Determination of Reservoir Pore Volume From Pressure Buildup Tests Using Type Curves

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**ABSTRACT**

Accurate estimation of reservoir drainage volume still remains an important goal in modern pressure transient analysis, primarily due to its direct impact on volumetric analysis and reserve estimation. This paper presents a new method to determine reservoir pore volume, along with flow capacity (kh), from pressure buildup tests by focusing on the use of pressure derivative and pressure integral-derivative functions correlated against dimensionless time based on drainage area (tDk).

We have developed new sets of type curves for unfractured wells in homogeneous, bounded circular and rectangular reservoirs, with no-flow and constant pressure outer boundaries. We have also studied the influence of producing time on pressure buildup tests for different reservoir and well configurations. We conclude that the reservoir must be produced to steady- or pseudosteady-state in order for the buildup response to be interpretable.

The primary result of this paper is our demonstration that bounded reservoir systems exhibit unique buildup characteristics during boundary-dominated flow. In addition, we have shown that this behavior can be correlated in terms of reservoir drainage volume (or area).

Finally, we have used the derivative of the traditional Matthews-Brons-Hazebroek pressure, P_MBH, to estimate the time to achieve pseudosteady-state flow conditions. We provide a table of these times along with values from previous studies.

**INTRODUCTION**

Type curve analysis has long been the modern convention for the interpretation of pressure transient test data.1-4 Along with the "traditional" dimensionless pressure and pressure derivative functions, we also consider the pressure integral and integral-derivative functions for analysis of transient and boundary-dominated buildup test data.

A typical pressure buildup response for a single well in a bounded reservoir includes the following three flow regimes:

- The so-called "early time" region is typically characterized by wellbore storage and skin effects. Limited analysis—no reservoir information, only well-related parameters.
- The "middle time" region which exhibits unique features indicative of the reservoir model. Analysis provides estimates of permeability and skin factor.
- The "late time" region, which shows the influence of boundary effects. Previous methods for the analysis of buildup data required knowledge of reservoir volume and flow capacity—which are our objective parameters.

While the use of type curves developed for infinite-acting reservoirs has become standard practice, these methods cannot be used to describe, even qualitatively, the reservoir boundary condition(s). In fact, the current practice for the analysis of late-time pressure buildup test data is to use the simulation and regression options in commercial well test analysis packages. This is often a tedious process which involves a trial-and-error approach, with little guidance as to how to start or proceed. A direct approach for the analysis of pressure buildup data which exhibits boundary effects has been lacking.

The pioneering work on pressure analysis of bounded circular reservoirs was introduced by Muskat in 1937. Muskat proposed a straight-line method for the estimation of drainage area, as well as reservoir flow capacity. For "late-time" data, a semilog plot of the logarithm of Δp, where Δp = P_wf,Δt=0 vs. shut-in time should yield a straight line with the correct choice of average drainage area pressure. Muskat's method is rigorously valid for circular reservoirs, although it has been successfully applied to symmetric reservoirs of other shapes. This method neglects the affect of drawdown transients on the shut-in pressure response, and analysis is frequently marked by difficulties in precise identification of the correct semilog straight line due to non-uniqueness.

References and illustrations at end of paper.
Many authors have since followed up on this work by suggesting similar straight-line approaches using, basically, superposition of the line source solution to the radial diffusivity equation. The advantages of applying the derivative response for analysis of "late-time" data have been demonstrated by Prouho and Lilley. These authors observed that using the pressure derivative method to analyze data in the boundary-dominated flow region allowed for the identification of unique signals from reservoirs with different configurations.

The approach outlined in this work is different because we propose the use of pressure derivative and pressure integral-derivative type curves based on Green's function line source solutions for bounded reservoirs with different shapes. The solutions have been developed from point source solutions introduced by Ozkan and Raghavan.

In particular, when the producing time, $t_p$, is less than the time required to reach pseudosteady- or steady-state, it is possible (if the drainage shape is known) to correctly analyze the buildup data using these solutions. In such cases, most of the straightline methods actually break down due to their inherent assumption of boundary-dominated behavior prior to shut-in. We do not endorse such practices and recommend a sufficient drawdown period prior to shut-in to eliminate ambiguities in the matching process. The derivative of the Matthews-Brons-Hazenbroek pressure function, $P_{MBH}$, has been used to estimate the dimensionless producing time, $t_{PD}$, required to establish boundary-dominated flow conditions for various reservoir config-urations with no-flow as well as constant pressure outer boundaries.

The difficulties associated with reliance on boundary-dominated data for improved reservoir analysis are a common problem in practical pressure transient testing. Besides the obvious economic constraints that negate the use of long shut-in periods required for rigorous analysis of many lower permeability formations, there are technical problems associated with the detection of small pressure changes that occur during boundary-dominated flow with high levels of precision. Even for cases in which such problems do not exist (high permeability reservoirs), there is still the need for a rigorous analysis procedure to correctly predict reservoir size, configuration, and location of the well with respect to the reservoir boundaries. This paper presents a direct, non-iterative graphical approach for estimating reservoir drainage volume.

In this paper, we have observed similar trends in reservoir responses when comparing type curves for circular reservoirs with those of other symmetric shapes. This behavior helps explain the success of applying circular reservoir solutions for pressure transient analysis of symmetric reservoirs with different shapes. In the case of non-symmetric reservoirs, a method has been devised to isolate the boundary-dominated behavior, enabling us to use the flow characteristics of the transition region to diagnose reservoir shape and configuration.

**MATHEMATICAL SOLUTIONS**

The reservoir model used here conforms to the usual idealizations employed in well testing:
- Fully penetrating unfractured well in a homogeneous, bounded reservoir
- Constant rate drawdown prior to shut-in
- Slightly compressible, chemically non-reactive fluid of constant viscosity

Compressible fluids (gas reservoirs) are easily analyzed with the use of appropriate transform functions:
- Pressure-squared, or
- Real gas pseudo-pressure, and
- Real gas pseudo-time

Two types of outer boundary conditions are considered in this work:
- No-flow
- Constant pressure

**No-Flow Outer Boundary Reservoirs**

The Laplace transform solution for a radial reservoir has been widely reported in the literature as

$$\hat{p}_D(t_D,t) = \frac{K(\mu D)}{\sqrt{\pi t}} \int_{0}^{t_D} \frac{I_0(\lambda u) - I_1(\lambda u)}{\lambda} d\lambda$$

Real time solutions were obtained from Eq. 1 through numerical Laplace transform inversion using the Gaver-Stehfest algorithm. Applying the method of superposition to this drawdown solution, the buildup pressure solutions are computed as

$$p_D(t_D,t_P) = p_{wD}(t_P) - p_{wD}(t_P + \Delta t_D) \quad \text{(2)}$$

The derivative of Eq. 2 is given by (see Appendix A)

$$p_D'(t_D) = \frac{\Delta t_D}{t_D} \left[p_{wD}(t_D) - p_{wD}(t_D + \Delta t_D)\right] \quad \text{(3)}$$

and the pressure integral-derivative is calculated from

$$p_D(t_D) = \Delta t_D \int_{0}^{t_D} p_{wD}(\Delta t) \, d\Delta t$$

where

$$p_{wD}(t_D) = \frac{1}{t_D} \left[p_{wD}(t_D) - p_{wD}(t_D + \Delta t_D)\right] \quad \text{(5)}$$

Using the expression for shut-in pressure, the pressure integral-derivative function is given by

$$p_D(t_D) = \left[p_{wD}(t_D) - p_{wD}(t_D + \Delta t_D)\right] \frac{t_D}{\Delta t_D} \quad \text{(6)}$$

These results are plotted versus dimensionless time based on drainage area, $A_D$, in Fig. 1.

Rectangular reservoirs were also studied extensively in this work. Solutions for rectangular reservoirs are developed from the instantaneous point source solutions given in Ref. 12. This "point" source solution is given by

$$p_D(x_D,y_D) = \frac{1}{x_D y_D} \left\{ \cosh(u_D y_D) + \cosh(u_D y_D) \right\}$$

where

$$y_D = y + \frac{y_D}{x_D} \quad \text{(7)}$$

and

$$y_D = y + \frac{y_D}{x_D} \quad \text{(8a)}$$

also

$$y_D = y + \frac{y_D}{x_D} \quad \text{(8b)}$$
\[ \varepsilon_x = \sqrt{u + \frac{k^2 \pi^2}{x_D^2}} \]  
\[ \varepsilon_x = \sqrt{u + \frac{n^2 \pi^2}{h_D^2}} \]  
\[ \varepsilon_{k,n} = \sqrt{u + \frac{k^2 \pi^2}{x_D^2} + \frac{n^2 \pi^2}{h_D^2}} \]

The remaining parameters are defined in the Nomenclature.

Continuous line source solutions have been derived by integrating Eq. 7 over the perforated length of the well, \( h_D \). This solution is given by

\[
\bar{p}_D(x_D,y_D,u) = \frac{\pi}{\varepsilon_x u} \left\{ \left[ \frac{\cosh(\varepsilon_x y_D)}{\sinh(\varepsilon_x y_D)} \right] + \sum_{k=1}^{\infty} \cos\left(\frac{k\pi x_D}{x_D}\right) \left[ \frac{\cosh(\varepsilon_x y_D)}{\sinh(\varepsilon_x y_D)} + \frac{\cosh(\varepsilon_x y_D)}{\sinh(\varepsilon_x y_D)} \right] \right\} \]

\[ \cdots \cdots \cdots \cdots (10) \]

Similar to Eq. 1, Eq. 10 was inverted using the Gaver-Stehfest algorithm, where Eq. 2a is used to compute the buildup response. Eq. 2a is given as

\[
p_D(x_D,y_D,t_D) = p_{wD}(t_D) + p_{wD}(t_D) + p_{wD}(t_D) \cdots \cdots (2a) \]

The pressure derivative and pressure integral-derivative were computed using Eqs. 3 and 6. The results for a well centered in a closed square reservoir are plotted in Fig. 2. Fig. 3 shows pressure solutions of the square reservoir case compared with responses given by Earlougher et al.17

At this point, we note that although the number of terms used for the infinite sum in Eq. 10 does influence the pressure solution, it does not influence the derivative and integral-derivative responses as long as enough terms are used to compute a stable solution.

The different solutions shown in Figs. 1 and 2 indicate that the significance of producing time on the overall buildup pressure behavior cannot be overemphasized. General solutions of the different reservoir systems have been plotted in Fig. 4. The curves assume pseudosteady-state conditions in the reservoir prior to the buildup test. The derivative of the Matthews-Brons-Hazebroek pressure, \( p_{MRH} \) has been presented in Figs. 5 and 6, from which approximate dimensionless times for the start of pseudosteady-state flow conditions (\( t_p \)DA) are estimated. These values are summarized in Table 1.

Comparison of Figs. 1 and 2 indicates that radial and rectangular reservoir models exhibit similar pressure responses. This demonstrates that circular reservoir solutions may be utilized for the analysis of pressure response in square reservoirs -- a fact that may be extended to other reservoirs with symmetric geometries.

**Constant Pressure Outer Boundary Reservoirs**

The same approach used for the development of type curves for completely closed reservoirs was used for reservoirs with constant pressure outer boundaries. The Laplace domain solution for radial flow in a homogeneous reservoir is given by

\[
\bar{p}_D(r_D,u) = \frac{K_0(\bar{u}r_D)}{K_1(\bar{u}r_D) + \bar{u}K_1(\bar{u}r_D)} \left[ \bar{u}K_1(\bar{u}r_D) + \bar{u}L_1(\bar{u}r_D) \right] \cdots \cdots (11) \]

Fig. 7 is a graphical representation of the real space solution of Eq. 11. These buildup solutions were obtained using Eq. 2, while the pressure derivative and pressure integral-derivative were computed from Eqs. 3 and 6, respectively. The approximate dimensionless time to reach steady-state from the drawdown solution was estimated to be 0.3. As can be seen, the reservoir responses vary for a dimensionless producing time, \( t_p DA \), less than the time required to establish boundary-dominated flow conditions (\( t_p DA = t_p DA \)), but stabilize beyond this period. Type curves have been presented in Fig. 6 for selected \( t_p DA \) values less than 0.3.

Line source solutions for rectangular reservoirs are also developed by integrating the corresponding instantaneous point source solution along the entire perforated height, \( h_D \). The result is

\[
\bar{p}_D(x_D,y_D,u) = \frac{2\pi}{\varepsilon_x u} \sum_{k=1}^{\infty} \frac{\sin(\frac{k\pi x_D}{x_D})}{\sinh(\frac{k\pi x_D}{x_D})} \left[ \frac{\cosh(\varepsilon_x y_D) - \cosh(\varepsilon_x y_D)}{\sinh(\varepsilon_x y_D)} \right] \cdots \cdots (12) \]

Real time solutions for a square reservoir are shown in Fig. 8. Comparison of Figs. 7 and 8 shows virtually no difference in the buildup responses of circular and square reservoirs, confirming the use of type curves based on radial flow behavior for the analysis of symmetric reservoirs of different shapes. The general buildup solutions of these reservoirs for cases in which \( t_p DA \) is greater than or equal to \( t_p DA \) have been plotted in Fig. 9. \( p_{MBH} \) plots were also used to estimate the approximate dimensionless time required to establish steady-state flow conditions in these reservoirs. The results are summarized in Table 1.

**TYPE CURVES FOR BOUNDARY-DOMINATED FLOW**

In this section we present the new type curves for the analysis of buildup responses from bounded reservoirs. They are summarized in Figs. 10 and 11 for the no-flow and constant pressure solutions, respectively. It is immediately obvious that the circular reservoir solutions are not included in these plots. As has been shown previously in Figs. 1 and 2, as well as in Figs. 5 and 6, circular reservoir solutions directly overlay not only the square reservoir responses, but other symmetric reservoir systems as well.

The key to the type curve development in this work is the ability to isolate boundary-dominated flow responses for all the reservoir systems. In order to minimize ambiguities and eliminate non-uniqueness in the matching process, we have unified the boundary-dominated responses by plotting pressure derivative responses versus \( t_p DA \) instead of using the original pressure solutions shown in Figs. 4 and 9 for the no-flow and constant pressure outer boundary conditions, respectively. Here \( t_p DA \) is a "shape dependent scale factor" and values for the various reservoir/wellbore configurations are presented in Figs. 10 and 11. By fixing the early-time (transient) and late-time (boundary-dominated) data during the matching process, the different characteristics of the middle-time (transition) region provide us with an important diagnostic tool for determination of reservoir system configuration.

We emphasize the fact that the type curve matching approach presented in this work is based on pressure derivative analysis. The pressure integral-derivative has been provided here to improve the accuracy of the matching process, especially when analyzing noisy field data. A separate pressure derivative plot for reservoirs with no-flow outer boundaries is shown in Fig. 12. Knowledge of initial pressure or shut-in pressure is not required, as is the case for previous methods, however, constant rate production during the drawdown period prior to shut-in is a requirement.

These type curves were developed from analytical solutions under the assumption that \( t_p DA \) is greater than or equal to the corresponding time required to establish boundary-dominated flow. While it is almost impossible to satisfy this requirement in every case, type curves of the kind given in Figs. 1 and 2, or Figs. 7 and 8 can be generated when the reservoir/well...
configuration is known. The required drawdown solutions have been provided by Eqs. 1, 10, 11, and 12, with the corresponding buildup pressure functions obtained from Eqs. 2, 2a, 3, and 6. In addition, drawdown type curves have been provided in Fig. 13. A unique match on any of the curves provides values for the simultaneous calculation of reservoir flow capacity and pore volume.

**TYPE CURVE ANALYSIS RELATIONS**

In this section we present an integrated buildup analysis procedure as well as various relations used for estimating formation properties. It is important that any non-uniqueness be removed during the type curve matching process, therefore, we will attempt to show that most of the ambiguities can be eliminated by simultaneously matching both the pressure derivative and pressure integral-derivative curves on the new type curves.

The following steps are recommended for consistent analysis

- If possible, produce the well to pseudosteady- or steady-state for reservoirs with constant pressure outer boundaries
- Use available geological information to establish whether the reservoir is homogeneous or naturally fractured
- Perform transient type curve analysis using early-time and middle-time data together with semilog analysis
- Perform post-transient type curve analysis by matching middle- and late-time data to establish reservoir configuration and volume
- Validate the entire buildup test by comparison to simulated results. The Laplace space solution for reservoirs with wellbore storage and skin may be computed from the drawdown solutions provided above using the van Everdingen and Hurst equation

\[ \bar{p}_{wd}(u, C_D, s) = \frac{\bar{u}_D + s}{u(1 + u C_D (\bar{u}_D + s))} \]  \hspace{1cm} (13)

**Transient Analysis: Type Curve for an Unfractured Well in a Homogeneous, Infinite-Acting Reservoir with Wellbore Storage and Skin Effects**

We will use the "Bourdet-Gringarten" type curve to analyze the transient portion of our pressure buildup test data. This type curve is used to estimate the following parameters

- Formation permeability, \(k\)
- Dimensionless wellbore storage coefficient, \(C_D\)
- Pseudoradial flow skin factor, \(s\)

**Reservoir flow capacity, \(kh\), is given by**

\[ k h = 141.2 \frac{q Bu}{\left( \frac{P_{wp}}{\Delta P_{hun} MP} \right)} \]  \hspace{1cm} (14)

From Eq. 14, formation permeability, \(k\), is given by

\[ k = 141.2 \frac{q Bu}{\left( \frac{P_{wp}}{\Delta P_{hun} MP} \right)} \]  \hspace{1cm} (15)

where \((\Delta P)_{hun}\) is either the pressure drop, pressure drop derivative, pressure drop integral, or the pressure drop integral-derivative function. Although any or all of these functions can be matched simultaneously, we have chosen to use only the pressure drop and pressure drop derivative functions in this work.

**Dimensionless wellbore storage coefficient, \(C_D\)**

\[ C_D = 0.0002637 \left( \frac{\Delta P_{hun} MP}{q Bu} \right) \]  \hspace{1cm} (16)

For a drawdown test, \(\Delta P_{hun}\) represents ordinary time. For a pressure buildup test, \(\Delta P_{hun}\) is either shut-in time or effective time. In this work, we assume that boundary-dominated flow conditions were attained prior to shut-in, so the use of shut-in time is rigorously justified.

**Pseudoradial flow skin factor, \(s\)**

\[ s = \frac{1}{2} \ln \left( \frac{C_D}{C_D + \frac{2}{MP}} \right) \]  \hspace{1cm} (17)

**Post-Transient Analysis: Type Curve for an Unfractured Well in a Homogeneous, Bounded Reservoir with Wellbore Storage and Skin Effects**

For this analysis, we will use the new type curves developed for pressure buildup responses in closed rectangular reservoir systems (Fig. 10 or Fig. 11 depending on the outer boundary condition). These type curves have been verified for use in the determination of drainage pore volume from pressure buildup response and can be used to estimate the following parameters

- Formation permeability, \(k\)
- Reservoir drainage area, \(A\)
- Reservoir pore volume, \(V_p\)

The properties are estimated using the relations given below.

**Formation permeability, \(k\)**

\[ k = 141.2 \frac{q Bu}{h} \left( \frac{P_{wp}}{\Delta P_{hun} MP} \right) \]  \hspace{1cm} (18)

where in this case we use only the pressure drop derivative or the pressure drop integral-derivative function for \((\Delta P)_{hun} MP\).

**Reservoir drainage area, \(A\)**

\[ A = \frac{0.0002637}{k} \left( \frac{\Delta P_{hun} MP}{q Bu} \right) \]  \hspace{1cm} (19)

**Reservoir drainage volume, \(V_p\)**

\[ V_p = \frac{4}{5} A \frac{h}{5.615} \]  \hspace{1cm} (20)

**SEMilog ANALYSIS RELATIONS**

In this section, we provide the appropriate relations for analyzing pressure and pressure integral data from buildup tests using the Miller-Dyer-Hutchinson (MDH) semilog method. The MDH method is a simplified approach which ignores prior production history. However, the method has shown to be rigorously accurate for the analysis of pressure buildup test data for cases in which the reservoir was produced to pseudosteady-state flow conditions prior to shut-in. Since our goal is to characterize pressure buildup tests where boundary effects are evident, the MDH approach of analyzing shut-in pressure or pressure drop with respect to ordinary time is appropriate by definition.

**Pressure Analysis Relations**

The pressure-time behavior for a buildup test sequence following a constant (or nearly constant) rate drawdown test in a homogeneous and infinite-acting reservoir is usually modeled by the so called "log time approximation." This relation serves as the motivation for plotting pressure versus the logarithm of time and provides the theory for graphically interpreting straight line trends in the data. The pertinent relations for estimating formation properties from "semilog" analysis are given below.

The formation permeability, \(k\), is given by

\[ k = 162.6 \frac{q Bu}{h} \]  \hspace{1cm} (21)
and the pseudoradial flow skin factor, \( s \), is

\[
s = 1.1513 \left( \frac{P_{wsl,1hr} - P_{wf,\Delta t=0}}{m} \right) - \log \left( \frac{k}{\phi \mu c_f R_0^2} \right) + 3.2275
\]

(23)

The skin factor estimate requires the pressure at 1 hr, \( P_{wsl,1hr} \), from the semilog straight line or its extrapolation, and this relation also requires the pressure at shut-in, \( P_{wf,\Delta t=0} \).

Pressure Integral Analysis Relations

We use the pressure integral function for two primary reasons. First, the pressure integral provides smoother data functions for analysis. However, we would be remiss in not acknowledging that the pressure integral does delay, or even mask certain features because of its nature to smooth the data with respect to time. Second, the pressure integral provides a complementary analysis to the pressure and pressure derivative analyses. This approach of using multiple data functions provides more confidence in our analysis results and decreases the chance of misinterpretation.

The pressure integral function has been given in Eq. 5. We define the pressure drop integral function, which is given as

\[
\Delta p_i = \int_0^t \Delta p(t) \, dt
\]

(24)

where the pressure drop definitions are

\[
\Delta p = P_{wsl} - P_{wf,\Delta t=0}
\]

(25)

\[
\Delta p_i = P_{wsl} - P_{wf,\Delta t=0}
\]

(26)

The computational issues concerning the pressure integral functions are discussed in Ref. 20.

We can analyze the pressure integral data function using conventional semilog analysis techniques. The analysis relations for estimating formation properties from a plot of the pressure integral versus the logarithm of time are given below.

We use exactly the same relation for estimating the formation permeability from the straight line portion of the data on a semilog plot (assuming such a trend exists). The permeability relation is given by Eq. 22.

A slight modification is required in the skin factor calculation. The skin factor relation for pressure integral analysis is given as

\[
s = 1.1513 \left[ \frac{P_{wsl,1hr} - P_{wf,\Delta t=0}}{m} \right] - \log \left( \frac{k}{\phi \mu c_f R_0^2} \right) + 3.2275 + \frac{1}{2}
\]

(27)

The skin factor estimate requires the value of the pressure integral function at 1 hr, \( P_{wsl,1hr} \), as read from the semilog straight line or its extrapolation. As before, this relation for the skin factor also requires the value of the pressure at shut-in, \( P_{wf,\Delta t=0} \).

APPLICATION OF THE NEW TYPE CURVES

The type curves have been applied extensively for the analysis of simulated pressure buildup responses, as well as to actual field data cases. Two independent commercial models have been used to simulate the pressure transient data. Fig. 14 shows a plot of pressure buildup data for three example wells on the derivative type curves. These are vertical, unfractured wells producing from a homogeneous formation in a reservoir with a drainage area of approximately 80 acres.

The first well located in the center of a 2x1 rectangular reservoir was simulated with Petroleum Workbench. The second well located at the center of a 4x1 rectangular reservoir, and the last well centered in a 8x1 rectangular reservoir, were both simulated with PanSystem.

Field Example Cases

In this section, we present the analyses of the two field cases considered in this study. The first case is a data set from Lee (Table 2), and the second case is a data set obtained from Proano and Lilley (Tables 3 and 4). These cases illustrate typical boundary-dominated responses which are likely to be ignored in routine well test analyses. The wells were initially produced at a fairly constant rate for some arbitrary period of time and then shut in for pressure buildup.

Case 1: (Lee Example 2.2)

This is a vertical, unfractured oil well producing from a fairly homogeneous formation of medium permeability above the bubble point pressure. The well is centered in a bounded square drainage area of approximately 160 acres. Average reservoir and fluid properties are summarized below.

Reservoir, Fluid Property and Production Data:

<table>
<thead>
<tr>
<th>Reservoir Properties:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wellbore radius, ( r_w )</td>
</tr>
<tr>
<td>Net pay thickness, ( h )</td>
</tr>
<tr>
<td>Porosity, ( \phi )</td>
</tr>
<tr>
<td>Drainage Area, ( A )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fluid Properties:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil formation volume factor, ( B_o )</td>
</tr>
<tr>
<td>Oil viscosity, ( \mu_o )</td>
</tr>
<tr>
<td>Total compressibility, ( c_t )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Production Parameters:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production rate, ( q_o )</td>
</tr>
<tr>
<td>Pressure at shut-in, ( P_{wf,\Delta t=0} ) (corrected)</td>
</tr>
</tbody>
</table>

Semilog Analysis Results: (Fig. 15)

As shown in Fig. 15, the semilog plot is dominated by wellbore storage effects, however, the derivative response of the log-log plot indicates the possibility of a brief period of radial flow was found to start at \( \Delta t = 20 \) hrs. Using the results of the straight-line portion of the data yields the following formation parameters

<table>
<thead>
<tr>
<th>Pressure Analysis:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{wsl,1hr} )</td>
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<tr>
<td>( P_{wsl,1hr} )</td>
</tr>
<tr>
<td>( m )</td>
</tr>
<tr>
<td>( m )</td>
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<td>( k )</td>
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<tr>
<td>( k )</td>
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<tr>
<td>( s )</td>
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<td>( s )</td>
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</tbody>
</table>

Pressure Analysis Calculations:

Permeability: (Semilog and type curve values are forced)

The analysis relation for the formation permeability, \( k \), is given by Eq. 22, that is

\[
k = 162.6 \frac{(250 \text{ STB/D})(1.136 \text{ RB/STB})(0.80 \text{ cp})}{(70.0 \text{ psia/cycle})(69 \text{ ft})}
\]

or

\[
k = 7.649 \text{ md}
\]

Skin Factor:

The analysis relation for the skin factor, \( s \), is given by Eq. 23

Solving for the skin factor, \( s \), with our data we obtain

\[
s = 5.816
\]

or

\[
s = 5.186
\]
Pressure Integral Analysis Calculations:

Permeability: (Semilog and type curve values are forced)
The analysis relation for the formation permeability, \( k \), is exactly the same as that for the pressure analysis, which is given by Eq. 28. Since we have forced the slopes to be equal, when solving for the permeability, \( k \), for this case we obtain exactly the same value as for pressure analysis

\[
k = 162.6 \frac{250 \text{ STB}/\text{D} \times (1.136 \text{ RB}/\text{STB} \times (0.80 \text{ cp})}{(70.0 \text{ psia/cycle} \times (69 \text{ ft})}
\]
or
\[k = 7.649 \text{ md}\]

Skin Factor:
For pressure integral analysis, the relation for skin factor, \( s \), is given by Eq. 28. Solving for \( s \) gives

\[
s = 1.1513 \left[ \frac{(d252 \text{ psia} - 3561.9 \text{ psia})}{(70.0 \text{ psia/cycle})} \right] - 1.1513 \log \left[ \frac{(0.039)(0.80 \text{ cp})(17.0 \times 10^{-6} \text{ psia}^{-1})(0.198 \text{ ft})^2}{(7.649 \text{ md})} \right] + 1.1513(3.2275) + \frac{1}{2}
\]
or
\[s = 5.707\]

Results of Transient Flow Type Curve Analysis: (Fig. 16)
Transient analysis of the early- and middle-time data is not difficult for this well since it is possible to identify the unit slope of the wellbore storage region. The 0.5 line match with middle-time data is also evident. A match on \( C_D^{2s} = 6 \times 10^8 \) stem provided the following parameters.

Type Curve Match: Bourdet-Gringarten Type Curve—Well in an Infinite-Aging Homogeneous Reservoir with Skin and Wellbore Storage Effects

Matching Parameters: \( C_D^{2s} = 6 \times 10^8 \)

\[\frac{[\Delta D]}{[D]}_\text{MP} = 1.0 \quad \frac{[\Delta P]}{[P]}_\text{MP} = 0.058 \text{ hr} \]

\[\frac{[\Delta P]}{[P]}_\text{MP} = 1.0 \quad \frac{[\Delta P]}{[P]}_\text{MP} = 60.80 \text{ psi}\]

Analysis Results:

\[k = 7.649 \text{ md}\]
\[C_D = 5626\]
\[s = 5.789\]

Calculations:

Permeability:
Solving the pressure match point relation for permeability, \( k \), using Eq. 18 gives

\[k = 141.2 \frac{250 \text{ STB}/\text{D} \times (1.136 \text{ RB}/\text{STB} \times (0.80 \text{ cp})}{(69 \text{ ft}) \times (60.80 \text{ psi})}
\]
or
\[k = 7.649 \text{ md}\]

Dimensionless Wellbore Storage Coefficient:
Solving for the dimensionless wellbore storage coefficient, \( C_D \), we have

\[C_D = 0.0002637 \frac{(7.649 \text{ md})}{(0.039)(0.80 \text{ cp})(17.0 \times 10^{-6} \text{ psia}^{-1})(0.198 \text{ ft})^2}
\]
or
\[C_D = 5626\]

Skin Factor:
The skin factor is computed in this particular case as

\[s = \frac{1}{2} \ln \left[ \frac{6 \times 10^8}{5626} \right]
\]
or
\[s = 5.789\]

Results of Post-Transient Flow Type Curve Analysis: (Fig. 17)
Fig. 17 presents analysis of the buildup response on the bounded reservoir type curves. We realize that wellbore storage has distorted most of the early-time data. However, the middle and late-time regions are quite distinct. This enabled us to fix the boundary-dominated well response both on the pressure derivative and pressure integral-derivative curves (the derivative match is shown in Fig. 17). We find that the behavior of the derivative response matched the solution of a well centered in a square reservoir.

Type Curve Match: Boundary Flow Buildup Type Curve—Vertical Well in Homogeneous Rectangular Reservoir with Closed Boundaries

Type Curve Matched: Well Centered in a Square \( ID = 1 \)

\[\frac{[\Delta D]}{[D]}_\text{MP} = 1.0 \quad \frac{[\Delta P]}{[P]}_\text{MP} = 1600 \text{ hr} \quad [P_{wD}]_\text{MP} = 1.0 \quad \frac{[\Delta P]}{[P]}_\text{MP} = 60.80 \text{ psi}\]

Pressure Derivative Analysis:

\[k = 7.649 \text{ md}\]
\[A = 6,084,589 \text{ ft}^2 \text{ or } 139.7 \text{ acres} \]
\[V_p = 2.92 \times 10^6 \text{ reservoir barrels}\]

Calculations:

Permeability: (Forced with previous analyses)
Solving the permeability relation for this case (which is forced to agree with previous analyses), we obtain

\[k = 141.2 \frac{250 \text{ STB}/\text{D} \times (1.136 \text{ RB}/\text{STB} \times (0.80 \text{ cp})}{(69 \text{ ft}) \times (60.80 \text{ psi})}
\]
or
\[k = 7.649 \text{ md}\]

Reservoir Drainage Area:
Using the computed permeability value, we estimate the drainage area, \( A \), to be

\[A = 0.0002637 \frac{(7.649 \text{ md})}{(0.039)(0.80 \text{ cp})(17.0 \times 10^{-6} \text{ psia}^{-1})}
\]

\[\times \frac{1600 \text{ hr}}{[1.0]_\text{MP}} \times \frac{1}{(ID = 1)}
\]
or
\[A = 6,084,589 \text{ ft}^2 = 139.7 \text{ acres}\]

Pore Volume:
The reservoir drainage volume, \( V_p \), for this particular case is estimated as

\[V_p = \frac{(0.039)(69 \text{ ft})(6,084,589 \text{ ft}^2)}{5.615}
\]
or
\[V_p = 2.92 \times 10^6 \text{ reservoir barrels}\]

Summary for Example 1: (Fig. 18)
The integrated approach adopted for the analysis of pressure buildup data has enabled us to achieve very consistent and quite
confident estimates of formation parameters. The computed parameters were verified by simulating the full buildup response, as shown in Fig. 18. The match is quite good despite the limited and scattered data available.

A comparison of our results with those provided in Ref. 23 is shown below.

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<tr>
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<th>Ref. 23</th>
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<td>Wellbore Storage, $C_D$</td>
<td>5880.0</td>
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<td>Flow Capacity, $kh$</td>
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<td>527.78 md-ft</td>
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<tr>
<td>Permeability, $k$</td>
<td>7.65 md</td>
<td>7.649 md</td>
</tr>
<tr>
<td>Drainage Volume, $V_p$</td>
<td>3.34x10^6 bbls</td>
<td>2.92x10^6 bbls</td>
</tr>
<tr>
<td>Drainage Area, $A$</td>
<td>160 acres</td>
<td>139.7 acres</td>
</tr>
</tbody>
</table>

Case 2: (Proano and Lilley\textsuperscript{11} - Well 1)

This data case is for a vertical, unfractured oil well producing from a homogeneous, high permeability formation. The well is thought to be located at the center of a bounded square reservoir. The well was produced for 24,717 hrs and shut-in for 11,845 hrs. Average reservoir and fluid properties are shown below.

Reservoir, Fluid Property and Production Data:

Reservoir Properties:
- Wellbore radius, $r_w$ = 0.51 ft
- Net pay thickness, $h$ = 120 ft
- Porosity, $\phi$ = 0.11 (fraction)
- Drainage Area, $A$ (previous estimate) = 2341.6 acres

Fluid Properties:
- Oil formation volume factor, $B_o$ = 1.21 RB/STB
- Oil viscosity, $\mu_o$ = 2.40 cp
- Total compressibility, $c_t$ = 5.0x10^-6 psi\textsuperscript{-1}

Production Parameters:
- Production rate ($q_{up}$), $q_o$ = 6304.0 STB/D
- Pressure at shut-in, $P_{wfs,\Delta t=0}$ = 3888.89 psia

Semilog Analysis Results: (Fig. 19)

Preliminary semilog analysis of the buildup data was conducted for both pressure and pressure-integral functions. The results are summarized as follows.

Pressure Analysis:  
\[ P_{ws,1hr} = 4064.5 \text{ psia} \]  
\[ P_{wfs,1hr} = 4058.3 \text{ psia} \]  
\[ m = 13.70 \text{ psi/cycle} \]  
\[ k = 1810.5 \text{ md} \]  
\[ s = 7.283 \]  
\[ s = 7.262 \]

Results of Transient Flow Type Curve Analysis: (Fig. 20)

Unlike Case 1, for which the early- and middle-time data quality was fairly good (very few distorted middle and late-time data), Case 2 data has a large degree of early-time data scatter. The early-time data are almost impossible to analyze due to wellbore storage distortions, however, the middle- and late-time data quality is acceptable.

Match point values were obtained, from which permeability, dimensionless wellbore storage coefficient, and skin factor were computed. A summary of the analysis is as follows.

Type Curve Match: Bourdet-Gringarten Type Curve--Well in an Infinite-Acting Homogeneous Reservoir with Skin and Wellbore Storage Effects

Matching Parameters: $C_{PE} = 2x10^9$
\[ [\Delta P]/[C_D]_{MP} = 1.0 \]  
\[ [P_{wd}]_{MP} = 1.0 \]  
\[ [\Delta T]/[T_D]_{MP} = 1.7x10^{-4} \text{ hr} \]  
\[ [\Delta P]/[P_{wd}]_{MP} = 11.9 \text{ psi} \]  

Analysis Results:
\[ k = 1810.5 \text{ md} \]  
\[ C_D = 236.4 \]  
\[ s = 7.975 \]

Results of Post-Transient Flow Type Curve Analysis: (Fig. 21)

Fig. 21 shows the post-transient analysis of the Proano and Lilley\textsuperscript{11} data using the new type curves. In this case, the problems associated with the distorted transient data mentioned above became irrelevant. The middle- and late-time regions were matched on the solution stem of a well centered in a square reservoir. The match point parameters are presented below.

Type Curve Match:
- Boundary Flow Buildup Type Curve--Vertical Well in Homogeneous Rectangular Reservoir with Closed Boundaries
- Type Curve Matched: Well Centered in a Square $r_{DA} = 1$
\[ [\Delta T]/[T_D]_{MP} = 1.0 \]  
\[ [\Delta P]/[P_{wd}]_{MP} = 11.9 \text{ psi} \]

Pressure Derivative Analysis:
\[ k = 1810.5 \text{ md} \]  
\[ A = 66,695,364 \text{ ft}^2 \text{ or } 1531.1 \text{ acres} \]  
\[ V_p = 156.8x10^6 \text{ reservoir barrels} \]

Summary for Example 2: (Fig. 22)

We found excellent agreement between our results for permeability and skin factor and those presented in Ref. 11. We believe that our estimates for drainage volume and area represents an improvement over the previous analysis results. The match of simulated test results and the original data functions is shown in Fig. 22. The comparison of our results with those obtained in Ref. 11 are as follows.

<table>
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<tr>
<th></th>
<th>Ref. 11</th>
<th>This Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well. Storage Coef., $C_D$</td>
<td>687.0</td>
<td>236.4</td>
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<td>Skin Factor, $s$</td>
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<td>7.975</td>
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<tr>
<td>Flow Capacity, $kh$</td>
<td>215,000 md-ft</td>
<td>217,260 md-ft</td>
</tr>
<tr>
<td>Permeability, $k$</td>
<td>1791.7 md</td>
<td>1810.5 md</td>
</tr>
<tr>
<td>Drainage Volume, $V_p$</td>
<td>239.7x10^6 bbls</td>
<td>156.8x10^6 bbls</td>
</tr>
<tr>
<td>Drainage Area, $A$</td>
<td>2341.6 acres</td>
<td>1531.1 acres</td>
</tr>
</tbody>
</table>

SUMMARY AND CONCLUSIONS

We have developed and verified a new set of type curves for the analysis of the boundary-dominated portion of pressure buildup test data. We have successfully applied these type curves to simulated pressure buildup data, as well as to field data cases.

The developments and observations we have made in this work lead us to the following conclusions:

1. The type curves provide direct estimates of formation flow capacity and reservoir pore volume using the pressure derivative or integral-derivative functions. Provided that sufficient data are available in the transition region between the transient and boundary-dominated flow regimes, we can also infer the reservoir shape and the configuration of the well relative to the reservoir boundaries.

2. A relatively severe limitation of this approach is that the well must be produced to pseudosteady-state (no-flow boundary) or steady-state (constant pressure boundary) flow conditions prior to shut-in. We strongly recommend that an adequate producing time be provided during test design to enable pseudosteady-state or steady-state conditions to be established in the reservoir prior to shut-in.

3. We have defined a "shape dependent scale factor," $T_D$, for normalizing the $T_D$ scale as a means to create a series of
unified stems for a variety of reservoir/well configurations (e.g., Figs. 10-12). This approach should reduce confusion in matching data and improve estimates of reservoir property.

4. We believe that these new type curves can be used to distinguish boundary effects from transient characteristics such as wellbore phase redistribution and naturally fractured reservoir behavior.

NOMENCLATURE

**General**

- \( A \) = area, \( \text{ft}^2 \)
- \( B \) = formation volume factor, RB/STB
- \( c_t \) = total system compressibility, psia\(^{-1} \)
- \( h \) = total formation thickness, ft
- \( h_p \) = dimensionless length of perforated interval
- \( I_0((x)) \) = modified Bessel function of the first kind, zero order
- \( I_1((x)) \) = modified Bessel function of the first kind, first order
- \( k \) = average permeability, md
- \( K_0((x)) \) = modified Bessel function of the second kind, zero order
- \( K_1((x)) \) = modified Bessel function of the second kind, first order
- \( m \) = slope of semilog straight line, psi/cycle
- \( P_D \) = dimensionless pressure
- \( P_{Di} \) = dimensionless pressure integral function defined in Eq. 5
- \( p_{m/b} \) = Matthews-Brons-Hazebrock pressure derivative
- \( q \) = production rate, STB/D
- \( r \) = radial distance, ft
- \( r_{dp} \) = dimensionless external radius
- \( s \) = skin factor
- \( t \) = time, hr
- \( t_{DA} \) = dimensionless time based reservoir drainage area
- \( t_{DF} \) = reservoir 'shape dependent scale factor'
- \( u \) = Laplace transform variable.
- \( x \) = distance in x-direction, ft
- \( y \) = distance in y-direction, ft
- \( z \) = distance in z-direction, ft

**Subscript**

- \( D \) = dimensionless
- \( e \) = external
- \( s \) = shut-in (buildup responses)
- \( w \) = wellbore

**Greek Symbols**

- \( \phi \) = porosity, fraction
- \( \mu \) = fluid viscosity, cp.
- \( \Delta \) = change in a variable

ACKNOWLEDGMENTS

The authors would like to thank Tom Mireles for diligently reviewing this manuscript. We acknowledge the support of the Fina Oil and Chemical Co. and the United States Department of Energy (DOE) for financial assistance provided through the DOE Class II Oil Program.

We also acknowledge the Department of Petroleum Engineering at Texas A&M University for providing computing facilities and resources for the preparation of this manuscript.

REFERENCES


APPENDIX A

The buildup pressure solution is obtained from superposition of the drawdown pressure response, and given by

\[ p_D(t_D) - p_wD(t_D) = p_D(t_D) - p_wD(t_D) \]

\[ + \frac{\partial}{\partial \Delta D} \left[ p_wD(t_D) \right] \int_0^{\Delta D} p_D(t_D) \, d\Delta D \] ..........................(A-1)

The "well testing" derivative of Eq. A-1 is given by

\[ p_D' (\Delta D) = \Delta D \frac{\partial}{\partial \Delta D} \left[ p_wD (\Delta D) \right] \]

\[ = \Delta D \left[ \frac{\partial}{\partial \Delta D} \left[ p_wD (t_D) \right] - \frac{\partial}{\partial \Delta D} \left[ p_D (t_D + \Delta D) \right] + \frac{\partial}{\partial \Delta D} \left[ p_wD (\Delta D) \right] \right] \] ..........................(A-2)

The first term in Eq. A-2 is zero, therefore we are left with

\[ p_D' (t_D) = - \Delta D \frac{\partial}{\partial \Delta D} \left[ p_wD (t_D + \Delta D) \right] + \Delta D \frac{\partial}{\partial \Delta D} \left[ p_wD (\Delta D) \right] \] ..........................(A-3)

Using the chain rule of differentiation for the first term in Eq. A-3 and the fact that

\[ \frac{\partial}{\partial \Delta D} \left[ (t_D + \Delta D) \right] = \frac{\partial}{\partial \Delta D} \left[ \Delta D \right] - 1 \] ..........................(A-4)

we rewrite Eq. A-3 as

\[ p_D' (t_D) = - \Delta D \frac{\partial}{\partial \Delta D} \left[ p_wD (t_D + \Delta D) \right] + \Delta D \frac{\partial}{\partial \Delta D} \left[ p_wD (\Delta D) \right] \]

or

\[ p_D' (t_D) = - \Delta D \frac{\partial}{\partial \Delta D} \left[ p_wD (t_D + \Delta D) \right] + \Delta D \frac{\partial}{\partial \Delta D} \left[ p_D (\Delta D) \right] \] ..........................(A-5)

By definition, the pressure integral-derivative function is given by

\[ p_D' (\Delta D) = \Delta D \frac{\partial}{\partial \Delta D} \left[ p_D (\Delta D) \right] \]

\[ = \Delta D \frac{\partial}{\partial \Delta D} \int_0^{\Delta D} p_D (\Delta D) \, d\Delta D \]

or

\[ p_D' (\Delta D) = p_D (\Delta D) - p_D (\Delta D) \] ..........................(A-6)

Note that

\[ p_D (\Delta D) = \Delta D \int_0^{\Delta D} p_D (\Delta D) \, d\Delta D \]

\[ = \Delta D \int_0^{\Delta D} p_D (t_D) \, d\Delta D = \Delta D \int_0^{\Delta D} p_D (t_D + \Delta D) \, d\Delta D \]

\[ + \Delta D \int_0^{\Delta D} p_D (\Delta D) \, d\Delta D \] ..........................(A-7)

Using a variable of substitution, we have

\[ \tau = (t_D + \Delta D) \] then \[ d\tau = d\Delta D \]

where the limits of integration become

\[ \Delta D = 0, \tau = t_D \]

\[ \Delta D = \Delta D, \tau = t_D + \Delta D \]

Substituting these relations into Eq. A-7 we obtain

\[ p_D (\Delta D) = p_wD (t_D) - \frac{1}{\Delta D} \int_0^{\Delta D} p_D (\tau) \, d\tau \]

\[ + \Delta D \int_0^{\Delta D} p_wD (\Delta D) \, d\Delta D \] ..........................(A-8)

Expanding Eq. A-8 gives

\[ p_D (\Delta D) = p_wD (t_D) \]

\[ - \frac{1}{\Delta D} \int_0^{\Delta D} p_D (\tau) \, d\tau + \int_0^{\Delta D} p_wD (\tau) \, d\tau \]

\[ + \Delta D \int_0^{\Delta D} p_wD (\Delta D) \, d\Delta D \] ..........................(A-9)

Completing the integration in Eq. A-9 gives

\[ p_D (\Delta D) = p_wD (t_D) - \frac{1}{\Delta D} \int_0^{\Delta D} p_D (\tau) \, d\tau \]

\[ + \frac{1}{\Delta D} \int_0^{\Delta D} p_wD (\tau) \, d\tau \]

\[ + \Delta D \int_0^{\Delta D} p_wD (\Delta D) \, d\Delta D \] ..........................(A-10)

Substituting Eqs. A-1 and A-10 into Eq. A-6 yields the pressure integral-derivative function for the pressure buildup case

\[ p_D (\Delta D) = \frac{p_D (\Delta D)}{\Delta D} - \frac{p_D (t_D + \Delta D)}{\Delta D} \]

\[ - \frac{p_wD (t_D + \Delta D) - p_D (t_D + \Delta D)}{\Delta D} \]

\[ - \frac{1}{\Delta D} \int_0^{\Delta D} p_wD (\tau) \, d\tau \] ..........................(A-10)

**TABLE 2**

<table>
<thead>
<tr>
<th>Reservoir, Fluid Property and Production Data:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir Properties:</td>
</tr>
<tr>
<td>Wellbore radius, ( r_w )</td>
</tr>
<tr>
<td>Net pay thickness, ( h )</td>
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<td>Porosity, ( \phi )</td>
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<td>Drainage Area, ( A ) (previous estimate)</td>
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<td>Fluid Properties:</td>
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<td>Total compressibility, ( c_t )</td>
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<td>Production Parameters:</td>
</tr>
</tbody>
</table>

SPE 29584
Production rate, \( q_o \)
Pressure at shut-in, \( p_{wf, At=0} \) (corrected) = 250 STB/D

\( p_{wf, At=0} \) (corrected) = 3561.9 psia

**Pressure-Time Data:**

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<th>Shut-in Pressure (psia)</th>
</tr>
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<td>4074.75</td>
</tr>
<tr>
<td>11.845</td>
<td>4074.82</td>
</tr>
</tbody>
</table>

**TABLE 3**

**Reservoir, Fluid Property and Production Data:**

**Reservoir Properties:**
- Wellbore radius, \( r_w \) = 0.51 ft
- Net pay thickness, \( h \) = 120 ft
- Porosity, \( \phi \) = 0.11 (fraction)
- Drainage Area, \( A \) (previous estimate) = 2341.6 acres

**Fluid Properties:**
- Oil formation volume factor, \( B_o \) = 1.21 RB/STB
- Oil viscosity, \( \mu_o \) = 2.40 cp
- Total compressibility, \( c_i \) = 5.0 x 10^{-6} psi^{-1}

**Production Parameters:**
- Production rate, \( q_{last} \), \( q_o \) = 6304.0 STB/D
- Pressure at shut-in, \( p_{wf, At=0} \) = 3888.89 psia

**Table 3** (Cont'd)

**Production History:**

<table>
<thead>
<tr>
<th>Elapsed Time (hrs)</th>
<th>Flow Rate (STB/D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
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<tr>
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<td>15.900</td>
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</tbody>
</table>

**TABLE 3**

**Case 2 -- Proano and Lilley**

**Example 1**
TABLE 1
Dimensionless Time Based on Drainage Area Required to Reach Pseudosteady-State Flow Conditions
(Steady-State for Reservoirs with Constant Pressure Outer Boundaries)

<table>
<thead>
<tr>
<th>Reservoir Type</th>
<th>Dimensionless Time to Pseudosteady-State, $t_{DAPSS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No-Flow Outer Boundaries</td>
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<tr>
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<td>Ref.25</td>
</tr>
<tr>
<td><img src="image1" alt="Circle" /></td>
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</tr>
<tr>
<td><img src="image2" alt="Square" /></td>
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</tr>
<tr>
<td><img src="image3" alt="Square with Cross" /></td>
<td>0.6</td>
</tr>
<tr>
<td><img src="image4" alt="Square with Circle" /></td>
<td>0.5</td>
</tr>
<tr>
<td><img src="image5" alt="Square with Circle and Cross" /></td>
<td>0.2</td>
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<tr>
<td><img src="image6" alt="Square with Cross and Circle" /></td>
<td>1.0</td>
</tr>
<tr>
<td><img src="image7" alt="Square with Cross and Circle" /></td>
<td>0.8</td>
</tr>
<tr>
<td><img src="image8" alt="Square with Four Circles" /></td>
<td>0.7</td>
</tr>
<tr>
<td><img src="image9" alt="Square with Four Circles and Cross" /></td>
<td>2.5</td>
</tr>
<tr>
<td><img src="image10" alt="Square with Four Circles and Cross" /></td>
<td>3.0</td>
</tr>
<tr>
<td><img src="image11" alt="Square with Eight Circles" /></td>
<td>---</td>
</tr>
<tr>
<td><img src="image12" alt="Square with Eight Circles and Cross" /></td>
<td>---</td>
</tr>
<tr>
<td><img src="image13" alt="Square with Eight Circles and Cross" /></td>
<td>---</td>
</tr>
</tbody>
</table>
Figure 1 - Buildup Type Curve Plot of Dimensionless Pressure Derivative and Integral-Derivative Functions for a Well Centered in a Bounded Circular Reservoir With a No-Flow Outer Boundary.

Figure 2 - Buildup Type Curve Plot of Dimensionless Pressure Derivative and Integral-Derivative Functions for a Well Centered in a Bounded Square Reservoir With No-Flow Outer Boundaries.
Figure 3 - Dimensionless Pressure Functions for Various Observation Points in a Closed Square Reservoir with the Producing Well at the Center of the Reservoir. Comparison of the Ozkan and Raghavan Solution with the Results of Earlougher, et al.
Figure 4 - Pressure Buildup Type Curves for Rectangular Reservoirs. Dimensionless Pressure Derivative and Integral-Derivative Responses Shown as Functions of Dimensionless Time Based on Drainage Area.
Figure 5 - Dimensionless Matthews-Brons-Hazebroek pressure derivative functions for a well at various positions in an 1x1 (square) and 2x1 rectangular reservoir. The pseudosteady-state flow condition is indicated by the $p'_{DMBH}$ functions converging to unity.

Figure 6 - Dimensionless Matthews-Brons-Hazebroek pressure derivative functions for a well at various positions in an 4x1 and 8x1 rectangular reservoir. The pseudosteady-state flow condition is indicated by the $p'_{DMBH}$ functions converging to unity.
Figure 7 - Buildup Type Curve Plot of Dimensionless Pressure Derivative and Integral-Derivative Functions for a Well Centered in a Bounded Circular Reservoir With a Constant Pressure Outer Boundary.

Figure 8 - Buildup Type Curve Plot of Dimensionless Pressure Derivative and Integral-Derivative Functions for a Well Centered in a Bounded Square Reservoir With a Constant Pressure Outer Boundary.
Figure 9 - Pressure Buildup Type Curves for Constant Pressure Rectangular Reservoirs. Dimensionless Pressure Derivative Responses are Shown as Functions of Dimensionless Time Based on Drainage Area.
Figure 10 - Pressure Buildup Type Curves for Rectangular Reservoirs, Normalized Time Scale. Dimensionless Pressure Derivative and Integral-Derivative Responses are Shown as Functions of Dimensionless Time Based on Drainage Area, Normalized by a Geometric Factor Related to Reservoir Shape and Well Location.
Figure 11 - Pressure Buildup Type Curves for Constant Pressure Rectangular Reservoirs, Normalized Time Scale. Dimensionless Pressure Derivative Responses are Shown as Functions of Dimensionless Time Based on Drainage Area, Normalized by a Geometric Factor Related to the Reservoir Shape and Well Location.
Figure 12 - Pressure buildup type curves for rectangular reservoirs, normalized time scale. Dimensionless pressure derivative responses are shown as functions of $\Delta t_D^4/t_D$.

Figure 13 - Pressure drawdown type curves for closed (no-flow) reservoirs. Dimensionless pressure derivative responses shown as functions of dimensionless time based on drainage area.
Figure 14 - Verification of Pressure Buildup Type Curves for Rectangular Reservoirs Using Commercial Reservoir Simulation Programs.
Figure 15 - Semilog Plot and Analysis for Pressure Buildup Example Case 1.

Figure 16 - Transient Type Curve Analysis for Pressure Buildup Example Case 1.
Figure 17 - Post-Transient Type Curve Analysis for Pressure Buildup Example Case 1.

Figure 18 - Simulated Performance for Pressure Buildup Example Case 1.
Figure 19 - Semilog Plot and Analysis for Pressure Buildup Example Case 2.

Figure 20 - Transient Type Curve Analysis for Pressure Buildup Example Case 2.
Figure 21 - Post-Transient Type Curve Analysis for Pressure Buildup Example Case 2.

Figure 22 - Simulated Performance for Pressure Buildup Example Case 2.