Analysis of Gas Well Reservoir Performance Data Using B-Spline Deconvolution

Abstract
This work presents the practical application of the recently developed B-spline based deconvolution methodology to analyze variable-rate/variable pressure drop well performance data from gas wells. As deconvolution provides the corresponding constant rate pressure drawdown response for a well/reservoir system which is affected by variable flowrates, we intend to use deconvolution of production data to identify the reservoir model and perform an analysis for estimating reservoir properties and reservoir volume.

In this work we apply our B-spline based deconvolution methodology to production pressure and flowrate history data (which are typically available on a daily or monthly basis — all field cases in this work (except Case 3) consider daily production data measurements). For this work, we apply our method using traditional gas well test data, as well as regularly measured gas well production data. We also demonstrate the appropriate handling of input data (particularly pressure test data and production data) to ensure stable/accurate deconvolution results.

The application cases in this work should be considered typical for a reservoir or production engineer, and we would expect similar performance/robustness of our methodology as it becomes a common analysis practice.

Objectives
The following objectives are proposed for this work:

- To apply and extend the recent B-spline based deconvolution methodology to analyze variable-rate/variable pressure drop gas well performance data.
- To identify the critical issues which affect the success of deconvolution methodology when applied to production data.
- To state specific recommendations for practice and/or future work.

Introduction
As orientation, we note that the conventional analysis of well test data involves the analysis of "high-frequency" pressure buildup data — specifically, the derivative of the pressure drop function with respect to the logarithm of time — using "superposition" or specialized time transforms. We can also perform a similar (albeit much simplified) approach for production data (i.e., boundary-dominated flow data). However, the purpose of using deconvolution is to "extract" the equivalent constant rate pressure drop (or pressure) function and to avoid the use of such "specialized" functions as described above. In short, the primary signature used to classify/establish the reservoir model is the constant-rate drawdown pressure behavior of a well/reservoir system — and the goal of any deconvolution algorithm is to extract that signature with as little corruption as possible in the "extraction" process.

All of which leads us to our present work for the analysis/interpretation of gas well production data using deconvolution. In many ways the gas well performance problem is prototypical — we generally have production rates and pressures available on a per well basis (due to regulations, data collection practices, or both). These data are often measured daily, but, unfortunately we are often faced with surface pressure measurements of unknown quality (gas flowrates are typically accurate, although some "manifold averaging" often occurs as well). Put simply, although the data are not ideal, the gas well performance scenario is "data rich" compared to most other cases (oil, volatile oil, gas condensate, commingled production, etc.).

Most deconvolution methods have been developed and applied to deconvolve "ideal" data (e.g., idealized pressure drawdown/buildup test sequences, monotonic or functional production rate decline sequences, etc.). Very few of these deconvolution methods perform well in practice due to the ill-conditioned nature of the deconvolution problem, which means that small changes in the input data (rate and pressure signals) cause large variations in the deconvolved, equivalent constant-rate pressures.

Recent developments by von Schroeter et al. and Levitan et al. make deconvolution a potentially useful tool in well test analysis — specifically because these deconvolution algorithms attempt to constrain errors in the rate and pressure signals. We have recently developed a new deconvolution algorithm which is based on using a B-spline representation for the derivative of the unknown constant-rate drawdown pressure response. In particular, we use a weighted sum of B-
splines with logarithmically-distributed knots to approximate the equivalent constant rate pressure function.

Our approach (refs. 5 and 6) uses B-splines, numerical inversion of the Laplace transform, and regularization (indirectly by the number of knots used in the selected B-spline and directly by penalizing the non-smoothness of the logarithmic derivative of the reconstructed constant-rate response). We have applied our new method to several field cases and we have shown that this new method can tolerate relatively large errors in the input pressure and flowrate data functions, suggesting a broad applicability in well test/production data analysis.

From a practical standpoint — when considering production data — there are several factors which can (and will) produce poor deconvolution results. The most common factors are incorrect initial pressure and inconsistent (i.e., uncorrelated) rate and pressure data. As simple as these factors are, these can render any production data analysis method ineffective — particularly deconvolution.

In this paper we present the following gas well performance cases:

Case 1: Synthetic Gas Well — Simple homogeneous, circular reservoir case — example used to provide a straightforward demonstration of the deconvolution methodology.

Case 2: Rocky Mountain (US) Gas Well — Fractured well (high conductivity vertical fracture), low permeability and fair productivity (transient flow only).

Case 3: Canada Gas Well — Incomplete pressure history, short drawdown sequence (completely distorted by wellbore storage).

Case 4: South Texas (US) Gas Well — Unfractured well in an over pressured gas reservoir with condensate production (not addressed).

Case 5: North Texas (US) Gas Well — Fractured well (apparent high conductivity vertical fracture), low permeability and low productivity (water loading (not addressed)).

Deconvolution Theory

Duhamel’s principle states that the observed pressure drop is the convolution of the input rate function and the derivative of the constant-rate pressure response (at $t=0$ the system is assumed to be in equilibrium (i.e., $p(r,t=0) = p_0$)). For reference, the convolution integral is defined as:

$$\Delta p_w(t) = \frac{1}{q_{ref}} \int_0^t \Delta p_w' (\tau) q(t-\tau) \, d\tau$$ ............................ (1)

The goal of variable-rate deconvolution is to estimate the unit (i.e., constant) rate reservoir response $[\Delta p_w(t)]$ using observed measurements of pressure drop $[\Delta p_w(t)]$ and flowrate $[q(t)]$. This objective leads us to an inverse problem in which the time-dependent "input signal" (constant-rate pressure response $[\Delta p_w(t)]$) must be extracted from the output signal (pressure drop response distorted by the specified variable-rate profile$[\Delta p_w(t)]$) and the input variable-rate profile $[q(t)]$.

When nonlinearities are present (e.g. the gas flow case) then the superposition principle is not explicitly applicable (hence, nor would deconvolution be applicable). We can use the pseudopressure/pseudotime transform (equivalent liquid case) as a means to "linearize" the problem so that convolution or deconvolution methods can be applied. Specific to this work, we utilize the pseudopressure transform for all cases, and we utilize the (material balance) pseudotime function for the cases which exhibit full boundary-dominated flow behavior (i.e., Cases 4 and 5).

B-Spline Based Deconvolution Method

In this section we provide a very brief summary of the B-spline-based deconvolution method. The reader is referred to references 5 and 6 for further detail.

We represent $\Delta p'_w(t)$ (i.e., the derivative of the unknown constant-rate drawdown pressure response $[\Delta p_w(t)/dt]$) as a weighted sum of B-splines of degree 2, defined over logarithmically evenly-spaced knots:

$$\Delta p'_w(t) = \sum_{i=l}^u c_i B_i^2(t)$$ .................................................. (2)

($l =$ index of 1st spline knot, $u =$ index of last spline knot)

Substituting Eq. 2 into Eq. 1, we have:

$$\Delta p_w(t) = \frac{1}{q_{ref}} \int_0^t \sum_{i=l}^u c_i B_i^2(t) q(t-\tau) \, d\tau$$ .......................... (3)

For the case of a discrete flowrate function, we use piecewise constant, piecewise linear, or any other appropriate representation for which the Laplace transform can be easily obtained. Taking the Laplace transform of the B-splines, we then apply the convolution theorem in the Laplace domain and we calculate the sensitivities (i.e., B-spline coefficients) of the observed pressure response with respect to the B-spline weights by numerical inversion of the Laplace transform.

Finally, the sensitivity matrix is used in conjunction with a least-squares criterion to yield the B-spline coefficients (and the constant-rate response (i.e., the spline formulation)). In the case of high levels of data noise we employ constraints on the change of logarithmic derivative of the B-splines between the knot locations (i.e., the regularization process).

Case 1: Synthetic Gas Well

In this section we generate a synthetic gas test case for comparison of deconvolution methods with conventional PTA/PA analysis methods — as well as to identify the factors which can contribute to poor deconvolution results. For our purposes we select a circular homogeneous reservoir model (see Table 1 for details) and we assume a variable-rate production history — the required (variable-rate) pressure response is generated by numerical solution of the specified flow history using the gas reservoir model.

Fig. 1 presents the production history along with the simulated (variable-rate) pressure history, which serve as the input data for deconvolution.
Table 1 — Reservoir and fluid properties for Case 1 (Synthetic Gas Well).

<table>
<thead>
<tr>
<th>Reservoir Properties:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wellbore radius, $r_w$</td>
<td>0.3 ft</td>
</tr>
<tr>
<td>Net pay thickness, $h$</td>
<td>30 ft</td>
</tr>
<tr>
<td>Formation permeability, $k$</td>
<td>33.3 md</td>
</tr>
<tr>
<td>Formation compressibility, $c_t$</td>
<td>$3 \times 10^{-6}$ psi$^{-1}$</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
<td>0.10 (fraction)</td>
</tr>
<tr>
<td>Reservoir outer boundary radius, $r_e$</td>
<td>4000 ft</td>
</tr>
<tr>
<td>Initial reservoir pressure, $p_i$</td>
<td>5000 psia</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Well Properties/Parameters:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wellbore storage coefficient, $C_s$</td>
<td>0.1 bbl/psi</td>
</tr>
<tr>
<td>Skin factor, $s$</td>
<td>0.05 (dim-less)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fluid Properties at Initial Reservoir Conditions:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid viscosity, $\mu_{gi}$</td>
<td>0.0265 cp</td>
</tr>
<tr>
<td>Formation volume factor, $B_{gi}$</td>
<td>0.00068 RB/SCF</td>
</tr>
<tr>
<td>Fluid compressibility, $c_{gi}$</td>
<td>$1.29 \times 10^{-4}$ psi$^{-1}$</td>
</tr>
<tr>
<td>Reservoir Temperature, $T_r$</td>
<td>212 Deg F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Production Parameters:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference production rate, $q_{ref}$</td>
<td>1000 MSCF/D</td>
</tr>
</tbody>
</table>

(Reference rate = equivalent constant rate for a given case)

We selected the rate profile in Fig. 1 to mimic typical pressure drawdown/buildup test sequences. The well is shut-in for 12 hours at the end of the test and the shut-in pressure data are used in conventional pressure buildup analysis.

We note that our deconvolution methodology performs very well in this case. In this case we do not perform deconvolution to remove the effects of wellbore storage — deconvolution has been used to only to convert the variable-rate pressure performance to an equivalent constant rate pressure profile.

![Figure 1 — Input data for Case 1 (Synthetic Gas Well).](image)

Fig. 2 presents the deconvolution results for the entire test sequence along with the pressure response generated using a constant rate numerical simulation of our gas reservoir model.

For Case 1, the well is shut-in for a final 12-hour pressure buildup (PBU) test, and it is difficult (if not impossible) to interpret reservoir boundaries using the pressure buildup data. On the other hand, deconvolution reveals all flow regimes for the entire test sequence. Reservoir boundaries are apparent in the deconvolved pressure derivative response — which allows estimation of reservoir volume and boundary orientation.

We have previously discussed the factors which can produce poor deconvolution results. In this section we address how these factors affect the deconvolution results. We first consider the factors that affect the deconvolution results. We first con-
sider the initial pressure variable. In our B-spline based
deconvolution algorithm, the initial reservoir pressure is an input
parameter that must be defined by the user.

Our tests prove that B-spline based deconvolution algorithm
is sensitive to the value of initial pressure. An incorrect estimate
of the initial pressure produces artifacts in the deconvolved
responses — particularly in well testing derivative. These
artifacts can be in the form of oscillations or false features
(straight lines, certain derivative behavior, etc.) — particularly
at late times (for our experiments). Fig. 4 illustrates the
deconvolution results generated using three different values of
initial reservoir pressure.

Deconvolution is performed with the exact rate history in the
previous portions of this example. Our next step is to assign
an erroneous value to an arbitrary rate (say the 20th rate value)
and then test the deconvolution process using this altered rate
history. Such an experiment is shown in Fig. 5.

The true value of the initial reservoir pressure is 5000 psia.
We first use 5005 psia as the input initial reservoir pressure
in deconvolution and we see distortions in well testing derivative
developing during the middle time (or transient radial flow)
regime. Increasing this estimate to 5010 psia worsens the
performance of the deconvolution procedure, the variations in
the deconvolved pressure derivative function are amplified
over the scenario where 5005 psia was assumed.

We conclude that the correct estimate of initial pressure must
be available for B-spline based deconvolution algorithm —
particularly for the analysis/interpretation of long-term
production data.

The next critical variable to be considered is that of the
accuracy of the production history. Using our B-spline based
deconvolution algorithm, discrete rate data can be approximat-
ed by several functional forms such as constant piecewise,
linear piecewise or linear combination of exponential func-
tions, etc. In addition, we can solve the deconvolution pro-
blem in "events" (where an event could contain several rates
of a specific type — e.g., discrete steps). In this "event"
approach, we could use arbitrary segments of the production
history (i.e., the events) where the rate function is approxi-
mated separately within each segment.

In Fig. 5 we note that the altered rate history yields artifacts in
the deconvolved well testing derivative function where in this
particular experiment the artifact is mostly an oscillation about
the correct trend (which is a reasonable expectation).

As with any deconvolution process, the pressure and rate data
must be consistent (i.e., correlated) for the B-spline based
deconvolution algorithm to perform successfully. It is also
worth mentioning that when there are inconsistencies in the
input data for deconvolution, artifacts are seen (or will start to
develop) approximately at the same intervals. This is due to
global nature of the deconvolution process — i.e., if there is an
issue with the input pressure/rate data, then this issue will
appear in the deconvolved responses (particularly in well
testing derivative) throughout the entire test sequence (not
simply in the period where the error occurred).

In this synthetic example we have attempted to illustrate how
B-spline deconvolution is affected by specific factors (in this
case the initial reservoir pressure and a (slightly) erroneous
rate profile). Obviously errors exist in practice, and we
encourage care and vigilance regarding deconvolution —
specifically with respect to the interpretation of the various
artifacts which may result. Effort should be devoted to data
preparation and "correlation" — and it may also be necessary
to employ a constraint/filter approach in the deconvolution
process.

We discuss the use of such a constraint technique (i.e., regu-
larization) in refs. 5 and 6, and we note that we have not
employed regularization in the B-spline deconvolution process
in this example. Had we done so, the "artifacts" would have
been minimized — but it is unlikely that regularization would
completely mitigate the artifacts and oscillations which we
caused by seeding errors into the deconvolution process.
Therefore, we strongly recommend performing deconvolution without regularization — at least initially, as a means of possibly identifying artifacts and possible causes. After which, regularization could be used to obtain the most reliable deconvolution results.

In the next section we apply our methodology to several field examples — and in these cases we do employ regularization where necessary to resolve artifacts, where any artifacts are presumed to have been introduced by errors in the input data.

Field Examples

Our field examples consist of three distinct groups:

- Rocky Mountain (US) Gas Well (fractured well in a low permeability reservoir).
- Canada Gas Well (incomplete pressure history, short drawdown sequence).
- Long-term production data (South and North Texas (US) gas wells, various characteristics and issues).

These examples illustrate the use of B-spline deconvolution for analyzing various events (production/shut-in sequences), and should be considered typical of cases that could be encountered in field operations.

Case 2: Rocky Mountain Gas Well

This case is an example of a fractured gas well in a low permeability formation which has been produced for 164 days and we note that the well was shut-in for 5 days during the production sequence (no reason was given — see Fig. 6 for the production history). Although the reservoir is low permeability, one expectation from the operator is that the well has not been produced to boundary-dominated flow condition — one objective of our analysis/interpretation is to verify/dispute this expectation.

Figure 6 — Input rate and pressure history for Case 2 (Rocky Mountain (US) Gas Well).

Our cursory review of Fig. 6 suggests that the data quality seems good (possibly very good), except for the early time pressure/rate data (well cleanup is the likely culprit). Our first effort is to consider the pressure buildup (PBU) test in isolation from the production history, this is shown in Fig. 7. In Fig. 7 we find that the PBU data are significantly affected by wellbore storage, and we note that fracture flow just begins to develop (in the vicinity of $\Delta t=10$ hr). Also, from the PBU data (again, in isolation from the production history) there is no evidence of reservoir boundaries. We can proceed and attempt to match the PBU data with an appropriate reservoir model, but as we would have no clear evidence regarding the reservoir boundaries, this analysis may be inconclusive at best. In such cases (long production history, short PBU), deconvolution should be used to reveal as much insight as possible to for the entire data sequence.

Figure 7 — Pressure buildup data — log-log diagnostic plot (Rocky Mountain (US) Gas Well).

Before performing deconvolution we should decide which part of the data will be the input for deconvolution (e.g., only the production sequence, only the PBU test data, or the entire data sequence). The rate and pressure data appear to be consistent/correlated (where consistency/correlation is used to describe the condition in which a change in rate yields an expected change in pressure and vice-versa).

Figure 8 — Case 2: Deconvolved responses (for the entire test sequence — 164 days) and model match.

As required by deconvolution (to be rigorous) we must use the entire rate history (as the input for deconvolution) in isolation from the production history, this is shown in Fig. 8.
We could use other functions (e.g., piecewise exponential segments), but we believe that the quality of the rate data and the character of the rate and pressure data warrant the use of piecewise constant rate segments to represent the rate history. Similarly, we believe that the all of the pressure data are sufficiently accurate to be used for deconvolution. Or, we may choose to perform deconvolution using only the "high-quality" pressure buildup data (since the well is shut-in, the pressure data should be unaffected by instantaneous rate changes). In the case of using the PBU test data, the deconvolved responses would be defined from the start of production to the end of shut-in period (based, as always on the entire rate history). Since these data are taken only on a daily basis, we believe it is possible for this sampling to cause issues/errors in the deconvolved responses (i.e., artifacts).

The deconvolution results are generated using the entire production sequence (164 days) and these results (pseudopressure drop and pseudopressure drop derivative) are shown in Fig. 8 (a log-log diagnostic plot). The wellbore storage distortion effects remain (recall that we did not account for wellbore storage in our deconvolution (i.e., we used only surface rates)), and we also see the emergence of the half-slope trend in the deconvolved well testing derivative function (signature of an infinite conductivity vertical fracture). The deconvolved responses do not provide evidence of any reservoir boundary(s), which confirms the low-permeability nature of the reservoir. However, for completeness we have modeled 2 cases — that of an infinite-acting reservoir and a reservoir with rectangular boundaries. Neither model is confirmed absolutely, but we believe that our use of these models helps to define possible configurations and may assist in reservoir development decisions.

Case 3: Canada Gas Well

In this example we consider a well test sequence conducted on a gas well where the duration of the entire test sequence is on the order of 40 hours. The well test consists of two parts — 24 hours of variable rate production and 16 hours of shut-in. The duration of the test sequence may seem short, but in this case we will demonstrate that, using deconvolution, we can extract considerable information from these data. Fig. 9 presents the pressure and rate history for this case.

The quality of these data is probably sufficient for deconvolution and analysis (although 40 hours of data is hardly optimal). We also note that the pressure data are only available for the latest production data (i.e., the last 6 hours) as well as for the pressure buildup (PBU) portion of the test sequence. We can not estimate the initial reservoir pressure from the early time pressure trend, but we can (and do) estimate the initial reservoir pressure from the pressure buildup trend. In Fig. 10 we present the raw pressure buildup data for this well on a log-log diagnostic plot — clearly, the entire PBU sequence is dominated by wellbore storage and skin effects.

As these data are dominated by wellbore storage distortion, it is very difficult (if not impossible) to make any interpretation (quantitative or qualitative) from this data set. These data are a candidate for deconvolution, but as we are using only surface rate data, the best we can hope for is to "extract" the underlying reservoir model, but this model will include wellbore storage and skin effects. Resolution of reservoir shape or drainage volume is not likely to be possible given the short duration of these data — and in fact, we may not see evidence of the underlying reservoir model (even transient flow) if the wellbore storage effects are completely dominate.

We have few choices regarding which part of the data set can be used for deconvolution due to limited pressure data. As the pressure data are limited to a small portion of the pressure drawdown sequence, and all of the pressure buildup sequence, we will focus only on the pressure buildup (PBU) data. As in our previous deconvolution efforts in this work, the rate profile will be modeled considering piecewise constant functions (see Fig. 9).

Our deconvolution results for this case are shown in Fig. 11. As an estimate of the initial reservoir pressure was not available, we iterated using the deconvolution procedure in order to estimate this variable — our best estimate of the initial reservoir pressure is 2900 psia. We also use regularization to minimize deconvolution artifacts, and we note "endpoint" effects at early times — even after regularization.
From the results shown in Fig. 11, we can conclude that these data are essentially dominated (completely) by wellbore storage and skin effects. We have imposed an infinite-acting homogeneous reservoir model (for radial flow) including skin and wellbore storage effects as shown in Fig. 11. This model is primarily for orientation, as the data trends on Fig. 11 do not warrant such scrutiny — this is a case of wellbore storage domination, the only remedies are: bottomhole shut-in during the pressure buildup or measurement of sandface flowrates.

Case 4: South Texas Gas Well

In this case we apply our B-spline based methodology to analyze and interpret traditional (long-term) gas production data. Production data are considered to be "low frequency/low resolution" data — typically production data are not measured at downhole conditions, or even with automated sensors.

Our goal in these applications is to "convert" the entire production sequence (including poor quality data) using deconvolution into an equivalent constant flowrate pressure response. We confirm our results using an approximate form of the material balance deconvolution procedure, which requires a log-log diagnostic plot of the rate normalized pseudopressure \( \frac{\Delta p}{q_g} \) versus material balance time \( \frac{G_p}{q_g} \). We then compare our deconvolution results and the approximate material balance deconvolution results, and we include a reservoir model established using the deconvolved data.

Case 4 is a gas well which has been produced for 5 years — the daily production and pressure data are shown in Fig. 12. We presume (with some editing and downsampling) that these data are appropriate for B-spline deconvolution.

In Fig. 12 we note several major fluctuations in the rate and pressure profiles, as well as several shut-ins. The data set for this case includes 5 years of daily pressure and rate measurements — as such, the data set must be downsampled and, in particular, we have approximated the flowrate data using a linear combination of exponential functions (excluding shut-ins). In Fig. 13 we present the deconvolved responses from the B-spline deconvolution method and using approximate material balance time (for gas). These deconvolution results are compared to an appropriate reservoir model (unfractured well in a homogeneous bounded rectangular reservoir) — we note good agreement.
Case 5: North Texas Gas Well

Our final case of production data analysis is for a gas well in a low permeability reservoir which has been produced without significant interruptions for approximately 10 years. The low gas flowrates indicate that this is a low-productivity reservoir, and the fluctuations in the pressure/rate data are due to water load-up in these wells (see Fig. 14). However, the overall data quality is good, the flowrate and pressure data are reasonably well correlated. Due to data volume (almost 10 years of daily data) the flowrate data are approximated by linear combinations of exponentials. In addition, this well was hydraulically fractured. Lastly, there are line pressure effects which are evident at late times (>60,000 hr) — these are not artifacts, but actual backpressure constraints on the production system.

In Fig. 15 we again compare our B-spline deconvolution results with a traditional production analysis based on an approximate material balance time deconvolution. Artifacts caused by inconsistencies are seen at early times in the well testing pressure derivative function (again we believe) due to sparse/erratic data. The late time well testing pressure derivative function trend confirms that boundary-dominated flow regime has been reached (unit slope behavior in the pseudopressure drop and derivative functions). The solution for a vertical well with a finite-conductivity fracture in a homogeneous reservoir with a closed circular boundary matches the deconvolved pressure response functions reasonably well, which also confirms our analysis.

Based on the evidence we have presented in this work, we believe that B-spline deconvolution can be successfully applied to production data (regularly measured flowrate and pressure data). Processing "low quality" production data with deconvolution can provide additional insight into reservoir performance analyses, but we note that there are costs associated with such processes/analyses — setup, regularization, and refinement are not trivial tasks.

It is worth noting that numerous other cases of long-term flowrate and (surface) pressure data have been successfully analyzed/interpreted using the proposed B-spline deconvolution technique — for both oil and gas well cases. Our goal in this paper was to provide a convincing argument that B-spline deconvolution should be more widely utilized for the analysis of production data.

Conclusions

In this work we have applied our B-spline deconvolution methodology to various classes of variable rate/pressure problems — and we note that several issues have arisen — these issues are:

1. (Issue) Inconsistent production data (i.e., rates and pressure data which are not correlated — indicating one or both variables are incorrect). This is very common in practice and must be noted as a potential pitfall for any deconvolution process.

2. (Issue) The initial reservoir pressure — this is a non-trivial issue from the perspective of the basis functions (i.e., pressure drop), but we must also recognize that the flowrate and pressure history must have an appropriate "starting point" — errors in the initial reservoir pressure are common, but not insurmountable in the context of analysis/interpretation.

In this work we have experienced reasonable success pertaining to the application of B-spline deconvolution to production/well test data, and we state the following conclusions:

1. (Conclusion) B-spline based deconvolution is an innovative and robust mechanism for the analysis of gas well reservoir performance data.

2. (Conclusion) Data reduction/data editing are required for the application of B-spline based deconvolution to production data. Particular care must be given to the approximation function(s) used for the flowrate profile, these should provide an accurate representation of the flowrate data with minimal sampling.

3. (Conclusion) Regularization (constraint-based processing) is a fact of life for B-spline or other deconvolution methods to be effectively applied to production data.
Recommendations for Practice/Future Work

The following recommendations have evolved from this work:

1. (Practice) Efforts should be made to acquire the best possible production rate and pressure data. While measured downhole rates and pressures are not economically feasible in general, this would (of course) be the best possible scenario. In practice, automated recording systems should be employed, and a preferred minimum sampling rate would be on the order of minutes.

2. (Future Work) Efforts should be made to modularize this methodology as a "filter" or data processing technique. Particularly from the standpoint of an individual analyst — this methodology must be reduced to a relatively simple processing approach.

3. (Future Work) "Downsampling" of data for B-splines is a potential limitation of this methodology, for both pressure transient test and production data analysis. Work should continue on techniques which could consider (much) larger quantities of data (i.e., an arbitrary quantity of data), but from a practical standpoint, this will be a very significant endeavor based on present computational tools and solution methods.

Nomenclature

B-Spline Deconvolution Variables

- $B^k_{ij}(t)$: $k$-th degree B-spline starting at $b^i$, dimensionless
- $c_i$: Vector of unknown coefficients, psi/hr
- $\Delta p_u(t)$: Derivative of $\Delta p(t)$, psi/hr [$d\Delta p(t)/dt$]
- $\Delta p(t)$: Constant (unit) rate pressure drawdown, psi
- $\Delta p_r(t)$: Variable-rate pressure drawdown, psi
- $q(t)$: Rate, MSCF/D (gas)
- $q_{ref}$: Reference rate (constant or unit-rate), MSCF/D

Field Variables

- $B_{gi}$: (Initial) Gas formation volume factor, RB/SCF
- $c_f$: Formation compressibility, psi$^{-1}$
- $c_{gi}$: (Initial) Gas compressibility, psi$^{-1}$
- $c_t$: Total system compressibility, psi$^{-1}$
- $C_s$: Wellbore storage coefficient, RB/psi
- $F_c$: Fracture conductivity, md-ft
- $G_p$: Cumulative gas production, MSCF
- $h$: Net pay thickness, ft
- $k$: Formation permeability, md
- $p(t)$: Pressure, psia
- $p_i$: Initial reservoir pressure, psia
- $\Delta p$: Pressure drawdown (with respect to $p_i$), psi
- $q_g$: Gas rate, MSCF/D
- $r_e$: Reservoir outer boundary radius, ft
- $r_w$: Wellbore radius, ft
- $s$: Skin factor, dimensionless
- $t$: Flowing time, hr
- $\Delta t$: Shut-in time, hr
- $T_r$: Reservoir Temperature, Deg F

Greek Variables

- $\phi$: Porosity, fraction
- $\mu_i$: (Initial) Gas viscosity, cp
- $\tau$: Dummy variable

SI Metric Conversion Factors

- $\text{cp} \times 1.0 = \text{Pa} \cdot \text{s}$
- $\text{ft} \times 3.048 = \text{m}$
- $\text{md} \times 9.869 \text{E-04} = \mu \text{m}^2$
- $\text{psi} \times 6.894 \text{E+00} = \text{kPa}$
- $\text{bbl} \times 1.589 \text{E-01} = \text{m}^3$

*Conversion factor is exact.

References