Abstract

This study introduces a novel approach to model the hydraulic fractures in a shale reservoir using a common stochastic method called “random-walk.” The goal of this work is to capture part of the “complexity” of a fracture/fracture network that has been generated by a hydraulic fracturing treatment and to attempt to characterize this fracture network using reservoir performance signatures.

The steps involved in this work are:

- Stochastic generation of a “random-walk” fracture pattern constructed as a scaled numerical model.
- Assessment of the “random-walk” fracture using sensitivity analyses which consider the following elements:
  - The tortuosity (i.e., the actual length to ideal length ratio)
  - The tendency to branch (or split).
  - The number of branching stages — the number of branches was held constant for a given set of cases.
- Comparison of the mass rate and beta mass rate-derivative performance of the various “random-walk” fracture cases compared to the “standard” model of a planar fracture.

The primary results of this work are:

- Generation of pressure distributions (maps) at given times (i.e., “time slices”) to qualitatively assess each complex-pattern during transient production. The pressure distribution figures (i.e., maps) are used to qualitatively determine the presence of fracture interference(s) and to identify a time interval where those interferences occur.
- Creation of a graphical correlation of reservoir performance in terms of cumulative recovery as a function of the fracture volume and “fracture complexity” (i.e., the number of branches).
- Creation of an empirical correlation between the number of branches in a given fracture pattern and the value of the mass rate beta-derivative during transient flow (we observed that the mass rate beta-derivative is essentially constant during transient flow regardless of the fracture network...
configuration, as such this constant value of the mass rate beta-derivative was selected for correlation).

This work provides an alternative description of hydraulic fractures in unconventional shale-gas reservoirs which, in concept, captures the complexity of the hydraulic fracture as a stochastic fracture network.

Early-time rate performance is believed to be an indicator of the geometry of the hydraulic fracture pattern. A fracture with a higher level of “complexity” yields higher values of mass rate beta-derivative when the fractures components are interfering with each other. Therefore, mass rate curves could be used as a diagnostic tool that helps the identification of the fracture geometric features.

Introduction
In the context of unconventional reservoirs, we are faced an increasing complexity of reservoir geometries which do not necessarily have analytical solutions — however; the ever-increasing capacity of computational resources has encouraged the use of numerical simulators to solve ever more complex problems in flow geometry, phase behavior, as well as fundamental flow behavior at the nano-scale.

In conventional and tight gas reservoirs the hydraulic fractures created by fluid injection are modeled by a single plane propagating from the point of fluid entry. This has been proven to be reasonably accurate in small-scale and so-called “mine-back” tests. However, with development of ultra-low permeability reservoirs and in particular, shales, the guiding concept has changed from “planar” fractures to much more “complex fractures,” often thought (or assumed) to follow the natural planes of weakness in shales. There are also other concepts whereas shale reservoirs are often over-pressured and require very high fracturing pressures, we may also be creating primarily vertical fractures (as theory suggests), but also fractures with a horizontal fracture component.

In addition, with the wide use of microseismic monitoring of hydraulic fracture treatments, we tend to observe quite complex patterns of the microseismic events, suggesting that we are creating quite complex hydraulic fractures geometries. Our hypothesis is that a somewhat random fracture pattern is created by hydraulic fracturing in shales, and we recall that our goal is to attempt to quantify the behavior of these conceptual fracture patterns for comparison with base models (i.e., the planar fracture) as well as actual field performance (discussed, but not pursued in this work).

Guiding Assumptions
The two-dimensional fracture propagation models proposed by Daguier et al. 1996; Mastorakos et al. 2003 use a probabilistic approach for a tensile (mode I) fracture growth. Although this method was applied to the evolution of micro-cracks near the crack tip, a similar methodology is followed to numerically generate hydraulically induced fractures of a larger extent. This approach assumes that a fracture grows from an initial point (hydraulic fluid injection zone for example) into a media where heterogeneities are uniformly-spatially distributed. In simple terms, we assume that the $xy$-plane contains a series of discrete integer points $(x, y) \in \mathbb{Z} \times \mathbb{Z}$, and a fracture can grow from a point to the next when the minimum critical fracturing stress $\sigma_c$ is locally exceeded. Considering that the mechanical properties are distributed in the two-dimensional porous media according to given probabilistic laws, the fracture growth direction could be randomly directed according to those distributed laws. As a first approach, we suppose that hydraulic fractures grow orthogonally to the least principal stress direction and that their tendency to deviate from that plane is controlled by the distribution of heterogeneities (assumed uniform). Under these assumptions, the probability for the fracture to deviate in a given direction is constant. The process is mathematically known as a “random-walk” path.
To summarize, the assumptions of this work are:

- Uniform distribution of heterogeneities that cause a variation of geomechanical properties such as:
  - In-situ stress field
  - Fracture initiation pressure
  - Elastic moduli (Young’s moduli and Poisson’s ratios)
- No interaction with natural fractures:
  - Natural fractures are not modeled
  - The pattern shape does not depend on the arrangement of natural fractures
- Fractures are modeled in a 2-D plane (invariance of the structure vertically along the z-axis)

The complex fracture geometries that will be generated will help us to quantify and assess the effect of each geometric feature (tortuosity, splitting, orientation of branches, and number of branches) on the flow behavior. As validation, we compare the pressure and rate profiles generated using the random-walk fracture patterns with the base case of a planar fracture. Our goal is to assess the hypothesis that a single plane fracture will overestimate the production performance in shale gas reservoir — and to understand the circumstances where the planar fracture case may have value as well as limitations.

**Mathematical Formulation**

The fracture trajectory is a 2-dimensional discrete density function, $F(x,y)$, which is defined as:

$$
\begin{align*}
F(x,y) &= \begin{cases} 
1 & \text{if } (x,y) \in \text{Fracture} \\
0 & \text{if } (x,y) \in \text{Matrix}
\end{cases} 
\end{align*}
$$

The function is assumed to start from the origin:

$$F(x_0,y_0) = F(0,0) = 1$$

Then, we iteratively define the function for a certain step $n \in \mathbb{N}$:

$$
\begin{align*}
F(x_{n+1},y_{n+1}) &= F(x_n + 1,y_n) \text{ with a probability } P_{x+1} \\
F(x_{n+1},y_{n+1}) &= F(x_n,y_n + 1) \text{ with a probability } P_{y+1} \\
F(x_{n+1},y_{n+1}) &= F(x_n,y_n - 1) \text{ with a probability } P_{y-1}
\end{align*}
$$

The probabilities to progress (grow) in each of the specific directions verify:

$$P_{x+1} + P_{y+1} + P_{y-1} = 1$$

The following distributions are used to generate a random fracture path:

$$
\begin{align*}
P_{x+1} &= \frac{1}{3} \\
P_{y+1} &= \frac{1}{3} \\
P_{y-1} &= \frac{1}{3}
\end{align*}
$$

**Figure 1** shows the possible progression paths starting from a current point (8,3) shown in blue; 3 different options of propagations are shown in red. **Figure 2** shows different “possible” realizations using the same growth probabilities and the same number of iterations $N = 1000$. One of the main advantages of a probabilistic method is the ability to generate a wide range of possible realizations. The observed patterns of fractures usually include branches which intersect with the networks of natural fractures.
Constraining the Fracture Network to Preferential Growth Directions

The defined probabilities govern the direction of the fracture. For example, if the probabilities to progress along the y-axis are equal:

\[ P_{y+1} = P_{y-1} \]  

(6)

Then the fracture is expected to oscillate around the x-axis and the expected value of the \( y_n \) coordinates of the points where the density function is a fracture is zero:

\[ E(y_n) = 1 \times P_{y+1} + (-1) \times P_{y-1} = 0. \]  

(7)

However, when the fracture is chosen to have a preferential progression toward a positive value of \( y>0 \), then \( P_{y+1} \) is slightly higher than \( P_{y-1} \)

\[ P_{y+1} > P_{y-1} \]  

(8)

In this case, the expected value of \( E(y_n) \) is positive, and the fracture will have the tendency to grow with a positive slope. For the same number of iterations \( N=1000 \), the upper section of Figure 3 shows that the first fracture tends to remain parallel to the x-axis (between \( y = -15 \) and \( y = 15 \)) when the probabilities to grow in the positive and negative y are equal \( P_{y+1} = P_{y-1} \). In the lower section of Figure 3, the probability for the fracture to go in the positive y-direction is slightly higher \( (P_{x+1} = P_{y+1} = \frac{2}{5}, P_{y-1} = \frac{1}{3}) \), and there is a steady increase (positive slope) of the fracture.
Bifurcation (Splitting) of a Given Fracture

The observation of fracture patterns in shale (or in other materials) show that the fracture does not only necessarily propagate in a linear fashion. While this process could presumably be described using a fully coupled geomechanical and flow simulation, we will instead investigate the possibility of having what we refer to as “branched fractures” or fractures that have one or more bifurcation (splitting) stages.

This branching character can be captured by the random-walk process. At some randomly generated step, the fracture has the tendency to split between “upper” and “lower” branches. Starting from that point, the propagation continues as if we had to separate fractures whose propagation is governed by the same probability laws. In Figure 4 we show different realizations generated using the same growth probabilities and the same number of iterations \((N = 100)\) and for which a bifurcation occurs after a randomly generated step \((1 < k < N)\).

For \(1 < i < k\):

\[
\begin{align*}
P_{x+1} &= \frac{1}{2} \\
P_{y+1} &= \frac{1}{4} \\
P_{y-1} &= \frac{1}{4}
\end{align*}
\]  

(9)

For \(k < i < N\):

\[
\begin{align*}
P_{x+1} &= \frac{2}{5} \\
P_{y+1} &= \frac{2}{5} \text{ for the upper branch and } P_{y+1} = \frac{1}{5} \text{ for the lower branch} \\
P_{y-1} &= \frac{2}{5} \text{ for the upper branch and } P_{y-1} = \frac{2}{5} \text{ for the lower branch}
\end{align*}
\]  

(10)

A multi-branching process is prescribed to generate our most complex fracture patterns (limited to 4 in this study for computational purposes).

Figure 5 illustrates different “possible” realizations using the same growth probabilities and the same number of iterations \((N = 1000)\) with 4 random bifurcation stages for a randomly generated number of
steps for each branch. After each bifurcation, the upper and lower branches progress according the probability law defined previously (for a single branch).

**Generation of the Fracture Network for Reservoir Simulation**

**Stochastic Fracture Generation**

The computed fracture patterns which is mainly a series of points in 2-D space are input into a numerical simulator by transforming these points into a “grid mesh” (a segmentation of the reservoir into prescribed 3-dimensional blocks (or grids). In most cases (and this work) the mesh is generated by considering that each point to the center of a grid-cell.

Based on a rectangular grid we use the MeshModifier code to create the grid matrix grid. We have chosen to use a centimeter scale as our largest grid size to ensure proper representation of the fracture network. To scale our problem we assume that the fracture network will reside in a sub-region of the reservoir ($\Delta x = \Delta y = 2.5m$, $\Delta z = 20m$), beyond the “fracture network” region we use geometrically increasing grid-sizes to achieve better computation efficiencies.

For reference, the dimensions of our synthetic reservoir are:

$$\Delta x = \Delta y = \Delta z = 20m$$ (11)

**Figure 6** shows a gridded fracture network on a 2-D grid ($N_x = N_y = 250, N_z = 1$) for a 3-branched fracture network.

**Figure 7** shows the entire reservoir 2-D grid ($N_x = 300, N_y = 350, N_z = 1$), the region where the fractures are assumed to grow is highlighted.
Only a restricted area in our synthetic reservoir contains the stochastic fracture network patterns \((x_F = y_F = 2.5\text{m}, z_F = 20\text{m})\) with a refined grid cell size \((dx = dy = 0.01\text{m}, dz = 20\text{m})\). Consequently, the total number of grid blocks in that area is

\[
N_T = N_x \times N_y \times N_z = 250 \times 250 \times 1 = 62500
\]  

As mentioned in previous sections, we elected to use this “scaled model” to evaluate the behavior of the stochastic fracture networks. We recognize that our “scaled model” has limitations, but we believe that this model accurately represents all of the expected behaviors in the fracture/reservoir system. As illustrated in Figure 7, the grid blocks size is chosen to be progressively (geometric increase) coarsening out of the fractured region in order to reduce the computational requirements of the simulator. For this configuration, our number of grid-blocks is:

\[
N_T = N_x \times N_y \times N_z = 300 \times 350 \times 1 = 105000
\]  

Using the MeshModifier code, only the “fractured reservoir” region is modified, the “unfractured reservoir” region is maintained at a constant grid configuration. All the generated fractures that are used in this research are given in Appendix A.

**Reservoir Simulation of the Generated Fractures**

A modified version of the single-phase, non-isothermal numerical simulator “Flow Transport Simulator” (or FTSim) has been used for the numerical modeling required in this work. The reliability and accuracy of the FTSim simulator has been tested as a part of this research; for more detail the reader may refer to Mhiri (2014).

Our guiding constraint for this work is that we are only considering the 2-D fracture network case (constant fracture height). The “wellbore” is taken as the base cell of the fracture network (defined as the origin point) as shown by the orange line in Figure 8. Our goal is to identify the fundamental nature of flow for a random-walk fracture pattern and we believe that 2-dimensional flow behavior is sufficient for our needs. Extensions to the 3-dimensional case would (likely) be more useful for understanding geomechanical behavior as well as potentially addressing layering issues.
The geometry of this synthetic shale reservoir is a small-scale, square-shaped rectangular. Although this is a synthetic case, parameters such as fracture permeability, matrix porosity and fracture porosity are thought to be representative of the Barnett Shale (Texas) (Houzé et al. 2010; Shelley et al. 2010). The data used are summarized in Table 1.

The MeshModifier code provides the construction of rectangular Cartesian grids that describe the flow models that we investigate in the next chapter. The shape and the geometric features of the fracture patterns are thought to have an influence on the flow behavior (rate and pressure behavior). The code is used to generate specific sets of fracture patterns (83 patterns are investigated and compared to the planar fracture model). The pattern properties that will be varied are the following:

- The fracture tortuosity (controlled by the ratio of the probabilities \( \frac{P_{x+1}}{P_{x+1} + P_{y-1}} \)).
- The occurrence of “branching.”
- The orientation of the branches (controlled by the ratio of probabilities in the y-direction \( \frac{P_{y+1}}{P_{y-1}} \)).
- The total number of branches.

![Figure 8—3-D view of a stochastic pattern network of fractures.](image)

**Table 1—Parameters for the Synthetic Shale Reservoir Considered in this Study.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>SI Units</th>
<th>Field Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fracture width, ( W_f )</td>
<td>1 cm</td>
<td>0.0328 ft</td>
</tr>
<tr>
<td>Reservoir thickness, ( h )</td>
<td>20 m</td>
<td>65.62 ft</td>
</tr>
<tr>
<td>Matrix permeability, ( k )</td>
<td>( 5 \times 10^{-20} ) m(^2)</td>
<td>( 5 \times 10^{-3} ) mD</td>
</tr>
<tr>
<td>Fracture permeability, ( k_f )</td>
<td>( 5 \times 10^{-11} ) m(^2)</td>
<td>( 5 \times 10^{4} ) mD</td>
</tr>
<tr>
<td>Matrix porosity, ( \Omega )</td>
<td>0.04</td>
<td>4 percent</td>
</tr>
<tr>
<td>Fracture porosity, ( \Omega_f )</td>
<td>0.50</td>
<td>50 percent</td>
</tr>
<tr>
<td>Initial reservoir pressure, ( p_i )</td>
<td>( 2.5 \times 10^7 ) Pa</td>
<td>3626 psi</td>
</tr>
<tr>
<td>Wellbore pressure, ( P_f )</td>
<td>( 1.25 \times 10^7 ) Pa</td>
<td>1813 psi</td>
</tr>
<tr>
<td>Fractured region length, ( x_F )</td>
<td>2.5 m</td>
<td>8.2 ft</td>
</tr>
<tr>
<td>Fractured region width, ( y_F )</td>
<td>2.5 m</td>
<td>8.2 ft</td>
</tr>
<tr>
<td>Reservoir length, ( x_R )</td>
<td>20 m</td>
<td>65.62 ft</td>
</tr>
<tr>
<td>Reservoir width, ( y_R )</td>
<td>20 m</td>
<td>65.62 ft</td>
</tr>
</tbody>
</table>
Then, the following workflow is applied to each of the generated fracture network:

- Monitor the pressure change around the fracture using a 2-D pressure profile.
- Use the pressure profile to identify the range of time that is impacted by the shape of the fracture.
- Compare the mass production rate curves of the stochastic-patterns to the planar fracture model.
- Assess the performance of the stochastic patterns (in terms of cumulative recovery).

As noted, the rate and pressure performance of the stochastic fracture networks are compared with those for the (single) planar fracture case (our “standard” case). Based on observations of the pressure distributions, we concluded that the effect of the pattern would “disappear” after a relatively short production period (less than 4 days in this case); primarily due to the reservoir size, recall that this is a “scaled” experiment.

Most simulations were targeted for 100 days (which is an extremely long time for this configuration), and as we noted that most features are observed on the order of 4 days many of the simulations were terminated in this timeframe to reduce computational expense.

The investigation of the different stochastic fracture network patterns allow us to assess the effect of each fracture property on the performance of the reservoir — specifically, tortuosity, occurrence of branching, orientation of branches and number of branches. One of the practical goals of this work is observe the behavior of complex fracture networks on production rate performance — and as a simplistic conclusion, we can state that the fracture networks we have generated in this work provide analogous production signatures to those observed in the field. This observation could have a significant impact on the analysis and interpretation of production performance as well as the evaluation of reserves.

Methodology to Assess the Performance of the Stochastic Patterns

The following provides an in-depth analysis of stochastic “random-walk” fracture patterns created in this work using numerical simulation (FTSim) via comparison of computed mass rate and pressure functions. Each fracture network is modeled and then the results from each model are compared in groups (by number of fracture branches) as well as to the single-planar fracture case (which is our standard for comparison).

Map the Pressure Change to Identify the Flow Regimes

In this paper we extract the “pressure maps” at different time values for each fracture network. Each simulation case is generated for at least 100 days of production at a constant wellbore pressure \( p_{wf} = 1.25 \times 10^7 \text{Pa} = \frac{p_i}{2} \). The 100-day production period was shown (via modeling) to be sufficient for this synthetic shale matrix permeability (50 nD) where we would definitely observe transient flow features (as we desire) and for some cases we can observe severe interference (in a pressure sense) of the created fracture networks. Given the reservoir size, the matrix permeability, and the various fracture networks, we established that we should begin to see interference/depletion effects at times on the order of 100 days. In short, our approach is based on the observation of transient flow behavior of the fracture network, where we evaluate the mass rate and \( \beta \)-derivative of the mass rate trends versus production time on a “log-log” style plot.

Specific to the generation and review of the pressure maps for this work, we note that we do indeed observe fracture interference effects, and we note that these interference effects are somewhat unique to a given stochastic fracture network patterns. For reference regarding all of our pressure maps, the displayed variable \( f \) is the \( p_f/p_i \) ratio. For a given time, the reservoir pressure, \( p_r \), is given as:

\[
p_f = \frac{p_i}{2} < p_r < p_i \tag{14}
\]
Therefore we expect the function \( f \) to be:

\[ 0.5 < f < 1 \quad (15) \]

The study of the pressure distributions for a given fracture network provides more insight than the mass production rate in isolation, but obviously we cannot measure pressure maps in the field. As with most tasks in reservoir engineering we use what we can measure as a proxy for what we cannot — so we will have to infer fracture interference, reservoir depletion, etc. from the production rate profiles.

**Reservoir Performance (Mass Production Rate and \( \beta \)-Derivative)**

In this section we present the introduction of the analysis/interpretation using the mass production rate and \( \beta \)-derivative functions. We present different groups of stochastic fracture networks, focusing (primarily) on the constant portion of the mass production rate \( \beta \)-derivative function as our diagnostic. Recall that for a single-planar fracture our expectation is “linear flow” (which has a constant \( \beta \)-derivative value of 0.5 (or 1/2)). For the various fracture network cases we would expect a \( \beta \)-derivative value greater than 0.5 (as a given fracture network should be more productive than the single planar fracture case). As such, we group cases according to the number of bifurcations (\( i.e., \) splits or branches), and we find that most cases for a given number of branches do “cluster” around a distinct (constant) value of the \( \beta \)-derivative.

The \( \beta \)-derivative of the rate function is defined as

\[ \beta = \frac{d\beta q(t)}{dt} = \frac{|d(\log(q))|}{d(\log(t))} \quad (16) \]

Where for all cases in this work \( q \) is the mass rate in Kg/s and \( t \) is the time in s. All the mass production decline rate plots and \( \beta \)-derivative plots are given in Appendix B.

**Single-Planar Fracture Case (Benchmark/Comparative Standard)**

As explained earlier, the single-planar fracture case is used to compare the performance of each of the stochastic fracture networks. As this work is based on a synthetic shale gas reservoir model, we must define the basic hydraulic fracture characteristics (summarized in Table 2) as well as the properties of the synthetic shale gas reservoir (\( i.e., \) the reservoir matrix), which are summarized in Table 3.

### Table 2—Fracture Parameters for the Synthetic Shale Gas Reservoir Case.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>SI Units</th>
<th>Field Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fracture half-length, ( x_f )</td>
<td>2.25 m</td>
<td>7.38 ft</td>
</tr>
<tr>
<td>Fracture width, ( 1 ) cm</td>
<td>0.0328 ft</td>
<td></td>
</tr>
<tr>
<td>Fracture thickness, ( 20 ) m</td>
<td>65.62 ft</td>
<td></td>
</tr>
<tr>
<td>Fracture permeability, ( 5 \times 10^{-11} ) m²</td>
<td>( 5 \times 10^4 ) mD</td>
<td></td>
</tr>
<tr>
<td>Fracture porosity, ( 0.50 )</td>
<td>50 percent</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3—Reservoir Parameters for the Synthetic Shale Gas Reservoir Case.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>SI Units</th>
<th>Field Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir length, ( x_R )</td>
<td>20 m</td>
<td>65.62 ft</td>
</tr>
<tr>
<td>Reservoir width, ( y_R )</td>
<td>20 m</td>
<td>65.62 ft</td>
</tr>
<tr>
<td>Reservoir thickness, ( h )</td>
<td>20 m</td>
<td>65.62 ft</td>
</tr>
<tr>
<td>Shale permeability, ( k )</td>
<td>( 5 \times 10^{-20} ) m²</td>
<td>( 5 \times 10^{-7} ) mD</td>
</tr>
<tr>
<td>Shale porosity, ( \varphi )</td>
<td>0.04</td>
<td>4 percent</td>
</tr>
<tr>
<td>Shale compressibility, ( C_f )</td>
<td>( 10^{-9} ) Pa⁻¹</td>
<td>( 6.9 \times 10^{-6} ) psi⁻¹</td>
</tr>
</tbody>
</table>
In Figure 9 we present the schematic of the single-planar hydraulic fracture in the reduced-scale reservoir. As we limit our study to 2 dimensions (constant reservoir and fracture thickness), all of our presentations are made relative to a 2-dimensional view of the reservoir. The wellbore is at the base of the hydraulic fracture and is held at a constant flowing bottomhole pressure\( (p_{wf} = 2.5 \times 10^7 Pa)\).

The use of pressure maps is a somewhat qualitative exercise, but this effort provides significant insight into the change of the pressure distribution in the reservoir. Our primary goal is to use pressure maps to identify/understand different flow regimes that occur during production. For the case of a single-planar fracture, a series of time-dependent pressure maps are shown in Figure 10. As the single-planar fracture case is well-established in the literature, the flow regimes for this case are well understood. For example, in Figure 10 we note that:

- Initial linear flow from the fracture to the wellbore, duration is only a few seconds.
- A transition period that occurs fracture to matrix linear flow as the fracture depletes.
- Linear flow in the matrix commences until fracture interference and/or boundary-dominated flow.
Pressure maps are given in Appendix C for a selected number of cases. For further detail, the reader may refer to Mhiri (2014).

The mass production rate profile for the case of a single-planar fracture is shown in Figure 11. We note that the linear portion of the data on this plot (roughly 50 s to 100,000 s) align to a half-slope straight line (-1/2 slope on the logarithm of mass production rate versus logarithm of production time). The primary utility of the β-derivative is that this function “automatically” identifies power-law flow regimes such as linear flow. As shown in Figure 11 the β-derivative profile confirms the half slope (β-derivative = 0.5) during the linear flow regime (roughly 50 s to 100,000 s).

As comment, the behavior between $10^{-2}$ sec $> t > 10^{0}$ sec is an “artifact” of the fracture (or fracture system) unloading (a sort of depletion), and we note the strong influence of this behavior in Figure 11 (mass production rate and β-derivative profile). This feature is also seen for the stochastic fracture network cases, and (again) is simply seen as an “artifact” of fracture depletion. For reference, other investigations also show this behavior (e.g., Olorode et al., 2013).

**Numerical Simulation and Analyses of the Fracture Patterns**

**Non-Branched Fracture Patterns**

A set of 3 different Mono-branched fracture cases are created according to the probabilities provided in Table 4. These probabilities were selected so that we can observe the effect of the fracture tortuosity (“crookedness”) on the flow behavior.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$P_{x+1}$</th>
<th>$P_{y+1}$</th>
<th>$P_{y-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non branched fracture 1</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Non branched fracture 2</td>
<td>0.3</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>Non branched fracture 3</td>
<td>0.1</td>
<td>0.45</td>
<td>0.45</td>
</tr>
</tbody>
</table>

These cases presented in Table 4 are shown in Figure 12 and we clearly note the tortuous nature of these fractures compared to the single-planar fracture case. This tortuosity is controlled by the value of $[P_{x+1}/(P_{y+1}+P_{y-1})]$ the higher this ratio is the more likely the fracture will have the tendency to oscillate.
in the y-direction and increase the fracture tortuosity. The length of the fractures (along the x-direction) has been constrained to be the same as for the plane fracture case $x_f = 2.25m$.

The performance of these patterns is analyzed and compared to the plane fracture case. Figure 13 shows that the tortuous and more voluminous fractures exhibit an early time linear fracture flow ($t 10^{-2}$ s). After the “fracture depletion” transition zone the trends converge toward the linear flow (half-slope) trend typically observed for the single-planar fracture case. In Figure 13 we confirm that the $\beta$-derivative function exhibits a near-constant value (approximately 0.5) during the apparent linear flow period.

Figure 13—Mass rate and $\beta$-derivative of the mass rate for a synthetic shale gas reservoir produced from 3 non-branched fracture patterns and compared to the planar fracture case.
Stochastic Fracture Networks — Mono-Branched Fracture Patterns

A set of 5 different Mono-branched fractures is created following the probabilities described in Table 5. The probabilities were selected so that we can analyze the effect the bifurcation occurrence (extent of $F_0$); and also the distance between the two sub-fractures $F_{01}$ and $F_{02}$ which is controlled by the ratio $[P_{y+1}/P_{y-1}]$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$P_{x+1}$</th>
<th>$P_{y+1}$</th>
<th>$P_{y-1}$</th>
<th>$P_{x+1}$</th>
<th>$P_{y+1}$</th>
<th>$P_{y-1}$</th>
<th>$P_{x+1}$</th>
<th>$P_{y+1}$</th>
<th>$P_{y-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mono-branched fracture 1</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Mono-branched fracture 2</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
<td>0.5</td>
<td>0.35</td>
<td>0.15</td>
<td>0.5</td>
<td>0.15</td>
<td>0.35</td>
</tr>
<tr>
<td>Mono-branched fracture 3</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
<td>0.5</td>
<td>0.35</td>
<td>0.15</td>
<td>0.5</td>
<td>0.15</td>
<td>0.35</td>
</tr>
<tr>
<td>Mono-branched fracture 4</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
<td>0.5</td>
<td>0.45</td>
<td>0.05</td>
<td>0.5</td>
<td>0.05</td>
<td>0.45</td>
</tr>
<tr>
<td>Mono-branched fracture 5</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
<td>0.4</td>
<td>0.3</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The generated mono-branched patterns are shown in Figure 14, we can notice that the first case has a very low opening between the two sub-fractures $F_{01}$ and $F_{02}$; this opening is progressively increased in the other patterns (Case 2, Case 3 and Case 4). Additionally, the occurrence of the bifurcation on the case 2 pattern is delayed compared to the other patterns.

The shapes variation between the mono-branched fracture aims to capture the effect of different geometric settings of the fractures. The pressure distributions for each of the simulated patterns allow us
to identify the period of flow where the two sub-fractures $F_{01}$ and $F_{02}$ start to exhibit an effect on the mass decline rate. The analysis of the pressure distribution shows that the interferences between different sub-fractures occur before $10^5 S$ of production ($t > 10^5$ s). The diagnosis of the mass production curve during that particular period will be critical for the branch-dependent characterization of the mass decline rate curves. From Figure 15 we can notice that between $50S$ and $10^5 S$ (approximately) the mass production rate curve exhibits a linear behavior with a slope slightly superior to the planar fracture case; Figure 15 confirms this observation as the $\beta$-derivative has a constant value higher on average than the 0.5 value of the planar fracture.

![Figure 15](image_url)

**Figure 15**—Mass rate and $\beta$-derivative of the mass rate for a synthetic shale gas reservoir produced from 5 mono-branched fracture patterns and compared to the planar fracture case.

The mono-branched fracture of case 1 and 2 seem to have a similar performance as the plane fracture after a relatively short time of production of around $50S$. This effect is due to the fact that the first case pattern has its two sub-fractures $F_{01}$ and $F_{02}$ very close so that they almost behave as a single fracture; it can be somehow “seen” as a non-branched fracture pattern. The mono-branched fracture of case 2 has very limited two sub-fractures, their effect is barely noticed and their interferences occur rather quickly. However, the mono-branched fracture of case 3 has an earlier occurrence of the fracture split and the interferences effect seem to take place for a longer time according to the mass production rate curve of Figure 15.

Figure 15 gives a more detailed insights on the variations of the mass production rate curve for the mono-branched fracture patterns. Apart from the patterns 1 and 2 that converge rather quickly toward a linear half-slope behavior; the mono-branched fracture patterns have a $\beta$-derivative slightly higher on average than 0.5 during the production period where the interferences take place (approximately between $10^2 S$ and $10^5 S$). The interferences are confirmed through the analysis of the pressure change maps.

We can also observe that the first mono-branched pattern is very close to the planar fracture case as its two sub-fractures remain very close; it somehow behaves like a non-branched pattern. The second pattern is also “poorly” performing because of the limited extent of its sub-fractures. This might partly explain the fact that the 1st and the 2nd mono-branched patterns have mass production rate curves that are close to the single-plane fracture mass production rate curve. In both cases, the interference effect is too small to be observed macroscopically on the production indicator plots.

**Stochastic Fracture Networks — Dual-Branched Fracture Patterns**

Similarly to the mono-fractured pattern cases, we generated 5 different patterns of fractures that have two stages of bifurcations. Table 6 summarizes the parameters that are used to constrain those fractures.
Basically, there are no variations between the different patterns; the effects that will be observed are due to the differences between different realizations of the same constrained structure. Additionally to the analysis of the effect of tortuosity, the branches spacing and the placement of the bifurcation we want to analyze the effect that the number of bifurcations stages or branches has on the production. Figure 16 shows the different patterns that are investigated in this paragraph.

| Table 6—Statistical Parameters for the Dual-Branched Fracture Pattern Case. |
|-------------------|-------------------|-------------------|-------------------|
| Parameter         | $F_a$             | $F_{a1}$, $F_{a2}$, $F_{a3}$ | $F_{a1}$, $F_{a2}$, $F_{a3}$ |
|                   | $P_{x1}$, $P_{y1}$, $P_{z1}$ | $P_{x1}$, $P_{y1}$, $P_{z1}$ | $P_{x1}$, $P_{y1}$, $P_{z1}$ |
| Dual-branched fracture 1 | 0.5 | 0.25 | 0.25 | 0.4 | 0.4 | 0.2 | 0.4 | 0.2 | 0.4 |
| Dual-branched fracture 2 | 0.5 | 0.25 | 0.25 | 0.4 | 0.4 | 0.2 | 0.4 | 0.2 | 0.4 |
| Dual-branched fracture 3 | 0.5 | 0.25 | 0.25 | 0.4 | 0.4 | 0.2 | 0.4 | 0.2 | 0.4 |
| Dual-branched fracture 4 | 0.5 | 0.25 | 0.25 | 0.4 | 0.4 | 0.2 | 0.4 | 0.2 | 0.4 |
| Dual-branched fracture 5 | 0.5 | 0.25 | 0.25 | 0.4 | 0.4 | 0.2 | 0.4 | 0.2 | 0.4 |

We notice that the length of each sub-fracture is totally random; however the fracture is constrained to remain in the grid-refined reservoir area. The pressure profile change of different patterns in the pressure maps show that the effect of fracture interferences seems to be occurring between $10^5$ and $10^5$. That particular period is analyzed on the mass production rate and $\beta$-derivative curves. A particular signature starts to appear in the mass production rate curve of Figure 17. In fact, the half-slope linear behavior tends to become a higher degree linear slope. The $\beta$-derivative curve is slightly higher than 0.5,
and it seems to be considerably higher than the derivatives for the mono-branched fracture patterns. This effect shall be studied later in further depth with more simulated patterns.

Stochastic Fracture Networks — Tri-Branched Fracture Patterns

5 different tri-branched fracture patterns are generated in this section. The obtained 5 structures are displayed in Figure 18. The reader can notice that the tri-branched pattern have a high enough number of branches that makes it more probable to have an intersection between different branches in a limited area. The same generation probabilities were used to generate those different patterns Table 7. The impact of the presence of 3 stages of branching starts is confirming the observation that we saw in the previous paragraph.
The investigation of the production curves of Figure 19 shows that between $10^2 S$ and $10^5 S$ the flow exhibits a linear behavior with a similar slope for different tri-branched patterns. This period of flow occurs when fractures are interfering (according to pressure profiles). The fractures are somehow defining “channels” which edges produce at a constant pressure, the $\beta$-derivative is therefore higher than for a planar fracture and for lower degree branched fracture that contains less “channels”.

![Figure 18—5 tri-branched fracture patterns (cases 1 to 5).](image)

**Table 7—Statistical Parameters for the Tri-Branched Fracture Pattern Case.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$F_0$</th>
<th>$F_{01}$, $F_{011}$, $F_{21}$, $F_{0111}$, $F_{021}$, $F_{0211}$</th>
<th>$F_{02}$, $F_{012}$, $F_{022}$, $F_{0112}$, $F_{0122}$, $F_{0212}$, $F_{0222}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tri-branched fracture 1</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Tri-branched fracture 2</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Tri-branched fracture 3</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Tri-branched fracture 4</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Tri-branched fracture 5</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

The investigation of the production curves of Figure 19 shows that between $10^2 S$ and $10^5 S$ the flow exhibits a linear behavior with a similar slope for different tri-branched patterns. This period of flow occurs when fractures are interfering (according to pressure profiles). The fractures are somehow defining “channels” which edges produce at a constant pressure, the $\beta$-derivative is therefore higher than for a planar fracture and for lower degree branched fracture that contains less “channels”.
Even though the simulated tri-branched patterns are quite geometrically different from each other, the mass production rate curves of Figure 19 display a similar behavior between $10^2$s and $10^5$s. Those interferences effect which have also been observed for a lower number of branches could be characterized by the slope of the mass production rate curve between $10^2$s and $10^5$s, this slope value, being 0.5 for the planar fracture, seems to be significantly higher for the simulated set of tri-branched fractures. On Figure 19, the $\beta$-derivative for the tri-branched pattern is 0.65 on average.

**Stochastic Fracture Networks — Quad-Branched Fracture Patterns**

Similarly to the Tri-branched fracture pattern cases, 5 different quad-branched fracture patterns are generated in this section. The obtained 5 structures are displayed in Figure 20, as we observed for the tri-branched fractures, the quad-branched patterns are dense enough for the branches to intersect between each other. The same generation probabilities were used to generate those different patterns Table 8. The impact of the presence of 4 stages of branching starts is confirming the observation that we saw in two previous sections.
In Figure 21 we can observe an early behavior of the mass production rate curve that exhibits a linear flow behavior with a characteristic slope clearly superior to 0.5. Furthermore, the identified approached value of 0.7 for the derivative between $10^2S$ and $10^5S$ confirms the increasing tendency of the slope in the mass production rate curve. In other words, the early time behavior of a mass production rate curve is impacted by the number of branches for a given set of stochastically generated fractures as the number of “channels” inside those fractures increase.
Similarly to the mono-branched, dual-branched and tri-branched fracture patterns, we observe a similar behavior for all of the quad-branched pattern during the phase where the fractures are interfering. Those interferences are confirmed by the observation of the pressure change maps. They generally occur between $10^2$ and $10^5$. During that same period, we can identify a signature slope in Figure 21 that characterizes all the quad-branched patterns. The slope value is better read on the $\beta$-derivative plots of Figure 21.

**Impact on Reservoir Performance**

Following the individual analysis of different generated fracture patterns, we started looking at the patterns as a whole. We noticed that some aspects characterize the different groups of generated patterns, as reservoir performance. Figure 22 presents the cumulative mass recovery at 100 days as a function of fracture volume for the stochastic fracture network patterns created in this work. We clearly note a correlation of the number of branching stages on cumulative mass recovery. While we should not generalize these observations, we should expect such correlations based on the number of branching stages and the total fracture volume per case. The dashed-black line is imposed as a “baseline” for each given fracture network case (i.e., this would be the base correlation of recovery and fracture volume. The individual trend for each stochastic fracture network case suggests a correlation with fracture network “complexity” (i.e., complexity would be a function of the number of branches and the fracture volume).
Correlation of the $\beta$-derivative Performance

A set of 83 simulated fracture network patterns are generated using the “random walk” process described above. All the simulated patterns of mass rate decline and $\beta$-derivative of the mass rate curves are given in Appendix B.

The average values of the $\beta$-derivative based on the mass production rate for all of the modeling cases are summarized in Table 9. In addition, we can assume that the minimum $\beta$-derivative ($0.5$) exists for cases where there are no branches. The maximum value of the $\beta$-derivative would be $1$ (i.e., unity), which corresponds to “depletion behavior” where the entire volume is fractured and we achieve a (very) high equivalent permeability value for the fractured reservoir volume. Physically, when the number of branches is infinitely high, the reservoir volume is essentially filled with fractures and the reservoir effectively becomes a tank (i.e., the pseudosteady-state or boundary-dominated condition exists).

<table>
<thead>
<tr>
<th>Number of branching stages</th>
<th>Average $\beta$-derivative between $1 \times 10^2s$ and $1 \times 10^5s.$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.56</td>
</tr>
<tr>
<td>2</td>
<td>0.59</td>
</tr>
<tr>
<td>3</td>
<td>0.64</td>
</tr>
<tr>
<td>4</td>
<td>0.70</td>
</tr>
</tbody>
</table>

In Figure 23 we present the average value of the $\beta$-derivative derived from the cases for a given number of fracture branches as a function of the number of branches present in the created stochastic fracture network. In order to have a functional representation (for possible use as a design tool), we developed the following model of the $\beta$-derivative as a function of the number of stochastic fracture branches:
Where the $a$, $b$, and $c$-coefficients are defined for our work using regression analysis. One possible consequence of the trend shown in Figure 23 is that this result could be used to interpret the geometry of an in-situ created/enhanced fracture network using the performance signature of the mass production rate curve (specifically, the $\beta$-derivative). Another conclusion that can be drawn from the trend shown in Figure 23 is that when we are investigating the behavior of $\beta$-derivative in practice (i.e., using field data) we could be observing the combined effect of natural and hydraulically created/enhanced fractures. This hypothesis implies that we should utilize microseismic monitoring as a means of trying to characterize the behavior of the created/enhanced fracture system and the pre-existing natural fracture network.

**Discussion**

The understanding of in-situ fracture geometries is essential to the accurate prediction of rate and pressure performance as well as reserves estimations for shale gas reservoirs. Therefore, we must consider that each geometrical feature which has an influence on the performance of the reservoir should be properly assessed and accounted for in a numerical simulation. This research proves that the following factors govern reservoir performance for created/enhanced fracture networks:

- Fracture tortuosity.
- The placement of the branching or bifurcation in the fracture.
- The orientation of the branches.
- The number of branching stages.

Each of these factors has an influence on the rate and pressure performance for a stimulated reservoir system. Although not an objective of this study, it is clear that we must establish how hydraulic fractures intersect/connect with the different types of fractures which were not been modeled in this study (i.e., natural, secondary and radial fractures). The connected fracture network results from the interaction between hydraulic fracture and the activation of other fracture networks present in the reservoir. Cipolla et al. (2011) attempted to quantify this complexity via reservoir modeling. However, one could (and probably should) argue that there is no unique work-path to construct a fracture network model that accurately matches the microseismic data (particularly in light of the fact that we are not certain what physical processes the recorded microseismic events actually represent).
The stochastically created fracture networks used in this research are all based on the same fundamental “random walk” algorithm, and this could be construed as a limitation. Although our investigation was not “exhaustive” due to the tedious nature of the simulations (and their associated computational costs), we are very confident that the results of this research are both representative and reproducible. As we have successfully deployed the β-derivative function as our diagnostic for interpreting the nature of a given fracture network, we must acknowledge that this is an “aggregate” tool because there are an infinite number of stochastic fracture patterns that could generate a given production performance signature.

Summary and Conclusions

Summary

In this work we generated 83 different stochastic (“random-walk”) fracture patterns generated to re-present distinct hypothetical complex fracture networks which are created during hydraulic fracture stimulation. The (mass) production rate behavior and the pressure distributions for these fracture patterns were assessed using the FTSim code, a single-phase, single-component numerical reservoir simulator that was developed at the Department of Petroleum Engineering at Texas A&M University.

Using these stochastic (“random-walk”) fracture pattern cases, we identified the effect of different geometric properties of the fractures on the flow behavior using the “β-derivative” of the mass flowrate generated by the simulator (we identified a specific, characteristic value of the “β-derivative” function for each set of “fracture branches”). We also utilized the pressure distribution maps created for each case to identify the periods when the “branches” for given fracture network were “interfering” (i.e., the pressure distributions intersect and aggregate).

As noted, the (mass) rate-time and (mass) rate-time β-derivative curves generated using the FTSim code were used to identify the signatures of different fracture patterns that were investigated. Using the β-derivative curves, an empirical model was created to predict the (constant) value of the β-derivative function for a given number of branching stages in a particular fracture pattern. We believe that this is a diagnostic feature that may be useful for assessing “stimulation efficiency” from field performance data. In a practical sense, assuming the observation of a particular (constant) value of the β-derivative obtained from production rate data, we can infer the number of fracture branching stages for a given well case.

Conclusions

- Early-time rate performance is an indicator of the geometry of the hydraulic fracture pattern.
- The β-derivative (mass rate) is observed to be directly related to the number of branching stages.
- The β-derivative (mass rate) for the single-plane hydraulic fracture is 1/2 (one-half) — where this behavior confirms the assumed theoretical performance for this case (i.e., non-interfering, linear flow).
- In practice, we can infer that for cases where the β-derivative is approximately equal to 1/2, then linear flow essentially exists for all simple (i.e., near-planar) fracture patterns.
- In terms of cumulative gas recovery, the more complex stochastic fracture patterns yield the highest recoveries.
- While we observe a unique performance of the β-derivative function for a given set of fracture branch patterns, we acknowledge that there are an infinite number of patterns which could yield a similar performance signature.

NOMENCLATURE

- \( \sigma_{\text{max}} \) = Maximum principal stress, Pa or MPa [Force/Surface]
- \( \sigma_c \) = Minimum critical fracturing stress, Pa or MPa [Force/Surface]
(x, y, z) = First, second and third coordinate in an orthogonal Cartesian coordinate system, m [Length]

\( F_{(x,y)} \) = Discrete density function that describes the fracture [dimensionless]

\( n \) = Integer that describes the iterative definition of the density function

\( N \) = Integer that describes the total number of iteration that defines the density function

\( k \) = Random integer that describes the number of iterations after which the fractures splits

\( P_{x+1} \) = Probability for the fracture to progress along the positive x direction by 1 increment [dimensionless]

\( P_{y+1} \) = Probability for the fracture to progress along the positive y direction by 1 increment [dimensionless]

\( P_{y-1} \) = Probability for the fracture to progress along the negative y direction by 1 increment [dimensionless]

d_x = Grid size along the x-direction, m [Length]

d_y = Grid size along the y-direction, m [Length]

d_Z = Grid size along the z-direction, m [Length]

\( \Delta_x \) = Reservoir size along the x-direction, m [Length]

\( \Delta_y \) = Reservoir size along the y-direction, m [Length]

\( \Delta_z \) = Reservoir size along the z-direction, m [Length]

\( N_x \) = Number of grid-blocks in the x-direction

\( N_y \) = Number of grid-blocks in the y-direction

\( N_z \) = Number of grid-blocks in the z-direction

\( N_t \) = Total of grid-blocks

\( x_f \) = Fracture half-length, m [Length]

\( w_f \) = Fracture width, m [Length]

\( h \) = Reservoir thickness, m [Length]

\( k \) = Matrix permeability, m^2 [Length^-2]

\( k_f \) = Fracture permeability, m^2 [Length^-2]

\( \phi \) = Matrix porosity (fraction) [fraction]

\( \phi_f \) = Fracture porosity (fraction) [fraction]

\( C_f \) = Formation compressibility, Pa^-1 [Length Time^2]

\( T_f \) = Initial reservoir temperature, Deg C [Temperature]

\( p_i \) = Initial reservoir pressure, Pa [Mass/(Length Time^2)/Mass]

\( p_f \) = Wellbore pressure, Pa [Mass/(Length Time^2)]

\( r_{we} \) = Equivalent well radius, m [Length]

\( x_F \) = Fractured region length, m [Length]

\( y_F \) = Fractured region width, m [Length]

\( Z_F \) = Fractured region thickness, m [Length]

\( x_R \) = Reservoir length, m [Length]

\( y_R \) = Reservoir width, m [Length]

\((x_n, y_n)\) = First and seconds coordinate of an orthogonal Cartesian system at the \( n \)th iteration, m [Length]

\( E(y_n) \) = Expected value of \( y_n \), m [Length]

\( x_{max} \) = Maximum extent in the x-direction of a stochastic fracture, m [Length]

\( x_{max1} \) = Maximum extent in the x-direction of a stochastic fracture after \( N \) iterations, m [Length]

\( F_0 \) = Stochastic fracture branch before any bifurcation stage

\( F_{01} \) = Upper Branch of the bifurcation of \( F_0 \) (1st bifurcation stage)

\( F_{02} \) = Lower Branch of the bifurcation of \( F_0 \) (1st bifurcation stage)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{011}$</td>
<td>Upper Branch of the bifurcation of $F_{01}$ (2nd bifurcation stage)</td>
</tr>
<tr>
<td>$F_{012}$</td>
<td>Lower Branch of the bifurcation of $F_{01}$ (2nd bifurcation stage)</td>
</tr>
<tr>
<td>$F_{021}$</td>
<td>Upper Branch of the bifurcation of $F_{02}$ (2nd bifurcation stage)</td>
</tr>
<tr>
<td>$F_{022}$</td>
<td>Lower Branch of the bifurcation of $F_{02}$ (2nd bifurcation stage)</td>
</tr>
<tr>
<td>$F_{0111}$</td>
<td>Upper Branch of the bifurcation of $F_{011}$ (3rd bifurcation stage)</td>
</tr>
<tr>
<td>$F_{0112}$</td>
<td>Lower Branch of the bifurcation of $F_{011}$ (3rd bifurcation stage)</td>
</tr>
<tr>
<td>$F_{0121}$</td>
<td>Upper Branch of the bifurcation of $F_{012}$ (3rd bifurcation stage)</td>
</tr>
<tr>
<td>$F_{0122}$</td>
<td>Lower Branch of the bifurcation of $F_{012}$ (3rd bifurcation stage)</td>
</tr>
<tr>
<td>$F_{0211}$</td>
<td>Upper Branch of the bifurcation of $F_{021}$ (3rd bifurcation stage)</td>
</tr>
<tr>
<td>$F_{0212}$</td>
<td>Lower Branch of the bifurcation of $F_{021}$ (3rd bifurcation stage)</td>
</tr>
<tr>
<td>$F_{0221}$</td>
<td>Upper Branch of the bifurcation of $F_{022}$ (3rd bifurcation stage)</td>
</tr>
<tr>
<td>$F_{0222}$</td>
<td>Lower Branch of the bifurcation of $F_{022}$ (3rd bifurcation stage)</td>
</tr>
<tr>
<td>$q(t)$</td>
<td>Mass rate at a time $t$, kg.s$^{-1}$ [Mass/Time]</td>
</tr>
<tr>
<td>$d_q q(t)$</td>
<td>$\beta$- derivative of $q(t)$ [dimensionless]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\beta$- derivative of $q(t)$ [dimensionless]</td>
</tr>
<tr>
<td>$p_t$</td>
<td>Pressure at time $t$, Pa [Mass/(Length Time$^2$)]</td>
</tr>
<tr>
<td>$a$</td>
<td>First calibration constant for the $\beta$-derivative empirical match [dimensionless]</td>
</tr>
<tr>
<td>$b$</td>
<td>Second calibration constant for the $\beta$-derivative empirical match [dimensionless]</td>
</tr>
<tr>
<td>$c$</td>
<td>Third calibration constant for the $\beta$-derivative empirical match [dimensionless]</td>
</tr>
<tr>
<td>$N_b$</td>
<td>Number of branching stages in the stochastic fractures [dimensionless]</td>
</tr>
</tbody>
</table>

References


Olorode, O., Freeman, C.M., Moridis, G. et al 2013. High-Resolution Numerical Modeling of Complex and Irregular Fracture Patterns in Shale-Gas Reservoirs and Tight Gas Reservoirs. DOI: 10.2118/152482-PA

Appendix A

x-y Fracture Maps (various fracture patterns)

Provided in this appendix for the convenience of the reader all the hydraulic fracture patterns that were used in this paper.

Figure A.1—3 non-branched fracture patterns (Cases 1 to 3).
Figure A.2—5 mono-branched fracture patterns (Cases 1 to 5).

Figure A.3—5 mono-branched fracture patterns (Cases 6 to 10).
Figure A.4—5 mono-branched fracture patterns (Cases 11 to 15).

Figure A.5—5 mono-branched fracture patterns (Cases 16 to 20).
Figure A.6—5 *dual-branched* fracture patterns (Cases 1 to 5).

Figure A.7—5 *dual-branched* fracture patterns (Cases 6 to 10).
Figure A. 8—5 dual-branched fracture patterns (Cases 11 to 15).

Figure A. 9—5 dual-branched fracture patterns (Cases 16 to 20).
Figure A. 10—5 *tri-branched* fracture patterns (Cases 1 to 5).

Figure A. 11—5 *tri-branched* fracture patterns (Cases 6 to 10).
Figure A. 12—5 tri-branched fracture patterns (Cases 11 to 15).

Figure A. 13—5 tri-branched fracture patterns (Cases 16 to 20).
Figure A. 14—5 quad-branched fracture patterns (Cases 1 to 5).

Figure A. 15—5 quad-branched fracture patterns (Cases 6 to 10).
Figure A. 16—5 quad-branched fracture patterns (Cases 11 to 15).

Figure A. 17—5 quad-branched fracture patterns (Cases 16 to 20).
Appendix B

Mass Rate and $\beta$-derivative Plots (various fracture patterns)

The mass rate decline curves and the $\beta$-derivative of the mass rate are provided in this Appendix for various fracture networks.

Figure B. 1—Mass rate and $\beta$-derivative of the mass rate for a synthetic shale gas reservoir produced from 20 mono-branched fracture patterns and compared to the planar fracture case.

Figure B. 2—Mass rate and $\beta$-derivative of the mass rate for a synthetic shale gas reservoir produced from 20 dual-branched fracture patterns and compared to the planar fracture case.
Figure B. 3—Mass rate and $\beta$-derivative of the mass rate for a synthetic shale gas reservoir produced from 20 tri-branched fracture patterns and compared to the planar fracture case.

Figure B. 4—Mass rate and $\beta$-derivative of the mass rate for a synthetic shale gas reservoir produced from 20 quad-branched fracture patterns and compared to the planar fracture case.
Appendix C

Time-Slice Pressure Maps (various fracture patterns)

The pressure “time-slice” maps for the various hydraulic fracture patterns are provided in this Appendix for reference.

Figure D. 1—Planar fracture pressure maps — $1 \times 10^3$s, $1 \times 10^2$s, $1 \times 10^3$s, $1 \times 10^4$s and $1 \times 10^5$s.
Figure D. 2—Non-branched fracture pressure maps — \(1 \times 10^1\) s, \(1 \times 10^2\) s, \(1 \times 10^3\) s, \(1 \times 10^4\) s and \(1 \times 10^5\) s.

Figure D. 3—Mono-branched fracture pressure maps — \(1 \times 10^1\) s, \(1 \times 10^2\) s, \(1 \times 10^3\) s, \(1 \times 10^4\) s and \(1 \times 10^5\) s.
Figure D. 4—Dual-branched fracture pressure maps — $1 \times 10^4$s, $1 \times 10^5$s, $1 \times 10^6$s, $1 \times 10^7$s and $1 \times 10^8$s.

Figure D. 5—Tri-branched fracture pressure maps — $1 \times 10^4$s, $1 \times 10^5$s, $1 \times 10^6$s, $1 \times 10^7$s and $1 \times 10^8$s.
Figure D. 6—Quad-branched fracture pressure maps — $1 \times 10^1$s, $1 \times 10^2$s, $1 \times 10^3$s, $1 \times 10^4$s and $1 \times 10^5$s.