Estimating the Stabilized Deliverability of a Gas Well Using the Rawlins and Schellhardt Method: An Analytical Approach

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ABSTRACT

This paper introduces a direct method to use the results of Rawlins and Schellhardt analysis to derive the constants "C" and "n" in the Rawlins and Schellhardt gas well deliverability equation. The motivation for this effort is the need to report the results of Rawlins and Schellhardt analysis to regulatory agencies, and the widespread use of their deliverability equation by engineers. We present a detailed procedure which shows how these results can be applied to deliverability forecasting. This paper includes an illustrative example in which the new method is applied to field data from the literature. This example presents comparisons between Houpeurt and Rawlins and Schellhardt analyses and shows the correlation between the two methods.

INTRODUCTION

The purpose of deliverability testing is to determine a gas well's production capabilities under specific reservoir conditions. A common productivity indicator obtained from these tests is the absolute open flow (AOF) potential, which is defined as the maximum rate at which a well could flow against a theoretical atmospheric backpressure at the sandface. Although in practice the well cannot produce at this rate, the AOF is often used by regulatory agencies for establishing field proration schedules and setting maximum allowable production rates for individual wells.

A number of testing techniques have been developed to assess a gas well's deliverability characteristics. Flow-after-flow tests are conducted by producing the well at a series of different flow rates and measuring the stabilized bottomhole flowing pressures. Each flow rate is established in succession without an intermediate shut-in period. The primary limitation of these tests is the long time required to reach stabilization in low permeability reservoirs. Consequently, the isochronal and modified isochronal tests were developed to shorten test times.

An isochronal test is conducted by alternatively producing the well, then shutting it in and allowing it to build up to the average reservoir pressure prior to the beginning of the next flow period. The modified isochronal test is conducted similarly, except

duration of the shut-in times often is not long enough to reach the true average reservoir pressure in the well's drainage area. Although isochronal and modified isochronal tests were developed to circumvent the long flow times required in low permeability reservoirs, these tests may still require a single, stabilized flow period at the end of the test in order to estimate the stabilized producing capacity of the well.

The conventional deliverability test analysis technique was proposed by Rawlins and Schellhardt. They observed that a log-log plot of the difference between the squares of the average reservoir pressure and the bottomhole flowing pressure against gas flow rate can be represented by a straight line defined by

\[ q_g = C (p_a^2 - p_{wf}^2)^n \]  

where \( C \) is defined as the stabilized performance coefficient, and \( n \) is the reciprocal of the slope of the straight line. Extrapolation of this line to the difference between the squares of the average reservoir pressure and the bottomhole flowing pressure equal to atmospheric pressure defines the AOF.

Eq. 1 was developed empirically from the observation of a number of gas well tests. Extrapolation of Eq. 1 over large variations in pressure can result in incorrect estimates of the AOF. Subsequent theoretical developments by Houpeurt have shown that a more accurate analysis for gas flow is possible with

\[ p_a^2 - p_{wf}^2 = a q_g + b q_g^2 \]  

where the flow coefficients, \( a \) and \( b \), are defined by

\[ a = \frac{1.422 \times 10^6 \mu_k T D}{k_g h} \left[ \frac{1.151 \log \left( \frac{10.06 A}{C_A k_w} \right)}{4} \right] \]  

\[ b = \frac{1.422 \times 10^6 \mu_k T D}{k_g h} \]  

Eq. 2 is a solution to the diffusivity equation for radial flow. Although the Houpeurt equation has a theoretical basis and is rigorously correct, the more familiar but empirically based Rawlins and Schellhardt equation continues to be used, indeed favored, by the natural gas industry. Consequently, we have combined the two analysis techniques and have developed a more
accurate version of the Rawlins-Schellhardt method which can be used in deliverability forecasting. Our technique can be used to estimate the stabilized performance coefficient, $C$, without requiring stabilized flowing conditions and is especially useful for analyzing isochronal and modified isochronal tests without a stabilized flow period. This is a simple method which requires only data from a modified isochronal test to develop a performance prediction for the well without a priori estimates of reservoir properties.

DEFINITIONS AND THEORETICAL DEVELOPMENT

Our deliverability test analysis technique is derived by equating derivatives of $\log (q_g)$ with respect to $\log (\Delta p)$ from both the Rawlins-Schellhardt and Houpeurt equations. A similar method was used by Brigham, Duong, and Poettmann and Kazemi to develop equations in terms of pressures-squared for estimating reservoir properties from gas deliverability tests. Because of the pressure-dependent gas properties, the pressure-squared forms of the deliverability equations (Eqs. 1 and 2) often are inaccurate at high pressures. Therefore, in the subsequent derivation, we use the pseudopressure transformation introduced by Al-Hussainy, et al.\textsuperscript{17}

$$p_p = 2 \int_{p_b}^{p} \frac{\rho \ dp}{\mu_s(p) \frac{\partial}{\partial p}} \tag{5}$$

Our method is also applicable, however, for deliverability equations written with pressure-squared as the dependent variable.

In terms of pseudopressure, the Rawlins and Schellhardt equation becomes

$$q_g = C [p_p(p_b) - p_p(p_w)]^n \tag{6}$$

Similarly, the Houpeurt equation is

$$p_p(p_b) - p_p(p_w) = a q_g + b q_g^2 \tag{7}$$

where the flow coefficients, $a$ and $b$, are defined by

$$a = 1.422 \times 10^6 \frac{r}{k_s h}$$

$$b = 1.422 \times 10^6 \frac{TD}{k_s h} \tag{8}$$

Taking the logarithm of both sides of Eq. 6 yields

$$\log (q_g) = \log (C) + n \log [p_p(p_b) - p_p(p_w)] \tag{9}$$

Rearranging Eq. 10 and solving for $n$ shows that $n$ is the slope of a log-log plot of $q_g$ vs. $q_p$. Alternatively, $n$ can be expressed as the derivative of $\log (q_g)$ with respect to $\log (\Delta p)$:

$$n = \frac{d \log (q_g)}{d \log [p_p(p_b) - p_p(p_w)]} = \frac{1}{q_g} \frac{d q_g}{d \log [p_p(p_b) - p_p(p_w)]} \tag{11}$$

Similarly, taking the logarithm of both sides of Eq. 7 yields

$$\log [p_p(p_b) - p_p(p_w)] = \log (a q_g + b q_g^2) \tag{12}$$

Differentiating $\log (\Delta p)$ with respect to $q_g$ gives

$$\frac{d \log (p_p(p_b) - p_p(p_w))}{d q_g} = \frac{1}{a q_g + b q_g^2} \frac{d [a q_g + b q_g^2]}{d q_g} = \frac{a + 2b q_g}{a q_g + b q_g^2} \tag{13}$$

Substituting Eq. 13 into Eq. 11 yields an equation for $n$ in terms of the Houpeurt flow coefficients:

$$n = \frac{1}{q_g} \frac{d q_g}{d \log (\Delta p)} = \frac{a + 2b q_g}{a q_g + b q_g^2} \tag{14}$$

To develop an expression for the performance coefficient, $C$, in terms of the Houpeurt flow coefficients, we combine Eqs. 10 and 12 to obtain

$$C = \frac{q_g}{a q_g + b q_g^2} = \frac{q_g}{a q_g + b q_g^2}^{1-n} \tag{15}$$

Eq. 15 is similar to a result derived by Poettmann and Kazemi. We show the applications and importance of this development in the procedure in the next section of this paper. The flow rate required in Eq. 15 is defined by solving Eq. 14 for the gas flowrate, $q_g$:

$$q_g = \frac{a (1 - n)}{b (2n - 1)} \tag{16}$$

Implicit in our derivation is the assumption of radial flow of a single-phase gas in a homogeneous, isotropic reservoir. For naturally fractured reservoirs, our method, like conventional deliverability analysis techniques, is valid only after the matrix-fracture system has begun to behave like a single, homogeneous unit. Similarly, our method is valid only after pseudoradial flow is exhibited in hydraulically fractured wells. We also assume wellbore storage effects are negligible.

DELIBERABILITY TEST ANALYSIS

Application of our method assumes that the slope, $1/n$, of the empirical deliverability plot remains constant with time. This assumption implies that, if we can calculate values of $a$ and $b$ (Eqs. 8 and 9, respectively) for given reservoir properties, we also can calculate a flow rate with Eq. 16. We then substitute this flow rate into Eq. 15 and calculate a stabilized $C$ value, and assuming a constant value for $n$, calculate the AOF:

$$AOF = C [p_p(p_b) - p_p(p_w)]^n \tag{17}$$

To apply our new deliverability analysis technique to field data, we present an analysis procedure below. We will then apply this procedure to a field example.

General Analysis Procedure. We recommend the following procedure to analyze isochronal and modified isochronal tests using our technique. Although presented in terms of pseudopressures, this procedure also is applicable with the pressure-squared variables.

1. Plot $\Delta p_p = p_p(p_b) - p_p(p_w)$ vs. $q_g$ on log-log graph paper for the measured flow data.
2. For each flow time, construct the best fit line through the data points. Typically, some of the early data points will not agree with the general trend of the data, so these points should be ignored in all subsequent analyses.
3. Determine the deliverability exponent, $n$, for each best-fit line by least-squares regression analysis using the following equation:

$$n = \frac{1}{q_g} \sum_{j=1}^{N} \left( \log q_g \log \Delta p_p \right) - \sum_{j=1}^{N} \log q_g \sum_{j=1}^{N} \left( \log \Delta p_p \right)$$

$$n = \frac{1}{q_g} \sum_{j=1}^{N} \left( \log q_g \log \Delta p_p \right) - \sum_{j=1}^{N} \log q_g \sum_{j=1}^{N} \left( \log \Delta p_p \right) \tag{18}$$

4. For $M$ isochronal tests, compute the arithmetic average deliverability exponent, $n$. 

...
5. Calculate the theoretical values of the Houpeurt coefficients, \(a\) and \(b\) (Eqs. 8 and 9, respectively), using values of permeability, skin, and the non-Darcy flow coefficient obtained from buildup or drawdown tests on the well, or in absence of these tests, from the modified isochronal test analysis methods presented by Brar and Aziz.\(^9\)

6. Calculate the rate (Eq. 16) at which the change in pressure occurs in the well and skin factor, given below were obtained from the extended flow point. We do not show the conventional methods presented by Brar and Aziz.\(^9\)

7. Calculate the stabilized \(C\) value using Eq. 15 and the result of Step 6 (Eq. 16).

8. Using \(\bar{n}\) from Step 4 and \(C\) from Step 7, calculate the AOF with Eq. 17.

Field Example. This field example is a modified isochronal test from Well 8, analyzed by Brar and Aziz.\(^9\) The well and test data are given below in Tables 1-5. To compare our method with the extended flow point, Table 6 gives a comparison of our results with the results we obtained from the modified isochronal test data, as Brar and Aziz showed. Permeability and skin factor are not necessary if Houpeurt analysis is performed, since the values of \(a\) and \(b\) can be obtained directly from the test data, rather than from theory, Eqs. 3 and 4. These reservoir properties obtained from the drawdown test are used only for comparison in this example.

Table 6 gives a comparison of our results with the results we obtained using the extended flow point. We do not show the conventional modified isochronal test analysis with the extended flow point here.
In this example, we will first illustrate the Brar and Aziz analysis technique; then we will use values of \(a\) and \(b\) from that method to illustrate our new procedure, the stabilized "C" method.

**Analysis Using Brar and Aziz Method**

1. Plot \(\frac{\Delta P_p}{q_g} = \frac{\Delta P_p}{q_g} = \frac{1}{q_g} \frac{P_{pw} - P_p}{P_{pw} - P_{pw} - P_p} \) versus flow rate, \(q_g\), on Cartesian coordinate paper for the measured isochronal flow data. This plotting function is calculated below for each rate.

<table>
<thead>
<tr>
<th>Time, hr</th>
<th>(q_g)</th>
<th>(\Delta P_p/q_g), psia/(cp-MMscf/D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>31.612</td>
<td>1.202 x 10^6</td>
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<tr>
<td>4.0</td>
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</tr>
<tr>
<td>6.0</td>
<td>392 x 10^6</td>
<td>1.248 x 10^6</td>
</tr>
</tbody>
</table>

2. The deliverability plot is shown below in Fig. 1. We now construct best fit lines through the modified isochronal data points for each time. The data points at the second rate (44.313 MMscf/D) do not fit on the same straight line as the other three points. These points at the second rate are therefore ignored in the subsequent calculations.

![Fig. 1 - Brar and Aziz Data Plot for Field Example 1](image)

3. Determine the slopes of the lines, \(b\), for each time by least-squares regression analysis on the first, third and fourth points using Eq. 20.

\[ b = \frac{N \sum_j^{N} \left( \frac{\Delta P_p}{q_g} \right) - \frac{\sum_j^{N} q_g \sum_j^{N} \left( \frac{\Delta P_p}{q_g} \right)}{N} \right)^2}{N \sum_j^{N} \left( q_g \right)^2 - \left( \frac{\sum_j^{N} q_g}{N} \right)^2} \]

For \(t=3.0\) hr

<table>
<thead>
<tr>
<th>Point</th>
<th>(q_g)</th>
<th>(q_g^2)</th>
<th>(\Delta P_p)</th>
<th>(\frac{\Delta P_p}{q_g})</th>
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<td>3</td>
<td>56.287</td>
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<td>8.920 x 10^7</td>
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</tr>
<tr>
<td>4</td>
<td>70.265</td>
<td>4,937</td>
<td>1.323 x 10^8</td>
<td>1.883 x 10^6</td>
</tr>
</tbody>
</table>

\[ b_1 = \frac{3 \left( 2.595 \times 10^6 \right) - \left( 158.164 \right) \left( 4.670 \times 10^6 \right)}{3 \left( 9.104 \times 10^3 \right) - \left( 158.164 \right)^2} \]

\[ b_1 = 1.737 \times 10^4 \text{ psia}^2/(\text{cp-MMscf/D}^2) \]

For \(t=4.0\) hr

<table>
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<tr>
<th>Point</th>
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<td>4,937</td>
<td>1.402 x 10^8</td>
<td>1.995 x 10^6</td>
</tr>
</tbody>
</table>

\[ b_2 = \frac{3 \left( 2.736 \times 10^6 \right) - \left( 158.164 \right) \left( 4.934 \times 10^6 \right)}{3 \left( 9.104 \times 10^3 \right) - \left( 158.164 \right)^2} \]

\[ b_2 = 1.760 \times 10^4 \text{ psia}^2/(\text{cp-MMscf/D}^2) \]

For \(t=5.0\) hr

<table>
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<th>Point</th>
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<th>(q_g^2)</th>
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</tr>
<tr>
<td>4</td>
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<td>4,937</td>
<td>1.454 x 10^8</td>
<td>2.069 x 10^6</td>
</tr>
</tbody>
</table>

\[ b_3 = \frac{3 \left( 2.831 \times 10^6 \right) - \left( 158.164 \right) \left( 5.111 \times 10^6 \right)}{3 \left( 9.104 \times 10^3 \right) - \left( 158.164 \right)^2} \]

\[ b_3 = 1.782 \times 10^4 \text{ psia}^2/(\text{cp-MMscf/D}^2) \]

For \(t=6.0\) hr

<table>
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<th>Point</th>
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<th>(q_g^2)</th>
<th>(\Delta P_p)</th>
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<td>4,937</td>
<td>1.490 x 10^8</td>
<td>2.121 x 10^6</td>
</tr>
</tbody>
</table>

\[ b_4 = \frac{3 \left( 2.891 \times 10^6 \right) - \left( 158.164 \right) \left( 5.220 \times 10^6 \right)}{3 \left( 9.104 \times 10^3 \right) - \left( 158.164 \right)^2} \]

\[ b_4 = 1.815 \times 10^4 \text{ psia}^2/(\text{cp-MMscf/D}^2) \]
4. Determine the average value for the slope by taking an arithmetic average of the \( b \) values calculated at each time.

\[ b = \frac{1.774 \times 10^4 \text{ psia}^2/(\text{cp-MMscf}/\text{D}^2)}{23440} \]

5. Calculate the transient deliverability line intercepts, \( a_t \), for each isochronal line using Eq. 21.

\[
 a_t = \frac{\sum_{j=1}^{N} \left( \frac{\Delta p}{q_j} \right) \sum_{j=1}^{N} (q_j)^2 - \sum_{j=1}^{N} q_j \sum_{j=1}^{N} \left( \frac{\Delta p}{q_j} \right)}{N \sum_{j=1}^{N} (q_j)^2 - \left( \sum_{j=1}^{N} q_j \right)^2}
\]

For \( t=3.0 \) hr

<table>
<thead>
<tr>
<th>point</th>
<th>( q_t )</th>
<th>( q_t^2 )</th>
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<th>( \Delta p_p/q_{q_2} )</th>
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<td></td>
<td>158.164</td>
<td>9,104</td>
<td>2.595 \times 10^8</td>
<td>4.670 \times 10^6</td>
</tr>
</tbody>
</table>

\[ a_{q4} = \left( 4.670 \times 10^6 \right) \left( 9.104 \times 10^3 \right) - \left( 158.164 \right) \left( 2.595 \times 10^8 \right) \]

\[ a_{q4} = 6.411 \times 10^5 \text{ psi}^2/(\text{cp-MMscf}/\text{D}) \]

For \( t=4.0 \) hr

<table>
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<tr>
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<td>158.164</td>
<td>9,104</td>
<td>2.736 \times 10^8</td>
<td>4.934 \times 10^6</td>
</tr>
</tbody>
</table>

\[ a_{q2} = \left( 4.934 \times 10^6 \right) \left( 9.104 \times 10^3 \right) - \left( 158.164 \right) \left( 2.736 \times 10^8 \right) \]

\[ a_{q2} = 7.166 \times 10^5 \text{ psi}^2/(\text{cp-MMscf}/\text{D}) \]

For \( t=5.0 \) hr

<table>
<thead>
<tr>
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<td>1.454 \times 10^8</td>
<td>2.069 \times 10^6</td>
</tr>
<tr>
<td></td>
<td>158.164</td>
<td>9,104</td>
<td>2.831 \times 10^8</td>
<td>5.111 \times 10^6</td>
</tr>
</tbody>
</table>

\[ a_{q3} = \left( 5.111 \times 10^6 \right) \left( 9.104 \times 10^3 \right) - \left( 158.164 \right) \left( 2.831 \times 10^8 \right) \]

\[ a_{q3} = 7.640 \times 10^5 \text{ psi}^2/(\text{cp-MMscf}/\text{D}) \]

6. Plot a graph of \( a_t \) versus \( \log t \). Draw the best-fit line through the data. Note that all of the data lie on the same straight line, therefore all points should be used in the linear regression analysis.

7. Calculate slope, \( m' \), and intercept, \( c' \), of the best-fit line of the plot of \( a_t \) versus \( \log t \). The least squares equations to calculate slope and intercept are given by Eqs. 22 and 23, respectively.

\[
m' = \frac{\sum_{j=1}^{N} (a_j \log t_j) - \left( \sum_{j=1}^{N} a_j \right) \sum_{j=1}^{N} (\log t_j)}{\sum_{j=1}^{N} (\log t_j)^2 - \left( \sum_{j=1}^{N} \log t_j \right)^2}
\]

\[
c' = \frac{\sum_{j=1}^{N} (a_j \log t_j)^2 - \left( \sum_{j=1}^{N} a_j \log t_j \right) \sum_{j=1}^{N} (\log t_j)}{\sum_{j=1}^{N} (\log t_j)^2 - \left( \sum_{j=1}^{N} \log t_j \right)^2}
\]

\[ m' = \frac{4(1.880 \times 10^6) - (2.905 \times 10^6)(2.556)}{4(1.684) - (2.556)^2} \]

\[ m = 4.674 \times 10^5 \text{ psia}^2/(\text{cp-MMscf/D})/\text{cycle} \]

\[ c' = \frac{(2.905 \times 10^6)(1.684) - (1.880 \times 10^6)(2.556)}{4(1.684) - (2.556)^2} \]

\[ c' = 4.276 \times 10^5 \text{ psia}^2/(\text{cp-MMscf/D}) \]

8. Calculate the formation permeability to gas using Eq. 24 and the slope of the semilog straight line calculated from Eq. 22.

\[ k_g = \frac{1.632 \times 10^6 T}{m' h} \quad (24) \]

\[ k_g = \frac{1.632 \times 10^6 (718)}{4(4.674 \times 10^5) (454)} \]

\[ k_g = 5.52 \text{ md} \quad (k_g = 4.23 \text{ md from Drawdown}) \]

9. Calculate the skin factor from Eq. 25

\[ s = 1.15 \left[ \frac{c'}{m'} \log \left( \frac{k_g}{\mu_g C_i A} \right) + 3.23 \right] \quad (25) \]

\[ s = 1.15 \left[ \frac{4.276 \times 10^5}{4.674 \times 10^5} \log \left( \frac{5.52}{0.0675(0.023)(0.000169)(0.2615)^2} \right) + 3.23 \right] \]

\[ s = -5.0 \quad (s = -5.2 \text{ from Drawdown}) \]

10. Determine the stabilization time of the well using Eq. 26 for a symmetrical, non-circular drainage area. The well is centered in a square drainage area with a shape factor of 30.88.

\[ t_s = \frac{3.791 \phi \mu_g C_i A}{k_g C_i A} \quad (26) \]

\[ t_s = 3.791(0.0675)(0.023)(0.000169)(640 \times 43560) \]

\[ t_s = 162.7 \text{ hr} \]

11. Determine the stabilized deliverability line intercept, \( a \), using the stabilization time and the semilog slope, \( m' \), and intercept, \( c' \), in Eq. 27.

\[ a = m' \log (t_s) + c' \quad (27) \]

\[ a = 4.674 \times 10^5 \log (162.7) + (4.276 \times 10^5) \]

\[ a = 1.461 \times 10^5 \text{ psia}^2/(\text{cp-MMscf/D}) \]

12. The pseudopressure at base pressure is 2003.8 psia^2/cp.

13. We now calculate the AOF potential using Eq. 28, the average \( b \) value, and the stabilized \( a \) value calculated from Eq. 27.

\[ \text{AOF} = \frac{a + \sqrt{a^2 + 4 b [p_p(P) - p_p(14.65)]}}{2b} \quad (28) \]

\[ \text{AOF} = \frac{(1.461 \times 10^4) + \sqrt{(1.461 \times 10^4)^2 + 4(1.774 \times 10^7)(1.049 \times 10^5) - 2.003.8}}{2(1.774 \times 10^7)} \]

\[ \text{AOF} = 205.5 \text{ MMscf/D} \]

Analysis Using The Stabilized "C" Method

1. Plot \( \Delta p_p = p_p (p_m) - p_p (p_w) \) versus flow rate, \( q_t \), on log-log graph paper for the measured modified isochronal flow data.

2. The deliverability plot is shown below. Note that the first data point does not follow the trend of the higher rate points, and should be ignored.

3. We may now determine the deliverability exponent, \( n \), for each line by least-squares regression analysis on the last three points, using Eq. 29.

\[ n = \frac{N \sum_{j=1}^{N} (\log q \log \Delta p_p) - \sum_{j=1}^{N} \log q \sum_{j=1}^{N} (\log \Delta p_p)}{N \sum_{j=1}^{N} (\log \Delta p_p)^2 - \left( \sum_{j=1}^{N} (\log \Delta p_p) \right)^2} \quad (29) \]

For \( r = 3.0 \text{ hr} \)

<table>
<thead>
<tr>
<th>Time, hr</th>
<th>( q_t )</th>
<th>( \Delta p_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>3.800 \times 10^7</td>
<td>5.720 \times 10^7</td>
</tr>
<tr>
<td>4.0</td>
<td>4.100 \times 10^7</td>
<td>5.990 \times 10^7</td>
</tr>
<tr>
<td>5.0</td>
<td>4.300 \times 10^7</td>
<td>6.270 \times 10^7</td>
</tr>
<tr>
<td>6.0</td>
<td>4.400 \times 10^7</td>
<td>6.330 \times 10^7</td>
</tr>
</tbody>
</table>

Fig. 3 - Data Plot for Stabilized "C" Analysis for Field Example 1

<table>
<thead>
<tr>
<th>Time, hr</th>
<th>( q_t )</th>
<th>( \Delta p_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>5.6287</td>
<td>70.265</td>
</tr>
<tr>
<td>4.0</td>
<td>9.240 \times 10^7</td>
<td>1.402 \times 10^8</td>
</tr>
<tr>
<td>5.0</td>
<td>9.470 \times 10^7</td>
<td>1.454 \times 10^8</td>
</tr>
<tr>
<td>6.0</td>
<td>9.610 \times 10^7</td>
<td>1.490 \times 10^8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time, hr</th>
<th>( q_t )</th>
<th>( \Delta p_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>31.612</td>
<td>44.313</td>
</tr>
<tr>
<td>4.0</td>
<td>44.313</td>
<td>3.800 \times 10^7</td>
</tr>
<tr>
<td>5.0</td>
<td>5.720 \times 10^7</td>
<td>5.990 \times 10^7</td>
</tr>
<tr>
<td>6.0</td>
<td>6.270 \times 10^7</td>
<td>6.330 \times 10^7</td>
</tr>
</tbody>
</table>
\[ n_1 = \frac{3 (41.6872) - (5.2436)(23.8294)}{3 (189.3466) - (23.8294)} = 0.5502 \]

For \(r = 4.0\) hr

\[
\begin{array}{c|cccc}
\text{point} & \log q_g & \log \Delta p_p & (\log \Delta p_p)^2 & \log q_g \log \Delta p_p \\
\hline
2 & 1.6645 & 7.7973 & 60.7979 & 12.8383 \\
3 & 1.7504 & 7.9637 & 63.6214 & 13.9617 \\
4 & 1.8467 & 8.1662 & 66.6280 & 15.0739 \\
\hline
5 & 2.2436 & 23.8898 & 90.3091 & 41.7932 \\
\end{array}
\]

\[ n_2 = \frac{3 (41.7932) - (5.2436)(23.8898)}{3 (190.3091) - (23.8898)} = 0.5423 \]

For \(r = 5.0\) hr

\[
\begin{array}{c|cccc}
\text{point} & \log q_g & \log \Delta p_p & (\log \Delta p_p)^2 & \log q_g \log \Delta p_p \\
\hline
2 & 1.6465 & 7.8014 & 60.8618 & 12.8450 \\
3 & 1.7504 & 7.9827 & 63.7325 & 13.9729 \\
4 & 1.8467 & 8.1872 & 66.8012 & 15.0984 \\
\hline
5 & 2.2436 & 23.9352 & 191.0473 & 41.8739 \\
\end{array}
\]

\[ n_3 = \frac{3 (41.8739) - (5.2436)(23.9352)}{3 (191.0473) - (23.9352)} = 0.5486 \]

For \(r = 6.0\) hr

\[
\begin{array}{c|cccc}
\text{point} & \log q_g & \log \Delta p_p & (\log \Delta p_p)^2 & \log q_g \log \Delta p_p \\
\hline
2 & 1.6465 & 7.8014 & 60.8618 & 12.8450 \\
3 & 1.7504 & 7.9827 & 63.7325 & 13.9729 \\
4 & 1.8467 & 8.1872 & 66.8012 & 15.0984 \\
\hline
5 & 2.2436 & 23.9573 & 191.3865 & 41.9113 \\
\end{array}
\]

\[ n_4 = \frac{3 (41.9113) - (5.2436)(23.9573)}{3 (191.3865) - (23.9573)} = 0.5375 \]

We now calculate the average slope, \(\bar{n}\), to be

\[ \bar{n} = \frac{0.5502 + 0.5423 + 0.5486 + 0.5375}{4} = 0.54 \]

4. Determine the theoretical value of the Houpeurt coefficient \(a\) using permeability and skin factor values calculated previously using Brar and Aziz analysis \((k = 5.52\, \text{md}, s = -5.0)\) in Eq. 30. We can use the average value for the coefficient \(b\) obtained from Brar and Aziz analysis.

\[
a = \frac{1.422 \times 10^6}{k_gh} \left[ 1.151 \log \left( \frac{10.06A}{C_A} \right)^{2/4} + s \right] \]  

\[
b = \frac{1.422 \times 10^6}{k_gh} \left[ 1.151 \log \left( \frac{10.06(640 \times 43560)}{30.8828 \times (0.2615)^2} \right)^{2/4} - s \right]
\]

5. Calculate the rate at which the change in pseudopressure determined using the Rawlins and Schellhardt equation is equal to the change in pseudopressure determined using the Houpeurt equation. We use the average slope of the deliverability plot, and the theoretical \(a\) and \(b\) values in Eq. 14.

\[ q_g = \frac{a(1 - n)}{b(2n - 1)} \]  

\[ q_g = \frac{1.467 \times 10^6 (1 - 0.54)}{1.774 \times 10^6 (2(0.54) - 1)} = \frac{475.49}{475.80} \cdot \text{MMscf/D} \]

6. Calculate the stabilized \(C\) value using Eq. 15.

\[ C = \frac{q_g}{a(q_g + bq_g)^n} \]  

\[ C = \frac{475.49}{[(1.467 \times 10^6)(475.49) + (1.774 \times 10^6)(475.49)^2]^{0.54}} = 2.843 \times 10^3 \]

7. Substitute the stabilized \(C\) value from Step 6 into Eq. 17 to calculate the AOF of the well. Use the average value of \(n\) calculated in Step 3.

\[ \text{AOF} = C \left( \frac{p_p(b) - p_p(14.65)}{p_p(1.049 \times 10^9 - 2003.8)} \right) \]  

\[ \text{AOF} = 2.843 \times 10^3 \left[ 1.049 \times 10^9 - 2003.8 \right]^{0.54} \]

\[ \text{AOF} = 211.4 \text{ MMscf/D} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Stabilized C</th>
<th>Brar &amp; Aziz Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n)</td>
<td>0.54</td>
<td>---</td>
</tr>
<tr>
<td>(C)</td>
<td>2.843 \times 10^{-3}</td>
<td>---</td>
</tr>
<tr>
<td>(a, \text{psia}^2/(\text{cp-MMscf/D}))</td>
<td>1.467 \times 10^6</td>
<td>1.461 \times 10^6</td>
</tr>
<tr>
<td>(b, \text{psia}^2/(\text{cp-MMscf/D}^2))</td>
<td>1.774 \times 10^4</td>
<td>1.774 \times 10^4</td>
</tr>
<tr>
<td>AOF, MMscf/D</td>
<td>211.4</td>
<td>205.5</td>
</tr>
</tbody>
</table>

**Table 6 - Comparison of Results**

**Analysis of Transient Deliverability Data**

**Analysis of Stabilized Deliverability Data**

**SUMMARY AND CONCLUSIONS**

In summary, this paper has presented a method for gas well deliverability forecasting in which the results of the more rigorous Houpeurt analysis can be written in terms of the more familiar Rawlins and Schellhardt equation, including the familiar \(C\) and \(n\) parameters. Although refs. 5-7 identify the correlating results (Eqs. 15 and 16) for relating Houpeurt analysis and Rawlins and Schellhardt analysis, these references do not address the use of these correlating results for deliverability forecasting. Nor do these references illustrate the linkage between Houpeurt analysis
and Rawlins and Schellhardt analysis with a detailed, illustrative example.

We have presented the Houpeurt analysis (in terms of pseudopressure) for a case from the literature and verified that the Houpeurt/Rawlins and Schellhardt correlation is valid and should yield accurate performance predictions as well as satisfy regulatory requirements which require analysis results from the Rawlins and Schellhardt method.

In addition, we have drawn the following conclusions regarding this work:

1. An analytical relationship between the more rigorous Houpeurt relation (Eq. 7) and the conventional Rawlins and Schellhardt relation (Eq. 6) has been developed (Eqs. 15 and 16).

2. The relationship between the Houpeurt analysis and Rawlins and Schellhardt analysis has been illustrated and verified using a field case from ref. 9. The results for deliverability forecasts, in this particular case, were within 3 percent.

NOMENCLATURE

- \( a \) = Houpeurt flow coefficient, \((\text{psia}^2/\text{cp})/(\text{MMscf/D})\) for calculations in terms of pseudopressure or \((\text{psia}^2/\text{cp})\) for calculations in terms of pressure-squared
- \( a_t \) = transient deliverability coefficient, \((\text{psia}^2/\text{cp})/(\text{MMscf/D})\) for calculations in terms of pseudopressure or \((\text{psia}^2/\text{cp})\) for calculations in terms of pressure-squared
- \( A \) = drainage area of well, \(\text{ft}^2\)
- \( AOF \) = absolute open flow potential, \(\text{MMscf/D}\)
- \( b \) = Houpeurt flow coefficient, \((\text{psia}^2/\text{cp})/(\text{MMscf/D})^2\) for calculations in terms of pseudopressure or \((\text{psia}^2/\text{cp})\) for calculations in terms of pressure-squared
- \( c \) = intercept of \(a_t\) or \((a_t + bq)\) vs. \(\log(t)\) plot, \((\text{psia}^2/\text{cp})/(\text{MMscf/D})\) for calculations in terms of pseudopressure or \((\text{psia}^2/\text{cp})\) for calculations in terms of pressure-squared
- \( c_f \) = formation compressibility, \(\text{psia}^{-1}\)
- \( c_g \) = gas compressibility, \(\text{psia}^{-1}\)
- \( c_t \) = total system compressibility, \(\text{psia}^{-1}\)
- \( c_w \) = water compressibility, \(\text{psia}^{-1}\)
- \( C \) = Rawlins and Schellhardt stabilized performance coefficient, \((\text{MMscf/D})/(\text{psia}/\text{cp})\) for calculations in terms of pseudopressure or \((\text{MMscf/D})/(\text{psia}/\text{cp})\) for calculations in terms of pressure-squared
- \( C_A \) = well drainage area shape constant or factor
- \( D \) = non-Darcy flow constant, \(\text{D/MMscf}\)
- \( h \) = net formation thickness, \(\text{ft}\)
- \( k_g \) = effective permeability to gas, \(\text{md}\)
- \( m' \) = slope of \(a_t\) or \((a_t + bq)\) vs. \(\log(t)\) plot, \((\text{psia}^2/\text{cp})/(\text{MMscf/D})\) per cycle for calculations in terms of pseudopressure or \((\text{psia}^2/\text{cp})/(\text{MMscf/D})\) per cycle for calculations in terms of pressure-squared
- \( n \) = exponent of the Rawlins-Schellhardt equation and reciprocal slope of the line on a log-log deliverability plot
- \( p \) = pressure, \(\text{psia}\)
- \( p_a \) = atmospheric pressure, \(\text{psia}\)
- \( p_b \) = base pressure, \(\text{psia}\)
- \( p_r \) = real gas pseudopressure, \(\text{psia}^2/\text{cp}\)
- \( p_r(p) \) = average reservoir pseudopressure, \(\text{psia}^2/\text{cp}\)
- \( p_r(p_{mf}) \) = flowing sandface pseudopressure, \(\text{psia}^2/\text{cp}\)

REFERENCES