A Quadratic Cumulative Production Model for the Material Balance of an Abnormally Pressured Gas Reservoir

F.E. Gonzales, D. Ilk, and T.A. Blasingame, SPE, Texas A&M U.

Abstract

The premise of this work is the concept, development, and application of an approximate relation for the material balance of abnormally pressured gas reservoirs. In particular, the proposed approximation is formulated directly from the rigorous material balance for the case of an abnormally pressured gas reservoir. Our "quadratic cumulative production" result is given by:

\[ \frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \left( \frac{1}{G} - \omega \right) G_p - \frac{\omega}{G} G_p^2 \right] \]

where \( \omega \) is defined as a function of \( G_p \) .......................................................... (1)

The \( \omega \)-function is defined by the "cumulative effective compressibility" \( [\tilde{\tau}_e(p)] \) and represents the influence of abnormal pressure. If we assume \( \omega \)-constant (but this does not mean that \( \tilde{\tau}_e(p) \) is constant), then we obtain the "quadratic" form:

\[ \frac{p}{z} \approx \frac{p_i}{z_i} - \alpha G_p - \beta G_p^2 \]

where \( \alpha = \frac{1}{G} - \omega \) and \( \beta = \frac{\omega}{G} \) .................................................................................. (2)

Eq. 2 is the base definition of the "quadratic cumulative production" relation (\( \omega \) is constant; therefore, \( \alpha \) and \( \beta \) are constant). We have shown this relation to be an extraordinarily accurate approximation of the rigorous material balance for the case of a dry gas reservoir with abnormal pressure effects.

Eq. 2 is suited not only for use as a characteristic model, but also for use as a data analysis mechanism (i.e., in this work we use Eq. 2 to develop a suite of analysis plots, plotting functions, a type curve, etc.). We also address the case of \( \omega \) being a linear function of \( G_p \) (as opposed to \( \omega \) being constant) — which results in a formulation where \( p/z \) is cubic in terms of \( G_p \).

In this work, we provide the following new results:

- The "quadratic cumulative production" model for the material balance behavior of abnormally pressured gas reservoir.
- A suite of 6 (six) plotting functions based on the \( p/z - G_p \) material balance model.
- A suite of 4 (four) \( \omega - G_p \) performance plots which are used to calibrate the analysis process.
- A new type curve \( p_{\omega} = [p/z; p/z; p/z] \) versus \( G_{\omega} = [G_p/G] \), based on the new \( p/z - G_p \) material balance model.

Objectives

The primary objectives of this work are:

- To develop a quadratic formulation of the rigorous material balance for the case of an abnormally pressured gas reservoir in terms of cumulative gas production (derivation is given in Appendix A).
- To develop plotting functions for the analysis of reservoir performance behavior based on the quadratic cumulative production formulation of the rigorous material balance for the case of an abnormally pressured gas reservoir. We note that we use 10 (ten) specific data plotting functions as part of this work — others are available, but we favor the 10 functions selected due to consistency and data representation/visualization that these functions provide. These functions are derived in Appendix B.
- To develop and validate a dimensionless "type curve" solution based on Eq. 1 and an auxiliary function (i.e., the so-called "pressure integral" of Eq. 1 based on \( G_p \)) (Appendix C).
- To validate and demonstrate the plotting functions and associated analysis relations using simulated reservoir performance cases, as well as various field performance cases.
Introduction

The rigorous material balance for the case of an abnormally pressured gas reservoir was developed by Fetkovich et al. (Fetkovich et al. 1998) and is given as:

\[
\frac{P}{\varepsilon} [1 - \bar{\epsilon}(p)(p_i - p)] = \frac{P_i}{z_i} \frac{1}{G} \left[ G_p - G_{inj} + W_p R_{sw} + \frac{5.615}{B_g} (W_p B_w - W_{inj} B_w - W_e) \right] \tag{3}
\]

Where Fetkovich et al. define the "effective compressibility" function, \(\bar{\epsilon}(p)\), as:

\[
\bar{\epsilon}(p) = \frac{1}{1 - S_{wi}} \left[ S_{wi} \bar{\epsilon}_w + \bar{\epsilon}_f + M (\bar{\epsilon}_w + \bar{\epsilon}_f) \right] \tag{4}
\]

The "nonpay/aquifer contribution ratio" (M) is defined by:

\[
M = \frac{V_{pNNP} + V_{pAQ}}{V_{pR}} \tag{5}
\]

For our work (and in general for the case of an abnormally pressured gas reservoir), M is assumed to be negligible. We recommend that formulations which include the M-parameter should be developed and applied only for the case where it is strongly believed that a "nonpay" or aquifer contribution of energy exists.

We will generally assume the cumulative water compressibility term (\(\bar{\epsilon}_w\)) to be constant, but we also acknowledge that there is no real loss of generality to retain a pressure-dependent relation for the cumulative water compressibility term. In contrast, we will generally consider that the cumulative formation compressibility (\(\bar{\epsilon}_f\)) is pressure-dependent, and we will attempt to estimate (\(\bar{\epsilon}_f\)) using Eq. 4.

Considering the case where \(G_{inj} = W_{inj} = W_p = W_e = 0\), we obtain the common form of the (dry) gas material balance relation for the case of "abnormal pressure" effects. This result is given as:

\[
\frac{P}{\varepsilon} [1 - \bar{\epsilon}(p)(p_i - p)] = \frac{P_i}{z_i} \left[ 1 - \frac{G_p}{G} \right] \tag{6}
\]

Gan and Blasingame (Gan and Blasingame 2001) utilized Eq. 6 to develop a sequence of spreadsheet-based analyses for estimating the gas-in-place, \(G\), as well as the pore volume compressibility function, \(\varepsilon\). The premise of the Gan and Blasingame approach is that two linear trends are often observed on a plot of \(p/z\) versus \(G_p\) for the case of an abnormally pressured gas reservoir — the first trend is the "abnormal" pressure trend, and the second is the "normal" pressure (or depletion) trend.

The first trend (i.e., the "abnormal" pressure trend) is given by:

\[
\frac{p}{z} = \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G_{app}} \right] \tag{7}
\]

The second trend (i.e., the "normal" pressure (or depletion) trend) is given as:

\[
p/z = \frac{(p/z)_A}{1 - G_p/G} \left[ 1 - \frac{G_p}{G} \right] \tag{8}
\]

A schematic plot of \(p/z\) versus \(G_p\) for the case of an abnormally pressured gas reservoir is shown in Fig. 1.

Gan and Blasingame applied this methodology to several cases of simulated reservoir performance, and as many field cases that could be found in the literature or from industry sources. The proposed methodology was shown to be robust and accurate for virtually all cases. The limitation of this approach (and of all existing analyses for abnormally pressured gas reservoirs) is that the only indication of "abnormal pressure" behavior is the observed decline in the \(p/z\) versus \(G_p\) performance from an apparent linear trend. In other words, no methodology exists in practice which can be used to verify the influence of abnormal pressure prior to some indication in \(p/z\) versus \(G_p\) performance.

We do note that the Moran and Samaniego (Moran and Samaniego 2000) approach — i.e., the use of the \(d(p/z)/dG_p\) function does hold some utility in being able to distinguish "normal" and "abnormal" pressure behavior uniquely — however, this method is not well suited to field use due to the behavior of the \(d(p/z)/dG_p\) function derived from field performance data.
It can be argued that the magnitude of reservoir pressure compared to the hydrostatic gradient can indicate abnormal pressure behavior [Prasad and Rogers (1987)] — however, predicting the onset of "normal pressure" behavior is not possible based solely on $p/z$ versus $G_p$ performance. Gan and Blasingame did propose a series of diagnostic checks to establish the existence of abnormal pressure effects, as well as provide an approximate correlation for the onset of "normal pressure" behavior. However, these are simply supplemental mechanisms to augment the proposed "two straight line $p/z$ analysis."

Our remaining discussions of material balance models will simply address other methods that have been proposed in recent times, and give a brief perspective on the utility of such methods. Yale et al [Yale et al (1993)] proposed a modified formulation of a material balance that is analogous in approach to that of Fetkovich et al — although Yale et al used a formulation in terms of formation volume factors to represent the various energy components, whereas Fetkovich et al use the "cumulative compressibility" approach.

We recommend the Fetkovich et al formulation — but we note that Yale et al also provide a significant body of data concerning the estimation (and correlation) of the instantaneous formation compressibility, $c_f$. This is a major contribution, and this work should not be overlooked. Yale et al produced a "type curve" for formation compressibility that is shown in Fig. 2. We believed that this work could help to orient analysis in the case of abnormally pressured gas reservoirs.
Ambastha [Ambastha (1993)] proposes and validates a "type curve" concept for a \( p_D \) (dimensionless \( p/z \) function) versus \( G_p \) (dimensionless \( G_p \) function) for the case of an abnormally pressured gas reservoir. This is a significant innovation — unfortunately, the format of the type curve causes the data to be skewed to a relatively perspective view. We have proposed an alternative type curve in this present work, and we recommend our format as it provides better resolution of the model and data functions.

As noted above, Moran and Samaniego provide an innovative and rigorous approach for the analysis of \( p/z-G_p \) performance which utilizes the concept of \( \frac{dp(z)}{dG_p} \) (and other derivative functions). This work could (and probably should) be seen as a breakthrough analysis technique — it is proposed as an analog to derivative analyses used in well testing and the theoretical aspects of this approach are well-founded. Unfortunately, the quality of \( p/z \) data is almost always inadequate for such analyses — and added to this issue that of data quantity (typically less than 10 \( p/z-G_p \) points are available for a given reservoir). In that light, the Moran and Samaniego method becomes an approach that is theoretically sound, but impractical for most field applications.

The motivation for the present work was the recognition that the Moran and Samaniego approach has the ability to provide "early" insight into "abnormal pressure" effects. We note that the Gan and Blasingame approach, while useful in concept and application, could be improved upon given a single (simple) model function (as opposed to using two models (i.e., the abnormal and normal pressure straight-line \( p/z \) versus \( G_p \) trends)). Gan and Blasingame do provide a single model which uses the unit-step function as switch (triggered by the \( p/z \) inflection value \( (p/z)_{\log} \)) — however, this model is empirical in development and application and we only reference its existence for completeness.

Given these motivations, we proceeded to develop the general \( p/z-G_p \) approximation as well as the "quadratic" \( (p/z-G_p^2) \) and "cubic" \( (p/z-G_p^3) \) approximations. The major results of this development are summarized below, and the details of this development are provided in Appendix A. We provide the development of an approximate formulation of the rigorous material balance for the case of an abnormally-pressured gas reservoir in terms of cumulative gas production and an auxiliary function \( \omega \). In short, we condense the contribution of the abnormal pressure component into the \( \omega \)-function.

As noted earlier in this work, we use Eq. 2 (i.e., the quadratic cumulative production model) as a basis to develop plotting functions which are used for the analysis of \( p/z \) versus \( G_p \) data. The development of these plotting functions is discussed in the next section.

**Development of the Quadratic Cumulative Production Model for the Material Balance of an Abnormally Pressured Gas Reservoir**

**Model Development:** The most relevant issue to consider regarding the validity of this work is that we have utilized the Fetkovich *et al* material balance formulation and we have established an approximating condition that permits us to formulate an explicit, closed form approximation to the Fetkovich *et al* material balance in terms of \( p/z \) and \( G_p \).

We have systematically established the stated approximating condition, and while our simplified material balance model may not be considered exact, we will show that the approximating condition is essentially universal (i.e., it was shown to be valid for every case we considered). Equally important is the observation that our new simplified material balance relation gave the correct estimates of gas-in-place for every case considered — and the model was shown to be tuned to performance data using at stages as small as 5-10 percent depletion (i.e., \( G_p/G<0.10 \)).

Our starting point is the Fetkovich *et al* material balance formulation where \( G_{n}=W_{in}=W_i=W_e=0 \), and we only consider the effect of the cumulative effective compressibility function, \( \bar{\tau}_c(p) \) — this form is given as:

\[
\frac{p}{z}[1-\bar{\tau}_c(p)(p_i-p)] = \frac{p_i}{z_i} \left[ 1-\frac{G_p}{G} \right] \tag{6}
\]

*Defining the \( \omega \)-function in terms of the \( \bar{\tau}_c(p) \) function, we have*

\[
\omega G_p = \bar{\tau}_c(p)(p_i-p) \tag{9}
\]

*Isolating the \( \omega \)-function, we have*

\[
\omega = \frac{1}{G_p} \bar{\tau}_c(p)(p_i-p) \tag{10}
\]

Alternatively, we can also define the \( \omega \)-function using the \( p/z, p/z_0, G, \) and \( G_p \) terms as given in Eq. 6, this effort yields

\[
\omega = \frac{1}{G_p} - \frac{p_i}{p/z} \left[ \frac{1}{G_p} - \frac{1}{G} \right] \tag{11}
\]

Using the form of the \( \omega \)-function given by Eq. 11, and assuming that we knew the gas-in-place \( (G) \) as well as the other variables, we could use Eq. 11 to establish if abnormal pressure behavior existed in this system (i.e., if \( \omega \neq 0 \)). In simple terms, Eq. 11 provides us a *mechanism* to estimate \( \omega \) and to evaluate the influence of abnormal pressure effects. We have
defined the \( \omega \)-function as a mechanism to assess the abnormal pressure behavior if the reservoir system. In Appendix A we provide a complete development and discussion of the rationale we use to establish the character of the \( \omega \)-function and the \( \bar{c}_e(p) (p_i - p) \) product.

In this section we will only focus on the behavior of the \( \omega \)-function relative to the validation of Eq. 2. As such, we present plots of \( \omega \) versus \( G_j/G \) for two sample cases. We have chosen to plot \( \omega \) versus \( G_j/G \) (rather than \( G \)) so that we can establish the validity of the \( \omega \) profile as a function of the depletion in the reservoir (i.e., \( G_j/G \)).

The first case is "Case 1," a numerical simulation case using an input \( c_f \) profile obtained from Fetkovich et al [see Gonzalez (2003)]. The second case is "Case 2," the classic literature example for an abnormally pressured gas reservoir — the "Anderson L" case [Duggan (1971)]. In Fig. 3 we present a plot of the \( \omega \) versus \( G_j/G \) profile for "Case 1" (the numerical simulation case). We note development of a clear linear trend in the \( \omega \) versus \( G_j/G \) data function — validating our hypothesis to some degree. We have constructed model trends for the \( \omega \)=constant and the \( \omega \)=linear cases — and both model trends appear to be relevant. We have elected to place the model trend for the \( \omega \)=constant case in the middle of the \( \omega \) versus \( G_j/G \) trend. This appears to be a good balance and suggests that our concept (i.e., \( \omega \approx \) constant) is reasonable — particularly considering application to field data.

The second case is "Case 2," the Anderson L field data case (South Texas, USA) [Duggan (1971)]. For this case, we present a plot of the \( \omega \) versus \( G_j/G \) in Fig. 4. In this case we prefer to place the constant \( \omega \) trend towards the top of the \( \omega \) versus \( G_j/G \) data distribution; as this placement represents a balance of the analysis on this particular plot, as well as a match of the various data functions for this case using the other data plots which are being simultaneously matched (the other plots are not shown, but all analyses are performed simultaneously in a spreadsheet). We also note that the apparent linear trend of \( \omega \) versus \( G_j/G \) is both reasonable and consistent. Our primary concern is the viability of this analysis plot for field data, and, at least in this particular case, we would say that the \( \omega \)—\( G_j/G \) methodology is both sound and accurate.

\[ \omega = \frac{1}{G_p} - \left[ \frac{\rho}{\rho_i} \right] \left[ \frac{1}{G_p} \right] - \frac{1}{G} \]

\[ \omega = 0.000058 - 0.0005491 \left( \frac{G_p}{G} \right) \left[ \frac{1}{BSCF} \right] \]

\[ \omega \approx 0.000042 \left[ \frac{1}{BSCF} \right] \] (estimate)

Figure 3 — Behavior of the \( \omega \)-function versus \( G_j/G \) for a simulated dry gas reservoir case. Note that in this case the maximum depletion of the model is approximately 70 percent — which justifies the (relatively) poor agreement of the \( \omega \)constant model with the data trend.

The "proof" of Eq. 2 (i.e., the quadratic cumulative production model) lies in the validity of the constant \( \omega \)-value as determined from the previous analysis (i.e., Figs. 3 and 4). Substituting \( \omega = 0.00042 \) 1/BSCF for "Case 1" into Eq. 2, we obtain the \( p/\rho \) versus \( G_p \) profile shown in Fig. 5. Similarly, for "Case 2" (Anderson L Field) we substitute \( \omega = 0.00529 \) 1/BSCF into Eq. 2 and generate the profile shown in Fig. 6. We note an extraordinarily accurate fit of the data trends in both Figs. 5 and 6, which we believe validates our concept for the \( p/\rho \)—\( G_p^2 \) material balance relation (i.e., Eq. 2).
Figure 4 — Behavior of the \( \omega \)-function versus \( G_p/G \) for the “Anderson L” field case example [Duggan, (1971)] (South Texas, USA). Note that this case shows an apparent depletion of about 50 percent — this is a possible explanation for the reasonably good correlation of data with both the constant and linear-\( \omega \) models.

Figure 5 — \( p/z \) versus \( G_p \) plot for the simulated performance of an abnormally pressured gas reservoir (variable compressibility only) (dry gas reservoir case).
In summary, we have established a group of diagnostic plots based on the characteristics of the effective compressibility-pressure drop term, \( \bar{\varepsilon}_c(p)(p_1 - p) \) (defined using Eq. 6), as well as the \( \omega \)-function (defined using Eq. 11). We recognize that there may be a slight bit of confusion between the new \( \omega \)-function and the \( \bar{\varepsilon}_c(p)(p_1 - p) \) function (we recall that \( \omega = (1/G_p)\bar{\varepsilon}_c(p)(p_1 - p) \) — however, we believe it is necessary to maintain the problem in terms of the original Fetkovich et al variable (i.e., \( \bar{\varepsilon}_c(p)(p_1 - p) \)) for the purpose of establishing the validity of Eq. 2 (see Appendix A for specific details). Further, we also believe it is necessary to establish the \( \omega \)-function as an independent variable for our approximate material balance relation which represents the case of a reservoir that exhibits abnormal pressure effects.

One purpose of using the \( \omega \)-function is to create a sequence of diagnostic plots which can be used to validate the existence of abnormal pressure effects in reservoir performance data. As a reference, we present the following list of "diagnostic plots" developed in this work based on the \( \bar{\varepsilon}_c(p)(p_1 - p) \) function or the \( \omega \)-function:

- Log-log plot: \( \bar{\varepsilon}_c(p)(p_1 - p) \) vs. \( G/J \) (power law trend)
- Cartesian plot: \( 1/(1 - \bar{\varepsilon}_c(p)(p_1 - p)) \) vs. \( G/J \) (linear trend)
- Cartesian plot: \( \omega \) vs. \( G_p \) (\( \omega = \) constant or \( \omega(G) \) linear)
- Cartesian plot: \( \omega \) vs. \( G/J \) (\( \omega = \) constant or \( \omega(G,J) \) linear)

**Plotting Functions for Data Analysis:** In this section we focus on the presentation and implementation of plotting functions developed using Eq. 2 (the specific plotting functions are developed in Appendix B, in full detail). We present an inventory of the plotting functions (PF) we have selected to use in this work in Table 1.

Each of these 6 (six) plotting functions (PF1-PF6) is used to provide unique insight into the character of the basis function (Eq. 2) — we present an illustrative example of these functions in the next section.

It is tempting to presume that we could extend the "quadratic" plotting functions (PF1, PF3, and PF5) to some other format and that these functions could yield independent estimates of gas-in-place. We have not pursued this effort (nor have we pursued independent analysis of the "linear" plotting function (PF2, PF4, and PF6)) — primarily because our "analysis" goal is to achieve a consistent estimate of gas-in-place for all of the data functions (including the diagnostic plots, the new \( p_{pi}-G_{pi} \) type curve, and the \( p/z-G_p \) summary plots). We achieve this goal by performing a dynamic, simultaneous match of all data functions in a spreadsheet program — using a single set of control parameters.
In Fig 7 (PF₁) we note that the "quadratic" characteristic behavior of PF₁ is clearly evident — we also note excellent agreement in the data and model functions. Similarly, in Fig 8 (PF₂) we find that PF₂ does exhibit the expected linear trend against Gₚ — however, we also note a characteristic oscillation of the data about the linear model trend. We will note that we believe that this oscillation in the data function is likely to be a legitimate feature of this numerical simulation data — we do not believe this behavior to be an artifact. Recall that our base model (Eq. 2) is approximate, and this oscillation merely proves that our model (and its related plotting functions) is approximate.

<table>
<thead>
<tr>
<th>Plotting Functions (PF)</th>
<th>Character</th>
<th>Fig.</th>
</tr>
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<tbody>
<tr>
<td>1. Δ(p/z) vs. G_p</td>
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<tr>
<td>2. Δ(p/z)/G_p vs. G_p</td>
<td>Linear</td>
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<tr>
<td>3. [ \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p ] vs. G_p</td>
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</tr>
<tr>
<td>4. [ \frac{1}{G_p^2} \int_0^{G_p} \Delta(p/z) dG_p ] vs. G_p</td>
<td>Linear</td>
<td>10</td>
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<tr>
<td>5. Δ(p/z) - [ \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p ] vs. G_p</td>
<td>Quadratic</td>
<td>11</td>
</tr>
<tr>
<td>6. [ \frac{1}{G_p} \int \Delta(p/z) dG_p ] vs. G_p</td>
<td>Linear</td>
<td>12</td>
</tr>
</tbody>
</table>

On the other hand, based on extensive application to field data [Gonzalez, (2003)], we can state that essentially none of the field data cases exhibit such oscillations (we only observe random variations about the trend as one might expect from field data). We present (PF₃) in Fig. 9 and we note very good agreement of the data and model functions. Similarly, we note in Fig. 10 (PF₄) the same behavior that we observed in Fig. 8 (i.e., oscillation of the data functions about a linear model trend).

Case 1 — Simulated Dry Gas Reservoir

Quadratic G_p Material Balance Relation for Abnormally Pressured Gas Reservoirs

Presentation Plot: Δ(p/z) — G_p Format

![Plot of Δ(p/z) vs. G_p (PF₁) — Case 1.](image-url)
Case 1 — Simulated Dry Gas Reservoir

Quadratic $G_p$ Material Balance Relation for Abnormally Pressured Gas Reservoirs

Presentation Plot: $\Delta(p/z)/G_p$ — $G_p$ Format

Governing Relation:
$$\Delta(p/z)/G_p = \alpha G_p + \beta G_p^2$$

Model Trend:
$$\Delta(p/z)/G_p = 7.523 \text{ [psi/BSCF]} + 0.7123 \text{ [psi/BSCF]^2} G_p \text{ [BSCF]}$$

Legend: Case 1 — Simulated Dry Gas Reservoir
- $\Delta(p/z)/G_p$ Data ($p_i = 7255.42 \text{ psia}$)
- $\Delta(p/z)/G_p$ Model (Quadratic $G_p$ form)
Gas-in-Place, $G = 686.4 \text{ BSCF}$ (input)

Figure 8 — Plot of $\Delta(p/z)/G_p$ vs. $G_p (PF_2)$ — Case 1.

Case 1 — Simulated Dry Gas Reservoir

Quadratic $G_p$ Material Balance Relation for Abnormally Pressured Gas Reservoirs

Presentation Plot: $\int[\Delta(p/z), G_p]/G_p$ — $G_p$ Format

Governing Relation:
$$\int[\Delta(p/z), G_p]/G_p = (1/2)\alpha G_p + (1/3)\beta G_p^2$$

Model Trend:
$$\int[\Delta(p/z), G_p]/G_p$$
$$= (1/2)7.523 \text{ [psi/BSCF]} G_p \text{ [BSCF]}$$
$$+ (1/3)0.7123 \text{ [psi/BSCF]^2} G_p^2 \text{ [BSCF]^2}$$

Legend: Case 1 — Simulated Dry Gas Reservoir
- $\int[\Delta(p/z), G_p]/G_p$ Data ($p_i = 7255.42 \text{ psia}$)
- $\int[\Delta(p/z), G_p]/G_p$ Model (Quadratic $G_p$ form)
Gas-in-Place, $G = 686.4 \text{ BSCF}$ (input)

Figure 9 — Plot of $\int_0^{G_p} \Delta(p/z) dG_p$ vs. $G_p (PF_3)$ — Case 1.
Figure 10 — Plot of \( \frac{1}{G_p} \int_0^{G_p} \Delta p(z) dG_p \) vs. \( G_p (PF_4) \) — Case 1.

Figure 11 — Plot of \( \Delta (p/z) - \frac{1}{G_p} \int_0^{G_p} \Delta p(z) dG_p \) vs. \( G_p (PF_5) \) — Case 1.
In Fig. 11 (PF5) we note good agreement of the model and data, and we will comment that this is a "difference function," as such we would expect that any minor deviations observed in previous plotting functions may be exaggerated (as we note in the middle of the quadratic trend). Likewise, in Fig. 12 (PF6) we observe a reasonable match of the (oscillating) data function about the linear model trend — as in Fig. 11, (PF6) is also a "difference function" and we would expect some exaggeration of features observed on the plotting functions presented earlier.

We again comment that all of the analyses shown in this sequence (i.e., the generation of the model function) were performed as a simultaneous match of all plots using control values of the gas-in-place and other parameters (we also note that the gas-in-place parameter dominates the analysis).

New Type Curve for Material Balance Analysis: In Appendix C we develop and present a new type curve solution that includes the dry gas material balance (no abnormal pressure effects), as well as the case of a gas reservoir with abnormal pressure effects as represented by Eq. 2.

We note that Eq. 2 is also valid for the case of no abnormal pressure effects (i.e., ω=0). Beginning with Eq. 2, we have

$$\frac{p}{z} \approx \frac{p_i}{z_i} - \alpha G_p - \beta G_p^2$$

where $\alpha = \frac{1}{G} - \omega$ and $\beta = \frac{\omega}{G}$ .................................................................................. (2)

In developing a "type curve" solution we must resort to "dimensionless" variables — therefore, we employ the following dimensionless variables in this effort:

$$\omega_D = \omega G$$ .................................................................................. (12)

$$p_D = \left[1 - \frac{p/z}{p_i/z_i}\right] = \left[\frac{p_i/z_i - p/z}{p_i/z_i}\right]$$ .................................................................................. (13)

$$G_{pD} = \frac{G_p}{G}$$ .................................................................................. (14)
Substituting Eqs. 12, 13, and 14 into Eq. 2, we obtain the "dimensionless" form of the "quadratic cumulative production" material balance relation for an abnormally pressured gas reservoir:

\[ p_D = (1 - \omega_D)G_{pD} + \omega_D G_{pD}^2 \]

Defining the \( p_{Di} = \frac{1}{G_{pD}} \int_0^{G_{pD}} p_D dG_{pD} \) "dimensionless pressure integral" function — and substituting Eq. 15 into this definition yields:

\[ p_{Di} = (1 - \omega_D) \frac{1}{2} G_{pD} + \frac{1}{3} G_{pD}^2 \]

We present the "abnormal pressure gas material balance type curve" in Fig. 13, where we note that we have used both the "dimensionless" pressure and pressure integral functions, \( p_D \) and \( p_{Di} \) (respectively) on this type curve.

Figure 13 — \( p_D \) and \( p_{Di} \) versus \( G_{pD} \) "type curve" plot for the "quadratic cumulative production" material balance relation for an abnormally pressured gas reservoir.
Reviewing Fig. 13 we note that our definition of \( p_D \) (and \( p_{Di} \)) has yielded very good model trends — and we suggest that this type curve should be both straightforward and consistent as a data analysis tool. In this work we have not employed Fig. 13 as an independent data analysis tool, but rather, as a component in the dynamic matching methodology we implemented in a spreadsheet program.

It is worth noting that we did use the type curve as a "start-up" analysis mechanism in our implementation (i.e., we used this plot to start or orient our analysis), and we also used Fig. 13 routinely as a "check" for our other analyses (\( \omega_D \) is estimated independently on this plot, regardless of how the plot is implemented for analysis). In summary, the type curve plot has been shown to be extremely valuable in the sequence for our analysis and interpretation of abnormally pressured gas reservoirs.

As a final comment, we will note that, on occasion, the \( p_D \) (and \( p_{Di} \)) data function at small values of \( G_{np} \) (maybe the first 1-2 points) appears to "curl" slightly (generally upwards). Based on our definition of the \( p_D \) variable (Eq. 14), this feature appears to be related to an incorrect estimate of initial reservoir pressure (\( p_i \)), which yields an incorrect estimate of \( p_i/z_i \). This "early data" issue was seldom significant, and the remainder of any given data set tended to match the type curve extremely well.

We are reluctant to recommend that the \( p_i/z_i \) estimate be corrected based on the curl of the \( p_D \) — \( G_{np} \) data function at small values of \( G_{np} \) because such actions may propagate through the remainder of the analyses. We will state that we believe that it is both appropriate and prudent that such corrections be made — however; we will leave such decisions to those who apply this methodology — with our noted caveat that any "correction" of \( p_i/z_i \) will affect the other analyses used in our methodology.

**Example Analysis of Field Data — Anderson L Reservoir (Case 2):**

We note that all of the analyses for this case were obtained exactly as described above — a set of reservoir parameters was optimized using a dynamic/simultaneous analysis of the entire dataset within a spreadsheet computer program. As we have stated earlier in this work, we do not recommend that this analysis sequence by automated in any fashion — user input and control are critical as a regression algorithm may lead to pursue solutions which are non-optimal at best, or physically inconsistent at worst.

Our primary goal in this presentation of results for the "Anderson L" Reservoir case is to demonstrate conclusively that underlying "quadratic cumulative production" material balance approximation is accurate and robust — we believe this is clearly proven by the evidence presented for this case.

**Figure 14** — Base plot of \( p/z \) vs. \( G_p \) — Case 2.
Case 2 — Anderson L (SPE 02938)

Quadratic $G_p$ Material Balance Relation for Abnormally Pressured Gas Reservoirs

Presentation Plot: $\Delta(p/z) - G_p$ Format

Figure 15 — Plot of $\Delta(p/z)$ vs. $G_p (P_F)$ — Case 2.

Figure 16 — Plot of $\Delta(p/z)/G_p$ vs. $G_p (P_F)$ — Case 2.
Case 2 — Anderson L (SPE 02938)

Quadratic $G_p$ Material Balance Relation for Abnormally Pressured Gas Reservoirs

Presentation Plot: Integral[$\Delta(p/z), G_p] / G_p$ — $G_p$ Format

Governing Relation:
\[
\text{Integral}[\Delta(p/z), G_p] / G_p = (1/2)\alpha G_p + (1/3)\beta G_p^2
\]

Model Trend:
\[
\text{Integral}[\Delta(p/z), G_p] / G_p = (1/2) 54.8992 \text{ [psi/BCSF]} G_p \text{ [BCSF]} \\
+ (1/3) 0.0071 \text{ [psi/BCSF²]} G_p^2 \text{ [BCSF²]}
\]

Legend: Case 2 — Anderson L (SPE 02938)

- Integral[$\Delta(p/z), G_p] / G_p$ Data ($p_i = 6602.08$ psia)
- Integral[$\Delta(p/z), G_p] / G_p$ Model (Quadratic $G_p$ form)

Gas-in-Place, $G = 73.5$ BSCF

Figure 17 — Plot of \( \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p \) vs. $G_p (P_i)$ — Case 2.

Case 2 — Anderson L (SPE 02938)

Quadratic $G_p$ Material Balance Relation for Abnormally Pressured Gas Reservoirs

Presentation Plot: Integral[$\Delta(p/z), G_p] / G_p^2$ — $G_p$ Format

Governing Relation:
\[
\text{Integral}[\Delta(p/z), G_p] / G_p^2 = (1/2)\alpha + (1/3)\beta G_p
\]

Model Trend:
\[
\text{Integral}[\Delta(p/z), G_p] / G_p^2 = (1/2) 54.8992 \text{ [psi/BCSF]} G_p \text{ [BCSF]} \\
+ (1/3) 0.0071 \text{ [psi/BCSF²]} G_p^2 \text{ [BCSF²]}
\]

Legend: Case 2 — Anderson L (SPE 02938)

- Integral[$\Delta(p/z), G_p] / G_p^2$ Data ($p_i = 6602.08$ psia)
- Integral[$\Delta(p/z), G_p] / G_p^2$ Model (Quadratic $G_p$ form)

Gas-in-Place, $G = 73.5$ BSCF

Figure 18 — Plot of \( \frac{1}{G_p^2} \int_0^{G_p} \Delta(p/z) dG_p \) vs. $G_p (P_i)$ — Case 2.
Figure 19 — Plot of \( \Delta(p/z) - \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p \) vs. \( G_p \) — Case 2.

Figure 20 — Plot of \( \frac{1}{G_p} \left[ \Delta(p/z) - \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p \right] \) vs. \( G_p \) — Case 2.
Case 2 — Anderson L (SPE 02938)
Validation of New Material Balance Concept for Abnormally Pressured Gas Reservoirs (Variable Compressibility Only)
Validation Plot for the $c_e(p)(p_f-p)$ Function

Figure 21 — Plot of $c_e(p)(p_i - p)$ vs. $G_p/G$ (validation plot) — Case 2.

Figure 22 — Plot of $1/(1 - c_e(p)(p_f - p))$ vs. $G_p/G$ (validation plot) — Case 2.
Case 2 — Anderson L (SPE 02938)
Validation of New Material Balance Concept for Abnormally Pressured Gas Reservoirs (Variable Compressibility Only)
Validation Plot ($G_p$ Format) for the $\omega$-Parameter

**Legend:**
- Case 2 — Anderson L (SPE 02938)
- $\omega$-Parameter calculated from MBE
- $\omega$-Parameter — constant
- $\omega$-Parameter — linear form in $G_p$

Gas-in-Place, $G = 73.5$ BSCF

Figure 23 — Plot of $\omega$ vs. $G_p$ (validation plot) — Case 2.

Figure 24 — Plot of $\omega$ vs. $G_p/G$ (validation plot) — Case 2.
Case 2 — Anderson L (SPE 02938)

Dimensionless $p/z$ Functions Versus the Dimensionless Cumulative Gas Production
Abnormally Pressured Gas Reservoir Case (Variable Compressibility Only)

![Diagram]

Figure 25 — Plot of dimensionless $p/z$ functions vs. $G_{pD}$ (type curve plot) — Case 2 ($\omega_D=0.40$, $G=73.5$ BSCF, $\omega=0.00544$).

In Fig. 14 we present the "base plot" of the $p/z-G_p$ functions for the "Anderson L" Reservoir case with the final result imposed on the $p/z$ vs. $G_p$ data. The most striking feature is the confirmation of the quadratic cumulative production behavior. Obviously, the "calibration" of the model and the $p/z-G_p$ data shown in Fig. 14 is the product of the dynamic/simultaneous analyses performed using the spreadsheet module (as are all of the results obtained in this work).

Figs. 15-20 provide the matches for plotting functions 1-6 (respectively) — and we note excellent agreement in all plots and for all trends. In Figs. 21-24 we present the "validation" plots using the $\omega$-function — each of these plots illustrates very good agreement (in a practical sense) with the data for the "Anderson L" Reservoir case. The "type curve" plot for this case (Fig. 25) is particularly striking as this plot also confirms the quadratic cumulative production character of this data set.

In Fig. 26 we present the $p/z$ vs. $G_p$ data with all results overlain — we observe an extraordinary match and we confirm the validity of the quadratic cumulative production model. Lastly, in Fig. 27, we using the Fetkovich et al approach to "reverse calculate" the $c_f(p)$ profile so as to confirm the abnormally pressured reservoir behavior — this effort was also successful.
Figure 26 — Comparison plot of \( p/z \) vs. \( G_p \) (all results) — Case 2.

Figure 27 — Plot of pore volume compressibility computed using Fetkovich, et al approach and compared to laboratory data — Case 2 (Anderson L Reservoir (assumed \( S_w = 0.25 \))).
Summary and Conclusions

Summary:
We believe that the "quadratic cumulative production" model (i.e., the \( \frac{p'}{z}G_p^2 \) material balance model) derived in this work is the most appropriate approximation available for the case of an abnormally pressured gas reservoir. Our result is given as:
\[
\frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \frac{1}{G} \alpha \frac{G_p - G_p^2}{G_p} \right]
\]

We have derived the \( \frac{p'}{z}G_p^2 \) approximation directly from the rigorous gas material for the case of an abnormally pressured gas reservoir (see Appendix A).

In order to deploy our new \( \frac{p'}{z}G_p^2 \) approximation as an analysis methodology, we developed 6 new data plotting functions (see Appendix B), as well as several new diagnostic plots. These plots/plotting functions provide a direct means of estimating the \( \omega \)-function and the gas-in-place (\( G \)) as part of an interactive analysis (in our case, we implemented these processes in a commercial spreadsheet package). We also developed a "type curve" solution using Eq. 1 (see Appendix C), where this type curve formulation is used to validate the other analyses directly (without iteration).

We have validated all of these methodologies using synthetic and field data cases — and all cases have been successfully analyzed/interpreted using our developments based on Eq. 1.

Conclusions:
1. The quadratic cumulative production model (Eq. 1) is very-well suited to the analysis and interpretation of material balance performance data obtained from abnormally pressured gas reservoirs. We will note that this relation should NEVER be implemented using regression. Instead, the interactive analysis methods discussed in this work should be used to interpret and analyze data.
2. The 6 plotting functions proposed for implementation of Eq. 1 (see Appendix B) have been successfully demonstrated and validated for use with material balance data from abnormally pressured gas reservoirs.
   As noted earlier, the "plotting functions" are used to diagnose the relevance of the quadratic cumulative production model. These functions and the associated plots are used to ensure that the \( \omega \)-function and the gas-in-place (\( G \)) are estimated in a consistent and robust manner.
3. In addition to the "plotting functions," we also have four "performance" plots which are based on rigorous material relation for an abnormally pressured gas reservoir. These plots are:
   (Plot1): \( \tau_e'(p)(p_i - p) \) versus \( G_p \) .......(log-log)
   (Plot2): \( \frac{1}{1 - \tau_e'(p)(p_i - p)} \) versus \( G_p \) .... (Cartesian)
   (Plot3): \( \omega \) versus \( G_p \) .... (Cartesian)
   (Plot4): \( \omega \) versus \( G_p / G \) .... (Cartesian)
   The performance plots have been validated as mechanisms which can be used to "guide" the analysis, as well as provide a visual correlation of the \( \omega \)-function.

Recommendations:
We put forth the following recommendations as mechanisms to extend this research work:
1. External Drive Energy: The extension of this methodology for cases of external drive energy (e.g., water influx, gas injection, etc.) should be considered.
2. Additional Validation: Efforts on validation of this approach using additional field cases and numerical simulation studies based on laboratory-derived \( c_f(p) \) profiles should be continued.
Nomenclature

Field Variables: (Pressure, Formation, and Fluid Properties)

- $B_w$ = Water formation volume factor, RB/STB
- $B_g$ = Gas formation volume factor, RB/MSCF
- $B_{gi}$ = Initial gas formation volume factor, RB/MSCF
- $c_e$ = Instantaneous effective compressibility, 1/psi
- $\bar{c}_e$ = Cumulative effective compressibility, 1/psi
- $c_i$ = Instantaneous pore volume compressibility, 1/psi
- $\bar{c}_f$ = Cumulative pore volume compressibility, 1/psi
- $c_w$ = Instantaneous water compressibility, 1/psi
- $\bar{c}_w$ = Cumulative total water compressibility, 1/psi
- $G$ = Gas-in-place, MSCF (or BSCF)
- $G_{app}$ = Apparent gas-in-place, MSCF (or BSCF)
- $G_{inj}$ = Injected gas volume, MSCF (or BSCF)
- $G_p$ = Cumulative gas production, MSCF (or BSCF)
- $M$ = Ratio of aquifer to reservoir volume, dimensionless
- $p$ = Average reservoir pressure, psia
- $p_i$ = Initial reservoir pressure, psia
- $R_{sw}$ = Solution gas water ratio, SCF/STB
- $S_{wi}$ = Irreducible water saturation, dimensionless
- $W_e$ = Cumulative water influx, RB
- $W_{inj}$ = Cumulative water injection, STB
- $W_p$ = Cumulative water production, STB
- $z$ = Gas compressibility factor, dimensionless
- $z_i$ = Gas compressibility factor at $p_i$, dimensionless
- $\Delta p$ = Pressure drop, psia
- $\omega$ = Abnormal pressure model parameter

Dimensionless Variables:

- $G_{PD}$ = Dimensionless cumulative gas production
- $p_D$ = Dimensionless $p/z$ function
- $p_{Di}$ = Dimensionless $p/z$ integral function
- $\omega_D$ = Dimensionless abnormal pressure model parameter

Greek Symbols:

- $\alpha$ = Parameter, 1/BSCF
- $\beta$ = Parameter, 1/BSCF$^2$
- $\phi$ = Porosity, fraction
- $\omega$ = Parameter, 1/BSCF

Subscript:

- $e$ = Effective
- $i$ = Initial

References


Appendix A: Development and Validation of a Simplified Model for the Material Balance of Abnormally Pressured Gas Reservoirs (*"Quadratic Cumulative Production" Model)

The general material balance relation for an abnormally pressured gas reservoir is given as follows by Fetkovich et al [Fetkovich et al (1998)]:

\[ \frac{P}{z_i} [1-\tau_e(p)(p_i - p)] = \frac{p_i}{z_i} \frac{1}{G} \left[ G_p - G_{inj} + W_p R_{sw} + \frac{5.615}{B_g} (W_p B_w - W_{inj} B_w - W_e) \right] \] .......................... (A.1)

Where Fetkovich et al define the "effective compressibility" function, \( \tau_e(p) \), as follows:

\[ \tau_e = \frac{1}{(1-S_W)} [S_W \tau_w + \tau_f + M(\tau_w + \tau_f)] \] .......................... (A.2)

Setting \( G_{inj} = W_{inj} = W_p = W_e = 0 \), we obtain the common form of the gas material balance relation (i.e., we are only interested in the issue of "abnormal pressure" effects):

\[ \frac{P}{z_i} [1-\tau_e(p)(p_i - p)] = \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right] \] .......................... (A.3)

Fetkovich et al proposed Eq. A.1 (or its abbreviated form, Eq. A.3) based on the concept that the "cumulative compressibility" function (i.e., \( \tau_e(p) \)) provides a better representation of the "abnormal pressure" effects observed in gas reservoir performance behavior (caused primarily by pore and water compressibility). Eq. A.3 has become the generally accepted reference model for the behavior of abnormally pressured gas reservoirs, although we do note that the implementation proposed by Fetkovich, et al is tedious and is over-constrained by data requirements.

Our goal is to utilize Eq. A.3 and develop an appropriate (and accurate) approximation for \( p/z - G_p \) behavior such that simplified performance models and plotting functions can be proposed and utilized.

The first step is to isolate the \( \tau_e(p) \) function in Eq. A.3 — solving for the \( \tau_e(p)(p_i - p) \) and \( \tau_e(p) \) terms in Eq. A.3 gives us the following forms:

\[ \tau_e(p)(p_i - p) = 1 - \frac{p_i}{z_i} \frac{1 - \frac{G_p}{G}}{p_i} \] .........................................................(A.4a)

\[ \tau_e(p) = \left[ 1 - \frac{p_i}{z_i} \left( 1 - \frac{G_p}{G} \right) \right] \frac{1}{(p_i - p)} \] .........................................................(A.4b)

We note that Fetkovich et al use these results (in particular, Eq. A.4b) as a "matching" function for comparison with results generated using Eq. A.2. In contrast, we are interested in the behavior of the \( \tau_e(p)(p_i - p) \) function (Eq. A.4a) — and, in particular, isolating the behavior of \( \tau_e(p)(p_i - p) \) versus \( G_p \). We propose that the following approximate model be used for the behavior of \( \tau_e(p)(p_i - p) \) versus \( G_p \):

\[ \tau_e(p)(p_i - p) \approx \omega G_p \] .........................................................(A.5)

We can readily note from Eq. A.4a that the \( G_p \) function exerts a strong influence on the \( \tau_e(p)(p_i - p) \) function — however, it is neither intuitive (nor obvious) that Eq. A.5 represents a valid model for the behavior of the \( \tau_e(p)(p_i - p) \) function.

We use synthetic and field data as a mechanism to validate our concept and establish Eq. A.5 as a viable model for the \( \tau_e(p)(p_i - p) \) function. We provide 2 validation cases in this Appendix — the first case is a dry gas reservoir case simulated using the \( c(p) \) function provided in Fig. 3 of the Fetkovich et al work [Fetkovich et al (1998)] and the second case is the "Anderson L" reservoir case [Duggan (1971)] which is regarded as a literature standard for field cases of abnormally pressured gas reservoirs.

In Fig. A.1 we present the behavior of \( \tau_e(p)(p_i - p) \) versus the \( G_p/G \) functions for the synthetic (dry gas) case on a log-log format and we note that a strong "power law" trend is evident throughout the range of \( G_p \) values (in particular, for very large levels of depletion). We also note a similar behavior of these functions for the "Anderson L" field data case as shown in Fig. 21 — in fact, the match appears very strong considering the quantity and quality of the data.
At this point we note that the concept model given by Eq. A.5 is an intermediate step — we are actually interested in the behavior of the function $1/[1 - \tilde{c}_e(p)(p_i - p)]$ as this is the multiplier for the right-hand-side (RHS) of the material balance equation. As such, we propose the following model for the $1/[1 - \tilde{c}_e(p)(p_i - p)]$ function:

$$1/[1 - \tilde{c}_e(p)(p_i - p)] \approx 1 + \xi G_p$$  \hspace{1cm} (A.6)

In this work, we could have simply proceeded directly to the $1/[1 - \tilde{c}_e(p)(p_i - p)]$ function, as opposed to investigating the standard $\tilde{c}_e(p)(p_i - p)$ function — however; it is our contention that Eq. A.5 can be used to establish Eq. A.6 (via expansion of the $1/[1 - \tilde{c}_e(p)(p_i - p)]$ term, using a geometric series based on Eq. A.5). Recalling the definition of a geometric series, we have:

$$1/[1 - x] \approx 1 + x + x^2 + x^3 + \ldots \quad (-1 < x < 1)$$  \hspace{1cm} (A.7)

A single-term expansion of Eq. A.7 can be written as:

$$1/[1 - x] \approx 1 + x \quad (-1 < x < 1)$$  \hspace{1cm} (A.8)

Substituting $x = \tilde{c}_e(p)(p_i - p)$ into Eq. A.8 gives us:

$$1/[1 - \tilde{c}_e(p)(p_i - p)] \approx 1 + \tilde{c}_e(p)(p_i - p) \quad (\tilde{c}_e(p)(p_i - p) < 1)$$  \hspace{1cm} (A.9)

Substituting the $\tilde{c}_e(p)(p_i - p) \approx \omega G_p$ term into the right-hand-side (RHS) of Eq. A.9 yields:

$$1/[1 - \tilde{c}_e(p)(p_i - p)] \approx 1 + \omega G_p \quad (\omega G_p < 1)$$  \hspace{1cm} (A.10)
Comparing Eqs. A.6 and A.10 we find that the two forms are equivalent (i.e., \( \zeta = \omega \)) — which is not a rigorous proof of Eq. A.5, but this result does directionally confirm our concept model for the form given by Eq. A.6. In Fig. A.2 we present the function \(1/[1 - \tau_e(p)(p_i - p)]\) versus \(G_p/G\) on a Cartesian grid for the simulated performance case — we note a very reasonable match of the data and model functions, where this comparison suggests that Eq. A.6 is an appropriate model for the behavior of the \(1/[1 - \tau_e(p)(p_i - p)]\) function.

We present the results for the Anderson L field case in Fig. 22 and we confirm a very strong agreement of this data and the proposed model (Eq. A.6).

Recalling the original material balance (neglecting \(G_{inj}, W_{inj}, W_p,\) and \(W_e\)) — i.e., Eq. A.3, we have:

\[
\frac{p}{z}[1 - \tau_e(p)(p_i - p)] = \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right]
\]

(A.3)

For completeness, we need to establish the identity for the \(1/[1 - \tau_e(p)(p_i - p)]\) function — to do so we solve Eq. A.3 for the \(1/[1 - \tau_e(p)(p_i - p)]\) function. This gives:

\[
1/[1 - \tau_e(p)(p_i - p)] = \frac{1}{p_i/z_i} \left[ \frac{G_p}{G} \right] \left[ 1 - \frac{G_p}{G} \right]
\]

(A.11)

---

**Case 1 — Simulated Dry Gas Reservoir**

**Validation of New Material Balance Concept for Abnormally Pressured Gas Reservoirs (Variable Compressibility Only)**

**Validation Plot for the \(1/[1-c_e(p)(p_r-p)]\) Function**

---

**Legend:**

- Case 1 — Simulated Dry Gas Reservoir
- \(1/[1-c_e(p)(p_r-p)]\) Data
- \(1/[1-c_e(p)(p_r-p)]\) Model
- Gas-in-Place, \(G = 688.4\) BSCF (input)

\[
1/[1-c_e(p)(p_r-p)] - 1 + \omega DG_p/G
\]

\[
\omega_D = (0.00042 1/BSCF)(688.4\ BSCF)
\]

\[
= 0.2883
\]

\(\omega_D = \omega G\)

---

**Governing Relation:**

\[
1/[1-c_e(p)(p_r-p)] = 1/[1-\tau_e(p)(p_i - p)]
\]

---

**Fig. A.2 — Behavior of the \(1/[1 - \tau_e(p)(p_i - p)]\) function versus \(G_p/G\) for a simulated dry gas reservoir case. The \(c_e(p)\) function (i.e., the instantaneous pore volume compressibility) used in this case was obtained from Fig. 3 of the Fetkovich et al work [Fetkovich et al (1998)]. The purpose of this plot is to validate the linearity of the \(1/[1 - \tau_e(p)(p_i - p)]\) function.**

We now present the development of our approximate material balance relation for dry gas reservoirs experiencing abnormal pressure effects which result from pressure-dependent pore volume compressibility (no water influx and no water injection/production terms are considered in this development). To begin, we divide through Eq. A.3 by the \([1 - \tau_e(p)(p_i - p)]\) term to yield a more convenient form of the material balance equation for this case. This gives:

\[
\frac{p}{z} = \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right] \left[ 1 - \tau_e(p)(p_i - p) \right]
\]

(A.12)
Substituting Eq. A.10 into Eq. A.12 we have:

\[ \frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right] (1 + \omega G_p) \] ................................................................. (A.13)

Expanding terms in the right-hand-side (RHS) of Eq. A.13, we obtain:

\[ \frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \left( 1 - \frac{G_p - \omega G_p}{G} \right) \right] \] ................................................................. (A.14)

At this point we consider the \( \omega \)-function to simply be an arbitrary constant required by the model. We have made no attempts to quantify the \( \omega \)-function, other than to provide the definition of the \( (\omega G_p = \bar{c}_e(p)(p_i - p)) \), \( \omega \)-function — where we note that we will generally consider \( \omega \) to be a function, but the specific goal of this particular derivation is to establish the relevance of \( \omega \) as a constant (i.e., a parameter).

Substituting the definition of the \( \omega \)-function (Eq. A.5) into the material balance relation (specifically, the form given by Eq. A.4a), we have the following definition of the \( \omega G_p \) product:

\[ \omega G_p = \frac{1}{G_p} \left[ 1 - \frac{G_p}{G} \right] \left( \frac{p_i}{z_i} \right) \] ................................................................. (A.15)

Isolating the \( \omega \)-function, we have:

\[ \omega = \frac{1}{G_p} - \frac{p_i}{z_i} \left[ 1 - \frac{1}{G} \right] \] ................................................................. (A.16)

From inspection of Eq. A.16 we note that the \( \omega \)-function is actually a function (i.e., there is no obvious indication that \( \omega \) would be constant) — however, the only mechanism we can use to prove our contention that the \( \omega \)-function is at least "approximately" constant is to consider synthetic and field data estimations of this function.

As such, the \( \omega \)-function is plotted against \( G_p \) in Figs. 3 and 4 (in the main text) for the case of simulated gas reservoir performance and the Anderson L field case [Duggan (1971)], respectively.

We note in Fig. 3 (the simulated data case where \( G_{p,in}/G \approx 0.7 \)) that our estimate of \( \omega = 4.2 \times 10^{-4} \) BSCF is a reasonable average of the values for \( 0 < G_p < 500 \) BSCF — although we acknowledge that this estimate is certainly open to other interpretations. For the Anderson L reservoir case (Fig. 4) we find an extraordinary match of our estimate \( (\omega = 4.5 \times 10^{-3} \) BSCF) compared to the data trend — in this case \( G_{p,in}/G \approx 0.5 \) — which probably accounts for the much better performance of the data for this case (as compared to the simulated data case).

Revisiting Eq. A.14 and expanding the right-hand-side (RHS) term, we have:

\[ \frac{p}{z} \approx \frac{p_i}{z_i} - \left( 1 - \frac{G_p}{G} \right) \frac{G}{G_p} \left( \frac{p_i}{z_i} \right) - \frac{\omega G_p}{G} \frac{p_i}{G} \] ................................................................. (A.17)

Or, using shorthand notation, Eq. A.17 becomes:

\[ \frac{p}{z} \approx \frac{p_i}{z_i} - \alpha G_p - \beta G_p^2 \] ................................................................. (A.18)

Where the \( \alpha \) and \( \beta \) coefficients are given by:

\[ \alpha = \frac{1}{G} - \omega \frac{G_p}{G} \frac{p_i}{z_i} \] ................................................................. (A.19)

\[ \beta = \frac{\omega G_p}{G} \frac{p_i}{G} \frac{z_i}{z_i} \] ................................................................. (A.20)

Eq. A.18 is the most basic building block of this work — our goal is to utilize this model to serve as an appropriate approximation for the rigorous gas material balance for the case of an abnormally pressured reservoir (i.e., Eq. A.3). From this point forward we will consider Eq. A.18 to be valid as a material balance model — and, as such, we will use Eq. A.18 as the basis for plotting functions, reserve calculations, etc — specifically for the case of an abnormally pressured gas reservoir.

As an aside, we will note that in Figs. 3 and 24 (main text) the \( \omega \)-function does indicate a slightly decaying linear trend. The most important conclusion that can be derived from Figs. 3 and 24 (main text) is that the behavior of the \( \omega \)-function is unique and can be approximated as a constant or as a linear function of \( G_p \) (or \( G/G \)). This conclusion forms the basis for the development of the "quadratic" and "cubic" cumulative production relations for gas material balance (the quadratic relation is given by Eq. A.18 and the cubic relation is derived below).
Presuming a linear trend for the \( w - G_p \) behavior we have:

\[
\omega \equiv a - b G_p \tag{A.21}
\]

Substitution of Eq. A.21 into Eq. A.13 gives us:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right] \left[ 1 + (a - b G_p) G_p \right] \tag{A.22}
\]

Expanding terms on the right-hand-side (RHS), we have:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right] \left[ 1 + a G_p - b G_p^2 \right]
\]

\[
\approx \frac{p_i}{z_i} \left[ 1 + a G_p - b G_p^2 \right] \left[ \frac{G_p}{G} + \frac{a}{G} G_p - \frac{b}{G} G_p^2 \right]
\]

Or, finally, we have:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \frac{1}{G} \right] G_p \left[ \frac{a}{G} + b \right] G_p^2 + \frac{b}{G} G_p^3 \tag{A.23}
\]

The general form of Eq. A.23 is given by:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} - \hat{a} G_p - \hat{b} G_p^2 + \hat{c} G_p^3 \tag{A.24}
\]

Where the coefficients \( \hat{a} \), \( \hat{b} \), and \( \hat{c} \) are defined as:

\[
\hat{a} = \left[ \frac{1}{G} - a \right] \frac{p_i}{z_i} \tag{A.25a}
\]

\[
\hat{b} = \left[ \frac{a}{G} + b \right] \frac{p_i}{z_i} \tag{A.25b}
\]

\[
\hat{c} = \frac{b}{G} \frac{p_i}{z_i} \tag{A.25c}
\]

We believe that the implementation of Eq. A.24 (the "cubic" cumulative production relation for gas material balance) would be problematic and generally less stable than the "quadratic" material balance model (i.e., Eq. A.18). Hence, our focus in this work will be the development of plotting functions and analysis relations for Eq. A.18.

The practical application of the "quadratic" and "cubic" cumulative production relations for gas material balance (Eqs. A.18 and A.24) is the comparison of these models to \( p/z \) versus \( G_p \) data. We provide 2 example applications of the "quadratic" and "cubic" relations — a synthetic gas reservoir performance case and the "Anderson L" gas reservoir field case [see Figs. 6 and A.3 (main text)].

In Fig. A.3 we provide data for the synthetic data case and we immediately note (as before) that this case has experienced significant reservoir depletion. The initial linear trend is that of the "apparent" gas-in-place model, which has been the starting point of traditional analyses of material balance data obtained from abnormally pressured gas reservoirs. We do not utilize the "apparent" gas-in-place trend in this work and only note its presence for reference.

The specific trends of interest shown on Fig. A.3 are those trends given by the constant "\( w \)" model (Eq. A.18) and the linear "\( w \)" model (Eq. A.24). We have previously referred to Eq. A.18 as the "quadratic" cumulative production relation for gas material balance (abnormally pressured reservoir case), and Eq. A.24 is the "cubic" \( G_p \) relation. For reference, the governing relation for the \( w \)-function is given by Eq. A.16:

\[
\omega \equiv \frac{1}{G_p} \left[ \frac{p_i/z_i}{p/z} - 1 \right] \tag{A.16}
\]
And the \( \frac{p}{z} - G_p \) model in terms of \( \omega \) is given by Eq. A.13:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right] (1 + \omega G_p) \quad \text{.................................................................} \quad (A.13)
\]

Case 1 — Simulated Dry Gas Reservoir

Validation of New Material Balance Concept for Abnormally Pressured Gas Reservoirs (Variable Compressibility Only)

\( p/z \) Plot — \( G_p \) Format

![Graph](image)

**Legend:**

- Case 1 — Simulated Dry Gas Reservoir
- \( p/z \) Data (\( p_f = 7255.42 \text{ psia} \))
- \( p/z \) Model (\( \omega = 0.00042 \))
- \( p/z \) Model (\( \omega = 0.0058 - 0.0005491 (G_p/G) \))
- \( p/z \) Model (\( G_{\text{Apparent}} = 1007 \text{ BSCF} \))
- Gas-in-Place, \( G = 686.4 \text{ BSCF} \) (input)

**Governing Relation:**

\[
(p/z) = (p_i/z_i) [1 - (G_p/G)] (1 + \omega G_p)
\]

**Cumulative Gas Production (\( G_p \)), BSCF**

\( p/z \) versus \( G_p \) plot for the simulated performance of an abnormally pressured gas reservoir (variable compressibility only) (dry gas reservoir case).

Since these are validation cases and we have all of the relevant data, we note that the "\( \omega \)" models shown in Fig. A.3 were "tuned" for this case using a variant of Fig. 3 (i.e., \( \omega \) is plotted versus \( G_p \) (as opposed to \( G_p/G \)). Specifically, we do not advocate the use of Eqs. A.18 and A.24 as regression models — but rather, we will use Eq. A.18 to derive plotting functions to estimate the relevant model parameters (including the \( \omega \)-function — see Appendix B).

In this particular case we note that using a constant value of the \( \omega \)-function is fairly straightforward — specifically the influence of a particular \( \omega \)-value can be established by sampling (or using trial and error). In comparison, the "best fit" of the \( \omega - G_p \) data [see Fig. 3 (main text)] yields a reasonable match of the \( p/z - G_p \) data, as projected on Fig. A.3.

For practical applications we will not have the "answer" (i.e., \( G \)) at our disposal and construction of Fig. 3 will not be possible (as a direct analysis technique), so this issue is more of a concept/validation concern. We note that multiple optimizations of the linear \( \omega - G_p \) trend are possible, and we must simply accept that Eqs. A.18 and A.24 (i.e., the final forms using the constant and linear \( G_p \) models for the \( \omega \)-function) are viable approximations which can be used as general tools for the material balance analysis of abnormally pressure gas reservoir systems.

Similar conclusions are made for the "Anderson L" (South Texas, USA) field case [Duggan (1971)]. In particular, we note the coordination between the analyses of the \( \omega \)-function on Fig. 4 (main text) and the corresponding summary plot of \( p/z \) versus \( G_p \) shown in Fig. 6 (main text).

Based on our observations, we could suggest that the data for the Anderson L case performs somewhat better than the synthetic reservoir performance case. The most likely reason is that the Anderson L case appears to have a maximum depletion of 50 percent (\( G_p/G \approx 0.5 \)), as opposed to the synthetic case where the maximum depletion approaches 75 percent. Another (perhaps more practical) reason is that the Anderson L case is derived using actual field performance data — normal fluctuations in performance may give the impression that the behavior better fits our assumptions (see Fig. 4 (main text) — where the dominant data trend could easily be assumed to be constant).

As with the synthetic reservoir performance case, we again note that the models presented in Fig. 6 (main text) are tied directly to the \( \omega - G_p \) behavior presented in Fig. 4 (main text). Specifically, the "\( \omega \)" models presented in Fig. 6 have not been tuned using statistical regression — but rather, using a combination of specialized plotting functions (given in Appendix
Appendix B: Development of Plotting Functions for the "Quadratic Cumulative Production" Form of the Material Balance Relation for Abnormally Pressured Gas Reservoirs

The base relation for this work is the "quadratic cumulative production" relation for the material balance of abnormally pressured gas reservoirs (derived specifically for the pressure-dependent pore volume compressibility case). This result was derived in Appendix A and is given as:

\[
\frac{\Delta p}{z} \approx \frac{P_i}{z_i} - \alpha G_p - \beta G_p^2 \tag{A.18}
\]

**Data Plot:** \(\Delta p/z\) versus \(G_p\) [Eq. A.18] (quadratic trend)

Where the \(\alpha\) and \(\beta\) coefficients are given by:

\[
\alpha = \left[ \frac{1}{G} - \omega \right] \frac{P_i}{z_i} \tag{A.19}
\]

\[
\beta = \frac{\omega}{G} \frac{P_i}{z_i} \tag{A.20}
\]

Graphically, Eq. A.18 yields a quadratic (concave downward) trend on the traditional \(\Delta p/z\) versus \(G_p\) plot. At early times Eq. A.18 appears to be linear (near \(\Delta p/z\)) and at late times (as \(p\) decreases rapidly), Eq. A.18 is dominated by the quadratic term and the \(\Delta p/z\) trend decreases rapidly away from the "apparent" linear \(\Delta p/z\) trend.

As we begin to define our plotting functions, for convenience, we define the "\(\Delta p/z\) difference function" as:

\[
\Delta(p/z) = \frac{P_i}{z_i} - \frac{p}{z} = \alpha G_p + \beta G_p^2 \tag{B.1}
\]

**Data Plot:** \(\Delta(p/z)\) versus \(G_p\) [Eq. B.1] (quadratic trend)

Dividing through Eq. B.1 by the cumulative gas production \(G_p\), we have:

\[
\frac{\Delta(p/z)}{G_p} = \alpha + \beta G_p \tag{B.2}
\]

**Data Plot:** \(\Delta(p/z)/G_p\) versus \(G_p\) [Eq. B.2] (linear trend)

While we are intrigued by the work of Moran and Sameniego [Moran and Sameniego (2000)] — which uses the derivative of the \(p/z\) behavior with respect to \(G_p\) — we believe that application of this approach will always be limited due to data quality and quantity.

However, we believe that the development of "auxiliary" \(p/z\) functions is a practical necessity for the analysis of reservoir performance data which are affected by abnormal pressure effects. Therefore, we propose a series of "integral" functions based on Eq. B.2 as auxiliary functions for the purpose of data analysis (specifically, data plotting functions).

Integrating Eq. B.2 with respect to \(G_p\) yields:

\[
\int_0^{G_p} \Delta(p/z) dG_p = \int_0^{G_p} [\alpha G_p + \beta G_p^2] dG_p = \frac{\alpha}{2} G_p^2 + \frac{\beta}{3} G_p^3 \tag{B.3}
\]

**Data Plot:** \(\int_0^{G_p} \Delta(p/z) dG_p\) versus \(G_p\) [Eq. B.3] (cubic trend)

Dividing Eq. B.3 by \(G_p\) yields the primary "integral" function of interest:

\[
\frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p = \frac{\alpha}{2} G_p + \frac{\beta}{3} G_p^2 \tag{B.4}
\]

**Data Plot:** \(\frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p\) vs. \(G_p\) [Eq. B.4] (quadratic trend)
Dividing Eq. B.3 by \( G_p^2 \) yields an auxiliary "integral" function:

\[
\frac{1}{G_p^2} \int_0^{G_p} \Delta(p/z)\,dG_p = \frac{\alpha}{2} + \frac{\beta}{3} G_p \tag{B.5}
\]

Data Plot: \( \frac{1}{G_p^2} \int_0^{G_p} \Delta(p/z)\,dG_p \) vs. \( G_p \) [Eq. B.5] (linear trend)

Subtracting Eq. B.4 from Eq. B.1 gives us:

\[
\Delta(p/z) - \frac{1}{G_p^2} \int_0^{G_p} \Delta(p/z)\,dG_p = \left[ \alpha G_p + \beta G_p^2 \right] - \left[ \frac{\alpha}{2} G_p + \frac{\beta}{3} G_p^2 \right] = \frac{1}{2} \alpha G_p + \frac{2}{3} \beta G_p^2 \tag{B.6}
\]

Data Plot: \( \Delta(p/z) - \frac{1}{G_p^2} \int_0^{G_p} \Delta(p/z)\,dG_p \) vs. \( G_p \) [Eq. B.6] (quadratic trend)

Dividing Eq. B.6 by \( G_p \) yields the "integral-difference" formulation, that, at least in concept, is analogous to the "derivative" functions proposed by Moran and Sameniego [Moran and Sameniego (2000)]. This result is given as:

\[
\frac{1}{G_p} \left[ \Delta(p/z) - \frac{1}{G_p^2} \int_0^{G_p} \Delta(p/z)\,dG_p \right] = \frac{1}{2} \alpha + \frac{2}{3} \beta G_p \tag{B.7}
\]

Data Plot: \( \frac{1}{G_p} \left[ \Delta(p/z) - \frac{1}{G_p^2} \int_0^{G_p} \Delta(p/z)\,dG_p \right] \) vs. \( G_p \) [Eq. B.7] (linear trend)

Our analysis procedure employs all of the designated plots (i.e., Eqs. A.18, B.1 to B.7). The procedure is implemented using a spreadsheet approach where the coefficients \( p/z, \alpha, \) and \( \beta \) are specified — \( p/z \) is typically known (or can be estimated) as the initial condition while \( \alpha \) and \( \beta \) are estimated by trial and error. The \( \alpha \) and \( \beta \) parameters can be tied to a single "\( \omega \)" value (per Eqs. A.19 and 20) or these coefficients can be estimated independently (again, by trial and error).

We suggest that most engineers should use a spreadsheet approach which requires hand manipulation of the coefficients. We NEVER recommend statistical optimization of the parameters using regression methods as this activity can (and will) lead to estimates of parameters which are physically inconsistent (e.g., negative values).

**Appendix C: Development of a Type Curve Solution for the Analysis of \( p/z-G_p \) Data for the Case of an Abnormally Pressured Gas Reservoir Using the "Quadratic Cumulative Production" Form of the Material Balance Relation**

As with previous developments, the base relation for the work in this Appendix is the "quadratic cumulative production" relation for the material balance of abnormally pressured gas reservoirs (i.e., the form given by Eq. A.14).

\[
\frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \frac{1}{G} \left[ G_p - \frac{G_p^2}{G} \right] \right] \tag{A.14}
\]

The objective of this Appendix is to use Eq. A.14 to develop a dimensionless cumulative production relation — from which a graphical "type curve" solution can be constructed. Defining a dimensionless "\( \omega_p \)" function, we have:

\[
\omega_p = \omega G \tag{C.1a}
\]

Or, in terms of the \( \omega \)-function, we obtain:

\[
\omega = \omega_p / G \tag{C.1b}
\]

Substituting Eq. C.1b into Eq. A.14 yields:

\[
\frac{p}{z} = \frac{p_i}{z_i} \left[ 1 - \left( 1 - \omega_p / G \right) G_p - \frac{\omega_p^2}{G} \right] \tag{C.2}
\]
Rearranging terms on the right-hand-side (RHS) of Eq. C.2 gives us:

\[
\frac{p}{z} = \frac{p_i}{z_i} \left[ 1 - (1 - \omega_D) \left( \frac{G_p}{G} \right) - \omega_D \left( \frac{G_p}{G} \right)^2 \right] \tag{C.3}
\]

Further rearranging terms Eq. C.3, we have:

\[
\left[ 1 - \frac{p}{z} \left( \frac{p_i}{z_i} \right) \right] = (1 - \omega_D) \left( \frac{G_p}{G} \right) - \omega_D \left( \frac{G_p}{G} \right)^2 \tag{C.4a}
\]

or,

\[
\left[ \frac{p_i}{z_i} \left( \frac{p_i}{z_i} \right) - \frac{p}{z} \right] = (1 - \omega_D) \left( \frac{G_p}{G} \right) - \omega_D \left( \frac{G_p}{G} \right)^2 \tag{C.4b}
\]

From Eqs. C.4a and C.4b, we observe the following "intuitive" dimensionless variables:

\[
p_D = \left[ 1 - \frac{p}{z} \left( \frac{p_i}{z_i} \right) \right] = \left[ \frac{p_i}{z_i} \left( \frac{p_i}{z_i} \right) - \frac{p}{z} \right] \tag{C.5}
\]

\[
G_{pD} = \frac{G_p}{G} \tag{C.6}
\]

Substituting Eqs. C.5 and C.6 into Eq. C.4a (or C.4b), we obtain the "dimensionless" form of the "quadratic cumulative production" material balance relation for an abnormally pressured gas reservoir:

\[
p_D = (1 - \omega_D) G_{pD} + \omega_D G_{pD}^2 \tag{C.7}
\]

In order to develop an "auxiliary" function to aid in this analysis, we define the "dimensionless pressure integral" function as follows for this case:

\[
p_{Di} = \frac{1}{G_{pD}} \int_0^{G_{pD}} p_D dG_{pD} \tag{C.8}
\]

Substituting Eq. C.7 into Eq. C.8, we have:

\[
p_{Di} = \frac{1}{G_{pD}} \int_0^{G_{pD}} [(1 - \omega_D) G_{pD} + \omega_D G_{pD}^2] dG_{pD} \tag{C.9}
\]

Completing the integration in Eq. C.9 yields:

\[
p_{Di} = \frac{1}{G_{pD}} \left[ (1 - \omega_D) \frac{1}{2} G_{pD}^2 + \omega_D \frac{1}{3} G_{pD}^3 \right] \tag{C.10}
\]

Multiplying through by the 1/$G_{pD}$ term, we have:

\[
p_{Di} = (1 - \omega_D) \frac{1}{2} G_{pD} + \omega_D \frac{1}{3} G_{pD}^2 \tag{C.10}
\]

In Fig. 13 (main text) we have previously presented the "type curve" based on Eqs. C.7 and C.10 for the "dimensionless" pressure and pressure integral functions, $p_D$ and $p_{Di}$, respectively.

In order to use Fig. 13, we define the data function defined by Eq. C.5 for $p_D$ as follows:

\[
(p_D)_{data} = 1 - \frac{p}{z} \left( \frac{p_i}{z_i} \right) \tag{C.11}
\]

We will use the definition of the "pressure integral" function (Eq. C.8) to develop an expression for computing the data function for $p_{Di}$. Recalling Eq. C.8, we have

\[
p_{Di} = \frac{1}{G_{pD}} \int_0^{G_{pD}} p_D dG_{pD} \tag{C.8}
\]

Defining a variable of substitution, $G_p = G_{pD} (i.e.,$ using Eq. C.6), we have

\[
G_p = G_{pD}
\]

\[
G_p = G_{pD}
\]
Taking the derivative of this variable of substitution, we obtain:

\[ dG_p = G dG_{pD} \]

Or, solving for \( dG_{pD} \), we have

\[ dG_{pD} = \frac{1}{G} dG_p \] .................................................................(C.12)

Evaluating the limits of the new variable of substitution, \( G_p \), we obtain:

\[ \begin{align*}
  \text{at } G_{pD} = 0: & \quad G_p = 0 \\
  \text{at } G_{pD} = G_p: & \quad G_p = G_p
\end{align*} \]

Substitution of Eq. C.13 into Eq. C.12, and using the new limits (in terms of \( G_{pD} \)), we obtain:

\[ p_{Di} = \frac{G}{G_p} \int_0^{G_p} p_{Di} \frac{1}{G} dG_p \]

Canceling the \( G \) terms, we have:

\[ (p_{Di})_{\text{data}} = \frac{1}{G_p} \int_0^{G_p} (p_{Di})_{\text{data}} dG_p \] .................................................................(C.13)

Where Eq. C.13 is the "data formulation" for \( p_{Di} \), and we note that \( (p_{Di})_{\text{data}} \) is used as a variable in the type curve matching process.
Appendix D: Supplementary References for the Material Balance of an Abnormally Pressured Gas Reservoir

Material Balance: Abnormally Pressured Gas Reservoirs


Case Histories: Abnormally Pressured Gas Reservoirs


Origin of Pressure: Abnormally Pressured Gas Reservoirs


