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Simplified Correlations for Hydrocarbon Gas Viscosity and Gas Density — Validation and Correlation of Behavior Using a Large-Scale Database

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Abstract

The focus of this work is the behavior of gas viscosity and gas density for hydrocarbon gas mixtures. The viscosity of hydrocarbon gases is a function of pressure, temperature, density, and molecular weight, while the gas density is a function of pressure, temperature, and molecular weight. This work presents new approaches for the prediction of gas viscosity and gas density for hydrocarbon gases over practical ranges of pressure, temperature and composition. These correlations can be used for any hydrocarbon gas production or transportation operations.

In this work we created an extensive database of measured gas viscosity and gas density (>5000 points for gas viscosity and >8000 points for gas density). This database was used to evaluate existing models for gas viscosity and gas density. In this work we provide new models for gas density and gas viscosity, as well as optimization of existing models using this database.

The objectives of this research are:

- To create a large-scale database of measured gas viscosity and gas density data which contains all of the information required to establish the applicability of various models for gas density and gas viscosity over a wide range of pressures and temperatures.
- To evaluate a number of existing models for gas viscosity and gas density.
- To develop new models for gas viscosity and gas density using our research database — these models are proposed, validated, and presented graphically.

For this study, we created a large-scale database of gas properties using existing sources available in the literature. Our data-base includes: composition, viscosity, density, temperature, pressure, pseudoreduced properties and the gas compressibility factor. We use this database to evaluate the applicability of the existing models used to estimate hydrocarbon gas viscosity and gas density (or more specifically, the z -factor). Finally, we provide new models and calculation procedures for estimating hydrocarbon gas viscosity and we also provide new optimizations of the existing equations-of-state (EOS) typically used for the calculation of the gas z -factor.

Introduction

Hydrocarbon Gas Viscosity

NIST — SUPERTRAP Algorithm: The state-of-the-art mechanism for the estimation of gas viscosity is most likely the computer program *SUPERTRAP* developed at the U.S. National Institute of Standards and Technology¹ (NIST). *SUPERTRAP* was developed from pure component and mixture data, and is stated to provide estimates within engineering accuracy from the triple point of a given substance to temperatures up to 1,340.33 deg F and pressures up to 44,100 psia. As the *SUPERTRAP* algorithm requires the composition for a particular sample, this method would not be generally suitable for applications where only the mixture gas gravity and compositions of any contaminants are known.

Carr, et al. Correlation: Carr, et al.² developed a two-step procedure to estimate hydrocarbon gas viscosity. The first step is to determine the gas viscosity at atmospheric conditions (*i.e.*, a reference condition). Once estimated, the viscosity at atmospheric pressure is then adjusted to conditions at the desired temperature and pressure using a second correlation. The gas viscosity can be estimated using graphical correlations or using equations derived from these figures.

Jossi, Stiel, and Thodos Correlation: Jossi, et al.³ developed a relationship for the viscosity of pure gases and gas mixtures which includes pure components such as argon, nitrogen, oxygen, carbon dioxide, sulfur dioxide, methane, ethane, propane, butane, and pentane. This "residual viscosity" relationship can be used to predict gas viscosity using the "reduced" density at a specific temperature and pressure, as well as the molecular weight. The critical properties of the gas (specifi-

cally the critical temperature, critical pressure, and critical density) are also required.

Our presumption is that the Jossi, *et al.* correlation (or at least a similar type of formulation) can be used for the prediction of viscosity for pure hydrocarbon gases and hydrocarbon gas mixtures. We will note that this correlation is rarely used for hydrocarbon gases (because an estimate of the critical density is required) — however; we will consider the formulation given by Jossi, *et al.* as a possible model for the correlation of hydrocarbon gas viscosity behavior.

The "original" Jossi, *et al.* correlation proposed for gas viscosity is given by:

$$\left[(\mu_g - \mu^*) \xi + 10^{-4} \right]^{\frac{1}{4}} = f(\rho_{r,JST}) \quad (1)$$

where:

$$f(\rho_{r,JST}) = 0.1023 + 0.023364 \rho_{r,JST} + 0.058533 \rho_{r,JST}^2 - 0.040758 \rho_{r,JST}^3 + 0.0093324 \rho_{r,JST}^4 \quad (2)$$

$$\xi = \frac{\frac{1}{T_c^6}}{\frac{1}{M_w^2} \frac{2}{p_c^3}} \quad (3)$$

and,

$$\begin{aligned} \rho_{r,JST} &= \rho/\rho_c, \text{ JST Reduced density, dimensionless} \\ \rho &= \text{Density at temperature and pressure, g/cc} \\ \rho_c &= \text{Density at the critical point, g/cc} \\ T_c &= \text{Critical temperature, deg K} \\ p_c &= \text{Critical pressure, atm} \\ M_w &= \text{Molecular weight, lb/lb-mole} \\ \mu_g &= \text{Gas viscosity, cp} \\ \mu^* &= \text{Gas viscosity at "low" pressure, cp} \end{aligned}$$

The Jossi, *et al.* correlation is shown in **Figs. 1** and **2**. Jossi, *et al.*³ reported approximately 4 percent average absolute error and also stated that *this correlation should only be applied for values of reduced density (ρ_r) below 2.0*. The behavior of the "residual" gas viscosity function is shown in **Figs. 1** and **2**.

Lee, Gonzalez, and Eakin, Correlation: The Lee, *et al.*⁵ correlation evolved from existing work in the estimation of hydrocarbon gas viscosity using temperature, gas density at a specific temperature and pressure, and the molecular weight of the gas. This correlation is given by:

$$\mu_g = 10^{-4} K \exp(X \rho^Y) \quad (4)$$

where:

$$K = \frac{(9.379 + 0.01607 M_w) T^{1.5}}{209.2 + 19.26 M_w + T} \quad (5)$$

$$X = 3.448 + \left[\frac{986.4}{T} \right] + 0.01009 M_w \quad (6)$$

$$Y = 2.447 - 0.2224 X \quad (7)$$

and,

$$\begin{aligned} \rho &= \text{Density at temperature and pressure, g/cc} \\ M_w &= \text{Molecular weight of gas mixture, lb/lb-mole} \\ T &= \text{Temperature, deg R} \\ \mu_g &= \text{Gas viscosity at temperature and pressure, cp} \end{aligned}$$

Lee, *et al.*⁴ reported 2 percent average absolute error (low pressures) and 4 percent average absolute error (high pressures) for hydrocarbon gases where the specific gravity is below 1.0. For gases of specific gravity above 1.0 this relation is purported to be "less accurate."

The range of pressures used by Lee, *et al.*⁵ in the development of this correlation is between 100 and 8,000 psia and the temperature range is between 100 and 340 deg F. This correlation can also be used for samples which contain carbon dioxide — (in particular for carbon dioxide concentrations up to 3.2 mole percent). **Fig. 3** shows the behavior of the Gonzalez, *et al.*⁵ data (natural gas sample 3) compared to the Lee, *et al.*⁴ hydrocarbon gas viscosity correlation.

Hydrocarbon Gas Density

A practical issue pertinent to all density-based gas viscosity models is that an estimate of gas density must be known. Although there are many equation of state (EOS) correlations for gas density (or more specifically, the gas z -factor) we found that these EOS models do not reproduce the measured gas densities in our database to a satisfactory accuracy. This observation led us towards an effort to "tune" the existing models (refs. 6-8) for the z -factor using the data in our database.

For reference, the definition of gas density for real gases is given by:

$$\rho = \frac{1}{62.37} \frac{p}{z} \frac{M_w}{RT} \quad (\rho \text{ in g/cc}) \quad (8)$$

where:

$$\begin{aligned} \rho &= \text{Density at temperature and pressure, g/cc} \\ p &= \text{Pressure, psia} \\ M_w &= \text{Molecular weight, lb/lb-mole} \\ z &= z\text{-factor, dimensionless} \\ T &= \text{Temperature, deg R} \\ R &= \text{Universal gas constant, 10.732 (psia cu ft)/(lb-mole deg R)} \end{aligned}$$

$$62.37 = \text{Conversion constant: } 1 \text{ g/cc} = 62.37 \text{ lbm/ft}^3$$

The real gas z -factor is presented as an explicit function of the pseudoreduced pressure and temperature as predicted by the "Law of Corresponding States"⁹ (see **Figs. 4-6**, where we use the data of Poettmann and Carpenter¹⁰). It is important to note that EOS models are implicit in terms of the z -factor, which means that the z -factor is solved as a root of the EOS. This must be considered in the regression process — the regression formulation must include the solution of the "model" z -factor as a root of the EOS.

Dranchuk-Abou-Kassem,⁶ Nishiumi-Saito,⁷ and Nishiumi⁸ provide EOS representations of the real gas z -factor. In particular, the Dranchuk-Abou-Kassem result is based on a Han-Starling form of the Benedict-Webb-Rubin equation of state (EOS) and is considered to be the current standard for the prediction of gas density.

z -Factor Model: Dranchuk-Abou-Kassem (ref. 6)

The DAK-EOS⁶ is given by:

$$z = 1 + \left[A_1 + \frac{A_2}{T_r} + \frac{A_3}{T_r^3} + \frac{A_4}{T_r^4} + \frac{A_5}{T_r^5} \right] \rho_r + \left[A_6 + \frac{A_7}{T_r} + \frac{A_8}{T_r^2} \right] \rho_r^2 - A_9 \left[\frac{A_7}{T_r} + \frac{A_8}{T_r^2} \right] \rho_r^5 + A_{10} (1 + A_{11} \rho_r^2) \frac{\rho_r^2}{T_r^3} \exp(-A_{11} \rho_r^2) \quad \text{..... (9)}$$

where:

- z = z -factor, dimensionless
- T_r = Reduced temperature, dimensionless
- $\rho_r = \rho/\rho_c$, Reduced density, dimensionless
- ρ = Density at temperature and pressure, g/cc
- ρ_c = Critical density, g/cc ($z_c=0.27$)

We must note that the *definition* of critical density is a matter of some debate — in particular, how is critical density estimated when this is also a property of the fluid? As such, we use the "definition" of ρ_c as given by Dranchuk and Abou-Kassem⁶:

$$\rho_c = z_c \frac{p_r}{z T_r} \quad (\text{where } z_c = 0.27)$$

and the "original" parameters given by Dranchuk and Abou-Kassem (ref. 6) for hydrocarbon gases are:

$$\begin{array}{ll} A_1 = 0.3265 & A_7 = -0.7361 \\ A_2 = -1.0700 & A_8 = 0.1844 \\ A_3 = -0.5339 & A_9 = 0.1056 \\ A_4 = 0.01569 & A_{10} = 0.6134 \\ A_5 = -0.05165 & A_{11} = 0.7210 \\ A_6 = 0.5475 & \text{..... (10)} \end{array}$$

The Dranchuk-Abou-Kassem (DAK-EOS) is compared to the data of Poettmann and Carpenter¹⁰ in **Figs. 7-9**. We note that the "original" DAK-EOS agrees quite well with the data trends, and would, in the absence of data to the contrary, seem to be adequate for most engineering applications. However, we would like to extend the range of this relation as well as provide a more statistically sound correlation of the EOS (*i.e.*, add more data to the regression process).

z -Factor Model: Nishiumi-Saito (ref. 7)

The Nishiumi-Saito model (NS-EOS)⁷ adds a few more terms to the original Dranchuk-Abou-Kassem expression, and is given by:

$$\begin{aligned} z = 1 + & \left[A_1 - \frac{A_2}{T_r} - \frac{A_3}{T_r^3} - \frac{A_4}{T_r^4} - \frac{A_5}{T_r^5} \right] \rho_r^2 \\ & + \left[A_6 - \frac{A_7}{T_r} - \frac{A_8}{T_r^2} - \frac{A_9}{T_r^5} - \frac{A_{10}}{T_r^{24}} \right] \rho_r^2 \\ & + A_{11} \left[\frac{A_7}{T_r} + \frac{A_8}{T_r^2} + \frac{A_9}{T_r^5} + \frac{A_{10}}{T_r^{24}} \right] \rho_r^5 \\ & + \left[\frac{A_{12}}{T_r^3} + \frac{A_{13}}{T_r^9} + \frac{A_{14}}{T_r^{18}} \right] \rho_r^2 (1 + A_{15} \rho_r^2) \exp(-A_{15} \rho_r^2) \end{aligned} \quad \text{..... (11)}$$

where:

- z = z -factor, dimensionless
- T_r = Reduced temperature, dimensionless
- $\rho_r = \rho/\rho_c$, Reduced density, dimensionless
- ρ = Density at temperature and pressure, g/cc
- ρ_c = Critical density, g/cc ($z_c=0.27$)

Our perspective in utilizing the NS-EOS is that this relation is purported to provide better performance in the vicinity of the critical isotherm — which is traditionally a region where the DAK-EOS has been shown to give a weak performance. We will compare the performance of the DAK and NS-EOS relations in detail once these relations are regressed using our gas density (z -factor) database.

Correlation of Hydrocarbon Gas Viscosity

In this section we provide comparisons and optimizations of existing correlations for hydrocarbon gas viscosity (refs. 3 and 4), as well as a new correlation for hydrocarbon gas viscosity that is implicitly defined in terms of gas density and temperature. Our approach is to use an extensive database of gas viscosity and gas density data, derived from a variety of literature sources (refs. 11-15). This database contains 2494 points taken from pure component data and 3155 points taken from mixture data. More data were available in the literature — however, the data points chosen for this study satisfy the following criteria:

- Temperature is greater than 32 deg F.
- Density measurement is available for each measurement of gas viscosity. (This criterion was not applied for selection of points for the correlation of gas viscosity at one atmosphere.)
- Gas composition must be representative of a natural gas (*e.g.*, data for binary mixtures containing decane were excluded (ref. 4)).
- Liquid or liquid-like (*i.e.*, unusually high) viscosities were excluded from consideration.

Gas Viscosity: Jossi, *et al.* (ref. 3)

In this section we test the performance of the Jossi, *et al.* model for gas viscosity against viscosity values from our database. We then propose a "refitted" form of the Jossi, *et al.* model where the coefficients of the original model were adjusted using regression to better match the viscosity values provided in our database. **Fig. 10** shows the results of the original Jossi, *et al.* model when applied to our database — we note that there are significant departures from the 45 degree straight line (conformance to this straight line would indicate perfect agreement between the measured and calculated gas viscosity values). There are 2494 *pure component* data points given on this plot, where this data match has an average absolute error of 5.26 percent in the prediction of gas viscosity. We wish to note that our database does include gas viscosity values measured at reduced density values greater than 2.0. We note that such points were not included in the original study by Jossi, *et al.*

In an attempt to minimize the error between our data and the Jossi, *et al.* model, we refitted the coefficients of the Jossi, *et al.* model using non-linear regression techniques. The generic form of the Jossi, *et al.* model is written in the form:

$$\left[(\mu_g - \mu^*) \xi + 10^{-4} \right]^{\frac{1}{4}} = f(\rho_{r,JST}) \dots\dots\dots (12)$$

where:

$$f(\rho_{r,JST}) = f_1 + f_2 \rho_{r,JST} + f_3 \rho_{r,JST}^2 + f_4 \rho_{r,JST}^3 + f_5 \rho_{r,JST}^4 \dots\dots\dots (13)$$

$$\xi = \frac{T_c^{e_1}}{M_w^{e_2} p_c^{e_3}} \dots\dots\dots (14)$$

The "optimized" coefficients obtained from the "refitting" the Jossi, *et al.* model are:

$$\begin{aligned} f_1 &= 1.03671\text{E-}01 & f_2 &= 1.31243\text{E-}01 \\ f_3 &= 1.71893\text{E-}02 & f_4 &= -3.12987\text{E-}02 \\ f_5 &= 8.84909\text{E-}03 \\ e_1 &= -1.21699\text{E-}01 & e_2 &= 3.91956\text{E-}01 \\ e_3 &= -1.50857\text{E-}01 \end{aligned} \dots\dots\dots (15)$$

and the variables are defined in the same fashion as the original correlation proposed by Jossi, *et al.* — where the most important issue is that this correlation is limited to *pure component* data. This means that the critical density is directly tied to the component — no alternate definition is permitted.

The performance of our "optimized" version of the Jossi, *et al.* model for gas viscosity is shown in **Fig. 11**. This plot shows better conformance to the 45 degree line than the original Jossi, *et al.* model shown in **Fig. 10**. The average absolute error for the optimized Jossi, *et al.* model is 4.43 percent (as

compared to 5.26 percent for the original Jossi, *et al.* model). For reference, Jossi, *et al.* reported an average absolute error of 4 percent when they presented their model (fitted to a variety of fluids — including non-hydrocarbon samples). It is relevant to note that Jossi, *et al.* used a relatively small database of pure component data.

We must also note that our optimization of this model requires the gas viscosity at one atmosphere (μ^*) — in this case we used an independent correlation for μ^* based on the relevant data from our database. This correlation for μ^* is an independent development and is discussed in a later section.

Gas Viscosity: Lee, *et al.* (ref. 4)

The Lee, *et al.* model for gas viscosity was utilized in a similar manner to the Jossi, *et al.* model — *i.e.* the performance of the original model was first assessed using our database, and then the coefficients of this relation were optimized using the database of gas viscosity. We note that in this work we have utilized data for both *pure components* and *gas mixtures*.

Fig. 12 shows the performance of the Lee, *et al.* model on 4909 points from our database. This figure shows that gas viscosity is *under predicted* by the Lee, *et al.* model at the higher end of the gas viscosity scale. The average absolute error associated with the comparison of this model with our viscosity database is 3.34 percent.

The coefficients of the Lee, *et al.* model were then optimized using the gas viscosity database in order to improve the performance of the model. These results are shown in **Fig. 13**. For the optimization the Lee, *et al.* relation, the correlation model was cast in the following form:

$$\mu_g = 10^{-4} K \exp(X \rho^Y) \dots\dots\dots (16)$$

where:

$$K = \frac{(k_1 + k_2 M_w) T^{k_3}}{k_4 + k_5 M_w + T} \dots\dots\dots (17)$$

$$X = x_1 + \left[\frac{x_2}{T} \right] + x_3 M_w \dots\dots\dots (18)$$

$$Y = y_1 - y_2 X \dots\dots\dots (19)$$

The optimized coefficients for this model are:

$$\begin{aligned} k_1 &= 1.67175\text{E+}01 & k_2 &= 4.19188\text{E-}02 \\ k_3 &= 1.40256\text{E+}00 & k_4 &= 2.12209\text{E+}02 \\ k_5 &= 1.81349\text{E+}01 \\ x_1 &= 2.12574\text{E+}00 & x_2 &= 2.06371\text{E+}03 \\ x_3 &= 1.19260\text{E-}02 \\ y_1 &= 1.09809\text{E+}00 & y_2 &= -3.92851\text{E-}02 \end{aligned} \dots\dots\dots (20)$$

The average absolute error for this "optimized" model is 2.29 percent. Lee, *et al.* reported average absolute errors of 2 to 4 percent for their original model — where we recall that the

original Lee, *et al.* correlations were generated using a less comprehensive database.

Gas Viscosity: Proposed "Implicit" Model

Our correlation work based on a "non-parametric" regression algorithm¹⁶ shows that gas viscosity is primarily a function of the following variables:

- Gas viscosity at 1 atm,
- Gas density, and
- Temperature.

We found that pressure and molecular weight could be discarded as explicit variables for this model (these are included implicitly in the gas density function). We then developed a new gas viscosity model, which is simply a generic expansion of the Jossi, *et al.* model using additional temperature and density dependent terms.

The relationship between the residual viscosity function (*i.e.*, $\mu_g - \mu_{1atm}$) and the gas density appears to be *univariate*, as shown in **Fig. 14**. A log-log plot of the residual viscosity data from our database shows significant scatter at low densities — where this behavior reveals a strong dependence of gas viscosity on temperature at low densities (see **Fig. 15**).

By observation we found that the "uncorrelated" distribution of data formed in the low-density range is directly related to temperature. We propose a rational polynomial model in terms of gas density with temperature-dependent coefficients and used nonlinear regression to fit our proposed model to temperature and gas density data. This model is given as:

$$\mu_g = \mu_{1atm} + f(\rho) \quad (21)$$

$$f(\rho) = \frac{a + b\rho + c\rho^2 + d\rho^3}{e + f\rho + g\rho^2 + h\rho^3} \quad (22)$$

$$a = a_0 + a_1T + a_2T^2 \quad (23)$$

$$b = b_0 + b_1T + b_2T^2 \quad (24)$$

$$c = c_0 + c_1T + c_2T^2 \quad (25)$$

$$d = d_0 + d_1T + d_2T^2 \quad (26)$$

$$e = e_0 + e_1T + e_2T^2 \quad (27)$$

$$f = f_0 + f_1T + f_2T^2 \quad (28)$$

$$g = g_0 + g_1T + g_2T^2 \quad (29)$$

$$h = h_0 + h_1T + h_2T^2 \quad (30)$$

The numerical values for the parameters of our proposed "implicit" model for gas viscosity (Eqs. 21 to 30) are given as follows:

$$\begin{aligned} a_0 &= 9.53363E-01 & a_1 &= -1.07384E+00 \\ a_2 &= 1.31729E-03 \\ b_0 &= -9.71028E-01 & b_1 &= 1.12077E+01 \\ b_2 &= 9.01300E-02 \end{aligned}$$

$$\begin{aligned} c_0 &= 1.01803E+00 & c_1 &= 4.98986E+00 \\ c_2 &= 3.02737E-01 \\ d_0 &= -9.90531E-01 & d_1 &= 4.17585E+00 \\ d_2 &= -6.36620E-01 \\ e_0 &= 1.00000E+00 & e_1 &= -3.19646E+00 \\ e_2 &= 3.90961E+00 \\ f_0 &= -1.00364E+00 & f_1 &= -1.81633E-01 \\ f_2 &= -7.79089E+00 \\ g_0 &= 9.98080E-01 & g_1 &= -1.62108E+00 \\ g_2 &= 6.34836E-04 \\ h_0 &= -1.00103E+00 & h_1 &= 6.76875E-01 \\ h_2 &= 4.62481E+00 \end{aligned} \quad (31)$$

A total of 4909 points were used in the regression calculation of these parameters (2494 *pure component* data and 2415 *gas mixture* data). The performance of the model is shown in **Fig. 16**. We note excellent agreement with the 45 degree straight-line trend. The average absolute error for this model as compared to our database is 3.05 percent. We also note there are non-hydrocarbon components such as carbon dioxide (0.19 to 3.20 percent), nitrogen (0.04 to 15.80 percent) and helium (0.03 to 0.80 percent) present in some of the gas mixtures used to develop these correlations.

Gas Viscosity: Hydrocarbon Gas Viscosity at 1 Atmosphere

In order to utilize both new and existing correlations for gas viscosity, it is imperative that we estimate the viscosity of a hydrocarbon gas mixture at 1 atm. We propose a new correlation for this purpose — where this correlation is given only as a function of the temperature (in deg R) and the gas specific gravity (as a surrogate for molecular weight of the mixture). The generic form of this relation is given by:

$$\ln(\mu_{1atm}) = \frac{a_0 + a_1 \ln(\gamma_g) + a_2 \ln(T) + a_3 \ln(\gamma_g) \ln(T)}{1 + b_1 \ln(\gamma_g) + b_2 \ln(T) + b_3 \ln(\gamma_g) \ln(T)} \quad (32)$$

In this correlation we used 261 data points for the gas viscosity at 1 atm where 135 of these are pure component data and 126 are gas mixture data. This new correlation gives an average absolute error of 1.36 percent. **Fig. 17** illustrates the comparison of the calculated gas viscosity at 1 atm and the measured gas viscosity at 1 atm.

The numerical values of the parameters obtained for the new gas viscosity model for viscosity at 1 atm model (Eq. 32) are given by:

$$\begin{aligned} a_0 &= -6.39821E+00 & a_1 &= -6.045922E-01 \\ a_2 &= 7.49768E-01 & a_3 &= 1.261051E-01 \\ b_1 &= 6.97180E-02 & b_2 &= -1.013889E-01 \\ b_3 &= -2.15294E-02 \end{aligned} \quad (33)$$

Correlation of Hydrocarbon Gas Density (z-factor)

Gas Density: Dranchuk-Abou-Kassem (DAK-EOS)

In this section we present the results of our regression work where we fitted the DAK-EOS to our gas density database. This was a multi-step process where we first perform regression of the DAK-EOS onto the "standard" and pure component databases. The "standard" database is a tabular rendering of the Standing and Katz z -factor chart. These data are presumed to accurately represent an "average" trend according to the "Law of Corresponding States".

After the "calibration" of the EOS, we can then use the mixture (and pure component) data to establish correlations for pseudocritical properties (we must correlate pseudocritical temperature and pressure because we will not be able to estimate these parameters independently — recall that we presume we have only the mixture gravity of the gas, not a full compositional analysis).

The "first step" regression (EOS to database) is shown on **Fig. 18**, where we note a very strong correlation. The associated plots for comparing the models and data for this case are shown in **Figs. 19-21** — we also observe a strong correlation (with only minor errors) near the critical isotherm. Coefficients for the DAK-EOS obtained from regression (using the Poettmann-Carpenter¹⁰ "standard" database) are:

$$\begin{aligned} A_1 &= 3.024696\text{E-}01 & A_7 &= -1.118884\text{E+}00 \\ A_2 &= -1.046964\text{E+}00 & A_8 &= 3.951957\text{E-}01 \\ A_3 &= -1.078916\text{E-}01 & A_9 &= 9.313593\text{E-}02 \\ A_4 &= -7.694186\text{E-}01 & A_{10} &= 8.483081\text{E-}01 \\ A_5 &= 1.965439\text{E-}01 & A_{11} &= 7.880011\text{E-}01 \\ A_6 &= 6.527819\text{E-}01 & & \end{aligned} \quad \dots\dots\dots (34a)$$

The average absolute error associated with this case is 0.412 percent (5960 data points). For reference, the original work by Dranchuk and Abou-Kassem (ref. 6) was based on a database of 1500 points and yielded an average absolute error of 0.486 percent.

Coefficients for the DAK-EOS regression using the "combined" database (Poettmann-Carpenter¹⁰ data and pure component data) are:

$$\begin{aligned} A_1 &= 2.965749\text{E-}01 & A_7 &= -1.006653\text{E+}00 \\ A_2 &= -1.032952\text{E+}00 & A_8 &= 3.116857\text{E-}01 \\ A_3 &= -5.394955\text{E-}02 & A_9 &= 9.506539\text{E-}02 \\ A_4 &= -7.694000\text{E-}01 & A_{10} &= 7.544825\text{E-}01 \\ A_5 &= 2.183666\text{E-}01 & A_{11} &= 7.880000\text{E-}01 \\ A_6 &= 6.226256\text{E-}01 & & \end{aligned} \quad \dots\dots\dots (34b)$$

For this case we obtained an average absolute error of 0.821 percent (8256 points). We note that this error is higher than error we obtained using the Poettmann-Carpenter¹⁰ "standard" database — however, this error is (certainly) still acceptable.

We now pursue the "second step" of this development by applying the *optimized* DAK-EOS on our mixture and pure component database (for the z -factor) as a mechanism to de-

velop relations for estimating the pseudocritical temperature and pressure.

In **Fig. 22** we present the calculated versus measured values of z -factor for the "mixtures/pure component" calibration. We note that 6032 data points were used (gas mixtures and pure component samples), and that we achieved an average absolute error of 3.06 percent for this case. In performing this regression, we simultaneously defined the new mixture rules for the DAK-EOS (*i.e.*, correlations of pseudocritical temperature and pressure as quadratic polynomials as a function of the gas gravity).

Figs. 23 and 24 present the results of the optimized DAK-EOS for the z -factor coupled with the optimized quadratic relations used to model the pseudocritical temperature and pseudocritical pressure (as a function of gas specific gravity). The optimized quadratic equations for the pseudocritical temperature and pressure of a given sample are given in terms of the gas specific gravity as follows: (DAK-EOS case only)

$$p_{pc} = 725.89 - 70.27\gamma_g - 9.05\gamma_g^2 \quad \dots\dots\dots (35)$$

$$T_{pc} = 40.39 + 549.47\gamma_g - 94.01\gamma_g^2 \quad \dots\dots\dots (36)$$

where,

$$\begin{aligned} p_{pc} &= \text{Pseudocritical pressure, psia} \\ T_{pc} &= \text{Pseudocritical temperature, deg R} \\ \gamma_g &= \text{Gas specific gravity (air = 1.0)} \end{aligned}$$

Eqs. 35 and 36 were calibrated using the DAK-EOS (and the coefficients for the DAK-EOS were taken from Eq. 34b). For the optimized DAK-EOS based on our research database, we note that only the combination of Eqs. 9, 34b, 35, and 36 can be used to estimate the z -factor for gas mixtures.

In summary, we have recalibrated the DAK-EOS against three databases — the Poettmann-Carpenter¹⁰ data (5960 points), an extended database which includes the Poettmann-Carpenter¹⁰ data and additional pure component data (8256 points), and a database of pure component and mixture data (6032 points).

In the first two cases we provide new coefficients to replace the original DAK-EOS (which was similarly defined by the original authors using pure component data). The average absolute errors for these cases were 0.486 percent and 0.821 percent, respectively.

Lastly, we applied the optimized DAK-EOS based on the "combined" database (Poettmann-Carpenter¹⁰ data and pure component data) (*i.e.*, the combination of Eqs. 9 and 34b) for the case of gas mixtures and developed new models for the pseudocritical pressure and pseudocritical temperature as functions of gas gravity. This model resulted in an overall average absolute error of 3.06 percent for z -factors estimated using the DAK-EOS (and the quadratic polynomials for T_{pc} and p_{pc}).

Gas Density: Nishiumi-Saito (NS-EOS)

This section follows a procedure similar to the previous work which provided new forms of the DAK-EOS. The first step

was to "refit" the coefficients of the NS-EOS model using the Poettmann-Carpenter¹⁰ database.

The results from this regression are:

$$\begin{aligned} A_1 &= 2.669857\text{E-}01 & A_9 &= -2.892824\text{E-}02 \\ A_2 &= 1.048341\text{E+}00 & A_{10} &= -1.684037\text{E-}02 \\ A_3 &= -1.516869\text{E+}00 & A_{11} &= 2.120655\text{E+}00 \\ A_4 &= 4.435926\text{E+}00 & A_{12} &= -5.046405\text{E-}01 \\ A_5 &= -2.407212\text{E+}00 & A_{13} &= 1.802678\text{E-}01 \\ A_6 &= 6.089671\text{E-}01 & A_{14} &= 8.563869\text{E-}02 \\ A_7 &= 5.174665\text{E-}01 & A_{15} &= 4.956134\text{E-}01 \\ A_8 &= 1.296739\text{E+}00 & & \end{aligned} \quad (37a)$$

The average absolute error achieved in this regression was 0.426 percent (5960 points), which is slightly higher than the DAK-EOS result for the same case (0.412 percent).

The refitting procedure was also performed on the extended database of the Poettmann-Carpenter¹⁰ data and pure component data (8132 points). We note that we used fewer data for the regression as compared to the same case for the DAK-EOS (8256 points) — we found it necessary to delete certain extreme points in this regression, particularly values near the critical isotherm. The regression coefficients for this case are:

$$\begin{aligned} A_1 &= 4.645095\text{E-}01 & A_9 &= -1.941089\text{E-}02 \\ A_2 &= 1.627089\text{E+}00 & A_{10} &= -4.314707\text{E-}03 \\ A_3 &= -9.830729\text{E-}01 & A_{11} &= 2.789035\text{E-}01 \\ A_4 &= 5.954591\text{E-}01 & A_{12} &= 7.277907\text{E-}01 \\ A_5 &= 6.183499\text{E-}01 & A_{13} &= -3.207280\text{E-}01 \\ A_6 &= 4.109793\text{E-}01 & A_{14} &= 1.756311\text{E-}01 \\ A_7 &= 8.148481\text{E-}02 & A_{15} &= 7.905733\text{E-}01 \\ A_8 &= 3.541591\text{E-}01 & & \end{aligned} \quad (37b)$$

The average absolute error for this model was 0.733 percent, which is somewhat better than the DAK-EOS result for the same case (0.821 percent).

In a similar manner to the DAK-EOS case, we also considered gas mixtures by developing new relations for the pseudo-critical pressure and temperature for use with the NS-EOS. The results for this case are:

$$p_{pc} = 621.81 + 81.09\gamma_g - 56.51\gamma_g^2 \quad (38)$$

$$T_{pc} = 46.91 + 542.86\gamma_g - 93.14\gamma_g^2 \quad (39)$$

The performance for the NS-EOS (using the coefficients from Eqs. 37a and 37b) is shown in **Figs. 25 to 27** (NS-EOS case only). The results for the "mixture" case are shown **Fig. 28**, and we note that this version of the NS-EOS has an average absolute error of 2.55 percent (with a database of 5118 points) and is uniquely defined by Eqs. 11, 37b, 38, and 39.

We note that we again used fewer data in this regression than the corresponding case for the DAK-EOS (5118 points for the NS-EOS and 6032 points for the DAK-EOS). This was necessary due to poor regression performance of the T_{pc} and p_{pc} parameters — and, as before, we removed extreme values

— those near the critical isotherm, high pressure/high temperature data, and cases of very high molecular weight. We appreciate that this issue may cause concerns — however, based on our procedures and vigilance in the regression process, we remain confident that the T_{pc} and p_{pc} correlations for this case (*i.e.*, NS-EOS) are both accurate and robust.

Conclusions

The following conclusions are derived from this work:

- The new correlations presented in this work for gas viscosity, z -factor, and gas viscosity at 1 atm are appropriate for applications in petroleum engineering.
- The original Jossi, *et al.*³ and Lee, *et al.*⁴ correlations for gas viscosity appear to yield acceptable behavior compared to our database, the average absolute errors (AAE) for these correlations are as follows:

$$\begin{aligned} \text{—Jossi, et al. original:}^3 & \quad \text{AAE} = 5.26 \text{ percent} \\ \text{—Lee, et al. original:}^4 & \quad \text{AAE} = 3.34 \text{ percent} \end{aligned}$$

However, the "refits" of these correlations (using our research database) exhibit significantly better representations of the data:

$$\begin{aligned} \text{—Jossi, et al. "refit:"} & \quad \text{AAE} = 4.43 \text{ percent} \\ \text{—Lee, et al. "refit:"} & \quad \text{AAE} = 2.29 \text{ percent} \end{aligned}$$

For reference, the Jossi, *et al.* correlation was fit using pure component data only (2494 points) — and can only be applied to pure component data (this is a requirement of the Jossi, *et al.* formulation). The Lee, *et al.* correlation was fit using both pure component and gas mixture data (4909 points), and should be considered appropriate for general applications.

- Our new "implicit" viscosity correlation (given as a function of density) works well for pure gases and for gas mixtures over a wide range of temperatures, pressures, and molecular weights. The average absolute error for the new "implicit" viscosity correlation is 3.05 percent for our combined database of pure component and natural gas mixture data (4909 total points).
- Our new correlation for gas viscosity at 1 atm gave an average absolute error of 1.36 percent based on 261 data points (135 pure component data and 126 gas mixture data).
- Although carbon dioxide, nitrogen, and helium were present in some of the gas mixtures, the new gas viscosity correlations match our research database very well — and, by extension, these correlations should work well (without correction) for practical applications where relatively small amounts of non-hydrocarbon impurities are present.
- The original work by Dranchuk and Abou-Kassem (DAK-EOS) for the implicit correlation of the real gas z -factor used 1500 data points and gave an average absolute error of 0.486 percent.⁶ Refitting the DAK-EOS to our research database we considered two cases — the "standard" database given by Poett-

mann and Carpenter¹⁰ (5960 points) and the "standard and pure component" database (the Poettmann and Carpenter data combined with the pure component data) (8256 points).

The average absolute errors (AAE) for the DAK-EOS correlations are:

- DAK-EOS "standard" AAE = 0.412 percent
- DAK-EOS "standard/pure" AAE = 0.821 percent

We performed a similar effort with the Nishiumi and Saito EOS⁷ (NS-EOS) using the same databases as for the DAK-EOS and obtained the following results:

- NS-EOS "standard" AAE = 0.426 percent
- NS-EOS "standard/pure" AAE = 0.733 percent

- For the case of gas mixture densities, we developed quadratic formulations to represent the pseudocritical temperature and pressure as functions of the gas specific gravity. A combined database of pure component and gas mixture data was used in this optimization.

Using the "optimized" DAK-EOS as a basis, we obtained an average absolute error of 3.06 percent (6032 data points) for the gas mixture correlation. Proceeding in a similar fashion using the "optimized" NS-EOS, we obtained an average absolute error of 2.55 percent (5118 data points).

Recommendations and Future Work

1. Further work should include investigations of the explicit effects of non-hydrocarbon components such as water, nitrogen, carbon dioxide, and hydrogen sulfide on both gas viscosity and gas density (*i.e.*, the gas z -factor).
2. This work could be extended to consider density and viscosity behavior of rich gas condensate and volatile oil fluids — however, we are skeptical that any sort of "universal" viscosity relation can be developed.

Nomenclature

AAE	= Absolute error, percent
p	= Pressure, psia
p_c	= Critical pressure, atm
p_{pc}	= Pseudocritical pressure, psia
p_{pr}	= Pseudoreduced pressure, dimensionless
p_r	= Reduced pressure, dimensionless
M_w	= Molecular weight, lbm/lb-mole
T	= Temperature, deg F
T_c	= Critical temperature, deg K
T_{pc}	= Pseudocritical temperature, deg R
T_{pr}	= Pseudoreduced temperature, dimensionless
T_r	= Reduced temperature, dimensionless
R	= Universal gas constant, 10.732 (psia cu ft)/(lb-mole deg R)
z	= z -factor, dimensionless
ρ	= Density, g/cc
ρ_r	= Reduced density, dimensionless

$\mu_{1\text{atm}}$	= Gas viscosity at 1 atm, cp
μ^*	= Gas viscosity at low pressures used by Jossi, <i>et al.</i> ⁴ , cp
μ_g	= Gas viscosity, cp
γ_g	= Gas specific gravity (air=1.0), dimensionless
y_{N_2, CO_2, H_2S}	= Mole fraction of the non-hydrocarbon component (fraction)

Subscripts

c	= critical value
pc	= pseudocritical value
r	= reduced variable
pr	= pseudoreduced variable

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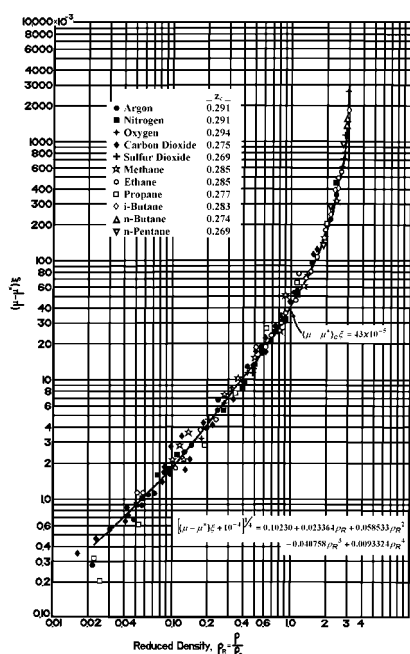


Figure 1 – The "residual" gas viscosity function versus reduced density for different pure substances of similar molecular weights (Jossi, *et al.*³).

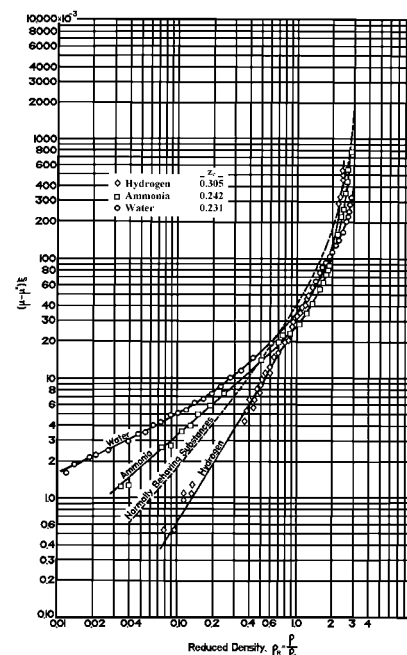


Figure 2 – The "residual viscosity" function versus reduced density for different pure components — note the effect of temperature at low reduced densities (Jossi, *et al.*³).

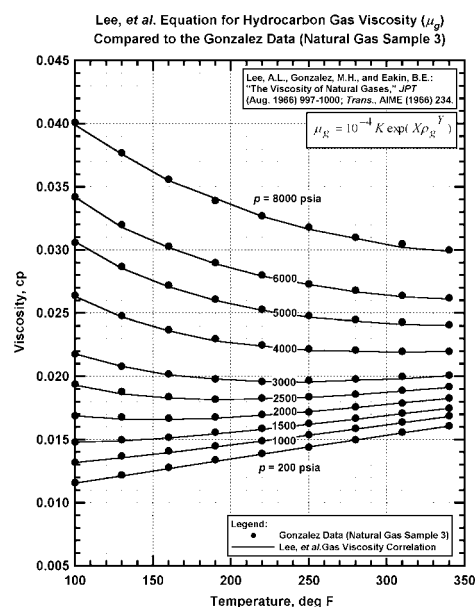


Figure 3 – Gas viscosity versus temperature for the Gonzalez, *et al.*⁵ data (natural gas sample 3) compared to the Lee, *et al.*⁴ hydrocarbon viscosity correlation.

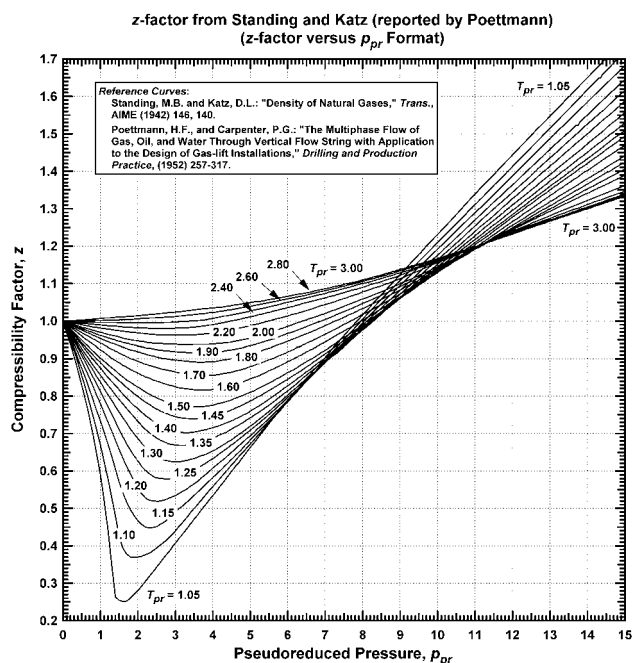


Figure 4 – Real gas z-factor, as attributed to Standing and Katz,⁹ plotted as a function of the pseudoreduced pressure (data of Poettmann and Carpenter¹⁰).

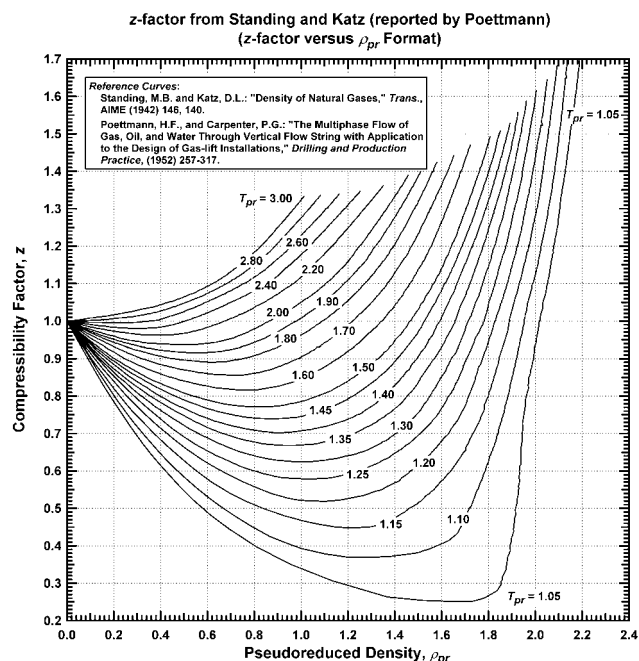


Figure 6 – Real gas z-factor, as attributed to Standing and Katz,⁹ plotted as a function of the pseudoreduced density function (data of Poettmann and Carpenter¹⁰).

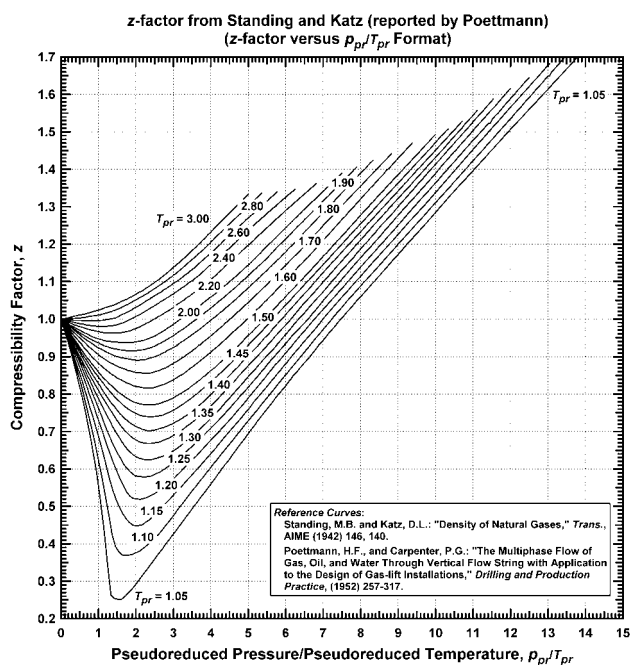


Figure 5 – Real gas z-factor, as attributed to Standing and Katz,⁹ plotted as a function of the pseudoreduced pressure divided by the pseudoreduced temperature (data of Poettmann and Carpenter¹⁰).

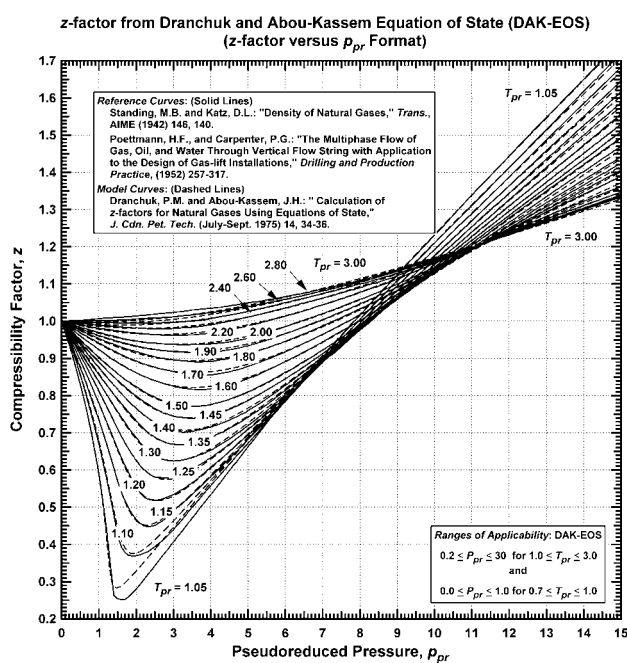


Figure 7 – Real gas z-factor, as attributed to Standing and Katz,⁹ plotted as a function of the pseudoreduced pressure (data of Poettmann and Carpenter¹⁰) compared to the original DAK-EOS (coefficients from Eq. 10).

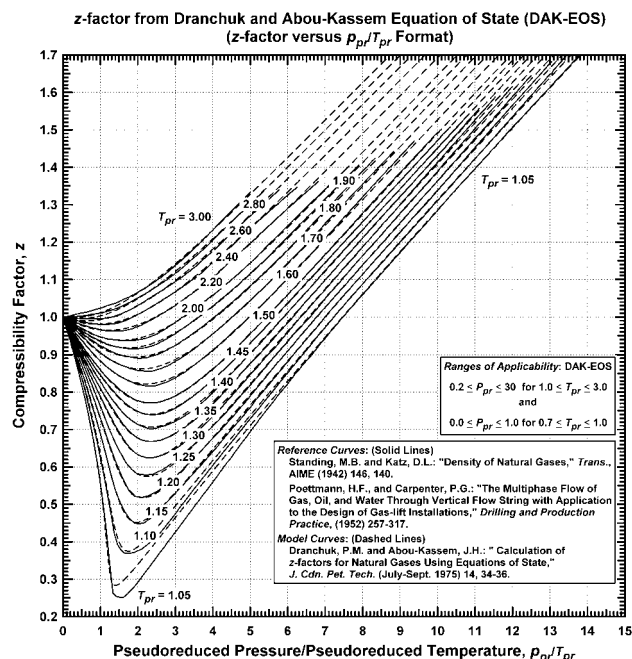


Figure 8 – Real gas z-factor, as attributed to Standing and Katz,⁹ plotted as a function of the pseudoreduced pressure divided by the pseudoreduced temperature (data of Poettmann and Carpenter¹⁰) compared to the original DAK-EOS (coefficients from Eq. 10).

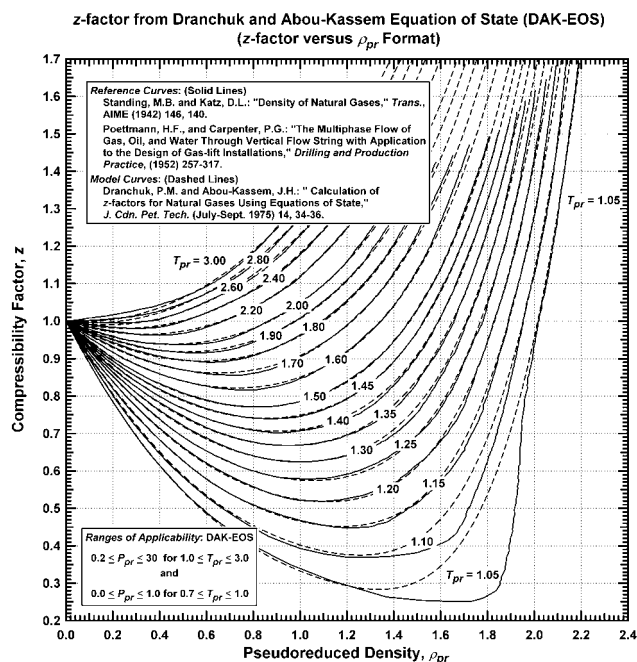


Figure 9 – Real gas z-factor, as attributed to Standing and Katz,⁹ plotted as a function of pseudoreduced density (data of Poettmann and Carpenter¹⁰) compared to the original DAK-EOS (coefficients from Eq. 10).

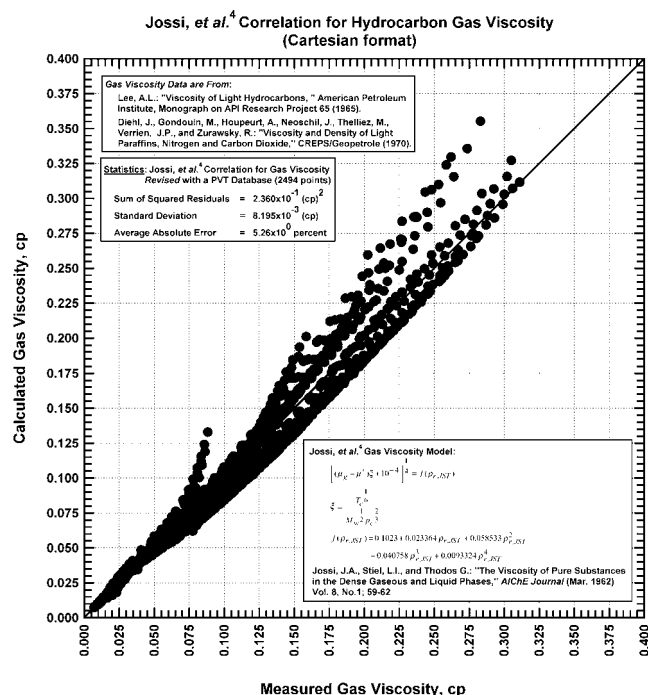


Figure 10 – Jossi, Stiel, and Thodos⁴ correlation for hydrocarbon gas viscosity tested with our database (Cartesian format).

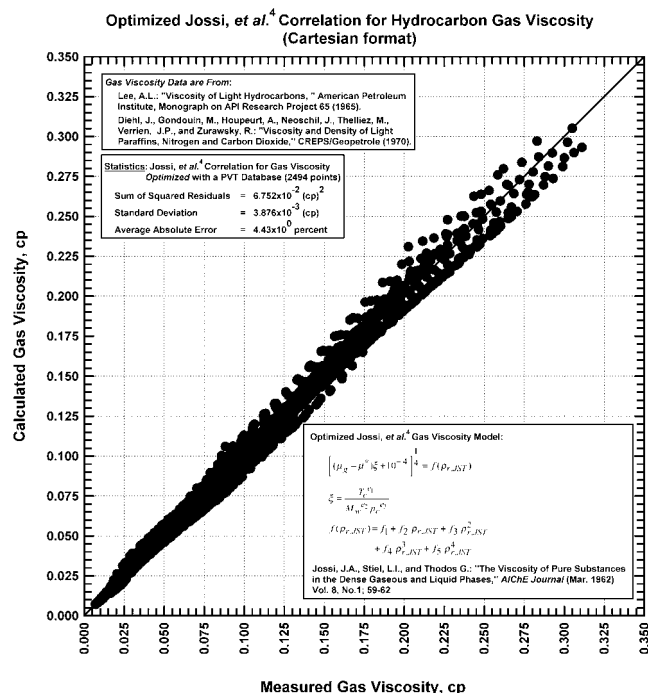


Figure 11 – Optimized Jossi, Stiel, and Thodos⁴ correlation for hydrocarbon gas viscosity optimized using our database (Cartesian format).

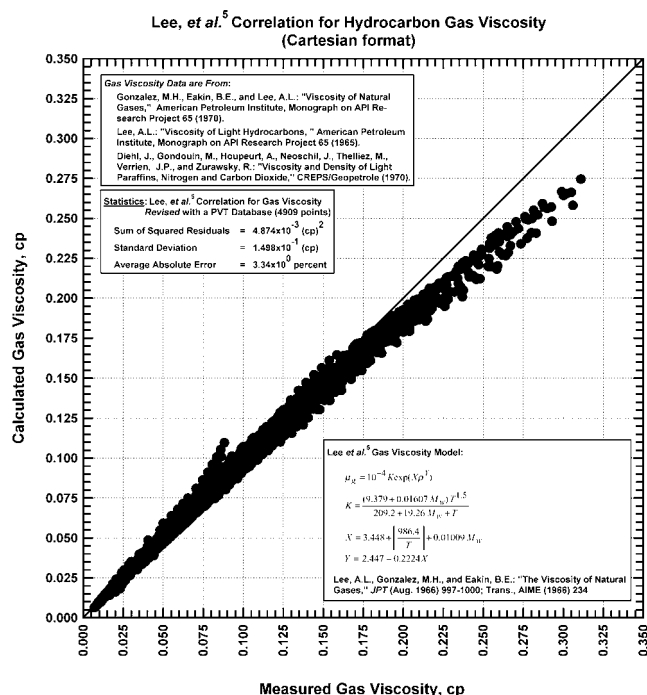


Figure 12 – Lee, Gonzalez, and Eakin⁵ correlation for hydrocarbon gas viscosity tested with our database (Cartesian format).

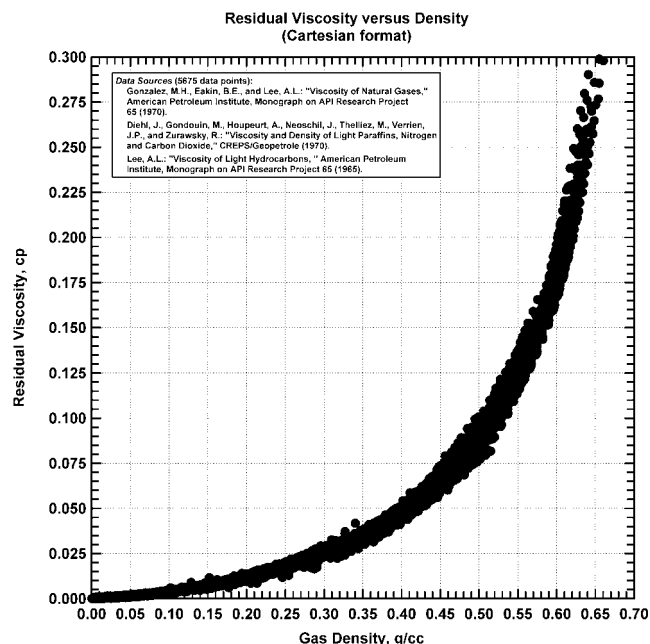


Figure 14 – Cartesian plot of the residual viscosity versus density for hydrocarbon gases.

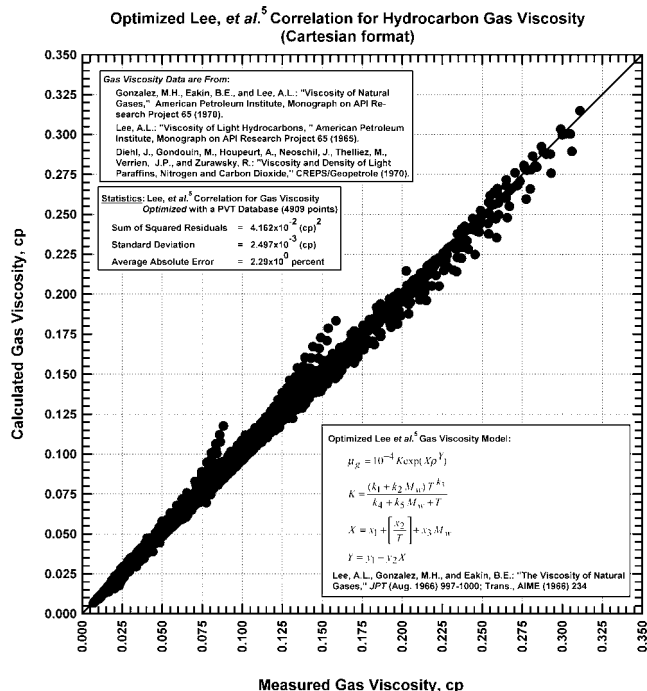


Figure 13 – Optimized Lee, Gonzalez, and Eakin⁵ correlation for hydrocarbon gas viscosity optimized using our database (Cartesian format).

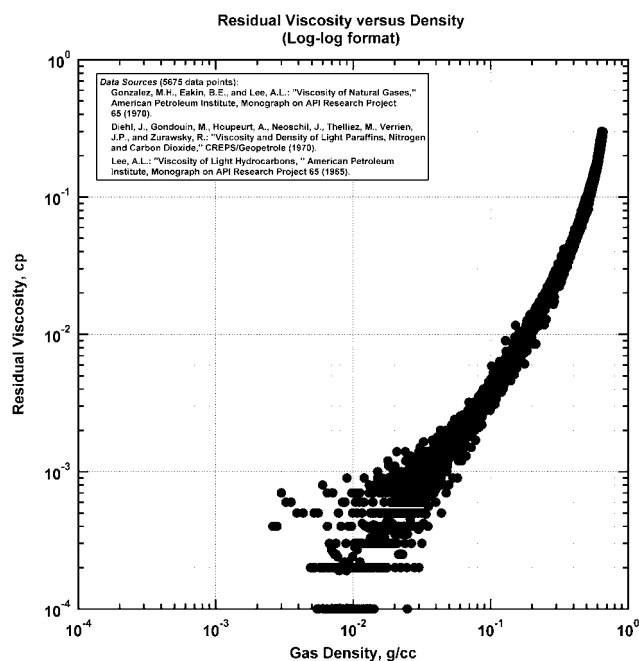


Figure 15 – Log-log plot of residual viscosity versus density for hydrocarbon gases.

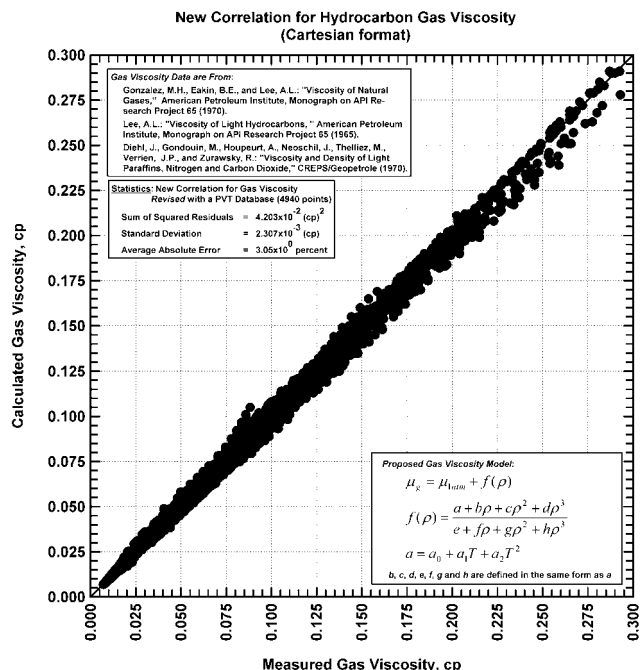


Figure 16 – Cartesian plot of the calculated versus the measured viscosity for hydrocarbon gases, the viscosity is calculated using the proposed implicit model for gas viscosity (in terms of gas density and temperature).

z-factor from the Optimized Dranchuk-Abou-Kassem Equation of State (DAK-EOS)
(Calculated z-factor versus Measured z-factor Format)

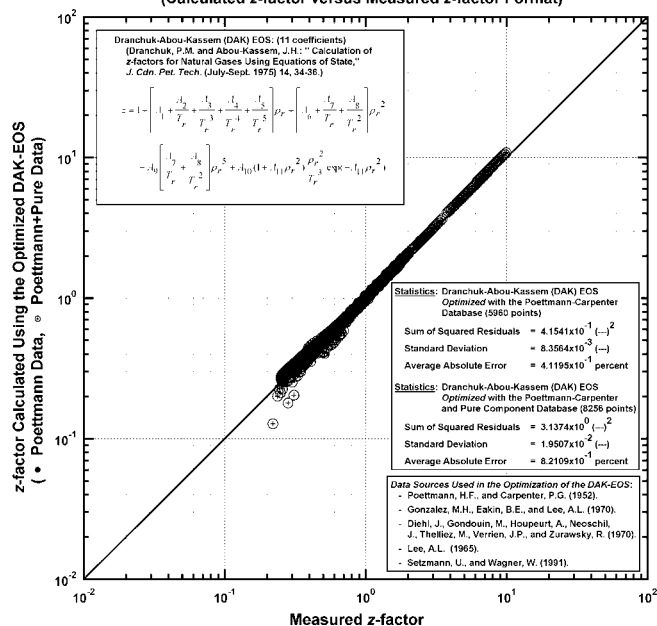


Figure 18 – Log-log plot of the calculated versus the measured z-factor — the z-factor is calculated using the optimized Dranchuk and Abou-Kassem⁶ EOS (coefficients from Eqs. 34a and 34b).

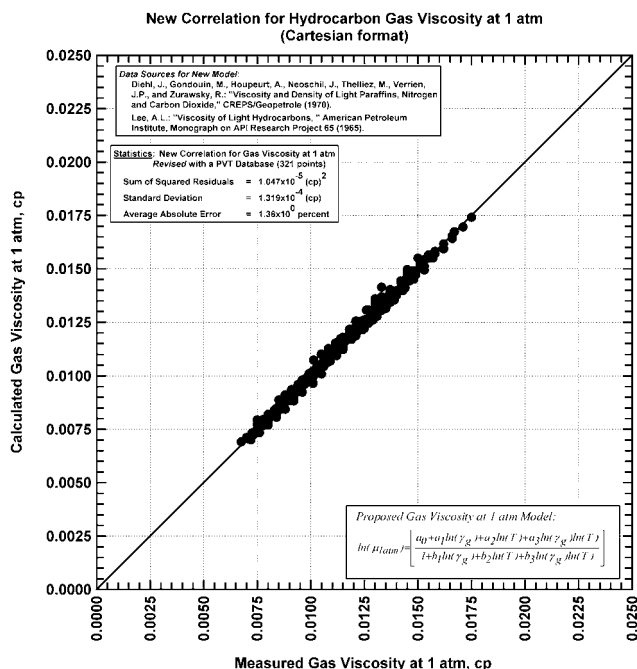


Figure 17 – Cartesian plot of the calculated versus the measured gas viscosity at 1 atm, the gas viscosity at 1 atm is calculated using the new rational polynomial model (Eq. 32).

z-factor from the Optimized Dranchuk-Abou-Kassem Equation of State (DAK-EOS)
(z-factor versus p_{pr} Format)

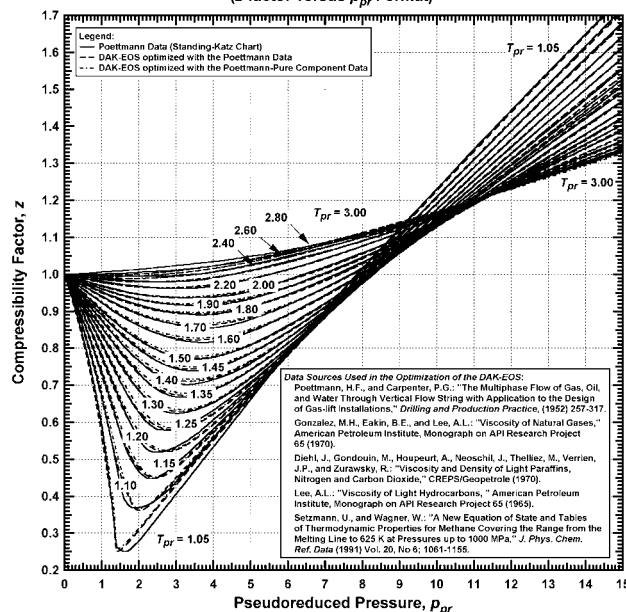


Figure 19 – Real gas z-factor, as attributed to Standing and Katz,⁹ plotted as a function of the pseudoreduced pressure (data of Poettmann and Carpenter¹⁰) compared to the optimized DAK-EOS (coefficients from Eqs. 34a and 34b).

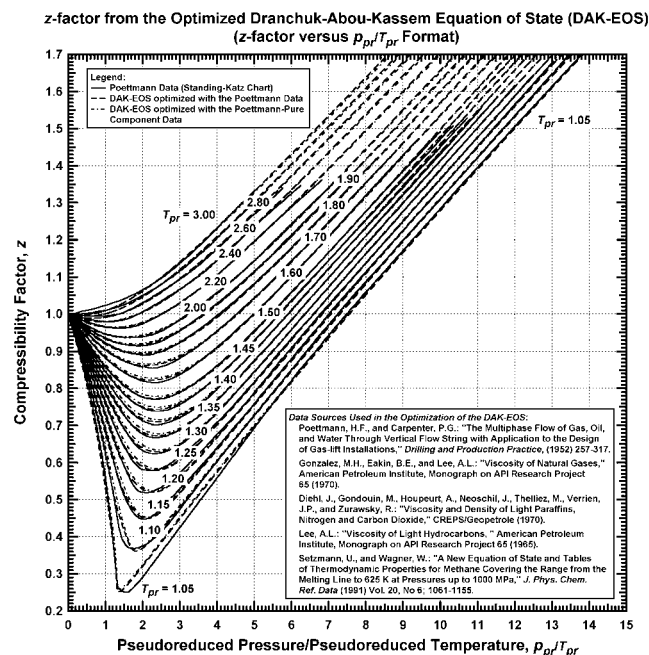


Figure 20 – Real gas z-factor, as attributed to Standing and Katz,⁹ plotted as a function of the pseudoreduced pressure divided by the pseudoreduced temperature (data of Poettmann and Carpenter¹⁰) compared to the optimized DAK-EOS (coefficients from Eqs. 34a and 34b).

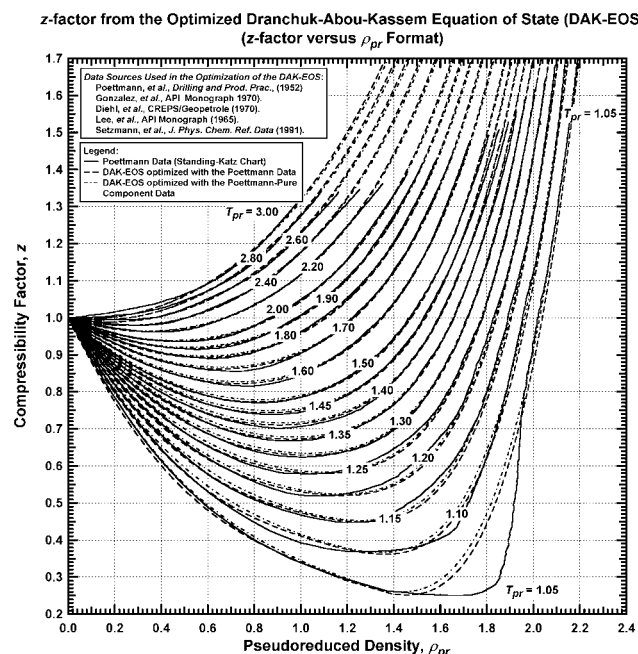


Figure 21 – Real gas z-factor, as attributed to Standing and Katz,⁹ plotted as a function of pseudoreduced density (data of Poettmann and Carpenter¹⁰) compared to the optimized DAK-EOS (coefficients from Eqs. 34a and 34b).

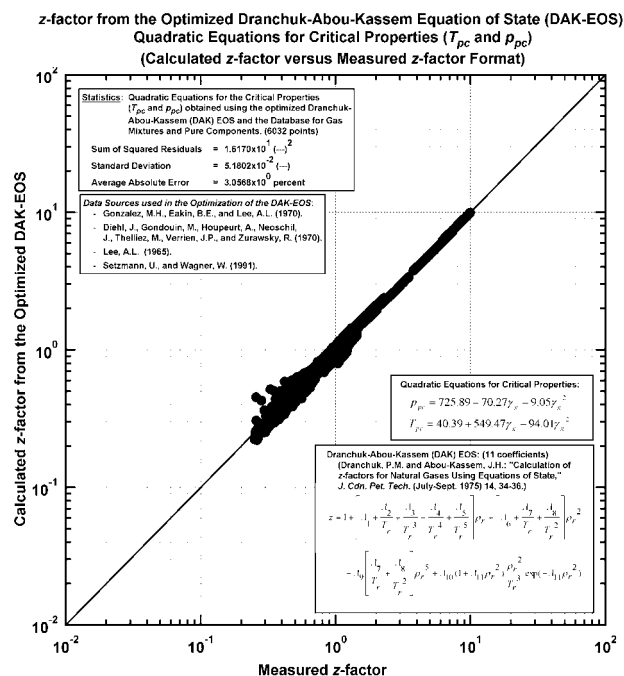


Figure 22 – Log-log plot of the calculated versus the measured z-factor, the z-factor is calculated using the optimized DAK-EOS and the new quadratic equations for the pseudocritical properties (coefficients from Eqs. 34b, 35, and 36).

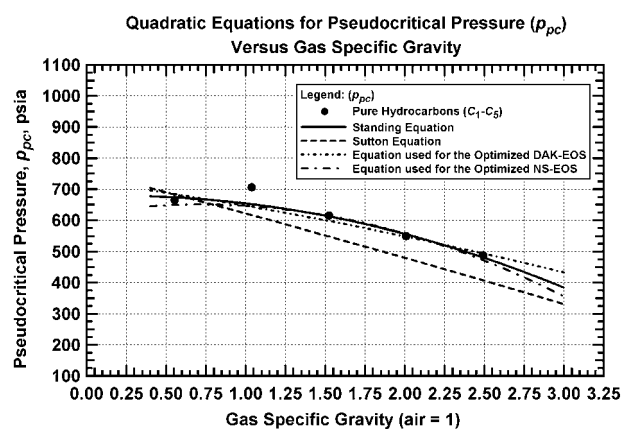


Figure 23 – Pseudocritical pressure behavior predicted from correlations (including pure component data) — the new correlations for p_{pc} are derived from the optimized DAK-EOS and the optimized NS-EOS.

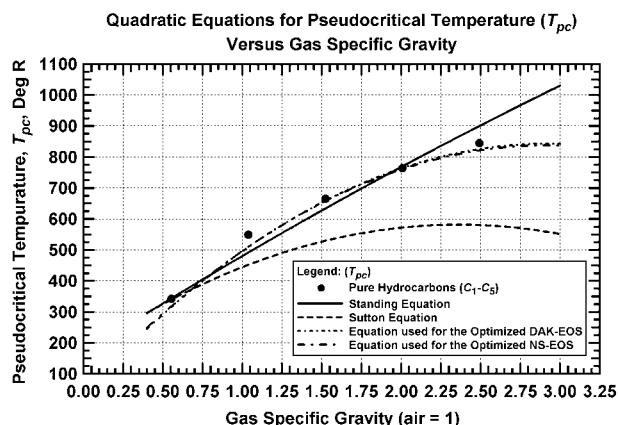


Figure 24 – Pseudocritical temperature behavior predicted from correlations (including pure component data) — the new correlations for T_{pc} are derived from the optimized DAK-EOS and the optimized NS-EOS.

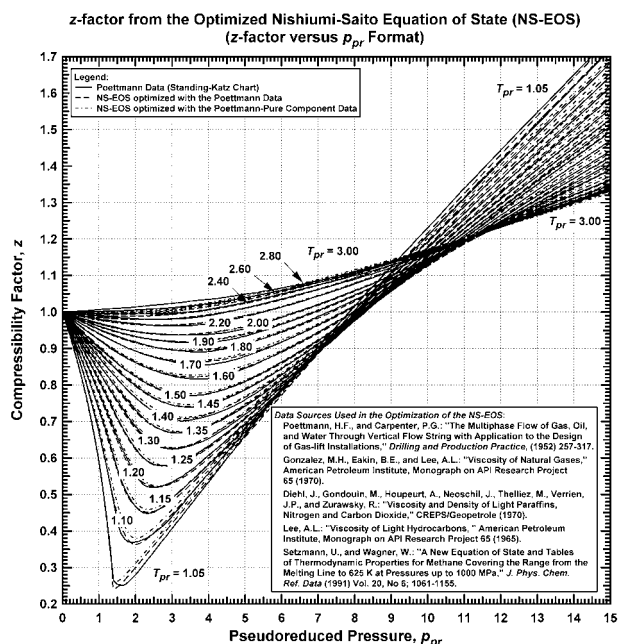


Figure 25 – Real gas z-factor, as attributed to Standing and Katz,⁹ plotted as a function of the pseudoreduced pressure (data of Poettmann and Carpenter¹⁰) compared to the optimized NS-EOS (coefficients from Eqs. 37a and 37b).

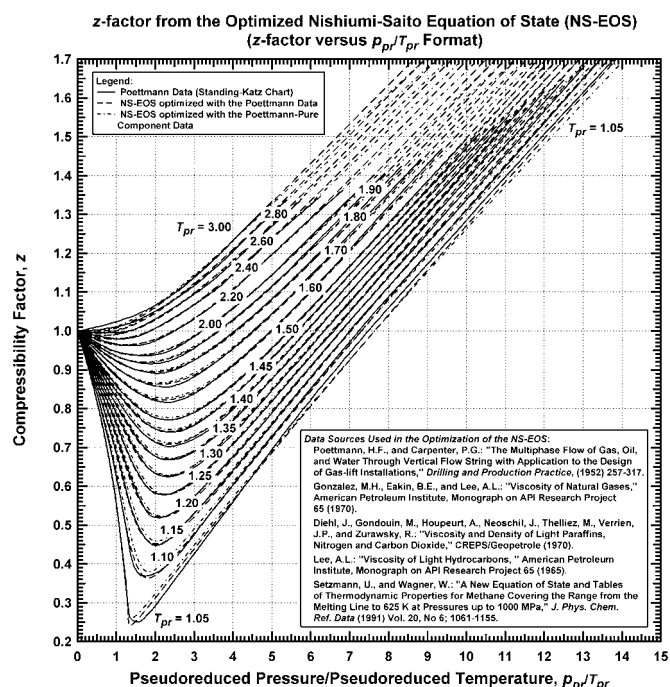


Figure 26 – Real gas z-factor, as attributed to Standing and Katz,⁹ plotted as a function of the pseudoreduced pressure divided by the pseudoreduced temperature (data of Poettmann and Carpenter¹⁰) compared to the optimized NS-EOS (coefficients from Eqs. 37a and 37b).

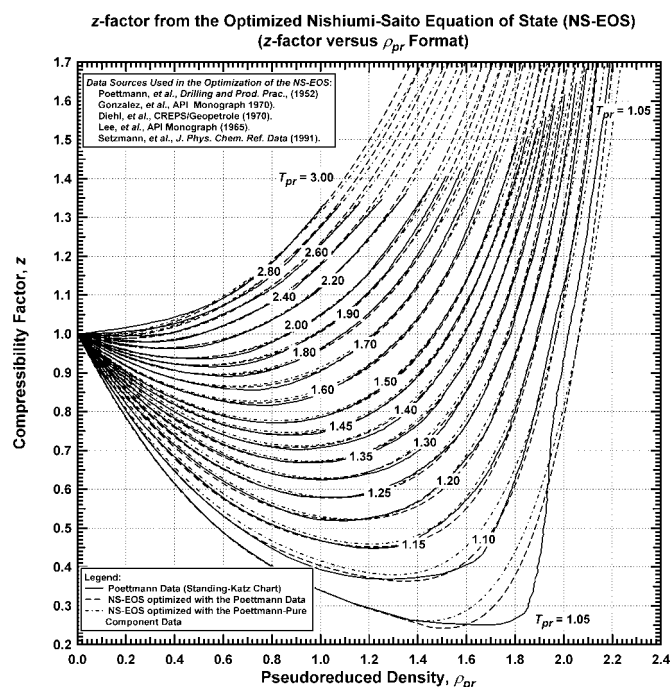


Figure 27 – Real gas z-factor, as attributed to Standing and Katz,⁹ plotted as a function of pseudoreduced density (data of Poettmann and Carpenter¹⁰) compared to the optimized DAK-EOS (coefficients from Eqs. 37a and 37b).

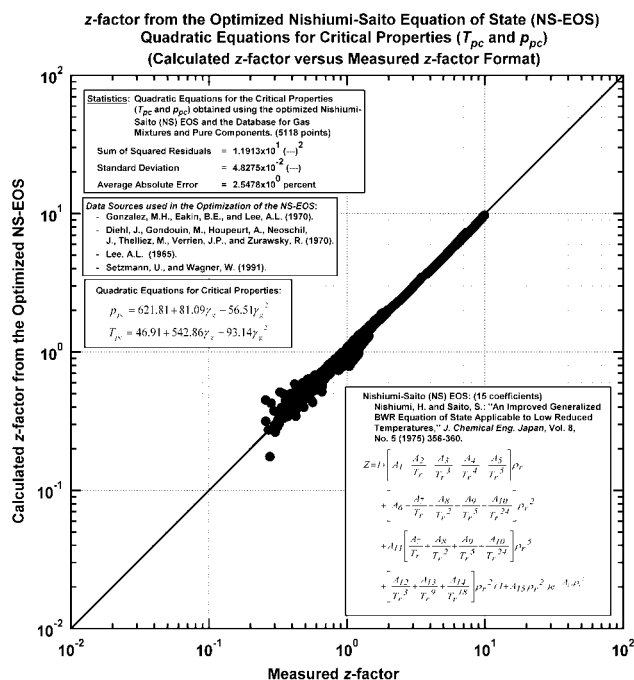


Figure 28 – Log-log plot of the calculated versus the measured z-factor, the z-factor is calculated using the optimized NS-EOS and the new quadratic equations for the pseudocritical properties (coefficients from Eqs. 37b-39).