Abstract

The analysis/interpretation of wellbore storage distorted pressure transient test data remains one of the most significant challenges in well test analysis. Deconvolution (i.e., the "conversion" of a variable-rate distorted pressure profile into the pressure profile for an equivalent constant rate production sequence) has been in limited use as a "conversion" mechanism for the last 25 years. Unfortunately, standard deconvolution techniques require accurate measurements of flowrate and pressure — at downhole (or sandface) conditions. While accurate pressure measurements are commonplace, the measurement of sandface flowrates is rare, essentially non-existent in practice.

As such, the "deconvolution" of wellbore storage distorted pressure test data is problematic — in theory, this process is possible, but in practice, without accurate measurements of flowrates, this process can not be employed. In this work we provide explicit (direct) deconvolution of wellbore storage distorted pressure test data using only those pressure data.

The value of this work is that we provide explicit tools for the analysis of wellbore storage distorted pressure data — specifically, we utilize the following techniques:

- Russell method (1965) (very approximate approach).
- "Beta" deconvolution (1950s and 1980s).
- "Material Balance" deconvolution (1990s).

Each method has been validated using both synthetic data and literature field cases and each method should be considered valid for practical applications (the Russell method was not used).

Our primary technical contribution in this work is the adaptation of various deconvolution methods for the explicit analysis of an arbitrary set of pressure transient test data which are distorted by wellbore storage — without the requirement of having measured sandface flowrates.

Objectives

The objective of this work is to provide a comprehensive study of the analytic techniques that can be used to explicitly deconvolve wellbore storage distorted well test data using only the given pressure data and the well/reservoir information.

Introduction

Previous Work: For the elimination of wellbore storage effects in pressure transient test data, a variety of methods using different techniques have been proposed. An approximate "direct" method by Russell (1966) "corrects" the pressure transient data distorted by wellbore storage into the equivalent pressure function for the constant rate case. Despite its simplicity, it has several shortcomings such as limited accuracy and erroneous skin factor estimation. In short, the Russell (1966) method should not be used.

Rate normalization techniques [Gladfelder et. al., (1955), Fetkovich and Vienot (1984)] have also been employed to correct for wellbore storage effects and these rate normalization methods were successful in some cases. The most appropriate application of rate normalization is its use for pressure transient data influenced by continuously varying flowrates. The application of rate normalization requires the sandface rate measurements and generally yields a shifted results trend that has the correct slope, but incorrect intercept in a semilog plot (incorrect skin factor).

Johnston (1992) showed that "material balance deconvolution" is a practical approach for the analysis of pressure transient data distorted by wellbore storage effects. In particular, this approach remedies the issue of a poor skin factor estimate that is typically obtained using rate normalization. Material balance deconvolution is also thought to require continuously varying sandface flowrate measurements. We will show that sandface flowrates can be approximated from the observed pressure data.

Essentially, rate normalization techniques are restricted when the lack of rate measurement exists. van Everdingen (1953) and Hurst (1953) demonstrated empirically that the sandface rate profile can be modeled approximately using an exponential relation for the duration of wellbore storage distortion during a pressure transient test. The van Everdingen/Hurst exponential rate model is given in dimensionless form as:

\[ q_D(t_D) = 1 - e^{-\beta t_D} \]  

(during wellbore storage distortion) ...(1)
Further, van Everdingen and Hurst showed that the "rate-time" relationship during afterflow (for a pressure buildup test) or unloading (in a pressure drawdown test) is a function of the pressure drop change with respect to time and a relatively constant wellbore storage coefficient.

Based on a material balance in the wellbore, the sandface flow-rate can be calculated by the following relation given in dimensionless form:

\[ q_D(t_D) = 1 - C_D \frac{dp_D}{dt_D} \]  
\[ \beta \]

Where we note that in the development of wellbore storage models/solutions (e.g., type curves), we always assume a constant wellbore storage coefficient.

Eqs. 1 and 2 laid the groundwork for "\( \beta \)-deconvolution" — Joseph and Koereritz (1982) and Kuchuk (1987) applied "\( \beta \)-deconvolution" for the analysis of wellbore storage distorted pressure transient data. In Appendix B, we provide a detailed derivation of the "\( \beta \)-deconvolution" relations that we use in our application. The \( \beta \)-deconvolution formula, which computes the undistorted pressure drop function directly from the wellbore storage affected data, is given as:

\[ p_sD(t_D) = p_{wD}(t_D) + \frac{1}{\beta} \frac{dp_{wD}(t_D)}{dt_D} \]  
\[ \beta \]

We note that Eq. 3 is only valid when the sandface flowrate profile follows an exponential trend as prescribed by Eq. 1. In this work, our objective is to generalize Eq. 3 by treating \( \beta \) as a variable \( \beta(t_0) \), rather than a constant. We develop several schemes to perform "\( \beta \)-deconvolution" directly using pressure derivative and/or pressure integral and integral-derivative data. We describe these schemes in detail in Appendix C.

Once we obtain the \( \beta(t_0) \) function, we utilize Eq. 3 as the mechanism for directly estimating the "undistorted pressure drop" function. The obvious advantage of "\( \beta \)-deconvolution" is that the wellbore storage effects are eliminated using only the given pressure data.

**Methodology**

This work was put forth as an attempt to provide a set of simple, explicit deconvolution formulas that could be used on wellbore storage distorted pressure transient test data. We evaluated a very old "correction" method by Russell (1966) and found this method to be unacceptable for all applications. We also evaluated the "material balance deconvolution" [Johnston, (1992)] for the purpose of evaluating pressure transient test data without any sandface rate information. This approach was successful and should be considered sufficiently accurate to be used as a practical tool for field applications.

The other "major" method considered was the direct \( \beta \)-deconvolution algorithm modified to estimate the \( \beta \)-parameter from pressure rather than flowrate data as originally proposed by van Everdingen (1953) and Hurst (1953). The modification of the \( \beta \)-deconvolution algorithm (given only in terms of pressure variables) was also successful.

**Russell Method** (1966): The pressure "correction" function given by Russell (1966) is given as:

\[ \frac{[p_{ws}(\Delta t) - p_{wf}(\Delta t = 0)]}{1 - \frac{1}{C_2 \Delta t}} = f(\Delta t = 1 \text{ hr}) + m_{sl} \log(\Delta t) \]

Where the \( C_2 \)-term is derived rigorously using Russell's assumptions of the system. The \( C_2 \)-term is used as an arbitrary constant to be optimized. In short, the Russell method has an elegant mathematical formulation, but ultimately, we believe that this formulation does not represent the wellbore storage condition, and hence, we do not recommend the Russell method under any circumstances.

**Material Balance Deconvolution**: The relations for the deconvolution of wellbore storage distorted well test data using material balance deconvolution are provided in Appendix D. The material balance-based, material balance time function for the pressure buildup case is given as:

\[ \Delta t_{mb,BU} = \frac{N_p_{wbs,BU}}{1 - q_{wbs,BU}} = \frac{\Delta t - \frac{1}{m_{wbs}} \Delta p_{ws}}{1 - \frac{1}{m_{wbs}} \frac{d}{d\Delta t}[\Delta p_{ws}]} \]

And the wellbore storage-based, rate-normalized pressure drop function for the pressure buildup case is given as:

\[ \Delta p_{s,BU} = \frac{\Delta p_{ws}}{1 - q_{wbs,BU}} = \frac{1}{1 - \frac{1}{m_{wbs}} \frac{d}{d\Delta t}[\Delta p_{ws}]} \]

In the material balance deconvolution formulation the \( \Delta t_{mb,BU} \) is used as a time function, in whatever fashion is required — plotting data functions, modeling, etc. And the \( \Delta p_{s,BU} \) function is used as a pressure drop function, in any appropriate manner that pressure drop would be employed.

**\( \beta \) Deconvolution**: We also present the application of our new \( \beta \)-deconvolution algorithm derived from wellbore-storage distorted pressure functions. The final result developed for application in our present work is given by:

\[ \Delta p_s = \Delta p_w + \frac{\Delta p_{wd}}{(\Delta p_w - \Delta p_{wd})} \Delta p_{wid} \]

Where, for the pressure buildup case, we have:

\[ \Delta p_w = p_{ws} - p_{wf}(\Delta t = 0) \]  
(pressure drop)

\[ \Delta p_{wd} = \Delta t \frac{d\Delta p_w}{d\Delta t} \]  
(pressure drop derivative)

\[ \Delta p_{wi} = \frac{1}{\Delta t} \int_{0}^{\Delta t} \Delta p_w d\tau \]  
(pressure drop integral)

\[ \Delta p_{wid} = \Delta t \frac{d\Delta p_{wi}}{d\Delta t} \]  
(pressure drop integral-derivative)

Of the methods reviewed/developed in this work, we believe that our modifications of the "material balance deconvolution" approach and the \( \beta \)-deconvolution algorithm will perform well in field applications. We note that both of these methods have been specifically recast for the analysis of wellbore storage distorted pressure transient test data.
Example Applications

Synthetic Example: In this example we provide a synthetic case for a well in an infinite-acting reservoir, with wellbore storage effects. In this example case the dimensionless wellbore storage coefficient ($C_{fD}$) is set at $1 \times 10^6$, and the results of this model are shown by the solid red line in Fig. 1. The "no storage" solution is shown as the solid black line in Fig. 1.

![Figure 1](image1.png)

**Figure 1** – Synthetic example using various deconvolution techniques (infinite-acting reservoir case with wellbore storage effects).

In this example we present the performance of the various deconvolution techniques in Fig. 1, and we provide a synopsis of the performance of each technique below.

- **Rate Normalization**: In this case the rate normalization process yields excellent results. We can see that the deconvolved response lies below the "no storage" response, but we conclude that this is a very good performance for the rate normalization method.

- **Material Balance Deconvolution**: The material balance deconvolution technique performs extremely well in this case, with minor discrepancies at the start of the data set and at the point where the wellbore storage and no wellbore storage solutions merge. This performance is very strong, and suggests that, based on the simplicity of the material balance deconvolution method, this is probably the most practical approach for the analysis of pressure transient test data distorted by wellbore storage.

- **β-Deconvolution**: The β-deconvolution technique is also a very strong performer in this case — most likely due to the analytic nature of the "data" (i.e., the dimensionless pressure and auxiliary functions). In other words, the fact that we used the analytical (i.e., exact) solutions in this process most likely accounts for the remarkable success of the β-deconvolution technique for this example.

Field Example: This example is taken from literature (Bourdet, 1989). In this case we provide the explicit deconvolution of field data using the methods presented in this work. The data are taken from a pressure buildup test and should be considered reasonably well behaved for field data. The deconvolution "conversion" results are shown in Fig. 2 (semilog format) and Fig. 3 (log-log format) — different plotting formats (semilog and log-log) are used to emphasize the character in the data.

The most positive aspect of the application of the explicit deconvolutions methods in this example is that we gain approximately 1.5 log cycles of results which can be analyzed using conven-

![Figure 2](image2.png)

**Figure 2** – (Semilog Plot) Bourdet (1989) field example using various deconvolution techniques (infinite-acting reservoir case with wellbore storage effects).

For the Bourdet (1989) case we have reviewed the given data and believe that the data are of sufficient quality to provide a reasonably competent deconvolution using explicit methods (i.e., rate normalization, material balance deconvolution, and β-deconvolution). We note that these data are clearly distorted (if not dominated) by wellbore storage effects.

- **Rate Normalization**: From Figs. 2 and 3 we note that the rate normalization profile is more stable than the β-deconvolution profile, but is not as accurate as the material balance deconvolution profile. In particular, the rate normalization profile is slightly unstable at early times.

- **Material Balance Deconvolution**: The response of the material balance deconvolution method as shown in Figs. 2 and 3 appears to be the most accurate deconvolution. We will note that we encountered negative values in the material balance time function.
(due to the negative "rates" computed from the wellbore storage-distorted data — these negative rates also affected the rate normalization and β-deconvolution results, as indicated by the off-trend performance at early times).

- **β-Deconvolution:** The β-deconvolution results shown in Figs. 2 and 3 are reasonably stable, and suggest a good performance of this approach for this data set. We had hoped for more stability in the β-deconvolution at early times, but all of the explicit deconvolution methods were affected at early times for this case.

As closure commentary regarding this example, we believe that this example does indicate success for the methods employed. Obviously the degree of success for any particular case will rely on the quality and relevance of the data. As for a general recommendation, we encourage vigilance in data acquisition, and care in the application of the methods used in this work. While these methods are theoretically supported, these methods are highly susceptible to data errors and bias.

**Summary**

We summarize this work as follows — the expectation of success for the deconvolution of pressure transient test data using explicit deconvolution techniques (rate normalization, material balance deconvolution, and β-deconvolution) must be tempered with the knowledge that we create an inherent bias when we do not use the rate profile — but rather, we infer the rate profile from a wellbore storage model imposed (in some manner) on the pressure data.

Having made those qualifying comments, we should also recognize that theory of each method does provide some confidence that these methods should perform well in practice. The primary concern must be the quality and relevance of the pressure data.

**Recommendations for Future Work**

The future work on this topic should consider mechanisms for further improvements in the material balance deconvolution and β-deconvolution methods as these methods are applied to wellbore storage distorted well test data.

**Nomenclature**

**Dimensionless Variables**

- $C_D$ = dimensionless wellbore storage coefficient
- $t_D$ = dimensionless time
- $p_D$ = dimensionless pressure
- $p_{RD}$ = dimensionless constant rate pressure
- $p_{sD}$ = dimensionless pressure distorted by wellbore storage
- $q_D$ = dimensionless rate
- $\beta$ = "beta-deconvolution" variable (dimensionless)

**Field Variables**

- $B_o$ = oil formation volume factor, vol/vol
- $c$ = fluid compressibility, vol/vol/psi
- $C_2$ = arbitrary constant, hr$^{-1}$
- $h$ = net pay thickness, ft
- $k$ = formation permeability, md
- $m_{wbs}$ = slope of wellbore storage dominated regime, psi/hr
- $N_p$ = cumulative oil production, vol
- $p$ = reservoir pressure, psia
- $p_{wfs(0)}$ = wellbore pressure at the time of shut-in, psia

$q$ = volumetric production rate, STB/D

$r$ = radial distance, ft

$s$ = skin factor, (dimensionless)

$u$ = Laplace variable

$t$ = producing time, hr

$\Delta t$ = shut-in time, hr

**Greek**

- $\gamma$ = Euler’s constant, 0.557216 …
- $\beta_i$ = "beta-deconvolution" variable, hr$^{-1}$
- $\mu$ = viscosity, cp
- $\rho$ = fluid density, lb/cu ft

**Subscripts**

- $a$ = after production period
- $d$ = "well-testing" pressure derivative
- $D$ = dimensionless quantity
- $f$ = pressure in the formation
- $i$ = initial reservoir conditions
- $n$ = index number
- $w$ = conditions at wellbore radius

**Superscripts**

- $'$ = derivative of a function.
- $i$ = integral of a function.

**References**


Appendix A — Russell method for "correction" of well test data distorted by wellbore storage (Russell, 1966)

The purpose of this Appendix is to summarize the work of Russell (1966) regarding an approximation to "correct" well test data distorted by wellbore storage. We begin by noting that this method does not provide results which can be considered useful in the context of modern well test analysis and interpretation methods.

As a starting point, we consider the well/reservoir configuration as defined by Russell for this case — schematic for this case is shown in Fig. A.1:

Russell made the following assumptions in the derivation of his wellbore storage "correction" solution:

— Completely penetrating well in an infinite reservoir.
— Slightly compressible liquid (constant compressibility).
— Constant fluid viscosity.
— Single-phase liquid flow in the reservoir.
— Gravity and capillary pressure neglected.
— Constant permeability.
— Horizontal radial flow (no vertical flow).
— Ideal gas (for the gas cushion in the well).

Although the Russell method was derived from analytical considerations, the problem actually solved is a variation of the true wellbore storage problem, derived using Russell's representation of the gas and liquid volume in the wellbore as the "wellbore storage" term. This formulation is not based on the same physics as the wellbore storage problem where the wellbore production (at the start of production or shut-in) is inversely proportional to the compressibility of the fluids in the wellbore (or the influence of a rising/falling liquid level).

In short, Russell (1966) approximated the wellbore storage concept in order to develop his "storage" function, presumably for the correction of wellbore storage distortion in pressure buildup tests. In field units, Russell's wellbore storage correction is given as:

\[
p_{WS}(\Delta t) - p_{wf}(\Delta t = 0) = 162.6 \frac{qB \mu}{kh} \left[ 1 - \frac{1}{C_2 \Delta t} \right] \left[ \log(\Delta t) + \log \frac{k}{\phi \mu \rho_{c} r_{w}^2} - 3.23 + 0.87 \right]
\]

Where the \( C_2 \)-term is defined as:

\[
C_2 = 0.00528 \frac{k}{r_{w}^2 \mu} \left[ \rho_g + \frac{1}{L} p_{wf}(\Delta t = 0) \right]
\]

Combining Eqs. A-1 and A-2 into a plotting function format, we obtain:

\[
\frac{[p_{WS}(\Delta t) - p_{wf}(\Delta t = 0)]}{\left[ 1 - \frac{1}{C_2 \Delta t} \right]} = f(\Delta t = 1 \text{ hr}) + m_{sl} \log(\Delta t)
\]

Russell treated the \( C_2 \)-term as an arbitrary constant to be optimized for analysis — in other words, the \( C_2 \)-term is the "correction" factor for the Russell method. As prescribed by Russell, the \( C_2 \)-term is obtained using a trial-and-error sequence which yields a straight line when the left-hand-side term of Eq. A-3 is plotted versus \( \log(\Delta t) \). Where the general form of the y-axis correction term prescribed by Eq. A-3 is:

\[
y = \left[ p_{WS}(\Delta t) - p_{wf}(\Delta t = 0) \right] \left[ 1 - \frac{1}{C_2 \Delta t} \right]
\]

A schematic of the Russell method is shown in Fig. A.2, where we note Russell's interpretation of the effect of the \( C_2 \)-term (i.e., where \( C_2 \) is too large and \( C_2 \) is too small).
And the skin factor can be estimated using:

\[ s = 1.151 \left[ \frac{f (\Delta t = 1 \text{ hr})}{m_{sl}} - \log \frac{k}{\phi \kappa c_m} + 3.23 \right] \quad \text{...(A-5)} \]

Russell (1966) also proposed a methodology to obtain the "extrapolated" pressure using the results of his correction procedure. We chose not to demonstrate this methodology, the interested reader is referred to Russell (1966) for more detail.

We present two example cases to demonstrate the shortcomings of the Russell method (lack of accuracy, limited range of application). The first example is for "Well B," an example taken from the original Russell reference [Russell (1966)]. The second example is taken from data in the reference paper by Meunier, et al. (1985).

Example 1: (Well B, Wilcox Sand formation) Russell presented the data and analysis for the "Well B" case as a "typical" example application of his wellbore storage correction method. We have reproduced this example and extended the results by presenting a large set of values for the \( C_2 \)-term to illustrate the influence of this term on the performance of the Russell correction.

For our reproduction of this case, we use \( C_2 = 2.0, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4, 3.8 \text{ hr}^{-1} \) in Eq. A-4, and we plot the results of this exercise on Fig. A.3. The value of the \( C_2 \)-term for which most of the points form a straight line (\( y \) versus \( \log (\Delta t) \)) is \( 2.8 \text{ hr}^{-1} \), and we obtain a straight-line slope (\( m_y \)) of about 70 psi/log cycle. A comparison of our results and those obtained by Russell is shown below.

<table>
<thead>
<tr>
<th>Conventional Analysis*</th>
<th>Russell Correction</th>
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</thead>
<tbody>
<tr>
<td>( p_{ws} ) versus ( \log (\Delta t) )</td>
<td>Eq. A-4 versus ( \log (\Delta t) )</td>
</tr>
<tr>
<td>Analysis ( m_{sl} ) (psi/log cycle)</td>
<td>Russell ( m_{sl} ) (psi/log cycle)</td>
</tr>
<tr>
<td>Russell (1966) 70</td>
<td>67 ( (C_2=3.0 \text{ hr}^{-1}) )</td>
</tr>
<tr>
<td>This Study 70</td>
<td>70 ( (C_2=2.8 \text{ hr}^{-1}) )</td>
</tr>
</tbody>
</table>

* Conventional analysis based on using the \( p_{ws} \) vs. \( \log (\Delta t) \) for data which are not affected by wellbore storage effects. The "conventional" straight-line trend is constructed using the data in the region of \( 10 < \Delta t < 40 \) hours.

As shown in Fig. A.3, our selection of \( C_2 = 2.8 \text{ hr}^{-1} \) as the approximate best fit value appears to be the case for which the Russell correction yields an apparent straight line trend. Russell (1966) noted that \( C_2 = 2.75 \text{ hr}^{-1} \) "might well have been chosen instead [of 3.0]."

Example 2: The following example is the field case given by Meunier et al., (1985). We have applied the Russell "correction" method in this example and used several values for the \( C_2 \)-term to illustrate the influence of this term on the performance of the Russell correction. We use \( C_2 = 9.0, 10.0, 11.0, 11.5, 11.9, 12.5, 13.5, 14.5 \text{ hr}^{-1} \) and we present our results in Fig. A.4. We obtained a slope value (\( m_{sl} \)) of about 53 psi/log cycle using the "best fit" value of the \( C_2 \)-term 11.9 hr

In the analysis of Meunier et al. (1985), value of the slope was reported as 57 psi/log cycle using the "sandface rate convolution" method.

Figure A.4 – Afterflow analysis, Meunier et al., (1985) data set
Approximate best fit obtained using \( C_2 = 11.9 \text{ hr}^{-1} \).

If we consider the performance of the Russell method objectively as applied to the data of Meunier et al. (1985), we would conclude that the "corrected" pressures (the symbols in Fig. A.4) are of little practical use. Obviously such data could not be used for pressure derivative analysis — even if we could accept the (very) approximate straight-line (i.e., the corrected data) such data would yield very erroneous pressure derivative profiles.

Appendix B — Derivation of the \( \beta \)-deconvolution formulation

We note that the lack of accuracy in flowrate measurements (when these exist) narrows the range of application of Gladfelter deconvolution method (i.e., rate normalization). Van Everdingen (1953) and Hurst (1953) (separately) introduced an exponential model for the sandface rate during the wellbore storage distortion period of a pressure transient test. The exponential formulation of the flowrate function is given as:

\[ q_D(t_D) = 1 - e^{-\beta_D} \quad \text{...(B-1)} \]

Eq. (B-1) is based on the empirical observations made by Van Everdingen and Hurst and as extended by others such as Kuchuk (1987) and Joseph and Koederitz (1982).
Recalling the convolution theorem, we have:

\[ p_{wD}(t_D) = \int_0^t q_D(\tau)p_{sD}(t_D - \tau) d\tau \]  \hspace{1cm} (B-2)

Taking the Laplace transform of Eq. B-2 yields:

\[ \overline{p}_{wD}(u) = u\overline{q}_D(u)\overline{p}_{sD}(u) \]  \hspace{1cm} (B-3)

Rearranging Eq. B-3 for the equivalent constant rate pressure drop function, \(\overline{p}_{sD}(u)\), we obtain:

\[ \overline{p}_{sD}(u) = \overline{p}_{wD}(u)\frac{1}{u\overline{q}_D(u)} \]  \hspace{1cm} (B-4)

The Laplace transform of the rate profile (Eq. B-1) is:

\[ \overline{q}_D(u) = \frac{1}{u} - \frac{1}{u + \beta} \]  \hspace{1cm} (B-5)

Substituting Eq. B-5 into Eq. B-4, and then taking the inverse Laplace transformation of this result yields the "beta" deconvolution formula:

\[ p_{sD}(t_D) = p_{wD}(t_D) + \frac{1}{\beta}\frac{dp_{wD}(t_D)}{dt_D} \]  \hspace{1cm} (B-6)

Where we note that Eq. (B-6) is specifically valid only for the exponential sandface flowrate profile given by Eq. B-1. This may present a serious limitation in terms of practical application of the \(\beta\)-deconvolution method.

To alleviate the issue of the exponential sandface flowrate, we propose that Eq. B-6 be solved for the \(\beta\)-term. Once this identity is established, we will then develop methods for estimating the \(\beta\)-term from data. After that we will use the identity (Eq. B-6) to estimate the pressure drop function for a constant production rate. Solving Eq. B-6 for the \(\beta\)-term, we have:

\[ \beta = \frac{1}{p_{sD}(t_D) - p_{wD}(t_D)} \frac{dp_{wD}(t_D)}{dt_D} \]  \hspace{1cm} (B-7)

Or, multiplying through Eq. B-7 by the \(C_D\)-term, we have

\[ \beta C_D = \frac{1}{p_{sD}(t_D) - p_{wD}(t_D)} C_D \frac{dp_{wD}(t_D)}{dt_D} \]  \hspace{1cm} (B-8)

Recalling the definition of the wellbore storage model, we have:

\[ q_D(t_D) = 1 - C_D \frac{dp_{wD}(t_D)}{dt_D} \]  \hspace{1cm} (B-9)

Assuming wellbore storage domination (i.e., \(q_D \approx 0\)) at early times, then Eq. B-9 becomes:

\[ C_D \frac{dp_{wD}(t_D)}{dt_D} \approx 1 \text{ (early time)} \]  \hspace{1cm} (B-10)

Separating and integrating Eq. B-10 (our early time, wellbore storage domination result), we have:

\[ p_{wD}(t_D) \approx \frac{t_D}{C_D} \text{ (early time)} \]  \hspace{1cm} (B-11)

Substituting Eqs. B10 and B-11 into Eq. B-8, we obtain:

\[ \beta C_D = \frac{1}{p_{sD}(t_D) - \frac{t_D}{C_D}} \text{ (early time)} \]  \hspace{1cm} (B-12)

Eq. B-12 suggests that we can "correlate" the \(\beta C_D\)-product with \(t_D/C_D\) — this observation becomes the basis for our use of these plotting functions to compare the \(\beta\)-deconvolution relations. The "master" plot of the \(\beta\)-deconvolution function for the case of a single well in an infinite-acting, homogeneous reservoir is derived using Eq. B-8 and is shown in Fig. B.1.

Appendix C — Derivation of the coefficients for \(\beta\)-deconvolution

\(\beta\)-Deconvolution — Derivative Approach: Although our stated goal is to develop a deconvolution approach which does not use the pressure derivative function, we can at least develop such a methodology as it may be of practical use in the future. Considering this problem only in terms of dimensionless solutions (and variables), we propose to use the derivative of the \(p_{wD}(t_D)\) function as a mechanism to compute the rate function (in our case the \(\beta(t_D)\) function from the van Everdingen (1953) and Hurst (1953) exponential approximation for sandface flowrate). Recalling this exponential rate model, we have:

\[ q_D(t_D) = 1 - e^{-\beta(t_D)t_D} \]  \hspace{1cm} (C-1)

Taking the time derivative of Eq. C-1 gives:

\[ \frac{dq_D}{dt_D} = \beta(t_D)e^{-\beta(t_D)t_D} \]  \hspace{1cm} (C-2)

Where the \(\beta(t_D)\)-term is defined as:

\[ \beta(t_D) = \beta(t_D) + \beta(t_D) \]  \hspace{1cm} (C-3)

Recalling the definition of the wellbore storage model, we have:

\[ q_D(t_D) = 1 - C_D \frac{dp_{wD}}{dt_D} \]  \hspace{1cm} (C-4)

Taking the time derivative of Eq. C-4 gives:

\[ \frac{dq_D}{dt_D} = -C_D \frac{dp_{wD}}{dt_D} = -C_Dp_{wD} \]  \hspace{1cm} (C-5)
Equating Eqs. C-2 and C-5 gives
\[ C_D p_{wD}''(t_D) = C_D \frac{d^2 p_{wD}}{dt_D^2} = -b(t_D)e^{-\beta(t_D)y_D} \] .......................... (C-6)

Equating Eqs. C-1 and C-4 gives
\[ e^{-\beta(t_D)y_D} = C_D \frac{dp_{wD}}{dt_D} = C_D p_{wD}(t_D) \] .......................... (C-7)

Combining Eqs. C-6 and C-7, and solving for \( b(t_D) \)
\[ b(t_D) = -\frac{p_{wD}}{p_{wD}} \]
\[ = -\frac{1}{t_D} \frac{p_{wD}}{p_{wD}} \]
\[ = \beta(t_D) + \beta'(t_D)y_D \] .......................... (C-8)

Where the \( p_{wD} \) and \( p_{wDd} \) terms are defined as:
\[ p_{wD} = t_D \frac{dp_{wD}}{dt_D} \] .......................... (C-9)
\[ p_{wDdd} = t_D^2 \frac{d^2 p_{wD}}{dt_D^2} \] .......................... (C-10)

We can use Eq. C-8 to determine \( \beta(t_D) \) and \( \beta'(t_D) \) — a graphical representation of this technique is shown in Fig. C.1.

**Figure C.1** – \( \beta \)-deconvolution via the derivative approach — \( \beta(t_D) \) and \( \beta'(t_D) \) determination technique.

The intercept and slope values \([\beta(t_D) \text{ and } \beta'(t_D)]\) could be approximated by numerical methods such as least squares — we do not suggest that this approach is functional, we simply present the details for possible use in the future.

**\( \beta \)-Deconvolution — Integral Approach:** In this case, we assume \( \beta(t_D) = \beta \) (constant) for the purposes of integration and differentiation. We will use integrals and integral-difference (derivative) functions to estimate \( \beta(t_D) \).

Recalling Eq. C-7, we have:
\[ C_D p_{wD}(t_D) = e^{-\beta(t_D)y_D} \] .......................... (C-11)

Assuming \( \beta(t_D) = \beta \) (constant), and integrating Eq. C-7 with respect to \( t_D \), we obtain
\[ C_D p_{wD}(t_D) = \frac{1}{\beta} \left[ 1 - e^{-\beta_D} \right] \] .......................... (C-12)

Integrating Eq. C-11 with respect to \( t_D \) yields
\[ C_D p_{wD}(t_D) = \frac{1}{\beta} \left[ \frac{1}{2} - e^{-\beta_D} \right] \] .......................... (C-13)

Where the \( p_{wD}(t_D) \) function is given by:
\[ p_{wD}(t_D) = \int_0^{t_D} p_{wD}(\tau)d\tau \] .......................... (C-14)

Substituting Eq. C-11 into Eq. C-12, we obtain
\[ C_D p_{wD}(t_D) = \frac{1}{\beta} \left[ \frac{1}{2} - C_D p_{wD}(t_D) \right] \] .......................... (C-15)

Dividing through Eq. C-14 by \( t_D \) gives
\[ C_D p_{wD}(t_D) = \frac{1}{\beta} \left[ \frac{1}{2} - C_D p_{wD}(t_D) \right] \] .......................... (C-16)

Where the \( p_{wD}(t_D) \) function in Eq. C-15 is given by:
\[ p_{wD}(t_D) = \int_0^{t_D} p_{wD}(\tau)d\tau \] .......................... (C-17)

Taking the derivative of Eq. C-15 with respect to \( t_D \) yields:
\[ C_D \frac{dp_{wD}}{dt_D} = \frac{1}{\beta} \left[ \frac{1}{2} - C_D p_{wD}(t_D) \right] \] .......................... (C-18)

Dividing through by \( C_D \), and multiplying both sides by \( t_D^2 \)
\[ p_{wD}(t_D) = \frac{1}{\beta} \left[ \frac{1}{2} - C_D p_{wD}(t_D) \right] \] .......................... (C-19)

Where the \( p_{wD}(t_D) \) function in Eq. C-18 is given by:
\[ p_{wD}(t_D) = \int_0^{t_D} p_{wD}(\tau)d\tau \] .......................... (C-20)

**Figure C.2** – \( \beta \)-deconvolution via the integral-derivative approach (approximation of \( \beta \) using Eq. C-20). (for wellbore storage effects in a single well in an infinite-acting, homogeneous reservoir).
Solving Eq. C-18 for \( \beta \) gives us:

\[
\beta(t_D) \approx \beta = \frac{1}{t_D} \left[ \frac{P_{WD}(t_D) - P_{WDd}(t_D)}{P_{WDid}(t_D)} \right] \quad \text{.......................... (C-20)}
\]

Where we assume \( \beta \approx \beta_D \). Eq. C-18 is compared to the analytical formulation for \( \beta \) (Eq. B-7) in Fig. C.2 \( (\beta_D \text{ versus } t_D/C_D) \) — and we note a very good correlation at "early" values of \( t_D/C_D \) (which is where wellbore storage effects are most important).

\[
\beta(t_D) \approx \beta = \frac{1}{t_D} \left[ \frac{P_{WD}(t_D) - P_{WDd}(t_D)}{P_{WDid}(t_D)} \right] \quad \text{.......................... (C-20)}
\]

Recasting Eq. C-20 into any consistent set of units, we have the following results for the "field units" form of the \( \beta \)-parameter, which we will express as \( \beta_f \). The result for \( \beta_f \) is.

\[
\beta_f = \frac{1}{t} \left( \frac{\Delta p_w - \Delta p_{wd}}{\Delta p_{wd}} \right) \quad \text{(pressure drawdown)} \quad \text{........... (C-21a)}
\]

\[
\beta_f = \frac{1}{\Delta t} \left( \frac{\Delta p_w - \Delta p_{wd}}{\Delta p_{wd}} \right) \quad \text{(pressure buildup)} \quad \text{........... (C-21b)}
\]

Where the \( \Delta p_w, \Delta p_{wd}, \Delta p_{wd}, \text{ and } \Delta p_{wd} \) functions are defined as:

\[
\Delta p_w = p_i - p_{wf} \quad \text{(pressure drawdown)} \quad \text{........... (C-22a)}
\]

\[
\Delta p_w = p_{ws} - p_{wf} (\Delta t = 0) \quad \text{(pressure buildup)} \quad \text{........... (C-22b)}
\]

\[
\Delta p_{wd} = \int_0^t \frac{d\Delta p_w}{dt} \quad \text{(pressure drawdown)} \quad \text{........... (C-23a)}
\]

\[
\Delta p_{wd} = \int_0^\Delta t \frac{d\Delta p_w}{d\Delta t} \quad \text{(pressure buildup)} \quad \text{........... (C-23b)}
\]

\[
\Delta p_{wf} = \frac{1}{\Delta t} \int_0^\Delta t \Delta p_{wd} d\tau \quad \text{(pressure drawdown)} \quad \text{........... (C-24a)}
\]

\[
\Delta p_{wf} = \frac{1}{\Delta t} \int_0^\Delta t \Delta p_{wd} d\tau \quad \text{(pressure buildup)} \quad \text{........... (C-24b)}
\]

\[
\Delta p_{wd} = \int_0^\Delta t \frac{d\Delta p_{wd}}{d\Delta t} dt \quad \text{(pressure drawdown)} \quad \text{........... (C-25a)}
\]

\[
\Delta p_{wd} = \int_0^\Delta t \frac{d\Delta p_{wd}}{d\Delta t} dt \quad \text{(pressure buildup)} \quad \text{........... (C-25b)}
\]

The \( \Delta p_i \) functions for the pressure drawdown and buildup cases are defined in field units as: (based on Eq. B-6)

\[
\Delta p_s = \Delta p_w + \frac{1}{\beta_f} \frac{d\Delta p_w}{dt} \quad \text{(pressure drawdown)} \quad \text{........... (C-26a)}
\]

\[
\Delta p_s = \Delta p_w + \frac{1}{\beta_f} \frac{d\Delta p_w}{d\Delta t} \quad \text{(pressure buildup)} \quad \text{........... (C-26b)}
\]

Substituting the for \( \beta_f \) definitions (Eqs. C-21a and C-21b) into the appropriate \( \Delta p_i \) functions (Eqs. C-26a and C-26b) gives the final "field" relation for \( \beta \)-deconvolution using the "integral-derivative" approach (a single relation is obtained for both the pressure drawdown and pressure buildup cases).

\[
\Delta p_s = \Delta p_w + \frac{1}{\beta_f} \frac{d\Delta p_w}{dt} \quad \text{.......................... (C-20)}
\]

or

\[
\Delta p_s = \Delta p_w + \frac{\Delta p_{wd}}{\Delta p_w - \Delta p_{wd}} \quad \text{........... (C-27)}
\]

**Appendix D — Material balance deconvolution relations for pressure drawdown and buildup test analysis (wellbore storage rate profiles).**

Material balance deconvolution is an extension of the rate normalization method. Johnston (1992) defines a new \( x \)-axis plotting function (material balance time) which provides an approximate deconvolution of the variable-rate pressure transient. There are numerous assumptions associated with the "material balance deconvolution" methods — one of the most widely accepted assumptions is that the rate profile must change smoothly and monotonically. In practical terms, this condition should be met for the wellbore storage problem.

The general form of material balance deconvolution is provided for the pressure drawdown case in terms of the material balance time function and the rate-normalized pressure drop function. The material balance time function is given as:

\[
t_{mb} = \frac{N_p}{q} \quad \text{.......................... (D-1)}
\]

The rate-normalized pressure drop function is given by:

\[
\frac{\Delta p}{q} = \frac{(p_i - p_{wf})}{q} \quad \text{.......................... (D-2)}
\]

The wellbore storage rate function for the pressure drawdown case, \( q_{wbs,DD} \), is given as:

\[
q_{wbs,DD} = 1 - \frac{d}{m_{wbs} d\Delta t} [\Delta p_{wf}] \quad \text{.......................... (D-3)}
\]

The wellbore storage rate function for the pressure buildup case, \( q_{wbs,BU} \), is given as:

\[
q_{wbs,BU} = \frac{1}{m_{wbs}} \frac{d}{d\Delta t} [\Delta p_{ws}] \quad \text{.......................... (D-4)}
\]

Where the wellbore storage "slope" is defined as:

\[
m_{wbs} = \frac{qB}{24C_s} \quad \text{.......................... (D-5)}
\]

And the pressure drop terms are defined as:

\[
\Delta p_{wf} = p_i - p_{wf} \quad \text{.......................... (D-6)}
\]

\[
\Delta p_{ws} = p_{ws} - p_{wf}(\Delta t = 0) \quad \text{.......................... (D-7)}
\]

The wellbore storage cumulative production function for the pressure drawdown case, \( N_{p,wbs,DD} \), is given as:

\[
N_{p,wbs,DD} = 1 - \frac{d}{m_{wbs} d\Delta t} [\Delta p_{wf}] \quad \text{.......................... (D-8)}
\]

The wellbore storage cumulative production function for the
The wellbore storage-based, rate-normalized pressure drop function for the pressure buildup case is given as:

$$\Delta p_{w, BU} = \frac{1}{1 - \frac{1}{m_{wbs}} \frac{d}{d\Delta t} \left[ \Delta p_{ws} \right]} \Delta p_{ws} \quad \text{(D-13)}$$

Plotting the rate-normalized pressure function versus the material balance time function (on log ($t_{mb}$) scales) shows that the material balance time function does correct the erroneous shift in the semilog straight-line obtained by rate normalization. We believe that the material balance deconvolution technique is a practical approach (perhaps the most practical approach) for the explicit deconvolution of pressure transient test data which are distorted by wellbore storage and skin effects.