CORRELATION OF BUBBLEPOINT PRESSURES FOR RESERVOIR OILS--A COMPARATIVE STUDY

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Abstract

None of the currently proposed correlations for bubblepoint pressure are particularly accurate.

Knowledge of bubblepoint pressure is one of the important factors in the primary and subsequent developments of an oil field. Bubblepoint pressure is required for material balance calculations, analysis of well performance, reservoir simulation, and production engineering calculations.

In addition, bubblepoint pressure is an ingredient, either directly or indirectly, in every oil property correlation. Thus an error in bubblepoint pressure will cause errors in estimates of all oil properties. These will propagate additional errors throughout all reservoir and production engineering calculations.

Bubblepoint pressure correlations use data which are typically available in the field; initial producing gas-oil ratio, separator gas specific gravity, stock-tank oil gravity, and reservoir temperature. The lack of accuracy of current bubblepoint pressure correlations seems to be due to an inadequate description of the process -- in short, one or more relevant variables are missing in these correlations.

We considered three independent means for developing bubblepoint pressure correlations. These are (1) non-linear regression of a model (traditional approach), (2) neural network models, and (3) non-parametric regression (a statistical approach which constructs the functional relationship between dependent and independent variables, without bias towards a particular model).

The results, using a variety of techniques (and models), establish a clear bound on the accuracy of bubblepoint pressure correlations. Thus, we have a validation of error bounds on bubblepoint pressure correlations.

Introduction

Correlations of the bubblepoint pressures of reservoir oil commonly use reservoir temperature, stock tank oil gravity, separator gas specific gravity, and solution gas/oil ratio at the bubblepoint as independent variables. These are the only data commonly available from field observations.

This paper presents a comparative analyses of the correlation of bubble-point pressures for reservoir oils using:

1. Standing-type models fitted using non-linear regression.

2. Non-parametric regression. This approach yields a "non-parametric" (point-to-point) regression of the data, but can also be adapted to provide functional approximations for the transforms of the individual variables.

3. Neural network modeling.

A data set consisting of data from 728 laboratory reservoir fluid studies (PVT studies) was used to create these three models for prediction of reservoir bubblepoint pressure. The ranges of data in this data set maybe found in Table 1.

Standing-Type Models:

A Standing-type model was used to correlate this database. In particular, we present the work of Velarde, where the modification of Standing's model given by Petrosky was used. This modified Standing-type relation is given by:

\[ p_b = a_1 \left[ \frac{R_g^{a_2} + a_3}{T_b \gamma_g^{a_4}} \right]^{10} \] .............................. (1)

where,

\[ X = \left( T^{a_6} \right)^{a_7} \left( a_8 AP^{a_9} \right) \] .............................. (2)
This model was fitted to the data set with non-linear regression, the following result was obtained:

\[ P_b = 1091.47 \left[ R_{sb}^{0.081465} \gamma_g^{-0.161488} \times 10^X - 0.740152 \right]^{-5.354891} \]

where,

\[ X = \left( 0.013098 T^{0.282372} - 8.2 \times 10^{-6} API^{2.176124} \right) \]

The Velarde relation (Eqs. 3 and 4) reproduced the bubblepoint pressures from the data set used in its creation to an average error of 0.6% and an average absolute error of 11.5%. Fig. 1 is a comparison of bubblepoint pressures predicted by the Velarde relation with the experimental data.

Non-Parametric Regression: (GRACE algorithm\(^4,5\))

Parametric regression—that is, regression where a prescribed model is fitted to data, is a robust and effective mechanism for representing a data function. However, it provides little insight into the interrelation of the independent variables, nor does it provide a "global" minimum expected error of the dependent and independent variables.

The non-parametric regression approach proposed by Breiman and Friedman,\(^4\) and refined by Xue,\ et al.,\(^5\) provides exactly such a "non-biased" mechanism for the purpose of establishing the minimum error relationship between the dependent and independent variables. The method of Alternating Conditional Expectations (ACE)\(^4\) is based on the concept of developing an optimal transformation of each dependent - and independent variables. -The method of establishing the minimum error relationship between the dependent and independent variables, this approach does not provide a computation (i.e., predictive) model. Figs. 2 through 5 show the optimal transformations of the individual data for the four independent variables. The optimal transform of the data for the dependent variable, bubblepoint pressure, is given in Fig. 6. This "non-parametric" regression results in a reasonable match of calculated and measured bubblepoint pressures. Fig. 7 shows this comparison of calculated and measured bubblepoint pressures for the non-parametric optimization.

The ACE and GRACE algorithms do not provide a predictive model, i.e., they do not result in equations. However, simple quadratic polynomials can be fitted to the optimal data transforms, resulting in:

\[ z_1 = 1.2206 - 1.8029 x 10^{-2} (API) - 3.8682 x 10^{-4} (API)^2 \]

\[ z_2 = 3.8649 - 6.5080 \gamma_g + 1.9670 x (\gamma_g)^2 \]

\[ z_3 = -7.7161 + 2.3604 \ln (T_{res}) - 1.6678 x 10^{-1} \ln (T_{res})^2 \]

\[ z_4 = -1.0056 x 10^1 + 1.6664 \ln (R_{sb}) \]

The inverse transform is defined as:

\[ z_0 = z_1 + z_2 + z_3 + z_4 \]

And the inverse transform, in terms of \(\ln (P_b)\), is given by:

\[ \ln (P_b) = 7.8421 + 4.9506 x 10^{-1} \ln (z_0) - 2.5726 x 10^{-3} \ln (z_0)^2 \]

The application of these quadratic polynomials to the optimal transforms determined by non-parametric regression does not seriously degrade the quality of the fit to the bubblepoint pressures. Eqs. 7 through 12 reproduce the bubblepoint pressures in the data set to an average error of 3.1 percent and an average absolute error of 11.8 percent. Fig. 8 gives a comparison of bubblepoint pressures calculated with
Validation of the Predictive Models

An independent data set consisting of data from 547 PVT studies was used to test the three models. None of these data were included in the 728 sample data set used in developing the models. Table 4 gives the ranges of data in the independent data set. Notice that the maxima and minima in the several variables are very similar in the two data sets (Table 1, Table 4).

The predictive abilities of the three models were tested using the independent variables from the independent data set and comparing the calculated values of bubblepoint pressure with the measured values. Table 5 gives the results. The non-linear regression and non-parametric models worked reasonably well on the independent data set—retaining their accuracy of approximately 13 percent average absolute error. The average absolute error of the neural network model increased from 6 percent (for the "original" data) to 25 percent for the independent data set. Figs. 10 through 12 show the calculated-measured comparison. Especially noteworthy is the comparison between Figs. 9 and 12 which illustrates that a trained neural network will not necessarily provide accurate predictions for data not involved in the "training."

Several other bubblepoint pressure correlations from the literature were tested with the independent data base. Table 6 and Figs. 13 through 15 show how predictions of the Standing, Vasquez and Beggs, and Kartoatmodjo and Schmidt equations (all non-linear regression type models) compare with the measured bubblepoint pressures from the independent data base.

Other proposed bubblepoint pressure correlations were also tested with the independent data set. In each case the average absolute error was larger than those given in Table 6. Unfortunately, the best results for any correlation, whether prepared by non-linear regression, non-parametric regression, or neural network, can only predict bubblepoint pressure (given the four commonly available independent variables) to an average absolute error of about 13 percent. This means that errors of 25 percent or greater are possible for a given situation.

Bubblepoint pressure is used, either directly or indirectly, in all oil property correlations. Thus the errors in estimates of bubblepoint pressure will propagate throughout all estimates of other fluid properties such as oil formation volume factor, oil viscosity, oil density, etc.

Correlations for these other oil fluid properties are reasonably accurate given accurate values of bubblepoint pressure. It appears that an accurate bubblepoint pressure correlation is not possible (given the usually available input data). Thus, two options are available: regular measurement of average reservoir pressure (pressure buildup tests, etc.) or obtaining a representative sample of the original reservoir fluid and measuring bubblepoint pressure and other properties in the laboratory.

Conclusions

1. The best possible correlations of bubblepoint pressure; given the usual input data; solution gas-oil ratio at bubblepoint, stock-tank oil gravity, separator gas specific gravity, and reservoir temperature; are accurate to an average absolute error of about 13 percent. This means that predicted values of bubblepoint pressure could be in error by 25 percent or more in some instances.
2. Errors this large will cause unacceptably high errors in the prediction by correlation of the other oil fluid properties of interest: oil formation volume factor, oil density, oil viscosity, and oil compressibility.
3. The only options currently available for obtaining accurate values of bubblepoint pressure are either regular field measurement of average reservoir pressures or laboratory measurement with a sample representative of the original reservoir oil.
Nomenclature

API = stock-tank gravity, °API

\( R_{sb} \) = solution gas-oil-ratio at the bubble-point, scf/STB

\( T \) = reservoir temperature, °F

\( X \) = temporary variable used in Petrosky2 correlation

\( x_n \) = independent variables, various units

\( y \) = dependent variable

\( z_n \) = transforms of the independent are dependent variables

\( \gamma_g \) = gas specific gravity, air = 1

References


### Table 1 - Ranges of Data in Data Set Used to Develop \( p_b \) Correlations (728 data sets)

<table>
<thead>
<tr>
<th>( p_b )</th>
<th>( T )</th>
<th>( R_{sb} )</th>
<th>( \text{API} )</th>
<th>( \gamma_g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>&lt;</td>
<td>6700</td>
<td>&lt;74</td>
<td>&lt;327</td>
</tr>
<tr>
<td>10</td>
<td>&lt;</td>
<td>1870</td>
<td>&lt;12</td>
<td>&lt;55</td>
</tr>
<tr>
<td>12</td>
<td>&lt;</td>
<td>1.367</td>
<td>(air = 1)</td>
<td></td>
</tr>
</tbody>
</table>
The neural network model has the following form:

\[ P_{\text{oi, normalized}} = \{w3 \cdot \tanh^{-1}[w2 \cdot \tanh^{-1}(w1 \cdot z_{\text{normalized}} + b1)] + b2\} + b3 \]

And the equation that used for normalized the input and output variables during the training process was:

\[ z_{\text{normalized}} = 2 \cdot z_i \cdot (z_{\text{MIN}})/(z_{\text{MAX}} - z_{\text{MIN}}) \]

Where \( z_i \) represents each of the input variables:

\( Z(1) = \) reservoir temperature, \( Z(2) = \) API, \( Z(3) = \) gas gravity, and \( Z(4) = \) GOR.

The maximum and minimum values used were:

<table>
<thead>
<tr>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir temperature</td>
<td>327</td>
</tr>
<tr>
<td>API</td>
<td>55</td>
</tr>
<tr>
<td>Gas gravity</td>
<td>1.367</td>
</tr>
<tr>
<td>GOR</td>
<td>1870</td>
</tr>
<tr>
<td>Pob</td>
<td>6700</td>
</tr>
</tbody>
</table>

The values for the bias (b1, b2, and b3) and for the weights (w1, w2, and w3) are:

\[ b1 = \{-3.28314879863207, 0.80243017013076, 1.08076979327138, -2.83265706363712\} \]

\[ b2 = \{1.92765824477335, 0.379736126901044, -0.0897286359210449, -1.7822122415044, -1.61216953831698\} \]

\[ b3 = -0.39379012457655 \]

\[ w1 = \{-0.795250144548125, -1.09209856833608, -0.97316703561262, 2.25817004067108, 0.0113831825310405, -0.0220112913878158, -0.0755903427477344, 0.820670010457579, -0.0443089522480293, 0.465761383477398, 0.897960024862847, 0.10774608592107, -0.132352081142938, 0.0374516997795222, 2.82233244250817, -1.05099952370368, 1.6134423199094, -1.56719989187548, -0.595446499130364, 0.07808285404462807\} \]

\[ w2 = \{-1.09646815273972, -0.854864648278981, -1.1247171493734, -0.138020648591696, 0.759786020717426, 0.077022164481693, -0.67113407900328, 0.51944826341125, 1.2240050238773, -1.37221570727192, 1.47930665237534, 1.138985511497154, 1.41822922194687, 0.197907870996054, 0.720664068001338, -0.62378427591244, -1.2491523164169, 1.95526556348521, -0.400971121833293, -0.0963149005694486, -1.12898030283311, -0.795224736531393, -1.0962449663587, 0.081550991824799, 0.20377122773846\} \]

and

\[ w3 = \{-0.25763606379853, -0.102717707525673, 2.50555221016993, -0.334182893301272, -2.688487244623\} \]

| Table 3 - Comparison of Results of the Three Prediction Models with Measured Bubblepoint Pressure for the Data Base Used in the Developments of the Models (the 728 Sample data set) |
|---------------------------------|-----------------|-----------------|
|                                | Average Error in \( p_b \) percent | Average Absolute Error in \( p_b \) percent |
| Non-linear regression-Standing-type equation | 0.6 | 11.4 |
| Non-parametric regression | 3.1 | 11.8 |
| Neural network | -0.3 | 6.0 |
Table 4 - Ranges of Data in the Independent Data Set Used to Validate the \( p_b \) Correlations

<table>
<thead>
<tr>
<th>( p_b )</th>
<th>( T )</th>
<th>( R_{sh} )</th>
<th>( \gamma )</th>
<th>( \text{API} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>121 psia</td>
<td>88°F</td>
<td>12 scf/STB</td>
<td>0.555</td>
<td>14.4 API</td>
</tr>
</tbody>
</table>

Table 5 - Comparison of Results of the Three Prediction Models with Measured Bubblepoint Pressures for the Independent Data Base (the 547 Sample data set)

<table>
<thead>
<tr>
<th>Prediction Model</th>
<th>Average Error in ( p_b ) percent</th>
<th>Average Absolute Error in ( p_b ) percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-linear regression-Standing-type equation</td>
<td>12.8</td>
<td></td>
</tr>
<tr>
<td>Non-parametric regression</td>
<td>13.0</td>
<td></td>
</tr>
<tr>
<td>Neural network</td>
<td>25.3</td>
<td></td>
</tr>
</tbody>
</table>

Table 6 - Comparison of the Results of Various Bubble-Point Pressure Correlations With Data From Independent Data Base

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Average Error in ( p_b ) percent</th>
<th>Average Absolute Error in ( p_b ) percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standing(^1)</td>
<td>-6.1</td>
<td>13.2</td>
</tr>
<tr>
<td>Vasquez and Beggs(^7)</td>
<td>2.8</td>
<td>13.1</td>
</tr>
<tr>
<td>Kartoatmodjo and Schmidt(^2)</td>
<td>-2.0</td>
<td>14.2</td>
</tr>
</tbody>
</table>

Fig. 1.- Comparison of calculated and measured bubblepoint pressures for Velarde \( p_b \) correlation. Correlation data set.

Fig. 2.- Grace optimal transformation of oil gravity (°API).
Fig. 3. - Grace optimal transformation of reservoir temperature, $T$.

Fig. 4. - Grace optimal transformation of gas specific gravity (air=1.0).

Fig. 5. - Grace optimal transformation of gas-oil-ratio ($R_{go}$).

Fig. 6. - Grace optimal transformation of bubblepoint pressure, $P_b$.

Fig. 7. - Comparison of calculated and measured bubblepoint pressures for Grace $P_b$ correlation-no model. Correlation data set.

Fig. 8. - Comparison of calculated and measured bubblepoint pressures for Grace $P_b$ correlation-quadratic equation model. Correlation data set.
Comparison of calculated and measured bubblepoint pressures for neural network $p_b$ correlation. Correlation data set.

Fig. 9.

Comparison of calculated and measured bubblepoint pressure for Velarde $p_b$ correlation. Independent data set.

Fig. 10.

Comparison of calculated and measured bubblepoint pressure for non-parametric $p_b$ correlation. Independent data set.

Fig. 11.

Comparison of calculated and measured bubblepoint pressures for Standing $p_b$ correlation. Independent data set.

Fig. 12.

Comparison of calculated and measured bubblepoint pressure for Vasquez and Beggs $p_b$ correlation. Independent data set.

Fig. 13.

Fig. 14.
Fig. 15. - Comparison of calculated and measured bubblepoint pressure for Kartoatmodjo and Schmidt $P_b$ correlation. Independent data set.