Abstract

The premise of this work is the development, validation, and application of a methodology to forecast production data in unconventional reservoirs where variable rate and pressure drop producing conditions are typically observed. In unconventional reservoirs, it is not common practice to maintain or even arrive quickly upon a constant flowing bottomhole pressure which is the primary assumption for the application of traditional time-rate decline curve analysis. As a result the application of traditional time-rate relations to these cases yields misleading results (i.e., EUR values) at best.

Our methodology involves the application of the rigorous convolution/superposition theory with the pressure drop normalized “empirical” rate decline relations as the “well/reservoir” model and the “measured/calculated” pressure drop data. Diagnostic plots of the loss ratio and the loss ratio derivative, or the so called “qDb” plots, are utilized to minimize the non-uniqueness inherent in the history matching process by providing guidance in the selection of rate decline model parameters. Following the selection of model parameters, the normalized rate decline relations are superposed with pressure (or pseudopressure) drop data to achieve a history match and, correspondingly, a production forecast.

Validation and application of the methodology is demonstrated using a variety of synthetic data generated to closely represent common operational practices in tight/shale oil and gas plays. A special emphasis is placed on handling non-linearities and performing sensitivities to account for future drawdown management. We show that the methodology effectively removes pressure variations from the rate history for each of the synthetic examples presented. We present our results using a variety of diagnostic and forecasting plots and both the magnitude of the EUR and the shape of the production profile for all cases.

Introduction

Decline curve analysis (DCA), or the extrapolation of only “time-rate” production data into the future, is one of the most commonly used techniques in the petroleum industry to estimate ultimate recovery (EUR) for producing wells. Essentially decline curve analysis refers to the calibration of a time-rate model, or decline curve, to a single well (or multiple wells) and extrapolating production to an abandonment limit to yield an estimate of ultimate recovery.
The simplicity of the method along with the fact that production from conventional reservoirs honors most of the underlying assumptions have led to Arps’ hyperbolic and exponential decline curve relations becoming the industry standard for production forecasts (Arps 1945). Johnson and Bollens (1927) laid the foundation for the exponential and hyperbolic relations when they introduced the loss ratio and the derivative of the loss ratio which are given below.

\[ D(t) = -\frac{1}{q(t)} \frac{dq(t)}{dt} \quad (1) \]
\[ \frac{1}{D(t)} = -\frac{q(t)}{dq(t)/dt} \quad (2) \]
\[ b(t) = \frac{d}{dt} \left[ \frac{1}{D(t)} \right] = -\frac{d}{dt} \left[ \frac{q(t)}{dq(t)/dt} \right] \quad (3) \]

The Arps’ exponential and hyperbolic equations were derived based on empirical observations of the above relations. For instance the exponential decline relation can be derived by assuming that the decline parameter or the loss-ratio (Eq. 1 or 2) is constant and solving the first order ordinary differential equation for rate. The exponential relation is given as:

\[ q(t) = q_i \exp[-D_i t] \quad (4) \]

Similarly, the hyperbolic equation can be derived by assuming that the derivative of the loss-ratio (Eq. 3), or “b-value”, remains constant and solving the second order ordinary differential equation. The hyperbolic relation is given below as:

\[ q(t) = \frac{q_i}{(1 + bD_i t)^{1/b}} \quad (5) \]

The foundation for this work begins with a discussion of the underlying assumptions governing the applicability of time-rate decline curve analysis. The most important assumption in relation to this work can be framed by considering the physics of fluid flow in a reservoir. Radial fluid flow as a function of time and position can be represented by the diffusivity equation given below:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial p}{\partial r} \right] = \frac{k}{\phi \mu c_o} \frac{\partial p}{\partial t} \quad (6) \]

In order to solve the above partial differential equation, a set of inner and/or outer boundary conditions and an initial condition must be specified. For instance the “constant-rate pressure” solution used extensively in pressure transient testing can be arrived at by assuming a constant rate inner boundary condition. Likewise, the “constant-pressure rate” solution can be derived by assuming a constant bottomhole pressure for the inner boundary condition. The “constant-pressure rate” solution forms the basis for both classic and modern decline curve analysis which inherently assumes a constant flowing bottomhole pressure throughout production.

The conditions for which Arps’ relations are applicable lead to the next considerations. While the exponential decline can be derived for the case of a closed reservoir under boundary dominated flow regime producing against a constant bottomhole pressure with the fluid above the bubble point, the applicability of the hyperbolic relation is less clear. Camacho and Raghavan (1991), Fetkovich (1990), and Ilk et al. (2009) have proposed attempts at validation of the hyperbolic equation for solution gas-drive, multi-layered gas, and gas flow during boundary dominated flow, respectively. An important assumption of each of the efforts was production under boundary dominated flow regime and a hyperbolic exponent, or b-value, between 0 and 1. This is in line with Arps’ original observation that the applicability of the hyperbolic equation was limited to reservoirs exhibiting boundary dominated flow.
The final considerations focus less on the foundations of the Arps’ equations themselves and more on their applicability in unconventional reservoirs (e.g., tight gas, shale gas, liquid-rich shales, and coalbed methane) which are the focus of this work. Ilk and Blasingame (2013) explain this by stating that we must be both realistic and practical when attempting to characterize production performance from systems where the permeability is on the order of 10–500 nd (or $10^{-6}$ to $500 \times 10^{-6}$ md) — the reservoir flow system is complex and although the induced (i.e. created) hydraulic fracture system enables (and dominates) the production performance, we have only the most rudimentary understanding of the flow structure in the hydraulic (and natural) fracture systems.

The ultra-low permeability nature of these reservoirs leads to extended periods of transient flow bringing the applicability of Arps’ equations into question. Indeed it has been well documented that the application of Arps’ hyperbolic equation to unconventional reservoirs yields $b$-values greater than 1 causing the decline model to be unbounded opening up the potential to significantly overestimate reserves (Rushing et al. (2007), Lee and Sidle (2010)). These revelations have led to the practice of “splicing” a terminal decline (i.e. exponential tail) onto the end of the hyperbolic decline model which was first proposed by Robertson (1988).

In light of the complex reservoir and flow conditions present for unconventional resource systems, various authors have derived additional time-rate relations in an attempt to better represent production behavior. For instance, Ilk et al. (2008, 2009) and Valkó (2009) independently introduced the Power-Law Exponential Model and Stretched Exponential model, while Clark et al. (2011) introduced the Logistic Growth Model and Duong (2011) contributed the Duong Model. At this time, each of the models summarized below can only be described as empirical while each tends to focus on a particular flow regime or characteristic behavior. These rate decline relations are given below:

\[
q(t) = \dot{q}_i \exp[-\tilde{D}_i t^n - D_{\alpha} t] 
\]  
\[
q(t) = \dot{q}_i \exp[\{-t / \tau\}^n] 
\]  
\[
q(t) = \frac{dG_p(t)}{dt} = \frac{K n_{LGM} a_{LGM} t^{(n_{LGM} - 1)}}{(a_{LGM} + t^{n_{LGM}})^2} 
\]  
\[
q(t) = q_t^{m_{paw}} \exp\left[\frac{a_{Daw}}{(1-m_{paw})}\left[\frac{1}{1-m_{paw}} - 1\right]\right] 
\]

One of the most significant limitations of time-rate (decline curve) analysis in unconventional reservoir systems is the assumption of a constant bottomhole pressure condition throughout the producing life of the well. Whereas this is often times not an issue for conventional systems, unconventional cases often exhibit variable pressure profiles as a result of changing operating conditions, choke management, and/or restricted rate production. The presence of a variable pressure profile is itself a violation of the decline curve theory for each of the models outlined above and, compounding the issue, wells often show extensive periods of constant production or very shallow decline profiles hampering the utility of traditional time-rate analysis as an effective tool for forecasting future production.

Ilk and Blasingame (2013) have previously demonstrated that superposition/convolution theory can conceptually be utilized with “empirical” rate decline relations to model production behavior for the case of variable pressure drop data. They presented field examples to validate the formulation and apply the methodology. However, they did not elaborate on the application and limitation of the methodology extensively examples and left these efforts for future work. To this end, the primary goal of this work is to continue Ilk and Blasingame work and provide theoretical background and discuss limitations of the methodology with numerical simulation (synthetic) examples.
Development of the Methodology

As a first step in the development of our methodology we will conceptually return to the radial flow diffusivity equation given by Eq. 6 and provide a brief discussion on model based production analysis.

\[
\frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial p}{\partial r} \right] = \frac{k}{\phi \mu c} \frac{\partial p}{\partial t} \quad (6)
\]

As with any second order partial differential equation, solutions to this equation require the selection of two boundary conditions and an initial condition which are associated with particular reservoir models and/or operating conditions. For this discussion we will concern ourselves with inner boundary conditions related to specific operating conditions, namely constant sandface rates and constant wellbore pressures, while we also assume that the system is at an equilibrium initial reservoir pressure at time \( t = 0 \) (i.e. \( p(r, t=0) = p_i \)).

Constant rate and pressure solutions to the diffusivity equation are typically solved to represent particular physical phenomena (fractured well, dual porosity, etc.), well configurations (vertical/horizontal wells, etc.), and reservoir boundaries. There is no shortage of these models in the petroleum engineering literature and a number of software applications are well suited to model based production and well test analysis. While analytical models can be derived according to a specific set of inner boundary condition assumptions, the question still remains concerning the handling of variable-rate/variable-pressure production.

This problem was first comprehensively addressed by van Everdingen and Hurst (1949) for the treatment of variable rate well-test data. They proposed the utilization of Duhamel’s principle (i.e., convolution integral) which, for the case of varying rates relevant to well testing applications, can be defined as the convolution of the input rate function and the derivative of a constant-rate pressure solution. Mathematical representations of Duhamel’s principal for continuous and discrete variations in rate are given below (Duhamel (1833)).

\[
\Delta p_{wf}(t) = \int_0^t q(t-\tau) p_{cr}(\tau) d\tau \quad (11)
\]

\[
\Delta p_{wf}(t) = \sum_{k=1}^n (q_k - q_{k-1}) (p_{cr}(t-t_{k-1})) \quad (12)
\]

In a similar manner we arrive at expressions for the rate solutions for variable pressure production. The solution for continuously varying pressure, which was presented as Eq. 11, and the solution for discrete pressure changes are shown below:

\[
q(t) = \int_0^t \Delta p_{wf} (t-\tau) q_{cr}(\tau) d\tau \quad (13)
\]

\[
q(t) = \sum_{k=1}^n (\Delta p_{wf,k} - \Delta p_{wf,k-1}) (q_{cr}(t-t_{k-1})) \quad (14)
\]

Eqs. 11-14 form the theoretical foundation for modern model based pressure transient and production analysis where dynamic model parameters are varied in an attempt to “history match” historical pressure data using superposed constant-rate pressure solutions and historical rate data by superposed constant-pressure rate solutions. A point not to be overlooked is the rigorous nature of the superposition principle meaning for exact inputs we will receive exact outcomes.

An additional point that warrants further discussion is the requirement of linearity when applying the superposition principle outlined above. For the case of slightly compressible liquid flow (i.e. undersaturated black oil reservoirs), direct application of Duhamel’s principle presumably satisfies the linearity condition, however non-linearities associated with gas flow, pressure dependent reservoir properties, and
non-Darcy flow may preclude direct application of the superposition principle. For the case of gas flow pressure and/or time transformations must be utilized in order to "linearize" the equations.

Wattenbarger and Ramey (1980) suggest that the pressure squared transformation, which assumes a real gas and a constant \( \mu z \) product, is applicable for pressures less than 2000 psi, however, care should be taken to qualify and quantify the range of pressure validity for real gas flow. To avoid potential misapplication, pseudo-pressure transformations as proposed by Al-Hussainy et al. (1966) can be used to linearize the equation. For the examples presented in this work the definition of normalized pseudopressure given in Eq. 15 was used to analyze gas wells. While a rigorous linearization requires a pseudotime transformation to adequately capture pressure dependent variations in gas viscosity and compressibility we will assume that the definition given below is adequate for our purposes.

\[
p_{pn} = \left( \frac{\mu_g z}{p} \right) \int_{p_0}^{p} \frac{p}{\mu_g z} dp
\]

In addition to nonlinearities present due to gas flow, various authors have diagnosed the presence of nonlinearities present in the form of pressure dependent permeability. Thompson et al. (2010) utilized specialized pseudopressure transformations to incorporate pressure dependencies when modelling overpressure reservoirs using analytical solutions while Okouma et al. (2011) studied their effect on EUR in Haynesville shale wells using numerical simulation. For our examples we will use a modified version of Eq. 15 given below as Eq. 16 where the pressure dependent permeability function will be defined by the exponential relation proposed by Yilmaz and Nur (1991) and defined by Eq. 17. The implications of each method will be visited for the validation of the methodology.

\[
p_{pn} = \left( \frac{\mu_g z}{kp} \right) \int_{p_0}^{p} \frac{k(p) p}{\mu_g z} dp
\]

As mentioned earlier, Ilk and Blasingame (2013) presented an initial field application of this methodology using only the modified hyperbolic and power-law exponential rate decline relations; however, in this study we will investigate the applicability of the methodology for each of the decline relations presented in Eqs. 7–10.

To this end, we can define the “constant-pressure modified hyperbolic”, “constant-pressure power-law exponential”, “constant-pressure stretched exponential”, “constant-pressure Duong”, and “constant-pressure logistic growth” decline relations as follows:

\[
q_{cp}(t) \approx \frac{q(t)}{\Delta p(t)} = \left( \frac{q_i}{\Delta p(t)} \right)_i \frac{1}{(1 + bD_i t)^{1/b}}
\]

\[
q_{cp}(t) \approx \frac{q(t)}{\Delta p(t)} = \left( \frac{\hat{q}}{\Delta p} \right) \exp[-\hat{D} t^\nu - D_z t]
\]

\[
q_{cp}(t) \approx \frac{q(t)}{\Delta p(t)} = \left( \frac{\hat{q}}{\Delta p} \right) \exp[-(t/\tau)^\eta]
\]
Now that we have defined our "constant-pressure proxies" using empirical decline relations, we can substitute them into the superposition formulation given by Eq. 14. The following five equations are the formulas used to generate and forecast the variable pressure production cases which are presented later in this work. Each corresponds in order to the relations presented above.

\[ q_{cp}(t) \approx q(t) / \Delta p(t) = (K / \Delta p)_t \frac{n_{LGM} a_{LGM} t^{(n_{LGM}-1)}}{(a_{LGM} + t^{n_{LGM}})^2} \]

\[ q_{cp}(t) \approx q(t) / \Delta p(t) = (q_1 / \Delta p) t^{m_{DG}} \exp \left[ \frac{a_{DG}}{(1 - m_{DG})} t^{(1-m_{DG})} - 1 \right] \]

The main point to address is how to establish the model parameters according associated with these equations. First of all, it is worthwhile to mention that the model parameters associated with the above equations are the same as those associated with the traditional time-rate equations introduced in the introduction section. The main differentiator is the use of of superposition to compute an equivalent constant pressure response. Accordingly we can determine our model parameters using diagnostic functions in a similar manner to the approach as that proposed by Ilk et al. (2008).

The calculation of the diagnostic signature is the first step of the proposed workflow and the definitions provided by Eqs. 28 and 29 can be used regardless of the pressure transformation assumption. The diagnostic signature and corresponding model match minimizes the non-uniqueness associated with blind matching of decline curve models, and once the model parameters are established a forecast can be generated according to any pressure scheme provided. Prior to demonstrating the methodology for the simulated cases, we outline a step-by-step procedure below.
Perform a comprehensive diagnostic analysis of the well (or group of wells) in question to quality control the data, understand the production history and any changes, and gather any subsurface information possible to guide the analysis.

First, assess whether any nonlinearities such as gas or multiphase flow, pressure dependent properties, non-Darcy flow, etc. are present and second, determine if the system can be linearized using an appropriate pseudo pressure and/or pseudo time transformation.

Calculate the \( D(t) \) and \( b(t) \) diagnostic data functions using the pressure drop normalized rate data (or pseudopressure drop normalized rate data as required).

Choose any appropriate rate decline relation.

Generate rate response by superposition of pressure drop data and the empirical rate decline model.

Generate a forecast of production performance (any pressure scheme can be used to account for future changes in pressure)

**Application of the Methodology using Synthetic Data**

In this section we validate and apply the proposed methodology with four simulated examples. The examples presented were constructed to represent well, reservoir, and fluid characteristics typically found in unconventional tight/shale oil and gas plays. Horizontal wells with multiple hydraulic fracture stages have become the development norm in the majority of the North American unconventional plays and, as such, a multi-fractured horizontal well model with infinite conductivity fractures was chosen as the well model for each case. Production histories from 183 days (= 6 months of production) to 500 days (3 years of production) with constant oil or gas rates and variable flowing pressures throughout were used in keeping with our goal of analyzing and forecasting wells where production practices prohibit the use of time-rate (decline curve) analysis.

When demonstrating the methodology it must be recognized that we are assuming that a pressure drop normalized empirical decline relation is an adequate proxy for a physical content pressure rate solution which allows us to apply the rigorous superposition principle to variable pressure production data. In other words, we assume that our pressure drop normalized decline relations serve as a “lumped parameter” approximations of specific characteristics of physical model solutions used in analytical and numerical production analysis. This assumption allows us to make quick forecast comparisons for proposed future operations using three or four decline curve model parameters as opposed to analyzing individual wells using more physically rigorous model based production analysis (i.e. analytical or numerical modelling) where data quality, time, and non-uniqueness may be a hindrance.

In addition to our assumption of an inherent physical meaning we are also assuming that our system is linear. In addition to gas flow, in many unconventional oil and gas plays it is highly suspected that their high pressure/high temperature (HP/HT) nature causes the presence of nonlinearities in the form of pressure dependent reservoir properties (geomechanical effects). As mentioned during the development of the methodology section of the work, we will use the pseudopressure transformation outlined in Eqs. 15 – 17 in an attempt to linearize our system and allow for the use of the superposition principle. The need for various transformations will be investigated in simulation example 3.
Another point that was addressed in depth by Ilk and Blasingame (2013) is the need for consistent data for the results of superposition to be meaningful. For this work we will be analyzing smooth synthetic data and accordingly data consistency will not be a factor, however, for field data diligent study of completion reports, production files, and the data itself as a part of a rigorous diagnostic analysis is required in order to ensure that the true reservoir behavior is being analyzed. This is especially relevant for unconventional reservoirs systems where production data is often inconsistent as a result of offset well production, changes in completions or surface equipment, liquid loading, etc. Ilk et al. (2011) provide a practical example of such a rigorous diagnostic workflow. Keeping the above assumptions in mind we will move on with our simulation examples.

Example 1: Multi-fractured Horizontal Gas Well

For this example 500 days of dry gas production from a multi-fracture horizontal well in a homogeneous reservoir was simulated. The only (expected) nonlinearity for this case is gas flow and as a result we will use pseudopressures calculated using Eq. 15 for this example. The data for this example is meant to be representative of a well exhibiting a long period of controlled drawdown (3 years) while maintaining rates...
at an almost constant level. The production data for this well is presented in Figure 1 and the reservoir, fluid, and well properties are provided in Table 1.

Following our proposed methodology, we calculate the pseudopressure drop normalized rate data using the initial reservoir pseudopressure and bottomhole flowing pseudopressures from the simulation. Note that in practice the lack of readily available bottomhole pressure data will require some means of converting flowing surface pressures to bottomhole conditions. We next calculate the $D$- and $b$- parameter data as a function of time according to Eqs. 28 and 29. For this work we calculate this data using the algorithm provided by Bourdet (1989) and, for a second resolution, a smoothing spline algorithm provided by
Pollock. A full discussion into various numerical differentiation algorithms is a subject of a future work. This data along with the diagnostic model fits for each decline relation are shown in Figure 2. We note that the diagnostic match for each relation is fairly good and the corresponding match on a semi-log plot of gas rate versus production time is shown in Figure 3. We see that each of the models captures the approximately constant 3,000 Mscf/D production trend the entire variable pressure history of the well. After reaching the end of the simulated data the last historic pressure value of 3,050 psi is used to extrapolate the models and forecast future production at which time the models behave according to their traditional time-rate formulation (i.e. constant bottomhole pressure assumption). Other plots, such as the log-log plot of pseudopressure drop normalized rate versus material balance time plot shown in Figure 4, can also be created in order to cross check the rate-time match and to assess flow regime changes using
rate decline relations. While also the subject of another work, supplementary plots such as these could be used to investigate correlating behavior between decline relations and analytical and/or numerical solutions.

Figs 5–7 show isolated plots of the modified hyperbolic history match, thirty year forecast, and pseudopressure drop normalized rate versus cumulative production plot respectively. As is often the case with unconventional wells early in the life of production, pressure drawdown is managed for some time until a constant line pressure is reached. Using traditional time-rate decline analysis it is not possible to incorporate additional drawdown (or changes in pressure profile) planned in the future. With our formulation, it is possible to include a planned future pressure extrapolation into the forecast. The dashed line displayed in each of the plots shows the application of the methodology including a planned future pressure extrapolation to a line pressure of 1,500 psi. As we can see, the inclusion of the pressure extrapolation causes a material impact on the thirty year EUR. Table 3 provides a summary of EUR values for each of the models fit for this example.

<table>
<thead>
<tr>
<th>Example 1</th>
<th>(q/(Δp)) or ((K/Δp)) (MSCF/D/psi) or (MSCF/psi)</th>
<th>(n) or (b) or (m) (dim.less)</th>
<th>(D_i) or (a) (1/D)</th>
<th>(D_{lim}) (percent/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M.HYP</td>
<td>1.3</td>
<td>2.25</td>
<td>0.020</td>
<td>7.0</td>
</tr>
<tr>
<td>PLE</td>
<td>13.5</td>
<td>0.12</td>
<td>1.800</td>
<td>N/A</td>
</tr>
<tr>
<td>SEM</td>
<td>18.4</td>
<td>0.11</td>
<td>0.001</td>
<td>N/A</td>
</tr>
<tr>
<td>DNG</td>
<td>1.8</td>
<td>1.07</td>
<td>0.970</td>
<td>N/A</td>
</tr>
<tr>
<td>LGM</td>
<td>2500</td>
<td>0.70</td>
<td>700</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Isolated plots displaying the same three plots for the power-law exponential (PLE), stretched exponential (SEM), duong (DNG), and logistic growth models (LGM) are displayed in Figs. 8–10, 11–13, 14–16, and 17–19 respectively. Through examination of the plots we can conclude that each of the empirical decline relations outlined in our introduction section can be applied using our methodology. We may also conclude that the pseudopressure transformations employed for this example adequately linearize the system which is evident by our match both on the diagnostic plots and our time-rate profile throughout the life of the well. A summary of the parameters associated with each model is included in Table 2.

Example 2: Multi-fractured Horizontal Gas Well with Pressure Dependent Permeability

For this example we again simulate production from a multi-fracture horizontal well in a homogeneous reservoir, however, for this case we include pressure dependent permeability effects in our model with permeability assumed to be the exponential function of pressure defined in Eq. 17. Due to the added nonlinearity (i.e. pressure dependent permeability or geomechanical effects) we will use the modified pseudopressure definition given by Eq. 16. For the pseudopressure calculations we will assume that the initial permeability and permeability modulus are known a priori which will not be the case for field production data. We will discuss the limitations of this assumption in the coming paragraphs. The data for this example is meant to be representative of HP/HT unconventional systems where operators controlled drawdown to maintaining rates and avoid pre-mature losses in productivity. The production data for this well is presented in Figure 20 and the reservoir, fluid, and well properties are provided in Table 4.

In order to move forward with our time-rate-pressure methodology we calculate the pseudopressure drop normalized rate data using the initial reservoir pseudopressure and bottomhole flowing pseudopressures from the simulation which were again calculated using Eqs. 16 and 17. It must be again noted that an a priori knowledge of the initial reservoir permeability and the permeability modulus, which governs the shape of the exponential function defining the $k(p)$ relationship, is required to calculate pseudopressures according to these equations. In the absence of additional information regarding the permeability
such as DFIT’s, core perms, or geomechanical lab reports this effectively adds another two “history match” parameters to each of the time-rate-pressure models. For this example we will limit ourselves to using the known simulation input for the pseudopressure calculations in an attempt to conceptually validate the pseudopressure transformation to match variable pressure production data. A full assessment of the inclusion of pressure dependent permeability for field production data is subject to another work.

The calculate $D(t)$ and $b(t)$ data along with the diagnostic model fits for each decline relation are shown in Figure 21. After using our diagnostic plotting functions to determine our model parameters we further adjust our initial production rates as needed to arrive at a model match for each decline relation which is shown in Figure 22. We see that each of the models adequately captures the approximately constant 2,000 Mscf/D production trend the entire variable pressure history of the well. After reaching the end of the simulated data the last historic pressure value of 5,545 psi is used to extrapolate the models and forecast future production at which time the models behave according to their traditional time-rate formulation (i.e.

![Figure 21](image1.png)  
**Figure 21**—Simulation Example 2 – $D$- and $b$- parameter versus time (computed using pseudopressure normalized rate data as input to the Bourdet and Smoothing Spline algorithm)

![Figure 22](image2.png)  
**Figure 22**—Simulation Example 2 – Time-Rate-Pressure history match plot (All Models)

![Figure 23](image3.png)  
**Figure 23**—Simulation Example 2 – Pseudopressure drop Normalized Rate versus Material Balance Time (All Models)

![Figure 24](image4.png)  
**Figure 24**—Simulation Example 1 – Time-Rate-Pressure history match plot with and without pressure extrapolations (Modified Hyperbolic Model)

![Figure 25](image5.png)  
**Figure 25**—Simulation Example 2 – Time-Rate-Pressure forecast plot with and without pressure extrapolations (Modified Hyperbolic Model)
constant bottomhole pressure assumption). An interesting note from this model trend, as well as other un-presented examples incorporating pressure dependent permeability, is that the distribution in 30 year EUR values across all of the models is quite narrow. As before we seek to validate our model match using a variety of plots such as the log-log plot of pseudopressure drop normalized rate versus material balance time plot shown in Figure 23 where we see that each of the models achieves an excellent match of the production history.
As before we provide isolated plots of the history match, thirty year forecast, and pseudopressure drop normalized rate versus cumulative production for the modified hyperbolic (HYP), power-law exponential (PLE), stretched exponential (SEM), duong (DNG), and logistic growth models (LGM) are displayed in Figs. 24–26, 27–29, 30–32, 33–35 and 36–38 respectively. The adequacy of the pseudopressure transformations in linearizing the system, again within the confines of the previously disclosed assumptions, is validated as each of the models is able to accurately capture the constant pressure production.
history as a result of the defined pressure model. The dashed line, representing an additional period of drawdown management, is extended to a constant bottomhole line pressure of just under 3,000 psi for this well. The model forecasts and the summary provided in Table 6 again show that additional drawdown has a material impact on both the magnitude of the 30 year EUR and the shape of the decline profile. Table 5 provides a summary of EUR values for each of the models fit for this example.
Isolated plots displaying the same three plots for the power-law exponential (PLE), stretched exponential (SEM), duong (DNG), and logistic growth models (LGM) are displayed in Figs. 8–10, 11–13, 14–16, and 17–19 respectively. Through examination of the plots we can conclude that each of the empirical decline relations outlined in our introduction section can be applied using our methodology. A discussion of the adequacy of the pseudopressure transformation will be the subject of the next simulation example. A summary of the parameters associated with each model is included in Table 2.
Example 3: Simulation Case Illustrating the Effect of Non-linearities

The goal of this example is to explore the effectiveness and necessity of using pseudopressure and pseudopressure with the inclusion of pressure dependent permeability when modeling systems with suspected nonlinearities. The same well, reservoir, and fluid settings and the resulting data described in simulation example 2 will again be used for this example. The first point to note is that this data was generated using a nonlinear numerical simulation with pressure dependent permeability included and we would expect to need pseudopressure transformations incorporating these nonlinearities to be a necessity.
To begin we will calculate our diagnostic data functions (i.e. \( D(t) \) and \( b(t) \)) using pressure drop normalized rate data as before, however, this time we will perform the calculation using three different pressure assumptions. The first used the normalized pseudopressure drop correcting for pressure dependent permeability (the same as Example 2), the second calculation used pseudopressures calculated according to Eq. 15, and the third used the raw bottomhole pressure data. The plot in the top left hand corner of Figure 39 shows an overlay of each of the diagnostic data streams. Only slight differences in
the diagnostic behavior using the pressure and pseudopressure transformations exist, however, when pressure dependent permeability is included a marked change in the diagnostic behavior occurs. It is worth it to note that the smoothing spline algorithm using the same amount of smoothing was used for each calculation to ensure consistency in the evaluation.

Table 5—Summary of decline model parameters for All Models (Example 2)

<table>
<thead>
<tr>
<th>Example 2</th>
<th>((q/\Delta p)_i) or ((K/\Delta p)_i) (MSCF/D/psi) or (MSCF/psi)</th>
<th>(n) or (b) or (m) (dim.less)</th>
<th>(D_i) or (\tau) or (a) (1/D)</th>
<th>(D_{lim}) (percent/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M.HYP</td>
<td>3.33</td>
<td>2.95</td>
<td>0.15</td>
<td>7.0</td>
</tr>
<tr>
<td>PLE</td>
<td>490</td>
<td>0.05</td>
<td>5</td>
<td>N/A</td>
</tr>
<tr>
<td>SEM</td>
<td>18400000000</td>
<td>0.0153</td>
<td>(1 \times 10^{-8})</td>
<td>N/A</td>
</tr>
<tr>
<td>DNG</td>
<td>3.35</td>
<td>1.02</td>
<td>0.77</td>
<td>N/A</td>
</tr>
<tr>
<td>LGM</td>
<td>35000</td>
<td>0.68</td>
<td>6000</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 6—Time-Rate-Pressure Analysis Results for All Models (Example 2)

<table>
<thead>
<tr>
<th>Example 2</th>
<th>EUR (w/o pressure extr) (BSCF)</th>
<th>EUR (w/pressure extr) (BSCF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M.HYP</td>
<td>7.71</td>
<td>13.05</td>
</tr>
<tr>
<td>PLE</td>
<td>9.30</td>
<td>14.51</td>
</tr>
<tr>
<td>SEM</td>
<td>9.56</td>
<td>14.86</td>
</tr>
<tr>
<td>DNG</td>
<td>9.38</td>
<td>14.62</td>
</tr>
<tr>
<td>LGM</td>
<td>9.31</td>
<td>14.53</td>
</tr>
</tbody>
</table>

The diagnostic behavior using the pressure and pseudopressure transformations exist, however, when pressure dependent permeability is included a marked change in the diagnostic behavior occurs. It is worth it to note that the smoothing spline algorithm using the same amount of smoothing was used for each calculation to ensure consistency in the evaluation.

The remaining plots in Figure 39 display the best diagnostic matches in order to establish the model parameters for each of the decline models. Because we are assuming that our pressure drop normalized decline models are adequate proxies for the physical reservoir behavior, a reasonable match of the diagnostic data should theoretically yield a match of our time-rate profile. Observing the \(D(t)\) and \(b(t)\) data in each of the plots in Figure 39 we note that both appear to be declining in a nearly linear (power-law)
fashion throughout the production history of this well. As a result of this characteristic behavior we chose to illustrate our time-rate history match using the power-law exponential decline model.

After establishing the $n$ and $D_i$ parameters for our power-law exponential model we tweak only the initial rate in order to arrive at the time-rate history match shown in Figure 40. It is immediately apparent that the only acceptable model match is the model including the effects of pressure dependent permeability in the pseudopressure transformation, which is the same as that provided in Figure 27. The other two models overlay each other, however they diverge significantly from the actual time-rate production data suggesting that their respective pressure assumptions did not adequately linearize the system for the application of superposition. The model parameters for each of the model matches are summarized in Table 7 below.
Conceptually this example illustrates the need to diligently address all non-linearities through the use of appropriate pseudopressure transformations. The impact of the initial permeability limitation is a point that needs to be investigated further in addition to the possibility of numerical discretization to include nonlinear behavior.

Example 4: Simulation Case with Variable Drawdown Magnitudes

This final simulation example aims to demonstrate a possibly practical application of our proposed methodology as a tool to compare the effect of future operational practices on the production forecast of a black oil well. The simulation settings are summarized below in Table 8 and an illustration of the data is shown in Figure 41. We ran the simulation for 183 days (≈ 6 months) according to a managed pressure drawdown schedule. The drawdown at the end of the simulation was approximately 3,290 psi or ≈ 35 percent which produced essentially constant rates around 400 Stb/D. The effect of different future drawdown assumptions on the model forecast will be studied as part of this example.

A question often asked, and one not addressed by empirical time-rate decline analysis, is how to forecast a scenario where constant rates prevail and a constant line pressure has not yet been reached. Even if a shallow decline trend where established inclusion of the effects of further drawdown is not a possibility and the forecast itself would be misleading. Of course, rigorous analytical or numerical model based analysis can be performed; however, data limitations and/or time constraints often make this an
unfeasible exercise. We propose our methodology as a quick way to study the effect of future drawdown strategies on the magnitude of the EUR and the shape of the production profile.

The methodology demonstrated for the previous simulation examples does not change, therefore we begin by calculating the $D(t)$ and $b(t)$ data using the pressure drop normalized rate data from the simulation. Since we are simulating black oil production we assume that the system is adequately linear using the untransformed raw pressure data. Figure 42 shows the diagnostic match for both the modified hyperbolic model and the power law exponential model. As a result of the apparent declining behavior of the diagnostic data led we conclude that the power-law exponential rate decline model was a more appropriate representation of the wells production history.
Figure 42—Simulation Example 4 – D- and b- paremeter versus time (computed using pressure normalized rate data as input to the Bourdet and Smoothing Spline algorithm)

Figure 43—Simulation Example 4 – Time-Rate-Pressure history match plot (Power-law Exponential Model)
After establishing our $n$ and $D_t$ parameters diagnostically we adjust our initial rate value to land upon the match shown in Figure 43. The model parameters associated with this match are also summarized in Table 9. As an initial forecast, and to provide a benchmark for our sensitivities to come later, we forecast the well assuming a no additional drawdown for the remainder of the life of the well ($p_{wf} = 5,710$ psi after 6 months of production) which yields an EUR value of approximately 794 Mstb.

As a sensitivity study we next examined the effect of various drawdown magnitudes followed by a constant line pressure after 4 years of production history. We assumed drawdown magnitudes ranging from 50 to 80 percent in 10 increments of 10. We first need to validate that our history match is still valid and that we are generating reasonable rate responses as a result of our pressure assumptions. We provide our history match visualization as Figure 44. It is evident, even on the narrow time frame shown on the plot, that the greater our drawdown the more sustained our rates tend to be which is consistent with our expectations.

Figure 45 shows the 30 year pressure and rate forecasts for each of the drawdown scenarios where we can clearly see the effect of the different scenarios in the rate profile. As before we can also create other plots to visualize our analysis such as the pressure drop normalized rate versus cumulative production plot shown in Figure 46. Finally we can also examine the effect on the magnitude of the EUR for various drawdown cases by plotting the cumulative production profiles as in Figure 47. From this plot, and the summary in Table 10, we can quickly see the substantial effect that the drawdown assumption has on the 30 year EUR value. For this case the difference between the base case to the 80 percent drawdown scenarios is nearly 1000 Mstb.

<table>
<thead>
<tr>
<th>Example 4</th>
<th>$(q/Dp)_{s}$ (STB/D/psi)</th>
<th>$n$ (dim.less)</th>
<th>$D_t$ (1/D)</th>
<th>$D_s$ (1/D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLE</td>
<td>1010000</td>
<td>0.029</td>
<td>14</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 9—Time-Rate-Pressure Analysis Results for the Power-law Exponential Model (Example 4)

Figure 44—Simulation Example 4 – Time-Rate-Pressure history match plot for various drawdown magnitudes (Power-law Exponential Model)
As a result of this simulation example it is suggested that our methodology could provide a quick means of both qualitatively and quantitatively assessing the effect of future operational strategies for wells exhibiting constant rate production profiles and not yet at a constant line pressure. This is especially helpful given the limiting assumptions (constant $p_{wf}$) of traditional time-rate analysis.

Figure 45—Simulation Example 4 – Time-Rate-Pressure forecast plot for various drawdown magnitudes (Power-law Exponential Model)

Figure 46—Pseudopressure drop Normalized Rate versus Cumulative Production for various drawdown magnitudes (Power-law Exponential Model)
Summary and Conclusions

This work presents the conceptual development and validation of a variable pressure decline curve methodology using a variety of synthetic data cases generated to reflect common operating conditions in unconventional oil and gas plays. The primary objective of the work was to develop a methodology to quickly forecast wells exhibiting extended periods of constant production as a result of managed pressure drawdown in wells prior to reaching a constant line pressure. Superposition of pressure drop normalized “empirical” decline curve with the variable pressure drop data were used to accomplish our objective with diagnostic plots (i.e. computed $D$- and $b$- parameter data versus time) used to guide the selection of model parameters.

In this work we have successfully applied the methodology to gas production with and without pressure dependent permeability and black oil production with sensitivities performed on future drawdown management. In addition to the successful application of the superposition methodology, we also showed a case where nonlinearities present in the system that requires the use of pseudopressure transformations to obtain a meaningful forecast. Finally, although we believe this work provides a solid conceptual foundation, some questions still remain to be answered. These questions are summarized as follows:

- Is there a way to linearize a system with pressure dependent permeability without requiring an a priori knowledge of the system permeability?

Table 10—Time-Rate-Pressure Analysis Results for All Models (Example 4)

<table>
<thead>
<tr>
<th>Example 4</th>
<th>EUR (MSTB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case (~35% drawdown)</td>
<td>794.1</td>
</tr>
<tr>
<td>50% drawdown at 4 years</td>
<td>1074.1</td>
</tr>
<tr>
<td>60% drawdown at 4 years</td>
<td>1283.9</td>
</tr>
<tr>
<td>70% drawdown at 4 years</td>
<td>1494.2</td>
</tr>
<tr>
<td>80% drawdown at 4 years</td>
<td>1705.8</td>
</tr>
</tbody>
</table>

Figure 47—Simulation Example 4 – Cumulative production profiles for various drawdown magnitudes (Power-law Exponential Model)
- How does this apply to field data? A comprehensive study across a variety of wells should be the subject of a future work.
- Is there any physical meaning which could be attributed to the rate decline relations? Are they statistical in nature? Can we correlate the model parameters to lumped well/reservoir parameters?

**Nomenclature**

- $a_{Dng}$ = Model coefficient for the Duong time-rate model, D$^{-1}$
- $a_{LGM}$ = Model coefficient for the Logistic Growth time-rate model, D$^{-1}$
- $b$ = Arps’ decline exponent (hyperbolic time-rate relation), dimensionless
- $c_f$ = Reservoir compressibility, psi$^{-1}$
- $c_t$ = Total system compressibility, psi$^{-1}$
- $D$ = Reciprocal of the loss ratio, D$^{-1}$
- $D_{lim}$ = Terminal decline constant for the exponential time-rate relation, D$^{-1}$
- $D_i$ = Initial decline constant for the exponential and hyperbolic time-rate relations, D$^{-1}$
- $\dot{D}_i$ = Decline coefficient for the Power-Law Exponential time-rate model, D$^{-1}$
- $D_\infty$ = Terminal decline coefficient for the Power-Law Exponential time-rate model, D$^{-1}$
- EUR = Estimated ultimate recovery, Bscf or Mstb
- $h$ = Net formation thickness, ft
- $\gamma_g$ = Specific gas gravity, dimensionless (air = 1)
- $\gamma_o$ = Specific oil gravity, dimensionless (water = 1)
- $\gamma$ = Permeability Modulus, psi$^{-1}$
- $k$ = Formation permeability, md
- $k_i$ = Initial Formation permeability, md
- $K$ = Carrying Capacity for the Logistic Growth time-rate model, Bscf or Mstb
- $(K/\Delta p)_i$ = Initial rate coefficient for the variable pressure logistic growth model (time-rate-pressure), Mscf/D/psi or Stb/D/psi
- $L_w$ = Horizontal Well length, ft
- $m_{Dng}$ = Time exponent for the Duong time-rate model, dimensionless
- $n$ = Time exponent for the Power-Law and Stretched Exponential time-rate models, dimensionless
- $n_f$ = Number of hydraulic fractures, dimensionless
- $n_{LGM}$ = Time exponent for the Logistic Growth time-rate model, dimensionless
- $\phi$ = Porosity, fraction
- $p$ = Pressure, psia
- $\Delta p_{wf}$ = Bottomhole pressure drop, psi
- $p_{cr}$ = Constant-rate pressure solution, psia/Stb/D or psia/Mscf/D
- $p_i$ = Initial pressure, psia
- $p_{sf}$ = Surface pressure, psia
- $q$ = Production rate, Mscf/D or Stb/D
- $q_{cp}$ = Constant-pressure rate solution, Mscf/D/psi or Stb/D/psi
- $q_i$ = Initial rate for the exponential and hyperbolic time-rate models, Mscf/D or Stb/D
- $\dot{q}_{i i}$ = Initial rate coefficient for the Power-Law and Stretched Exponential time-rate models, Mscf/D or Stb/D
- $q_1$ = Initial rate coefficient for the Duong time-rate model, Mscf/D or Stb/D
- $(q/\Delta p)_i$ = Initial rate coefficient for the variable pressure decline model (time-rate-pressure), Mscf/D/psi or Stb/D/psi
- $r$ = Distance in radial coordinates, ft
\begin{align*}
r_w & = \text{Wellbore radius, ft} \\
P_s & = \text{Oil saturation, fraction} \\
P_o & = \text{Oil saturation, fraction} \\
S & = \text{skin factor, dimensionless} \\
T_r & = \text{Reservoir Temperature, } \circ{\text{F}} \\
t & = \text{Production time, days} \\
\tau & = \text{Time coefficient for the Stretched Exponential time-rate model, D}^{-1} \\
\mu_g & = \text{Viscosity, cp} \\
x_f & = \text{Effective fracture half length, ft} \\
z & = \text{gas-law deviation factor, dimensionless}
\end{align*}

**References**


Pollock, D.S.G.. Smoothing with Cubic Splines. Available from the Department of Economics, Queen Mary College, University of London, Mile End Road, London E1 4 NS