Development and Application of the Multiwell Productivity Index (MPI)

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Summary
In this article we generalize the concept of the pseudosteady-state productivity index for the case of multiple wells producing from or injecting into a closed rectangular reservoir of constant thickness. The work complements the analytical study by Rodriguez and Cinco-Ley for systems produced at constant flowing pressures. Wells are represented by fully penetrating vertical line sources located arbitrarily in a homogeneous and isotropic reservoir. The multiwell productivity index (MPI) is a square matrix of dimension n, where n is the number of wells. The MPI provides a simple, reasonably accurate and fast analytical tool to evaluate well performance without dividing the cluster into single-well drainage areas. The MPI approach is used to obtain approximate analytical solutions for constant (but possibly different) wellbore flowing pressures, and to visualize the resulting pressure field. In addition, the skin factor trace technique is introduced as a tool to monitor a cluster of wells. The MPI technique is illustrated using a synthetic example taken from Ref. 2, as well as two field cases.

Introduction
Although it has been known from the inception of reservoir engineering studies that any change in the condition of a well (damage, stimulation, choke change) modifies the production characteristics of the well-reservoir system, only the pioneering work of Rodriguez and Cinco-Ley approached this problem using analytical techniques. The constant flowing wellbore pressure solution of Ref. 1 has been improved in several ways by Camacho-V. et al., including the possibility of allowing wells starting to produce at different times.

Similar to the work in Refs. 1 and 2, we consider liquid flow within a homogeneous reservoir of constant thickness. This work is, however, restricted to pseudosteady-state flow conditions. The results can be used for any type of wellbore condition so long as the pseudosteady-state approximation is acceptable.

Pseudosteady-state flow is the (idealized) finite-acting portion of the constant-rate solution for a bounded reservoir, and this lends itself to a simpler description. The physical meaning is that all elementary portions of the reservoir contribute to the overall production rate by the same amount. Depletion is a parallel shift of the pressure distribution with time.

While, formally, the pseudosteady state is a limiting case of the constant-rate solution, solution-gas drive reservoirs spend most of their life in a series of states, and closely resemble this flow condition.

The basic concept in this article is the multiwell productivity index (MPI) matrix, which relates the production rate vector to the pressure drawdown vector. The pseudosteady-state flow condition ensures the uniqueness of the MPI matrix for a given system. (Without the assumption of the pseudosteady-state flow condition, this uniqueness would be lost, even for a single-well system.)

Multiwell Productivity Index
We consider a rectangular homogeneous reservoir of uniform thickness, h, porosity, φ, permeability, k, and no-flow outer boundaries. The single-phase fluid viscosity, μ, and the total compressibility, c_t, are considered constant. The wells are represented by line sources.

The four corner points of the rectangle are located at (0, 0), (x, 0), (0, y), and (x, y). In any given time interval, the number of wells, n, their locations (x_wj, y_wj), the wellbore radii, r_wj, and the skin factors, s_j, are considered constant. Fig. 1 shows a schematic of the reservoir.

For the case where only one well is on production (say, the jth one), Ozkan (see also Ref. 6, page 107) gives the pressure distribution in the reservoir during the pseudosteady state as

$$\bar{p} - p(x,y) = \frac{a_1 \mu B}{2 \pi kh} \sum_{j=1}^{n} a[x_D, y_D - x_wD, y_wD] q_j,$$

where the influence function $a[x_D, y_D - x_wD, y_wD]$ is given by

$$a[x_D, y_D - x_wD, y_wD] = 2 \pi \left[ \frac{1}{3} - \frac{y_D}{y_{wD}} + \frac{y_{wD}^2}{2 y_{wD}^2} \right]$$

and

$$t_n = \frac{\cosh[m \pi (y_{wD} - y_D) - y_{wD}]}{\sinh[m \pi y_{wD}]}.$$

with $x_D$ and $y_D$ defined as $x/x_e$ and $y/y_e$, respectively. In the Appendix we introduce several computational simplifications to ensure a fast and reliable approximation of the infinite sum in Eq. 2. Also shown in the Appendix is the relation of the influence function to the Dietz' shape factor.

By superpositing, for the n-well system with n production/injection wells, we have

$$\bar{p} - p(x,y) = \frac{\alpha_1 \mu B}{2 \pi kh} \sum_{j=1}^{n} a[x_D, y_D - x_wD, y_wD, y_{wD}] q_j,$$

where the $q_j$ production rates are ‘‘constant’’ for the given time interval. All prior information is contained in the average pressure, $\bar{p}$, and, as such, we do not need to specifically account for the initial pressure distribution or the production history.

The two basic vector quantities we would like to relate are the pressure drawdown vector,

$$\bar{d} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} = \begin{bmatrix} \bar{p} - p_{w1} \\ \bar{p} - p_{w2} \\ \vdots \\ \bar{p} - p_{wn} \end{bmatrix},$$

and the surface production-rate vector,

$$\bar{q} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}. \quad (6)$$

Applying Eq. 4 for a point located at the circumference of wellbore i and taking into account the drop in pressure due to the skin factor, we obtain
The dimensionless MPI matrix \([J_0]\) plays the same role as the dimensionless productivity index,
\[
J_0 = 1 \left( \frac{3}{4} + s \right),
\]
(15)
for a single well located in the center of a circular reservoir.

Using vector notation, the overall production rate, \(q_T\), can be written as
\[
q_T = \overline{J}_o \overline{d},
\]
(16)
where the overall productivity index, \(\overline{J}_o\), is the vector obtained as the sum of the rows of the MPI matrix. The overall production rate is the scalar product with the drawdown vector. (The superscript \(T\) denotes “transposed,” and it is used here because vectors without the superscript \(T\) are treated as column vectors.)

**Applications**

**Constant Flowing Wellbore Pressure Solution.** The overall material balance for a slightly compressible liquid system is
\[
\frac{d\bar{p}}{dt} = \frac{\alpha_2}{c_i V_p} q_T.
\]
(17)
Substituting Eq. 16 into the material balance relation we obtain
\[
\frac{d\bar{p}}{dt} = c_2 - c_1 \bar{p},
\]
(18)
where
\[
c_1 = \frac{\alpha_2}{c_i V_p} \left( [1 \cdots 1] \overline{J}_o \right)
\]
and
\[
c_2 = \frac{\alpha_2}{c_i V_p} (\overline{J}_o \overline{d}_w).
\]
do not depend on time. The physical meaning of \(c_1\) is that it is a decline exponent (time\(^{-1}\)), and \(c_2/c_1\) is a limiting pressure, which is the weighted average of the constant flowing wellbore pressures, with the weights being the elements of the overall productivity index. If a specified flowing wellbore pressure is less than \(c_2/c_1\), the particular well will turn into an injector at late times, as explained in Ref. 1. By solving Eq. 18, we find that the average reservoir pressure follows an exponential decline,
\[
\bar{p} = \frac{c_2}{c_1} + \left( \bar{p}_i - \frac{c_2}{c_1} \right) \exp(-c_1 t).
\]
(19)

In Eq. 19 we use the average value of the initial pressure, where “initial” means the start of the pseudosteady-state flow condition, and time is measured from that point. Hence, there is no need to know the initial pressure distribution, and obviously it needs not be constant. This is an important distinction, because we may wish to use Eq. 19 for a series of pseudosteady-state flow conditions, where it is not reasonable to assume that the pressure distribution is constant at the beginning of each subsequent pseudosteady state.

By substituting Eq. 19 into Eqs. 5 and 11, the production rates are easily obtained. All time functions will be of the following form: a constant plus an exponential term with the same decline exponent. The constant will be zero for the overall production rate. This is in accordance with the results derived in Ref. 1 using a more rigorous approach.

Our solution procedure uses the concept of pseudosteady state for the constant flowing wellbore pressure case. There is an apparent contradiction in this approach, because the pseudosteady state is not possible with constant wellbore pressure (the wellbore pressures must decline at the same rate as the average reservoir pressure). Nevertheless, one can consider a series of time intervals: in each interval the wellbore pressures decline by a small value, then the production rates are reduced, and the wellbore pressures jump by a small value. Considering the whole process,
the wellbore pressures ‘‘scatter’’ a little around the constant values in a saw-like manner as shown schematically in Fig. 2. The power of the pseudosteady-state concept lies in the fact that such an approximation causes little error in the calculated production rates. (A less rigorous but more popular formulation of the same fact for a single-well configuration is that ‘‘constant rate’’ and ‘‘constant pressure’’ productivity indices are essentially the same.)

Visualization of the Pressure Field. Eq. 4 can be used to calculate pressures at any point in the reservoir once the wellbore production rates are known. Rates can be obtained as observations in the case of field data, but we can also use the solution procedure described above to generate rate profiles. In either case, Eq. 4 is used to compute the pressure distribution. The effectiveness of our new approach to create pressure maps may find several applications.

Solving for Skin Factor. Given long-term production data for a cluster of wells, (i.e., a time, pressure and rate stream for each well), we can analyze the performance history of each well with respect to the other wells. By rearranging Eq. 8, we express the skin factor vector as

\[ \vec{s} = [D_q]^{-1} \left( \frac{2 \pi k h}{\alpha \mu B} \delta - [A] \vec{q} \right), \]

(20)

where

\[ [D_q]^{-1} = \begin{bmatrix}
 q_1^{-1} & 0 & \ldots & 0 \\
 0 & q_2^{-1} & \ldots & 0 \\
 \vdots & \vdots & \ddots & \vdots \\
 0 & 0 & \ldots & q_n^{-1}
\end{bmatrix}, \]

(21)

When using Eq. 20, the average reservoir pressure required for calculating the pressure drawdown vector can be obtained using the cumulative production, pore volume, and total compressibility coupled with an overall material balance. Wellbore flowing pressures and production rates are assumed to be available, as well as the permeability, thickness, viscosity, and formation volume factor. An element of the skin factor calculated from Eq. 20 is not necessarily the mechanical skin factor of the individual well, but rather a suitable measure of the deviation of the performance of the well from an ideal one. Nevertheless, if plotted against time, the ‘‘skin factor trace’’ obtained may reveal differences in the behavior of the individual wells. At first, the rationale for such a suggestion is not obvious. For a single well, this would mean the calculation of the skin factor from pressure drawdown, the theoretical PI, and the production rate. Reservoir heterogeneity and nonuniform reservoir thickness will affect the results, and once the pressure drops below the bubblepoint, gas evolution and multiphase flow effects will distort the calculated values. Therefore, for a single-well, pseudosteady-state data do not allow us to separate phenomena related to the bulk of the reservoir from phenomena restricted to the vicinity of the well. The idea presented here applies only to the multiwell scenario. In that case, it can be assumed that phenomena related to the bulk of the reservoir affect all wells similarly. Therefore, if Eq. 20 is used to create a skin factor trace plot, comparison of the curves may reveal a potential problem for a particular well, where one or more wells behave differently from the other members of the cluster. One may argue that production decline curves of the individual wells can serve the same purpose. The skin factor trace technique is, however, more appropriate because it correctly accounts for the effects that originate from the interaction of neighboring wells.

Synthetic Example. In Ref. 2, a synthetic example is considered for a reservoir initially drained by two wells. After 100 days a third well (an infill) begins to produce. Reservoir, fluid properties, as well as other information are presented in Table 1.

We first wish to calculate the MPI matrix and the overall productivity index vector for the first 100 days. The influence of the first well on its perimeter, that is, the \( a_{11} \) element of the influence matrix, is calculated using the procedure described in the Appendix. The dimensionless coordinates are \( x_{w1}=2,333.33 \) ft, \( y_{w1}=3,000 \) ft, and \( r_{w1} = 0.00017857 \), therefore

\[ a_{11} = a[0.233315184, 0.36666667, 0.23333333, 0.36666667, 0.5] = 10.6867. \]

Repeating the calculations for the remaining three elements we obtain

![Flowing Wellbore Pressure](Image)

**Fig. 2**—Approximation of the constant flowing wellbore pressure condition by a series of pseudosteady states, conceptual explanation.
Adding the skin factor matrix,

\[
[D_s] = \begin{bmatrix} 10 & 0 \\ 0 & 0 \end{bmatrix}
\]

to matrix \([A]\), yields

\[
[A] + [D_s] = \begin{bmatrix} 20.6876 & -1.26321 \\ -1.2633 & 10.4619 \end{bmatrix}
\]

After matrix inversion (LU decomposition and back substitution) we obtain

\[
[J_0] = ([A] + [D_s])^{-1} = \begin{bmatrix} 0.0487 & 0.005881 \\ 0.005881 & 0.09623 \end{bmatrix}
\]

The MPI in STB/D/psi is

\[
[J] = \begin{bmatrix} 4.1266 & 0.4983 \\ 0.4983 & 8.1601 \end{bmatrix}
\]

and the overall productivity index vector (in the same units) is

\[
J^T = [4.6250, 8.6584, 13.2753]
\]

In this time interval, \(c_1 = 0.00395 \text{ day}^{-1}\) and \(c_2/c_1 = 1000 \text{ psi}\) at 500 days the average reservoir pressure is 1,054 psi. Our Fig. 3 shows similar production behavior to Fig. 5 of Ref. 2, except for the few days after the infill well was turned on to production. This small discrepancy is expected, because our solution assumes the pseudosteady state, whereas in Ref. 2 all flow regimes are accounted for.

Fig. 4 is the three-dimensional visualization of the pressure field for the synthetic example at the end of the first 100 day period. The pressure field is shown as both a surface and as a contour plot. The pressure distribution at the end of the second period is shown in Fig. 5. A comparison of Figs. 4 and 5 reveals the effect of the infill well on the pressure distribution.

We note that, whenever a negative skin factor is involved, the calculated near-wellbore pressure is not realistic. While this is the known limitation of the concept of negative skin factor, rather than the limitation of the proposed approach, it must be kept in mind when the calculated pressure field is interpreted.

Since this example is synthetic, the skin factor trace calculated from our solution, shown in Fig. 6, is almost trivial. As can be seen, the skin factor trace consists of three horizontal lines. It is
worth comparing the skin factor behavior of Well 1 with the production decline of the same well shown in Fig. 3. There is a change in the rate of decline of production at 100 days due to the appearance of the infill well. However, this behavior is not the result of the deterioration of the near-well conditions and indeed, it is “filtered out” from the skin factor trace by Eq. 20.

Analysis of Field Data

Because of the importance of this potential application, our field examples will focus on the analysis and interpretation of well performance. We consider two study areas from the North Robertson unit (NRU) in Gaines County, Texas. The North Robertson (Clear Fork) Field was developed on a nominal 40-acre well spacing beginning in 1956. The dominant reservoir producing mechanism for the original 141 wells was solution-gas drive. The initial reservoir pressure in the Lower Clear Fork was estimated to be 2,800 psia. The original oil in place was estimated to be approximately 215 MMSTB from decline curve analysis, with primary producing period between 1956 and 1987.

The Clear Fork formation is a shallow-shelf carbonate composed primarily of a massive dolomite section with varying degrees of anhydrite cement. The geologic setting at the time of deposition and subsequent diagenesis contributed to the heterogeneous nature of the formation. The wells were initially completed in Lower, Middle, and Upper Clear Fork, at measured depths of between 6,200 and 7,200 ft. The majority of the original completion intervals were in the Lower Clear Fork, which is considered the main pay interval. All the wells in the North Robertson unit were eventually hydraulically fractured, usually in two stages, a Lower/Middle Clear Fork fracture stimulation and a subsequent Upper Clear Fork fracture stimulation.

The oil flow rate data were allocated to individual wells on a tract basis. The wells were tested (for allocation purposes) on a semiannual basis. For that reason, we choose to average the production data over 6 month intervals, although the analysis could easily be done on a monthly or even hourly basis, if such data were available.

Although there are no flowing bottomhole pressure data available for the North Robertson wells, almost all the wells were placed on rod pump very soon after their initial completion. These wells typically “pump off” in anywhere from 2 to 12 weeks, depending on the flow capacity of the reservoir surrounding the individual wells. At which time the bottomhole producing pressure can be assumed to be approximately 100 psia (based on extensive fluid level measurements). Because we are using 6 month average production data, we can safely assume that for all cases $p_{wf}$ is approximately 100 psia. The rock and fluid property data for the NRU cases are shown in Table 2.

The average reservoir pressure trend for each of the study areas was estimated based on the initial reservoir pressure and numerous shut-in bottomhole pressure data recorded at the end of the primary producing period (June 1987).

Since these wells produce from a rather large gross reservoir interval ($\geq 1,000$ ft), it is often difficult to determine exactly how much of the gross interval actually contributes to production. By using the average reservoir pressure as an input variable, we determined the net thickness of the study area from material balance. We also note that these resulting net thickness values correlated well with the “reservoir quality” defined by previous geologic studies.

Field Example 1. The first field example consists of five wells in the south-central portion of the NRU shown in Fig. 7. This area is considered to be of moderate to poor reservoir quality. Using the average reservoir pressure at the end of primary production as an input parameter, we obtained an effective net pay thickness of 83 ft. The calculated original oil in place from the multiwell technique was 10.5 MMSTB, while the sum of the oil-in-place values for the individual wells from decline curve analyses yielded 6.7

![Fig. 6–Skin factor trace, synthetic example.](image)

![Fig. 7–Well configuration, field example No. 1.](image)

![Table 2—NRU (CLEAR FORK) ROCK AND FLUID PROPERTY DATA](table)
The discrepancy between the two values is likely a function of the reservoir area we used in our analysis and the result of the reservoir heterogeneity that exists within the Clear Fork interval. That is, each individual well does not completely contact all the oil within its drainage area. We would expect that in a less heterogeneous system these values would be much closer to one another and, in fact, the difference in the results between the two analysis techniques could be used to define the degree of reservoir heterogeneity.

The oil rates for individual wells and the calculated average reservoir pressure trend for the study area are shown in Fig. 8. The calculated skin factor traces for all wells are shown in Fig. 9. As expected, the multiwell skin factors tend to increase with time. The explanation for sharp decreases could be always found from well records and were the result of individual well remediations. The results for each individual well are shown in Figs. 10 through 14, where we have annotated these figures to indicate workovers and recompletions. The transient (early-time) skin factors from individual well decline curve analyses are shown for each well for comparison purposes (these are given as horizontal lines). We note good agreement between the multiwell skin factor trace and the decline curve skin factor, e.g., for early-time data in Fig. 10, for all times in Fig. 12, for late time data in Fig. 13, and for intermediate times in Fig. 14. For Fig. 11 the agreement is poor, indicating that the pseudosteady-state and transient analyses do not always agree. The ability of the skin factor trace to signal well problems is obvious for two wells, NRU 3003 and NRU 3006. These wells showed a more intense performance deterioration than the other wells in the cluster, and it is not by chance that those two wells were abandoned before the end of the primary production period. Looking only at the oil production rates (Fig. 8), the poor performance of those two wells becomes obvious much later than from Fig. 9. The pressure distribution at 6,000 days (Fig. 15) shows that, because of the high skin factors of the two problematic wells, the northern part of the study area was not being effectively produced at that time.

Field Example 2. Our second field example consists of five wells in the southeastern portion of the NRU (Fig. 16) in an area of moderate to high reservoir quality. Using the average reservoir pressure at the end of primary production as an input parameter, we obtained an effective net pay thickness of 123 ft. The calculated original oil in place from the multiwell technique was 16.8 MMSTB, while the sum of the oil-in-place values for the indi-
Fig. 10–Oil rate and skin factor trace, Well NRU 3003, field example No. 1.

Fig. 11–Oil rate and skin factor trace, Well NRU 3006, field example No. 1.

Fig. 12–Oil rate and skin factor trace, Well NRU 3007, field example No. 1.
Fig. 13—Oil rate and skin factor trace, Well NRU 3008, field example No. 1.

Fig. 14—Oil rate and skin factor trace, Well NRU 1702, field example No. 1.

Fig. 15—Pressure distribution at 6,000 days, field example No. 1.

Fig. 16—Well configuration, field example No. 2.
Individual wells from decline curve analyses was 11.6 MMSTB. The oil rates for the individual wells and the calculated average reservoir pressure for the study area are shown in Fig. 17. The calculated multiwell skin factor trace, shown in Fig. 18, indicates high completion efficiencies and only moderate or no damage evolution. Any significant improvement in the skin factor trace could be identified as a direct result of individual well workovers. The pressure distribution at 6,000 days (Fig. 19) shows that the wells depleted the cluster in an essentially balanced manner.

Conclusions
The multiwell productivity index provides a useful analytical tool which expands the capabilities of the reservoir/production engineer. Approximate analytical solutions can be easily generated, as can a pressure map for a cluster of wells. Based on the concept of the multiwell productivity index, we introduced a technique to analyze the performance behavior of a cluster of wells. The approach is best suited to analyze a moderate number of wells in areas that can be described by fairly homogeneous reservoir properties. Therefore, the scope of the proposed technique lies be-
between production decline curve analysis of individual wells and large-scale numerical flow simulation of entire fields. In the first field case we demonstrated that the multiwell skin factor trace may allow early detection of well performance problems at a time when visual examination of the production decline plot might yet be inconclusive. The analysis of the second field case concluded that within the particular study area the wells performed uniformly. Based on the field examples, we suggest that the technique can be incorporated into real time reservoir surveillance systems.

Nomenclature

\[ a \] = influence function  
\[ B \] = formation volume factor, RB/STB  
\[ B_0 \] = oil formation volume factor, RB/STB  
\[ C_A \] = Dietz shape factor (single well)  
\[ c_1 \] = decline constant, \( t^{-1} \), day\(^{-1} \)  
\[ c_2 \] = arbitrary constant, \( m/(L^2 t) \), psi/day  
\[ c_e \] = oil compressibility, \( L^3/tm \), psi\(^{-1} \)  
\[ c_s \] = total system compressibility, \( L^2/m \), psi\(^{-1} \)  
\[ h \] = formation thickness, L, ft  
\[ J \] = productivity index, \( L^4/t/m \), \( L^2/m \), (STB/D)/psi  
\[ J_0 \] = dimensionless productivity index  
\[ k \] = effective permeability, \( L^2 \), md  
\[ n \] = number of wells  
\[ N \] = number of terms in the summation  
\[ p \] = pressure, \( m/(L^2 t) \), psi  
\[ \bar{p} \] = material balance average reservoir pressure, \( m/(L^2 t) \), psi  
\[ \bar{p}_i \] = initial material balance average reservoir pressure, \( m/(L^2 t) \), psi  
\[ P_{wf} \] = flowing bottomhole pressure, \( m/(L^2 t) \), psi  
\[ q \] = oil flow rate, \( L^3/t \), STB/D  
\[ r_e \] = drainage radius for a single well, L, ft  
\[ r_w \] = wellbore radius, L, ft  
\[ x \] = skin factor  
\[ S_{T}, S_1, S_2, S_3 \] = terms in the summation  
\[ t \] = time, t, days  
\[ x_s \] = size of study area in the x direction, L, ft  
\[ y_s \] = size of study area in the y direction, L, ft  
\[ x_i \] = individual well x coordinate, L, ft  
\[ y_i \] = individual well y coordinate, L, ft  
\[ x_D \] = dimensionless x-coordinate well location  
\[ y_D \] = dimensionless y-coordinate well location  
\[ V_p \] = pore volume, \( L^3 \), ft\(^3 \)

Greek Letters

\[ \alpha_1 \] = conversion factor (for field units = 887.22, for consistent system = 1)  
\[ \alpha_2 \] = conversion factor (for field units = 5.615, for consistent system = 1)  
\[ \phi \] = porosity, fraction  
\[ \mu \] = fluid viscosity, \( m/(L t) \), cp  
\[ \gamma \] = Euler’s constant (0.577 215 66...)  
\[ \epsilon \] = small step, dimensionless

Vectors

\[ \vec{d} \] = drawdown vector, \( m/(L^2 t) \), psi  
\[ \vec{j}_0 \] = overall productivity index vector, \( L^4/m, (STB/D)/psi \)  
\[ \vec{q} \] = production rate vector, \( L^3/t \), STB/D  
\[ \bar{p}_{wf} \] = vector of flowing wellbore pressures, \( m/(L^2 t) \), psi

Matrices

\[ [A] = \text{influence matrix, dimensionless} \]  
\[ [D_s] = \text{diagonal matrix of skin factors, dimensionless} \]  
\[ [D_q] = \text{diagonal matrix of surface production rates, } L^3/t, \text{STB/D} \]  
\[ [J] = \text{productivity index matrix, } L^4/m, (STB/D)/psi \]  
\[ [J_0] = \text{dimensionless productivity index matrix} \]

Subscripts

\[ e \] = boundary value  
\[ i \] = initial value  
\[ j \] = well index  
\[ m \] = index in the summation  
\[ T \] = total

Superscript

\[ T = \text{transposed} \]

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References


Appendix: Calculation of the Influence Function

The procedure required to calculate the influence function is a slightly modified version of the procedure proposed in Ref. 5.

1. Making use of the symmetry of this problem, we calculate

\[ a[x_D, y_D; x_w, y_w; y_{wD}] = a[x_D, x_w, y_{wD}, y_D] \] if \(|x_D - x_w| < |y_D - y_{wD}|\),

\[ a[x_D, x_w, y_{wD}, y_D] \] otherwise.

(A-1)

2. Further use of symmetry allows us to exchange the role of x and y (if necessary):

\[ a[x_D, x_w, y_{wD}, y_D] \] if \(|x_D - x_w| > |y_D - y_{wD}|\),

\[ a[y_D, x_w, x_{wD}, y_{wD}] \] otherwise.

(A-2)

3. The function \( a^0 \) is defined by Eq. 4 as

\[ a[x_D, x_w, y_{wD}, y_D] = 2\pi y_{wD} \left( \frac{1}{3} - \frac{y_D}{y_{wD}} + \frac{y_D^2 + y_{wD}^2}{2y_{wD}^2} \right) + S_T, \]

(A-3)

where

\[ S_T = 2 \sum_{m=1}^{\infty} \frac{m \cos(m\pi x_D) \cos(m\pi x_w)}{m \pi x_D} \]

and

\[ \cos[m\pi(y_D - y_{wD})] + \frac{\cos[m\pi(y_D - y_{wD})]}{\sinh(m\pi y_{wD})}. \]

(A-5)
The infinite sum in Eq. A-4 can be replaced by a finite approximation consisting of three parts:

\[ S_T = S_1 + S_2 + S_3, \]  

(A-6)

where

\[ S_1 = 2 \sum_{n=1}^{N} \frac{t_n}{m} \cos[m \pi x_D] \cos[m \pi x_u D], \]  

(A-7)

\[ S_2 = -\frac{t_{N}}{2} \ln\left[\left(1 - \cos(\pi(x_D + x_u D))\right)^2 + \left(\sin(\pi(x_D + x_u D))\right)^2\right] \]

\[ -\frac{t_{N}}{2} \ln\left[\left(1 - \cos(\pi(x_D - x_u D))\right)^2 + \left(\sin(\pi(x_D - x_u D))\right)^2\right], \]  

(A-8)

and

\[ S_3 = -2t_{N} \sum_{n=1}^{N} \frac{1}{m} \cos[m \pi x_D] \cos[m \pi x_u D]. \]  

(A-9)

The first part is the usual finite approximation stopping after the Nth term in the summation. The second and third parts are obtained from the identity,

\[ \sum_{m=1}^{\infty} \frac{1}{m} \cos[m \pi x_D] \cos[m \pi x_u D] \]

\[ = -\frac{1}{4} \ln\left[\left(1 - \cos(\pi x_D + x_u D)\right)^2 + \left(\sin(\pi x_D + x_u D)\right)^2\right] \]

\[ -\frac{1}{4} \ln\left[\left(1 - \cos(\pi x_D - x_u D)\right)^2 + \left(\sin(\pi x_D - x_u D)\right)^2\right], \]  

(A-10)

and from the fact that \( t_n \) alone converges "fast." The advantage of this algorithm is that only a few hyperbolic functions have to be evaluated since the number of terms, N, is usually less than 100.

As a byproduct of this work, we obtain a convenient algorithm to calculate the Dietz'2 shape factors. Traditionally, when the Dietz shape factors are presented the dimensionless distances are based on \( y_xD \), therefore \( y_xD = 1 \) and \( x_xD \) is greater than or equal to unity. Based on the relation derived by Ozkan,5 the shape factor is expressed as

\[ \ln C_A = \ln \frac{4}{x_xD} - \gamma - 2 \ln \frac{\epsilon}{x_xD} \]

\[ -2 \ln \left[ \frac{x_xD + \epsilon}{x_xD} \frac{y_xD}{x_xD} \frac{y_xD}{x_xD} \right] \]

(A-11)

where the Euler constant is \( \gamma = 0.577 215 66,... \) and \( \epsilon \) is a suitable small positive number (e.g., \( 10^{-6} \)).

For example, for \( x_xD = 1 \), \( x_uD = 0.5 \), and \( y_uD = 0.5 \), our algorithm yields \( C_A = 3.430 14 \) and \( C_A = 30.8811 \) which is in excellent agreement with the known value (30.88). While our procedure does not provide new results relative to the generally used algorithm of Larsen,8 it is simpler in the sense that it does not require numerical evaluation of the exponential integral function.

**SI Metric Conversion Factors**

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Symbol</th>
<th>Value</th>
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<tr>
<td>acre</td>
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</tr>
<tr>
<td>bbl</td>
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<td>1.589733</td>
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<tr>
<td>cp</td>
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<tr>
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<td>( \times )</td>
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<tr>
<td>md</td>
<td>( \times )</td>
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<tr>
<td>psi</td>
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<tr>
<td>PSI</td>
<td>( \times )</td>
<td>1.450377</td>
</tr>
</tbody>
</table>

*Conversion factors are exact.*

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