Quote du Jour:  When I transfer my knowledge, I teach. When I transfer my beliefs, I indoctrinate.  

— Arthur Danto (1968)

**Topic:** Electrical Properties of Reservoir Rocks

**Objectives:** (things you should know and/or be able to do)

**Fundamentals:**
- Be familiar with conductors and non-conductors of electrical current in porous media.
- Be able to derive the following expression and define all terms:
  \[ R_o = \frac{R_w A L_p}{A_p L} \]
  Note that \( \phi = A_p \) and \( \tau = \frac{L_p}{L} \) where \( \tau \) is called "tortuosity" and is considered to be a measure of rock cementation and/or the pore throat distribution. \( \tau \) can not be measured directly and must be determined by calculation.
- Be familiar with the definition of the formation resistivity factor, \( F \), where \( F \) is defined as:
  \[ F = \frac{R_o}{R_w} = \frac{\tau}{\phi} \]
- Be familiar with the definition of the resistivity index, \( I \), where \( I \) is defined as:
  \[ I = \frac{R_t}{R_o} = \frac{R_t}{FR_w} \]

**Clean Sand Relations:**
- Be familiar with and be able to use the Archie and Humble equations to estimate porosity given the formation resistivity factor, \( F \):
  \[ F = \frac{R_o}{R_w} = \frac{a}{\phi^m} \]
- Be familiar with the Archie result for water saturation, \( S_w \), (fraction):
  \[ S_w = 1 - \frac{1}{n} = \frac{R_t}{R_o} \]
  Be able to determine the cementation factor and saturation exponent for clean (non-shaly) reservoir rocks using the standard log-log plotting approach, as well as the new "type curve" approach.

**Shaly Sand Relations:**
- Be familiar with the effect of shale content on the resistivity index/saturation relation. In particular, be familiar with using the Waxman and Smits model and the Schlumberger total shale model to yield the resistivity index/saturation relation.
- Be able to determine the saturation exponent for shaly reservoir rocks using regression analysis and type curves.
- Be familiar with the effect of shale content on the formation factor/porosity relation. In particular, be familiar with using the Waxman and Smits model and the "parallel resistance" shale model for the formation factor/porosity relation.
- Be able to determine the cementation factor for shaly reservoir rocks using regression analysis and type curves.
Lecture Outline:

- Development of saturated porous media relations
  - Resistance and resistivity
    - Water filled cube
    - Water filled porous media
  - Definition of the formation resistivity factor — Development of the Archie and Humble formation factor relations
  - Definition of the resistivity index — Development of the Archie saturation relation
- Developments using the Waxman and Smits model and the Schlumberger total shale model to yield a resistivity index/saturation relation.
  - Graphical analysis using type curves (direct plotting).
- Developments using the Waxman and Smits model and the "parallel resistance" shale model to yield a formation factor/porosity relation.
  - Graphical analysis using type curves (overlay matching).

Reading Assignment:

- Review attached notes.
  - Resistive Characteristics of Porous Media Theoretical and Experimental Considerations.
  - Additional Comments on the Resistivity (or Formation) Factor (F).
  - Formation Factor/Porosity Relations for Shaly Sands
Exercises: For your own practice/skills building—do **NOT** turn in!

- In each of these derivations/problems you are to work in complete detail and you must show all work.
  
  - Using fundamental principles, you are to derive the formation resistivity factor, \( F \), in terms of the "tortuosity," \( \tau \), where \( F \) is defined as:
    \[
    F = \frac{R_\phi}{R_w} = \frac{\tau}{\phi}
    \]

- You are to provide a critical and detailed review (at least 1 page) for the following paper(s):
  

For each paper you are to address the following questions: (Type or write neatly)

- **Problem:**
  - What is/are the problem(s) solved?
  - What are the underlying physical principles used in the solution(s)?

- **Assumptions and Limitations:**
  - What are the assumptions and limitations of the solutions/results?
  - How serious are these assumptions and limitations?

- **Practical Applications:**
  - What are the practical applications of the solutions/results?
  - If there are no obvious "practical" applications, then how *could* the solutions/results be used in practice?

- **Discussion:**
  - Discuss the author(s)'s view of the solutions/results.
  - Discuss your own view of the solutions/results.

- **Recommendations/Extensions:**
  - How could the solutions/results be extended or improved?
  - Are there applications other than those given by the author(s) where the solution(s) or the concepts used in the solution(s) could be applied?
Resistive Characteristics of Porous Media
Theoretical and Experimental Considerations
(from Petroleum Engineering 620 Course Notes — 1997)

Petroleum Engineering 620
Fluid Flow in Reservoirs
Fig. 6.23—Propagation of an electrical charge in a conductive material.

From: Jorden, J.R. and Campbell, F.L.: Well Logging II--Electric and Acoustic Logging, SPE Monograph Series, 10 (1986), Richardson, TX.
FORMATION FACTOR

- Formation factor is defined as the ratio of the resistivity of completely brine saturated rock to the resistivity of the saturating brine. Tests are conducted on 1" or 1 1/2" cylindrical plugs or whole core samples.

- Formation factor is, therefore, a function of porosity and the pore geometry of the rock. One expression commonly observed is:

\[ F = \frac{R_o}{R_w} = \frac{1}{\phi^m} \]

Where \( m \) is called the cementation factor and is equal to the slope of the line relating \( F \) to \( \phi \) on a log-log plot.

- In many formations a common relationship is:

\[ F = \frac{a}{\phi^m} \]

- Tests are conducted over several day increments until values stabilize.

- Samples should be selected to cover the porosity range noted in the core. This assures better definition of \( m \) and \( a \).

Fig. 6.28—Relationship between formation resistivity factor and porosity for idealized pore systems (after Towle).  

From: Jorden, J.R. and Campbell, F.L.: Well Logging II--Electric and Acoustic Logging, SPE Monograph Series, 10 (1986), Richardson, TX.
Fig. 6.24—Relationship between rock conductivity and saturating brine conductivity.

Fig. 6.25—Effect of vugginess on the electrical conductivities of water-bearing rocks.

From: Jorden, J.R. and Campbell, F.L.: Well Logging II--Electric and Acoustic Logging, SPE Monograph Series, 10 (1986), Richardson, TX.


Effect of Overburden Pressure on Formation Factor

At Zero Overburden
\[ F = \frac{1.0}{\phi^{1.99}} \]

At 5000 PSI Overburden
\[ F = \frac{1.0}{\phi^{2.23}} \]

Effect of Overburden Pressure on Formation Factor
For \( \phi = 15\% \)
Formation Factor Varies from 43.6 to 68.8
For \( F = 45 \)
\( \phi \) Varies from 15\% to 18\%

Porosity (\( \phi \)) : Fraction

RESISTIVITY INDEX

- Oil and gas are not electrical conductors. Their presence in an element of reservoir or in a core sample will reduce the mean cross-sectional area of the flow path for an electric current and increase the length of the flow path, thus increasing the resistivity.

- Resistivity Index is defined as the ratio of rock resistivity at any condition of gas, oil, and water saturation to its resistivity when completely saturated with water:

\[ I = \frac{R_t}{R_0} = S_w^{-n} \]

- Thus, the Resistivity Index is a function of water saturation. It is also a function of the pore geometry. The presence of cation-exchangeable clays (montmorillonites) often cause apparent low Resistivity Index values to be observed, particularly as water saturations decrease.

- The Resistivity Index exponent, n, is also influenced by confining or overburden pressures, and should be determined under overburden conditions where the rock is significantly susceptible to the effect.

- A minimum of three saturations, and preferably five, should be obtained on each sample. Individual sample I versus \( S_w \) plots, as well as composite plots of all samples are typically reported.

Fig. 6.30—Effect of clay content and hydrocarbon saturation on electrical conductivity of hydrocarbon-bearing rock.


Fig. 6.32—Effect of pyrite on conductivity of hydrocarbon-bearing rock.
Fig. 6.34—Effect of wettability on electrical conductivity of sandstones (after Keller). 

Fig. 6.35—Effect of wettability on electrical conductivity of carbonates (after Sweeney and Jennings). 

From: Jorden, J.R. and Campbell, F.L.: Well Logging II--Electric and Acoustic Logging, SPE Monograph Series, 10 (1986), Richardson, TX.
Resistivity Index (RI) vs Water Saturation for Range of Measured Saturation Exponents

RI = \frac{1.0}{S_w^n}

RI = \frac{1.0}{S_w^{2.0}}

n = 2.2

n = 1.6

Effect of "n" on Water Saturation
At RI = 30, S_w Varies From 12% to 21% Pore Space

$S_w = n \sqrt[n]{\frac{F \cdot R_w}{R_t}} = n \sqrt[n]{\frac{R_o}{R_t}}$

Rock Properties Influencing Calculated Water Saturations

CONDUCTIVE SOLIDS

- The clay minerals present in a natural rock act as a separate conductor and are sometimes referred to as "conductive solids". Actually, the water in the clay and the ions in the clay water act as the conducting materials. The effect of the clay on the resistivity of the rock is dependent upon the amount, type, and manner of distribution of the clay in the rock. A few minerals are conductive in their own right, but do not constitute a significant problem in formation evaluation. Clays result in a variable formation factor dependent on the water salinity (resistivity) used to saturate the core. Fresh water yields low F values and low m values.

![Diagram of Formation Factor vs. Brine Resistivity](image)

**Effect of Brine Resistivity on Formation Factor**

Additional Comments on the Resistivity (or Formation) Factor \( (F) \)
(from Petroleum Engineering 306 Course Notes -- 1991)
Definition of Resistivity, $R (\Omega \cdot m)$

$$R = \frac{r \cdot A}{L} \quad (1)$$

where,

$r = \text{resistance (what you would measure)} \quad \Omega$

$A = \text{cross-sectional area L to current flow} \quad m^2$

$L = \text{length of conductor} \quad m$

Definition of $R_c$

$$R_c = \frac{r_c \cdot A}{L} \quad (2)$$

where,

$r_c = \text{resistance of core at 100\% water saturation} \quad \Omega$

$A = \text{cross-sectional area of core face} \quad m^2$

$L = \text{length of core} \quad m$

Definition of $R_w$

$$R_w = \frac{r_w \cdot A_p}{L_p} \quad (3)$$

where,

$r_w = \text{resistance of the formation brine} \quad \Omega$

$A_p = \text{cross-sectional area of the container} \quad m^2$

$L_p = \text{effective length of the container} \quad m$

From the figure we note that

$A = \text{Area of plane MNOP} = \Delta y \cdot \Delta z \quad (4)$

$A_p = \text{Area available for fluid storage}$

or $A_p = \Delta y \cdot \Delta z + \frac{L}{L_p}$ \quad (5)

$\frac{L}{L_p}$ is used to yield a "volume" average cross-sectional area.

Fig. 6.23—Propagation of an electrical charge in a conductive material.
Definition of formation factor, $F$

At 100% saturation, $r_o = r_w$, which yields

$$\frac{R_o}{A} = \frac{R_w}{A_p} L_p$$  \hspace{1cm} (6)

Solving Eq.6 for $F = R_o / R_w$ gives

$$F = \frac{R_o}{R_w} = \frac{A}{A_p} \frac{L_p}{L}$$  \hspace{1cm} (7)

where $F$ is defined as the "formation resistivity factor". Combining Eqs. 4, 5, and 7 gives

$$F = \frac{\Delta \rho \Delta z}{L_p} \frac{L_p}{L} \frac{L_p}{L} \frac{L_p}{L}$$

or

$$F = \frac{1}{\phi} \left( \frac{L_p}{L} \right)^2$$  \hspace{1cm} (8)

where we now define the "tortuosity" as $\zeta = \left( \frac{L_p}{L} \right)^2$. This yields

$$F = \frac{\zeta}{\phi}$$  \hspace{1cm} (9)
Formation Factor/Porosity Relations for Shaly Sands
(from Petroleum Engineering 306 Course Notes — 1991)

Petroleum Engineering 620
Fluid Flow in Reservoirs
Waxman-Smits Shale Relation

Waxman-Smits (1968) give the following result for the conductivity of shaly sands:

\[
\frac{\kappa \cdot \sigma_0}{F_a} = \frac{1}{1 + a}
\]

(1)

where

- \(\sigma_0\) = Conductivity of core at \(s_w = 1\), (ohm-m)\(^{-1}\)
- \(F_a\) = Apparent formation water resistivity factor \((R_w / \sigma_w)\), dimensionless
- \(a\) = dimensionless
- \(\sigma_w\) = Shale parameter, dimensionless
- \(\kappa\) = formation water resistivity, ohm-m

\(B = (1 - 0.06 \exp(-0.77/\sigma_w))^{1/6}\), correlation variable, mol eq. liter\(^{-1}\)

\(Q_v\) = Volume concentration of clay exchange ions, mol eq. liter\(^{-1}\)

Using the definition of the formation resistivity factor, \(F = \frac{1}{\sigma_w} = \frac{1}{\kappa \cdot \sigma_0}\), gives

\[
\frac{1}{F} = \frac{1}{F_a} (1 + a)
\]

(2)

or

\[
F_a = (1 + a) F
\]

(3)

Recall the general model for \(F\) given by Archie

\[
F = c \phi^{-m}
\]

(4)

Combining Eqs. 3 and 4 gives

\[
F_a = c (1 + a) \phi^{-m} = c' \phi^{-m}
\]

(5)

where \(c' > c\), so a trend of \(F_a\) vs. \(\phi\) for a shaly sand should lie above a trend of \(F_a\) vs. \(\phi\) for a similar clean sand. We
will use the data given by Whitsman and Smith to verify or dispute this result. But first we will develop an alternate relation based on parallel resistance theory proposed by Wyllie.

Parallel Resistance Relation

Patnode and Wyllie (Trans ASME, 1950) proposed that clay acts like a parallel resistor in conjunction with the rock and formation brine. Recall Ohm's Law

\[ I = \frac{V}{r} \]  

for \( n \) resistors in parallel the total current \( I \), is equal to the sum of the currents in the loop. Also, the voltage across the loop, \( V \), is constant. This gives

\[ I_{\text{tot}} = I_1 + I_2 \]

or using Eq. 6

\[ \frac{V}{R_{\text{tot}}} = \frac{V}{r_1} + \frac{V}{r_2} \]

Eliminating the constant voltage term

\[ I = \frac{1}{R_{\text{tot}}} + \frac{1}{R_{\text{tot}}} \]

Substituting the apparent resistivity \( R_{\text{tot}} \) for \( R_{\text{tot}} \), and the clay and non-clay resistivities gives

\[ \frac{1}{R_{\text{tot}}} = \frac{1}{R_{\text{clay}}} + \frac{1}{R_{0}} \]

Multiplying through Eq. 8 by the resistivity of the formation brine, \( R_{\text{f}} \), gives
\[
\frac{R_w}{R_{o, x}} = \frac{R_w}{R_o} + \frac{R_w}{R_{o, x}}
\]

or

\[
\frac{1}{F_a} = b + \frac{1}{F}
\]  

where

\[
b = \frac{R_w}{R_{o, x}}
\]  

Solving Eq. 9 for the "apparent" formation resistivity factor, \( F_a \), gives

\[
F_a = \frac{1}{b + \frac{1}{F}}
\]

or

\[
F_a = \frac{F}{1 + bF}
\]

or

\[
F_a = \frac{F}{1 + \frac{R_o}{R_{o, x}}}
\]

Recall the general model for \( F \) given by Archie [Eq.(4)]

\[
F = c \phi^{-m}
\]

combining Eqs. 4 and 11 gives

\[
F_a = \frac{c}{1 + \frac{R_o}{R_{o, x}}} \phi^{-m} = c' \phi^{-m}
\]

Note \( c' = \frac{c}{1 + \frac{R_o}{R_{o, x}}} \), gives \( c > c' \) indicating that a trend of \( F_a \) vs. \( \phi \) for a shaly sand should lay below a trend for a clean sand. This contradicts the Waxman-Smits relation.
The only way to resolve this contradiction (if there is a resolution) is to compare clean and shaly formation data. Data from Waxman and Smits are plotted on the attached graph; the $c'$ and $m$ values are as follows:

<table>
<thead>
<tr>
<th>Case</th>
<th>Sample Set</th>
<th>$c'$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Clean Sandstone</td>
<td>0.95</td>
<td>2.4</td>
</tr>
<tr>
<td>2</td>
<td>Shaly Sandstone #1</td>
<td>0.92</td>
<td>2.4</td>
</tr>
<tr>
<td>3</td>
<td>Shaly Sandstone #2</td>
<td>1.28</td>
<td>1.8</td>
</tr>
<tr>
<td>4</td>
<td>Very Shaly Sandstone</td>
<td>1.8</td>
<td>2.0</td>
</tr>
<tr>
<td>5</td>
<td>Eocene Shaly Sandstone</td>
<td>0.88</td>
<td>1.8</td>
</tr>
<tr>
<td>6</td>
<td>Tertiary Shaly Sandstone</td>
<td>1.94</td>
<td>2.0</td>
</tr>
</tbody>
</table>

The only clear conclusion is that $c'_{shaly} > c'_{clean}$ for these samples. This does not mean that this conclusion is true in general. These data from Waxman & Smits suggest that the Waxman-Smits concept is more applicable to field data than the parallel resistance concept. The most important conclusion is that the Archie type model $(\sigma_r = c' \phi^{-m})$ appears to be valid for shaly sands -- which allows us to make calculations regardless of whose model for $c'$ is correct. It is also worth noting that, for the Waxman-Smits data, $m_{shaly} < m_{clean}$, as suggested in the course notes.
Waxman and Smits Data for Clean and Shaly Sands

Legend: (Waxman-Smits Results)
- "Clean" Sandstone
- "Shaly" Sandstone #1
- "Shaly" Sandstone #2
- "Very Shaly" Sandstone
- Eocene Shaly Sandstone
- Tertiary Shaly Sandstone

$\frac{F_{oR_n}}{R_w}$ vs Porosity ($\phi$), fraction
Plot of Formation Resistivity Factor/Porosity Relations for Clean Sands (No Shale)

Legend ($F = R_p/R_w$)
- $F_D = F/c = \phi^m$ (Archie, $c=1$)
- $F_D = F/c = \phi^{-2.15}$ (Humble, $c=-0.62$)
- $F_D$ (Perez-Rosales) = $F = 1 + 1.85(1-\phi)/(\phi-0.10)$
- $F_D$ (Maxwell) = $F = (3 - \phi)/(2\phi)$
- $F_D$ (Slawinski) = $F = (1.3219 - 0.3219\phi^2)/\phi$

Note:
The Maxwell and Slawinski equations are for unconsolidated spheres, while the Humble and Perez-Rosales equations are for consolidated sandstones.
Grid paper for $F=R_0/R_w$ versus Porosity($\phi$) type curve solution
Saturation/Resistivity Index Relations for Shaly Sands
(from Petroleum Engineering 306 Course Notes — 1991)

Petroleum Engineering 620
Fluid Flow in Reservoirs
Recall the Archie empirical result.

\[ I = R_t = \frac{5_w^{-n}}{R_w} \]  

(1)

Vasen and Smits (SPES, June 1968) developed a semi-analytical water saturation / resistivity relation for shaly sands. This relation is

\[ I = 5_w^{-n} \left[ \frac{1 + a}{1 + a/5_w} \right] \]  

(2)

where

\[ a = R_w B Q_v \]  

(3)

and

\[ R_w = \text{formation water resistivity, ohm-m} \]
\[ B = [1 - 0.6 \exp(-0.77/R_w)] 74.6, \text{correlation variable, mol-equiv-liter}^{-1} \]
\[ Q_v = \text{volume concentration of clay-exchange cations, mol-equiv-liter}^{-1} \]

Although Eq. 2 is theoretically rigorous, it is also difficult to solve for water saturation, $5_w$. Rearranging and solving for $5_w$ will illustrate this point

\[ 5_w^n \left( \frac{1 + a}{5_w} \right) = \frac{1+a}{I} \]

or

\[ 5_w^n + a 5_w^{n-1} = \frac{1+a}{I} \]  

(4)

Note that for arbitrary $n$-values Eq. 4 cannot be solved explicitly and some root solving method is required.
However, an explicit solution for Eq. 4 is highly desired. We resolve this dilemma by solving Eq. 4 for $n=2$, which yields

$$5w^2 + a5w = 1 + a$$

(5)

Applying the quadratic formula to Eq. 5, and only accepting the largest root gives

$$5w = \frac{1}{2} \left[ \sqrt{a^2 + 4(1+a)I} - a \right]$$

(6)

So we conclude, for practical purposes, that $n=2$ is sufficient. A "type curve" solution of Eq. 2 is attached. Note that the $a=0$ cases have several $n$-values and the $n=2$ cases have several $a$-values. This plot can be used to determine $n$ for the $a=0$ cases and/or $a$ for the $n=2$ cases, simply by plotting $I$ vs. $5w$ data directly on the plot and reading the appropriate $a$ or $n$-value. An example at the end of this development illustrates these determinations.

Other Considerations

At present we do not have a physical understanding of the "$a" coefficient in the Warman-Smith relation (Eqs. 2 and 3). One possible concept involves the use of the "total shale" equation developed by Schlumberger Research. This relation is

$$\frac{1}{R_i} = \frac{5w^2}{a_{eq}^2 \rho_w (1 - \varphi_{th})} + \frac{\varphi_{sh} \ 5w}{E_{sh}}$$

(7)
Multiplying thru Eq. 7 by \( R_e \) gives

\[
\frac{E_0}{R_e} = \frac{1}{I} = \frac{E_{shy}}{R_{shy}} \left( 1 + \frac{1}{(1 - V_{sh})} \right) \frac{5_w^2}{R_h}
\]

or since \( E_{shy} = \frac{E_{shy}}{R_{shy}} \), \( E_{clean} = \frac{E_{shy}}{R_{shy}} \), we have

\[
\frac{1}{I} = \frac{E_{shy}}{E_{clean}} \left( 1 + \frac{1}{(1 - V_{sh})} \right) \frac{5_w^2}{R_{shy}}
\]

Rearranging the Newman-Smith relation (Eq. 5) to be in the same form as Eq. 8, we have

\[
\frac{1}{I} = \frac{1}{1 + a} \left( \frac{5_w^2}{1 + a} + \frac{E_{shy}}{E_{clean}} \right) \frac{5_w}{R_{shy}}
\]

Equating coefficients in Eqs. 8; 9 we have

\[
\frac{1}{I} = \frac{E_{shy}}{E_{clean}} \left( 1 + \frac{1}{(1 - V_{sh})} \right)
\]

\[
\frac{1}{1 + a} = \frac{E_{shy}}{E_{clean}} \left( 1 - V_{sh} \right)
\]

\[
\frac{a}{1 + a} = \frac{E_{shy}}{E_{clean}} \frac{V_{sh}}{R_{shy}}
\]

Dividing Eq. 11 by Eq. 10 gives

\[
a = \frac{E_{shy}}{E_{clean}} \frac{V_{sh}}{(1 - V_{sh})}
\]

where,

- \( E_{clean} \) = formation resistivity factor for a clean sand
- \( E_{shy} \) = formation resistivity factor for a shaly sand
- \( R_0 \) = Resistivity at \( 5_w = 1 \) in the shaly sand, ohm-m
- \( R_{sh} \) = Resistivity at \( 5_w = 1 \) in a nearby shale, ohm-m
\( V_{sh} \) = shale fraction in the shaly sand

\( V_{sh} \) is determined from some total shale indicator. For instance, a Neutron-density log crossplot. Eq. 12 indicates a theoretical link between the Waxman-Smits model and reservoir rock properties. This vastly improves our confidence in the Waxman-Smits model. It is not clear that the result in Eq. 12 can be extended for cases where \( n \neq 2 \) for the Waxman-Smits model, Eq. 2.

Example - Determination of \( n \) coefficient for a clean sand.

Data from a dolomite with significant diagenetic in the form of calcite nuggets – this particular sample appears to be homogeneous with no calcite nuggets.

\[
\begin{array}{|c|c|c|}
\hline
\text{Sw, fraction} & \text{I} = K_{1}R_{0} & \rho_{e}, \text{psia (air-brine)} \\
\hline
0.156 & 1 & 2.72 \\
0.9 & 1.263 & 2.81 \\
0.645 & 1.595 & 11.24 \\
0.385 & 2.802 & 44.97 \\
0.308 & 3.963 & 101.2 \\
0.211 & 4.253 & 179.9 \\
\hline
\end{array}
\]

Plotting these data on the clean sand plot gives \( n \approx 1 \). See attached type curve for details.
Plot of Saturation/Resistivity Index Relations for Clean Sands (No Shale)

\[ I(S_w) = \frac{R}{R_0} = S_w^{-\alpha} \]

From this plot no. 1.
Plot of Saturation/Resistivity Index Relations for Clean Sands (No Shale)

\[ I(S_w) = \frac{R}{R_0} = S_w^n \]

- \( n = 1, 1.2, 1.4, 1.6, 1.8, 2, 2.2, 2.4 \)

- \( S_w \): Water Saturation

- \( R \): Resistivity

- \( R_0 \): Resistivity at zero water saturation
Plot of Saturation/Resistivity Index Relations for Clean and Shaly Sands (n=1)

Waxman-Smits Shale Equation:

\[
I(S_w) = \frac{R_w}{R_o} = S_w^{\frac{a}{((1-a)/(1+a/S_w))}}
\]

Rearranging for saturation (n=1 case) gives

\[
S_w = (1+a)/a - a
\]

Schlumberger "Total" Shale Equation:

\[
a = (R_w/R_o)(P_{clean}/P_{shaly})/(V_{shaly}/(1-V_{shaly}))
\]

where \(V_{shaly}\) is obtained from a Neutron-density log crossplot. The Schlumberger equation for "a" may not be valid for cases where \(n\) does not equal 2.

Legend:
- "Clean" Sand Curves \(a=0\), \(n\)-values as indicated
- "Shaly" Sand Curves \(n=1.0\), \(a\)-values as indicated
Plot of Saturation/Resistivity Index Relations for Clean and Shaly Sands (n=1.2)

Legend:
- Dashed "Clean" Sand Curves a=0, n-values as indicated
- Solid "Shaly" Sand Curves n=1.2, a-values as indicated

Waxman-Smits Shale Equation:
\[ f(S_w) = \frac{R_c}{R_o} = S_w^{-\frac{n}{(1+a)(1+a/S_o)}} \]
This case (n=1.2) cannot be solved explicitly for saturation.

Schlumberger "Total" Shale Equation:
\[ a = \frac{R_w/R_o}{(R_{clean}/R_{shale})(V_{sh}(1-V_{sh}))} \]
where \( V_{sh} \) is obtained from a Neutron-density log crossplot. The Schlumberger equation for "a" may not be valid for cases where n does not equal 2.
Plot of Saturation/Resistivity Index Relations for Clean and Shaly Sands (n=1.6)

Waxman-Smits Shale Equation:
\[ I(S_w) = R/R_o = S_w^{-a} \times (1+S_w)/(1+a/S_w) \]
This case (n=1.6) can not be solved explicitly for saturation.

Schlumberger "Total" Shale Equation:
\[ a = (R/R_o)^{(1-c)/(S_{shaly} S_{water})} \times (1-V_a)/(1-V_w) \]
where \( V_a \) is obtained from a Neutron-density log crossplot. The Schlumberger equation for "a" may not be valid for cases where n does not equal 2.
Plot of Saturation/Resistivity Index Relations for Clean and Shaly Sands (n=2)

Waxman-Smits Shale Equation:
\[ I(S_w) = \frac{R_S}{R_o} = S_w^{-\alpha} \left[ \sqrt{1 + a/1 + a/S_w} \right] \]
Rearranging for saturation (n=2 case) gives
\[ S_w = \left( \frac{1}{2} \right) \left( \sqrt{a^2 + 4(1 + a)/I(S_w)} - a \right) \]

Schlumberger "Total" Shale Equation:
\[ a = \frac{R_S}{R_o} \left( \frac{1}{F_{neutron} F_{density}} \left( V_a/(1-V_a) \right) \right) \]
where \( V_a \) is obtained from a Neutron-density log crossplot.
Plot of Saturation/Resistivity Index Relations for Clean and Shaly Sands (n=2.4)

Legend:
- "Clean" Sand Curves \(a=0\), \(n\)-values as indicated
- "Shaly" Sand Curves \(n=2.4\), \(a\)-values as indicated

Waxman-Smits Shale Equation:
\[I(S_w) = R_i/R_o = S_w^{-n} \frac{1}{1+a/S_w}\]
This case \((n=2.4)\) cannot be solved explicitly for saturation.

Schlumberger "Total" Shale Equation:
\[a = (R_i/R_o) \left( I_{\text{shale}} \right) \left( V_a/(1-V_a) \right)\]
where \(V_a\) is obtained from a Neutron-density log crossplot. The Schlumberger equation for "a" may not be valid for cases where n does not equal 2.
References — Electrical Properties of Reservoir Rocks:

The Electrical Resistivity Log as an Aid in Determining Some Reservoir Characteristics

By G. E. Archie*

(Dallas Meeting, October 1941)

The usefulness of the electrical resistivity log in determining reservoir characteristics is governed largely by: (1) the accuracy with which the true resistivity of the formation can be determined; (2) the scope of detailed data concerning the relation of resistivity measurements to formation characteristics; (3) the available information concerning the conductivity of connate or formation waters; (4) the extent of geologic knowledge regarding probable changes in facies within given horizons, both vertically and laterally, particularly in relation to the resultant effect on the electrical properties of the reservoir. Simple examples are given in the following pages to illustrate the use of resistivity logs in the solution of some problems dealing with oil and gas reservoirs. From the available information, it is apparent that much care must be exercised in applying to more complicated cases the methods suggested. It should be remembered that the equations given are not precise and represent only approximate relationships. It is believed, however, that under favorable conditions their application falls within useful limits of accuracy.

INTRODUCTION

The electrical log has been used extensively in a qualitative way to correlate formations penetrated by the drill in the exploitation of oil and gas reservoirs and to provide some indication of reservoir content. However, its use in a quantitative way has been limited because of various factors that tend to obscure the significance of the electrical readings obtained. Some of these factors are the borehole size, the resistivity of the mud in the borehole, the effect of invasion of the mud filtrate into the formation, the relation of the recorded thickness of beds to electrode spacing, the heterogeneity of geologic formations, the salinity or conductivity of connate water, and, perhaps of greatest importance, the lack of data indicating the relationship of the resistivity of a formation in situ to its character and fluid content.

On the Gulf Coast it is found that the effects of the size of the borehole and the mud resistivity are generally of little importance, except when dealing with high formational resistivities or extremely low mud resistivities. Fortunately, little practical significance need be attached to the exact values of the higher resistivities recorded. Low mud resistivities are not common, but when this condition is encountered it may be corrected by replacing the mud column. With the present advanced knowledge of mud control, invasion of mud filtrate into sands can be minimized, thereby increasing the dependability of the electrical log. The effect of electrode spacing on the recorded thickness of a bed is often subject to compensation or can be sufficiently accounted for to provide an acceptable approximation of the true resistivity of the formation. As development of a field or area progressively enhances the knowledge of the lithologic section, the resistivity values of the electrical log take on greater significance, ultimately affording acceptable interpretations. The salinity, and
therefore the conductivity, of the connate water associated with the various producing horizons may be determined with sufficient accuracy by the usual sampling procedure.

Determination of the significance of the resistivity of a producing formation as recorded by the electrical log appears, for the present at least, to rest largely with the application of empirical relationships established in the laboratory between certain of the physical properties of a reservoir rock and what may be termed a formation factor. It should be stressed at this point that numerous detailed laboratory studies of the physical properties of the formations in relation to the electrical measurements in question are essential to a reliable solution of the problems dealing with reservoir content. The purpose of this paper is to present some of these laboratory data and to suggest their application to quantitative studies of the electrical log. It is not intended to attempt to discuss individual resistivity curves and their application. The disturbing factors (borehole, bed thickness, and invasion) are discussed briefly only to indicate instances when they are not likely to affect the usefulness of the observed resistivity.

Resistivity of Sands When Pores Are Entirely Filled with Brine

A study of the resistivity of formations when all the pores are filled with water is of basic importance in the detection of oil or gas by the use of an electrical log. Unless this value is known, the added resistivity due to oil or gas in a formation cannot be determined.

The resistivities of a large number of brine-saturated cores from various sand formations were determined in the laboratory; the porosity of the samples ranged from 10 to 40 per cent. The salinity of the electrolyte filling the pores ranged from 20,000 to 100,000 milligrams of NaCl per liter. The following simple relation was found to exist for that range of porosities and salinities:

\[ R_e = F R_w \]  \hspace{1cm} [1]

where \( R_e \) = resistivity of the sand when all the pores were filled with brine, \( R_w \) = resistivity of the brine, and \( F \) = a “formation resistivity factor.”

In Figs. 1 and 2, \( F \) is plotted against the permeabilities and porosities, respectively, of the samples investigated. The data presented in Fig. 1 were obtained from consolidated sandstone cores in which the cementing medium consisted of various amounts of calcareous as well as siliceous materials. The cores had essentially the same permeability, parallel to and perpendicular to the bedding of the layers. All of the cores were from producing zones in the Gulf Coast region. Cores from the following fields were used: Southeast Premont, Tom Graham, Big Dome-Hardin, Magnet-Withers, and Sheridan, Texas; also La Fice, and Happytown, La. Fig. 2 presents similar data obtained from cores of a widely different sandstone; that is, one that had extremely low permeability values compared with those shown in Fig. 1 for corresponding porosities. These cores were from the Nacatoch sand in the Bellevue area, Louisiana.

From Figs. 1 and 2 it appears that the formation resistivity factor \( F \) is a function of the type and character of the formation, and varies, among other properties, with the porosity and permeability of the reservoir rock; many points depart from the average line shown, which represents a reasonable relationship. Therefore, individual determinations from any particular core sample may deviate considerably from the average. This is particularly true for the indicated relationship to permeability. Further, although the variation of \( F \) with porosity for the two groups of data taken from sands of widely different character is quite consistent, the effect
of variations in permeability on this factor is not so evident. Naturally the two relationships could not be held to apply with equal rigor because of the well established fact that permeability does not bear the same relation to porosity in all sands. From close inspection of these data, and at the present stage of the investigation, it would appear reasonably accurate to accept the indicated relationship between the formation resistivity factor and porosity. Thus, knowing the porosity of the sand in question, a fair estimate may be made of the proper value to be assigned to \( F \), based upon the indicated empirical relationship

\[ F = \theta^{-m} \]  

or from Eq. 1,

\[ R_s = R_a \theta^{-m} \]  

where \( \theta \) is the porosity fraction of the sand and \( m \) is the slope of the line representing the relationship under discussion.
From a study of many groups of data, $m$ has been found to range between 1.8 and 2.0 for consolidated sandstones. For clean unconsolidated sands packed in the laboratory, the value of $m$ appears to be about 1.3. It may be expected, then, that the loosely or partly consolidated sands of the Gulf Coast might have a value of $m$ anywhere between 1.3 and 2.

**Resistivity of Formations when Pores Are Partly Filled with Brine, the Remaining Voids Being Filled with Oil or Gas**

Various investigators—Martin,\(^1\) Jakosky,\(^2\) Wyckoff,\(^3\) and Leverett—have studied the variation in the resistivity of sands due to the percentage of water contained in the pores. This was done by displacing varying amounts of conducting water from the water-saturated sand with non-conducting fluid. Fig. 3 shows the relation which the various investigators found to exist between $S$ (fraction of the voids filled with water) and $R$ (the resulting resistivity of the sand) plotted on logarithmic coordinates. For water saturations down to about 0.15 or 0.20, the following approximate equation applies:

$$S = \left(\frac{R_n}{R}\right)^{1/n} \quad \text{or} \quad R = R_n S^{-n} \quad [4]$$

For clean unconsolidated sand and for consolidated sands, the value of $n$ appears to be close to 2, so an approximate relation can be written:

$$S = \frac{R_n}{\sqrt{R}} \quad [5]$$

or from Eq. 1,

$$S = \sqrt{\frac{R_n}{R}} \quad [6]$$

Since in the laboratory extremely short intervals of time were allowed for the establishment of the equilibrium conditions compared with underground reservoirs, there is a possibility that the manner in which the oil or gas is distributed in the pores may be so different that these relations derived in the laboratory might not apply underground.

**Fig. 3.—Relation of $S$ to $\frac{R}{R_n}$**

<table>
<thead>
<tr>
<th>Legend and Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curve</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>Wyckoff</td>
</tr>
<tr>
<td>Martin</td>
</tr>
</tbody>
</table>

Considerable encouragement on this point is established, however. For example, Eq. 4 appears to hold even though gas or oil is the nonconducting phase. Each probably assumes a different distribution in the pores, yet the resulting resistivity is not appreciably changed. Also, no great change is found in the average relation between the formation resistivity factor and porosity for changes in types of consolidated sandstones. This indicates that even though the oil or gas underground may fill the pore space in a different manner from that in the short-time laboratory experiments, the relationship expressed by Eq. 4 should apply equally well underground.

**Basic Resistivity Values to Be Obtained**

**in Estimating Fluid Content of a Sand**

The foregoing discussion indicates that the basic values to be obtained are: (1) the resistivity of the sand in question under-

\(^1\) References are at the end of the paper.
ground \( (R) \), and (2) the resistivity of the same sand when its pores are entirely filled with connate water \( (R_c) \).

The first value can be obtained from the electrical log when all factors can be properly weighed. The latter may also be obtained from the log when a log is available on the same horizon where it is entirely water-bearing. Of course, this is true only when the sand conditions, particularly porosity, are the same as at the point in question and when the salinity of the connate or formation water throughout the horizon is the same.

In a water-drive reservoir, or any reservoir where the connate water is in direct contact with the bottom or edge water, there should be no appreciable difference in the salinities through the horizon, at least within the limits set forth for the operation of Eqs. 1 and 4; that is, when the salinity of the connate water is over 20,000 mg. NaCl per liter and the connate water is over 0.15. In depletion-type reservoirs, or when connate water is not in direct contact with bottom or edge water, special means may have to be devised to ascertain the salinity of the connate water.

When it is not possible to obtain \( R_c \) in the manner described above, the value can be approximated from Eq. 3, \( \theta \) and \( m \) having been determined by core analyses and \( R_c \) by regular analyses.

### Calculation of Connate Water, Porosity and Salinity of Formation Water from the Electrical Log

The resistivity scale used by the electrical logging companies is calculated assuming the electrodes to be points in a homogeneous bed. Therefore, the values recorded must be corrected for the presence of the borehole, thickness of the layers in relation to the electrode spacing, and any other condition different from the ideal assumptions used in calculating the scale.

Consider a borehole penetrating a large homogeneous layer, in which case the electrode spacing is small in comparison with the thickness of the layer. If the resistivity of the mud in the hole is the same as the resistivity of the layer, there will be, of course, no correction for the effect of the borehole. If the resistivity of the mud differs from the resistivity of the layer, there will be a correction.

Table 1 shows approximately how the presence of the borehole changes the observed resistivity for various conditions. The third curve, or long normal, of the Gulf Coast is considered because this arrangement of electrodes gives very nearly a symmetrical picture on passing a resistive layer and has sufficient penetration in most instances to be little affected by invasion when the filtrate properties of the mud are suitable.

### Table 1.—Effect of Borehole on Infinitely Large Homogeneous Formation

<table>
<thead>
<tr>
<th>True Resistivity of Formation, Meter-ohms</th>
<th>Observed Resistivity on Electric Log</th>
<th>In an 8-In. Borehole</th>
<th>In a 15-In. Borehole</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Resistivity of Mud in Hole (at Bottom-hole Temperature) of</td>
<td>0.5 Meter-ohms</td>
<td>1.5 Meter-ohms</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>50</td>
<td>65</td>
<td>65</td>
<td>65</td>
</tr>
</tbody>
</table>

The values in Table 1 have been calculated assuming a point potential "pick-up" electrode 3 ft. away from a point source of current, other electrodes assumed to be at infinity, and it has been found that the table checks reasonably well with field observations. Checks were made by: (1) measuring the resistivity of shale and other cores whose fluid content does not change during the coring operation and extraction from the well; (2) measuring the resistivity of porous cores from water-bearing formations after these cores were
resaturated with the original formation water. Adjustment due to temperature difference, of course, is necessary before the laboratory measurement is compared with the field measurement.

**Table 2.—Effect of Formation Thickness, No Borehole Present**

<table>
<thead>
<tr>
<th>Layer between Large Shale Bodies Having Resistivity of 1.0 Meter-ohms</th>
<th>Thickness of Layer</th>
<th>Observed Resistivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 Ft.</td>
<td>16 Ft.</td>
<td>8 Ft.</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>11</td>
</tr>
</tbody>
</table>

The correction at the higher resistivities appears to be appreciable. However, in the Gulf Coast when the value of $R_e$ is low the correction is not so important. For example, assume a friable oil sand whose true resistivity is 50 meter-ohms and whose resistivity when entirely water-bearing is 0.50 meter-ohms; the connate water would occupy about 0.10 of the pore volume (Eq. 5). However, if the observed value on the log, 65 meter-ohms, were used without correcting for the borehole, the connate water would be calculated to occupy 0.09 of the pore volume. Therefore, although the effect of the borehole size and mud resistivity on the observed resistivity readings may be appreciable, the resultant effect on the calculated connate-water content of the sand is not important.

When the thickness of the formation is very large in comparison with the electrode spacing, there will, of course, be no correction to make for the thickness of the layer. However, when the thickness of the formation approaches the electrode spacing, the observed resistivity may be very different from the true value. Table 2 shows approximately what the third curve (long normal) of the Gulf Coast would read for certain bed thicknesses and resistivities. It is assumed that large shale bodies are present above and below the beds, at the same time neglecting the presence of the borehole and again assuming point electrodes.

**Fig. 4.—Electrical Log of An East Texas Well.**

Diameter of hole, 7 3/8 in.; mud resistivity, 3.4 at 85°F; bottom-hole temperature, approximately 135°F.

Tables 1 and 2 assume ideal conditions, so if the sand is not uniform, or if invasion affects the third curve, the observed resistivity values may deviate farther from the true value. The magnitude of the influencing factors, of course, will limit the usefulness of the observed resistivity value recorded on the log. Invasion of the mud filtrate is probably the most serious factor; however, as previously mentioned, it can often be controlled by conditioning the mud flush for low filtrate loss.

Fig. 4 shows a log of an East Texas well. The observed resistivity on the long normal curve for the interval 3530 to 3560 ft. is 62 meter-ohms, or, from Table 1, approximately 50 meter-ohms after correcting for the borehole. In this instance the mud resistivity at the bottom-hole temperature of 135°F is approximately 2.2 meter-ohms.
The interval is thick enough so that there should be no appreciable effect due to electrode spacing. The formation is more or less a clean friable sandstone, so Eq. 5 can be used to approximate the connate-water content. The formation resistivity factor for this sand is approximately 15, using Eq. 2 where \( \theta = 0.25 \) and \( n = 1.8 \). The resistivity of the formation water by actual measurement is 0.075 meter-ohms at a bottom-hole temperature of 135°F. Therefore, from Eq. 1, \( R_* \) for this sand is 15 \times 0.075 = 1.1 meter-ohms. This value checks reasonably well with the value recorded at 3623 to 3638 ft. on this log as well as on the many logs from this pool where the Woodbine sand is water-bearing; i.e., 0.9 to 1.5 meter-ohms. The close check obtained between the calculated and recorded resistivity of the water sand indicates that invasion is not seriously affecting the third curve. Solving Eq. 5, the connate water of the zone 3530 to 3560 ft. occupies approximately \( \sqrt{1.1 \times 0.15} = 0.15 \) of the pore volume. The accepted value assigned for the connate-water content of the East Texas reservoir is 17 per cent.

An electrical log of a sand in the East White Point field, Texas, is shown in Fig. 5. The observed resistivity at 4075 ft. is approximately 5 meter-ohms. The value of \( F \) for this sand by laboratory determination is 6. The sand is loosely consolidated, having 32 per cent porosity average. The resistivity of the formation water by direct measurement is 0.063 meter-ohms at the bottom-hole temperature of 138°F. Therefore, \( R_* = 6 \times 0.063 \) or 0.38 meter-ohms. This checks well with the value obtained by the electrical log between the depths of 4100 and 4120 ft., which is 0.40 (see amplified third curve). Therefore, invasion probably is not seriously affecting the third curve. From Tables 1 and 2 it appears that the borehole and electrode spacing do not seriously affect the observed resistivity at 4075 ft. The connate water is approximately \( \sqrt{0.38 
\\frac{5.0}{50}} \), or 0.27.

Other uses of the empirical relations may have occurred to the reader. One would be the possibility of approximating the maximum resistivity that the invaded zone could reach (when formation water has a greater salinity than borehole mud) by Eq. 1, where \( R_* \) would now be the resistivity of the mud filtrate at the temperature of the formation and \( F \) the resistivity factor of the formation near the borehole. By knowing the maximum value of resistivity that the invaded zone could reach, the limits of usefulness of the log could be better judged. For example, assume that a porous sand having an \( F \) factor of less than 15 was under consideration. If the mud filtrate resistivity were 0.5 meter-ohms, the resistivity of the invaded zone, if completely flushed, would be 15 \times 0.5 = 7.5. Thus the observed resistivity values of this sand up to approximately 7.5 meter-ohms could be due to invasion.
ACKNOWLEDGMENT

Cooperation of the Shell Oil Co., Inc., and permission to publish this paper are gratefully acknowledged. The resistivity measurements on the numerous cores were performed under the supervision of S. H. Rockwood and J. H. McQuown, of the Shell Production Laboratories.

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DISCUSSION

(H. F. Beardmore presiding)

S. W. Wilcox,* Tulsa, Okla.—This paper recalls some of my own observations on the correlation of the electrical resistance of earth materials with their other physical properties. While Geophysical Engineer for the Department of Highways, of the State of Minnesota, from 1933 until 1936, I was primarily engaged in conducting earth-resistivity surveys prospecting for and exploring sand and gravel deposits. This work was done by two field parties using equipment of the Gish-Rooney type, and was carried out in every part of the state, both winter and summer.

In brief, when a sand or gravel prospect was discovered, in any way, it was detailed by the resistivity survey to outline its extent and to locate test holes for field and laboratory sample analysis. This survey consisted of a grid of “steptraverses” of one or more electrode separations, and for each an “iso-ohm,” or equal resistance contour plan map, was drawn.

Several thousand earth-resistivity readings were taken over more than one hundred prospects. In some instances the test pitting was started before the completion of electrical survey and their findings were soon available for checking any suspected correlation theory and confirming what subsurface factors were being measured and how effectively.

From accepted earth-resistivity theory, it follows that within a definite sphere surround-

* Seismograph Service Corporation.
In practical terms, I found that an apparent resistance reading of 500 foot-ohms for a 20-ft. electrode separation recorded over ground or glacial moraines of southern Minnesota reliably suggested a deposit of sand or gravel worth further investigation. As a matter of record, prospecting in the part of the state where these materials are very scarce, less than 3 per cent of the test holes located on the geophysical information failed to yield granular materials of commercial quality and quantity for at least highway subgrade treatment. Varying the electrode interval gave additional confirmation as to the thickness of the deposit and very little else.

In connection with our field work, we made extensive laboratory studies, attempting to work out the relation between the moisture content of sand and gravel and its specific resistance. These apparently simple experiments were not of much help in clearing up my field interpretations. Several variables were very hard to control in the laboratory.

The analogy between this type of earth-resistivity mapping and electrologging is close. The first measures electrical impedance along a surface generally parallel to the bedding planes; the latter, up a borehole more or less perpendicular to them. The same general limitations and possibilities appear to be common to both methods. Obviously, controls for checking are easier to obtain for plan mapping than for well logging within the depth of effective penetration.

My interpretation problems appeared to be essentially similar to those of electrical well logging where the operator, after observing the character of the resistance and the self-potential curves, tells his client whether pipe should be set. The accuracy of his prediction is based largely on experience and not on slide-rule calculations.

Mr. Archie's paper suggests an experimental attack for expanding and improving the interpretation technique of electrical well logging. Any contribution of this nature that increases its effectiveness is of great value to the petroleum industry. I offer my own experiences and observations to emphasize that he has tackled a difficult research problem and wish him luck.

Dr. A. G. Loomis,* Emeryville, Calif.—In the laboratory, we take into account the variations in measured resistivities of sands and tap water by finding out the cause of the variations in resistivity. That is, if the tap water itself varied from day to day, its electrolyte content must vary from day to day and chemical analysis would indicate the change. If sands did not give consistent resistivity readings, the character of the sands (in other words, the formation resistivity factor) probably changed or the kind and amount of water contained in the sand must have varied.

* Shell Development Co.
INTRODUCTION TO PETROPHYSICS OF RESERVOIR ROCKS

G. E. ARCHIE
Houston, Texas

ABSTRACT

There is need for a term to express the physics of rocks. It should be related to petrology much as geophysics is related to geology. "Petrophysics" is suggested as the term pertaining to the physics of particular rock types, whereas geophysics pertains to the physics of larger rock systems composing the earth.

The petrophysics of reservoir rocks is discussed here. This subject is a study of the physical properties of rock which are related to the pore and fluid distribution. Over the past few years considerable study has been made of rock properties, such as porosity, permeability, capillary pressure, hydrocarbon saturation, fluid properties, electrical resistivity, self- or natural-potential, and radioactivity of different types of rocks. These properties have been investigated separately and in relation, one to another, particularly as they pertain to the detection and evaluation of hydrocarbon-bearing layers.

GENERAL

This paper is concerned with rocks and their fluids in situ, particularly for the detection and evaluation of hydrocarbon deposits penetrated by a bore hole. Fundamentally, therefore, the study is one of pore size distribution and fluid distribution of each phase (oil, gas, water) within the pores of the rock.

The subject must not be limited to permeable rocks containing hydrocarbons, but should include the impermeable layers and permeable layers containing water as well. This must be done in order to distinguish between them.

DISCUSSION

Rocks are heterogeneous. Therefore, the pore-size distribution as well as the fluid distribution within the pores may be complicated, particularly from the microscopic point of view. We are dealing with a heterogeneous material, together with a great many varying conditions within this heterogeneous material. This, no doubt, is the reason for early belief that a quantitative approach to the problem might never be attained.

When rocks are studied from a macroscopic viewpoint, however, a definite continuity is found. A correlation of rock properties has resulted in the discovery that definite relations or trends exist between rock characteristics. If pieces of rock representing each transition phase of a formation are studied, definite trends are noted. No matter how thoroughly a single piece of rock is studied, even on a microscopic scale, it is not possible to predict the properties of a formation as a whole. This should not be taken to mean that fundamental research on a microscopic scale is not of great importance in the study of rock porosity.

Though permeable rocks are, by nature, heterogeneous, their characteristics

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2 Shell Oil Company. The writer expresses his appreciation to the Shell Oil Company for permission to publish this paper and to the many authors having published information on the subjects covered here. An extensive bibliography would be needed to refer the reader to all pertinent articles.
follow definite trends when considering a formation as a whole. Relations between the basic rock pore properties may be indicated somewhat as follows.

\[
\begin{align*}
\text{Type of Rock} & \\
\text{Pore Size Distribution} & \quad \text{Water Saturation} \\
\text{Porosity} & \quad \text{Connate} \\
\text{Permeability} & \quad \text{Capillary Pressure} \\
& \quad \text{(Pore Interconnection)}
\end{align*}
\]

Type of rock, as here referred to, is a formation whose parts have been deposited under similar conditions and have undergone similar processes of later weathering, cementation or re-solution, as, for example, the upper Eocene Wilcox, lower Frio, Woodbine, or Bartlesville in a particular area.

The connecting lines are meant to portray the fact that a specific formation or rock type will have certain effective pore-size distributions which will produce a particular family of capillary pressure curves. The pore-size distribution controls the porosity and is related to the permeability and water saturation. Further, a certain rock will exhibit a relation between porosity and permeability.

**ROCK TYPE-POROSITY-PERMEABILITY RELATION**

A broad relationship exists between porosity and permeability of a formation. Figure 1 shows a plot of the measured values of these properties of cores from the upper Eocene Wilcox sandstone at Mercy, Texas, and the Nacatoch sandstone at Bellevue, Louisiana. The scattering is great, but it must be remembered that the only reason a trend exists at all is that the formation as a whole was deposited under a similar environment; individual parts (local environment) may differ from the whole. The trends shown in Figure 1 may be represented by a line. The average trend for different formations is shown in Figure 2. Note the paralleling of trends of different formations. (Only limited data are available in the high permeability range; therefore the lines have not been extended beyond the values shown.) Rocks indicated on the left, those having low porosity for a certain permeability, have relatively large pores, for example, the oolitic limestone. Those on the right have a high porosity for the permeability, indicating a smaller pore size, for example, the poorly sorted shaly sands of the Gulf Coast and the poorly sorted, shaly, calcareous Nacatoch sandstone.

It is interesting to find that the increase in permeability with increase in porosity is of the same general order for many of the formations. An increase in porosity of about 3 per cent produces a ten-fold increase in permeability. This is striking in view of the fact that the formations are widely different. For example,
Figs. 1 and 2.—Average relation between porosity and permeability for different formations.
the Smackover limestone, on the average, has a permeability of 1 millidarcy when the total porosity is only 7 per cent; the Nacatoch sandstone for the same permeability has an average total porosity of about 28 per cent.

It will be noted that some of the trends are not essentially parallel with the others: (1) Faluxy sandstone is believed to be of dune origin and has grains of uniform size; as a result, the average pore size is larger than for water-deposited sandstones and the permeabilities are higher, particularly in the lower range of porosity; (2) lower Wilcox has probably undergone considerable change since deposition (cementation of the fine pores and solution causing larger pores) and has a steeper trend than the other formations.

It appears that as diagenesis continues, the porosity-permeability trend moves toward the left. Compare the loose Gulf Coast sand trend with the harder sandstones where the smaller pores are absent and the larger pores are possibly enlarged.

Figure 2 indicates that an approximation of the probable permeability can be made when the porosity of a formation is known. It must be remembered, however, that the relation is an average of a large number of data from a formation and, when applying the relation to the same formation at a different locality, the assumption is made that the rock structure is similar.

If the type of void structure throughout the formation is similar, the relation is, of course, more significant. However, even at the same location, certain parts of a formation may deviate considerably from the average trend because the pore structure or type of rock changes appreciably. An example of this is shown in Table I. This small interval is purposely chosen from several hundred feet of Eocene Wilcox formation.

<table>
<thead>
<tr>
<th>Depth</th>
<th>Permeability</th>
<th>Porosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,459-51</td>
<td>1.0</td>
<td>14.1</td>
</tr>
<tr>
<td>10,451-52</td>
<td>1.0</td>
<td>17.1</td>
</tr>
<tr>
<td>10,452-53</td>
<td>0.5</td>
<td>15.1</td>
</tr>
<tr>
<td>10,453-54</td>
<td>0.8</td>
<td>15.0</td>
</tr>
<tr>
<td>10,454-55</td>
<td>2.3</td>
<td>16.3</td>
</tr>
<tr>
<td>10,455-56</td>
<td>1.1</td>
<td>16.1</td>
</tr>
<tr>
<td>10,456-57</td>
<td>3.4</td>
<td>16.0</td>
</tr>
<tr>
<td>10,457-58</td>
<td>0.1</td>
<td>11.2</td>
</tr>
<tr>
<td>10,458-59</td>
<td>72.6</td>
<td>14.6</td>
</tr>
<tr>
<td>10,460-61</td>
<td>82.0</td>
<td>14.0</td>
</tr>
<tr>
<td>10,462-63</td>
<td>45.2</td>
<td>12.9</td>
</tr>
<tr>
<td>10,464-64</td>
<td>140.0</td>
<td>14.6</td>
</tr>
<tr>
<td>10,464-67</td>
<td>22.4</td>
<td>10.4</td>
</tr>
</tbody>
</table>

The interval 10,458–64 feet has a porosity of about 14.5 per cent and a permeability of about 100 millidarcys, whereas the interval immediately above, from 10,450–58 feet, has a porosity of about 16 per cent and only 1 millidarcy permeability. The former interval is clean and well sorted and therefore not typical of the lower Wilcox. Therefore, this relation of porosity versus permea-
bility is only valid in a general way for a formation as a whole, and may not apply to a small integral part. The average data shown in Figure 2 does, however, help one visualize relative pore size of different types of formations.

ROCK TYPE—CAPILLARY PRESSURE—CONNATE WATER—PERMEABILITY RELATION

Air permeability of a dry rock sample is a measure of the average contributing effect of pores of all sizes. This average is not sufficient for complete analysis because the pore size must be considered in order to obtain permeability “in situ” (relative permeability) and fluid distribution. Capillary pressures of rocks help to analyze this average figure by obtaining what may be called effective pore-size distribution.

Capillary pressures of rocks have been discussed in the literature regarding connate water, and recently the relation between permeability and capillary pressure has been presented. In order to tie the captioned relation more closely to rock type, capillary-pressure curves of different types of permeable formations are presented; the pore structures are due to a wide variety of geologic processes, that is, (a) sedimentation with little alteration, (b) alteration by solution, (c) redeposition or cementation. A comparison of capillary-pressure curves for different rock types and for the same type with varying permeabilities may then be made.

The following types of permeable formations are presented.

1. Friable sandstone, having high permeability and well sorted grains (Pennsylvanian sand, Heflin, Oklahoma)
2. Friable sandstone, poorly sorted grains, grading to shaly sandstone in the low-permeability range; partly cemented (upper Eocene Wilcox formation, Mercy, Texas)
3. Friable sandstone, poorly sorted grains, shaly and calcareous; comparatively high porosity for permeability (Mancos formation, Bellevue, Louisiana)
4. Hard sandstone, heavily cemented and considerable resolution; comparatively low porosity for permeability (lower Eocene Wilcox formation, Sheridan, Texas)
5. Limestone, crystalline texture, rock material of original deposition, porosity being of secondary nature, consisting of small interconnected vugs due to solution (San Andres limestone, West Texas)
6. Limestone, finely granular to earthy texture, siliceous; comparatively high porosity for permeabilities (Devonian cherty limestone, West Texas)

In types 1, 2, and 3, the pore size, shape of pore, and interconnection of the pores are controlled mainly by the original deposition, altered little by later cementation or solution. Therefore, pore structure is due mainly to the manner in which the fragments were deposited, that is, the sorting action and packing of the grains due to wave action and later compaction. The pore structure in type 4 is due mainly to later cementation and solution; in other words, the pore structure originally due to sedimentation has been altered considerably. The effective pore structure in type 5 is due almost entirely to solution with some redeposition. The origin of 6 is controversial.

Families of capillary pressure curves for each of these types are shown in.

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Fig. 3.—Families of capillary pressure curves for some sandstones.
Fig. 4.—Families of capillary pressure curves for some limestones; also idealized curve.
Figures 3 and 4. These curves depict results obtained by injecting mercury into the cores. In order to show porosity in the graphs, the abscissae are "per cent bulk volume occupied by mercury." For example, it may be noted in Figure 3 that for sample No. 33795A, permeability 1625 millidarcys, the curve, with increasing pressure, approaches the abscissa 27.7, its effective porosity. These charts representing data from suites of cores show how the capillary-pressure curve, permeability, and porosity are related.

Certain general conclusions can be drawn by comparing the graphs. It can be seen that all of the samples having appreciable permeability exhibit a plateau, or seat, and a steep slope (idealized curve, Fig. 4). Examination of the capillary pressure curves will reveal that two straight lines (representing plateau and steep slope) can be drawn in and define the curves fairly well. The angle formed by extending these characteristic lines is useful. This angle "A" increases as the permeability of each type of rock decreases until one line appears and the plateau disappears.

The families of curves exhibit a striking similarity for the same permeabilities, regardless of type of formation or origin of porosity. Of course there are differences; for example, the rocks with high permeability for porosity exhibit much steeper steep slope (for example, San Andres limestone) because of a less amount of small pore space. Formations with comparatively low permeability for porosity, however, exhibit a more gentle steep slope because of the many small pores (Nacatoch and Devonian limestone). The curves of the Devonian limestone, which approaches chalk in texture, differ most from the others. It has a proportionally large amount of fine to very fine pores.

Photomicrographs of specimens of each type of rock investigated are presented in Plates I and II in order to show the visible difference in rock texture and pore structure. Notice the large pores in one of the San Andres limestone specimens, the medium pores in the sandstones and the very fine pores in the Devonian limestone and shaly sandstones. The rock textures include crystalline limestone, medium to very fine granular sandstones and very fine granular, almost chalky, Devonian limestone.

**OTHER ROCK PROPERTIES RELATED TO PORE-SIZE DISTRIBUTION**

If it were possible to measure the fundamental properties (exact pore size and fluid distribution) in situ of formations penetrated by the bore hole, the volume of the hydrocarbon in place and the productivity of the layer could be calculated. However, it is practically impossible as yet to get a direct measurement of the factors, porosity, permeability, hydrocarbon saturation, and thickness of the layer, in place, by coring or other physical measurements.

Complete recovery of cores can not be assured and all permeable cores recovered are invariably contaminated with the drilling fluid, or fluid conditions have changed because of pressure and temperature changes on bringing the core to the surface. Therefore, we must resort to indirect measurements, such as elec-
UPPER EOCENE WILCOX, MERCY TEXAS
FRIABLE SANDSTONE, POORLY SORTED GRAINS GRADING TO SHALY SANDSTONE 15X

LOWER EOCENE WILCOX, SHERIDAN TEXAS
HARD SANDSTONE, HEAVILY CEMENTED AND CONSIDERABLE RE-SOLUTION 15X
NACATOCH SANDSTONE, BELLEVUE LA. 15X
HIGH POROSITY FOR GIVEN PERMEABILITY

SAN ANDRES LIMESTONE, WEST TEXAS 10X

DEVONIAN CHERTY LIMESTONE, CROSSETT TEXAS 10X
COMPARATIVELY HIGH POROSITY FOR GIVEN PERMEABILITY
trical resistivity, self-potential, and neutron reaction, which can be recorded in a bore hole filled with mud. In order for these indirect measurements to be useful they must be directly related to the physical properties desired (porosity, permeability, and fluid saturation). Correlation of the indirect with the actual physical properties from a macroscopic point of view by actually testing numerous cores has led to the discovery that definite relationships or trends do exist.

**ROCK TYPE-SELF POTENTIAL-GROUND WATER SALINITY-PERMEABILITY RELATION**

Discussions in the literature have pointed out that the self potential is composed chiefly of two components: (1) the flow potential, and (2) the chemical potential. The flow potential is thought to be a smaller part of the total where

![Diagram](image)

Fig. 5.—Measurement of shale potential.

the ground waters are very saline. Actually, the flow potential in a well can be measured by recording the S.P. with pressure on the well head.

The chemical potential has been expressed by the equation

\[
\text{S.P.} = -K \log \frac{R_1}{R_2}
\]

where \(K\) depends mainly on the type of impermeable rock, and the resistivity ratio is that of the mud filtrate \(R_1\) and formation water \(R_2\). The difference between the S.P. recorded opposite an impermeable shale, for instance, and that recorded opposite an infinitely permeable clean sandstone may be expressed by


this equation. This difference can be measured in the laboratory by placing salt water at one end of a piece of shale and fresh water at the other end. Actually, in the laboratory it is difficult to measure the voltage thus generated by placing the electrodes in the salt and fresh water because of electrode potentials; therefore, the set-up shown in Figure 5 is used. It will be seen that each of the electrodes is now in the same solution, thus cancelling the electrode potential so bothersome in laboratory measurements. Note further that the potential recording at 1 is the “shale potential” plus the liquid contact potential \((D/B)\). The recorded potential is the result of:

\[
\begin{align*}
B/C & \text{ fresh water-shale contact} \\
C/D & \text{ shale-salt water contact} \\
D/B & \text{ salt water-fresh water liquid junction contact.}
\end{align*}
\]

Shale cores obtained from different formations have been tested by using different amounts of dissolved salts in the salt water. It appears that \(K\) in the for-

![Diagram](image)

**Fig. 6.** Qualitative relation between self potential and sand-shale section.

going equation takes on different values for different shales. For example, shales of the Eocene Wilcox have a \(K\) value of the order of 60, while that of some shallow Pennsylvanian shales may be as low as 25.

When the permeability of the formation is not high, however, the S.P. recorded opposite it in a bore hole is somewhat less. In the case of a sand and shale section, the relation between type of formation and self potential may be expressed by Figure 6. Actually, in some cases, the ordinate in Figure 6 may be replaced qualitatively, at least, with a permeability scale. For example, Figure 7-A shows the S.P. curve recorded opposite a section of the Eocene Wilcox formation. The interval 10,800 to 11,800 feet is a sandstone and shale section. The
sandstone is more or less the same type throughout, being poorly sorted, well cemented, and hard. Increasing amounts of argillaceous material are noted with decreasing permeabilities. The formation water is the same throughout this section, the mud in the bore hole at the time of the survey was uniform, and all the formations are water-bearing. The formation was extensively cored and the average permeability is indicated opposite the layers from which sufficient cores were recovered and analyzed.

The magnitude of the self-potential for the various layers is plotted versus the permeability in Figure 7-B. Note the apparent semi-logarithmic relation (similar to that shown in Figure 6).
It may be noted that there is considerable scattering of points and the relation between S.P. and permeability is qualitative in nature.

ROCK TYPE-ELECTRICAL RESISTIVITY-POROSITY-WATER SATURATION RELATION

The electrical resistivity of rocks when the pores are saturated with brine may be expressed by:

\[ R_o = F R_w \]

where \( R_o \) is the resistivity of the rock when saturated with brine (over about 10 grams per liter dissolved salts); \( F \) is the formation resistivity factor; and \( R_w \) is the resistivity of the brine. \( F \) is found to be related to the porosity and the type of rock.

\[ F = f^{-m} \text{ or } R_o = R_w f^{-m} \]

where \( f \) is the porosity of the rock and \( m \) is related to the type of rock.

Again it must be remembered that this equation represents a trend or an average line through a number of measured values (Fig. 8 and 9).

RELATION OF POROSITY TO FORMATION RESISTIVITY FACTOR

![Diagram](image)

**Fig. 8.—Resistivity factors of various sandstones versus porosity.**

The electrical resistivity of a rock when hydrocarbon-bearing may be expressed by:

\[ R = R_w S^{-n} \]

where \( S \) is the fraction of the voids filled with brine; \( R \) is the resistivity of the hydrocarbon-bearing rock; and \( n \) depends, apparently, on the type of rock.

---

Laboratory results on artificially saturated loose sand packs indicate $n=2$, (Figure 10-a), whereas measurements on consolidated sandstones naturally saturated (as withdrawn from the well) indicate $n=1.9$ (Fig. 10-b). The foregoing equation holds for water saturations down to about 10 per cent. Below this value there is some indication that the interconnection between the water is no
longer uniformly continuous and the resistivity increases more rapidly than indicated by the equation.

Actually, therefore, it may be said that the electrical resistivity of a hydrocarbon-bearing rock depends on the porosity, brine saturation, salinity of the brine, and type of rock.

**ROCK TYPE-NEUTRON REACTION-HYDROGEN CONTENT-POROSITY RELATION**

Several papers have been written on this subject, and it appears that the reaction from neutron bombardment of formations depends to a large extent on their
INTRODUCTION TO PETROPHYSICS OF RESERVOIR ROCKS

hydrogen content. The effect of rock type is apparently not known. The porosity must be considered, for this determines the actual void space available to contain hydrogen-bearing fluids. The relative relation between total porosity and response to neutron bombardment, using present field methods, is illustrated in Figure 11. The limestone zones are oil-bearing. Each point represents an average of several feet where the porosity was relatively uniform. The relation would be different if these zones were dry gas-bearing, for the hydrogen density of dry gas in the pores is much lower.

![Graph showing relation of total porosity to neutron response.]

**Fig. 11.—Response to neutron bombardment.**

SUMMARY OF INDIRECT PROPERTIES

Figure 12 illustrates diagrammatically how the various indirect properties are related to the desired reservoir properties. Some relationships are rigid and quantitative, while others are not easily predicted and are considered only qualitative. After a qualitative relation is studied further, other factors may be discovered which, when incorporated in the equation, put the relationship on a more quantitative basis. For example, the qualitative relation between porosity and permeability is made more quantitative by introducing pore size or the capillary-pressure curve. The solid lines in the chart indicate what are now known to be quantitative relationships, while the broken lines indicate qualitative relationships.

Figure 12 shows how intricately the various properties are related; all relations

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G. E. ARCHIE

INTERRELATION OF PHYSICAL PROPERTIES OF ROCKS

Fig. 12.

PETROPHYSICAL SYSTEM

Fig. 13.
are tied to the type of rock. The diagram shows the many possibilities that can arise in attempting to unravel the interrelationships in order to evaluate a hydrocarbon accumulation in situ. In actual practice, further complications arise due to practical difficulties, economic considerations, and the personal equation. For example, the presence of the bore hole itself, its geometry, and the fact that the bore hole must be filled with mud in order to drill the hole brings up many problems. The layers penetrated are not infinitely thick; therefore, boundary effects (thin layer effect) must be calculated and applied to all indirect measurements. Also, mud filtrate contaminates the permeable layers near the bore hole and it too must be considered. In fact, these practical complications are commonly the most difficult to interpret in attempting to detect and evaluate deposits penetrated in a bore hole.

CONCLUSIONS

In conclusion, a tentative petrophysical system, from the macroscopic viewpoint, is presented in Figure 13 for illustrative purposes. The system revolves mainly around pore-size distribution which defines the capillary-pressure curve, permeability, and porosity. The pore-size distribution does not necessarily define the type of rock, for actually several types of rock may have essentially the same pore-size distribution.

It is not meant that the mineral composition of the rock should be neglected in a study of these relationships. It must be recognized that the type of clay minerals present, for example, will no doubt play a greater role in future study.

It should also be mentioned that lithologic description of rocks is important. In fact, it should be broadened to express pore-size distribution as well as mineral distribution. It is felt that this is very important because drill-cuttings sample-logging (formation type) really is an integral part of this outline.

The relations between rock characteristics should be thought of as trends. Actually, these may be expressed by mathematical formulae; however, the formulae can not be applied in a rigid manner, as is done when considering the properties of homogeneous materials. It must be kept in mind that appreciable deviations from the average trend may occur. The less uniform the data, the less rigid will be the average relation, for some permeable rocks are more heterogeneous than others. The generally uniform types of permeable rocks are sandstones, oölitic limestones, and the so-called granular-appearing dolomites. The less uniform types are the so-called vesicular, vugular, or cavernous, or even fractured limestones and dolomites.

8 Schlumberger Company pamphlet, Resistivity Departure Curves (May, 1947).
10 Schlumberger Company pamphlet, op. cit.
FORMATION FACTORS OF UNCONSOLIDATED POROUS MEDIA: INFLUENCE OF PARTICLE SHAPE AND EFFECT OF CEMENTATION

M. R. J. WYLIE AND A. R. GREGORY, GULF RESEARCH AND DEVELOPMENT CO., PITTSBURGH, PA., MEMBERS AIME

ABSTRACT

The literature reveals that scant attention has been paid to the systematic experimental determination of the formation factors of unconsolidated porous media. No experiments appear to have been made on the effect of increasing cementation on the formation factor of an initially unconsolidated porous medium.

Measurements have accordingly been made of formation factors as a function of porosity for aggregates of spheres in the porosity range 12-56 per cent and of cubes, cylinders, discs and triangular prisms in the porosity range 30-45 per cent. The results are examined in the light of the theoretical equations of Clerk Maxwell, Fricke and Slawinski.

Aggregates of unconsolidated spheres and beach sands have been artificially cemented with silica and the formation factor-porosity relationships determined. A theory is outlined which seeks to explain the results obtained and which postulates that formation factor and porosity data for consolidated sandstones may be used to determine the original porosity of the unconsolidated sands from which the consolidated sandstones were derived. It follows also that for consolidated sandstones \( F = C \phi^n \) where \( F \) is formation factor, \( \phi \) fractional porosity and \( C \) and \( k \) are constants.

INTRODUCTION

All schemes of electric log interpretation implicitly or explicitly require the use of formation resistivity factors or, as they are now generally called, formation factors. The originator both of the concept of formation factor and of the use of formation factor data in electric log interpretation was Archie. More recently it has been suggested that formation factors are of value in describing fluid flow in porous media. A full discussion of this problem together with adequate references has been given by Wyllie and Spangler.

In spite of the theoretical and practical importance of the subject, there appear to be remarkably few basic formation factor data in the petroleum literature. A wider search reveals that this situation is not confined to the literature of the petroleum industry.

At various times over the last century, attempts have been made to compute the theoretical relationship between the formation factor and porosity of simple systems of unconsolidated particles. However, the formulae derived are not all identical and the experimental evidence adduced in support of each is not wholly convincing. Most of the experimental work has been carried out on systems of unconsolidated spheres, although some attention has been given to systems of spheroids and cylinders. Other simple shapes have received neither theoretical nor experimental investigation. The writers feel that a better understanding of the formation factors of complex systems of practical importance requires a firm foundation of reliable experimental data applicable to simple systems. Accordingly, they have reviewed the relevant literature and have determined experimentally the relationship between the formation factor and porosity of unconsolidated aggregates of particles of regular geometrical shapes. Some experiments have also been carried out to investigate the effect of cementation on the formation factor-porosity relationship.

PREVIOUS WORK

Although the term formation factor was not used, the relationship between the resistivity of a solution containing dielectric solids and the resistivity of a similar solution not containing solids is an old and important problem.

The formation factor of a system of dispersed spheres as a function of porosity was considered by Clerk Maxwell. Maxwell arrived theoretically at the following expression:

\[
F = \frac{3 - \phi}{2\phi}
\]

where \( F \) is formation factor and \( \phi \) fractional porosity.

Lord Rayleigh also examined the problem of the formation factor of systems of spheres and cylinders and concluded that for spheres Clerk Maxwell's expression gave a value of \( F \) which, at low porosities particularly, was too small. However,
Lord Rayleigh's expressions were extremely complex and were not supported by any experimental data.

Fricke's generalized Clerk Maxwell's treatment; he incidentally confirmed Equation (1) for spheres. His more general expression, applicable not only to spheres but also to oblate and prolate spheroids, was

\[ F = \frac{(x + 1) - \phi}{x\phi} \]

where \( x = 2 \) for spheres and \( x < 2 \) for spheroids. The greater the axial ratio of the spheroid, the smaller Fricke computed \( x \) to be. Fricke's mathematical treatment appears to indicate that in any randomly disposed aggregation of particles \( F \) will be a minimum, for any value of porosity, when the particles are spherical. Some experimental data were adduced by Fricke and by Fricke and Morse⁹ to support his mathematical conclusions. While the data appear to substantiate the approximate accuracy of Equation (2), they cannot be held to confirm it to any high degree of precision. Most of the data presented by Fricke and Fricke and Morse lead only to the conclusion that Equation (2) reflects rather well the relative change in \( F \) which accompanies a change in the porosity of a randomly disposed aggregate of particles of a particular shape.

Ślawinski⁷ has also discussed the relationship between \( F \) and \( \phi \) for aggregates of spheres both in contact and dispersed. The contribution of Ślawinski appears to have been rather neglected, possibly because the mathematical approach he used is so simple and crude as to be superficially unconvincing. Nevertheless, Ślawinski presents more experimental evidence than other workers to support his theoretical deductions.

For an aggregate of spheres which are in contact with one another Ślawinski derives

\[ F = \frac{(1.3219 - 0.3219 \phi)^3}{\phi} \]

Isolated measurements of the formation factors of unconsolidated systems of spheres and silica sands at various porosities have been reported in the literature. Notable among these are those given by Archie,¹ by Klinkenberg¹ and by Schofield and Dakshanamurti.⁸

**EXPERIMENTAL**

Preparation of Particles of Regular Shape

Glass or plastic spheres which have high sphericity may be obtained with a wide range of diameters. The separation of relatively non-spherical particles from particles of high sphericity may be achieved by making use of the greater ability of the latter particles to roll down a slightly inclined plane.

The procurement of accurately sized particles of non-spherical shape presents a problem of greater difficulty. In this work the problem was largely solved by utilizing an ingenious suggestion made to the writers by Forrest F. Versaw of the engineering division of these laboratories. Plastic (Lucite) rod of square, circular, triangular and other regular cross-sections may be purchased. Since these rods are manufactured by an extrusion process, their cross-sections are extremely uniform. To prepare, for example, cubes, it is then necessary to cut a rod of the square cross-section into lengths equal to the length of a side of the cross-section. At the suggestion of Versaw this was expeditiously achieved in the following manner. A die was prepared containing six square holes of a size just sufficient to pass the plastic rods. A flat plate was mounted behind the die at a distance from it approximately equal to the length of a side of the required cube. Six rods were then pushed through the holes in the die and against the plate and while in this position were cut by means of a thin-bladed circular saw mounted eccentrically. The eccentricity of the saw blade insured that cutting could occur during only approximately half the time required by the saw to make one complete revolution. During the non-cutting portion of the cycle, the rods were again pushed against the plate, this action leading to six more cubes being cut as the saw rotated. A jet of air was used to blow the cut particles from between the die and plate into a container. Utilizing this technique, it was easily possible, by applying steady pressure to the six rods, to cut cubes at the rate of some 200 a minute. The cut lengths, after initial adjustments of the saw blade and back plate had been made, were found to be reproducible to within ±0.001 in. Feathering of the cut edges was small and was removed by tumbling the particles in a ball-mill containing sawdust.

Cubes, cylinders and triangular prisms were prepared by this method using appropriate dies. Discs were obtained by stamping from plastic sheet.

The dimensions and other properties of all regular particles used, together with information bearing on the properties of some natural sands which were also investigated, are given in Table 1.

**Method of Measuring Formation Factor**

Formation factor measurements were carried out by packing the particles in plastic or glass cylindrical cells. Cells varied in size from a five in. diameter cylinder with a volume of 2,418 cu cm to a six mm diameter glass tube with a volume of 6.9 cu cm. The electrical resistance of the column filled with electrolyte only was measured first, followed by the resistance of the column packed with particles and saturated with the same electrolyte. The electrolyte used was either 0.1 N or 0.5 N potassium chloride. The ratio of the two measured resistances gives the formation factor directly.

The majority of the formation factor data for the unconsolidated particles were obtained using a five in. internal diameter cylindrical cell designed to utilize either a two-electrode or a four-electrode resistivity measuring system. It was found (Fig. 1) that the measurement was sensitive to the ratio of the diameter of the cell to that of the particle. A minimum ratio of 25 was found to be necessary. This measurement was easily satisfied when the five in. diameter cell was used. Porosities were determined from the weights and densities of particles used and the known volumes of the cells.

---

All resistances were first corrected to 25°C.

---

### FIG. 1 — EFFECT OF CONTAINER/PARTICLE SIZE RATIO ON FORMATION FACTORS MEASURED.
Table 1 — Summary of Properties of Particle Aggregates Used

<table>
<thead>
<tr>
<th>Particle</th>
<th>Material</th>
<th>Average Dimensions</th>
<th>Average Area</th>
<th>Average Volume</th>
<th>Density</th>
<th>Sphericity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spheres</td>
<td>Glass</td>
<td>3 mm dia.</td>
<td>0.2827 cm³</td>
<td>1.41 x 10⁻⁵ cm³</td>
<td>2.573</td>
<td>1.000</td>
</tr>
<tr>
<td>Spheres</td>
<td>Polystyrene</td>
<td>1.3 mm dia.</td>
<td>0.0531 cm³</td>
<td>1.15 x 10⁻⁵ cm³</td>
<td>1.644</td>
<td>1.000</td>
</tr>
<tr>
<td>3M Superbrute Beads</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade No. 5</td>
<td>Glass</td>
<td>0.711 mm dia.</td>
<td>1.58 x 10⁻⁵ cm³</td>
<td>1.88 x 10⁻⁵ cm³</td>
<td>2.99</td>
<td>1.000</td>
</tr>
<tr>
<td>Grade No. 7</td>
<td>Glass</td>
<td>0.470 mm dia.</td>
<td>6.04 x 10⁻⁶ cm³</td>
<td>5.43 x 10⁻⁶ cm³</td>
<td>2.893</td>
<td>1.000</td>
</tr>
<tr>
<td>Grade No. 9</td>
<td>Glass</td>
<td>0.279 mm dia.</td>
<td>2.44 x 10⁻⁶ cm³</td>
<td>1.14 x 10⁻⁶ cm³</td>
<td>2.87</td>
<td>1.000</td>
</tr>
<tr>
<td>Grade No. 12</td>
<td>Glass</td>
<td>0.127 mm dia.</td>
<td>5.07 x 10⁻⁷ cm³</td>
<td>1.07 x 10⁻⁷ cm³</td>
<td>2.77</td>
<td>1.000</td>
</tr>
<tr>
<td>Grade No. 19</td>
<td>Glass</td>
<td>0.033 mm dia.</td>
<td>3.42 x 10⁻⁸ cm³</td>
<td>1.88 x 10⁻⁸ cm³</td>
<td>2.39</td>
<td>1.000</td>
</tr>
<tr>
<td>Cubes</td>
<td>Lucite</td>
<td>0.1267 x 0.1264</td>
<td>9.61 x 10⁻⁴ in.²</td>
<td>2.03 x 10⁻⁴ in.²</td>
<td>1.176</td>
<td>0.805</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylinders</td>
<td>Lucite</td>
<td>0.1299&quot; dia.</td>
<td>7.79 x 10⁻⁶ in.²</td>
<td>1.674 x 10⁻⁶ in.²</td>
<td>1.176</td>
<td>0.875</td>
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<td></td>
<td></td>
<td>0.1264&quot; hgt.</td>
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<tr>
<td>Discs</td>
<td>Plastic</td>
<td>0.1522&quot; dia.</td>
<td>4.92 x 10⁻⁶ in.²</td>
<td>7.693 x 10⁻⁶ in.²</td>
<td>1.434</td>
<td>0.818</td>
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<tr>
<td></td>
<td></td>
<td>0.0564&quot; hgt.</td>
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<tr>
<td>Triangular Prisms</td>
<td>Lucite</td>
<td>0.2514&quot; length</td>
<td>11.24 x 10⁻⁶ in.²</td>
<td>1.837 x 10⁻⁶ in.²</td>
<td>1.177</td>
<td>0.644</td>
</tr>
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<td></td>
<td></td>
<td>0.1120&quot; hgt. A</td>
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<tr>
<td></td>
<td></td>
<td>0.1127&quot; hgt. B</td>
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<td></td>
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<tr>
<td></td>
<td></td>
<td>0.1127&quot; hgt. C</td>
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</tr>
<tr>
<td>Golf Coast Beach Sand</td>
<td>Silica</td>
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<td></td>
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<td>0.817</td>
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<tr>
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<td></td>
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<tr>
<td>Creek Sand</td>
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<td></td>
<td></td>
<td></td>
<td>2.65</td>
<td>0.819</td>
</tr>
</tbody>
</table>

Control of Porosity

When the formation factor-porosity relationship in any unconsolidated aggregate is determined, it is desirable to cover the widest possible porosity range compatible with the retention of contacts between particles. Graton and Fraser established six types of stable packing arrangements for spheres. These varied from a cubic packing with a porosity of 47.6 per cent to rhombohedral packing with a porosity of 25.95 per cent. Other definite geometrical arrangements yield porosities which lie between these two extremes. However, regular packings are anisotropic and in this work it was particularly desired that the packings achieved should always be isotropic.

When three mm glass spheres were used, it was found possible to cover a porosity range of only four per cent (36-40 per cent) by dint of jarring or vibrating the container in which the spheres were packed. By mixing spheres of different sizes, a wide range of porosities could be covered. Furnas developed a method for computing the proportion of particles of various sizes needed to produce a mixture of minimum porosity. In general, he found that minimum voids were obtained as the ratio between the sizes of particles used increased and, within limits, as the number of sizes increased. Although the formula developed by Furnas are not rigorous, they are approached quite closely in actual measurements made with spheres. In this work, the minimum ratio of the diameters of spheres used was 0.011. When a two-component mixture comprised of 71.3 per cent by volume three mm diameter spheres and 28.6 per cent by volume 0.033 mm diameter spheres was used, a minimum porosity of 16.25 per cent was achieved. This figure is in good agreement with the postulates of Furnas. However, when a third component, spheres of diameter 0.711 mm were added to the mixture (in the volumetric ratio three mm = 58.1 per cent, 0.711 mm = 23.5 per cent and 0.033 mm = 18.4 per cent) the porosity was decreased further to 12.4 per cent. This is not in accordance with the views of Furnas, who suggested that when the ratio of diameters is greater than 0.003 only two components are required to achieve a minimum porosity.

In the case of particles other than spheres, only one size was available and this fact necessarily reduced the range of porosity over which it was found possible to make measurements of formation factor. Nevertheless, as is noted below, the trend of the results for non-spherical particles parallels comparable data for spheres. There seems no reason to believe that an increase in the range of porosities of the packings of any of the non-spherical particles used would lead to anomalous data.

Cementation

It was not possible to reproduce any natural methods of cementation in the course of this work. Nevertheless, while realizing the limitations of the method used, it was considered worthwhile to cement artificially unconsolidated porous media and to determine the relationship between formation factor and porosity in such cemented media. Two types of unconsolidated packings were chosen for the experiments; glass spheres and a beach sand. By controlling the particle size distribution, it was possible to vary the original porosity of the unconsolidated packings of spheres before cementation was commenced. The cementing material was introduced by first saturating the unconsolidated particles with a hydrolyzed solution of tetraethyl orthosilicate. A stream of air was then blown through the saturated aggregate. The air led to the evaporation of alcohol and the formation of hydrated silica gel between the particles. This gel served to bind the particles together. The porous aggregate containing the hydrated silica gel was then heated to drive off the water; a pure silica deposit remained. A repetition of this series of operations served to decrease the porosity to any desired figure. In practice, it was necessary to confine the unconsolidated material in a Lucite cylinder of one in. internal diameter. Since the evaporation of alcohol takes place preferentially at the ends of the core, the silica deposit builds up rapidly at these two points. To achieve entirely homogeneous cementing does not seem possible, but a reasonable approximation to homogeneity may be attained if a core about 10 in. long is cemented and this core is cut into sections each of which is about }\frac{3}{4} in. long. It was found that sections which were taken from near the center of the original long core were reasonably homogeneous.

Formation factor measurements were made on the artificially cemented cores after they had been saturated with a 0.5 N KCl solution. The cores were clamped between platinum-platinum gauze electrodes which were backed with cotton compressed saturated with 0.5 N KCl solution. Porosities were calculated by weighing cores both dry and after they had been saturated with KCl solution. The resistivity of the saturating KCl solution was obtained from measurements made with a calibrated
dip cell. All measurements of cemented cores were made using the two-electrode method, and all resistivities were corrected to 25°C.

Sphericity Determinations

In order to classify the shape of the particles, sphericity calculations were made for all the non-spherical particles used. The sphericity of the large plastic particles of definite geometrical form was determined according to the definition of Wadell\(^6\) for true sphericity.

For sand grains of irregular shape, Wadell's method\(^6\) for determining the sphericity of particles was used. Fifty grains were chosen at random from representative sand samples for these measurements. The results of the sphericity measurements are shown in Table 1 and Fig. 4.

Unconsolidated Packings

Fig. 2 shows a comparison of the formation factors of all unconsolidated particle aggregates examined for the porosity range 0.30-0.50. Fig. 3 shows the data for spheres over the wider porosity range 0.10-0.56. In each figure curves corresponding to Equation (1) and Equation (3) are shown, together with a curve for the relationship \(F = \phi^{1.4}\). The latter expression, based on Archie's original work, is often quoted as the formation factor-porosity relationship for unconsolidated porous media.

Considering first the data of Fig. 2, it is evident that the Clerk Maxwell equation, (1), underestimates the formation factors of spheres in contact. The same is true, but to a lesser extent, of the Archie relationship \(F = \phi^{1.4}\). The Slawinski equation, (3), appears to fit the experimental data extremely well in the porosity range shown.

The data appear qualitatively to satisfy the general predictions of the Fricke equation, (2). That is, at any porosity the formation factor appears to be a minimum for spheres. The difference at any porosity between the formation factor of an aggregate of spheres and an aggregate of non-spherical particles may be as much as 20 per cent. The shape of the particle is clearly of importance in affecting the magnitude of this difference. An attempt to correlate the experimental data with the Wadell sphericities is made below.

In Fig. 3 it will be seen that the experimental data for spheres indicate that the Slawinski formula gives formation factors at porosities less than 0.20 which are too low. The Archie relationship fits the experimental data very well for \(0.25 > \phi > 0.10\).

Fig. 4 shows a plot of Wadell sphericity against formation factor for all particles used. The porosity chosen for plotting the figure is 0.35. Although the trend of the plotted curve indicates an increase of formation factor with decrease in Wadell sphericity, there is a considerable scatter of points. The scatter is least for the sands which are, of course, not composed of identical particles and which might, therefore, be presumed to have a tendency to fit an average curve.

Consolidated Packings

Fig. 5 shows the effect of cementation on the formation factor of packings of varying initial porosities and particle shapes. The very rapid increase in formation factor which ensues as porosity is decreased by the presence of cement is obvious.

It was found virtually impossible to produce an entirely homogeneous distribution of cement in the cores. In consequence the accuracy of the data in Fig. 5 is not as high as that of the data presented for the unconsolidated packings. Nevertheless, the experimental data appear to indicate that
the best relationship between formation factor and porosity is a straight line on double logarithmic paper. The line passes through an initial point defined by the formation factor and porosity of the unconsolidated aggregate which was cemented in a particular series of experiments. The data appear to indicate that for the particular type of cementing used the relationship between formation factor and porosity has the form

\[ F = C \phi^k \]  \hspace{1cm} (4)

where \( k = 4.2 \) and \( C \) is a constant controlled by the porosity and formation factor of the original unconsolidated aggregate before it was cemented.

**DISCUSSION**

The data obtained for the formation factor-porosity relationships of unconsolidated aggregates of particles have been shown in Figs. 2 and 3. From a practical point of view, especially in the application of formation factor data to electric log interpretation, the following points seem worthy of emphasis. The formation factor of an unconsolidated aggregate of particles is unquestionably a function of the average shape of the particles constituting the aggregate. At any porosity, in an isotropic packing, the formation factor is a minimum for a packing of spheres. For aggregates of sand grains such as are encountered in nature, the formation factor-porosity relationship will probably lie somewhat above the line shown on Fig. 3 for spheres. The actual position of the line will be dependent upon the sphericity of the sand grains but the data given for the beach sand may be regarded as reasonably typical. The fact that the points for triangular prisms do not parallel those for particles of other shapes is almost certainly a consequence of the tendency of aggregates of triangular prisms to orientate as their porosity is decreased.

No existing formula relating the formation factor and porosity of unconsolidated packings of particles fits all the experimental data found. For spheres Sławinski's formula, Equation (3) is excellent when \( 0.25 < \phi < 0.56 \). Archie's well-known relationship \( F = \phi^{-m} \) matches the experimental data reason-

ably well when \( 0.10 < \phi < 0.25 \). (For natural sand grains an exponent somewhat higher than 1.3 would be necessary; about 1.35 might be a good choice. However, the fact that most natural sedimentary sand beds are somewhat anisotropic may make this difference, in practice, largely an academic distinction.)

It is important to note that the Archie formula is only approximate and does not hold over the range of porosities which may be obtained with unconsolidated packings. More important is the fact that the experimental data indicate that even for unconsolidated packings the Archie formula, \( F = \phi^{-m} \) where \( m \) is a constant, is not accurate and is of no fundamental significance. This observation is important when consolidated porous media are considered. The erroneous belief that \( F = \phi^{-1.3} \) for unconsolidated porous media had fundamental significance appears to have led to the unwarranted assumption that \( F = \phi^{-m} \), with \( m \) a constant, for consolidated porous media of various kinds. This formula was challenged by Winsauer, Shearlin, Masson and Williams who suggested that \( F = C \phi^{-k} \), with \( C \) and \( k \) constant, was a better relationship. Winsauer et al. reached their conclusion empirically, but...
Formation Factors of Unconsolidated Porous Media. Influence of Particle Shape and Effect of Cementation

![Graph showing formation factor vs. porosity for consolidated Pennsylvanian sandstone.](image)

FIG. 6—FORMATION FACTOR-POREOSITY DATA FOR CONSOLIDATED PENNSYLVANIAN SANDSTONE, VELMA POOL, OKLAHOMA.

many experimental data support their formula, including data by Archie, as one of the writers has noted.

The relationship between formation factor and porosity in consolidated porous media is of the greatest practical importance in logging since consolidated porous media are more common than unconsolidated and a formula connecting \( F \) and \( \phi \) is essential if the step from an electrical parameter to a rock parameter is to be made. By and large the data obtained in this work tend to support formulae of the type \( F = C\phi^n \). They do not lend any credence to formulae of the type \( F = \phi^m \) except coincidentally as specific forms of the more general equation.

The writers are aware that the cementing process they used was highly artificial. Objections can be made on this score to the validity of the data obtained and their applicability to natural consolidated porous media. Nevertheless, because of the practical importance to geologists of the conclusions reached if their theory is correct, and because it affords opportunity for checking by other workers, the writers present a new theory to cover the many experimental data reported for the formation factor-porosity relationship in naturally consolidated sedimentary rocks.

**THEORY**

Archie was the first to demonstrate that if the measured formation factors and porosities of specimens of a consolidated sandstone formation were plotted on double logarithmic paper, a scatter of points was obtained through which it often appeared justifiable to draw a straight line. This observation has been confirmed by much subsequent work.

The human eye does not function logarithmically and core analysis, even under ideal conditions, is surprisingly inaccurate. It is desirable not to plot individual plug analyses. Instead, formation factors which are the averages of large numbers of individual measurements in porosity ranges of one or two porosity per cent should be plotted against the average porosities of the ranges. This method frequently gives rise to surprisingly good straight line plots and confirms the fact that the relationship between formation factor and porosity is of the form \( F = C\phi^n \). It reveals at the same time that \( F = \phi^m \) is a particular case of this relationship and one less often encountered than has perhaps been believed. Fig. 6 is an example of the method. In Fig. 6 the greatest weight is attached to those points which are of greatest statistical significance.

The regularity that nature evinces must have a rational explanation. It appears that an explanation which best fits all current data may be based on the well-known relationship between resistivity index and saturation in porous media. A review of this subject has been given elsewhere. It will suffice to say that it has been established that the relationship between resistivity index, \( I \), and fractional wetting phase saturation, \( S_w \), is of the form

\[
I = S_w^n \quad (5)
\]

The exponent \( n \) for any particular porous medium is a constant but has a magnitude which depends upon the manner in which the saturation \( S_w \) is varied. Fig. 7 shows this strikingly; in the same porous medium \( n \) can be 2.6 or 4.6 depending upon whether the saturation is varied by capillary imbibition or drainage. Other methods of changing the saturation, i.e., by flushing, are known to result in still other values of \( n \).

It has been shown from electrical and relative permeability data that the relationship between non-wetting phase resistivity index, \( I_n \), and fractional non-wetting phase saturation, \( S_n \), is also of the form of Equation (5). That is

\[
I_n = S_n^{n-n} \quad (6)
\]

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\[
I_n = S_n^{n-n} \quad (6)
\]
If \( n \) varies in Equation (5) so must \( k \) in Equation (6) since the two indices are interrelated. Thus, in the same porous medium \( k \) will vary widely depending upon the manner in which the non-wetting phase saturation is varied. In the cementing process used in this work \( k \) was 4.2; different cementing processes would presumably lead to other values of \( k \).

Now \( I_i = F/F_w \), where \( F \) is the formation factor of a sand at any stage of cementation and \( F_w \) is the formation factor of the sand before cementation commenced.

\[
I_i = F/F_w = 1/S_i \quad \text{from Equation (6)}
\]

\[
I_i = F/F_w = F\phi_i/\left(S_i\phi_i\right) = \frac{F\phi_i}{\phi_i} \quad \phi_i \quad \cdots \quad \cdots \quad \cdots \quad (7)
\]

In Equation (7) \( \phi_i \) is the porosity of the sand originally before cementation commenced, \( \phi \) the porosity of the partially cemented sand. The constant \( C \) is defined by the formation factor and porosity of the original unconsolidated sand (particle shape, sorting and degree of compaction) and the exponent \( k \) (type of cementing process).

Thus theory lends support to the view that the fundamental relationship between formation factor and porosity in consolidated sandstones is \( F = C\phi^k \). That is, a straight line is obtained when plotting appropriate formation factor and porosity data on double logarithmic paper. This straight line actually goes through an origin defined by the porosity and formation factor of the original unconsolidated sand before cementation commenced. It is possible to estimate the original porosity from diagrams of the type shown on Fig. 8. This diagram is plotted for particles which are perfect spheres. To illustrate its use, the data on Fig. 6 which give \( F = 0.65 \phi^{-1.9} \), may be introduced. As shown on Fig. 8, for \( C = 0.65 \), \( k = 1.91 \), the original porosity, \( \phi_o \), of this sand appears to have been about 44 per cent. If less spherical grains are assumed, \( \phi_o \) would be somewhat smaller.

**REFERENCES**


**ACKNOWLEDGMENTS**

The authors' thanks are due to Paul D. Foote, executive vice-president, Gulf Research and Development Co., for permission to publish this paper. They acknowledge also the assistance afforded by M. B. Spangler in the design of equipment and by Miss Shirley L. Kanaan in making measurements.

**DISCUSSION**

*By R. D. West, Trinidad Leaseholds, Ltd., Pointe-a-Pierre, Trinidad, B.W.I.*

Wyllie and Gregory say that, in general, the Archie relationship \( F = \phi^{-n} \) tends to underestimate the resistivity factor of an unconsolidated system of spheres for a given porosity between 0.30 and 0.50. The agreement is said to be good, how-
ever, when $\phi$ is between 0.25 and 0.10. The Archie relationship was found to underestimate the resistivity factor in the 0.30 and 0.50 range, and this suggests the exponent $m$ is numerically too small.

In this connection it is interesting to consider the simple case of the theoretical stable packings of equal sized spheres. Graton and Fraser\(^2\) list six such packing systems, although there are only four if orientation is ignored. The cubic system, with a porosity of 0.4764 and the rhombohedral system, with a porosity of 0.2595 are the isotropic end members, and the orthorhombic and the tetragonal-sphenoidal systems, with porosities of 0.3954 and 0.3019 respectively, are intermediate anisotropic members.

Considering first the two isotropic systems: Pirson\(^3\) has shown that the resistivity factor in the direction of any of the axes of the packing system is equal to 2.64 for the cubic system and 5.81 for the rhombohedral system. These two quantities are thus the resistivity factors for the two systems.

Assuming that the equation relating $F$ to $\phi$ is in the form $F = a \phi^b$, we have

$$2.64 = a \cdot (0.4764)^b$$
$$5.81 = a \cdot (0.2595)^b$$

Solving these simultaneous equations, we get $a = 1$ and $b = -1.3$. Here then are at least two points in the range 0.30 to 0.50 in which the Archie relationship is valid.

With the anisotropic orthorhombic packing, the resistivity factor is 3.38 along two axes of the packing system and 4.40 along the third. With the former resistivity factor, the exponent $m$ is $-1.3$, and with the latter it is $-1.6$. Thus in general, since the resistivity factor for this system will be somewhere in the range 3.38 to 4.40, the exponent will be somewhere in the range $-1.3$ to $-1.6$.

Examining the way in which the resistivity factor is calculated for these packing systems, we see that the essential variation is the integral, which has two varying factors, the shape of the grain and its limits in angular measure. In these cases, the grain shape is the same, but the limits of the integral decided by the nature of the packing, are different.

Considering mixtures of different sizes of spheres qualitatively we see that the grain shape is unaltered but the limits of the integral, by increased "nesting" are changed in such a way as further to increase the resistivity factor. According to Furnas,\(^4\) the porosity of a "normal" packing of spheres of different sizes is less than the porosity of any one of its size components. Thus, with different sizes of spheres we expect a higher resistivity factor and a lower porosity than with equal sizes of spheres.

If the increase in the one has a certain proportion to the decrease in the other, the relationship $F = \phi^{-1.3}$ may still hold. If, however, one changes at a different rate relative to the other, the value of the exponent would change. This fundamental point seems worthy of experimental examination.

From the above analysis one would expect the Archie relationship to be valid with isotropic mixtures of equal or nearly equal sized spheres from the lowest porosity obtainable (something below 0.25) up to a porosity of 0.4764. The Archie relationship would, in general, underestimate the resistivity factor for anisotropic mixtures of such spheres. Its validity for mixtures of spheres of considerably different sizes cannot yet be forecast.

If the mixtures of spheres used by Wyllie and Gregory to obtain porosities in the range 0.30 to 0.50 were isotropic, it seems that the Archie relationship should apply to this porosity range almost more than to the lower porosity range, in which the spheres were presumably of more widely different sizes.

Referring to Fig. 8 in Wyllie and Gregory's paper, it can be seen spheres approximate closest to the Archie relationship, then come shapes with some curved and some flat surfaces, then shapes with all flat surfaces. While an analysis of the effect of grain shape on the resistivity factor integral has not been made, it would seem that higher resistivity factors (and hence a numerically greater exponent for a given porosity) should result when there are sharp corners instead of only curved surfaces bounding the pore channels. This point, too, seems worthy of experimental examination.

Finally on a point of nomenclature: Archie defined a "formation resistivity factor." This has by some, including Wyllie, become shortened to "formation factor." There are, however, at least two important formation factors: the formation resistivity factor and the formation volume factor. Furthermore, it does not seem correct to talk of the "formation factor" of a mud slurry, but the "resistivity factor" of a mud slurry seems a reasonable quantity. We suggest that Wyllie change over from his "formation factor" to a "resistivity factor." It will also correspond better with his "textural factor" and others.

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AUTHORS' REPLY TO MR. WEST

West has added Pirson's calculated formation factors for certain regular packings of spheres in an endeavor to show that for at least two porosities in the range 0.30-0.50 the Archie relationship $F = \phi^{-1.3}$ is valid. In doing this West appears to have fallen into a common error. Any regular packing of spheres is not truly isotropic, i.e., in consequence of its very regularity it does not show the same formation factor if measured in any direction whatsoever. Only a truly random packing has this unique property, and it is specifically random packings that were discussed in the paper. Whether Pirson's expressions for the formation factor of certain regular packings calculated for definite directions of measurement would agree with appropriate experimental observations we are unable to say. However, there is a distinct possibility that they would not, since in making the calculations the lines of current flow appear to have been assumed parallel to each other under all circumstances. This can only be a first approximation to the true state of affairs.

The difficulties of computing conduction through the interstices of porous media, even those composed of particles of the simplest geometry, are enormous. This, of course, is the reason why, even today after a century of effort, no accurate general expression exists. The work of Fricke, to which reference has been made, may be cited to show the complexity of the mathematical procedures.

We are in complete sympathy with West's strictures regarding nomenclature. The only defense we can make is that we have followed current practice. The possibility of confusing formation factor with formation volume factor is, fortunately, rather remote. Unquestionably, however, resistivity factor would have been a less ambiguous shortened form of formation resistivity factor than the present formation factor. However, we might point out that at least formation factor is a form rather analogous to "cell factor" which is commonly used in physical chemistry. Perhaps the real reason for the use of the term is that $F$ was an unused symbol in the logging literature while $R$ might have been confused with resistivity. Alvo, in teaching, alliteration is the pedagogue's pride; $F$, for resistivity factor, would be regarded as thoroughly undesirable.

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Electrical Conductivities in Oil-Bearing Shaly Sands

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ABSTRACT

A simple physical model was used to develop an equation that relates the electrical conductivity of a water-saturated shaly sand to the water conductivity and the cation-exchange capacity per unit pore volume of the rock. This equation fits both the experimental data of Hill and Milburn and data obtained recently on selected shaly sands with a wide range of cation-exchange capacities.

This model was extended to cases where both oil and water are present in the shaly sand. This results in an additional expression, relating the resistivity ratio to water saturation, water conductivity and cation-exchange capacity per unit pore volume. The effect of shale content on the resistivity index-water saturation function is demonstrated by several numerical examples.

INTRODUCTION

A principal aim of well logging is to provide quantitative information concerning porosity and oil saturation of the permeable formations penetrated by the borehole. For clean sands, the relationships between measured physical quantities and porosity or saturation are well known. However, the presence of clay minerals greatly complicates log interpretation, particularly the electrical resistivity and SP logs, and considerably affects evaluation of hydrocarbon-bearing formations. The conductance and electrochemical behavior of shaly sands and their relation to log interpretation have been studied by many workers. Wylie and Lynch reviewed this work in some detail.

Virtually all laboratory measurements of electrical resistivity and electrochemical potential of shaly sands published to date are the work of Hill and Milburn. Their measurements were made on about 300 cores covering a large variety of sedimentary rocks, and a wide range of equilibrating NaCl solution concentrations. Hill and Milburn described their conductivity data by an empirical equation in which the shaly sand conductivity $C_s$ was expressed as a function of the solution conductivity $C_w$ and two parameters $b$ and $F_01$. The quantity $b$ was shown to be a measure of the effective clay content of the rock, being approximately proportional to the cation-exchange capacity of the rock divided by its pore volume. The latter ratio is designated as $Q_w$ in this paper and has the dimensions meg/ml or equiv/liter. $Q_w$ is identical with the term representing the concentration of fixed charges in the Meyer-Sievers and Teorell theory of permselective membrane behavior. $F_01$ is a formation resistivity factor referred to a hypothetical equilibrating solution resistivity of 0.01 ohm m at 25°C, where clay effects presumably are minimized. $F_01$ was correlated to porosity by an Archie-type equation. The Hill-Milburn equation describes their data with a standard deviation of approximately 1 percent and a maximum deviation of ±10 percent.

Shaly sands behave as permselective cation-exchange membranes, their electrochemical efficiencies increasing with increasing clay content. The electrochemical potential data of Hill and Milburn were expressed graphically, and demonstrate that the membrane efficiency (or cation transport number) of these sands is a function only of the $b$ value (i.e., $Q_w$) and the respective salt concentrations of the two solutions forming the liquid junction in the sand. The diffusion potentials are not dependent on $F_01$ or any parameter relating to the porosity or pore geometry of the rock.

The Hill-Milburn resistivity equation correctly predicts a decreasing sand conductivity $C_S$ with decreasing solution conductivity $C_w$. However, at some low value of $C_w$, the calculated $C_S$ vs. $C_w$ function goes through a minimum; with further decrease in $C_w$, the predicted sand conductivity increases sharply. As pointed out by Hill and Milburn, an increasing value of $C_S$ with decreasing $C_w$ is physically meaningless. Since this occurs below the range of practical values of $C_w$, the usefulness of the empirical equation is not affected when

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1References given at end of paper.

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JUNE, 1968
applied to water-saturated sands. However, as will be discussed later, the equation cannot be extended to oil-bearing shaly sands since the minimum in the $C_e$-$C_w$ curve then is shifted to higher $C_w$ values.

In this paper an equation for shaly sands is developed which relates the resistivity ratio to water saturation, water conductivity and an independently determined shaliness parameter, using a simple physical model and modern concepts of the nature of the electrical charge of clay minerals.

THE MODEL

The essential features of the shaly sand conductivity plots ($C_e$ vs $C_w$) are shown in Fig. 1. In the range of dilute equilibrating electrolyte solutions (from 0, to about 0.1 to 0.5 $m$ NaCl), the sand conductivity increases sharply with increasing solution concentration at a greater rate than can be accounted for by the increase in $C_w$. With further increase in equilibrating solution conductivity, the sand conductivity increases linearly.

The model consists of two resistance elements in parallel, one component consisting of the free electrolyte contained in the pore volume of the rock $C_{e0}$, and another resulting from the conductance contribution of the exchange cations associated with the clay $C_e$. We can write

$$C_{rock} = C_e + C_{e0} \quad \ldots \ldots \ldots (1)$$

and

$$C_e = xC_{e0} + yC_w \quad \ldots \ldots \ldots (2)$$

where $C_{e0}$, $C_e$ and $C_w$ are the specific conductances of core, clay exchange cations and equilibrating salt solution, respectively; and $x$ and $y$ are appropriate geometric constants. We assume that the electrolyte solution in the porous system has the same electrical conductivity as that for the equilibrating solution, i.e., Donnan effects are neglected.

The sharp increase in conductance with increasing concentration of electrolyte in the dilute range is attributed to an increasing exchange-cation mobility. Conductance due to cation exchange can proceed by cation migration in the electric field from one fixed exchange site on the clay particles to another, from assemblies of clay platelets to exchange sites on other assemblies and through the free electrolyte. An increase in clay content and available exchange sites with little or no free electrolyte present should decrease the energy required for electrical conduction and, consequently, result in an increase in exchange-cation mobility. Further, if increasing concentrations of salt solutions were introduced in the pores, there should be another and energetically easier path through the solution, again accompanied by a significant increase in exchange-cation mobility. At some relatively high concentration of equilibrating electrolyte solution, further increase in concentration should make little difference in exchange-ion mobility; i.e., this mobility should then reach a constant and maximum value. At higher electrolyte concentrations, the sand conductivity will increase linearly with increase in solution conductivity.

Various self-diffusion and activation energy measurements of ionic diffusion in three-dimensional matrices having fixed charge sites support these assumptions. Richman and Thomas reported values of the self-diffusion coefficients for Na⁺ ion in cylindrical rods of a synthetic ion-exchange resin. The concentration of counter ions $Q_e$ in this material varied from 1.5 to 1.7 equiv/liter. These authors observed that the diffusion coefficient $D$ increased by a factor of about 4.8 as the equilibrating electrolyte solution was varied from deionized water to 0.03 $N$ NaCl, and then remained constant as the concentration was increased further. Activation energies for self-diffusion of the Na⁺ ion decreased from about 10.3 to about 2.2 kcal mole⁻¹ as the external salt concentration was increased from 0.01 to 0.07 $N$.

Similar measurements were made in bentonite-water plugs by Lai and Mortland. Their data show a decrease in the activation energy for self-diffusion of the Na⁺ ion with increasing clay concentrations in salt-free bentonite plugs. Lai and Mortland also reported measurements of the self-diffusion of Na⁺ ion in bentonite plugs (0.5 gm clay/2 ml system) equilibrated with varying concentrations of NaCl solution. The diffusion coefficient increased from about 2.3 to $4.8 \times 10^{-6}$ sq cm sec⁻¹ when the solution concentration in equilibrium with the plug was increased from 0.01 to 1 $N$ NaCl.

Recently, Gasz reported measurements of the self-diffusion coefficients of sodium and chloride ions in a 3.75 percent Na-bentonite gel containing increasing amounts of NaCl. The self-diffusion coefficient for sodium ion increased from about $2.3 \times 10^{-6}$ to $8 \times 10^{-6}$ sq cm sec⁻¹ with increase in NaCl concentration from 0 to about 0.06 $N$; no significant change in $D$ was observed for the chloride ion. Calculated self-diffusion coefficients obtained from conductivity data on the same systems,

FIG. 1 — CORE CONDUCTIVITY ($C_e$) AS A FUNCTION OF EQUILIBRATING SOLUTION CONDUCTIVITY ($C_w$).
the use of a two-component model as in Eq. 1 and the Nernst-Einstein equation were in excellent agreement with the observed coefficients.

We assume next that the electric current transported by the countercations associated with the clay travels along the same tortuous path as the current attributed to the ions in the pore water. Thus, the geometric parameters $x$ and $y$ of Eq. 2 are assumed to be equal. By analogy with the idea of formation resistivity factor for clean sands,

$$ x = y = \frac{1}{F^*} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$ (3)

where $F^*$ is the shaly sand formation resistivity factor. We can then write

$$ C_e = \frac{1}{F^*} \left( C_b + C_e \right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$ (4)

For clean sands, $C_e = 0$, and $F^*$ reduces to $F$, the usual formation factor defined as $C_b/C_e$. This assumption is justified since the cation transport numbers are independent of any parameter relating to rock porosity or pore geometry, as indicated by the electrochemical data of Hill and Milburn. Various expressions for the cation transport number (and hence, electrochemical potentials) can be derived from conductivity equations based on a parallel conductance model, depending on the number of resistor elements considered. Under these conditions, the resulting cation transport numbers would not be independent of geometric “cell constant” parameters such as $x$, $y$, and $F^*$ unless these parameters are equal.

For the straight-line portion of the conductivity curve where a constant and maximum exchange-cation mobility has been assumed, we write

$$ C_e = \frac{\mu_{Na}^{ex}}{1000} Q_v = \frac{\lambda_{Na}^{ex}}{1000} Q_v \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$ (5)

where $C_e =$ specific conductance of the clay counterions, mho cm$^{-1}$

$\mathcal{F} =$ faraday

$\mu_{Na}^{ex} =$ maximum sodium exchange ion mobility, sq cm volc$^{-1}$ sec$^{-1}$

$Q_v =$ concentration of sodium exchange cations associated with the clay, equiv/liter

$\lambda_{Na}^{ex} =$ maximum equivalent ionic conductance of the sodium exchange ions, sq cm equiv$^{-1}$ ohm$^{-1}$

Since $C_e$ is a constant over this range of $C_w$ values, the slope of the straight-line portion of the conductivity curve is equal to the reciprocal of the formation factor $1/F^*$ (Fig. 1); the intercept $BC$, obtained by extrapolating the straight-line portion of the conductivity plot to the $C_e$ axis, is equal to $C_e/F^*$; and the line segment $AB$ is equal to $C_e$ or ($\lambda_{Na}^{ex} Q_v$)/1,000.

Required tests of the model are (1) demonstration of a proportionality between segments $AB$ obtained from $C_e - C_w$ plots, and $Q_v$ values determined independently by analytical methods for a variety of shaly sands, and (2) correlation of $F^*$'s obtained from slopes of $C_e - C_w$ plots to porosity and pore geometry of the rocks. $C_e$ is expressed as a volume conductivity in Eq. 5. This model cannot differentiate between a volume and a surface conductance mechanism for the clay countercations, since either consideration results in the volume concentration term $Q_v$ in Eq. 5. Considering the exchange ion contributions as a surface conductance, we obtain

$$ C_e = \frac{S_{Na}^{ex}}{1000} Q_v \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$ (6)

where $C_e$ is the specific surface conductance of the clay (ohm$^{-1}$) and $Q_v$ is the number of equivalents of exchange cation per unit of surface area. Further,

$$ C_e = S_{Na}^{ex} Q_v \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$ (7)

where $S$ is the specific surface area of the clay (sq cm/gm clay) and $g$ is the weight of clay per unit of rock pore volume (gm clay/ccm). Comparing Eqs. 5 and 7,

$$ Q_v S_g = \frac{Q_v}{1000} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$ (8)

Clearly the volume concentration term $Q_v$ and the associated volume conductivity are simpler to measure and use than the parameters of Eqs. 7 and 8 which are required to describe the surface conductivity mechanism, i.e., surface concentration of fixed charge sites, specific surface area of the clay, etc.

Returning to the curved portion of the conductivity curve in the low $C_w$ region, we assume an exponential rise of the countercation mobility up to its constant and maximum mobility at higher solution conductivities. This assumption is based on conductivity data for synthetic ion-exchange resin plugs, e.g., the work of Sauer et al. It may be tested by comparison with shaly sand conductivity data, and by extrapolation to $C_w = 0$ for comparison with literature data for clay conductivities.

The general equation for water-saturated shaly sands then becomes

$$ C_e = \frac{1}{F^*} \left( B Q_v + C_e \right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$ (9)

with

$$ B = \left[ 1 - \exp \left( -C_w/\gamma \right) \right] 0.001 \lambda_{Na}^{ex} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$ (10)

$C_e$ and $C_w$ have the units mho cm$^{-1}$. $B$ is introduced to represent the equivalent conductance of the countercations as a function of solution conductivity $C_w$ with units in mho sq cm/meq. The value of $y$ is determined by the rate of increase of the countercation mobility from that at zero water conductivity up to its constant value at the higher water conductivities.
The parameter \( a \) is determined by the mobility of the exchange cations at \( C_w = 0 \),

\[
\left( \lambda_a^m \right)' = \frac{1}{\lambda_a^m}, \ldots \ldots \ldots \ldots \ldots (11)
\]

where \( \left( \lambda_a^m \right)' \) is the equivalent ionic conductance of the exchange cations at \( C_w = 0 \).

**COMPARISON WITH EARLIER WORK**

Similar assumptions as to the equivalence of the cell factors for surface and bulk solution conductivity were made by Cremer and Ladelolou\textsuperscript{13-15} in their work on clay mineral suspensions and gels. Their data indicate that isocconductivity points exist for different clay mineral gel-salt systems. By extrapolating the straight-line portions of the \( C_q \) vs \( C_w \) plots for a series of gels consisting of varying percentages of the identical clay species, a common isocconductivity point is found. Cremer and Ladelolou observed that the conductivity at an isocconductivity point is independent of gel porosity. By comparison, they found that their form of the \( F-\phi \) function is in formal agreement with that derived from theories by Burger\textsuperscript{16} and Fricke,\textsuperscript{17} relating electrical conductance of stationary particles in dilute suspensions to particle shape factors and porosity. The Cremer-Ladelolou \( F-\phi \) relation clearly is not applicable to shaly sands; moreover, the accuracy of the Cremer-Ladelolou data does not permit calculation of reliable values of exchange-cation mobilities for clay plugs equilibrated at any salt concentration.\textsuperscript{18}

The proposed mechanism differs from that suggested by Wyllie and his co-workers,\textsuperscript{1,10,12} They represented the condution process by an equivalent resistor model containing three resistance components in parallel instead of two as suggested here. Two resistor elements are common to both models: the third consists of conductive solid and electrolyte solution elements in series. Contributions from this component can account for the sharp increase in the conductivity curves at low \( C_w \) values, rather than an increase in exchange-ion mobility. The Wyllie model requires four geometric parameters, three of which are independent. These can be obtained by appropriate manipulation of the conductivity equation under various limiting conditions.

Some comparison should be made with the work of de Witte.\textsuperscript{19} He concluded from empirical considerations that the conductivity behavior of water-bearing shaly sands could adequately be represented by the equation

\[
C_q = A_0 + B_0 C_w, \ldots \ldots \ldots \ldots \ldots (12)
\]

where \( A_0 \) and \( B_0 \) are constants. Eq. 12 is similar to that originally proposed by Patnode and Wyllie,\textsuperscript{20} and neglects the curved portion of the conductivity plots at low \( C_w \)'s. However, de Witte defined a "maximum formation factor", the limiting value of apparent formation factor as \( R_f \to 0 \), namely \( F_m = 1/B_q \). \( F_m \) is identical to \( F^* \) of this paper. de Witte also recognized a shaliness factor \( A_D/B_q \) which was "an absolute rock parameter ... independent of \( R_w' \). In this paper, \( A_D/B_q \) is equal to the maximum specific conductance of the exchange cations of the clay \( (\lambda_a^m Q_a) /1,000 \).

**EXPERIMENTAL PROCEDURES AND DATA**

Conductivity data for about 315 shaly sand cores have been published by Hill and Milburn.\textsuperscript{3} These data include independent cation-exchange capacity and \( Q_u \) determinations on 36 samples. Additional unpublished measurements including \( Q_u \) and conductivity data on some 54 cores, together with conductivity data on 167 other samples, were made available to us by H. J. Hill. The conductivity data generally include three to four experimental points in the straight-line portion of the curve \( (C_w > 0.05 \text{ mho cm}^{-1}) \) and one to two points in the curved region, but these usually were obtained at equilibrating solution conductivities greater than 7 m mho cm\(^{-1}\) (solution resistivities less than about 1.4 ohm m).

While a large variety of rock types is represented in Group 1, the distribution of \( Q_u \) values is weighted heavily in the range \( 0 \leq Q_u \leq 1 \) equiv/liter; this group contains about 94 percent of the total number of samples. Another set of shaly sands (Group 2) was selected on the basis of a wider distribution of \( Q_u \) values; with these samples, conductivity measurements at low equilibrating solution conductivities (down to about 2.1 m mho cm\(^{-1}\)) were emphasized. Conductivity data for Group 2 cores are probably the most accurate and complete with respect to range of \( Q_u \) and \( C_w \) values examined.

**GROUP 1**

Independent \( Q_u \) determinations, together with \( C_q \), \( C_w \) measurements, were described for a selected set of sands by Hill and Milburn.\textsuperscript{3} Petrophysical and conductivity data for this set are presented in Table 1. Similar measurements were carried out on 54 additional cores. A description of these samples is given in Table 2; petrophysical characteristics and conductivity data are summarized in Table 3. Table 4 gives general characteristics and petrophysical data for the remaining samples used in the conductivity studies.

Experimental procedures employed in these conductivity measurements were described by Hill and Milburn.\textsuperscript{3}

\( Q_u \) **Determinations**

Cation-exchange capacities (CEC) as determined by Hill and Milburn (Table 1) were measured chromatographically using ammonium acetate solutions. Further CEC determinations (Table 3) were made using a procedure suggested by Mortland and Mellor.\textsuperscript{21} This method requires repeated equilibration of the crushed rock sample with concentrated BaCl\(_2\) solution and washing to remove excess barium ions, followed by conductometric

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titration with standard MgSO₄ solution. Results are equivalent to the ammonium acetate method.

Core porosities were calculated from the following measurements made prior to the CEC determinations: (1) bulk volumes measured by mercury displacement and (2) grain volumes determined by buoyancy weightings under brine.

GROUP 2

Shaly Sand Samples

Shaly sand samples were selected which varied widely both in CEC and in manner of clay distribution. The range of CEC values extended from 0 to 1.5 meq/ml. Cores where 0.3 < CEC < 1.5 originated from a lower Tertiary horizon and contained fairly pure montmorillonite as the clay fraction, mainly

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<th>Suite</th>
<th>Sample Number</th>
<th>Permeability (%)</th>
<th>Air Permeability (cm)</th>
<th>CEC (equiv./liter)</th>
<th>Exp.</th>
<th>F*</th>
<th>C (Conductivity Units, mhos cm⁻¹)</th>
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1 Brine containing 12 g NaCl per liter.
2 At 300 psi differential pressure.
TABLE 2 — DESCRIPTION OF SAMPLES INVESTIGATED

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<th>Suite</th>
<th>Type of Formation</th>
<th>Description</th>
<th>Age and Locality</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>Clean sandstone</td>
<td>Clean, medium to fine grained (340 to 80 μ), friable quartz sand, major cementing material present: calcite and quartz</td>
<td>Micaceous, Weeks Island, La.</td>
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<tr>
<td>2</td>
<td>Shaly sandstone</td>
<td>Micaceous, shaly, fine grained to silty (140 to 65 μ), hard quartz sand, cementing material: chiefly calcite</td>
<td>Eocene, lower Wilcox sand, Sherman, Tex.</td>
</tr>
<tr>
<td>3</td>
<td>Shaly sandstone</td>
<td>Highly calcareous, shaly, medium to fine grained (255 to 10 μ) arkosic sandstone; cementing material: chiefly calcite</td>
<td>Oligocene, Frio sand, Seadrift, Tex.</td>
</tr>
<tr>
<td>4</td>
<td>Very shaly sandstone</td>
<td>Very shaly, very fine grained to silty (100 to 50 μ), consolidated quartz sand</td>
<td>Cretaceous, Taylor sand, Big Foot, Tex.</td>
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</table>

surrounding the grains and interstitial. The cores in which 0 < Q<sub>e</sub> < 0.5 came from an Eocene and an Albian horizon and contained mainly kaolinite and illite. In these cores, the clay occurs around the grains, but mainly in thin laminae. The compositions of the clay fractions (<2 μ) of the samples are given in Table 5.

Cylindrical cores 2.54 cm in diameter and 2 to 4 cm long were cut with their axes parallel to the bedding planes. After extraction with chloroform and ether, they were encased in epoxy resin. The cores then were equilibrated with a deaerated, saturated NaCl solution, and stored in this solution for several months before the experiments were started.

**Apparatus**

Electrical conductivities were measured with an accuracy of 0.1 percent using an impedance bridge. The cell was similar to that described by Hill and Milburn, contact between the platinitized platinum electrodes and the cores was achieved through the equilibrating electrolyte solution in the end compartments of the cell. The measuring frequency was 1,592 cycles/second. Tests indicated that measured conductivities were independent of frequency, at least from 50 to 20,000 cycles/second. Measurements were conducted at a constant temperature of 25 ± 0.05°C.

TABLE 3 — COMPOSITION OF CLAY FRACTION (<2 μ) OF SHALY SANDS

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<th>Illite (percent)</th>
<th>Chlorite (percent)</th>
<th>Montmorillonite (percent)</th>
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**Table 4 — Description and Petrophysical Characteristics of Rocks Investigated**

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<th>Age, Locality</th>
<th>Number of Samples</th>
<th>Conductivity Points per Sample</th>
<th>Range of Equilibrating NaCl Solutions (Normality)</th>
<th>Porosity Range (percent)</th>
<th>Permeability Range (md)</th>
<th>Range of F&lt;sup&gt;e&lt;/sup&gt; Values</th>
<th>Range of Q&lt;sub&gt;e&lt;/sub&gt; (equiv/liter)</th>
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<td>Pliocene, Ventura Field, Calif.</td>
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<td>0.06 - 3.06</td>
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<td>0.13-19</td>
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<td>Cretaceous, Viking Formation, Alto, Canada</td>
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<td>0.12 - 4.79</td>
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<td>0.6 - 664</td>
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</table>

**June, 1968**
The contribution of the electrode compartments was determined by measuring the conductivity of cells with the cores replaced by perspex rings having an internal diameter equal to the nominal diameter of the cores (2.54 cm) and varying in length from 1 to 5 cm. The "cell constant" of the electrode compartments was obtained by extrapolating the conductance data to zero length. The influence of small variations in core diameter on this cell constant was determined by measuring conductivities with perspex rings 3 cm long, but varying in internal diameter from 2.0 to 2.8 cm.

Cores with higher clay content ($Q_v$ greater than 0.2 meq/ml) were in danger of falling apart upon flushing with NaCl solutions of a concentration lower than 0.072 molal. To prevent their disintegration, these cores were fitted at each end with a plate of highly permeable, fired bentheim sandstone. These plates were 2.54 cm in diameter and a few millimeters thick and were cut from a long bar previously mounted in epoxide. They were glued to the cores at the resin rim at each axial end of the cores. Before mounting, the electrical cell constant of the plates was determined so that the appropriate corrections could be made for the electrical conductivity determinations.

**NaCl Solutions**

The salt used was Analar reagent NaCl, fused overnight at 500°C and stored until use in a desiccator in which CaCl$_2$ was the drying agent. Salt solutions were made up by weight.

The concentrations of the salt solutions in use were verified by conductivity measurements at 25°C. The accuracy of these electrical concentration determinations is somewhat better than 0.1 percent, except for concentrations higher than 4 molal. At these higher concentrations, the change of conductivity with concentration becomes increasingly smaller and accurate results cannot be expected. For solutions with concentrations greater than 4 molal, checks were made gravimetrically by evaporation of a known weight of solution.

**Cation-Exchange Capacity Measurements**

The determinations of CEC per unit pore volume of the shaly sand cores were made on pieces of rock adjacent to the places from where the cores had been removed because the method used was destructive. The cores were selected for the investigation from homogeneous portions of rock. Some duplicate determinations of exchange capacities were made at different adjacent pieces of rock, and the measure of agreement can be seen in Table 6. These cores have become valuable as a result of having undergone many electrochemical measurements; in order that they may be available for further work, some accuracy in the exchange capacities has been sacrificed. We do not believe, however, that this is a serious disadvantage since no systematic errors are introduced. The $Q_v$ values cited in the second sample set of Table 6 were used in further calculations since they were taken closer to the cores of interest.

### Measuring Procedure

As shaly sands may fall apart when kept in contact with brine of very low salinity, the investigation was started at high NaCl concentrations. At each concentration the cores were equilibrated by repeated flushing and storing in a desiccator filled with the appropriate salt solution. We considered the cores to be at equilibrium with the salt solution when, upon further flushing, the conductivity measurements scattered within the measuring accuracy of about 0.1 percent.

At high salt concentrations, equilibrium is reached very slowly. Equilibrating times of 2 to 3 months frequently were required. Nevertheless, in all cores except those from extremely clean sands, the first two or three conductivity points at descending NaCl concentration were found to be reproducible. When the measurements were repeated after the cores had been flushed and "equilibrated" again with brines of higher salinity, much higher conductivities were recorded. The shalier the sand, the more pronounced was this effect.

To investigate the phenomenon, a few cores with varying clay content were selected. A typical example is shown in Fig. 2. Here the conductivity of a moderately shaly core is given as a function of saturating brine conductivity for a sequence of

![Fig. 2 — Electrical Conductivity ($C_v$) of a Shaly Core as a Function of the Conductivity of the Equilibrating Brine ($C_m$).](image-url)

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observations obtained under equilibrating conditions as previously described. It is seen that reproducible data were obtained only from Point 7 on.

The reproducible data are considered now as equilibrium data. The clay of this particular core occurs mainly in laminae. Effects demonstrated in Fig. 2 are less pronounced when the clay is more homogeneously distributed, and also when the cores are first saturated with an NaCl solution of a concentration lower than about 1 molal. This behavior of the conductivity of shaly sands may be caused primarily by the almost complete dehydration of the clay due to the Soxhlet extraction procedure and the heating during encasing in epoxy resin. It is presumed from the above observations that rehydration of the clay, and therefore reactivation of the clay partitions, is an extremely slow process as far as saturated solutions are concerned because of their lower water activity. Grim\textsuperscript{22} reported that completely dehydrated montmorillonite is extremely difficult to re-equilibrate with water.

Results of the equilibrium conductivity measurements made at 25°C on the shaly sands equilibrated with NaCl solutions at 10 different concentrations varying from saturated solutions of 6.144 to 0.018 molal, are given in Table 7. Origin of the samples, their porosities, and $Q_v$ values also are in Table 7. Some representative plots of core conductivity $C_w$ vs electrolyte solution conductivity $C_v$ are presented in Fig. 3 for cores with different values of $Q_v$.

**DISCUSSION OF DATA**

Values of $F*$ and $\lambda_{Na}Q_v/1,000$ were calculated from the slopes and intercepts ($AB$, Fig. 1), respectively, of the straight-line portions of the conductivity plots. Extrapolations were made with Eq. 9 and the method of least mean squares. These calculated values of $\lambda_{Na}Q_v/1,000$ were plotted against the independently determined values of $Q_v$ for the 90 cores of Group 1 (Fig. 4). The data were fitted by a straight line passing through the origin and a method of reduced regression where both variables are considered subject to error. The correlation coefficient for the resulting relation

$$\left(10^{-3}\lambda_{Na}Q_v\right)_{cond} = 0.0383\left[Q_v\right]_{\text{final}}$$

was 0.981, with $R_{max}$ or $\lambda_{Na}$ equal to 38.3 sq cm equiv$^{-1}$ ohm$^{-1}$. Confidence limits of 10 and 90 percent (80 percent of the expected values lie between these limits) for $\lambda_{Na}$ were 35.9 and 39.6 sq cm equiv$^{-1}$ ohm$^{-1}$.

The method of least mean squares was used to calculate $F*$ and intercept $AB$ values from the conductivity data obtained with Group 2 cores. Values of $\lambda_{Na}$ were calculated from the adjacent $Q_v$ determinations for each of the individual samples (Table 8). The average value of $\lambda_{Na}$ for this set was 46.3 sq cm equiv$^{-1}$ ohm$^{-1}$, and the standard deviation was 15.8 sq cm equiv$^{-1}$ ohm$^{-1}$. The value of $\lambda_{Na}$ obtained from cores in Group 1 lies within the above standard deviation.

$F*$ values were plotted vs the Hill-Milburn $F_{01}$ values for the total of 553 cores of Group 1, and it is apparent that the two parameters are equivalent. The data were fitted by a straight line through the origin, and the slope was determined to be 1.031 by reduced regression methods; the 10 and 90 percent confidence limits of the slope were 1.023 and 1.039, respectively. The correlation coefficient for this relation was essentially unity.

The various relations between $F_{01}$ and porosity found for some four different formations by Hill and Milburn\textsuperscript{2} also must hold between $F*$ and porosity as well. Similar correlations are shown for Group 2 sands (Fig. 3). For the Eocene sand, $m = 1.74$; for the lower Tertiary sand, $m = 2.43$. In both sets, a few cores obviously do not belong to the same group with similar pore geometries. We have no detailed explanation for these deviations.

We conclude that the formation resistivity factor $F*$, defined as the reciprocal of the slope of a plot of $C_o$ vs $C_w$, is related to porosity according to Archie's\textsuperscript{6} first empirical equation

$$F* = \varphi^{-m}$$

Similarly, other forms of Eq. 14, as suggested by
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<td>$\lambda_{eq}/1000$ (mho cm²/meq)</td>
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Values between parentheses considered not reliable because of extreme deviation from the rest of the assembly, probably due to $Q_v$ determination. They were not used in calculating the average.

For $C_w > 0.060$ mho/cm, $\delta = 1.$
Winsauer et al.,23 can be used to relate $F^*$ to porosity.

The two major tests of the proposed conductivity model, as discussed previously, are met.

Conductivity data obtained for Group 2 cores are quite detailed in the curved portion of the $C_\nu - C_w$ plots (2.085 < $C_w$ < 52.49 mho cm$^{-1}$) and permit calculation of the increase in counterion mobility with increasing equilibrating electrolyte concentrations within this concentration range. The increase in $\mu_{Na}$ has been represented by a simple empirical relation (Eq. 10) for $B$ as a function of $C_w$ and two constants $a$ and $y$.

$$\delta = [1 - a \exp \left(-C_w / \gamma \right)] . . . . . (15)$$

$$B = \delta \lambda_{Na}^e / 1000 . . . . . . . . . (16)$$

The equivalent conductance of the clay counterions is denoted by $B$, with $\lambda_{Na}^e / 1000$ representing $B$ at its maximum value ($\delta = 1$) in the straight-line portion of the $C_\nu - C_w$ curves.

Values of $\delta$, at each value of $C_w$, and for each core, were calculated from the ratio of the measured $C_\nu$ values to the hypothetical values ($C_\nu'$) of $C_\nu$, calculated from the straight-line extrapolation at the same $C_w$ (Fig. 1). Since the same formation factor $F^*$ applies throughout the entire electrolyte concentration range,

$$r = \frac{C_\nu}{C_\nu'} = \frac{(\delta_\lambda_{Na}^e Q_r) / 1000}{\lambda_{Na}^e Q_r} , . . . . . (17)$$

thus,

$$\delta = (r - 1) \frac{(1000) C_\nu}{\lambda_{Na}^e Q_r} + r . . . . . . (18)$$

Values of $\delta$, together with their average values and standard deviations, are given as a function of $C_w$ in Table 8.

Values of $B$ as a function of $C_w$ were calculated according to Eq. 16 using average values of $\delta$ and $\lambda_{Na}^e$ (Fig. 6). The empirical equation representing $B$ as a function of $C_w$ at 25°C is found to be

$$B = [1 - 0.6 \exp \left(-C_w / 0.013 \right)] 0.046$$

mho cm$^2$ mo$^{-1}$ . . . . . . . . . . . . . . . . . . . . . . . . . (19)

and, as shown in Fig. 6, provides a reasonable fit for these data. Eqs. 9, 10 and 19 describe available shaly sand data about as well as does the Hill-Milburn equation.

Extrapolation to $C_w = 0$ using Eq. 19 results in a calculated value for the equivalent conductance of the sodium exchange ion ($\lambda_{Na}^{ex}$) of 18.4 sq cm equiv$^{-1}$ ohm$^{-1}$. This is substantially equal to the value of equivalent conductance of sodium exchange ions of about 18 sq cm equiv$^{-1}$ ohm$^{-1}$ which can be deduced from conductivity measurements of sodium montmorillonite-distilled water gels at 25°C reported by Van Olphen and Waxman.24 While this agreement is quite comforting, some caution should be observed since the calculated value of ($\lambda_{Na}^{ex}$) is obtained by extrapolating an empirical relation beyond observed data.

The assumption in the model of an exponential increase of counterion mobility with increase in solution conductivity is justified by comparison with shaly sand conductivity data and by the apparent agreement of extrapolated conductivity at $C_w = 0$ with clay gel conductivities.

A further general criterion for the model follows from the thermodynamic relation between electrical transport numbers calculated for conditions of nonzero electric current flux and zero concentration gradient, and diffusion potentials measured under conditions of nonzero concentration gradient and zero electric current flux. It should be possible to

![Graph showing Shaly Sands Formation Resistivity Factor ($F^*$) vs Porosity ($\phi$).](image)

**FIG. 5 — SHALY SANDS FORMATION RESISTIVITY FACTOR ($F^*$) VS POROSITY ($\phi$).**

**FIG. 6 — EQUIVALENT CONDUCTIVITY ($B$) OF THE COUNTERIONS ASSOCIATED WITH CLAY AS A FUNCTION OF EQUILIBRATING ELECTROLYTE CONDUCTIVITY ($C_w$).**

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obtain expressions for the cation and anion transport numbers from the conductance model. These expressions then can be introduced into the equation for the diffusion potential generated by a liquid junction in the shaly sand. This is discussed by Smits\textsuperscript{11} who compared experimental diffusion potentials in shaly sands with calculated potentials based on theory, including parameters developed from the conductance model. Excellent agreement was obtained between both sets of potentials.

**CONDUCTIVITY EQUATION FOR OIL-BEARING SHALY SANDS**

The conductivity equation for water-bearing shaly sands is extended to describe the conductivity of shaly sands containing both oil and brine.

When water saturation $S_w$ is less than unity, the exchange ions associated with the clay become more concentrated in the remaining pore water. This concentration $Q_w$ is related to $Q_v$ and $S_w$ according to

$$Q'_w = Q_v / S_w$$  \hspace{1cm} (20)

where $Q'_w$ is the effective concentration of exchange ions at $S_w > 0$. We assume that the mobility of the exchange ions is unaffected by the partial replacement of water. Then the conductivity of the counterions is given by $BQ'_w / S_w$ (mho cm$^{-1}$). This assumption was based initially on field evidence which indicates that the SP deflection opposite oil-bearing rocks is reduced when compared with the SP response opposite water-bearing sections of the same reservoir. Reduction of the total electrical potential is attributed to an increase in permselective membrane efficiency of the oil-bearing sands relative to the efficiency of the same sands at $S_w = 1$. The diffusion potential across these sands comprises one of the components of the total SP response. This increase in the membrane efficiency of the sands is associated with an increased effective clay content which must be due to a decrease in the volume of pore space filled with water. A similar suggestion was made earlier by Hill and Milburn\textsuperscript{3} and later by de Witte.\textsuperscript{19} The equivalent Hill-Milburn assumption is

$$b' = b / S_w$$  \hspace{1cm} (21)

recalling that the $b$ parameter is approximately proportional to $Q_v$.

**LABORATORY EVIDENCE FOR $Q'_w$ ASSUMPTION**

Laboratory data obtained by McLaughlin\textsuperscript{25} support the assumption stated in Eq. 20. These data consist of diffusion potential measurements using shaly sands at conditions of water saturation equal to and less than unity. The respective membrane efficiencies are interpreted from the measured emf's using Fig. 13 of Ref. 3, and are expressed in terms of the $b$ and $b'$ parameters.

A group of plugs were cut from cores taken from a formation in the Gohlke field, Texas. Diffusion potentials in the samples were measured using two NaCl solutions having concentrations equal to the original mud filtrate and formation water, respectively. These potentials were measured both at $S_w = 1$ and at residual oil conditions.

As indicated above, values of $b'$ were determined at $S_w$ from the emf measurements. Similarly, $b$ values were determined for these sands at $S_w = 1$. A plot of $b$ vs $b'S_w$ is given in Fig. 7. Considering the experimental problems inherent in the determination of $S_w$, comparison of $b$ values with values of $b'S_w$ shows remarkably good agreement.

The assumption of Eq. 20 rules out the use of the Hill-Milburn equation to describe the conductance behavior of oil-bearing shaly sands. With decrease in $S_w$ and the accompanying increase in $Q'_w$ or $b'$, the minima in the calculated conductivity curves are shifted to quite high values of $C_w$, well within the range of practical field values. Thus, depending on the original values of $b$ and the particular values of $S_w$, substantial portions of the conductivity curves will indicate increasing sand conductivities with decreasing $C_w$'s (at constant $Q_v$ values) and will be physically meaningless.

We further assume that the conductivity of an oil-bearing shaly sand can be described by an equation analogous to Eq. 9 for water-saturated sands,

$$C_t = \frac{1}{G^*} \left( C_w + BQ'_w / S_w \right)$$  \hspace{1cm} (22)

Here $C_t$ is the specific conductance of a partially water-saturated sand and $G^*$ is a geometric factor, being a function of porosity, water saturation and pore geometry, but independent of clay content ($Q_v$). $G^*$ is expected to increase with decreasing $S_w$ since the water-filled pore space is decreasing, apart from complications due to distribution of the two phases in the pore network of the rock.

For clay-free or "clean" sandstones, $F^*$ and $G^*$ reduce to

![FIG. 7 — EFFECT OF OIL SATURATION ON DIFFUSION POTENTIALS OF SHALY SANDS.](image-url)
\[ F = \frac{C_\text{w}}{C_\text{o}} = \frac{R_\text{w}}{R_\text{o}} \ldots \ldots \ldots (23) \]

and

\[ G = \frac{C_\text{v}}{C_\text{t}} = \frac{R_\text{t}}{R_\text{v}} \ldots \ldots \ldots \ldots (24) \]

The resistivity ratio \( I \) is defined as

\[ I = \frac{R_\text{t}}{R_\text{o}} = \frac{C_\text{v}}{C_\text{t}} = \frac{G}{F} \ldots \ldots \ldots (25) \]

and according to Archie's second empirical relation, \( I = S_\text{w}^{-n} \) (\( = G/F \)) \ldots \ldots \ldots \ldots \ldots \ldots \ldots (26)

The value of \( n \) may differ from the value of \( m \) in Eq. 14.

By analogy, we define

\[ G/F = S_\text{w}^{-n} \ldots \ldots \ldots \ldots \ldots \ldots (27) \]

where \( n \) is the exponent for a hypothetical rock having the same pore geometry as the shaly sand, but whose clay content is inactive, i.e., \( n \) is independent of \( Q_\text{w} \). For the limiting case \( Q_\text{w} = 0 \) or \( C_\text{w} = \infty \), Eq. 27 reduces to Eq. 26.

Combining Eqs. 9, 22, 25 and 27, we obtain for the resistivity ratio

\[ I = S_\text{w}^{-n} \left( \frac{C_\text{v} + BQ_\text{w}}{C_\text{w} + BQ_\text{w} / S_\text{w}} \right) \ldots \ldots \ldots (28) \]

Eq. 28 can be expressed in terms of water resistivity

\[ I = S_\text{w}^{-n} \left( \frac{1 + R_\text{w} BQ_\text{w}}{1 + R_\text{v} BQ_\text{v} / S_\text{v}} \right) \ldots \ldots \ldots \ldots (29) \]

where the units of water resistivity \( R_\text{w} \) and the term \( BQ_\text{w} \) are ohm m and (ohm m)\(^{-1}\), respectively. For \( Q_\text{w} \) expressed as meq/m or equiv/liter, the numerical value of \( B \) in Eq. 28 is

\[ B = [1 - 0.6 \exp(-0.77/R_\text{w})]^{4.6} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (30) \]

Figs. 8 through 11 demonstrate the effects of variation of clay content, water salinity and pore geometry on the \( I-S_\text{w} \) relationship, as predicted by Eq. 29. We observed that even small amounts of clay have a marked effect on the resistivity ratio. Significant departures from the simple \( I-S_\text{w} \) relation (Eq. 26) are noted with increasing water resistivity at constant \( Q_\text{w} \); further, \( \log S_\text{w} \) is nonlinear with respect to \( \log I \).

As a final example, an illustration of the influence of clay in a reservoir sand on electric log interpretation is given. We assume a resistivity ratio of \( I = 4 \) obtained from log readings, and estimate the
value of \( n^* \) to be 2. For a clean sand, these values lead to an oil saturation \( S_o = 0.50 \) by the Archie relation (Eq. 26). The influence of various degrees of shaliness associated with different water resistivities for this example is shown in Fig. 12. Oil-in-place estimates are increased considerably from 33 to 47 percent with increasing values of \( Q_w \) above the clean sand estimate in certain fresh water zones.

CONCLUSIONS

An equation has been developed, based on a physical model, that relates the electrical conductivity of a water-saturated shaly sand to the water conductivity and the clay content of the sand. Clay contents are expressed by the cation-exchange capacities per unit pore volume of the rocks which can be determined by independent analyses. Various criteria established to test the model are satisfied by clay sand conductivity data. These data further permit determination of sand conductivities and clay counterion mobilities down to very low equilibrating electrolyte solution conductivities.

The model is extended to describe the conductivity of oil-bearing shaly sands. An expression is developed which relates the resistivity ratio to water saturation, water resistivity and CEC per unit pore volume of rock. This equation generally predicts higher oil-in-place estimates than are obtained from the usual clean sand relations.

NOMENCLATURE

\( a \) = dimensionless constant, Eqs. 10 and 11
\( A_D \) = constant in de Witte's equation\(^{19} \) (Eq. 12 in this paper
\( B_D \) = constant in de Witte's equation\(^{19} \) (Eq. 12 in this paper
\( b \) = Hill-Milburn parameter\(^{3} \) related to effective clay content of shaly sand at \( S_w = 1 \)
\( b' \) = Hill-Milburn parameter\(^{3} \) related to effective clay content in oil-bearing shaly sand
\( B \) = equivalent conductance of clay exchange cations (sodium) as a function of \( C_w \) at 25°C (Eq. 10), mho sq cm meq⁻¹
\( C_e \) = conductance contribution of clay exchange cations to shaly sand conductivity, ohm⁻¹ or mho
\( C_o \) = specific conductance of clay exchange cations, mho cm⁻¹
\( C_{e1} \) = conductance contribution of bulk electrolyte solution to conductivity of shaly sand, ohm⁻¹ or mho
\( C_{o1} \) = hypothetical specific conductance of shaly sand at low values of \( C_w \) defined in Eq. 17, mho cm⁻¹
\( C_s \) = specific surface conductance of clay, ohm⁻¹ or mho
\( C_p \) = specific conductance of a partially water-saturated sand, mho cm⁻¹
\( C_w \) = specific conductance of aqueous electrolyte solution, mho cm⁻¹
\( D \) = self-diffusion coefficient, sq cm sec⁻¹
\( F_{01} \) = Hill-Milburn formation resistivity factor\(^{3} \) for shaly sands
\( F \) = formation resistivity factor for clean sand
\( F^* \) = formation resistivity factor for shaly sand (this paper)
\( \mathcal{F} \) = faraday
\( g \) = weight of clay per unit of rock pore volume, gm clay/cu cm
\( G \) = formation resistivity factor for partially water-saturated clean sand, Eq. 24
\( G^* \) = formation resistivity factor for partially water-saturated shaly sand
\( I \) = resistivity index, Eq. 25
\( m \) = porosity exponent or lithology factor, Eq. 14
\( m_{NaCl} \) = molal concentration of NaCl solution, mol NaCl/1,000 gm water
\( N \) = aqueous electrolyte solution concentration, Normality, moles solute/liter solution
\( n \) = Archie saturation exponent for clean sands, Eq. 26
\( n^* \) = saturation exponent for shaly sand
\( Q_s \) = surface concentration of clay exchange cations, equiv cm⁻²
\( Q_v \) = volume concentration of clay exchange cations, meq ml⁻¹ or equiv liter⁻¹
\( Q_{v1} \) = volume concentration of clay exchange cations in oil-bearing shaly sand (Eq. 20), meq ml⁻¹ or equiv liter⁻¹
\( r \) = ratio of measured to hypothetical shaly sand conductivities at identical \( C_w \)'s, Eq. 17
\( R_o \) = resistivity of sand 100 percent saturated with aqueous salt solution, ohm m

FIG. 12 — OIL-IN-PLACE ESTIMATES AS A FUNCTION OF CLAY CONTENT OF SAND AND FORMATION WATER RESISTIVITY (ASSUMED CONDITIONS, \( l = 4 \), \( n^* = 2 \)).
\( R_s \) = resistivity of a partially water-saturated sand, ohm m

\( R_w \) = resistivity of equilibrating aqueous salt solution, ohm m

\( S \) = specific surface area of clay, sq cm/gm clay

\( S_o \) = fractional oil saturation

\( S_w \) = fractional water saturation

\( x \) = geometric cell constant for shaly sand, Eq. 2

\( y \) = geometric cell constant for shaly sand, Eq. 2

\( \delta \) = constant defined by Eq. 10, mho cm\(^{-1}\)

\( \chi_{Na}^e \) = maximum equivalent ionic conductance of sodium exchange cations associated with clay (25C), sq cm equiv\(^{-1}\) ohm\(^{-1}\)

\( \chi_{Na}^e' \) = equivalent ionic conductance of sodium exchange cations associated with clay (25C) at C_0 = 0, sq cm equiv\(^{-1}\) ohm\(^{-1}\)

\( \mu_{Na}^e \) = maximum sodium exchange ion mobility (25C), sq cm volt\(^{-1}\) sec\(^{-1}\)

\( \phi \) = porosity

**ACKNOWLEDGMENTS**

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Theoretical and Experimental Bases for the Dual-Water Model for Interpretation of Shaly Sands

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Abstract

A simple petrophysical model proposed by Waxman and Smits (WS)\(^1\) in 1968 and Waxman and Thomas (WT)\(^2\) in 1972 accounts for the results of an extensive experimental study on the effects of clays on the resistivity of shaly sands. This model has been well accepted by the industry despite a few inconsistencies with experimental results. It is proposed that these inconsistencies resulted from the unaccounted presence of salt-free water at the clay/water interface. Electrochemistry indicates that this water should exist, but is there enough to influence the results? Both a theoretical study and reinterpretation of Waxman-Smits-Thomas data show that there is.

The corresponding new model starts from the Waxman and Smits concept of supplementing the water conductivity with a conductivity from the clay counterions. The crucial step, however, is equating each of these conductivity terms to a particular type of water, each occupying a representative volume of the total porosity. This approach has been named the "dual-water" (DW) model because of these two water types—the conductivity and volume fraction of each being predicted by the model.

The DW model has been tested on most of the core data reported in Refs. 1 and 2. The DW concept is also supported by log data\(^3\) and has been successfully applied to the interpretation of thousands of wells. However, the scope of this paper remains limited to the theoretical and experimental bases of the DW model.

The Petrophysical DW Model

The purpose of this model is to account for the resistivity behavior of clayey sands. For petrophysical considerations, a clayey formation is characterized by its total porosity, \(\phi_t\); its formation factor, \(F_0\); its water saturation, \(S_{wt}\); its bulk conductivity, \(C_i\); and its concentration per unit PV of clay counterions, \(Q_v\). The formation behaves like a clean formation with identical parameters \(\phi_t, F_0,\) and \(S_{wt}\) but containing a water whose conductivity, \(C_{we}\), differs from the bulk formation water.**

Neither the type of clays nor their distribution influences the results. Since the formation obeys Archie's laws,

\[
C_i = \left[ \frac{S_{wt}}{F_0} \right] C_{we}. \quad \text{(1)}
\]

The clayey sand equivalent water conductivity, \(C_{we}\), can be considered a mixture of two waters.

1. A clay water surrounds the clay particles but has a conductivity independent of the type and amount of clay. Its conductivity, \(C_{cw}\), comes exclusively from the clay counterions. The volume fraction of clay water, \(V_{cw}\), is directly proportional to the counterion concentration, \(Q_v\).

\[
V_{cw} = V_Q Q_v, \quad \text{(2)}
\]

where \(V_Q\) is the amount of clay water associated with 1 unit (meq) of clay counterions.

2. The water further away from the clay is called far water. Its conductivity, \(C_{fw}\), and ionic concentration correspond to the salinity of bulk-formation water. The volume fraction of this water, \(V_{fw}\), is the balance between the total water content and the clay water.

\[
V_{fw} = V_w - V_{cw} = \phi_t (S_{wt} - V_Q Q_v). \quad \text{(3)}
\]

The implicit assumption is that the far water is displaced preferentially by hydrocarbons.

The two water conductivities are combined as their volumetric weighted averages to give the equivalent fluid conductivity of the formation as

\[
C_{we} = \frac{(S_{wt} - V_Q Q_v) C_{fw} + V_Q Q_v C_{cw}}{S_{wt}}, \quad \text{(4)}
\]

from which

\[
C_i = \frac{S_{wt}}{F_0} \left[ C_w + \frac{V_Q Q_v}{S_{wt}} (C_{cw} - C_w) \right]. \quad \text{(5)}
\]

*Now retired.

**\(F_0\) is defined as the formation factor of a rock of pore geometry identical to that of the clayey sand but in which the clays have been replaced by a nonconductive mineral free of electrical surface effect. Accordingly, \(F_0\) behaves like the formation factor of a clean formation.

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In water-saturated formations, in which \( S_{sw} = 1 \), these expressions are simplified to:

\[
C_{sw} = (1 - \nu Q_v) C_w + \nu Q_v C_{cw} \tag{6}
\]

and

\[
C_0 = \frac{1}{F_0} [(1 - \nu Q_v) C_w + \nu Q_v C_{cw}] \tag{7}
\]

Water saturation in Eq. 5 is computed as a fraction of total porosity. Because \( S_{sw} \) includes the clay water, sandy sands may have high water saturation and still produce water-free hydrocarbons. A better clue to the nature of production may be given by \( S_{fw} \), the fraction of porosity \( \phi_{fw} \) filled by far water:

\[
S_{fw} = \frac{V_{fw}}{\phi_{fw}} \tag{8}
\]

\( \phi_{fw} \), comparable to effective porosity, is given by

\[
\phi_{fw} = \phi_s - \nu Q_v \phi_s \tag{9}
\]

so that

\[
S_{fw} = \frac{S_{sw} - \nu Q_v}{1 - \nu Q_v} \tag{10}
\]

Therefore, the values of \( \nu Q_v \) and \( C_{cw} \) must be known at formation temperature for the interpretation model to be solved. The determination of these two parameters is discussed later.

**Theory of the DW Model**

Clays are usually sheet-like particles characterized by a very large surface compared to their volume (Fig. 1a). These clay-sheets are made by the stacking of two or three layers of aluminum octahedra or silica tetrahedra lattices. There is often an excess of negative electrical charges inside the clay-sheet frequently caused by the substitution of Mg \(^{2+} \) for some Al \(^{3+} \) in the octahedral lattice. Compensation for this local electrical unbalance is necessary to maintain the electrical neutrality of the clay particle. The compensating agents are positive ions, or countercations, clinging to the surface of the clay-sheets in the dry state (Fig. 1b). The resulting positive surface charge, equal to the internal negative deficit, is a constant characteristic of the clay called cation exchange capacity (CEC). It is expressed in milli-equivalent (meq) per 100 grams [3.5 oz] of dry material.

When the clay particles are immersed in an electrolytic solution, the Coulomb forces keeping the positive countercations at the clay surface are drastically reduced by the high dielectric permittivity of water. The countercations leave the clay surface and move relatively freely in the water (Fig. 1c), contributing to the electrical conductivity of the water and thereby increasing the rock conductivity. This added conductivity is proportional to the concentration of countercations in water. It is therefore logical to express the surface charge in terms of countercation concentration in the PV or \( Q_v \). It is expressed in meq/mL of total porosity.

Because of the mobility of clay countercations in water, they may be replaced charge-to-charge by any other cation in the electrolyte, provided the electrical balance is maintained. Refs. 1 and 2 deal exclusively with rocks where countercations and cations are Na \(^+ \), as does this paper.

Despite their mobility, clay countercations do not distribute themselves uniformly in the water. They remain attracted near the clay surface by the negative charge they neutralize, but they are kept at some distance by water molecules adsorbed at the clay surface or surrounding the countercations. As a result, they are separated from the clay by a thin water layer that ions cannot penetrate.

The existence of this ion-free water around clay particles should not come as a surprise. The fact that the conductivity or chlorinity of water in some shales is lower than that in surrounding sands \(^{5,6} \) could be related to it, although ion-rich shale waters have also been reported. \(^{7} \)

More conclusively, it has been shown experimentally that fresh waters are squeezed out from saltwater clays at very high pressure. \(^{8} \) In addition, the "swelling clays" are the most frequent manifestations of the expansion of the ion-free layer under a change of water salinity.

**The Diffuse Layer and Outer Helmholtz Plane (OHP)**

If one accepts as a first approximation that ions and water molecules have negligible dimensions, the distribution of Na \(^+ \) and Cl \(^- \) ions in the vicinity of the clay surface would be as predicted by the Gouy model, described in electrochemistry text books, and illustrated in Fig. 2. The solid and dashed lines represent the local concentrations of Na \(^+ \) and Cl \(^- \) ions, respectively, at distance \( x \) from the clay surface.

The zone near the clay surface where Na \(^+ \) exceeds Cl \(^- \) constitutes the "diffuse layer." Its thickness is

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characterized by the parameter $x_d$ related theoretically to the salt concentration, $<n>$, in the bulk water at 25°C (77°F) by

$$x_d = 3.06 \sqrt{\frac{1}{\gamma <n>}}, \quad ................. (11)$$

where $x_d$ is in angstroms (Å), $<n>$ in mol/dm$^3$ [mol/mL], and $\gamma$ is the NaCl activity coefficient.

According to the Gouy model, the diffuse layer is equivalent to (1) a layer of thickness, $x_d$, which contains the Na$^+$ cations necessary to balance the charge deficiency of the clay but none of the Na$^+$ or Cl$^-$ ions of the salt dissolved in the water; and (2) the zone beyond $x_d$, with the salt concentration of the bulk water.

The definition of the clay-water zone would be complete if it were not for the finite dimensions of ions and water molecules. Actually, the Na$^+$ counterions of the diffuse layer are kept some distance away from the clay surface by the layer of water molecules adsorbed on the clay surface and a shell of hydration water molecules surrounding the sodium ion (Fig. 3).

At their closest approach, the Na$^+$ ions are located with their centers lying on what is called the OHP at a distance $x_H$ from the surface (Fig. 3).

From Fig. 3 and Ref. 9, the distance of the OHP is

$$x_H = 2r_w + \sqrt{3}r_w + r_{Na} = 6.18 \text{ Å},$$

where $r_w$, the radius of water molecule, equals 1.4 Å, and $r_{Na}$, the radius of the Na ion, is 0.96 Å at room temperature.

The water salinity at which the Gouy layer thickness, $x_d$, matches the OHP distance, $x_H$, may be computed from Eq. 11 to be $<n_1> = 0.35$ mol/dm$^3$ [0.35 mol/mL]. The corresponding value for $\gamma_1$, the activity coefficient, at this salinity is 0.71. Two situations are then possible.

1. The water salinity exceeds $<n_1>$. Then all the counterions are located on the OHP. The zone of salt exclusion is limited by the OHP and its thickness, $x_H$, is about 6.2 Å at room temperature, and is independent of bulk water salinity: $x_d = x_H$.

2. The salinity is less than $<n_1>$. Then, there is truly a diffuse layer. The thickness of the zone of salt exclusion, $x_d$, is given by the Gouy diffuse layer model and is now salinity-dependent.

In general, we may write

$$x_d = \alpha x_H,$$

where $\alpha$, derived from Eq. 11, is

$$\alpha = \sqrt{\frac{\gamma_1 <n_1>}{\gamma <n>}}, \quad ................. (12)$$

Of course, $\alpha = 1$ when the water salinity exceeds $<n_1>$.

Volume of Clay-Water

To determine the volume occupied by salt-free water, one needs to know the clay surface area in contact with the water. This surface of the clay is actually related to the clay charge.

It has been recognized for some time that the surface charge density of most clay minerals is relatively constant. This point has been investigated with somewhat diverging results. For example, the average of 25 results from Refs. 13 through 16 leads to an area of 675 m$^2$/meq [7,266 sq ft/meq] when specific area is greater than 100 m$^2$/g [1,076 sq ft/g] and 580 m$^2$/meq [6,243 sq ft/meq] when specific area is less than 100 m$^2$/g [1,076 sq ft/g]. On the other hand, Patchett finds for 109 shale samples an area of 580 m$^2$/meq [6,243 sq ft/meq] when specific area is greater than 100 m$^2$/g [1,076 sq ft/g] and 800 m$^2$/meq [8,611 sq ft/meq] when specific area is less than 100 m$^2$/g [1,076 sq ft/g]. Patchett’s results do not apply too well here. They deal with shale and the measured area includes the contribution of the silt fraction, particularly significant when the specific area is small.
Conductivity of Clay-Water

The water near the clay contains only the clay counterions with pore space concentration \(Q_v\). Their concentration in clay-water is then

\[
Q_{cw} = \frac{Q_v \phi_i}{V_{cw}} \quad \text{.................................. (15)}
\]

Let \(\beta\) be the counterion equivalent conductivity. The clay water conductivity is then

\[
C_{cw} = \frac{\beta Q_v \phi_i}{V_{cw}} = \frac{\beta}{V_{cw}} = \frac{\beta}{v_Q} \quad \text{.................................. (16)}
\]

which is independent of the amount and type of clay.

In a NaCl environment where sodium CEC predominates, \(\beta\) and \(C_{cw}\) are universal parameters that depend mainly on temperature and somewhat on salt concentration. With nonsodium CEC, encountered in some shales, they also depend on the type of counterion. The value of \(\beta\) is derived from core measurements later.

Effect of Temperature

Counterion Equivalent Conductivity \(\beta\). Temperature decreases the water viscosity, which affects, in the same manner, the mobility of any ion in the water. It is also likely to lower the weak bond of counterions with the clay. Accordingly, \(\beta\) should increase as fast as or somewhat faster than the conductivity of a NaCl solution.

Clay-Water Thickness. For salinity greater than \(x_{H_1}\), the clay-water thickness is \(x_{H}\) distance of OHP. Temperature decreases the average residence time of the adsorbed water molecules on the clay by decreasing the strength of dipolar bond, thus decreasing the effective thickness of the adsorbed water layer. Consequently, \(v_Q\) should decrease with temperature. Assuming the average residence time to be inversely related to the frequency of jumps of adsorbed water molecules, one may use the rate process theory\(^9\) to predict the shape of the temperature effect:

\[
v_Q = \frac{1}{f} = \frac{e^{-\Delta G_0/RT}}{T} \quad \text{.................................. (17)}
\]

where \(T\) is absolute temperature and \(\Delta G_0\) is free energy change from adsorbed to free-water molecule state.

For salinity less than \(x_{H_1}\) the clay water has the thickness of the diffuse layer, shown by its theory\(^9\) to vary with \(T^{\frac{1}{2}}\) (\(T\) being the absolute temperature).

\[
(v_Q)_{T} = (v_Q)_{T_0} \left[ \frac{T}{T_0} \right]^{\frac{1}{2}} \quad \text{.................................. (18)}
\]

From 25°C to 200°C [77°F to 392°F], \(v_Q\) should increase by 26%.

This discussion has presented the physical foundation of the petrophysical model. The existence of clay water...
has been justified and its petrophysical parameters have
been identified. The actual values remain to be deter-
mined experimentally.

The Experimental Approach

The experimental data come primarily from the pub-
lished work of Waxman and Smits1 and Waxman and
Thomas.2 In addition, Shell Development has allowed
us to use some unpublished data concerning the effect of
temperature and saturation studied in Ref. 2.

Quantification of \( v \beta \), \( \beta \), and \( C_{cw} \)

Waxman and Smits' experiments on water-saturated
clayey sands established that the relation between \( C_0 \)
(shaly sand conductivity) and \( C_w \) (bulk water conduc-
tivity) has the characteristic shape shown in Fig. 5.

For bulk water NaCl salinity in excess of 0.40
mol/dm\(^3\) [0.40 mol/L] (25°C [77°F]), corresponding to
\( C_w > C_1 \), the shaly sand conductivity \( C_0 \) follows a
straight line of slope \( s \). This line intercepts the \( C_w \) axis at
a value (\( -C_x \)). Thus, for values of \( <n> \) greater than
0.40,

\[
C_0 = \alpha (C_w + C_x). \quad \cdots \quad (19)
\]

In this high-salinity range, \( \alpha = 1 \) and the DW model conduc-
tivity, \( C_0 \), given by Eq. 7, can be written in a form
similar to Eq. 19:

\[
C_0 = \frac{1 - v \beta Q_v}{C_w} \left( C_w + \frac{v \beta Q_v}{1 - v \beta Q_v} C_{cw} \right). \quad \cdots \quad (20)
\]

In this form Eqs. 19 and 20 may be compared term-by-
term.

Intercept

\[
C_x = \frac{v \beta Q_v}{1 - v \beta Q_v} C_{cw} = -\frac{\beta Q_v}{1 - v \beta Q_v}. \quad \cdots \quad (21)
\]

The right-hand term is obtained by application of Eq. 16.
The intercept magnitude, \( C_x \), is a quadratic function of
\( Q_v \) with parameters \( v \beta \) and \( \beta \). These parameters can be
determined by curve fitting Eq. 21 to experimental core
data.

For doing so, the \( C_x \) intercepts for the 86 samples of
Group 1 of Ref. 1 were recomputed from the basic \( C_0 \)
conductivity data. The results are presented on Fig. 6.
The vertical bars represent the uncertainty caused by
departures of the \( C_0 \) data from a straight line at high
salinities. Curves 1 and 2 are best fits of Eq. 21 for the
majority of the data and for the most southeasterly data,
respectively.

From curve 1

\[
v \beta = 0.30 \text{ dm}^3/\text{meq} [0.30 \text{ mL}/\text{meq}]
\]

and

\[
\beta = 2.05 \text{ (S/m)/(meq/cm}^3\text{)} \quad [2.05 \text{ (mho/m)/(meq/cm}^3\text{)}].
\]

From curve 2

\[
v \beta = 0.225 \text{ cm}^3/\text{meq} [0.225 \text{ mL}/\text{meq}]
\]

and

\[
\beta = 2.50 \text{ (S/m)/(meq/cm}^3\text{)} \quad [2.50 \text{ (mho/m)/(meq/mL)}].
\]

Curve Fit 1, which is statistically the most significant,
gives a value of \( v \beta = 0.30 \) close to the theoretical 0.28
from Eq. 13. Another independent estimate of \( v \beta \) is given under Temperature Effect. It confirms this choice.
\( C_{cw} \) is computed from Eq. 16, leading to \( C_{cw} = 6.8
\text{ S/m} [6.8 \text{ mho/m}].

Some comments should be made at this point on the
values found for \( \beta \) and \( v \beta \) for Na\(^+\) is found to be 2.05
(S/m)/(meq/cm\(^3\)) [2.05 (mho/m)/(meq/mL)]. This is
between the conductivity of Na\(^+\) of montmorillonite in
distilled water\(^1\) 1.8 (S/m)/(meq/cm\(^3\)) [1.8
(mho/m)/(meq/mL)] and the one of Na\(^+\) ions in saline
solution of same ionic strength as clay-water 2.75
(S/m)/(meq/cm\(^3\)) [2.75 (mho/m)/(meq/mL)].
Concerning $\nu Q_v$, it should be noted that it imposes a limit to the value of $Q_v$, to be expected in shaly formations. According to Eq. 2, and with the obvious condition $Q_T \gg V_{cw}$:

$$Q_v \approx \frac{1}{\nu Q_v^\beta}.$$

Thus, $Q_v$ should be less than 3.3 in formations at 25°C [77°F] and 5.5 in formations at 200°C [392°F]. Most $Q_v$, reported in the literature respect these limits although these truly apply only to fully wet clays. When part of the clay surface loses its water (desiccation, grain to grain contact, etc.), the corresponding counterions become fixed on the dry surface (see Theory of Dual Water) and lose their electrical effect. However, these counterions are still counted in the CEC measured chemically. This leads to apparent $Q_v$, that are excessively high and $\beta$ that seem abnormally low.

The exceptional values measured by Kern et al. on samples from low-porosity gas sands $^{18}$ ($Q_v = 20$, $\beta = 0.5$) illustrate an extreme case of this situation. Still, it may happen to a lesser degree on usual rock samples. A way around this problem is to measure an effective $Q_v$, corresponding only to the mobile (conductive) counterions. $Q_v$ is then obtained from $C_T$ through Eq. 21 using $\beta$ and $\nu Q_v$ at the $C_T$ temperature. Actually this "electrical" $Q_v$ has been preferred to the chemical one in most computations in this study. Additional justifications for this practice are given in the last section of this paper.

Slope. From Eqs. 19 and 20

$$z = \frac{1}{F_0} = \frac{1 - \nu Q_v}{F_0} \hspace{1cm} \text{(22a)}$$

or the inverse form

$$F_0 = F^*(1 - \nu Q_v) \hspace{1cm} \text{(22b)}$$

In Eq. 22b, $(1 - \nu Q_v)$ corrects for the presence of the clay-water and converts the measured formation factor, $F^*$, to the idealized one, $F_0$.

Accordingly, $F_0$ should be less distorted by clay content, and therefore more closely related to Archie’s relation:

$$F = \phi_i^{-m} \hspace{1cm} \text{(23)}$$

This deduction can be verified on the 136 samples of Refs. 1 and 3 as shown on Figs. 7 and 8. Figs. 7a and 7b are plots of the formation factor against $\phi_i$ for WS and DW models, respectively. It is seen that the large scatter of the WS plot (Fig. 7a) has been reduced by a factor of two by the clay-water correction (Fig. 7b). Figs. 8a and 8b plot the values of Archie’s exponent, $m$, computed from Eq. 23, against the shaliness factor $Y = Q_v \phi_i/(1 - \phi_i)$ or clay counterions per unit volume of

*Peter Day of Schlumberger-Doll Research has demonstrated that further improvements are obtained by correcting $F_0$ for the tortuosity added by the clay.

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solid. The $m$ of WS (Fig. 8a) appears more clay-dependent, whereas the $m$ of DW remains close to two even at high shaliness. The fact that the formation factor of the DW model is more closely related to the one of Archie’s is of practical importance in log interpretation, for which the porosity is usually the source of the formation factor.

Curvature of the $C_0$ Line

An inevitable consequence of the DW model is that the $C_0$ line should be curved downward at salinities below $<n_1>$. Clay-water-layer expansion at lower salinities displaces some of the far water without ionic compensation.

Data from Group 2 of Ref. 1 and Fig. 9, showing a typical sample from this group, show that a curve exists below $<n> = 0.4$ mol/dm$^3$ [0.4 mol/L]. This is close enough to the theoretical value $<n_1> = 0.35$ mol/dm$^3$ [0.35 mol/L] to assume that this curvature could be the predicted one and that the actual value of $<n_1>$ should be

$$<n_1> = 0.40 \text{ mol/dm}^3 [0.40 \text{ mol/L}].$$

Quantitatively, the formation conductivities $C_{0f}$ computed from Eqs. 7, 12, and 14 were compared to the measured conductivities $C_{0m}$ for the 15 most significant samples of group 2, Ref. 1 (see Appendix). The average of ratios $C_{0m}/C_{0f}$ are shown in Table 1.

It is seen that clay-water expansion explains quantitatively the $C_0$ curvature to any $C_0$ down to 1 S/m [1 mho/m]. This is illustrated on Fig. 9. This limit covers most cases of interpretation. The additional drop of $C_{0m}$ below 1 S/m [1 mho/m] must be attributed to other causes, probably to the decrease of counterion mobility in dilute solution as suggested by Waxman.\(^{1,17}\) Then, Table 1 would lead to approximately

$$\beta_{df} = \beta (1 - 0.4 e^{-2C_0}). \quad \text{.................(24)}$$

Temperature Effect

This study is based on the conductivity measurements on the nine shaly sands described in Table 4 of Ref. 2. The procedure used to transform it into intercept $C_0$ and slope $1/F^*$ at each temperature is described in the Appendix. $F^*$ and $C_0$ values are listed in Tables A-1 and A-2.
TABLE 1—CURVATURE CORRECTION FACTOR, \(C_w\) (mho/m)

<table>
<thead>
<tr>
<th>(C_{2w})</th>
<th>(C_{1w})</th>
<th>(C_{3w})</th>
<th>(C_{4w})</th>
<th>(C_{5w})</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.25</td>
<td>2.8</td>
<td>1.5</td>
<td>0.78</td>
<td>0.41</td>
</tr>
<tr>
<td>1.0</td>
<td>1.01</td>
<td>1.0</td>
<td>0.91</td>
<td>0.82</td>
</tr>
</tbody>
</table>

TABLE 2—\(v^*\) vs. TEMPERATURE (°C)

<table>
<thead>
<tr>
<th>(v^*_{av})</th>
<th>(80)</th>
<th>(140)</th>
<th>(200)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v^*_{av})</td>
<td>0.30</td>
<td>0.259</td>
<td>0.223</td>
</tr>
<tr>
<td>(v^*_{av}(T))</td>
<td>0.30</td>
<td>0.254</td>
<td>0.219</td>
</tr>
</tbody>
</table>

The effect of temperature on \(F^*\) can be verified qualitatively in Table A-1 or on Fig. 10, where the changes in \(F^*\) between two arbitrary temperatures are plotted against the shaliness \(Q_v\) of the samples.

Despite some large dispersion from sample to sample, Fig. 10 shows clearly that \(F^*\) is essentially temperature-independent in nonshaly formation \((Q_v = 0)\) and that the change due to temperature increases with shaliness; this is in agreement with Eq. 22. Similar results were also reported by Kern, \(^19\) in which a different approach was used to quantify the variations in \(F^*\).

The variations of \(v^*_{av}\) with temperature were computed from Table A-1 using Eq. 22, under the assumptions that \(F_0\) is invariant and that \(v^*_{av}(22)\) is 0.30. The average values of \(v^*_{av}\), computed from \(\Sigma(v^*_{av} \cdot Q_v)/\Sigma Q_v\), are listed in Table 2 and displayed in Fig. 11.

\[
v^*_{av}(T) = 0.30 \times \frac{295 + 25}{T_K + 25}, \quad \ldots (25)
\]

where \(T_K\) represents absolute (Kelvin) temperature. \(v^*_{av}(T)\) duplicates \(v^*_{Q_v}\) with only 2% error. Notice the close dependence of \(v^*_{av}\) on the reciprocal of \(T_K\) as expected from the rate process theory.

**Counterion Conductivity.** The increase of intercept \(C_x\) (Table A-2) reflects the increase of \(\beta\) with temperature since

\[
C_x = \frac{\beta Q_v}{1 - v^*_{av} Q_v}.
\]

A value of \((\beta Q_v)\) was computed for each \(C_x\), using the \(v^*_{av}\) values determined in the previous subsection. This was the basis for a weighted average \(\beta_{av}\) computed at each temperature. Results are normalized in Table 3 and Fig. 12.

\(\beta_{av}\) is well approximated by the function

\[
\beta T_{-c} = 2.05 \frac{T_{-c} + 8.5}{22 + 8.5}, \quad \ldots (26)
\]

where \(T_{-c}\) is temperature in °C. It can be compared with the conductivity \(C_{Na}\) of Na ion in salt solution of same ionic strength as clay-water listed on the final line of the table.

Clearly Na\(^+\) ions from clays or saline solutions exhibit similar dependence on temperature above 80°C [176°F] (water viscosity), with the counterion conductivity growing slightly faster at lower temperatures.

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TABLE 3—$\beta$ vs. TEMPERATURE (°C)

<table>
<thead>
<tr>
<th></th>
<th>22</th>
<th>80</th>
<th>140</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_v$</td>
<td>2.05</td>
<td>5.86</td>
<td>10.3</td>
<td>13.7</td>
</tr>
<tr>
<td>$\beta_{e_0}$</td>
<td>2.05</td>
<td>5.86</td>
<td>10.0</td>
<td>14.0</td>
</tr>
<tr>
<td>$C_{ke}$</td>
<td>2.73</td>
<td>6.57</td>
<td>10.5</td>
<td>13.5</td>
</tr>
</tbody>
</table>

These results are in fair agreement with those reported by Kern et al., who found the counterion conductivity to follow the same variation as NaCl solution.

Clay-Water Conductivity, $C_{cw}$. Entering Eqs. 25 and 26 into Eq. 16 leads to

$$C_{cw} = 7.10^{-4}(T_{w} + 8.5)(T_{e} + 298). \quad (27)$$

As is apparent in Fig. 13, $C_{cw}$ increases faster than $C_{w}$ with temperature. The decrease of the clay-water volume increases the concentration in counterions.

New Estimate of $\nu_B$ at 22°C [72°F]. Previous estimates of $\nu_B$ were derived from either chemistry principles or $C_{w}$ intercept data. A new estimate will be derived now from an independent source: the $F^*$ values (Table A-1).

According to Eqs. 22 and 26,

$$\left(\frac{1}{F^*}\right) = \left(\frac{1}{F_0}\right) + \frac{1}{T_{K} + 25} \cdot \frac{(\nu_B(22)(295+25)Q_v)}{F_0}, \quad (28)$$

with appropriate substitutions

$$Z_i = B_i + A_i \cdot X_i;$$
$$Z_i = 1/F^*;$$
$$X_i = (T_{K} + 25);$$
$$B_i = 1/F_0;$$
$$A_i/B_i = [(\nu_B(22)(320)Q_v). \quad (29)$$

$A_i$ and $B_i$ determined from regression analysis, together with the raw data, are listed in Table A-1 for 11 core samples.

An estimate of $\nu_B(22)$ could be made for each core from

$$\nu_B(22) = \frac{(A_i/B_i)}{320Q_v}, \quad (30)$$

To obtain a meaningful average value for $\nu_B(22)$ we need to do better than simply take the average value of $\nu_B(22)$, since such an average will reflect the considerable uncertainty in computing $(A_i/B_i)$ for samples

Fig. 12—Change in $F^*$ due to temperature vs. $Q_v$.

Fig. 13—$C_{cw}$ vs. temperature.

with low values of $Q_v$; instead we use the form

$$\nu_B(22) = \frac{\Sigma(A_i/B_i)}{320\Sigma Q_v} = \frac{916}{320 \times 10.8} = 0.27 \text{ meq/dm}^2 \text{ [meq/mL].} \quad (31)$$

This new and independent estimate compares well with the previous ones of 0.28 (chemistry) and 0.30 (intercept).

**Comparison With Core Data.** The conductivity at 200°C [392°F] of the samples studied by WT have been computed according to the dual-water temperature correction model. The results are presented in Table 4 as

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TABLE 4—TEMPERATURE CORRECTION TEST

<table>
<thead>
<tr>
<th>Core</th>
<th>(Q_x)</th>
<th>Waxman</th>
<th>Dual Water</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>2830C</td>
<td>0.64</td>
<td>4.71</td>
<td>4.94</td>
<td>5.08</td>
</tr>
<tr>
<td>2797G</td>
<td>0.90</td>
<td>4.81</td>
<td>5.20</td>
<td>5.07</td>
</tr>
<tr>
<td>2798B</td>
<td>0.99</td>
<td>4.85</td>
<td>5.29</td>
<td>5.00</td>
</tr>
<tr>
<td>2830A</td>
<td>1.15</td>
<td>4.90</td>
<td>5.48</td>
<td>5.57</td>
</tr>
<tr>
<td>Shale 1</td>
<td>2.0</td>
<td>5.16</td>
<td>6.81</td>
<td>8.60</td>
</tr>
<tr>
<td>Shale 2</td>
<td>2.1</td>
<td>5.19</td>
<td>7.02</td>
<td>8.30</td>
</tr>
</tbody>
</table>

Comparison of DW and WS Models

The additional data released to us include the measurements of the conductivities of the core samples used by WT in their saturation experiments. These data, given in Tables A-3 and A-5, offer the opportunity to make a comparison between the DW and WS models.

Variance of Models From \(C_0\) (Water-Saturated) Measurements. The test covers 52 \(C_0\) measurements distributed over 12 rock samples. The model parameters have been determined by best fitting to experimental conductivities on the basis of percentage accuracy. The basic conditions have been maintained that the sum of the percentage errors for any fit is nil and the variance is minimum. For the DW model, \(F_0\) and \(Q_x\) were optimized simultaneously for a best fit with conductivities.

For the WS model, a first fit WS (1) was obtained optimizing \(F^*\) but keeping the \(Q_x\) determined by WT from the \(C_0\) conductivity data (Ref. 2, Table 2). Because of the poor quality of results, another fit, WS (2) was also made, optimizing both \(F^*\) and \(Q_x\) simultaneously as was done for the DW model.

Best estimates of \(Q_x\), \(F_0\), and \(F^*\) are given in Table A-4 for the 12 rock samples. They define the rock model parameters used in the saturation study discussed later.

The variance between the corresponding models and measured conductivities has been used to quantify the quality of fit with experimental \(C_0\). The smaller the variance, the better the fit. Variance computations, expressed in percentages, are summarized in Table 5.

WS (1) exhibits the greatest variance. Apparently the \(Q_x\) determined by WT does not fit the \(C_0\) data well. The DW model gives the best fit; better than WS (2).

Archie's Formation Factor and Saturation Exponents. It is interesting to compare the values of \(m\) and \(n\) obtained for the 12 rock samples according to the three models. \(m\) and \(n\) are computed from

\[
F = \phi_i^{1-m} \text{ and } C_i = \frac{S_w^n}{F_0^n} \text{,} C_w^{\text{eq}}.
\]

The derivation of \(n\) is more complex because each rock sample is characterized by many computed \(n\) values, each one coming from an experiment where either the saturation or \(C_w\) has been changed. The average \(\bar{n}\) obtained for each sample, and the variance from \(\bar{n}\), or the corresponding standard deviation, \(\sigma_n\), will characterize the rock sample saturation exponent.

\(m\), \(\bar{n}\), and standard deviation from \(\bar{n}\) are listed for the three models of the 12 rock samples in Table A-6. This sample-by-sample comparison of \(m\) and \(\bar{n}\) may be of interest to the log analyst.

If it is true, as suggested by Waxman, that the stability of \(n\) in relation to \(\bar{n}\) is a good criterion of the quality of the model, then the DW model seems better, as seen from Table 6. Its variance is one-tenth of the WS (1) and one-third of the WS (2).

The average of \(m\) and \(\bar{n}\) over the 12 rock samples are given in Table 7. One may try and express \(n\) as a function of \(m\). An expression of this type is given under \(\bar{n} = f(m)\) in Table 7. The standard deviation of \(n\) and \(f(m)\) are rather large particularly for Model WS (1). There is apparently no significant advantage in using \(\bar{n} = f(m)\) instead of \((\bar{n})_{av}\).

<table>
<thead>
<tr>
<th>Number of Measurements</th>
<th>DW Model</th>
<th>WS (1) Model</th>
<th>WS (2) Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>3218C</td>
<td>6</td>
<td>38.8</td>
<td>247.1</td>
</tr>
<tr>
<td>3279B</td>
<td>4</td>
<td>1.2</td>
<td>209.1</td>
</tr>
<tr>
<td>3281</td>
<td>4</td>
<td>4.3</td>
<td>333.1</td>
</tr>
<tr>
<td>495C</td>
<td>6</td>
<td>15.4</td>
<td>93.8</td>
</tr>
<tr>
<td>521C</td>
<td>6</td>
<td>20.4</td>
<td>33.6</td>
</tr>
<tr>
<td>3280B</td>
<td>3</td>
<td>26.7</td>
<td>9.0</td>
</tr>
<tr>
<td>3282C</td>
<td>3</td>
<td>1.8</td>
<td>105.9</td>
</tr>
<tr>
<td>512C</td>
<td>6</td>
<td>30.8</td>
<td>77.8</td>
</tr>
<tr>
<td>3227A</td>
<td>5</td>
<td>12.0</td>
<td>10.0</td>
</tr>
<tr>
<td>3229B</td>
<td>5</td>
<td>33.4</td>
<td>24.5</td>
</tr>
<tr>
<td>5311B</td>
<td>2</td>
<td>2.4</td>
<td>2.4</td>
</tr>
<tr>
<td>3140A</td>
<td>2</td>
<td>0.0</td>
<td>57.3</td>
</tr>
<tr>
<td>Total Variance</td>
<td>52</td>
<td>185</td>
<td>1198</td>
</tr>
<tr>
<td>Standard Deviation (%)</td>
<td></td>
<td>1.9</td>
<td>4.8</td>
</tr>
</tbody>
</table>

162=http://www.spe.org/petroleument./
The $Q_r$ Evaluation Problem. Table 8 compares the electrical $Q_r$ of the three models to the chemical $Q_r$. The $Q_r$ of WS (1), published in Ref. 2, is remarkably similar to chemical $Q_r$; however, this is at the expense of poor fit to $C_p$ measurements and large dispersion of $n$ exponent. $Q_r$ of WS (2) and DW agree very well in view of the large differences in the basic parameters of two models. They both read significantly in excess of chemical $Q_r$ (+25%).

A similar trend was noticed with the sample set of the temperature study. These differences cannot be explained by clay dehydration, but they illustrate the limitations of the correlation between chemical and electrical $Q_r$. They suggest that $Q_r$ inferred from electrical measurements, whether conductivity, electrochemical potential, or dielectric constant are likely to be more appropriate to the interpolation of resistivity logs for saturation.

Conclusions

The conductivity model of shaly formations proposed by Waxman and Smits has been modified to take into account the exclusion of salt around clay particles. It was shown that, conductivity-wise, a shaly formation behaves as if it were clean, but contains a water that is a mixture of formation and so-called "clay" waters. The conductivity of the clay-water is quasuniversal and depends mainly on temperature. The WS data were reinterpreted in this light to obtain the parameters of this model. Improved coherence and fit with experimental data were obtained with this DW model.

Perhaps more significantly, the DW model has brought unification and simplification to log interpretation by putting the evaluation of shaly formations into the usual framework where resistivity is analyzed in terms of total porosity and saturation.

Nomenclature

- $A_i = \sqrt{2}B_i$
- $A_{sp}$ = specific area of clay surface, m$^2$/g [sq ft/oz]
- $A_v$ = clay surface area per unit of PV, m$^2$/cm$^3$ [sq ft/cm$^3$]
- $B = $ equivalent conductivity of compensating counterions, WS model (S/m)/(meq/cm$^3$) [(mho/m)/(meq/mL)]
- $B_i = 1/F_0$
- $B_0 = $ constant value of $B$ at high water salinities, $B_0 = C_t/Q_r$
- $C_{cw} = $ conductivity of clay-water, S/m [mho/m]
- $C_t = $ true conductivity of (hydrocarbon-bearing) formation, S/m [mho/m]
\[ C_T = \text{total conductivity, S/m [mho/m]} \]
\[ C_w = \text{(formation) water conductivity, S/m [mho/m]} \]
\[ C_{we} = \text{effective conductivity of water in a shaly sand, S/m [mho/m]} \]
\[ C_x = \text{Cw curve extrapolated to Cw axis (Cx=Bw/Qw), S/m [mho/m]} \]
\[ C_0 = \text{conductivity of 100% water-saturated formation, S/m [mho/m]} \]
\[ C_{oc} = \text{computed formation conductivities, S/m [mho/m]} \]
\[ C_{om} = \text{measured formation conductivities, S/m [mho/m]} \]
\[ C_1 = \text{the Cw value above which C0 varies linearly with Cw, S/m [mho/m]} \]
\[ \text{CEC = cation-exchange capacity (dry rock), meq/100 g [meq/3.5 oz]} \]
\[ F = \text{resistivity formation factor (for a clean sand), F=R_0/R_{we}} \]
\[ F_0 = \text{resistivity formation factor for a shaly sand as used in DW shaly-sand model} \]
\[ F^* = \text{resistivity formation factor for a shaly sand as used in WS shaly-sand model, F^*=R_0/R_{we}} \]
\[ \Delta G_0 = \text{free energy change} \]
\[ m = \text{exponent in Archie F=phi relationship for clean sands} \]
\[ n = \text{saturation exponent} \]
\[ <n> = \text{salt concentration in water, mol/dm}^3 \text{[mol/L]} \]
\[ <n>_1 = \text{salt concentration above which alpha=1, mol/dm}^3 \text{[mol/L]} \]
\[ Q_v = \text{concentration of clay counterions per unit PV, meq/cm}^3 \text{[meq/mL]} \]
\[ (Q_v)_el = \text{value of Qv computed from conductivity measurements, meq/cm}^3 \text{[meq/mL]} \]
\[ r_{Na} = \text{radius of Na}^+ \text{ion, Å} \]
\[ r_{w} = \text{radius of water molecule, Å} \]
\[ R_0 = \text{resistivity of formation 100% saturated with water (R_0=1/C_0), Ω-m} \]
\[ R_t = \text{true resistivity of (hydrocarbon-bearing) formation, Ω-m} \]
\[ R_{we} = \text{(formation) water resistivity, Ω-m} \]
\[ s = \text{slope of Cw vs. Cw curve for Cw greater than C1} \]
\[ S_{fw} = \text{fraction of "far-water" pore space that is water filled} \]
\[ S_{sw} = \text{water saturation in volume fraction of total porosity} \]
\[ S''_w = \text{water saturation to nth power} \]
\[ T_{c,c} = \text{temperature in °C} \]
\[ T_{K} = \text{temperature in K} \]
\[ T_0 = \text{reference temperature, K} \]
\[ V_{fw} = \text{volume fraction of far water} \]
\[ V_{Q} = \text{volume of clay-water per counterion at 22°C [72°F] when alpha=1, cm}^3 \text{[meq/mL]} \]
\[ V_{Q} = \text{volume of clay-water per counterion at 22°C [72°F] when alpha=1, cm}^3 \text{[meq/mL]} \]
\[ V_{Q} = \text{volume of clay-water per counterion at 22°C [72°F] when alpha=1, cm}^3 \text{[meq/mL]} \]
\[ V_{Q} = \text{volume of clay-water per counterion at 22°C [72°F] when alpha=1, cm}^3 \text{[meq/mL]} \]
\[ V_{Q} = \text{volume of clay-water per counterion at 22°C [72°F] when alpha=1, cm}^3 \text{[meq/mL]} \]
\[ V_{Q} = \text{volume of clay-water per counterion at 22°C [72°F] when alpha=1, cm}^3 \text{[meq/mL]} \]
\[ V_{Q} = \text{volume of clay-water per counterion at 22°C [72°F] when alpha=1, cm}^3 \text{[meq/mL]} \]
\[ V_{Q} = \text{volume of clay-water per counterion at 22°C [72°F] when alpha=1, cm}^3 \text{[meq/mL]} \]
\[ x_d = \text{thickness of diffuse layer, Å} \]
\[ x_H = \text{distance of OHP from clay surface, Å} \]
\[ y = \text{amount of clay normalized to fraction of solids volume, Y=Q_v/(1-φ_f)} \]
\[ Z_t = 1/F^* \]
\[ α = \text{expansion factor for diffuse layer} \]
\[ β = \text{equivalent conductivity of sodium counterions, DW model at 22°C [72°F], (S/m)[meq/cm}^3 \text{[meq/mL]} \]
\[ β_{av} = \text{value of β for low-salinity solutions, (S/m)[meq/cm}^3 \text{[meq/mL]} \]
\[ β_{cal} = \text{value of β at temperatures T, °C} \]
\[ γ = \text{NaCl activity coefficient} \]
\[ ρ = \text{specific clay-area coefficient, m}^2 \text{[sq ft/meq]} \]
\[ φ_f = \text{total porosity, fraction} \]

Acknowledgments

We thank Shell Development Co. for making available additional experimental data concerning the dependence of conductivity on saturation and temperature. We acknowledge the contribution of Peter Day in optimizing the F vs. φ relationship. We thank Ernie Finklea for his patient help in the preparation of this paper for publication.

References


APPENDIX

Correlation of $F$ vs. $\varphi$ and $m$ vs. Shaliness

The origin of the sample data is as follows.

<table>
<thead>
<tr>
<th>Shale</th>
<th>22</th>
<th>50</th>
<th>80</th>
<th>110</th>
<th>140</th>
<th>170</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>2853 F</td>
<td>2858 G</td>
<td>2927</td>
<td>2937</td>
<td>2830 C</td>
<td>2797 C</td>
<td>2799 B</td>
</tr>
<tr>
<td>A</td>
<td>173</td>
<td>159</td>
<td>142</td>
<td>129</td>
<td>119</td>
<td>109</td>
<td>93.7</td>
</tr>
<tr>
<td>B</td>
<td>142.3</td>
<td>12.52</td>
<td>22.5</td>
<td>32.5</td>
<td>26.5</td>
<td>32.5</td>
<td>32.5</td>
</tr>
</tbody>
</table>

Curve of $Q_s$ Line

This study is based on the samples of Table 7, Ref. 1, which satisfy the criterion

$$0.5Q_s < \langle Q_s \rangle < 2Q_s$$

Namely, these are 4, 5, 11, 12, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27.

**Temperature Study**

The raw data for the nine samples of Table 4 of Ref. 2 consisted of 35 $Q_s$ measurements per sample, at seven temperatures and five salinities. These data were smoothed by fitting a monotonic curve over the seven data points for a given salinity and the origin.

The next step was to define the $(C_9, C_{10})$ straight line from these data by linear regression on the three points of highest conductivity at each temperature. Each line has been characterized by its intercept $C_9$ and $F^*$ (slope $^{-1}$), which constitute the basic material for this study (Tables A-1 and A-2).

The two shales of Table A-2 are described in Table 5 of Ref. 2. Their $F^*$ have been computed from their conductivity at 22°C [72°F] and the ratio of conductivity vs. temperature of Fig. 8, Ref. 2.

$$\left( F^* \right) = A \left( \frac{1}{T_0} \right)$$

Comparison with WS Model

Table A-3 Experimental values of $R_0$,

Table A-4 Best-fit models of $C_9 = 1/R_0$,

Table A-5 Experimental resistivity index and saturation.

Table A-6 Formation factors and saturation exponents.

Tables A-3 and A-5 are experimental data provided by Shell Development Co.

**TABLE A-1—F* VS. TEMPERATURE**

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>$22$</th>
<th>$50$</th>
<th>$80$</th>
<th>$110$</th>
<th>$140$</th>
<th>$170$</th>
<th>$200$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F^*$</td>
<td>2853 F</td>
<td>2858 G</td>
<td>2927</td>
<td>2937</td>
<td>2830 C</td>
<td>2797 C</td>
<td>2799 B</td>
</tr>
<tr>
<td>$A_1$</td>
<td>2858</td>
<td>2858</td>
<td>2858</td>
<td>2858</td>
<td>2858</td>
<td>2858</td>
<td>2858</td>
</tr>
<tr>
<td>$B_1$</td>
<td>2858</td>
<td>2858</td>
<td>2858</td>
<td>2858</td>
<td>2858</td>
<td>2858</td>
<td>2858</td>
</tr>
</tbody>
</table>

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### TABLE A-2—INTERCEPTS $C_r$ AT $T_{SC}$

<table>
<thead>
<tr>
<th>$(O_{2})_w$</th>
<th>22</th>
<th>50</th>
<th>80</th>
<th>110</th>
<th>140</th>
<th>170</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>2683 F</td>
<td>0.16</td>
<td>0.561</td>
<td>0.849</td>
<td>1.20</td>
<td>1.49</td>
<td>1.78</td>
<td>2.08</td>
</tr>
<tr>
<td>2886 G</td>
<td>0.16</td>
<td>0.730</td>
<td>0.995</td>
<td>1.39</td>
<td>1.78</td>
<td>1.92</td>
<td>2.27</td>
</tr>
<tr>
<td>2927</td>
<td>0.28</td>
<td>0.710</td>
<td>1.16</td>
<td>1.59</td>
<td>2.04</td>
<td>2.68</td>
<td>3.41</td>
</tr>
<tr>
<td>Ohio E</td>
<td>0.33</td>
<td>1.60</td>
<td>2.34</td>
<td>2.99</td>
<td>3.58</td>
<td>4.10</td>
<td>4.74</td>
</tr>
<tr>
<td>2937</td>
<td>0.66</td>
<td>2.31</td>
<td>3.67</td>
<td>5.05</td>
<td>6.72</td>
<td>8.37</td>
<td>9.92</td>
</tr>
<tr>
<td>2830 C</td>
<td>1.06</td>
<td>3.10</td>
<td>5.50</td>
<td>8.60</td>
<td>11.4</td>
<td>14.4</td>
<td>16.8</td>
</tr>
<tr>
<td>2797 G</td>
<td>1.39</td>
<td>4.15</td>
<td>7.52</td>
<td>12.0</td>
<td>16.2</td>
<td>20.2</td>
<td>23.9</td>
</tr>
<tr>
<td>2799 B</td>
<td>1.28</td>
<td>4.14</td>
<td>7.65</td>
<td>11.7</td>
<td>16.0</td>
<td>19.2</td>
<td>21.9</td>
</tr>
<tr>
<td>2820 A</td>
<td>1.32</td>
<td>4.93</td>
<td>8.33</td>
<td>12.3</td>
<td>15.6</td>
<td>18.9</td>
<td>21.7</td>
</tr>
</tbody>
</table>

Note: $(O_{2})_w$ are computed from intercept at 200°C.

### TABLE A-3—EXPERIMENTAL VALUES OF $R_g (\Omega \cdot m)$

<table>
<thead>
<tr>
<th>0.0495</th>
<th>0.0806</th>
<th>0.099</th>
<th>0.0987</th>
<th>0.125</th>
<th>0.165</th>
<th>0.327</th>
<th>0.985</th>
<th>1.04</th>
</tr>
</thead>
<tbody>
<tr>
<td>3218 C</td>
<td>3.63</td>
<td>4.27</td>
<td>5.27</td>
<td>6.63</td>
<td>10.8</td>
<td>23.1</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>3279 B</td>
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<td>2.01</td>
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<td>1.88</td>
<td>2.01</td>
</tr>
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<td>3226 B</td>
<td>9</td>
<td>1.83</td>
<td>1.64</td>
<td>0.12</td>
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<td>1.81</td>
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<td>3301 B</td>
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<td>1.82</td>
<td>1.85</td>
<td>0.06</td>
<td>1.83</td>
<td>1.91</td>
<td>0.04</td>
<td>1.83</td>
<td>1.89</td>
</tr>
<tr>
<td>3140 A</td>
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<td>1.83</td>
<td>1.78</td>
<td>0.07</td>
<td>1.96</td>
<td>2.02</td>
<td>0.06</td>
<td>1.89</td>
<td>1.86</td>
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### SI Metric Conversion Factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>Conversion</th>
<th>Unit</th>
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<tbody>
<tr>
<td>°F</td>
<td>°C</td>
<td></td>
<td>5 (°F−32)/1.8</td>
</tr>
<tr>
<td>ft</td>
<td>m</td>
<td></td>
<td>× 0.3048</td>
</tr>
<tr>
<td>mL</td>
<td>cm³</td>
<td></td>
<td>× 1.0 *</td>
</tr>
<tr>
<td>mho</td>
<td>$S$</td>
<td></td>
<td>× 1.0 *</td>
</tr>
<tr>
<td>oz</td>
<td>g</td>
<td></td>
<td>× 2.83452 E+01</td>
</tr>
</tbody>
</table>

- Conversion factor is exact.

Salinity and Saturation Effects on Shaly Sandstone Conductivity

M. Argaud, Elf Aquitaine; H. Giouse, Gaz de France; and C. Staley, J. Tomanic, and K. Winkler, Schlumberger-Doll Research

ABSTRACT

Although a considerable amount of effort has been expended on the subject, the interpretation of shaly sandstone resistivities often proves problematic. Since the amount of high quality experimental data is somewhat limited, a joint research project was conducted to acquire more laboratory data and to evaluate the data in terms of new and existing theoretical models.

This project consisted of two sets of experiments:

• conductivity measurements on fully brine saturated rock samples investigating the effect of changes in salinity and/or type of cations;

• resistivity index measurements at decreasing water saturation and various salinities achieved by capillary drainage.

The second set of experiments will be presented in this paper. Supporting data consist of mercury porosimetry, SEM analysis, and QO as measured by chemical titration, membrane potential and CQ0 vs CW intercepts.

Our data on resistivity index show that commonly used shaly-sand models such as Waxman-Smits or Dual-Water have difficulty predicting saturation when clay conductivity is dominant. This can occur when clay content is high or when brine conductivity is low. Models that incorporate additional geometrical information about the conducting paths, such as the Schwartz-Sen model or the DC model of Giouse and Argaud, do a better job of predicting the experimental results.

INTRODUCTION

The interpretation of shaly sandstone resistivities has long been an important problem in log analysis. As a result, a large number of resistivity-salinity and resistivity-saturation formulas have been developed by log analysts. A good review is given by Worthington. Interpretation becomes particularly troublesome whenever clay conductivity becomes a significant portion of the total rock conductivity. This can occur in rocks having large clay content or in rocks having low brine conductivity. Since very few laboratory data exist relevant to these conditions, we present in this paper new laboratory measurements of resistivity index vs water saturation (F vs S0). We used a capillary equilibrium technique similar to that of Waxman and Thomas, except for the addition of a 4-terminal resistivity measurement. We used rocks having various clay contents and ran experiments using different brine salinities.

Rather than attempt to evaluate the 30+ models described by Worthington, we will instead focus upon the three most widely used conductivity models (Archie, Waxman-Smits, Dual-Water) and two new models recently proposed by Schwartz and Sen and by Giouse and Argaud (called the DC model). The WS and DW models are especially concerned with the electrochemical effects of clays, while the SS and DC models also include geometrical effects of the different conduction paths in a rock.

The well-known Archie expression generally applies only to clay-free sands and is given by:

\[ C_t = S_w^n \phi^m C_w \]  

(1)

where \( C_t \) is the rock conductivity, \( S_w \) is the water saturation, \( \phi \) is porosity and \( C_w \) is the brine conductivity. \( n \) and \( m \) are generally referred to as the saturation exponent and cementation exponents, respectively. \( m \) is usually defined by Archie's other expression:

\[ F = \frac{C_{w}}{C_0} = \phi ^{m} \]  

(2)

where \( F \) is the formation factor and \( C_0 \) is the rock conductivity when \( S_w=1 \).

The Waxman-Smits equation includes a correction for clay conductivity and is given by:
\[ C_s = S_w^m \Phi_m \left( C_w + \frac{BQ_v}{S_w} \right) \]  
where \( Q_v \) is the cation exchange capacity (CEC) per unit pore volume and B is the equivalent conductance of the clay counterions:

\[ B = 4.6 \left[ 1 - 0.6 e^{(C_m/1.3)} \right] \]  

The Dual-Water model has a more complicated clay correction term given by:

\[ C_t = S_w^m \Phi_m \left[ C_w \left( 1 - \frac{\alpha V Q_v}{S_w} \right) + \frac{BQ_v}{S_w} \right] \]  
where \( \beta \) is the conductivity of the clay bound water and \( \alpha V \) is the thickness of the bound water layer.

The Schwartz-Sen model assumes different tortuositities for the surface conduction path and the pore conduction path and arrives at:

\[ C_t = S_w^m \left[ C_w \left( 1 + \frac{1.93mQ_v}{S_w} \right) \right] + 1.3^m \Phi_m Q_v \]  
The constants in the SS model were derived by fitting a more general form of this equation (with \( S_w = 1 \)) to the Waxman-Smits data set.

The DC model was derived rather differently than the above models by interpreting the curvature often seen in \( I_x \) vs \( S_w \) data as being caused by the intersection of two line segments. The initial segment corresponds to desaturation of the connected macroporosity. The later segment corresponds to additional desaturation of the microporosity contained in clays, rough surfaces, etc. The two segments meet at a critical saturation termed \( S_{w1} \). Each saturation range is described by a different equation:

\[ \text{for } S_{w1} < S_w < 1 : \quad C_{lt} = S_w^m \Phi_m \left( C_w + C_e \right) \]  

\[ \text{for } S_w < S_{w1} : \quad C_{t2} = S_w^m \Phi_m \left( C_w + S_w^{1/2} C_e \right) \]  

where \( C_e = \text{excess conductivity associated with the clay conductor is equivalent to WS's } BQ_v \) term.

This model is developed further below.

**EXPERIMENTAL PROCEDURES**

Because the project involved two different laboratories, the procedures differ slightly. We therefore identify techniques and data with (E) indicating Elf and (S) indicating Schlumberger wherever appropriate.

We used the equilibrium capillary drainage technique with brine as the wetting fluid and oil as the displacing fluid. The oils were Soltrol 130 (E) and dodecane (S). The most important technical problems to consider are:

- The choice of the semi-permeable material to be set at the downstream face of the core. Either ceramics or cellulose membranes were used. They both have advantages and disadvantages. The main problem is to detect when they begin leaking at high capillary pressures.

The electrical resistivity measurements. Except for the very first measurements (E): samples M1.1, L3.2 which used 2-terminals, the electrical resistivities were measured using a 4-electrode system (AMNB) where current electrodes A and B are steel platinums contacting the end faces of the core, and potential electrodes M and N are silver (S) or Arap Ni-alloy (E) contacting the circumference of the core. The potential electrodes are molded into rubber or Vison sleeves that jacket the samples. Normally only the MN resistance is used for resistivity index computation. However, in some cases the AB values were used for comparison and detection of radial heterogeneity. In this case, the end faces of the sample were silver painted in order to minimize contact resistance.

- Capillary pressure maintenance and saturation determination. At (E), oil pressure was insured by a cascade of pressure air regulators pushing mercury and oil from a graduated glass tube into the sample. The water outlet was at atmospheric pressure in a constant level burette, with the usual precautions against evaporation (only metallic tubing, oil column above the water level, etc.). Water saturation was computed from the volume of brine expelled. At (S), a servo-controlled syringe pump is used to maintain capillary pressure as oil is injected into the sample. Brine output is collected as at (E). Saturation is measured both from the amount of brine collected and by measuring the volume of oil injected by the syringe. The overall duration of the experiment was typically two to three months.

- Saturation homogeneity and capillary pressure control. In contrast to routine porous plate \( I_x \) vs \( S_w \) measurements, the techniques used here allow the recording of data at arbitrary saturation intervals. However, not all of these points may be used to derive a resistivity/saturation relationship because of the possible saturation heterogeneity. If the saturation is not constant along a profile parallel to the centerlines, one gets an over-estimated value of Archie's \( n \) exponent, as has been recently shown by Lyle et al. If the saturation is not constant perpendicular to the centerlines, one gets an under-estimated \( n \) value. The latter case, which is rare, may occur in high permeability samples when the sample is horizontal due to gravity, or in strongly anisotropic rocks, where in the first stage of drainage oil invasion results in a cluster of invaded pores predominantly lying parallel to the sample's axis. A good example is given by sample GM2A*2 (Fig. 2). The first case is more common, due to static causes (a vertical, high permeability sample) or dynamic causes - because oil invasion takes place at an end face, the oil saturation profile shows, during the first capillary pressure step, a front moving towards the outlet. The symmetry in the saturation profile results in an excess \( n \) value. The effect is small in the case of low permeability samples where the front is not very steep and the saturation difference across the front is low (typically 100%-90%-10%), but is very important in high permeability samples where the front is more "piston-like" with typically \( S_w = 100%-65%-35\% \). Superimposed on that front effect at the first capillary pressure step is the effect of measuring the electrical resistance over only part of the sample, which produces a cross-over during that first step. Checking the electrical resistances of the AM, MN, NB slides allows real-time detection of the front's move, and no \( S_w \)-L pair should be kept for interpretation before resistance NB has increased roughly as much as AM. At the following capillary pressure steps, one might fear that again a front develops. However, numerical simulations (Argaud) show there is no front propagation. The oil saturation increases almost homogeneously through the core, except close to the downstream face.

In brief, the first part of the \( I_x \) vs \( S_w \) curve is questionable down to typically \( S_w = 90\% \) in low permeability samples, and to 65% in high permeability samples by undesired effects that prohibit the use of the experimental data in that zone for interpretation.

The use on XCT or NMR tomography to check the saturation homogeneity is highly desirable.
Defining the \( R_0 \) value. After saturating a core, and before starting a desaturation experiment, it is necessary to circulate some pore volumes of brine through the core and to wait for equilibrium between brine and clay minerals, exactly as for the \( C_0 \) vs \( C_w \) experiments. Failure to do so results in a wrong \( R_0 \) reference value needed to compute the resistivity index and a vertical shift in the curve (the extrapolated \( I_r \) value is not equal to unity at \( S_w=1 \)).

On all the figures of \( S_w \) vs \( I_r \) results, we prefer to show the actual experimental points, rather than the smooth curves presented by many investigators.

**Resistivity Index Results**

**Derivation of a New Formula**

The curves for samples M1, L3 and CON5 show at some intermediate saturation value a change (decrease) in slope not accounted for by either the WS or DW models. This limiting saturation can often be related to inflections in the mercury porosimetry curves. Decreases in the slope have previously been reported by Diedrich et al.\(^1\) who associated them with a rough coating of clays at the grain surfaces (in parallel with the free pore space), and by Swanson\(^1\), who explained them by the sequential desaturation of microporous chert clumps in series with the main pore system. In fact (Argaud\(^2\)), there is a symmetry in the heterogeneity effects associated with the sequential desaturation of two sub-media characterized by different capillary pressure curves (having thus different \( S_w \) values for any given \( P_L \) value). This symmetry is double, concerning either the type of grouping of the two sub-media (in series or in parallel) or the type of desaturation event (starting to intrude or stopping intrusion). Thus qualitatively, a negative bending (slope decrease) of the resistivity index curve may be related to either starting intrusion in a finer grained medium in series with the coarser medium which has already experienced desaturation (as proposed by Swanson\(^1\)) or by stopping desaturation in one of two sub-media associated in parallel and having started desaturation together. We believe that the latter explanation is the real one for shaly sandstone, as it also accounts for the higher "irreducible" saturations often associated with them.

Following an approach by Worthington & Pollall\(^1\), we modify WS to make tortuosity apparent:

\[
C_{\phi} = \frac{C_w S_w^n + C_\phi}{\nu S_w^n + \nu S_w^{n-1}}
\]

which has the following meaning. The free pore conductor volume fraction has dropped by \( S_w \) and its tortuosity has been increased from \( t \) at \( S_w=1 \) up to \( \nu S_w^{n-1} \) at \( S_w=1 \). The clay conductor has suffered no volume reduction and its tortuosity has increased by the same amount as that of the first conductor.

Now, let us assume that desaturation takes place sequentially, in two stages:

1. **Stage 1:** \( S_w < S_w < 1 \): down to some limiting saturation \( S_w \), both conductors undergo the same changes in conducting volume and tortuosity. This leads to a first stage formula:

\[
C_{\phi} = \frac{C_w S_w^n + C_\phi}{\nu S_w^n + \nu S_w^{n-1}}
\]

or alternatively:

\[
C_{\phi} = \frac{S_w^n (C_w + C_\phi) = S_w^n C_0}{F}
\]

that is, a true Archie behavior (constant \( n \), no clay-effect). For \( S_w=1 \), it reduces to:

\[
C_0 = \frac{(C_w + C_\phi) F}{S_w^n}
\]

2. **Stage 2:** \( S_w < S_w \): desaturation now leaves the clay conductor unaffected in both associated water content and tortuosity. Its conductivity term keeps a constant value, the one reached at \( S_w=S_w \), i.e. \( BQ_0 S_w^{n-1}/F \). The free pore conductor still undergoes desaturation, its conductivity term keeps decreasing in the same way as in Eq.(7). As a result, the total conductivity now becomes:

\[
C_{\phi} = \frac{C_w S_w^n + C_\phi S_w^{n-1} S_w}{F}
\]

This two-domain formula, initially introduced by Gioussi\(^2\) ("DD" model) with a fixed \( n \) value (independent of brine conductivity, depending solely on the pore structure) accounts for the change in slope at \( S_w=S_w \). It is generalized into the "DC" model (double conductor) to include the effect of varying \( n \) with brine salinity. In practice, mercury porosimetry can be used to estimate \( S_w \).

**Comparing the Models to Data**

To test the WS, DW and SS models we used the "chemical" (CEC-derived) \( Q_w \), since the models are based upon that measure. We did not try to fit the curves showing a positive departure between the measured and predicted values. The WS and DW models can be rewritten as a single formula for curve-fitting:

\[
I_r = \frac{S_w^n}{1 - \frac{1 + \frac{a}{S_w}}{1 + \frac{a}{S_w}}}
\]

with one adjustable parameter \( a \), and a dimensionless parameter \( n \) derived from \( Q_w \), \( C_w \) and counter-ion mobilities:

\[
WS: n = n - a \frac{BQ_0}{C_w}, \quad DW: n = n - \frac{a (BQ_0/C_w) - \sigma e \sigma Q_w}{Q_w}
\]

In the SS model, the cementation exponent \( m \) was determined from the slope of \( C_0 \) vs \( C_w \) and \( \phi \). Here again there is one adjustable parameter \( m \) when fitting the \( S_w \) vs \( I_r \) data. To test the ability of the WS, DW and SS models to account for changes in brine salinity (or equivalently changes in clay content), we used the high salinity data to determine \( n \) by non-linear regression. This \( n \) value was then used to compute the theoretical curve at low salinity (except for L3, for which the two salinities were very close to each other; here we fit the data sets independently).

For the DC model, one rewrites Eq. 7 and 8 as:

\[
S_w < S_w < 1: \quad I_r = S_w^n \quad \text{\quad (15)}
\]

\[
S_w < S_w \text{\quad (16)}
\]

Here there are three adjustable parameters : \( n \), \( S_w \) and the dimensionless parameter \( a = C_\phi / C_w \). In all three models (WS, DW, DC) the dimensionless parameter \( a \) appears with the same physical meaning - the conductivity ratio between surface conductivity and "free" water conductivity.
For all the models, non-linear regression was used over the $S_w$ saturation range where inhomogeneity effects were absent. The quality of the fit is expressed by the standard deviation $\delta$. Results are given in Table 2. When the parameter appears in parentheses, its value has been derived from other information (such as measured $m$ or $Q_c$). Otherwise the parameter value is derived from the non-linear regression.

Comparing the fits of the various models to the experimental data, it is clear that at high salinities, all of the models do a good job of fitting the data. However, at low salinities and low saturations, where surface conductivity dominates, only the SS and DC models do a good job of matching the data. The SS model requires the same inputs as do WS and DW, and can therefore be used wherever those models are currently being used.

The DC model matches the curves especially well. This is not surprising, since it is the only one that explicitly incorporates a slope change, and because three parameters were adjustable at each salinity without forcing a correspondence between high- and low-salinity parameters. When the salinity changes from high to low values, the $\alpha$ parameter increases as expected, except for M1 (probably due to the poorer quality of the data of M1*.1, a two electrode measurement) and AUS12 (experimental problem at $S_w$=0.3). Interestingly also, the n value is observed to decrease slightly, while, at least for good quality data, $S_w$ does not appear to change significantly. This could be salinity dependent appears at first sight surprising, and deserves additional theoretical work beyond the scope of this paper.

Two samples, GM10 and T1, show a positive departure from an Archie line. This is thought to be due to the presence of clays as isolated bunches instead of as continuous coverage, and of numerous smooth planar grain surfaces (quartz overgrowths).

CONCLUSIONS

1. A two-conductor formula has been proposed to explain resistivity index/saturation results observed in laboratory experiments and to account for the change of slope of the curves. During the first stage of desaturation, the two conductors (free pore space and clay-volume) are affected in the same way by desaturation, and a typical Archie's law results. At lower saturations, only the free pore space keeps desaturating, while the clay-volume remains unchanged. The associated "bound-water" is not only the diffuse layer in the sense of the DW model, but also incorporates capillary-held water associated with the clays that for some reason remains trapped. Thus, the conductivity term may be larger than the $BQ_c$ or $BQ_c/(1-o_{cl}Q_c)$ terms of the WS and DW models respectively. The limit between the two saturation domains may be roughly anticipated from the mercury porosimetry curve.

2. When repeating the resistivity index/desorption experiments with lower salinity brines, the n exponent may be lower, resulting from higher trapping of residual water associated with the clay conductor, and a higher connectivity of this conductor due to structural factors (clay swelling).

3. The so-called "clay-effect" (negative bending of the $I_p$ vs $S_w$ curve at low saturation) appears as a particular aspect of a more general problem - the transition from dominant pore space conduction to dominant grain surface conduction as saturation decreases. The latter may be seen as a competition between residual water connectedness, which is favored by continuous surface rugosity often but not always linked with clay minerals, and disconnectedness, which is favored by clean, smooth grain surfaces.

4. At high salinities, all of the models tested (WS, DW, SS, DC) do a good job of fitting the data. However, at low salinities and low saturations, where surface conductivity dominates, only the SS and DC models do a good job of matching the data. Both of these models incorporate geometrical effects of the surface conductor that become significant when surface conductivity dominates over free pore fluid conductivity.

REFERENCES


<table>
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<tr>
<th>Sample</th>
<th>Origin</th>
<th>ϕ</th>
<th>K (md)</th>
<th>$Ω_a$ (mec/ml)</th>
<th>Matrix</th>
<th>Clays</th>
</tr>
</thead>
<tbody>
<tr>
<td>L3</td>
<td>Rotilegend (The Netherlands)</td>
<td>0.122</td>
<td>2.82</td>
<td>0.039</td>
<td>Medium-grained sand, dominant quartz and dolomite cement, small neo-formed quartz crystals as pore-filling</td>
<td>Shredded illite as discontinuous pore-lining (1-5 μm) and books of kaolinite (3-10 μm).</td>
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<tr>
<td>M1</td>
<td>Cretaceous (offshore Congo)</td>
<td>0.165</td>
<td>2.71</td>
<td>0.163</td>
<td>Fine-grained (50-80 μm) micaceous and feldspathic sand; small neo-formed quartz grains</td>
<td>Pore-lining chlorite (1-5 μm) as important grain surface coverage. Bunches of illite on weathered feldspars.</td>
</tr>
<tr>
<td>T1</td>
<td>Cretaceous (offshore Gabon)</td>
<td>0.219</td>
<td>203</td>
<td>0.00 to 0.01</td>
<td>Medium-grained (60-150 μm) sand, dominant quartz, very little mica or feldspar, most grains show overgrowths with smooth planar surfaces</td>
<td>Isolated chlorite platelet bunches or books of kaolinite.</td>
</tr>
<tr>
<td>GM2A</td>
<td>Permian quarry rock (NE France)</td>
<td>0.210</td>
<td>114</td>
<td>0.193</td>
<td>Fine-grained (90-100 μm), micaceous and feldspathic sand. Quartz grains occasionally with smooth planar surfaces</td>
<td>Dominant illite as needles or shredded plates (1-3 μm) occasionally as continuous coverage; kaolinite fans (10 μm).</td>
</tr>
<tr>
<td>GM10</td>
<td>Permian quarry rock (NE France)</td>
<td>0.245</td>
<td>821</td>
<td>0.097</td>
<td>Medium-grained (100-150 μm) micaceous and feldspathic sand; quartz grains with smooth planar surfaces</td>
<td>Dominant kaolinite literally clogging some pores only (10-20 μm) and illite (shredded plates or needles (2-10 μm) only on weathered micas or feldspars.</td>
</tr>
<tr>
<td>CON5</td>
<td>N/A</td>
<td>0.149</td>
<td>7</td>
<td>0.25</td>
<td>Medium-grained (~100 μm) micaceous sand.</td>
<td>Considerable clay filling of kaolinite books and illite needles.</td>
</tr>
<tr>
<td>AUS12</td>
<td>N/A</td>
<td>0.200</td>
<td>678</td>
<td>0.07</td>
<td>Coarse-grained (200-400 μm) clean quartz sand; generally smooth grain surfaces</td>
<td>A few isolated patches of kaolinite books.</td>
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</tbody>
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### Table 2. Input parameters for resistivity index curves

<table>
<thead>
<tr>
<th>Case</th>
<th>$C_w$ (S/m)</th>
<th>$S_w$ range fitted (# of points fit)</th>
<th>$S_w$</th>
<th>model</th>
<th>Model parameters: $n$ = parameter fixed from measured $Q_i$, else parameter derived from regression fit</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>L3*1</td>
<td>2.824</td>
<td>0.46-0.77 (16)</td>
<td>0.051</td>
<td>WS</td>
<td>$(a=0.0765)$, $(m=1.996)$, $(n=1.539)$, $(s=0.1937)$, $(m=1.725)$</td>
<td>$n$=1.759, $S_w$=1.759, no inference from high salinity fit</td>
</tr>
<tr>
<td>L3*2</td>
<td>3.304</td>
<td>0.41-0.85 (22)</td>
<td>0.135</td>
<td>WS</td>
<td>$(a=0.0628)$, $(m=1.550)$, $(s=0.0117)$, $(m=1.186)$</td>
<td>$n$=1.486, $S_w$=1.756, for WS, DW and SS, $n$ derived from high salinity fit</td>
</tr>
<tr>
<td>M1*1</td>
<td>3.692</td>
<td>0.45-0.89 (35)</td>
<td>0.024</td>
<td>WS</td>
<td>$(a=0.2565)$, $(m=1.914)$, $(s=0.2373)$, $(m=2.195)$</td>
<td>$n$=1.115, $S_w$=1.825, for WS, DW and SS, $n$ derived from high salinity fit</td>
</tr>
<tr>
<td>M1*2</td>
<td>11.986</td>
<td>0.407-0.935 (20)</td>
<td>0.099</td>
<td>WS</td>
<td>$(a=0.0819)$, $(m=1.914)$, $(s=0.0117)$, $(m=2.195)$</td>
<td>$n$=1.115, $S_w$=1.825, for WS, DW and SS, $n$ derived from high salinity fit</td>
</tr>
<tr>
<td>T1</td>
<td>2.824</td>
<td>0.33-0.79 (15)</td>
<td>0.061</td>
<td>Archie</td>
<td>$(a=0.9794)$, $(m=1.920)$, $(s=0.1085)$, $(m=1.683)$</td>
<td>$n$=1.895, $S_w$=0.682, for WS, DW and SS, $n$ derived from high salinity fit</td>
</tr>
<tr>
<td>CON5*1</td>
<td>3.23</td>
<td>0.35-0.83 (127)</td>
<td>0.228</td>
<td>WS</td>
<td>$(a=0.9774)$, $(m=1.920)$, $(s=0.1085)$, $(m=1.683)$</td>
<td>$n$=1.895, $S_w$=0.682, for WS, DW and SS, $n$ derived from high salinity fit</td>
</tr>
<tr>
<td>CON5*2</td>
<td>0.354</td>
<td>0.28-0.87 (110)</td>
<td>0.014</td>
<td>WS</td>
<td>$(a=0.9774)$, $(m=1.920)$, $(s=0.1085)$, $(m=1.683)$</td>
<td>$n$=1.895, $S_w$=0.682, for WS, DW and SS, $n$ derived from high salinity fit</td>
</tr>
<tr>
<td>AUS12*1</td>
<td>3.23</td>
<td>0.20-0.60 (151)</td>
<td>0.259</td>
<td>WS</td>
<td>$(a=0.1113)$, $(m=1.793)$, $(s=0.0304)$, $(m=1.660)$</td>
<td>$n$=1.629, $S_w$=0.239, for WS, DW and SS, $n$ derived from high salinity fit</td>
</tr>
<tr>
<td>AUS12*2</td>
<td>0.354</td>
<td>0.333-0.70 (135)</td>
<td>0.028</td>
<td>WS</td>
<td>$(a=0.3533)$, $(m=1.502)$, $(s=0.3533)$, $(m=1.800)$</td>
<td>$n$=1.787, $S_w$=0.515, for WS, DW and SS, $n$ derived from high salinity fit</td>
</tr>
<tr>
<td>GM10*1</td>
<td>4.562</td>
<td>0.29-0.60 (21)</td>
<td>0.089</td>
<td>Archie</td>
<td>$(a=0.2495)$, $(m=1.784)$, $(s=0.0494)$, $(m=1.528)$</td>
<td>$n$=1.548, $S_w$=0.256, for WS, DW and SS, $n$ derived from high salinity fit</td>
</tr>
<tr>
<td>GM2A*1</td>
<td>4.562</td>
<td>0.18-0.6 (86)</td>
<td>0.096</td>
<td>WS</td>
<td>$(a=0.2495)$, $(m=1.784)$, $(s=0.0494)$, $(m=1.528)$</td>
<td>$n$=1.548, $S_w$=0.256, for WS, DW and SS, $n$ derived from high salinity fit</td>
</tr>
<tr>
<td>GM2A*2</td>
<td>1.079</td>
<td>0.22-0.49 (97)</td>
<td>0.028</td>
<td>WS</td>
<td>$(a=0.7931)$, $(m=1.784)$, $(s=0.3474)$, $(m=1.528)$</td>
<td>$n$=1.548, $S_w$=0.256, for WS, DW and SS, $n$ derived from high salinity fit</td>
</tr>
</tbody>
</table>
Fig 1 - Resistivity Index curve GM2A+1
C_w = 4.562 S/m

Fig 2 - Resistivity Index curve GM2A+2
C_w = 1.079 S/m
Fig 3 - Resistivity Index curve L3*1 * Cw= 2.824 S/m

Fig 4 - Resistivity Index curve L3*2 * Cw= 3.304 S/m
Fig 5 - Resistivity Index curve M1*1
Cw = 3.692 S/m

Fig 6 - Resistivity Index curve M1*2
Cw = 11.986 S/m
Fig 7 - Resistivity Index curve CON5*2
Cw = 0.354 S/m

Fig 8 - Resistivity Index curve CON5*1
Cw = 3.230 S/m
Fig 9 – Resistivity Index curve AUS12*2
Cw = 0.354 S/m

Fig 10 – Resistivity Index curve AUS12*1
Cw = 3.230 S/m