Fundamentals of Fluid Flow in Porous Media

At the end of this module, you will:

● Be familiar with the differential equations used for fluid flow in porous media.
  — The gas flow diffusivity equations (Eq. 5.39, 5.47, and 5.59).
  — The "liquid form" of the "dimensionless" diffusivity equation (Eq. 5.119).
● Be familiar with the solutions for these differential equations.
  — The solution for a well in an infinite-acting reservoir.
  — The solution for a well undergoing pseudosteady-state flow.
  — The solution for a well with wellbore storage and skin effects.
● Be familiar with the concept of the "radius of investigation."
● Be familiar with the "skin factor" used to represent non-ideal behavior.
● Be familiar with "wellbore storage" effects which can affect pressure behavior.
● Be familiar with the "Horner" approximation for production time in a buildup test.
● Be familiar with the van Everdingen-Hurst Solutions to the Diffusivity Equation.

Representative Elementary Volume


Porosity

Domain of microscopic inhomogeneity

Domain of possible macroscopic inhomogeneity

Inhomogeneous media

Homogeneous medium

Range for \( U_0 \)

\( U_{\text{min}} \) \hspace{2cm} \( U_{\text{max}} \)

Volume \( U \)
(Very) Basic Flow in Porous Media

\[ \frac{d\Delta P}{2 \rho L V^2} = f \]

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\[ R = \frac{dvp}{\mu} \]

Cornell and Katz Unified Flow Relation for Porous Media

Gas Diffusivity Equation (General Formulation)

- **Diffusivity Equations for a "Dry Gas":**

  - **General Form for Gas:**
    \[ \nabla \cdot \left[ \frac{p}{\mu_g z} \nabla p \right] = \frac{\phi c_t}{k} \frac{p}{z} \frac{\partial p}{\partial t} \n\]

  - **Diffusivity Relations:**
    - Pseudopressure/Time:
      \[ \nabla^2 p_p = \frac{\phi \mu_g c_t}{k} \frac{\partial p_p}{\partial t} \]
    - Pseudopressure/Pseudotime:
      \[ \nabla^2 p_p = \frac{\phi}{k (\mu_g c_t)} p_n \frac{\partial t_p}{\partial t_a} \]

  - **Definitions:**
    - **Pseudopressure:**
      \[ p_{pg} = \left[ \frac{\mu_g z}{p} \right] p_n \int_{p_{base}}^{p} \frac{p}{\mu_g z} dp \]
    - **Pseudotime:**
      \[ t_a = \left[ \mu_g c_t \right] p_n \int_{0}^{t} \frac{1}{\mu_g (p)c_t (p)} dt \]
Gas Diffusivity Equation (General Formulation)

\[ t_a = \left[ \mu_g c_t \right]_n \int_0^t \frac{1}{\mu_g(p)c_t(p)} \, dt \]

- "Dry Gas" Pseudotime Condition: (\( \mu_g c_g \) vs. \( p \))
  - Concept: IF \( \mu_g c_g \approx \) constant, THEN pseudotime NOT required.
  - \( \mu_g c_g \) is NEVER constant — pseudotime is always required (for liquid eq.).
  - However, can generate numerical solution for gas cases (no pseudotime).
Gas Diffusivity Equation (General Formulation)

"Dry Gas" — $p/(\mu_g z)$ vs. $p$

- "Dry Gas" PVT Properties: $(p/(\mu_g z)$ vs. $p$)
  - Basis for the "pressure" approximation (i.e., use of $p$ variable).
  - Concept: $(p/\mu_g z)$ = constant (*never valid*).
Gas Diffusivity Equation (General Formulation)

"Dry Gas" — $\mu_g c_g$ vs. $p$

- "Dry Gas" PVT Properties: ($\mu_g c_g$ vs. $p$)
  - Concept: If $\mu_g c_g \approx$ constant, pseudotime NOT required.
  - Readily observe that $\mu_g c_g$ is NEVER constant, pseudotime required.
Gas Diffusivity Equation (\(p^2\) Formulation)

- Diffusivity Equations for a "Dry Gas": \(p^2\) Relations
  - \(p^2\) Form — Full Formulation:
    \[
    \nabla^2 (p^2) - \frac{\partial}{\partial p^2} \left[ \ln(\mu g z) \right] \nabla (p^2)^2 = \frac{\phi \mu_g c_t}{k} \frac{\partial}{\partial t} (p^2)
    \]
  - \(p^2\) Form — Approximation:
    \[
    \nabla^2 (p^2) = \frac{\phi \mu_g c_t}{k} \frac{\partial}{\partial t} (p^2)
    \]
Gas Diffusivity Equation (\(p^2\) Formulation)

"Dry Gas" — \(\mu_g z\) vs. \(p\)

- "Dry Gas" PVT Properties: \((\mu_g z\) vs. \(p\))
  - Basis for the "pressure-squared" approximation (i.e., use of \(p^2\) variable).
  - Concept: \((\mu_g z)\) = constant, valid only for \(p<2000\) psia.
Gas Diffusivity Equation ($p^2$ Formulation)

$$p_{pg} = \left[ \frac{\mu_g z}{p} \right]_{p_n} \int_{p_{base}}^{p} \frac{p}{\mu_g z} dp$$

$\mu_g z$ versus $p$ (Cartesian Format) for 200 °F
(From Dranchuck EOS (z) and Lee, et al. ($\mu_g$) Correlations)

$\mu_g z$ versus $p$ (Log-Log Format) for 200 °F
(From Dranchuck EOS (z) and Lee, et al. ($\mu_g$) Correlations)

- "Dry Gas" PVT Properties: ($\mu_g z$ vs. $p$)
  - Concept: IF ($\mu_g z$) = constant, THEN $p^2$-variable valid.
  - ($\mu_g z$) ≈ constant for $p < 2000$ psia.
  - Even with numerical solutions, $p^2$ formulation would not be appropriate.
Gas Diffusivity Equation (p Formulation)

Diffusivity Equations for a "Dry Gas": p Relations

- **p Form — Full Formulation:**

\[
\nabla^2 p - \frac{\partial}{\partial p} \ln \left( \frac{\mu_g z}{p} \right) (\nabla p)^2 = \frac{\phi \mu_g c_t}{k} \frac{\partial p}{\partial t}
\]

- **p Form — Approximation:**

\[
\nabla^2 p = \frac{\phi \mu_g c_t}{k} \frac{\partial p}{\partial t}
\]
Gas Diffusivity Equation (p Formulation)

\[
ppg = \left[ \frac{\mu g z}{p} \right]_{p_n}^{p_{base}} \frac{p}{\mu g z} dp
\]

- "Dry Gas" PVT Properties: \((p/(\mu g z) vs. p)\)
  - Concept: IF \(p/(\mu g z) = \) constant, THEN \(p\)-variable is valid.
  - \(p/(\mu g z)\) is NEVER constant — pseudopressure required (for liquid eq.).
  - \(p\) formulation is never appropriate (even if generated numerically).
**Gas Diffusivity Equation (Multiphase Formulation)**

**Multiphase Case — p-Form Relations (Perrine-Martin)**

**Gas Equation:**

\[
\nabla \cdot \left[ \left[ \frac{k_g}{\mu_g B_g} + R_{so} \frac{k_o}{\mu_o B_o} + R_{sw} \frac{k_w}{\mu_w B_w} \right] \nabla p \right] = \frac{\partial}{\partial t} \left[ \phi \left[ \frac{S_g}{B_g} + R_{so} \frac{S_o}{B_o} + R_{sw} \frac{S_w}{B_w} \right] \right]
\]

**Oil Equation:**

\[
\nabla \cdot \left[ \frac{k_o}{\mu_o B_o} \nabla p \right] = \frac{\partial}{\partial t} \left[ \phi \frac{S_o}{B_o} \right]
\]

**Water Equation:**

\[
\nabla \cdot \left[ \frac{k_w}{\mu_w B_w} \nabla p \right] = \frac{\partial}{\partial t} \left[ \phi \frac{S_w}{B_w} \right]
\]

**Multiphase Equation:**

\[
\nabla^2 p = \phi \frac{c_t}{\lambda_t} \frac{\partial p}{\partial t} \quad \lambda_t = \frac{k_o}{\mu_o} + \frac{k_g}{\mu_g} + \frac{k_w}{\mu_w}
\]

**Compressibility Terms:**

\[
c_o = -\frac{1}{B_o} \frac{dB_o}{dp} + \frac{B_g}{B_o} \frac{dR_{so}}{dp}
\]

\[
c_w = -\frac{1}{B_w} \frac{dB_w}{dp} + \frac{B_g}{B_w} \frac{dR_{sw}}{dp}
\]

\[
c_g = -\frac{1}{B_g} \frac{dB_g}{dp}
\]

\[
c_t = c_o S_o + c_w S_w + c_g S_g + c_f
\]
Time-Pressure Schematic Plots

Semilog Plot

\[ p_{wf} \]

\[ \log t \]

Cartesian Plot

\[ p_{wf} \]

\[ t \]
Pressure Distributions — Solutions

All relations given in FIELD units.

Steady-State Solution:

\[ p_r = p_w + 141.2 \frac{qB\mu}{kh} \ln(r/r_w) \quad [p_r - p_{wf} \text{ form}] \]

Full Solution: \((q = \text{constant})\)

\[ p_D = \frac{1}{141.2} \frac{kh}{qB\mu} (p_i - p_r) \approx \frac{1}{2} E_1 \left[ \frac{r_D^2}{4t_D} \right] - \frac{1}{2} E_1 \left[ \frac{r_{eD}^2}{4t_D} \right] + 2 \frac{t_D}{r_{eD}^2} \exp \left[ \frac{-r_{eD}^2}{4t_D} \right] + \left[ \frac{r_D^2}{2r_{eD}^2} - \frac{1}{4} \right] \exp \left[ \frac{-r_{eD}^2}{4t_D} \right] \]

Transient Solution: \((q = \text{constant})\)

\[ p_D (r_D, t_D) = \frac{1}{2} E_1 \left[ \frac{r_D^2}{4t_D} \right] \]

Radius of Investigation:

\[ r_{inv} = 2.434 \times 10^{-2} \sqrt{\frac{k}{\phi \mu \nu t}} \]
Pressure Distributions — Transient Flow

Radial Pressure Distribution (Lee text Fig. 1.7)
Pressure Drawdown and Buildup Cases — E1(x) Solution

Pressure Distributions for Transient Radial Flow

- Note the effect of the drawdown.
- Note that the buildup pressure trends retrace last drawdown trend.
- Recall that all measurements are at the wellbore, we cannot "see" in the reservoir — our analyses are inferred from wellbore measurements.
**Pressure Distributions — Pseudosteady-State**

The physical concept of the PSEUDOSTEADY-STATE FLOW condition is defined as the condition where the pressure at all points in the reservoir changes at the same rate. Mathematically, this condition is given by:

\[
\frac{d}{dt} [p(r,t)]_r = \text{constant}
\]

Pseudosteady-State Flow — Summary of Relations

\((p_r - p_{wf})\) Flow Relations: (Circular Reservoir)

\[
p_r - p_{wf} = 141.2 \frac{qB \mu}{kh} \left[ \frac{r_e^2}{(r_e^2 - r_w^2)} \ln \left( \frac{r}{r_w} \right) - \frac{1}{2} \left( \frac{r_e^2 - r_w^2}{r_e^2 - r_w^2} \right) + s \right]
\]

\((\bar{p} - p_{wf})\) Flow Relations: (\(\gamma = 0.577216\) Euler's constant)

\[
\bar{p} = p_{wf} + 141.2 \frac{qB \mu}{kh} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} + s \right] 
\]

(Circular Reservoir)

\[
\bar{p} = p_{wf} + 141.2 \frac{qB \mu}{kh} \left[ \frac{1}{2} \ln \left( \frac{4A}{e^\gamma r_w^2 C_A} \right) + s \right] 
\]

(General Formulation)

Time-Dependent Pseudosteady-State Flow Relations:

\[
p_r = p_i - 141.2 \frac{qB \mu}{kh} \left[ \ln \left( \frac{r_e}{r} \right) + \frac{1}{2} \left( \frac{r_e^2 - r_w^2}{r_e^2 - r_w^2} - \frac{3}{4} \right) \right] - 5.615 \frac{qB}{V_p C_t} t
\]

\[
p_{wf} = p_i - 141.2 \frac{qB \mu}{kh} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} + s \right] - 5.615 \frac{qB}{V_p C_t} t
\]

**Pseudosteady-State Flow — Illustrative Behavior**

\[ r_{inv} = 2.434 \times 10^2 \sqrt[4]{\frac{k}{\phi \mu c}} t \]

- \( r_1 = 32.2 \text{ ft} \)
- \( r_2 = 88.4 \text{ ft} \)
- \( r_3 = 195 \text{ ft} \)
- \( r_4 = 413 \text{ ft} \)

(1) \( t_1 = 1.77 \text{ hr} \)
(2) \( t_2 = 13.3 \text{ hr} \)
(3) \( t_3 = 64.6 \text{ hr} \)
(4) \( t_4 = 291.7 \text{ hr} \)

Pressure Distribution during Constant Rate Transient Flow Drawdown

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*Reservoir Pressure Distribution — Constant Rate Transient Flow Drawdown.*

**Pseudosteady-State Flow — Illustrative Behavior**

Reservoir Pressure Distribution during Constant Wellbore Pressure Transient Flow Drawdown.

![Graph showing reservoir pressure distribution with various time intervals and corresponding radii](image)

- \( r_1 = 32.2 \text{ ft} \)
- \( r_2 = 88.4 \text{ ft} \)
- \( r_3 = 195 \text{ ft} \)
- \( r_4 = 413 \text{ ft} \)

(1) \( t_1 = 1.77 \text{ hr} \)
(2) \( t_2 = 13.3 \text{ hr} \)
(3) \( t_3 = 64.6 \text{ hr} \)
(4) \( t_4 = 291.7 \text{ hr} \)

\[
\frac{k}{\mu \phi c_i} = 2.434 \times 10^{-2} \sqrt{\frac{1}{t}}
\]

**Reservoir Pressure Distribution — Constant Wellbore Pressure Transient Flow Drawdown.**

Pseudosteady-State Flow — Illustrative Behavior

Reservoir Pressure Distribution During Constant Rate Post-Transient Flow Drawdown, Homogeneous Reservoirs.

Pseudosteady-State Flow — Illustrative Behavior

Reservoir Pressure Distribution During Constant Rate Post-Transient Flow Drawdown, Homogeneous Reservoirs

Reservoir Pressure Distribution — Constant Wellbore Pressure Post-Transient Flow Drawdown, Homogeneous Reservoirs.

Reservoir and Well Solutions

- **Reservoir Models:**
  - Unfractured Well
  - Fractured Well
  - Naturally Fractured Reservoir

- **Reservoir + Well Models:**
  - Unfractured Well: \( WBS + IARF \)
  - Pressure Buildup in a Rectangle
  - Linear (Sealing) Reservoir Boundaries
  - Fractured Well: no \( WBS \)
  - Fractured Well: \( WBS \)
  - Naturally Fractured Reservoir: Unfractured well
**Unfractured Well — Orientation and Solutions**

**Discussion:** Orientation and Solutions (Unfractured Wells)

- Pressure profile propagates radially away from well (homogeneous).
- Cylindrical source solution $\rightarrow$ finite wellbore.
- Line source solution $\rightarrow$ infinitesimal wellbore (i.e., a line).
**Discussion: Flow Regimes (Unfractured Wells)**

- **INFINITE-ACTING RADIAL FLOW (IARF)** is the most "popular" regime.
- **PSEUDOSTEADY-STATE (PSS) flow** → CLOSED BOUNDARIES.
- **STEADY-STATE (SS) flow** → CONSTANT PRESSURE (not realistic).

Schematic drawing of geometry and boundary conditions for radial flow, constant-rate cases.
Discussion: Skin Factor Concept (Unfractured Wells)

- Finite skin concept → zone of "altered" permeability near the well.
- Infinitesimal skin concept → mathematical convenience.
- Negative skin has mathematical (and physical) limitations.
Unfractured Well — Complete Flow Solution

Solution for all points in the reservoir:

\[ p_D(r_D, r_e D, t_D) \approx \frac{1}{2} E_1 \left[ \frac{r_D^2}{4 t_D} \right] - \frac{1}{2} E_1 \left[ \frac{r_e D^2}{4 t_D} \right] + 2 \frac{r_D^2}{r_e D^2} \exp \left[ \frac{-r_e D^2}{4 t_D} \right] + \left[ \frac{r_D^2}{2r_e D^2} - \frac{1}{4} \right] \exp \left[ \frac{-r_e D^2}{4 t_D} \right] \]
Unfractured Well — Transient Flow Solution

Solution at the Well (only):

\[ p_D(r_D = 1, s, t_D) = \frac{1}{2} E_1 \left( \frac{1}{4t_D} \right) + s \]

\[ \approx \frac{1}{2} \ln \left( \frac{4}{e^\gamma} t_D \right) + s \quad (\gamma = 0.577216 \ldots \text{Euler's Constant}) \]
Discussion: Flow Regimes

- **FORMATION LINEAR** flow **DOES NOT EXIST** (a few seconds at most).
- **FORMATION** linear flow $\rightarrow$ **High** fracture conductivity.
- **BILINEAR** flow $\rightarrow$ **Low** fracture conductivity.
Fractured Well — Fracture Damage Comparison

**Discussion:** Fracture Damage Comparison

- **Argument:** Finite conductivity can be modeled as damage...  
  *(false!)*
- "Fluid loss" damage is no referred to as "fracture face" skin.  
- "Choked fracture" damage is just a constant skin factor.  
  *(not correct)*
Fractured Well — Analytical Solution (Uniform Flux)

General (Uniform Flux) Solution: (Infinite Conductivity Solution; \( x_D \approx 0.732 \))

\[
p_D(t_{Dxf},|x_D|<1) = \frac{\sqrt{\pi} t_{Dxf}}{2} \left[ \text{erf}\left(\frac{1-x_D}{2 \sqrt{t_{Dxf}}} \right) + \text{erf}\left(\frac{1+x_D}{2 \sqrt{t_{Dxf}}} \right) \right] + \frac{1-x_D}{4} E_1 \left(\frac{(1-x_D)^2}{4t_{Dxf}}\right) + \frac{1+x_D}{4} E_1 \left(\frac{(1+x_D)^2}{4t_{Dxf}}\right)
\]

Short-Time Solution: Linear Flow

\[
p_{wD}(t_{Dxf}) = \sqrt{\pi} t_{Dxf}
\]

Long-Time Solution: Pseudoradial Flow (Infinite Conductivity Fracture)

\[
p_{wD}(t_{Dxf}) = \frac{1}{2} \left[ \ln(t_{Dxf}) + 2.20000 \right]
\]

(pseudoradial flow: <1% error, \( t_{Dxf} > 10 \))

Identities:

\[
\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2) \, dt
\]

[\text{erf}(0) = 0; \text{erf}(\infty) = 1; \text{erf}(-\infty) = -1]

\[
E_1(z) = \int_z^\infty \frac{e^{-t}}{t} \, dt
\]

\[
[E_1(z < 0.01) \approx \ln\left(\frac{1}{ze^\gamma}\right); E_1(\infty) = 0]
\]

\[(\gamma = 0.577216... \text{ Euler's constant})\]
Fractured Well — Transient Flow Solution

Real Domain: (uniform flux)

\[ P_D \left[ |x_D| \leq 1, y_D = 0, t_{Dx_f} \right] = \frac{1}{4} \int_{-1}^{+1} E_1 \left[ \frac{(x_D - x_{wD})^2}{4t_{Dx_f}} \right] dx_{wD} \]

\[ = \frac{\sqrt{\pi t_{Dx_f}}}{2} \left[ \text{erf} \left( \frac{(1-x_D)}{2\sqrt{t_{Dx_f}}} \right) + \text{erf} \left( \frac{(1+x_D)}{2\sqrt{t_{Dx_f}}} \right) \right] \]

\[ + \frac{(1-x_D)}{4} E_1 \left[ \frac{(1-x_D)^2}{4t_{Dx_f}} \right] + \frac{(1+x_D)}{4} E_1 \left[ \frac{(1+x_D)^2}{4t_{Dx_f}} \right] \]
Naturally Fractured Reservoirs — Fracture Patterns

Discussion: Fracture Patterns

- Fracture patterns are due to stress orientation.
- Large-scale fractures can yield tremendous productivity.
- Stress state changes during production (depletion) — re-fracture?

Fracture Pattern 1: \( \sigma_1, \sigma_3 \) acting in the bedding plane and \( \sigma_2 \) acting normal to the bedding plane (\( \sigma_1 \) — dip direction; \( \sigma_2 \) — strike direction). (Stearns, Courtesy AAPG.)

Fracture Pattern 2: \( \sigma_1, \sigma_3 \) acting in the bedding plane and \( \sigma_2 \) acting normal to the bedding plane (\( \sigma_1 \) — dip direction; \( \sigma_2 \) — strike direction). (Stearns, Courtesy AAPG.)

Various types of fractures generated by folding (courtesy of Leroy").


**Discussion: Fracture Models**

- Kazemi initially produced "slab" model using numerical simulator.
- De Swaan developed the solution for transient interporosity flow.
- Najurieta developed Laplace domain form of De Swaan result.

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Discussion: Warren and Root Model

- "Borrowed" (i.e., stolen) from Barenblatt and Zheltov.
- By far the most popular "heterogeneous" reservoir model.
- Some physical limitations, but its simplicity provides unique flexibility.
Naturally Fractured Reservoirs — W&R Model

Laplace Domain Solution:

\[ \bar{p}_D(u, r_D, \omega, \lambda, s) = \frac{1}{u} K_0(\sqrt{u f(u)} r_D) + \frac{s}{u} \quad \text{(Line Source Solution)} \]

\[ \approx \frac{1}{2u} \ln \left[ \frac{4}{e^r} \frac{1}{r_D^2} \frac{1}{u f(u)} \right] + \frac{s}{u} \quad \text{("Log" Approximation)} \]

\[ f(u) = \frac{\lambda + \omega(1 - \omega)u}{\lambda + (1 - \omega)u} \]

Real Domain Solution: (Derived from the Log Approximation Solution)

\[ p_D(t_D, r_D = 1, \omega, \lambda, s) \approx \frac{1}{2} \ln \left[ \frac{4}{e^r} t_D \right] - \frac{1}{2} E_1 \left[ \frac{\lambda}{\omega(1 - \omega)} t_D \right] + \frac{1}{2} E_1 \left[ \frac{\lambda}{(1 - \omega)} t_D \right] + s \]

\[ p_D'(t_D, r_D = 1, \omega, \lambda, s) \approx \frac{1}{2} + \frac{1}{2} \exp \left[ -\frac{\lambda}{\omega(1 - \omega)} t_D \right] - \frac{1}{2} \exp \left[ -\frac{\lambda}{(1 - \omega)} t_D \right] \]

(No Wellbore Storage)

Radius of Investigation (Transient Radial Flow)

\[
p_D(r_D, t_D) = \frac{1}{2} E_1 \left[ \frac{r_D^2}{4 t_D} \right]
\]

\[
\approx \frac{1}{2} \ln \left[ \frac{4}{e^\gamma} \frac{t_D}{r_D^2} \right] \quad (\gamma = 0.577216...\text{Euler's Constant})
\]

Solve for \( p_D(r_D, t_D) = 0 \), implies that \( p(r_{inv}, t) = p_i \)

\[
\frac{1}{2} \ln \left[ \frac{4}{e^\gamma} \frac{t_D}{r_{D,inv}^2} \right] = 0
\]

\[
\frac{4}{e^\gamma} \frac{t_D}{r_{D,inv}^2} = 1
\]

\[
r_{D,inv}^2 = \frac{4}{e^\gamma} t_D \text{ or } r_{D,inv} = \sqrt{\frac{4}{e^\gamma} t_D}
\]
**Radius of Investigation (Transient Radial Flow)**

\[
r_{D, inv} = \sqrt{\frac{4}{e^\gamma}} t_D
\]

where

\[
p_D = \frac{1}{141.2} \frac{kh}{qB\mu} (p_i - p_{wf})
\]

\[
t_D = 2.637 \times 10^{-3} \frac{k}{\phi \mu c_t r_w^2} t
\]

\[
r_D = \frac{r}{r_w}
\]

Solving using the dimensionless variables yields

\[
r_{inv} = \sqrt{\frac{4}{e^\gamma}} 2.637 \times 10^{-3} \sqrt{\frac{kt}{\phi \mu c_t}}
\]

\[
= 2.434 \times 10^{-2} \sqrt{\frac{kt}{\phi \mu c_t}}
\]
Radial Flow "Skin Factor"
(used to represent non-ideal behavior)
"Wellbore Storage" Effects

Under Construction
Rate Effects
(Horner Approximation and Recommendations for Well Rates)
van Everdingen-Hurst Solutions