Decline Curve Analysis for Unconventional Reservoir Systems — Variable Pressure Drop Case
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Abstract

The premise of this work is the development and application of a new methodology to forecast production data in unconventional reservoirs where variable rate and pressure drop data are typically observed throughout production. Decline curve analysis techniques for the estimation of ultimate recovery (EUR) require the constant bottomhole pressure condition during the producing life of the well — whereas it is not regular practice to maintain a constant bottomhole pressure profile throughout production in unconventional reservoirs. Therefore, the applicability of the time-rate decline relations is questionable, and methods to remove pressure variations from rate response are needed for generating future production forecasts.

From a conceptual viewpoint, we propose the utilization of the convolution/superposition theory along with the recently developed "empirical" time-rate equations, which are normalized by pressure drop data. In order to avoid non-uniqueness, a workflow is used where model parameters for the "normalized" decline curve equations are identified using diagnostic "qDb" plots. Normalized decline curve equations are then convolved with the pressure drop data to achieve a history match and to forecast production.

We provide demonstrative application of this technique using an example from an high pressure high temperature shale gas reservoir. For varying bottomhole pressure cases, we show that our proposed techniques effectively remove pressure variations from the rate history. We present the differences in computed EUR values using decline curve analysis with and without corrections for varying pressures. In addition, forecasts are generated using supplementary plots such as pressure drop normalized rate versus cumulative production.

Introduction

Decline curve analysis (DCA) is undoubtedly the commonly used technique to estimate ultimate recovery (EUR) of the producing wells in petroleum industry. In simple terms, decline curve analysis is regarded as the extrapolation/forecast of production (only "time-rate" data) into future. The use of only production data (i.e., time-rate data) to estimate ultimate recovery has been an industry practice since the introduction of the Manual for the Oil and Gas Industry under the Revenue Act of 1918 by the United States Internal Revenue Service (1919). Following that publication several authors began to publish their work on decline curve analysis and reserves estimates. For example, Lewis and Beal (1918) provide initial guidance on the extrapolation of future production of oil wells. Cutler (1924) presents a detailed review of the oil reserves estimation procedures in 1920's. Johnson and Bollens (1927) introduce the loss ratio and the loss ratio derivative definitions which lay the basis for the exponential and the hyperbolic rate decline relations. In his seminal work, Arps (1945) presents the derivation of empirical exponential and hyperbolic rate decline relations, which are still the widely utilized relations for production extrapolations of oil and gas wells. It is noted that Arps' decline curve relations can be considered as an industry standard and regularly used to forecast production in conventional reservoirs. However, it can also be stated that simplified time-rate (decline) models such as Arps' decline curves cannot accurately capture all elements of the performance behavior in unconventional reservoirs. This point will be elaborated later in this section, but we will now proceed with the fundamentals on decline curve analysis.
The starting point of our discussion is the basics regarding decline curve analysis. Essentially, when decline curve analysis is mentioned, intuitively we refer to a rate decline of a single well (or multiple wells). When the time-rate data of a well exhibits a decline behavior, then a rate-decline model can be calibrated to data for extrapolating production to yield estimated ultimate recovery at a specified time or at an abandonment rate. As mentioned earlier, several decline models exist in the literature and the most important point of the discussion is the nature and applicability of the decline curve analysis. In order to address this point, let’s take physics of fluid flow in reservoirs into consideration. One of the building blocks of reservoir engineering is the diffusivity equation, which represents fluid flow in a reservoir as functions of time and position. The diffusivity equation (a second order partial differential equation) is given below:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) = \frac{k}{\phi \mu c} \frac{\partial p}{\partial t} \quad \text{(in Darcy (or SI) units)} \tag{1}
\]

Eq. 1 can be solved with certain boundary conditions (i.e., inner boundary condition and outer boundary condition) and an initial condition (initial pressure). For our purposes in this paper, we focus on the solution of the diffusivity equation related with the \textit{inner boundary condition}. Depending on the assumption for the inner boundary condition, different solutions of the diffusivity equation are obtained. For instance, "constant-rate pressure" solution is obtained if the inner boundary condition is assumed as constant rate at the wellbore. "Constant-pressure rate" solution is obtained if we assume constant bottomhole pressure at the wellbore for inner boundary condition. Therefore, the main condition for decline curve analysis can be stated as the "constant bottomhole pressure" requirement throughout production, which yields a rate solution as a function of time for constant bottomhole pressure. In other words, all decline curve relations (whether analytical or empirical) assume constant bottomhole pressure condition.

As mentioned earlier, Arps’ relations have been the common choice for decline curve analysis in practice for a long time. The basic definitions related with Arps’ relations are given below:

\[
D(t) = -\frac{1}{q(t)} \frac{dq(t)}{dt} \quad \text{(Definition of the decline parameter)} \tag{2}
\]

\[
\frac{1}{D(t)} = -\frac{q(t)}{dq(t)/dt} \quad \text{(Definition of the loss-ratio)} \tag{3}
\]

\[
b(t) = \frac{d}{dt} \left[ \frac{1}{D(t)} \right] = -\frac{d}{dt} \left[ \frac{q(t)}{dq(t)/dt} \right] \quad \text{(Derivative of the loss-ratio)} \tag{4}
\]

According to these definitions, one can obtain the exponential and hyperbolic relations. Specifically, if Eq.2 or 3 (decline parameter or the loss ratio) is assumed constant, then the solution of the ordinary differential equation yields the exponential decline, which can be derived for the case of pseudosteady-state (or boundary-dominated) flow in a closed reservoir producing above bubble point pressure and being produced at a constant wellbore flowing pressure. The exponential relation is given as:

\[
q(t) = q_i \exp[-D_i t] \tag{5}
\]

Furthermore, hyperbolic rate decline relation is obtained if Eq. 4 (second order ordinary differential equation) is solved for the rate function. By definition the "b-value" is constant in the hyperbolic relation. Although hyperbolic relation is widely used, the conditions under which hyperbolic rate decline actually occur has been a topic of debate since its introduction more that 80 years ago. There have been various attempts to validate hyperbolic decline for the solution gas-drive (Camacho and Raghavan 1991), multi-layered gas reservoirs (Fetkovich 1990), and gas flow during boundary dominated flow regime (Ilk et al. 2009) and these efforts can be considered as successful up to a point, but all these derivations assume boundary dominated flow conditions and thus b-values have to be between 0 and 1. Most importantly, it has been suggested (by Arps and others) that the occurrence of boundary-dominated flow regime is the primary condition for the application of the hyperbolic relation, which is given below as:

\[
q(t) = \frac{q_i}{(1 + bD_i t)^{1/b}} \tag{6}
\]
For the case of unconventional reservoirs (e.g., tight gas, shale gas, liquid-rich shales, and coalbed methane), the starting point for any discussion of decline curve analysis must be an understanding that no simplified time-rate model can accurately capture all elements of the performance behavior. We must be both realistic and practical when attempting to characterize production performance from systems where the permeability is on the order of 10-500 nd (or 10x10^{-6} to 500x10^{-6} md) — the reservoir flow system is complex and although the induced (i.e., created) hydraulic fracture system enables (and dominates) the production performance, we have only the most rudimentary understanding of the flow structure in the hydraulic (and natural) fracture systems.

Application of the Arps' hyperbolic relation to unconventional reservoirs results in \( b \)-values greater than 1 (one) for almost all the cases due to the ultra-low permeability nature of unconventional reservoirs. This outcome has the significant potential to overestimate reserves if the ultimate extrapolation is not constrained by "splicing" a terminal exponential decline trend at some time in future. In the studies provided by Rushing et al. [2007] and Lee and Sidle [2010] these authors showed that the unconstrained use Arps' hyperbolic rate relation (particularly for cases where the \( b \)-values are greater than 1) can and almost always does yield significant overestimates of reserves.

As an attempt to better represent the general character of time-rate production data for a multiply-fractured horizontal well, which is considered as the standard well completion, in an ultra-low permeability reservoir, numerous authors have developed time-rate relations using certain specific bases in order to best represent a particular scenario. These developments include the following time-rate relations:

\[
q(t) = \hat{q}_i \exp[-\hat{D}_i t^n - D_\infty t]
\]

Power-Law Exponential Model [Ilk et al. (2008, 2009)]............... (7)

\[
q(t) = \hat{q}_i \exp[-(t/\tau)^n]
\]

Stretched Exponential Model [Valkó (2009)] ................................. (8)

\[
q(t) = \frac{dG_p(t)}{dt} = \frac{Kn LGM a_{LGM} t^{(n_{LGM} - 1)}}{[a_{LGM} + t^{n_{LGM}}]^2}
\]

Logistic Growth Model [Clark et al. (2011)] ................................. (9)

\[
q(t) = q_i t^{-m_{Dmg}} \exp\left[\frac{a_{Dmg}}{(1 - m_{Dmg}) \left[t^{(1-m_{Dmg})} - 1\right]}\right]
\]

Duong Model [Duong (2011)]................................................. (10)

Each relation has its own strengths, and in absolute fairness, at this time, each/all of these models can be only be described as empirical, there is no direct link with reservoir engineering theory, other than via analogy. For example, the Stretched Exponential model is essentially an infinite sum of exponentials, so in as an analog, the concept of adding the rigorous exponential decline to some limit could be thought to "define" this model. The Power-Law Exponential model is essentially the same as the Stretched Exponential model (except for a constraining variable \( D_\infty \)), and this relation was derived exclusively from the observed behavior of \( D(t) \) and \( b(t) \). At this point, we must assume that the proposed models are essentially empirical in nature, and generally center on a particular flow regime and/or characteristic behavior.

In this work our focus is decline curve analysis in unconventional reservoirs, where the applicability of decline curve analysis (or time-rate analysis) is questionable due to many reasons. Most importantly, changing operating conditions throughout the producing life of the well, restricted rate production or production curtailment present variable pressure profiles during production which conflict with the theory of decline curve analysis. In practice, this may not be an issue for conventional reservoirs where constant pressure production can quickly be achieved, but in the case of unconventional reservoirs, this could become a serious issue for recovery estimations. In fact we have observed numerous examples where rates are maintained constant (or no actual decline trend can be identified) for a considerable amount of time and wells could exhibit shallow pressure decline trends. Traditional decline curve analysis could be misleading for these cases. Therefore, our primary goal in this study is to integrate pressure data into decline curve analysis and then analyze and forecast production using time-pressure-rate formulations by utilizing "empirical" decline curve relations. We will present the conceptual development for this type of analysis in the next section.
Development of the Methodology

This section attempts to describe the conceptual development of our methodology. Once again we recall the solution of the diffusivity equation considering specific "inner boundary" conditions. The inner boundary conditions are generally associated with the well operational conditions — i.e., constant rate (wellbore) inner boundary or constant pressure (wellbore) inner boundary. In general, it can be suggested that well test (pressure transient) analysis is related with "constant rate" solutions of the diffusivity equation and decline curve analysis is associated with "constant pressure" solutions of the diffusivity equation.

For the case of variable rate or variable pressure inner boundary condition, Duhamel's principle states that the observed pressure drop is the convolution of the input rate function and the derivative of the constant-rate pressure solution — at \( t=0 \) the system is assumed to be in equilibrium (i.e., \( p(r, t=0) = p_i \)). Similarly, observed rate function is the convolution of the input pressure drop function and the derivative of the constant-pressure rate solution. For reference, the convolution integral is defined as:

\[
\Delta p_{wf}(t) = \int_{0}^{t} q(t-\tau)p_{cr}'(\tau) \, d\tau \quad \text{[Pressure solution for variable rate case]} \quad (11)
\]

\[
q(t) = \int_{0}^{t} \Delta p_{wf}(t-\tau)q_{cp}'(\tau) \, d\tau \quad \text{[Rate solution for variable pressure drop case]} \quad (12)
\]

Equations 11 and 12 can also be written in discrete form when a series of discrete changes in rate data and pressure drop data are present, as:

\[
\Delta p_{wf}(t) = \sum_{k=1}^{n} (q_k - q_{k-1})(p_{cr}(t-t_{k-1})) \quad \text{[Pressure solution for "discrete" variable rate case]} \quad (13)
\]

\[
q(t) = \sum_{k=1}^{n} (\Delta p_{wf,k} - \Delta p_{wf,k-1})(q_{cp}(t-t_{k-1})) \quad \text{[Rate solution for "discrete" variable pressure drop case]} \quad (14)
\]

van Everdingen and Hurst (1945) first initiated using Duhamel's principle (superposition) in analysis of variable rate well-test data. They utilized Duhamel's principle to obtain dimensionless wellbore pressure-drop function response for a continuously varying flow rate. The underlying idea was to introduce a method to convolve/superimpose the constant rate pressure function response with a continuous (smooth) rate profile to produce the variable rate wellbore pressure-drop function response. Theoretically, modern (model-based) pressure transient and production analysis methodologies are founded on the superposition principle where "constant rate" pressure solution is superposed with discrete changes in rate data (Eq. 13) to generate "variable rate" pressure drop data, and "constant pressure" rate solution is superposed with discrete changes in pressure drop data (Eq. 14) to generate "variable pressure" rate data.

Constant rate or constant pressure solutions are referred to as models — i.e., solutions which represent a physical condition, well configuration and reservoir boundary such as a finite conductivity fractured well solution or horizontal well with multiple fractures, etc. Petroleum industry is replete with models and current software tools are practical for the application of model-based analysis (superposition principle) to analyze pressure transient and production data. In essence, model parameters (e.g., permeability, fracture half-length, reservoir boundaries, etc.) are calibrated to generate relevant response function using superposition, and model response is then compared to actual data for history matching. This process is also called "forward modeling". Unfortunately, for the case of field data, it is inevitable that multiple models and various model parameters could yield similar (or same) history matches, and thus pointing out the issue of non-uniqueness. To deal with the issue of non-uniqueness, various diagnostic plots (e.g., "log-log" plot, Blasingame plot, flowing material balance plot, type curves, etc.) are simultaneously used throughout the history matching process to have more insight into model parameters.

Finally, it is worth to mention that the superposition principle is rigorous which means that for the exact inputs, we will obtain exact results. It is absolutely critical to mention that superposition principle requires "linearity" and for the case of non-linear conditions such as gas flow, multi-phase flow, pressure dependencies, etc., transformations (e.g., pseudopressure, pseudotime) are required to obtain "linearity" or numerical discretizations can be utilized. After describing superposition and its practical application, we will now present our concept of variable pressure decline curve analysis with superposition by
using empirical rate decline relations.

As mentioned earlier, our proposal for the variable pressure decline curve analysis is to use the empirical rate decline relations as the "constant-pressure" rate solution in Eq. 14 and to obtain the history match by using the discrete pressure drop changes. For example, the "constant-pressure power-law exponential" rate decline relation can be written along the lines of this definition as:

\[ q_{cp}(t) = (\hat{q} / \Delta p) \exp[-\hat{D}_i t^n - D_\infty t] \] ......................................................... (15)

Eq. 15 can be inserted into the superposition formula, Eq. 14, this gives:

\[ q(t) = \sum_{k=1}^{n} (\Delta p_{wf} - \Delta p_{wf}) \exp[-\hat{D}_i (t-t_{k-1})^n - D_\infty (t-t_{k-1})] \] ......................................................... (16)

Therefore, Eq. 16 can be used to compute/generate data which are affected by the changes in the pressure drop. We note again that any of the empirical rate decline relations can be substituted into Eq. 14 as the constant-pressure rate solution and then superposition can be used. At this point the key question to be asked is how to obtain the model parameters (such as b, D_i, n, etc.) of rate decline relations for the history matching process. For this purpose, we propose to use the \( D(t) \) and \( b(t) \) functions (Eq. 2 and Eq. 4) as we do for decline curve analysis with a modification. In this case, instead of using only time-rate data to compute \( D(t) \) and \( b(t) \), we will use pressure drop normalized rate data (i.e., \( q/\Delta p(t) \)). It is worth to note that pressure drop normalization serves only for an approximation, and it is not exact. For example, for the power-law exponential case, pressure drop normalized rate function can be expressed as

\[ q(t) / \Delta p(t) \approx (\hat{q} / \Delta p) \exp[-\hat{D}_i t^n - D_\infty t] \] ......................................................... (17)

Correspondingly, diagnostic "normalized" \( D(t) \) and \( b(t) \) functions can be formulated as:

\[ \hat{D}(t) \equiv -\frac{1}{q(t) / \Delta p(t)} \frac{d(q(t) / \Delta p(t))}{dt} \] ......................................................... (18)

\[ \hat{b}(t) \equiv \frac{d}{dt} \left[ \frac{1}{\hat{D}(t)} \right] \equiv \frac{d}{dt} \left[ \frac{q(t) / \Delta p(t)}{d(q(t) / \Delta p(t))/dt} \right] \] ......................................................... (19)

Based on the formulations above, model parameters associated with each rate decline relation can be calibrated using the diagnostic plots of \( D(t) \) and \( b(t) \). Once an initial estimate is obtained for the model parameters, further calibration may be required to achieve the history match. For the case of non-linearity such as gas flow, transformations are required to apply superposition principle. The following procedure can be followed to apply the methodology:

- Use any appropriate rate decline relation
- \( D(t) \) and \( b(t) \) data are calculated using pressure drop normalized rate data
- Establish decline curve model parameters using diagnostic plots (\( D(t) \) and \( b(t) \))
- Generate rate response by superposition of pressure drop data and the empirical rate decline model
- Generate forecast of production performance (a pressure scheme can be used to account for future changes in pressure)

In the next section, we will describe the application of this proposed methodology using field examples.
Application of the Methodology to a Field Example

In this section we present the application of the proposed methodology to a field example. Our rationale in the development of "variable pressure" decline curve analysis is essentially to analyze and forecast production when flowing pressures are not constant and the well production is relatively at an earlier stage (e.g., less than a year). To that end, the example that we present in this paper include gas production data for less than 200 days with declining pressure data. The example well presented in this study is producing in a high pressure/high temperature shale gas reservoir in North America.

First and foremost, we start with the assumptions for the application of the methodology. Our main assumption is that we are indirectly assuming that the "empirical" relations have some sort of "physical" meaning. In other words, the empirical rate decline relations can be considered as the "constant-pressure" rate solutions, and thus the superposition principle can be applicable. Although, there is no direct derivation of the rate decline relations found in the literature (except for the exponential which can be derived for slightly compressible fluid above the bubble point pressure case), it can be suggested that these relations are "lumped parameter" type models applicable to certain conditions (i.e., flow regimes, well configuration, etc.). The model parameters of each relation can be related to physical model parameters, and the paper by Ilk et al. (2011) presents an example of correlating decline curve model parameters to a physical solution.

We are also assuming that the system is linear — although this cannot be the case for our examples as it is highly suspected that pressure dependent properties (e.g., pressure dependent permeability, pressure dependent fracture conductivity, etc.) are present in the high pressure/high temperature shale gas reservoir from which our examples were taken. Although, we used pseudopressure transformation to account for the gas flow behavior, it can be possible that we have not completely linearized the system. Nevertheless, we will continue with only pseudopressure transformation, and in this paper we will only seek to validate our concept of superposition and variable-pressure rate decline curves. We will leave further verification for future study.

Another important point that is worth to mention is superposition principle is applicable when pressure and rate data are consistent. This means that if inconsistent data are present throughout production (e.g., increasing pressures and increasing rates, etc.), it is most likely that generated response would not be meaningful. Unfortunately, in the case of unconventional reservoir systems, there are a variety of sources which could create inconsistent rate and pressure data such as offset fracture interference, changes in tubulars, changes in surface equipment, liquid loading, effect of water rates, etc. In these cases, analyst must be vigilant and identify the issues prior to analysis — it is strongly recommended that a variety of diagnostic plots can be used for identifying the issues related with production data. If the issues related with production are not handled, application of superposition principle would not be meaningful.

Keeping these in mind, we start our proof of concept with the first example. The production data for this well include almost 170 days of daily rate and surface pressure data. Fig. 1 presents the production history of this well — it can clearly be seen that first 40 days of surface pressure data are not meaningful, therefore we are going to omit this data in our analysis. Other than that, it is observed that rates are maintained by controlling the drawdown yielding gradually decreasing pressure profile. Therefore, we can suggest that this example can be nominated as a good example of the proposed methodology.

In order to perform the methodology, an initial pressure estimate is required along with the pressure data. Ideally, bottomhole pressures are required for analysis and forecast. However, in this work we will use the flowing surface pressure data assuming that surface data are sufficient for analysis as this is a case of dry gas reservoir and surface to bottomhole conversion is not an issue. For our purposes, we will use the power-law exponential and modified hyperbolic rate decline relations to test the proposed methodology. It is possible to use the other available rate decline relations but again we leave this as an area for future study.

We compute the $D$-parameter and $b$-parameter data continuously from $q/\Delta p$ (time-rate-pressure) data as described earlier in Eqs. 18 and 19. Inconsistent and erroneous data could potentially affect the calculations of the $D$- and $b$- parameters. Although severe oscillations are observed in Fig. 2 (i.e., plot of computed $D$- and $b$- parameters versus time), we can still interpret the data and obtain model parameters of the corresponding rate decline relations. We then employ the superposition principle using first power-law exponential relation and next modified hyperbolic relation with the actual pressure drop data. It is again worth to mention that at this point we assume that the initial pressure is available to calculate the pressure drop data.

For the power-law exponential rate decline case, the history match is shown in Fig. 3. It can clearly be observed from the plot that the inconsistent data cannot be matched (especially, the early time surface pressure data). Overall, we have a good match of the data with the power-law exponential model. Another feature of the application of superposition is that we can extend pressures to an arbitrary pressure value in future. In other words, we do not have to assume constant pressure for the rate forecast. Instead we can extrapolate the line pressure if the actual pressure data are above the line pressure. Correspondingly, in our example, we extrapolate the pressure for the cast to a line pressure estimate of 1,000 psia for the forecast. In Fig. 3 and Fig. 4, we can see the effect of with and without pressure extrapolation on rate forecast. Specifically,
for the case of Well 1, the difference in the forecast is 0.84 Bscf for the power-law exponential model. Similarly, modified hyperbolic model is superposed with the pressure drop data and the history match and forecast are shown in Fig. 5 and Fig. 6. As observed, modified hyperbolic model yields very good match with rate data as well. It is noted that we seven (7) percent terminal decline to constrain the hyperbolic relation. Table 1 below summarizes the results of our analyses along with the model parameters:

<table>
<thead>
<tr>
<th>Well 1</th>
<th>EUR (BSCF)</th>
<th>n or b (dim.less)</th>
<th>$D_i$ (1/D)</th>
<th>$(q/\Delta p)_i$ (MSCF/D/psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLE (w $p_f$ extr)</td>
<td>5.82</td>
<td>0.194</td>
<td>1.96</td>
<td>1,114,608</td>
</tr>
<tr>
<td>PLE (w/o $p_f$ extr)</td>
<td>4.98</td>
<td>0.194</td>
<td>1.96</td>
<td>1,114,608</td>
</tr>
<tr>
<td>M.HYP (w $p_f$ extr)</td>
<td>5.75</td>
<td>1.0</td>
<td>0.115</td>
<td>115,057</td>
</tr>
<tr>
<td>M.HYP (w/o $p_f$ extr)</td>
<td>4.79</td>
<td>1.0</td>
<td>0.115</td>
<td>115,057</td>
</tr>
</tbody>
</table>

Since the pressure data involved in the analysis procedure, we can utilize various plots for more resolution such as logarithm of pressure drop normalized rate versus cumulative production plot and pressure drop normalized rate versus material balance time (on log-log scale) plot. For instance, logarithm of pressure drop normalized rate versus cumulative production formulation can serve as a "EUR" extrapolation plot. Fig. 7 presents the match for the power-law exponential case and Fig. 8 presents the case for the modified hyperbolic case with and without pressure extrapolation in the forecasts. The tail in the hyperbolic model corresponds to the terminal decline (i.e., exponential) portion of the equation. Furthermore, Fig. 9 presents the pressure drop normalized rate versus material balance time plot for the case of power-law exponential and modified hyperbolic rate decline relations and we observe very good matches of the data with models. This plot can help in identifying the flow regimes using rate decline relations, and might even indicate correlating behavior with model-based solutions (e.g., half slope, quarter slope, unit slope, etc.).

Finally, we utilize rate decline relations from traditional decline curve analysis point of view in Figs. 10 and 11 for power-law exponential case and in Figs. 12 and 13 for modified hyperbolic case. We note that for these cases only time-rate data are used for computing $D$- and $b$- parameters. Table 2 summarizes the model parameters for the results of time-rate analysis without considering pressure data. As can be observed, differences are present in the model parameters as well as in the EUR values. This is completely due to the fact that time-rate analysis is limited for constant pressure. In fact, our observations from other field examples, where constant pressures are observed throughout production, indicate that there is no difference between time-rate and time-rate-pressure analyses. To that end, it can be suggested that the importance of time-rate-pressure (or variable pressure rate decline analysis) is pronounced for the case of wells where pressures are above line pressure and changing with time.

<table>
<thead>
<tr>
<th>Well 1</th>
<th>EUR (BSCF)</th>
<th>n or b (dim.less)</th>
<th>$D_i$ (1/D)</th>
<th>$(q/\Delta p)_i$ (MSCF/D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLE (time-rate)</td>
<td>4.86</td>
<td>0.7</td>
<td>0.024</td>
<td>20,586</td>
</tr>
<tr>
<td>M.HYP (time-rate)</td>
<td>5.51</td>
<td>0.5</td>
<td>0.006</td>
<td>18,438</td>
</tr>
</tbody>
</table>

Summary and Conclusions

Summary: This work presents the conceptual development of a decline curve analysis methodology for the case of varying pressures. The primary objective of this work is to lay out the formulation of a variable pressure rate decline analysis methodology (or time-rate-pressure analysis) using "empirical" rate decline relations available in the literature. To that end, we propose to use the superposition principle to generate the rate response and then forecast production with pressure constraints. Diagnostic plots (i.e., computed $D$- and $b$- parameter versus time) are utilized for guidance on model parameters. However, in the proposed methodology diagnostic plots are computed by time-rate-pressure data.

We have shown in this work that to a certain extent, we can successfully apply superposition for variable pressure rate decline analysis and forecast production considering different pressure constraints. On the other hand, there are still key questions to be answered and we see this work as a start rather than an end of a research. These questions can be summarized as:
● How does the limitation regarding non-linearity affect superposition principle with empirical rate decline relation?
● Is it possible to apply the other rate decline relations to variable rate/pressure data?
● Is there a physical meaning which could be attributed to rate decline relations or if these relations have a statistical nature?
● How does the superposition principle work with synthetic data (i.e., simulated data for various well/reservoir configurations)?

These are the questions that we plan to work on for further study. As we have already mentioned before, this work should better be considered for a start of research.

References

Nomenclature

\( a_{Dng} \) = Model coefficient for the Duong time-rate model, \( D^{-1} \)
\( a_{LGM} \) = Model coefficient for the Logistic Growth time-rate model, \( D^{-1} \)
\( b \) = Arps' decline exponent (hyperbolic time-rate relation), (from time-rate formulation), dimensionless
\( \hat{b} \) = Arps' decline exponent (hyperbolic time-rate relation), (from time-rate-pressure formulation), dimensionless
\( \beta \) = "Beta" function (relates rate and derivative functions), dimensionless
\( c_t \) = Total compressibility, psi\(^{-1} \)
\( D \) = Reciprocal of the loss ratio (from time-rate formulation), \( D^{-1} \)
\( \hat{D} \) = Reciprocal of the loss ratio (from time-rate-pressure formulation), \( D^{-1} \)
\( D_{lim} \) = Terminal decline constant for the exponential time-rate relation, \( D^{-1} \)
\( D_i \) = Initial decline constant for the exponential and hyperbolic time-rate relations, \( D^{-1} \)
\( D_{\alpha} \) = Terminal decline coefficient for the Power-Law Exponential time-rate model, \( D^{-1} \)
\( \Delta p_{wf} \) = Bottomhole pressure drop, psi
\( EUR \) = Estimated ultimate recovery, BSCF
\( \phi \) = Porosity, fraction
\( G_p \) = Cumulative gas production, MSCF or BSCF
\( G_{p,\text{max}} \) = Maximum gas production (at a specified time limit), MSCF or BSCF
\( k \) = Formation permeability, md
\( K \) = Carrying Capacity for the Logistic Growth time-rate model, MSCF or BSCF
\( \mu \) = Viscosity, cp
\( n_{Dng} \) = Time exponent for the Duong time-rate model, dimensionless
\( n \) = Time exponent for the Power-Law and Stretched Exponential time-rate models, dimensionless
\( n_{LGM} \) = Time exponent for the Logistic Growth time-rate model, dimensionless
\( p \) = Pressure, psia
\( p_{cr} \) = Constant-rate pressure solution, psia/STB or psia/MSCF/D
\( p_i \) = Initial pressure, psia
\( p_{wf} \) = Surface pressure, psia
\( p_{of} \) = Bottomhole pressure, psia
\( q \) = Production rate, MSCF/D
\( q_{cp} \) = Constant-pressure rate solution, MSCF/D/psia or STB/D/psia
\( q_i \) = Initial rate for the exponential and hyperbolic time-rate models, MSCF/D
\( q_{il} \) = Initial rate coefficient for the Power-Law and Stretched Exponential time-rate models, MSCF/D
\( q_{it} \) = Initial rate coefficient for the Power-Law and Stretched Exponential time-rate models, MSCF/D
\( \frac{q}{\Delta p} \) = Initial rate coefficient for the Power-Law model (time-rate-pressure), MSCF/D/psi
\( q_{il} \) = Initial rate coefficient for the Duong time-rate model, MSCF/D
\( r \) = Distance in radial coordinates, ft
\( t \) = Production time, days
\( \tau \) = Time coefficient for the Stretched Exponential time-rate model, \( D^{-1} \)
Fig. 1 — Well 1 Production history plot -- Flowrate and surface pressure versus production time.

Fig. 2 — Well 1 $D$- and $b$-parameters versus time (computed using time-pressure-rate data).
Fig. 3 — Well 1 Production history match plot (power-law exponential rate decline model).

Fig. 4 — Well 1 Production history match plot (power-law exponential rate decline model) (late time).
Fig. 5 — Well 1 Production history match plot (modified hyperbolic rate decline model).

Fig. 6 — Well 1 Production history match plot (modified hyperbolic rate decline model) (late time).
Fig. 7 — Well 1 Pressure drop normalized rate and cumulative production plot (power-law exponential model).

Fig. 8 — Well 1 Pressure drop normalized rate and cumulative production plot (modified hyperbolic model).
Fig. 9 — Well 1 — Pressure drop normalized rate versus material balance time plot

Fig. 10 — Well 1 — Comparison between time-rate and time-rate-pressure rate decline analyses (power-law exp).
Fig. 11 — Well 1 -- Comparison between time-rate and time-rate-pressure rate decline analyses (power-law exp) late time.

Fig. 12 — Well 1 -- Comparison between time-rate and time-rate-pressure rate decline analyses (modified hyperbolic).
Fig. 13 — Well 1 -- Comparison between time-rate and time-rate-pressure rate decline analyses (modified hyperbolic) late time.