ + \frac{1}{p_{LD}(t) \beta C_D} [p_{LD}(t) - tf(t) \exp\left(-\frac{t}{\beta C_D}\right)] \] \hspace{1cm} (B-9)

Rearranging Eq. B-7 and solving for the exponential term gives
\[ \exp\left(-\frac{t}{\beta C_D}\right) = \frac{1}{C_D} \frac{p_{wCD}(t)}{p_{LD}(t)} \] \hspace{1cm} (B-10)

Combining Eqs. B-8 and B-10 gives
\[ \frac{d}{dt} [p_{wCD}(t)] = \frac{1}{C_D} \left[ 1 - \frac{p_{wCD}(t)}{p_{LD}(t)} \right] \] \hspace{1cm} (B-11)

Combining Eqs. B-7, B-9 and B-10 gives

\[ \frac{d}{dt} [p_{wCD}(t)] = \frac{p_{wCD}(t)}{p_{LD}(t)} \left[ \frac{p_{wCD}(t)}{p_{LD}(t)} \right] \]

\[ + \frac{1}{C_D} \left[ 1 - \frac{p_{wCD}(t)}{p_{LD}(t)} \right] \left[ 1 - \frac{p_{wCD}(t)}{p_{LD}(t)} \right] \]

\[ \frac{d}{dt} [p_{wCD}(t)] = \frac{1}{C_D} \left[ 1 - \frac{p_{wCD}(t)}{p_{LD}(t)} \right] \] \hspace{1cm} (B-12)

The purpose of obtaining the derivative function is to generate plotting functions for type curve analysis and for use in the computation of dimensionless sandface flow rates, \( q_{wCD} \), as given by Eq. A-2. Recalling Eq. A-2, we have
\[ q_{wCD}(t) = \frac{1}{C_D} \frac{d}{dt} [p_{wCD}(t)] \] \hspace{1cm} (A-2)

Combining Eqs. A-2 and B-8 gives
\[ q_{wCD}(t) = 1 - \exp\left(-\frac{t}{\beta C_D}\right) \] \hspace{1cm} (B-13)

which is of the form
\[ q_{wCD} = 1 - \exp(-\beta t) \] \hspace{1cm} (B-14)

where
\[ \beta = \frac{1}{\beta C_D} \] \hspace{1cm} (B-15)

Eq. B-14 was proposed originally by van Everdingen, and used by Ramey for well test analysis. van Everdingen proposed Eq. B-14 based on empirical observations of field data, whereas we propose Eq. B-14 based on analytical considerations. The significance of our derivation is not that it proves van Everdingen's observations, rather we show that the form of Eq. B-14 can be developed rigorously. The applicability of Eq. B-14 must be determined via comparison of Eqs. A-2 and B-14 using simulated data.

Similar results for \( q_{wCD} \) could be obtained by combining Eqs. A-2 and B-9 (or B-12). However, for the purposes of the present discussion, we only wished to verify the van Everdingen model for \( q_{wCD} \).

Appendix C: Derivation of Approximations for \( p_{wCD}(t) \) --

Case 2: \( p_{LD}(t) = a + bt \)

Surfing with the linear model
\[ p_{LD}(t) = a + bt \] \hspace{1cm} (C-1)

Taking the Laplace transform of Eq. C-1 gives
\[ \tilde{p}_{LD}(s) = \frac{a}{u} + \frac{b}{u^2} \] \hspace{1cm} (C-2)

Substituting Eq. C-2 into A-7 gives
\[ \tilde{p}_{wCD}(u) = \frac{a}{u} + \frac{b}{u^2} \] \hspace{1cm} (C-3)

Separating the equation in the numerator gives
\[ \tilde{p}_{wCD}(u) = \frac{a}{u+C_D u^2 + C_D b u} + \frac{b}{u^2} \] \hspace{1cm} (C-4)

\[ = \frac{a}{u(C_D u + C_D b)} + \frac{b}{u^2} \] \hspace{1cm} (C-4)

\[ = \frac{a}{C_D u + C_D b} + \frac{b}{u^2} \] \hspace{1cm} (C-4)

\[ = \beta \frac{1}{u(u+\theta)} + \theta \frac{1}{u^2} \] \hspace{1cm} (C-4)
where
\[ \alpha = 1 + \frac{Cp}{\frac{Cp}{a}} \]
\[ \beta = \frac{b}{Cp} \]
\[ \theta = \frac{b}{Cp} \]

The inverse Laplace transform of Eq. C-4 is
\[ P_{wCD}(t) = \frac{b}{\alpha} \left[ 1 - \exp(-\alpha t) \right] + \frac{\beta}{\alpha^2} \left[ \exp(-\alpha t) - \alpha t - 1 \right] \]  (C-5)

For convenience, we leave our result in the form given by Eq. C-5. Now we must consider a scheme to determine the coefficients \( a \) and \( b \) in the \( p_{CD}(t) \) model. Recall Eq. C-1
\[ p_{CD}(t) = a + b D \]  (C-1)

Differentiating Eq. C-1 with respect to \( t \) gives
\[ \frac{d}{dt} p_{CD}(t) = b \]  (C-6)

Combining Eqs. C-1 and C-5 and solving for the \( a \) coefficient gives
\[ a = p_{CD}(t) - t b \left( \frac{d}{dt} p_{CD}(t) \right) \]  (C-7)

In conclusion, we have developed Eq. C-5 based on the assumption of \( p_{CD}(t) \) behaving in a linear fashion, at least locally. Although a closed form derivative of Eq. C-5 could be developed, this expression is so complex (recall \( a, \beta, \theta \) are functions of time) that numerical differentiation would be more efficient.

Appendix D: Derivation of Approximations for \( p_{CD}(t) \) - Case 2: \[ p_{CD}(t) = a_{01} + a_{11} t + a_{21} t^2 \]  (D-1)

Starting with the quadratic model
\[ p_{CD}(t) = a_{01} + a_{11} t + a_{21} t^2 \]  (D-1)

Taking the Laplace transform of Eq. D-1 gives
\[ \bar{p}_{CD}(\alpha) = \frac{a_{01} + a_{11}}{\alpha^2 + \alpha + a_{21}} \]  (D-2)

Substituting Eq. D-2 into A-7 gives
\[ \bar{P}_{CD}(s) = \frac{a_{01} + a_{11} + a_{21}}{1 + C_D u^2 \left( \frac{a_{01}}{u^2} + \frac{a_{11}}{u} + a_{21} \right)} \]  (D-3)

Rearranging Eq. D-3 gives
\[ \bar{P}_{CD}(s) = \frac{a_{01} + a_{11} + a_{21}}{u^2 + \alpha u + \beta} + \frac{a_{21}}{u^2 (u^2 + \alpha u + \beta)} \]  (D-4)

where
\[ a_{21} = 2 a_{21}' \]
\[ b_{21} = 1 + C_D a_{21}' \]
\[ b_{21}' = C_D a_{21}' \]
\[ c_{21} = a_{21}' \]
\[ c_{21}' = a_{21} \]
\[ a = b_{21} \]
\[ \beta = b_{21}' \]

We will also make the following definitions for convenience
\[ \eta = \sqrt{\frac{\beta}{4} - \frac{\alpha^2}{4}} \]
\[ \zeta = a \]

And finally
\[ A = 1 + \frac{a}{2 \eta} \]
\[ B = \frac{a}{2 \eta} \]
\[ C = \frac{a}{2 \eta} \]

Taking the inverse Laplace transform of Eq. D-4 gives
\[ p_{CD}(t) = \frac{c_0 h_0 (t)}{c_1 f_1 (t) + c_2 f_2 (t)} \]  (D-5)

where
\[ h_0 (t) = A \left[ \exp(-B t) - \exp(-C t) \right] \]  (D-6)
\[ f_1 (t) = A \left[ \frac{1}{B} \left[ 1 - \exp(-B t) \right] - \frac{1}{C} \left[ 1 - \exp(-C t) \right] \right] \]  (D-7)
\[ f_2 (t) = A \left[ \frac{1}{B} \left[ 1 - \exp(-B t) \right] - \frac{1}{C} \left[ 1 - \exp(-C t) \right] \right] \]  (D-8)

Although Eq. D-5 is a bit tedious for hand calculations, it should be relatively easy to program into a calculator or spreadsheet application software package. We are, however, left with the problem of determining the coefficients in Eq. D-1. We recommend the use of a quadratic collocation polynomial over a 3-point grid. This computational procedure is initiated by calculating the collocation coefficients. For a 3-point grid, collocation coefficients are
\[ y_0 = p_{CD}(0) \]
\[ y_1 = p_{CD}(t_0) - p_{CD}(0) = \frac{c_0}{t_0} \]
\[ y_2 = p_{CD}(t_2) - y_0 - y_1 (t_2 - t_0) \]
\[ t_0 = t_0 \]
\[ t_1 = t_1 \]
\[ t_2 = t_2 \]

And the \( a \) coefficients are
\[ a_0 = y_0 - y_1 t_0 - y_2 t_0 t_1 \]
\[ a_1 = y_1 - y_2 (t_0 + t_1) \]
\[ a_2 = y_2 \]

The results presented in this work for the calculation of Eq. D-5 used the preceding method to compute the \( a \) coefficients.

Appendix E: Derivation of Explicit Formulae for the Computation of Wellbore Phase Redistribution Effects

The purpose of this derivation is to provide explicit means to compute wellbore phase redistribution effects. Previously, the effects have only been computed using Laplace space solutions. This appendix provides a rigorous derivation of convolution identities which use the dimensionless wellbore storage pressure, \( p_{CD}(t) \), and the dimensionless phase redistribution pressure, \( p_{CD}(t) \).

The dimensionless sandface flow rate, \( q_{CD} \), for this case is given by Faur13 as
\[ q_{CD} = 1 - C_D \left[ \frac{d}{dt} p_{CD}(t) - \frac{d}{dt} p_{CD}(t) \right] \]  (E-1)

The convolution integral for this case is
\[ P_{CD}(t) = \int_0^t \frac{d}{dt} \left( q_{CD}(t) \right) p_{CD}(t - \eta) \, d\eta \]  (E-2)
The Laplace transform of Eq. E-2 is
\[ \mathcal{F}_{wD}(u) = \mathcal{F}_{wCD}(u) \]  
(E-3)
Taking the Laplace transform of Eq. E-1 we have
\[ \mathcal{F}_{wCD}(u) = \frac{u}{uC_D \mathcal{F}_{wD}(u) - \mathcal{F}_{wCD}(u)} \]  
(E-4)
Rearranging Eq. E-3 gives
\[ \frac{\mathcal{F}_{wD}(u)}{\mathcal{F}_{wD}(u)} = \mathcal{F}_{wCD}(u) \]  
(E-5)
Rearranging Eq. E-4 gives
\[ u \mathcal{F}_{wCD}(u) = 1 - u^2 C_D [\mathcal{F}_{wD}(u) \cdot \mathcal{F}_{wCD}(u)] \]  
(E-6)
Equating Eqs. E-5 and E-6 gives
\[ \frac{\mathcal{F}_{wD}(u)}{\mathcal{F}_{wD}(u)} = 1 - u^2 C_D [\mathcal{F}_{wD}(u) \cdot \mathcal{F}_{wCD}(u)] \]  
(E-7)
Solving for \( \mathcal{F}_{wD}(u) \) gives
\[ \mathcal{F}_{wD}(u) = \frac{1}{\mathcal{F}_{wD}(u) + u^2 C_D} \]  
(E-8)
We recognize that Eq E-8 is the relation for wellbore storage that has been addressed in previous appendices.

Eq. E-7 can be rewritten as
\[ \mathcal{F}_{wD}(u) = \mathcal{F}_{wCD}(u) + \mathcal{F}_{wCD}(u) \]  
(E-9)
where
\[ \mathcal{F}_{wCD}(u) = \frac{1}{\mathcal{F}_{wD}(u) + u^2 C_D} \]  
(E-10)
The inverse Laplace transform of Eq. E-9 is given as
\[ \mathcal{F}_{wD}(u) = \mathcal{F}_{wCD}(u) + \mathcal{F}_{wCD}(u) \]  
(E-11)
Eq. E-11 suggests that we can express the effect of distortion due to wellbore phase redistribution as a component term added to existing wellbore storage solution. The application of this method will depend on our ability to obtain the inverse Laplace transform of Eq. E-10.

Taking the inverse Laplace transform, we obtain the following using the convolution identity
\[ \mathcal{F}_{wCD}(u) = C_D \int_0^t \frac{d^2}{dt^2} [\mathcal{F}_{wCD}(t)] \mathcal{F}_{wCD}(t - \tau) d\tau \]  
(E-12)

\[ \mathcal{F}_{wCD}(u) = \frac{u}{uC_D \mathcal{F}_{wD}(u) - \mathcal{F}_{wCD}(u)} \]  
(E-4)

Rearranging Eq. E-3 gives
\[ \frac{\mathcal{F}_{wD}(u)}{\mathcal{F}_{wD}(u)} = \mathcal{F}_{wCD}(u) \]  
(E-5)

Rearranging Eq. E-4 gives
\[ u \mathcal{F}_{wCD}(u) = 1 - u^2 C_D [\mathcal{F}_{wD}(u) \cdot \mathcal{F}_{wCD}(u)] \]  
(E-6)

Equating Eqs. E-5 and E-6 gives
\[ \frac{\mathcal{F}_{wD}(u)}{\mathcal{F}_{wD}(u)} = 1 - u^2 C_D [\mathcal{F}_{wD}(u) \cdot \mathcal{F}_{wCD}(u)] \]  
(E-7)

Solving for \( \mathcal{F}_{wD}(u) \) gives
\[ \mathcal{F}_{wD}(u) = \frac{1}{\mathcal{F}_{wD}(u) + u^2 C_D} \]  
(E-8)

We recognize that Eq E-8 is the relation for wellbore storage that has been addressed in previous appendices.

Eq. E-7 can be rewritten as
\[ \mathcal{F}_{wD}(u) = \mathcal{F}_{wCD}(u) + \mathcal{F}_{wCD}(u) \]  
(E-9)

where
\[ \mathcal{F}_{wCD}(u) = \frac{1}{\mathcal{F}_{wD}(u) + u^2 C_D} \]  
(E-10)

The inverse Laplace transform of Eq. E-9 is given as
\[ \mathcal{F}_{wD}(u) = \mathcal{F}_{wCD}(u) + \mathcal{F}_{wCD}(u) \]  
(E-11)

Eq. E-11 suggests that we can express the effect of distortion due to wellbore phase redistribution as a component term added to existing wellbore storage solution. The application of this method will depend on our ability to obtain the inverse Laplace transform of Eq. E-10.

Taking the inverse Laplace transform, we obtain the following using the convolution identity
\[ \mathcal{F}_{wCD}(u) = C_D \int_0^t \frac{d^2}{dt^2} [\mathcal{F}_{wCD}(t)] \mathcal{F}_{wCD}(t - \tau) d\tau \]  
(E-12)

or, alternatively
Figure 1 - Type curve plot of $P_{w,m}$ for a homogeneous reservoir. $P_{w,m}$ computed using constant $P_{w,o}$ assumption.
Figure 2 - Type curve plot of $p_{wCD}$ for a homogeneous reservoir. $p_{wCD}$ computed using constant $p_{ab}$ assumption.
Figure 3 - Type curve plot of $q_{w,CD}$ for a homogeneous reservoir. $q_{w,CD}$ computed using constant $P_{rf}$ assumption.
Figure 4 - Type curve plot of $p_{wCD}$ for a homogeneous reservoir. $p_{wCD}$ computed using linear $p_{eD}$ assumption.
Figure 5 - Type curve plot of $p_{wCD}'$ for a homogeneous reservoir. $p_{wCD}'$ computed using linear $p_{AD}$ assumption.
Figure 6 - Type curve plot of $q_{wCD}$ for a homogeneous reservoir. $q_{wCD}$ computed using linear $p_{sd}$ assumption.
Figure 7 - Type curve plot of $p_{wCD}$ for a homogeneous reservoir. $p_{wCD}$ computed using quadratic $p_{aD}$ assumption.
Figure 8 - Type curve plot of $p_{wCD}'$ for a homogeneous reservoir. $p_{wCD}'$ computed using quadratic $p_{esD}$ assumption.
Figure 9 - Type curve plot of $q_{wCD}$ for a homogeneous reservoir. $q_{wCD}$ computed using quadratic $p_{sD}$ assumption.
Figure 10 - Type curve plot of $p_{wCD}$ for a vertically fractured well (infinite conductivity fracture) in a homogeneous reservoir. $p_{wCD}$ computed using constant $p_{sD}$ assumption.
Figure 11 - Type curve plot of $p_{wCD}'$ for a vertically fractured well (infinite conductivity fracture) in a homogeneous reservoir. $p_{wCD}'$ computed using constant $p_{sd}$ assumption.
Figure 12 - Type curve plot of $q_{wCD}$ for a vertically fractured well (infinite conductivity fracture) in a homogeneous reservoir. $q_{wCD}$ computed using constant $p_{D}$ assumption.
Figure 13 - Type curve plot of $p_{wCD}$ for a vertically fractured well (infinite conductivity fracture) in a homogeneous reservoir. $p_{wCD}$ computed using linear $p_{sD}$ assumption.
Figure 14 - Type curve plot of $p_{wCD}'$ for a vertically fractured well (infinite conductivity fracture) in a homogeneous reservoir. $p_{wCD}'$ computed using linear $p_{ID}$ assumption.
Figure 15 - Type curve plot of $q_{w,c}^{CD}$ for a vertically fractured well (infinite conductivity fracture) in a homogeneous reservoir. $q_{w,c}^{CD}$ computed using linear $P_f$ assumption.
Figure 16 - Type curve plot of $p_{wCD}$ for a vertically fractured well (infinite conductivity fracture) in a homogeneous reservoir. $p_{wCD}$ computed using quadratic $p_{D}$ assumption.
Figure 17 - Type curve plot of $P_{w,c}$ for a vertically fractured well (infinite conductivity fracture) in a homogeneous reservoir. $P_{w,c}$ computed using quadratic $p$ assumption.
Figure 18 - Type curve plot of $q_{wCD}$ for a vertically fractured well (infinite conductivity fracture) in a homogeneous reservoir. $q_{wCD}$ computed using quadratic $p_0$ assumption.
Figure 19 - Type curve plot of $P_{\infty}$ approximations for a well in a naturally fractured reservoir (transient interporosity flow, $C_o=1 \times 10^5$, $S=10$, $\lambda=1 \times 10^{-6}$, and $\omega=1 \times 10^{-3}$).
Figure 20 - Type curve plot of $p_{wCD}$ approximations for a well in a naturally fractured reservoir (pseudosteady-state interporosity flow, $C_D=1\times10^5$, $S=10$, $\lambda=1\times10^{-6}$, and $\omega=1\times10^{-3}$).
Figure 21 - Comparison of numerical inversion solution and results computed using the explicit phase redistribution calculation. Line source (radial flow) solution \((C_D=10^2, C_{aD}=20, C_{\Phi D}=10^2, S=0)\).
Appendix C: Derivation of Approximations for $p_{wCD}(t_D) -$

Case 2: $p_{AD}(t_D) = a + bt_D$

Starting with the linear model

$$p_{AD}(t_D) = a + bt_D$$  \hspace{1cm} (C-1)

Taking the Laplace transform of Eq. C-1 gives

$$\bar{p}_{AD}(u) = \frac{a + \frac{b}{u}}{u^2}$$  \hspace{1cm} (C-2)

Substituting Eq. C-2 into A-7 gives

$$\bar{p}_{wCD}(u) = \frac{\left(\frac{a + \frac{b}{u}}{u^2}\right)}{1 + CDu^2\left(\frac{a + \frac{b}{u}}{u^2}\right)}$$  \hspace{1cm} (C-3)

Separating the equation in the numerator gives

$$\bar{p}_{wCD}(u) = \frac{a}{u + CDau^2 + CDbu} + \frac{b}{u^2 + CDau^3 + CDbu^2}$$

$$= \frac{a}{u(CDau + 1 + CDb)} + \frac{b}{u^2(CDau + 1 + CDb)}$$

$$= \frac{1}{CDu\left(u + \frac{1 + CDb}{CDa}\right)} + \frac{b}{u^2\left(u + \frac{1 + CDb}{CDa}\right)}$$

$$= \beta \frac{1}{u(u + a)} + \theta \frac{1}{u^2(u + a)}$$  \hspace{1cm} (C-4)

where

$$\alpha = \frac{1 + CDb}{CDa}$$

$$\beta = \frac{a}{CD} \times \phi = \frac{1}{CD}$$

$$\theta = \frac{b}{CD} \times \theta = \frac{b}{CD}$$

The inverse Laplace transform of Eq. C-4 is

$$p_{wCD}(t_D) = \frac{\beta}{\alpha} \left[1 - e^{-\alpha t_D}\right] + \frac{\beta}{\alpha^2} \left[\exp(-\alpha t_D) + \alpha t_D - 1\right]$$  \hspace{1cm} (C-5)

**Note corrections for Eqs. in Appendix C of SPE 21826.**

**This may be useful for Hwk 9. Sorry for any inconvenience - I forgot about these typos...**

Tom Blasingame 11 Dec 1998