Decline-Curve Analysis Using Type Curves—Case Histories

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Summary. Case-history studies demonstrate methods of analyzing rate-time data to determine reservoir variables and to predict future production. Constant-wellbore-pressure analysis techniques use existing \( q_{at}^{-1} t_{at} \) type curves and new \( q_{at}^{-1} t_{at} \) type curves from actual field data.

Introduction

Since Fetkovich’s \(^1\) original presentation in 1973, many successful applications have been made with declining rate-time data using the type-curve approach. Case-history studies of individual oil and gas wells, groups of wells in a field, and total fields are presented in this follow-up paper. Additional papers \(^2\)–\(^7\) dealing with the constant-wellbore-pressure solution (which also include the depletion period) have since been published to aid analysis and understanding of what is now called “advanced decline-curve analysis.”

In essence, decline-curve analysis is a forecasting technique: rate-time data are history-matched on an appropriate type curve, and then a forecast is made. Complex simulation studies proceed similarly. By using basic reservoir engineering concepts and knowledge, we know what direction to take, what type curve(s) to choose, and where the rate-time data should fit.

Decline-curve analysis must work because it is founded on basic fluid-flow principles—the same principles used in pressure-transient analysis. The problem most engineers have had and will continue to have with decline-curve analysis is bad, erratic, or insufficient data. Careful attention to obtaining accurate flow rates, flowing pressures, and downtime should help solve the problem. A good rate-time analysis not only will give the same results as conventional pressure-transient analysis, but also will allow a forecast to be made directly at no cost in lost production. For low-permeability stimulated wells, in particular, pressure-buildup testing could be eliminated in many cases as being of little value or economically unjustifiable because of the resulting production loss when compared with what can be obtained from properly conducted constant-wellbore-pressure drawdown tests.

Rate-Time Type-Curve Analysis Concepts

The Radial Flow Solution. The fundamental basis of advanced decline-curve analysis is an understanding of the constant-wellbore-pressure solutions and their corresponding log-log type-curve plots, which are the inverse of the constant-rate solution. Fig. 1 is a composite of the analytic constant-wellbore-pressure solution and the Arps \(^8\) exponential, hyperbolic, and harmonic decline-curve solutions on a single dimensionless type curve. The depletion stem values of \( b \) range between 0 (exponential) and 1 (harmonic), which are the normally accepted limits. The exponential-depletion stem (\( b = 0 \)) is common to the analytic solution and to the Arps equation.

Decline-curve dimensionless rate and dimensionless time in terms of reservoir variables are defined for the type curve as

\[
q_{at} = q_D \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{1}{2} \right] \quad \text{(1)}
\]

or

\[
q_{at} = \frac{q(t)}{kht(p_i - p_w)} \quad \text{or} \quad 141.2 \mu B \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{1}{2} \right]
\]

and

\[
t_{at} = \frac{0.00634k}{\phi(\mu_{eP})} \frac{1}{r_w^2} \left[ \frac{r_e^2}{r_w^2} - 1 \right] \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{1}{2} \right] \quad \text{(3)}
\]

or

\[
t_{at} = \frac{t_D}{\frac{r_e^2}{r_w^2} - 1} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{1}{2} \right] \quad \text{(4)}
\]

Published values of \( q_D \) and \( t_D \) for the infinite and finite constant-pressure solutions for single-phase radial flow were transformed into a defined decline-curve dimensionless rate and time, \( q_{at} \) and \( t_{at} \), by Eqs. 1 and 4. The values in Fig. 2 were used to generate Fig. 3, which is a plot of the decline-curve dimensionless rate and time, \( q_{at} \) and \( t_{at} \), for various values of \( r_e/r_w \), down to 0.1. The constant ½ was used in the final equations with \( p_i \) (after ½, ¾, and ¾ were tried), because a better correlation was obtained, particularly at small \( r_e/r_w \) stems; the constant-pressure-outerbounary case during the transient period also overlies the closed-outer-boundary-case type curve.

The plotted type curves (Figs. 1 and 3) generated from the exact \( q_{at}^{-1} t_{at} \) constant-wellbore-pressure solution are also exact by definition, although they were generated with the ½ value. The curves cannot be used by simply changing ½ to ¾. One can only back-calculate the correct \( q_{at}^{-1} t_{at} \) from these curves with the value of ½. The \( r_e/r_w \) stems were discontinued at a value of 10 because the correlation begins to break down as linear instead of radial flow develops—i.e., as \( r_e \) approaches \( r_w \).

In Fig. 1, note that a \( t_{at} \) between 0.2 and 0.3 separates the transient period from the depletion period. Fitting rate-time data to the Arps equation is valid only when depletion sets in and the transient period is over. If flowing pressures are available and are not reasonably constant but smooth and monotonically decreasing, the pressure-normalized rate, \( \log q/\Delta p \) vs. \( \log t \), should be used for analysis.

Rapidly declining rate data fitting the early transient \( r_e/r_w \) stems are characteristic of low-permeability stimulated wells and often result in a unique fit. Stimulation causes the rate data to appear
on a small $r_e/r_w$ stem, and the low permeability then allows them to remain on the stem for real-time periods. Data for high-permeability stimulated wells leave the transient stems and go to pseudosteady state almost immediately. Conversely, a well with a large positive skin producing at a truly constant wellbore pressure will yield very flat rate declines, indistinguishable from $b=0$ to $\infty$, and will also look like a constant-rate situation. Just as we make a log $\Delta P$-log $\Delta t$ type-curve plot to find the semilog straight line in pressure-buildup analysis, in decline-curve analysis we must make a log $q$-log $\Delta t$ type-curve plot of rate-time data to see whether the data are transient.

With regard to the $r_e/r_w$ transient stems, we will repeat a statement from the original paper: "Note from the composite curve [Fig. 1] that rate data existing only in the transient period of the constant terminal pressure solution, if analyzed by the empirical Arps approach, would require values of $b$ much greater than 1 to fit the data." The principal objective of that paper was the development of Fig. 1, which provided a method of analysis for transient data. Transient data should not be interpreted by the Arps equation.

Fig. 4 illustrates the effect of transposing the $r_e/r_w$ stem of 10, indicative of a low-permeability stimulated well response, and the $r_e/r_w$ stem of 10,000, indicative of a large positive skin or damaged well, to the depletion stage where the Arps equation is applicable. An equivalent Arps $b=10$ approximates the $r_e/r_w$ stem of 10,000; a $b=3$ approximates the $r_e/r_w$ stem of 10. They appear equivalent and on log-log type-curve matching would be indistinguishable if the Arps exponent $b$ were left unbounded. The same data, if fit on the transient portion to the left of $t_{dd}=0.2$ and then extrapolated, must ultimately go down a depletion stem. The same data fit to an Arps equation with $b>1$ will extrapolate to infinity with no rational basis of terminating the forecast. The exponent $b$ must be bound between 0 and 1.

If we rearrange Eq. 2, we can evaluate the productivity factor from the $q_{dd}-q(t)$ match point:

$$
\frac{kb}{\ln \left( \frac{r_e}{r_w} \right) - \frac{1}{2}} = \left[ \frac{141.2 \mu B}{P_1 - P_{wf}} \right] \left[ \frac{q(t)}{q_{dd}} \right]
$$

where $r_{w}^* = r_w e^{-z}$. The skin term can also include the effect of the shape factor $C_d$ (see Ref. 9).

Assuming that $(r_e/r_w)^2$ is large compared with 1 in the term $[(r_e/r_w)^2 - 1]$, reintroducing thickness, $h$, in the $t_{dd}$ equation, Eq. 3, thus $(kh/\phi h)$, and substituting Eq. 5 into Eq. 3, we can obtain the following equation in terms of the match point $q_{dd}-q(t)$ and $t_{dd}$:

$$
V_p = \pi r_e^2 h_o = \left[ \frac{\mu B}{(\mu c_v)(P_1 - P_{wf})} \right] \left( \frac{t}{t_{dd}} \right) \left( \frac{q(t)}{q_{dd}} \right)
$$

This equation gives the PV at the start of the decline analysis.
It must be pointed out that Eq. 6 is valid only for closed-outerboundary situations when the onset of depletion is indicated by the data showing evidence of starting down one of the depletion stems. In the case of water-drive reservoirs, there might be a sufficient delay in aquifer movement to detect depletion, which could then be evaluated. Transient data alone with no indication of depletion are not unique on the $q_{D}/q_{PD}$ type curve of Fig. 1 or 3. Data only in the transient stage could fit on every stem. This is more easily seen if transient data were fit only on the $q_{D}/t_{D}$ type curve (Fig. 2) to the left of $r_{c}/r_{w}=10$. Clearly, this portion of the curve is common to all $r_{c}/r_{w}$ depletion stems from 10 to infinity.

**Single-Vertical-Fracture Solution.** The 1975 Locke and Sawyer constant-wellbore-pressure, infinite-conductivity, vertical-fracture solution type curve (Fig. 5) begins, with regard to depletion stems, where the radial-flow solutions illustrated in Figs. 1 through 3 leave off. In terms of effective wellbore radius ($r_{wa}=L_{x}/2$), $r_{c}/r_{wa}$ of 10 approximately equals $L_{x}/L_{x}=5$, where the single-vertical-fracture solution more closely represents the physical situation.

With improved stimulation techniques, hydraulic fracture lengths, $L_{x}$, can and do start approaching $L_{x}$ for 5- and 10-acre [2- and 4-ha] spacings—i.e., $L_{x}/L_{x}=1$. The $r_{c}/r_{wa}$ stems could easily be extended to include values less than 10 with little loss in the type-curve evaluation accuracy; see Ref. 10 for $q_{D}/t_{D}$ values of $r_{c}/r_{w}$ less than 10.

In our experience, the basic type curves of Figs. 1 through 3 and 5 have solved most of our decline-curve analysis problems.

With regard to the early transient period, the dashed line of Fig. 6 illustrates the infinite-conductivity, vertical-fracture solution expressed in terms of $r_{wa}=L_{x}/2$—i.e., $t_{FD}$ converted to $t_{wa}$ by the following equation:

$$t_{waD}=t_{FD}$$

Rate-time data for a stimulated well can be matched and forecast on either the infinite-conductivity, vertical-fracture solution or the plane-radial-flow solution with $r_{wa}$ or effective wellbore radius for skin, with little practical difference in the resulting forecast. A rapid decline in rate with time usually identifies a stimulated low-permeability well. Data fit on a single-vertical-fracture solution do not identify fracture volume depletion or a naturally fractured reservoir. Whether or not an induced stimulation fracture is propagated down a natural fracture is irrelevant in identifying a naturally fractured system.

**Fig. 6—Comparison of dimensionless flow rate for plane radial flow and infinite-conductivity, vertical-fracture, constant-pressure solutions.**

**Fig. 7—Comparison of natural fracture and single-vertical-fracture models: (a) intense natural fracturing and matrix acid effect; (b) induced single vertical fracture from acid fracture.**
Fig. 8—\(q_o\) vs. \(t_d\) for constant-pressure production (after Da Prat et al.)*.

The question of whether naturally fractured reservoirs can initially have negative values of skin without stimulation needs to be addressed because the rapid decay in rate resulting from a negative skin effect can be and has been misinterpreted as identifying a naturally fractured reservoir. We will discuss this again later with examples. If we examine Fig. 7a, representing an intensely naturally fractured reservoir (similar to the Warren and Root* model), we see from the definition of skin that for a well drilled in a naturally fractured reservoir to have a negative skin, it must penetrate a region near the wellbore that has a permeability greater than that in the interwell region. The likelihood is remote that we could be so fortunate every time we drill a well in a naturally fractured hydrocarbon reservoir. Cutting a single vertical natural fracture with a vertical or subvertical well is equally remote. Wells drilled in a naturally fractured reservoir will initially have large positive skins because of heavy mud losses around the wellbore into the natural fractures. In an intensely naturally fractured limestone reservoir, an acid treatment generally removes the mud damage and results in good negative skins. Pressure-transient data obtained after such a stimulation fit the van Everdingen and Meyer* matrix-acid solution (a situation where the permeability near the wellbore is truly altered) as opposed to the single-vertical-fracture solution based on the model shown in Fig. 7b. In the case of the Greater Ekofisk development, the type-curve characteristics are noticeably different after stimulation. Data fit to the single-vertical-fracture solution usually result in a low fracture intensity index (FI), whereas data fit to the matrix-acid solution result in a high FI. The fracture volume associated with Fig. 7a will be connected to the wellbore; in contrast, any fracture volume associated with Fig. 7b may not be connected. In our experience, wells have never obtained a negative skin in a naturally fractured reservoir except after a stimulation treatment.

Naturally Fractured Reservoir (Warren and Root Model) Type Curves. Dual-porosity or naturally-fractured-reservoir type curves developed by Da Prat et al.* were a significant and timely contribution to decline-curve analysis concepts. The unsupported statement “fracture depletion” with rapidly declining rate-time data is widely used. Careless use of the word “fractured” when dealing with hydraulically fractured wells and the corresponding rapid decline in rate associated with these successful fracture jobs have helped perpetuate the fracture-volume-depletion myth.

On the basis of the Da Prat et al. naturally fractured (Warren and Root model), dual-porosity, constant-wellbore-pressure type curves, the only identifying characteristic is the double-exponential decline: depletion of the fracture volume followed by depletion of the matrix volume. Segments A to B in Fig. 8 represent fracture depletion, and segments B to C represent matrix depletion. This behavior is not equivalent to the two parallel straight lines from the constant-rate solution of Warren and Root* because theirs was an infinite-reservoir solution. Two semilog force-fits (double exponential) of early-time data can be and have been manufactured by unsuspecting engineers trying to smooth the rate-time data. There is no indication from the dual-porosity type curves of a \(b > 1\) any-

where, except again in the transient portion, which is identical in character to the homogeneous-reservoir solution. The transient portion can yield only apparent values of \(b > 1\). Also, the matrix-depletion stream can be so flat as to be easily misinterpreted as having a \(b\) much greater than 1 (see the displaced \(r_e/r_w\) stem of 10,000 in Fig. 4). Gas saturations within the matrix block for solution-gas-drive systems along \(w_d\) end effects (little or no oil flow from the matrix blocks) would appear as large positive skins with respect to oil flow, further creating long, flat, transient \(r_e/r_w\) stems. Simulation or removal of matrix skins is not possible.

Log-Log Decline-Curve Plot. If we closely examine Eqs. 2 and 3 expressing \(q_{AD}t_{AD}\) and consider the nature of the log \(q_{AD}\) vs. \(t_{AD}\) plot, we should recognize that real rate-time data, in any convenient units, when plotted as \(q_{AD} vs. t_{AD}\) can look exactly like one of the \(q_{AD}t_{AD}\) type-curve plots previously discussed. The data in rate and time will be shifted from the unit solution only by the coefficient of \(q\) and \(t\) in \(q_{AD}\) and \(t_{AD}\); respectively. Some basic reservoir knowledge usually suggests which type curve, and where on the curve, we should expect to obtain a match. By overlaying ratetime data on Fig. 1 or 3, for example, we can obtain a match of \(q_{AD}\), \(t_{AD}\), \(r_e/r_w\), \(s\), \(r_e\), or \(Pv\). In a given field, all wells should normally be expected to match the same depletion type curve, although skins could be different; the axis will be shifted in time and rate for each well only by the coefficient of \(q_{AD}\) and \(t_{AD}\). For \(q_{AD}\), the coefficient is

\[
\frac{141.2\mu B}{kh(p_i-p_{w})} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{1}{2} \right]
\]

and for \(t_{AD}\), the coefficient is

\[
\frac{0.0063k}{\phi(\mu c_i)r_w} \left[ \frac{1}{2} \left( \frac{r_e}{r_w} \right)^2 - 1 \right] \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{1}{2} \right]
\]

The overlaying technique is a fundamental concept that leads to the idea of developing a field type curve from rate-time data alone. The type curve so developed may or may not appear anything like an existing solution. Fig. 9 is the log \(q\)-log \(t\) plots of Wells A through D in the same field; the match results are given in Table 3 of Ref. 1. First, note that the data from the four wells overlay each other and have developed a single log-log type curve at least to the range
of existing data. Essentially, the shift in the rate axis reflects a different \( k_h \), while the shift in the time axis reflects a different \( k \) and \( r_w \). We could match and have matched this curve to a known analytic solution (Fig. 3). By knowing or estimating reservoir variables from some source, such as a complete buildup analysis or early-time rate-time decline analysis on at least one of the wells, we can back-calculate values of \( q_{dB} \) and \( t_{DB} \). Slight differences in \( r_w/r_w^0 \) stems appear to be reflected in the early transient period. The significance of the collapsed \( q_{dB}^{-1}t_{DB} \) plot (Fig. 3) is illustrated by this example.

The next logical step in the use of type curve and log-log plot concepts is the development of the total field type curve. Total field production can be considered as an “average well” times the number of wells. Wells overlap wells within a field with the same drive mechanism, so why shouldn’t field rate-time production data from the same formation with similar drive mechanisms overlaps other fields in the same formation? The reservoir and fluid variables \( [k, h, r_w (spacing), r_{w0}], \mu_o, B_o, P_i, P_{pwf}, \phi, s_w, \text{and } c_j \) can be different for all fields. This concept could be demonstrated with the development of the Monterey type curve from historical production data from several Monterey fields in California.

### Arps Limits of \( b \)

The Arps stems in Fig. 1, \( b = 0 \) to 1, combined with the analytic transient stems, deserve some discussion. First, the data used by Arps to develop and test his original equations were from real fields and wells. They indicate that real-world data most often do not follow the single-phase analytic solution for depletion, the \( b = 0 \) solution. The limits to \( b \) that he found by use of Cutler’s data were between 0 and 0.7, with over 90% of the cases having values less than 0.5: no case was found with a \( b \) in excess of 0.7. Arps’ own experience, however, indicated that \( b = 1 \) did occur, but only rarely.

If we consider the initial declining-rate period as nothing more than an extended drawdown test, then matching the early-time data on the rate-time type curve for reservoir parameter evaluation yields initial permeability, \( k_i \), and \( (\mu c_o) \), at \( t = 0 \), the start of the decline analysis. A value of \( b > 0 \) reflects changing values of \( (k_o/\mu_o B_o)_{p} \) and \( (\mu c_o)_{p} \), during reservoir depletion. For a given drive mechanism, \( k_i \), \( (\mu c_o) \), and \( b \) should be sufficient to describe a type curve for a given field or formation. Theoretically, an oil pseudo-pressure, \( P_{ps} \), and a pseudodimensionless time, \( t_{pDL} \), could be developed from a history-matched \( k_m = k_{m0} \) relationship to drive the rate-time data to the analytic solution \( b = 0 \); however, this is too complex a procedure and will find little practical use. Nevertheless, the pursuit may be worthwhile, perhaps leading to a better understanding of what causes the different values of \( b \).

Carter’s study of the effect of pressure level and drawdown on gas well rate-time behavior provides some insight into what causes \( b \) values to be greater than 0. Examination of his Fig. 1 shows that the early transient period is unaffected by a variation in \( (\mu c)_p \), while the depletion stems increase from \( b = 0 \) for \( \lambda = 1 \) to \( b = 0.3 \) for \( \lambda = 0.75 \), and \( b = 0.5 \) for \( \lambda = 0.55 \). Carter’s \( \lambda \) is defined as

\[
\lambda = \frac{\mu (p_i) c_o (p_i)}{\mu c_o} \quad \text{or} \quad \lambda = \left[ \frac{\mu (p_i) c_o (p_i)}{2} \right] \left[ \frac{P_{ps} - P_{pwf}}{P_i - P_{pwf}} \right]
\]

One could interpolate between his \( \lambda \) values by interpolating between the approximate \( b \) values.

A \( b > 0 \) for solution-gas-drive reservoirs should reflect an increasing total compressibility with increasing gas saturation. Later development of other supplemental drives—such as gravity segregation, limited water movement, late-time crossflow from nonwellbore productive layers, and hydrocarbon influx from the periphery of the reservoir—would tend to increase the value of \( b \).

### Decline-Curve Analysis Using Type Curves—Individual Well Cases

Well M-4X. Well M-4X was the fourth of five appraisal wells drilled on a carbonate Middle Cretaceous Mishrif structure in the Middle East. In Jan. 1974, data for production rate vs. time, obtained on a long-duration production test after two separate acid treatments totaling 14,000 gal [53 m³] 20% HCl, indicated a
Fig. 12—Well Edda 10X—graphic production forecast compared to an Arps regression fit of \( b = 2.5 \).

Fig. 13—MHC Well A—type-curve match comparison of data fit on both the radial-flow, constant-wellbore-pressure solution and the infinite-conductivity, vertical-fracture solution.

severe decline in production rate. The initial rate declined steadily from 2,361 to 1,045 BOPD [375 to 166 m³/d oil] after only 160 hours (6.7 days) of testing (see Fig. 10). This high initial decline rate was generally interpreted to be depletion. More specifically, it was interpreted as fracture and vug porosity depletion because some vugs and fractures were identified in the initial core description.

First and foremost about the test is that it was recognized as a true constant-wellbore-pressure test where the rate must necessarily decline with time. The constant wellhead flowing pressure observed during the test of 53 to 55 psi [365.4 to 379.2 kPa], coupled with the fact that the reservoir fluid was highly undersaturated and should then have an essentially constant oil head, resulted in a constant bottomhole flowing pressure (BHFP) during the entire test. A constant-wellbore-flowing-pressure analysis was made by type-curve matching the rate-time data in Fig. 10. The match shown in Fig. 10 was unique and conclusively established that the rate-time decline was a transient phenomenon and not depletion; i.e., the rate-time data fit the transient portion of the analytic type curve. Depletion would be identified by the rate-time data overlying an \( r_{p} / r_{w} \) exponential-depletion stem (analogous to the reservoir limit test).

Because the identifying mark of dual-porosity depletion is an exponential-depletion stem (depletion of the fracture/vugs) followed later by another exponential-depletion stem (depletion of the matrix), we clearly cannot attribute the well’s rapid decline in rate to the reservoir’s being naturally fractured or vuggy. The rapid transient decline rate of the well is the expected behavior of a successfully stimulated well of moderate to low permeability. One should look at the ratio \( k/\mu \) to see what is moderate or low permeability. For this well,

\[
\frac{k}{\mu} = 51 \text{ md} \quad \frac{2.3 \text{ cp}}{25 \text{ md/ cp} \ [\sim 25,000 \text{ md/ Pa} \cdot \text{s}].}
\]

For a gas reservoir of 0.02 cp [0.02 mPa·s], the same ratio of 25 would yield a permeability of 0.5 md. Both would exhibit similar transient behavior.

Using the rate-time data from the 7-day production test, we forecast the well’s future production rate as a function of time by drawing a line through the rate-time data overlaid on the uniquely matched portion of the type curve and down a presumed \( r_{p} / r_{w} \) stem for an assumed spacing, \( r_{e} \). At an appropriate rate, the BHFP was lowered to 500 psia [3447.4 kPa] by use of the superposition method given in Ref. 1.

Well Edda 10X. Well Edda 10X rate-time data taken in Nov. 1973 were obtained on the second appraisal well drilled in the Upper Cretaceous chalk reservoir of the Edda field, one of several fields located in the Greater Ekofisk development of the Norwegian North Sea. A drillstem test taken in Nov. 1973 after an acid fracture treatment, without the use of proppants, indicated a very severe decline in production rate during the drawdown test. The rate declined from 9,500 BOPD [1510 m³/d oil] at 1 hour to 4,600 BOPD [731 m³/d oil] after only 10 hours. The flowing tubing pressure varied from 896 psia [6177.9 kPa] at the beginning of the test to 751 psia [5178.1 kPa] at the end, essentially a constant-wellbore-pressure condition. Again, as in the Well M-4X test, the rate decline was incorrectly assumed to be either natural fracture volume depletion, because the Greater Ekofisk development reservoirs are known to be naturally fractured, or closure of the induced fracture as a result of pressure drawdown—both exotic and simplistic explanations.

Fig. 11 illustrates the type-curve match on the plane-radial-flow, constant-wellbore-pressure solution with another unique match on the transient or infinite-acting period. No exponential depletion, fracture volume depletion, or any other type of depletion is indicated. It is not possible to determine whether the reservoir is naturally fractured from the rate-time decline. Again, the rapid decline in rate is the expected behavior of a successfully stimulated low-permeability well.

An Arps depletion stem match of the data gives an apparent \( b = 2.5 \), which, of course, is invalid.

An evaluation of the \( q_{D} / I_{f} \) match and the results obtained from the pressure-buildup analysis are summarized in Table 1. The values of permeability and skin obtained from the rate-time drawdown analyses and the Horner buildup are essentially the same. To determine whether the reservoir was naturally fractured, a fracture index, \( I_{f} \), was calculated from a permeability value obtained from a matrix-plug permeability-permeability plot compared with a buildup or drawdown calculated permeability:

\[
I_{f} = \frac{k_{BU} \quad \text{or} \quad k_{DD}}{k_{(\phi-k)}} = \frac{0.9 \text{ md}}{0.66 \text{ md}} = 1.4. \quad \text{................. (9)}
\]

For this well, there appears to be little natural fracturing at this location because the index is 1.4. This is not the case, however, for most of the development wells drilled later in this field. We
TABLE 2—MHF GAS WELL A: COMPARISON OF PRODUCTION FORECASTS

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<th>Time (months)</th>
<th>Constant-Pressure Solution (Mscf/D)</th>
<th>Infinite-Conductivity Vertical-Fracture Solution (Mscf/D)</th>
<th>Ref. 13 Simulator Results (Mscf/D)</th>
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<td>20</td>
<td>71</td>
<td>64</td>
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will demonstrate this in a full-field rate-time analysis of the total field production.

Fig. 12 illustrates a one-well forecast that was made before any development drilling by extrapolating down an r_2/r_w stem for the premise well spacing. Future rates were read from the real-time scale on which the rate data were plotted. Also shown in Fig. 12 is an extrapolation of the same data fit on an Arps b = 2.5, which is clearly incorrect. Although one would not attempt an Arps equation fit and extrapolation on only 10 hours of production data, it serves as the only example in which, using transient data, false values of b > 1 would severely underestimate production.

To summarize, this example illustrated the ability to develop a sound technical decline-curve analysis prediction with basic reservoir engineering principles and only 10 hours of rate-time data.

MHF Well A. Agarwal et al. 1 presented 300 days of rate-time data for a massive hydraulically fractured (MHF) well. Fig. 13 illustrates a type-curve match of their data on the radial-flow, constant-wellbore-pressure solution and the infinite-conductivity, single-vertical-fracture, constant-wellbore-pressure solution. Clearly, all the data lie on the transient or infinite-acting period, and there is no evidence of depletion. An evaluation of the match points on the basic radial-flow, constant-wellbore-pressure solution yields k = 0.0081 md. This value is identical to the Agarwal et al. prefracture test result and is the same value obtained from matching on the vertical-fracture, constant-wellbore-pressure solution. Calculated values of skin or L_w are reasonably close. Table 2 lists the Agarwal et al. forecast results obtained when their type-curve analysis and reservoir and fluid properties were entered into their MHF simulator. Listed on the far left is the forecast read directly from the match on the basic radial-flow, constant-wellbore-pressure solution. The middle column is the forecast read directly from the match on the infinite-conductivity, vertical-fracture, constant-wellbore-pressure solution. Note the good agreement between the

Fig. 14—MHF Well A graphic production forecast.

Fig. 15—Type-curve fit of all Cullender's Gas Well 3 data in terms of C, pressure-normalized rate, on the infinite-conductivity, vertical-fracture, constant-wellbore-pressure solution.
TABLE 3—WEST VIRGINIA GAS WELL A

<table>
<thead>
<tr>
<th>Reservoir and Fluid Properties</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Gas specific gravity</td>
<td>0.57 (air = 1.00)</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.06</td>
</tr>
<tr>
<td>Water saturation</td>
<td>0.35</td>
</tr>
<tr>
<td>Original pressure, psia</td>
<td>4,175</td>
</tr>
<tr>
<td>Pressure at start of decline, psia</td>
<td>3,268</td>
</tr>
<tr>
<td>Viscosity at 3,268 psia, cp</td>
<td>0.0171</td>
</tr>
<tr>
<td>System compressibility at 3,268 psia, psi^{-1}</td>
<td>177 x 10^{-6}</td>
</tr>
<tr>
<td>Thickness, ft</td>
<td>47</td>
</tr>
<tr>
<td>Temperature, °F</td>
<td>70</td>
</tr>
<tr>
<td>Wellbore radius, ft</td>
<td>0.354</td>
</tr>
<tr>
<td>Rate before 106-day pressure buildup, Mscf/D</td>
<td>2,181</td>
</tr>
<tr>
<td>ΔP_p, psi²/ft³</td>
<td>774 x 10⁶</td>
</tr>
<tr>
<td>B_p at 3,268 psia, scf/ft³</td>
<td>208.8</td>
</tr>
<tr>
<td>B_g at 4,175 psia, scf/ft³</td>
<td>253.9</td>
</tr>
</tbody>
</table>

casting of rate-time decline would be grossly in error, even if we correctly estimated a stabilized backpressure curve position from reservoir variables. On the curve of Fig. 14, note the point at which the rate departs from the transient stem and starts down the depletion stem. This point represents the stabilized backpressure curve position. The rate at the given ΔP² would establish a point on the stabilized backpressure curve.

All early transient production higher than this stabilized rate would be completely ignored in a conventional deliverability rate-time forecast. Further complicating a conventional deliverability approach is the inability to get valid reservoir pressures from reasonable-duration pressure-buildup tests in such a low-permeability well to determine original gas in place from a p/²-vs.-G_p graph. Additional discussion of this point occurs later in the San Juan example.

To illustrate more clearly the shifting of the backpressure curve with time to the stabilized curve position, Cullender’s 14 Gas Well No. 3 backpressure curve coefficients—C values for a 214-hour flow and C values from a four-point isochores and 72-hour deliverability tests covering a period of 9 years—were plotted and matched on the infinite-conductivity, vertical-fracture, constant-wellbore-pressure solution (Fig. 15). Fig. 16 shows these same data plotted as a series of backpressure curves shifting with time. (See Table 3 of Ref. 14 for the complete set of the data.) This well was initially acid-fractured. Note the near-perfect fit of all the data on the infinite-conductivity, vertical-fracture, constant-wellbore-pressure solution with no indication of wellbore performance deterioration. Even after 9 years of production and a 106-psi [730.9-kPa] reservoir shut-in pressure decline, the 72-hour values of C fall on the original curve trace.

West Virginia Gas Well A. Well A is a low-permeability gas well located in West Virginia. It produces from the Oosonda gas chert that has been hydraulically fractured with 50,000 gal [189 m³] of 3% gelled acid and 30,000 lbm [13 608 kg] of sand. After initial completion, the well was placed on production for 200 days and then shut in for a 106-day pressure buildup in an attempt to obtain reservoir pressure. A conventional Horner analysis of the buildup data gave p_R = 3,268 psia [22 532 kPa], k = 0.082 md, and s = -5.4. A type-curve analysis of the same data indicated that the correct semilog straight line started at about 600 hours (25 days).

<table>
<thead>
<tr>
<th>TABLE 4—WEST VIRGINIA GAS WELL A: SENSITIVITY TO r_e/r_w</th>
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<tbody>
<tr>
<td>k, md-ft</td>
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<tr>
<td>V_e, 10⁶ ft³</td>
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<tr>
<td>r_e, ft</td>
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<tr>
<td>r_w, ft</td>
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<tr>
<td>G, at 3,268 psia, Bscf</td>
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<tr>
<td>G, at 4,175 psia, Bscf</td>
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<tr>
<td>Horner Analysis</td>
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<tr>
<td>p_R Basis</td>
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Fig. 17 is a log-log plot of monthly production data. These are raw monthly production data obtained directly from production files, not data from special tests. The 8 years of rate-time data were matched on the radial-flow, constant-wellbore-pressure solution (dashed line), the \( r_w/r_{w_{2}} \) exponential stem of 20, and \( b=0 \) to yield \( k=0.0651 \) md, \( s=-5.38 \), and \( r_w =1,547 \) ft [472 m]. The solid line through the same data shows a fit on the \( b=0.5 \) and Carter’s \( \lambda =0.55 \) constant-wellbore-pressure solution on the \( r_w/r_{w_{2}} \) stem of 20. Results from the match on a \( b=0.5 \) resulted in a \( k=0.0700 \) md, \( s=-5.17 \), and \( r_w =1,242 \) ft [379 m]. These results compare closely with those obtained from the match on Carter’s type curve, which gave \( k=0.0678 \) md, \( s=-5.17 \), and \( r_w =1,252 \) ft [382 m]. Carter’s \( \lambda \) was calculated to be 0.555 for this example. For the pressure ratio \( p_{wi}/p_{w_{2}} =500 \) psi/3,268 psi = 0.15 [3447 kPa/22,532 kPa], Figs. 8 and 9 of Ref. 1 also indicate that the expected depletion stem should have a \( b=0.5 \) for this gas well.

Occasional shut-ins and blows to the atmosphere for dewatering the wellbore did occur during the normal production of the well. One would expect a \( b=0.5 \) situation gradually to approach a \( b=0 \) performance because liquid loading occurs when the flow rate declines and the wellbore deteriorates with time.

**Evaluating the Match.** The previous discussion of the match to the \( r_w/r_{w_{2}} \) stem of 20 leaves some doubt as to whether the match to the 20 stem is unique or the best answer. We will investigate the sensitivity of results to stems of 10, 20, and 50 for the \( b=0 \) and \( b=0.5 \) match. To illustrate the complete evaluation of the matching technique, we will use the \( b=0.5 \) match (Fig. 17) to the composite type curve as an example. Table 3 lists all the pertinent reservoir variables for the well. For the match point of \( b=0.5 \), \( q_{dd} =0.58 \), \( q_{t} =1,000 \) Mcf/d [28.3103 std m³/d], \( t_{dd} =0.126 \), and \( \tau =1 \) days.

When Eq. 5 is expressed in terms of gas units and pseudopressure, \( p_{pp} \), the productivity factor is

\[
\frac{kh}{\ln \left( \frac{r_e}{r_{w_{2}}} \right) } = \frac{q_{t}}{q_{dd} T_{w_{2}} (p_{pp} - p_{pp_{f}})}
\]

\[= 1,000(14.7)(620)\]

\[= 19.87 \times 10^{-6} (520)(7.948 \times 10^{-5} - 208 \times 10^{3})\]

\[= 1,956 \text{ md-ft [0.599 md-ft]}
\]

Also expressing Eq. 6 in terms of gas units and pseudopressure, \( p_{pp} \), we have

\[V_{pp} = \pi r_{e}^{2} \phi h = \frac{2,000 p_{pp} T_{w_{2}} (p_{pp} - p_{pp_{f}})}{(\mu_{c}) T_{w_{2}} (p_{pp} - p_{pp_{f}})} \left( \frac{t}{t_{dd}} \right) \left( \frac{q_{t}}{q_{dd}} \right)
\]

\[= \frac{2,000(14.7)(620)}{0.0172(177 \times 10^{-6})(520)(7.948 \times 10^{-5} - 208 \times 10^{3})}
\]

\[\times \left( \frac{100}{0.126} \right) \left( \frac{1,000}{0.58} \right)
\]

\[= 20.36 \times 10^{6} \text{ ft}^{3} [0.5765 \times 10^{6} \text{ m}^{3}]
\]

and

\[r_e = \sqrt{\frac{V_{pp}}{\pi \phi h}} = \sqrt{\frac{20.36 \times 10^{6}}{3.1416(70)(0.06)}} = 1,242 \text{ ft [379 m]}
\]

The gas in place, \( G_{t} = 3,268 \text{ psia [22,532 kPa]} \), with \( p_{i} \) being the pressure at the start of the decline analysis, is then

\[G = V_{pp}(1 - S_{w}) B_{tg}
\]

\[= 20.36 \times 10^{6}(1 - 0.35)(208.8)
\]

\[= 2.763 \text{ Bscf [78.2 x 10^{6} std m^{3}]}
\]

For comparison purposes, \( G = 3,268 \text{ psia [22,532 kPa]} \) from the Carter type-curve match on \( \lambda =0.55 \) is 2,807 Bscf [79.4 x 10^{6} std m^{3}], while that obtained from the \( b=0 \) match is 4,286 Bscf [121.3 x 10^{6} std m^{3}].

Assuming little or no PV change resulting from rock and water expansion, at the original reservoir pressure of 4,175 psia [28,787 kPa], \( B_{g} = 253.9 \text{ scf/ft}^{3} [3 \text{ m}^{3}/\text{m}^{3}] \), an original gas in place, \( G_{i} \), was then calculated to be 3,360 Bscf [95.1 x 10^{6} std m^{3}]. The difference of 0.597 Bscf [17 x 10^{6} std m^{3}] compares well with the measured cumulative production of 0.580 Bscf [16 x 10^{6} std m^{3}] between the two average reservoir pressure intervals.

With the productivity factor calculated as 1.965 md-ft [0.599 md-ft] and \( r_e \) calculated as 1,242 ft [379 m], we can now investigate the sensitivity of \( k \) and \( s \) to \( r_w/r_{w_{2}} \) stems (Table 4). Regardless of the \( r_w/r_{w_{2}} \) stem chosen, once the depletion stem is established by the rate-time data in the composite type curve (Fig. 1), \( q_{dd}(t) \) and \( t_{dd}(t) \) are fixed and the PV is then fixed. The calculated initial permeability and skin are insensitive to the \( r_w/r_{w_{2}} \) stem selection. In this case, we have the 106-day pressure-buildup analysis run just before the start of the decline-curve analysis in which to compare. From the Horner results, the correct \( r_w/r_{w_{2}} \) stem appears to be \( \sim 20 \). If at early time we had taken more frequent, precise rates and flowing pressures, then we could have uniquely fit the stem. At the very least, \( kh \) and \( s \) could have been calculated from a short-duration, constant-wellbore-pressure test, as opposed to calculating \( kh \) and \( s \) from a long-duration build-up test. This was done for West Virginia Gas Well B [15*]; Fig. 18 compares the pressure-normalized, constant-wellbore-pressure analysis (the rate data alone had no character) and the corresponding Horner analysis results for this well. Therefore, one need not have shut in Gas Well A for 106 days to obtain \( kh \) and \( s \) for selecting the proper \( r_w/r_{w_{2}} \) stem.

Table 5 summarizes the pertinent results of matching the rate-time data on \( b=0 \), \( b=0.5 \), and Carter’s \( \lambda =0.55 \). By comparing the difference of 0.91 Bscf [25.8 x 10^{6} std m^{3}] between a calculated \( G_{i} =5.20 \text{ Bscf [147.2 x 10^{6} std m^{3}] at 4,175 psia [28,787 kPa]} \) and the \( G =4.29 \text{ Bscf [121.5 x 10^{6} std m^{3}] at 3,268 psia [22,532 kPa]} \) with the real cumulative production of 0.58 Bscf [16.4 x 10^{6} std m^{3}] between the two shut-in pressures, the \( b=0 \) as a match is ruled out and the \( b=0.5 \) must be selected.

San Juan Example. Early in 1982, a reservoir study of all wells completed in the Blanco Mesaverde pool in one entire township in Rio Arriba County, NM (San Juan basin), was initiated to quantify the reserve increase resulting from the infill drilling program begun in 1975. The initial spacing of 320 acres [130 ha] was halved through infill drilling.

The San Juan basin is located in northwestern New Mexico and extends into southwestern Colorado. The gas-producing formations in the basin are sandstones of Upper Cretaceous Age. The Mesaverde ranges in gross thickness from a few hundred to almost 1,800 ft [550 m]. Average porosity in the Mesaverde is about 10%, and the permeability ranges from 0.02 to 1.0 md. Initial pressure in the study area was about 1,200 psi [8274 kPa].

The area we investigated had 72 original wells, drilled two per section, plus 72 infill wells. Initial pressure for a majority of the infill wells was some 30 to 40% less than original reservoir pressure, indicating that drainage was occurring. A limited number of infill wells had initial pressures essentially equal to original pressure, indicating that drainage was not occurring at these locations.

The reserve determination method used in the past was the p/z plot. This method consisted of plotting annual 7-day shut-in pressures vs. cumulative gas produced. Although annual 7-day shut-ins were taken, the final shut-in pressure was well below the true static reservoir pressure. Because of the low-permeability stimulated character of the wells, the semilog straight line was seldom reached, making the determination of average reservoir pressure difficult. This essentially negated the p/z plot as a useful reservoir analysis tool until infill wells were drilled. Trend plotting of the short-duration shut-in and its corresponding p/z was used without recognizing the need to pass through the initial p/z value (see Fig. 19). The initial p/z value was ignored to prevent an apparent yearly increase in gas reserves.

A majority of the infill wells came in at pressures about 30 to 40% lower than the original reservoir pressure. In these cases, the
infill original pressure could be used to provide a pressure point on the plot of p/z vs. cumulative gas produced of the original well. A reserve figure was determined for the original well by passing a line from the initial p/z through the infill-well p/z value with an abandonment pressure of 100 psi [690 kPa] assumed. This method could not be used, however, when the infill initial pressure essentially equaled the original reservoir pressure. In such cases, the most practical method available to determine the reserve was decline-curve analysis.

Figs. 20 and 21 present the rate-time data and type-curve match of the original well, Well 58, and the offset infill well, Well 58A. The latter came in at near-original pressure. Data for both wells (as for all wells in the study area) were obtained from a commercially available data base. The character of the monthly rate-time data was erratic—a problem not uncommon in rate-time analysis—and average 6-month rates available from the data base were plotted at midpoint intervals as a form of data smoothing. The average 6-month data points appear as solid squares on the type-curve matches. Notice how the character of the production profile was enhanced by this smoothing technique.

A type-curve match of the rate-time data for Well 58 indicates the well to be on decline and going down a depletion stem. The calculated k and s are 0.36 md and −4.3, respectively, with a calculated drainage area of 225 acres [91 ha]. Although not apparent, a match on different r_e/r_w stems would result in essentially the same calculated value of r_e. The drainage radius is fixed once depletion is evident. Table 6 illustrates this point for Well 58 matched on r_e/r_w stems of 100 and 200. The rate-time data of Well 58A were matched to the Locke and Sawyer type curve and found to be entirely transient. The k and s were calculated to be 0.07 md and −6.4, respectively.

These results are consistent in that one would expect the infill well permeability to be less than the original well permeability, when the infill location had not been drained. The more negative skin

![Fig. 21—San Juan Gas Well 58A rate-time type-curve match of production data on the Locke and Sawyer type curve.](image)

**Table 6—San Juan Well 58: Sensitivity to r_e/r_w**

<table>
<thead>
<tr>
<th>r_e/r_w</th>
<th>k, md</th>
<th>s</th>
<th>r_e, ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.34</td>
<td>-4.3</td>
<td>1,765</td>
</tr>
<tr>
<td>200</td>
<td>0.40</td>
<td>-3.6</td>
<td>1,776</td>
</tr>
</tbody>
</table>

![Fig. 22—Single-well simulation study of rate reduction effect on 7-day annual shut-in pressures.](image)

factor for the infill well represents the improvement in hydraulic fracturing technology and design, because the original well was stimulated in the late 1950's. A production forecast for the infill well was developed by forecasting down a depletion stem of L_e/L_{ref}=3.2. This stem corresponds to the skin of −6.4 and a drainage area of 95 acres [38 ha] to keep the total drainage area of 320 acres [130 ha] whole.

A reserve forecast for all wells on decline in the study area was made from a rate-time analysis. These calculated reserves are within 9% of the p/z reserve determined with the infill-well original pressure. The reserve study of the township indicated that infill drilling resulted in an average reserve increase of 11%.

Reserve increases resulting from infill drilling have been postulated as a result of the flattening observed in the p/z trend after infill drilling. If total field production remains essentially constant (fixed market demand) before and after infill drilling, a reduction

![Fig. 23—San Juan field—well example of rate reduction effect on 7-day annual shut-in pressures.](image)

![Fig. 24—Semilog plot of production history of Gentry and McCray's Oil Well Example No. 2 data.](image)
in some of the original wells' flow rates will occur after the start of infill drilling. This reduction in flow rate will cause a corresponding increase in flowing pressure. When the well is next shut in for an annual 7-day test, the final buildup pressure will automatically reflect a higher final shut-in pressure than when the well was produced at the previously higher rate. The obvious limiting case is when a well is cut back to zero flow rate, and only then will 7-day shut-ins reflect true reservoir pressure. The rate-reduction effect is demonstrated by a two-dimensional (2D), single-well, transient gas model simulation and actual field data. Also, examination of pressure production data from low-permeability gas wells in fields not subject to infill drilling shows the same increased shut-in pressure trend as a result of reduction in output resulting from the current oversupply of gas.

The single-well simulation is based on an area of 320 acres [130 ha], a permeability of 0.05 md, and a 500-ft [152-m] infinite-conductivity vertical fracture corresponding to a 6.5 skin. In the simulation, the well produced at a constant rate of 240 Mscf/D [6.8 x 10^3 std m^3/d] for 10 years and then at 160 Mscf/D [4.53 x 10^3 std m^3/d] for 10 years. Annual 7-day shut-ins were simulated, and Fig. 22 presents the resulting plot of p/c vs. cumulative gas produced. Note the rise in 7-day p/c values when the rate reduction took effect. It is this rise that can be misinterpreted as a reserve increase. Fig. 23 is an actual well example illustrating the same problem. In this instance, the flow rates of the original wells were reduced because of the production of infill wells. Notice the effect of the reduced flow rate on the p/c plot, which is identical to that shown by the 2D, single-well model study.

In low-permeability reservoirs, transient effects may last for several years. Coupled with this problem are the extreme shut-in times required to establish usable average reservoir pressures and the difficulty in determining the stabilized backpressure curve. Decline type-curve analysis provides a reserve estimate and a production forecast, which can easily be updated without knowledge of reservoir pressures or the stabilized curve.

**Reported Cases of b > 1.** Gentry and McCray made a study attempting to determine why some wells exhibit decline-curve values of b > 1. The reservoir model described in their study did not include the effects of transient flow behavior. The rate-time data presented as Field Example No. 2 (Fig. 24) were plotted as log q-log t and yielded an almost perfect type-curve match of all the data on the Locke and Sawyer infinite-conductivity, single-vertical-fracture, constant-wellbore-pressure solution (see Fig. 25). Note that the first year of data is in the transient or finite-time period.

All the data were expected to match this type curve because the well was described as being completed in the Mississippi limestone and the producing formation was stated to be fractured, with a tight matrix. No evidence of a double-depletion exponential decline indicative of a naturally fractured reservoir appears in the log-log data plot. "Fractured" may have simply meant hydraulically fractured. In any case, a well completed in a limestone reservoir would probably have been stimulated. Because oil wells are generally drilled on small spacing, and because improved stimulation techniques result in hydraulic fracture lengths beginning to approach

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**Fig. 25**—Locke and Sawyer type-curve match of Gentry and McCray's Oil Well Example No. 2 data.

**Fig. 26**—Reinitialization of Gentry and McCray's Oil Well Example No. 2 data at 3 years matched to an exponential, b = 0.

**Fig. 27**—Dimensionless N_p/q_t plot for Gentry and McCray's Oil Well Example No. 2 data, indicating an apparent b between one and infinity.

**Fig. 28**—Wattenberg field—well data matched to the Locke and Sawyer type curve and compared to a regression fit using an Arps equation with b = 2.29.
spacing, \(L_{i1}/L_{i2} \rightarrow 1\). In the unique match of Fig. 25, \(L_{i1}/L_{i2} = 1.5\). Fig. 26 shows a match of the 2- to 6-year data, but they are reinitialized in time after depletion has clearly set in, at about 2 years. The data fit a decline with \(b = 0\) and can even be recognized as such on their original semilog plot. Fig. 27 shows the same rate-time data and their use of the Arps equation in a different form, leading to implied values of \(b\) between 1 and infinity. Once again, an attempt to fit transient-dominated data to the Arps depletion equation leads to apparent values of \(b > 1\).

One further example from the literature illustrates a fit of transient data to the Arps equation with a least-squares computer model to "precisely determine optimum values for the coefficients \(a\), \(b\), and \(q_{dc}\)." Fig. 28 is a log-log plot of and match of Well No. 2, Wattenberg field, an example from Ref. 17. The match is again made on the infinite-conductivity, vertical-fracture, constant-wellbore-pressure-solution type curve. The Wattenberg field produces from a tight gas sand and is developed on 160-acre [65-ha] spacing with hydraulic fracture lengths averaging at least 1,500 ft [457 m]. Good engineering judgment indicates that with an \(L_{i1} = 1,320\) ft [402 m] and a fracture length of 1,500 ft [457 m], the data should decline down an \(L_{i1}/L_{i2} \approx 1.0\) stem on the type-curve match, which it does after a nearly 5-year-long transient flow period. The "unique and unbiased" statistical extrapolation of the Arps equation fit of the data with \(b = 2.29\) yields a life of 1,042 years to an economic limit of 200 Mscf/month [5.66 \times 10^9 \text{ std m}^3/\text{month}], which is unreasonable. The extrapolation of the vertical-fracture-solution type curve on the basis of basic reservoir information with spacing and fracture lengths gives a rational answer.

According to Ref. 17, this application of the regression approach by use of the Arps equation was used on some 200 tight-gas wells. A correlation of \(b\) with fracture fluid volume on about 50 Wattenberg wells indicated that in nearly all cases, \(b > 1\) and was as high as 3.5 in one instance.

All cases we have seen where \(b > 1\) have been shown to be transient rate-time decline of low-permeability stimulated wells. Statistical approaches to decline-curve analysis that permit \(b > 1\), the recognized upper limit to the Arps equation, can lead to bad results and bad decisions. The normal range of apparent \(b\), from force fits of transient data to the Arps equation, appears to be between 2.2 and 2.5. To identify transient data and their end, a log-log plot of rate-time data must be made.

**Decline-Curve Analysis Using Type-Curves—Field Cases**

Field E. One of our earliest field type-curve analysis cases was a one-well field. The depletion mechanism was virtually a full bottomwater drive or, more specifically, a constant-pressure-outsideboundary case. To date, no analytic work concerning the expected value of \(b\) with water displacement processes has been done.

Field E is located in the Far East and produces from a carbonate Upper Miocene Kais reef. The reservoir is highly undersaturated with a producing gas/oil ratio (GOR) of \(\sim 3\) scf/bbl [\(-0.54\) std \(\text{m}^3/\text{m}^3\)] and a gravity of 47°API [0.79 \(\text{g/cm}^3\)], and is more than likely to be naturally fractured—a typical situation for developing a strong waterdrive. Only the upper 50% of the producing interval was perforated to avoid early water coning. The well was initially completed with 4 shots/ft [13 shots/m] and acidized with 6,100 gal [23 \(\text{m}^3\)] 30% HCl staged with ball sealers. The initial reservoir pressure was 2,921 psia [201,400 kPa]. As with most of the reef reservoirs in the area, later shut-in pressures return to within 10 to 20 psi [68.9 to 137.9 kPa] of the original reservoir pressure after a 24-hour shut-in.

Fig. 29 is a semilog plot illustrating the production performance of the field in terms of oil production, total fluid production, and WOR. Gas rates were so small that the GOR is not plotted. The initial oil rate decline basically coincides with increasing water production when, after 1 year, gas-lift facilities were installed for all fields in the area. Successful gas lifting in this field began in July 1979 and resulted in the first BHFP change and another decline period. Gas injection rates were increased twice more to lower the BHFP, resulting in yet two more decline periods. Fig. 30 is the same oil production rate data now placed on a log \(q_o\)-log \(t\) plot in preparation for type-curve analysis. Each of the three additional decline periods following increased gas injection rates were reinitialized in time and log \(q_o\)-log \(t\) plots made for each one. They all exactly overlapped the initial decline established during the natural-flow period. This should be expected from superposition principles; i.e., every new transient introduced into the well or field must go back and retrace the original \(q_o(t)\) curve (Fig. 31).

(Referring back to the Cullender Gas Well No. 3 data log \(q_o\)-log \(t\) plot in Fig. 15, note that the ten 72-hour annual tests taken be-
between 1945 and 1953 with up to 106 psi [730.9 kPa] of pressure depletion exactly overlie the original type-curve match.)

A match was made with the composite \( q_{AD}^{-1}q_{AD} \) type curve (Fig. 1) on a value of \( b=0.5 \). Evaluation of the match point yielded \( k=152 \) md and \( z=-4.1 \). This compares well with the initial well test Horner analysis of \( k=143 \) md and \( z=-3.9 \). Table 7 compares the results and lists the reservoir and fluid properties used for the well.

Fig. 31 shows the \( q^{-1}q \) match on the \( q_{AD}^{-1}q_{AD} \) type curve. If we now transfer the \( q_{AD}^{-1}q_{AD} \) axes onto the tracing paper of the real-time plot, we have a Kais Reef full waterdrive type curve that could be used to predict future performance with backpressure changes for any set of reservoir parameters \( (k, \phi, h, p_i) \) and fluid \( (\mu, B, c_i) \) parameters with any well spacing \( (r_w) \) and any skin effect \( (r_{sw}) \) of another Kais Reef reservoir having no performance data at all. This grid transfer is equivalent to back-calculating \( q_{AD}^{-1}q_{AD} \) values for each of the plotted rate-time data from the known reservoir variables listed in Table 7 and the results obtained from the pressure-buildup analysis.

**Edda Field.** The Edda field is the smallest of four overpressured volatile oil reservoirs within the Greater Ekofisk development in the Norwegian Sector of the North Sea. Table 8 lists the basic reservoir and fluid properties. Production is from seven wells completed in the Upper Cretaceous chalk (Maestrictian) or Tor formation. Slight to moderate natural fracturing is indicated from the F1 derived from the pressure-buildup analysis (see Table 9). Note that Well C-5 indicates no natural fracturing while Well C-2 has the highest \( f_0 \) of 28. All wells were acid-fractured without propants on completion. As indicated by the skin values listed in Table 9, all wells appear to have been successfully stimulated.

Fig. 32 illustrates the production performance of the field with time in terms of monthly average oil and gas production and GOR. Note that the field came on fairly rapidly, resulting in a classic field decline curve. The surface flowing pressure after the first few months has been virtually constant throughout the field's production history. The slight dips in production are a result of field shut-ins. Note also the slight production peaks that follow, discussed in detail later. Of special note is the flattening of the GOR curve starting in about mid-1982.

Fig. 33 is the total field oil production rate plotted in terms of measured daily production rate vs. time on a log \( q^{-1}q \) basis. To the best of our knowledge, this is the first time that daily production for a field was measured and available for decline-curve analysis over a 4-year period. Transient spikes after shut-ins are clearly visible and significant on this plot and are used in the decline analysis. There were several field shutdowns followed by an initial transient spike (flush production), as expected for such a low-permeability field with successfully stimulated wells. Theoretically, each of the transient spikes (if reinitialized in time) should retrace the field's initial \( q_{AD}^{-1}q_{AD} \) transient decline. Fig. 34 is a plot of two of the transient spikes obtained after extended shut-ins, and indeed they both virtually overlie the original field transient decline. The rate and time scales shown in Fig. 34 are for the initial production period only; the rate and time scales for Periods A and B have been omitted for clarity of presentation. Note that the field transient appears to end after about 100 days of production. Permeability, skin, and reservoir volumes were calculated for the total field match and each of the two production transients on an average-well basis also—I.e., \( q_f \) divided by the number of wells. Table 8 summarizes the
TABLE 9—EDDA: INITIAL-COMPLETION-WELL TEST RESULTS AND FRACTURE INTENSITY INDICES

<table>
<thead>
<tr>
<th>Well</th>
<th>$kh$ (md-ft)</th>
<th>$h$ (ft)</th>
<th>$k$ (md)</th>
<th>$s$</th>
<th>$\phi$ (%)</th>
<th>$k$ matrix ($\phi$-k plot)</th>
<th>$k$ well test</th>
<th>$k$ matrix ($\phi$-k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-2</td>
<td>2,028</td>
<td>110</td>
<td>18.4</td>
<td>-4.9</td>
<td>25.3</td>
<td>0.66</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>C-5</td>
<td>50</td>
<td>70</td>
<td>0.7</td>
<td>-4.0</td>
<td>23.8</td>
<td>0.50</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>C-9</td>
<td>1,119</td>
<td>120</td>
<td>9.3</td>
<td>-4.5</td>
<td>24.6</td>
<td>0.98</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>C-10</td>
<td>218</td>
<td>82</td>
<td>2.7</td>
<td>-4.7</td>
<td>24.0</td>
<td>0.50</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>C-11</td>
<td>510</td>
<td>90</td>
<td>5.7</td>
<td>-4.6</td>
<td>24.1</td>
<td>0.62</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>C-14</td>
<td>298</td>
<td>114</td>
<td>2.6</td>
<td>-3.0</td>
<td>22.6</td>
<td>0.40</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>C-15</td>
<td>201</td>
<td>115</td>
<td>1.7</td>
<td>-3.7</td>
<td>20.8</td>
<td>0.28</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>4,424</td>
<td>701</td>
<td>41.1</td>
<td>-29.4</td>
<td>165.2</td>
<td>3.44</td>
<td>74</td>
<td></td>
</tr>
</tbody>
</table>

Arithmetic Average Value* 632 100 5.9 -4.2 23.6 0.49 11

$k \approx 0.32$ md.

match results for $r_e/r_w = 50$ and $b = 0.6$. Reservoir pressures and gas saturations for total-compressibility calculations for the decline analysis of the transient spikes following shut-ins were estimated from a total field pressure, GOR, and a field-deliverability-matched, compositional material-balance study that is normally updated yearly. Note from the table the slightly declining values of effective permeability to oil values as a result of increasing gas saturation in the reservoir. The calculated reservoir PV's from all three matches are about the same.

A comparison of the calculated PV from the initial match of the total field rate-time data of $202.8 \times 10^4$ res bbl [32.2 $\times 10^6$ res m$^3$] with that obtained from the total field compositional material-balance performance-matched PV of $201.1 \times 10^6$ res bbl [32.0 $\times 10^6$ res m$^3$] is excellent.

Material-balance matching of the reservoir consisted of two distinct periods of interest with respect to the decline-curve analysis. The period before the GOR flattened out when forecasted gave rate-time data that fit a $b = 0.3$. The period after the flattening, because of more efficient indicated recovery (less gas voidage), now results in a $b = 0.6$, the same value of $b$ indicated from the decline-curve match. Al-Kasim,$^{19}$ using data we furnished and a one-dimensional radial model matched to the most current field performance data, also found an early $b = 0.3$, but $b = 0.7$ most of the time. Clearly, no double-depletion stems indicative of a dual-porosity, naturally fractured reservoir can be observed in the rate-time decline data.

**Evaluating the Match.** We will again illustrate the evaluation of the matching technique and investigate the sensitivity of results to $r_e/r_w$ stems of 20, 50, and 100 for $b = 0.6$, now for an oilfield case. For a match point of $b = 0.6$, $q(t) = 10,000$ STB/D [1590 stock-tank m$^3$/d] oil, $q_{dd} = 0.295$, $t = 100$ days, and $t_{op} = 0.29$.

With Eq. 5, the field productivity factor is

$$
\frac{kh}{\ln \left( \frac{r_e}{r_{w1}} \right) - \frac{1}{2}} = \frac{141.2(\mu_o B_o)}{(P_i - P_{w1})} \frac{q(t)}{q_{dd}}
$$

$$
\approx \frac{141.2(0.416)}{(6,300 - 1,500)} \left( \frac{10,000}{0.295} \right)
$$

$$
= 414.82 \text{ md-ft} [126.43 \text{ md} \cdot \text{m}].
$$

Because there are seven wells, $q(t)$ is divided by seven and the average well productivity factor is

$$
\frac{kh}{\ln \left( \frac{r_e}{r_{w1}} \right) - \frac{1}{2}} = 59.26 \text{ md-ft} [18.1 \text{ md} \cdot \text{m}].
$$

**Fig. 32**—Edda field production performance.

**Fig. 33**—Edda field type-curve match of daily oil production data on the $q_{dd}^{-1}t_{op}$ type curve.
Eq. 6 will yield the total field PV:

$$V_{PF} = \frac{1}{(\mu_c/\mu_w)(p_i-p_w)} \left( \frac{t}{q_f} \right) \left( \frac{q(t)}{q_o} \right)$$  \hspace{1cm} (6)$$

$$= \left( \frac{0.416}{(0.185 \times 27 \times 10^{-6})(6,300-1,500)} \right) \left( \frac{100}{0.29} \right) \left( \frac{10,000}{0.295} \right)$$

$$= 2.028 \times 10^6 \text{ res bbl} [3.22 \times 10^7 \text{ res m}^3].$$

$$r_{eF} = \sqrt{\frac{V_p \times 5.615}{\pi h \phi}}$$

$$= \sqrt{\frac{2.028 \times 10^6(5.615)}{\pi 100(0.236)}} = 3.919 \text{ ft} [1195 \text{ m}].$$

Then the oil in place, \( N_o \), at the start of the decline analysis when \( p_i = 6,300 \text{ psia} [43,439 \text{ kPa}] \) and \( B_o = 1.92 \) is

\[
N_o = \frac{V_{PF}(1 - S_w)}{B_o} = \frac{202.8 \times 10^6 \text{ bbl}(1 - 0.365)}{1.92} = 67.08 \text{ MMSTB} [10.7 \times 10^6 \text{ stock-tank m}^3].
\]

Because \( 1.2 \times 10^6 \text{ STB} [0.191 \times 10^6 \text{ stock-tank m}^3] \) had been produced before the start of the decline analysis, the original oil in place, \( N_{oir} \), is indicated to be 68.3 MMSTB \( [10.9 \times 10^6 \text{ stock-tank m}^3] \). The pressure and GOR history-matched compositional material-balance program obtained an \( N_i \) of 67.4 MMSTB \( [10.7 \times 10^6 \text{ stock-tank m}^3] \). The comparison is good.

Using the decline-curve analysis, \( N_i = 68.3 \) MMSTB \( [10.9 \times 10^6 \text{ stock-tank m}^3] \), and the cumulative recovery of 16.6 MMSTB \( [2.64 \times 10^6 \text{ stock-tank m}^3] \) to Jan. 1, 1984, results in a 24.3\% recovery to date. From the decline-curve projection to an economi-

---

**TABLE 10—EDDA SENSITIVITY TO \( r_{e}/r_{wq} \)**

<table>
<thead>
<tr>
<th>( r_{e}/r_{wq} )</th>
<th>Total Field</th>
<th>Average Well</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1,035.2</td>
<td>147.90</td>
</tr>
<tr>
<td>50</td>
<td>1,415.4</td>
<td>202.2</td>
</tr>
<tr>
<td>100</td>
<td>1,702.9</td>
<td>243.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( kh ), md-ft</th>
<th>Total Field</th>
<th>Average Well</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.35</td>
<td>14.15</td>
<td>20.02</td>
</tr>
<tr>
<td>10.35</td>
<td>17.03</td>
<td>24.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( V_p ), 10^6 bbl</th>
<th>Total Field</th>
<th>Average Well</th>
</tr>
</thead>
<tbody>
<tr>
<td>202.8</td>
<td>202.8</td>
<td>28.9</td>
</tr>
<tr>
<td>202.8</td>
<td>202.8</td>
<td>29.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( r_{wq} )</th>
<th>Total Field</th>
<th>Average Well</th>
</tr>
</thead>
<tbody>
<tr>
<td>195.95</td>
<td>78.38</td>
<td>29.62</td>
</tr>
<tr>
<td>195.95</td>
<td>39.19</td>
<td>14.81</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( s_F )</th>
<th>Total Field</th>
<th>Average Well</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5.34</td>
<td>-4.43</td>
<td>-3.73</td>
</tr>
<tr>
<td>-5.34</td>
<td>-3.73</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( N_i ), MMSTB</th>
<th>Total Field</th>
<th>Average Well</th>
</tr>
</thead>
<tbody>
<tr>
<td>67.1</td>
<td>67.1</td>
<td>9.6</td>
</tr>
<tr>
<td>67.1</td>
<td>67.1</td>
<td>9.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material-Balance</th>
<th>Total Field</th>
<th>Average Well</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{oi} ), MMSTB</td>
<td>68.3</td>
<td>9.8</td>
</tr>
<tr>
<td>68.3</td>
<td>9.8</td>
<td>9.8</td>
</tr>
</tbody>
</table>

\( r_{wq} = \sqrt{\text{number of wells} \times r_e^2} \)

\( = \sqrt{(10.354 \text{ ft})^2} = 0.8969 \text{ ft.} \)
Fig. 35—Comparison of calculated Edda field performance based on the average well determined from the $q_{w}$ vs. $t_{w}$ type-curve match with actual field performance.

Fig. 36—Clyde Cowden lease—production performance.

Fig. 37—Clyde Cowden lease—log-log plot of monthly oil production vs. time.

Fig. 38—Clyde Cowden lease—type-curve match of the primary production period.

Fig. 39—Clyde Cowden lease—type-curve match of the reinitialized waterflood decline period.

Fig. 40—Clyde Cowden lease—log-log overlay of primary and reinitialized waterflood periods on $b = 0.3$.

The indicated ultimate recovery of 22.6 MMSTB [$3.59 \times 10^8$ stock-tank barrels] will result in a 33.1% recovery.

Table 10 shows the effect of several $r_e / r_w$ stems on $k$ and $s$ for the total field match, and the results are also expressed as an average well—i.e., total field rate divided by number of wells. With the productivity factor and $r_e$ fixed, the effect of varying $r_e / r_w$ stems is illustrated for the total field and the average well. The arithmetic average skin $s = -4.2$, determined from individual well tests (Table 9), might be used to select an $r_e / r_w$ of 50.

To test the match, the average well ($k=2.02$ md, $s=-4.43$, $r_e / r_w=50$, $b=0.6$) was forecast in terms of $q_{AD}$ vs. $t_{AD}$ and superposition, which included all the shutdown periods. The result multiplied by seven wells is compared with the actual total field performance in Fig. 35.

**Clyde Cowden.** The Clyde Cowden lease is part of the Goldsmith (5,600-ft [1707-m]) field in Ector County, TX. The main development of the lease occurred between 1950 and early 1952, with 50 wells drilled on 40-acre [16-ha] spacing. An acid treatment of the dolomitic formation was a routine part of each completion. The dominant drive mechanism during the primary recovery period is understood to be a solution gas drive.

Fig. 36 shows the production history of the lease from 1949 to 1982. The primary decline period began in Oct. 1951 and ended with the beginning of the waterflood program in Aug. 1961. Between 1970 and 1972, most of the producing wells were hydraulically fractured. The secondary decline period began at about this time, in May 1971.

The 34-year production history shown in Fig. 36 is replotted on a log-log scale in Fig. 37. The primary recovery period (1951–61), matched to an $r_e / r_w$ stem of 1,000 and a $b=0.3$, is shown in Fig. 38. The reinitialized plot of the waterflood decline period is shown in Fig. 39. Here, too, the match is on an $r_e / r_w$ stem of 1,000 and $b=0.3$. The primary and secondary decline period log-log plots overlie each other (Fig. 40).
<table>
<thead>
<tr>
<th>TABLE 11—CLYDE COWDEN: RESERVOIR AND WELL DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_i$ (at discovery), psig</td>
</tr>
<tr>
<td>$p_i$ (match), psig</td>
</tr>
<tr>
<td>$p_{sat}$, psig</td>
</tr>
<tr>
<td>$c_s$ at 1,900 psig, $10^{-6}$ psi$^{-1}$</td>
</tr>
<tr>
<td>$\mu_s$ at 1,900 psig, cp</td>
</tr>
<tr>
<td>$B_o$ at 1,900 psig, RB/STB</td>
</tr>
<tr>
<td>$\mu B_0$ at 1,250 psig, cp-RB/STB</td>
</tr>
<tr>
<td>$\phi$</td>
</tr>
<tr>
<td>$S_{wi}$</td>
</tr>
<tr>
<td>$h$, ft</td>
</tr>
<tr>
<td>$r_o$, ft</td>
</tr>
</tbody>
</table>

Clyde Cowden: Analysis Results, Average Well
From Primary Decline Period Match

<table>
<thead>
<tr>
<th>Match Point</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$q(t)$, bbl/month oil</td>
<td>10,000</td>
</tr>
<tr>
<td>$t$, months</td>
<td>10</td>
</tr>
<tr>
<td>$r_w/r_{we}$</td>
<td>1,000</td>
</tr>
<tr>
<td>$q_{AD}$</td>
<td>0.11</td>
</tr>
<tr>
<td>$t_{AD}$</td>
<td>0.135</td>
</tr>
<tr>
<td>$b$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Average Well

| $kh$, md-ft | 1,519 |
| $k$, md | 32 |
| $V_p$, $10^6$ res bbl | 2.07 |
| $r_w$, ft | 717 |
| $r_{we}$ | 0.717 |
| $s$ | -0.8 |
| $N_s$ (average well), MMSTB | 0.97 |
| $N_u$ (base), MMSTB | 48.5 |

Produced before decline, MMSTB

| $N_u$ | 1.3 |
| $N_u$ (material-balance calculation), MMSTB | 49.8 |

![Fig. 41—Monterey constant-wellbore-pressure type curve.](image)

The results of type-curve analysis and relevant reservoir data are summarized in Table 11. The match-calculated average well permeability of 32 md is in good agreement with the permeability range of 20 to 30 md reported for the Upper Clearfork, and the negative skin of -0.8 is consistent with the perhaps only moderately effective acid treatment at completion.

With the method detailed earlier for Edda, the match-derived $N_u$ value of 49.8 MMSTB [$7.92 \times 10^6$ stock-tank m$^3$] agrees well with the 48 to 52 MMSTB [$7.63 \times 10^6$ to $8.27 \times 10^6$ stock-tank m$^3$] $N_u$ estimates from earlier material-balance calculations.

Production forecasts were made for the primary decline and for Waterflood Period 2 on the basis of the decline match of $b=0.3$ for both. A recovery of 9.8 MMSTB [$1.56 \times 10^6$ stock-tank m$^3$] (or 20% of 49.8 MMSTB $N_u$ [$7.92 \times 10^6$ stock-tank m$^3$]) is forecast for primary recovery; the primary-plus-waterflood forecast is 18.4 MMSTB [$2.93 \times 10^6$ stock-tank m$^3$] (or 37% of $N_u$). The incremental recovery of 8.6 MMSTB [$1.37 \times 10^6$ stock-tank m$^3$] resulting from waterflooding represents 88% of primary recovery.

That the primary decline and the waterflood decline appear to fall on the same decline stem, $b=0.3$, may be coincidental. While we can think of some possible explanation why this should occur, we can think of none why this must occur. As such, one of the main reasons for presenting this example is to encourage others to examine primary and waterflood histories for their decline exponents. We suppose that if we were asked to produce a waterflood forecast in 1 day given primary production history, we would as a first approximation forecast a waterflood decline on the same value of $b$ as that established during primary recovery.

![Fig. 42—Overlay of log-rate-vs.-log-time production data of five California Monterey producing fields.](image)

Monterey Type Curve. In an effort to determine the production performance characteristics of fields producing from the Monterey formation in California, production data from the Lompoc and Orcutt fields were obtained, and an average well rate-time curve was established for each field.

A log-log $t$ tracing-paper plot was made from the data for each of the fields. If these two separate log q-log $t$ plots are overlaid and the vertical and horizontal axes shifted as required, they exactly overlay. This was a surprise at first, yet was consistent with type-curve theory in that the plots should differ only by the coefficients involved in the definition of $q_{AD}$,$t_{AD}$—i.e., reservoir and fluid variables. Further, the drive mechanisms, relative permeability relationships, and natural fracturing characteristics also must be similar.

The question of the correctness of the representative average well was immediately raised. Good-quality, unambiguous data available on a single lease within the Orcutt field were also plotted log q-log $t$, and that plot also exactly overlaid both of the field plots. This appeared to verify the total-field-averaging technique used to obtain our average well plot for each field. Each plot by itself was limited in range; however, all three plots combined virtually doubled the total range of data.

Although suggested by theory, this was the first test of the concept that fields within the same formation having similar drive mechanisms should overlap each other, regardless of the fact that they may have totally different fluid and rock properties, well spacing, stimulation response, and reservoir pressures.

Fluid and rock properties, spacing, and reservoir and flowing pressures were then estimated for the Lompoc and Orcutt fields and a $q_{AD}$,$t_{AD}$ was calculated for each of the rate-time points to establish a dimensionless type curve for the Monterey formation. Our final dimensionless type curve, which we refer to as the Monterey type curve, is shown in Fig. 41. We further confirmed the type curve with the addition of rate-time data from the West Cat Canyon, the Santa Maria Valley, and the Zaca fields. Fig. 42 shows the complete five-field overlay. This type curve essentially matches the harmonic decline stem $b=1$. 

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The $q_{AD}t_{AD}$ type curve developed for the Monterey formation was converted to a $p_{AD}t_{AD}$ type curve to enable us to calculate how long a well or field could produce at a constant rate to a fixed flowing pressure before going on decline (see Fig. 43). The late-time portion of the $p_{AD}t_{AD}$ type curve appears to be a $1/2$ slope, a $\sqrt{t}$ relationship, which may or may not be significant. It is unlikely that linear flow is from the matrix blocks because the apparent linear flow period is too long and the matrix blocks are considered very small. The Monterey chert is generally overlain with a thick, very-low-permeability mudstone. Therefore, crossflow from it could be a possible explanation.

The field or formation type curve represents the ultimate use of reservoir or field analogy. The $q_{AD}t_{AD}$ Monterey type curve is readily applied on an average well basis to develop forecasts for varying reservoir properties and fluid properties, spacing, flowing pressure, and stimulation. Several wells with differing productivities could be used by proportioning total reservoir volume to correspond with each individual well productivity factor to arrive at an average well. Summing each well's forecast developed from the Monterey type curve would result in a more straightforward total field forecast.

Still further flexibility can be added by the use of two conventional one-cell, material-balance forecasts, which when combined match the Monterey type curve. Each material balance allows the inclusion of a drilling schedule, downtime, a relative permeability curve to predict gas rates, and oil and gas production limits. Two forecasts were generated with such a simple model, assuming there exists an intensely fractured area, or volume, and a slightly fractured or nonfractured area or volume with very contrasting deliverabilities. The contrasting deliverability areas can exist areally or vertically. (See Ref. 23 for the effect of hydrocarbon influx of various degrees from a low-permeability outer-boundary region.) The percentage of fractured-area volume to nonfractured-area volume was arrived at by trial and error to get a composite decline rate-time data match of the Monterey type curve. Each separate forecast had the typical solution-gas drive of $b=0.3$.

If one were to develop a sophisticated three-dimensional fracture model, it should first be history-matched to the Monterey type curve before being used to make production forecasts.

Fig. 44 is a semilog plot of the Monterey type curve. Of special note is the precipitous early decline, which later flattens. This behavior can easily be misinterpreted as indicating depletion of the fracture volume followed by flow from only the matrix blocks. The log-log plot of the Monterey type curve does not exhibit in any way the double depletion stems characteristic of the constant-wellbore-pressure solution of the dual-porosity Warren and Root model.

**Conclusions**

1. For decline-curve analysis, a log $q$-log $t$ plot should be made to identify transient data and/or depletion data. The plot should be reinitialized in time to eliminate any constant-rate production period.

2. The Arps equation must be applied only to rate-time data that indicate depletion. The limits of the value of $b$, when the Arps equation applies, are between 0 (exponential) and 1 (harmonic). A forced fit of transient data to the Arps equation results in apparent values of $b > 1$; generally, these false values of $b$ fall in the range of 2.2 to 2.5. Further, rapidly declining rate data are characteristic of low-permeability stimulated wells (apparent $b > 1$). Often a unique fit of such data can be obtained on the transient portion of a type curve. The misuse of the Arps equation with transient data generally results in overly optimistic forecasts and is technically incorrect.

3. With rate-time data, the double-depletion decline is the only indication of a dual-porosity system. The dual-porosity, constant-wellbore-pressure solution (Warren and Root model) does not anywhere indicate $b > 1$, except for the apparent $b$ values from the early transient period; the character of this period is identical to the homogeneous-reservoir solution.

4. The pore volume and reservoir parameters $kh$ and $s$ at the start of decline analysis can be calculated from a type-curve match once depletion is indicated by the data and the decline exponent $b$ is defined or reasonably estimated.

5. In low-permeability gas reservoirs, reserve estimates and production forecasts developed from rate-time data would be more appropriate than conventional curves of $p_r$ vs. cumulative production used with the calculated stabilized backpressure curve approach.

6. Superposition with $q_{AD}t_{AD}$ with values of $b > 0$ can be successfully applied to field problems.

7. Field or formation $q_{AD}t_{AD}$ type curves can be developed from basic reservoir data and declining rate-time production data.

8. Advanced decline-curve analysis should always be supported by and checked against already existing well and reservoir information.

**Recommendations**

An effort should be made to obtain accurate rate and surface flowing pressure data to improve the reliability of decline-curve analysis. If these data are taken frequently at early times while producing widely open, if possible, or at a fixed choke setting, then an evaluation of such initial reservoir parameters as $kh$ and $s$ can be made to assist in fixing the $r_i/r_w$ decline-curve stem once depletion sets in. The Edda case history is a classic example of what can be done with good-quality data taken frequently.

Dimensionless type curves to characterize the more important producing formations should be developed from the vast amount of existing field data.

Finally, some analytic work needs to be done with regard to determining what values of $b$ should result from a water displacement process. We need a stronger theoretical basis for waterflood decline-curve analysis.

**Nomenclature**

- $b$ = reciprocal of decline-curve exponent
- $B_g$ = gas FVF, surface vol/res vol
- $B_o$ = FVF, res vol/surface vol
- $c_g$ = gas compressibility, psi$^{-1}$ [kPa$^{-1}$]
- $c_i$ = total compressibility, psi$^{-1}$ [kPa$^{-1}$]
\[ C = \text{gas-well backpressure-curve coefficient} \]
\[ D_t = \text{initial decline rate, } t^{-1} \]
\[ e = \text{natural logarithm base, } 2.71828 \]
\[ G = \text{gas in place at start of decline analysis, surface-measured} \]
\[ G_i = \text{initial gas in place, surface-measured} \]
\[ G_p = \text{cumulative gas production, surface-measured} \]
\[ h = \text{thickness, ft [m]} \]
\[ I_f = \text{fracture index (Eq. 8)} \]
\[ k = \text{effective permeability, md} \]
\[ L_r = \text{reservoir half-length, ft [m]} \]
\[ L_{r_f} = \text{fracture half-length, ft [m]} \]
\[ n = \text{exponent of backpressure curve} \]
\[ N_p = \text{cumulative oil production, STB [stock-tank m³]} \]
\[ N_o = \text{oil in place at start of decline analysis, STB [stock-tank m³]} \]
\[ p_i = \text{initial pressure, at start of decline, psia [kPa]} \]
\[ p_{gb} = \text{gas pseudopressure, psi}^2/\text{cp [kPa}^2/\text{mPa·s}] \]
\[ p_{po} = \text{oil pseudopressure, psi/\text{cp [kPa}/\text{mPa·s}] } \]
\[ p_{wf} = \text{bottomhole flowing pressure, psia [kPa]} \]
\[ q_{AD} = \text{decline-curve dimensionless rate (Eq. 4)} \]
\[ q_d = \text{dimensionless rate} \]
\[ q(t) = \text{surface rate of flow at time } t \]
\[ r_e = \text{external-boundary radius, ft [m]} \]
\[ r_w = \text{wellbore radius, ft [m]} \]
\[ r_{wa} = \text{effective wellbore radius, ft [m]} \]
\[ s = \text{skin} \]
\[ S_w = \text{water saturation} \]
\[ t = \text{time, days for } t_D \]
\[ t_{AD} = \text{decline-curve dimensionless time} \]
\[ t_D = \text{dimensionless time} \]
\[ T = \text{reservoir temperature, °R [K]} \]
\[ V_p = \text{reservoir PV, ft}^3 \text{ or bbl (consistent units on } q \text{ and } B) \]
\[ z = \text{gas compressibility factor, dimensionless} \]
\[ \Delta = \text{change} \]
\[ \lambda = \text{type-curve parameter used to characterize gas well drawdown, dimensionless} \]
\[ \mu = \text{viscosity, cp [Pa·s]} \]
\[ \phi = \text{porosity, fraction of bulk volume} \]

Subscripts
- BU = buildup
- DD = drawdown
- F = field
- g = gas
- i = initial
- o = oil
- p = production

Superscript
- = average

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References

SI Metric Conversion Factors
- $\text{cp} \times 1.0 \times 10^8 \text{E} + 00 = \text{mPa·s}$
- $\text{ft} \times 3.048 \times 10^3 \text{E} - 01 = \text{m}$
- $\text{ft}^3 \times 2.831685 \times 10^2 \text{E} - 02 = \text{m}^3$
- $\text{°F} \times (\text{°F} - 32)/1.8 \times 10^4 \text{E} - 01 = \text{°C}$
- $\text{md-ft} \times 3.008142 \times 10^2 \text{E} - 03 = \text{md-m}$
- $\text{psi} \times 6.894757 \times 10^1 \text{E} + 00 = \text{kPa}$
- $\text{psi}^{-1} \times 1.450377 \times 10^2 \text{E} - 01 = \text{kPa}^{-1}$

*Conversion factor is exact.*