A Production-Based Method for Direct Estimation of Gas-in-place and Reserves

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SPE Members

Abstract

This paper presents a systematic method for the direct estimation of gas-in-place and reserves using only gas production data. The fundamental concept used in this work is "quadratic" rate-cumulative relation, which is given as:

\[ q_g = q_{gi} - D_i G_p + \frac{1}{2} D_i^2 G_p^2 \] .....................................................(1)

Where we define the decline constant, \( D_i \), for this case as:

\[ D_i = \frac{2q_{gi}}{\left[ 1 - \frac{p_{sat}/p_{gas}}{p_i/p_{gas}} \right]^2 G} \] .........................................(2)

The basis of our method is the development and application of several rate-time/cumulative production-time plotting functions which are derived using Eq. 1 (see Appendix A). Eq. 1 is itself derived from a rigorous coupling of the gas material balance equation for a volumetric dry gas reservoir and a modified version of the stabilized gas flow equation for a gas well producing at a constant bottomhole pressure. The proposed procedure is designed to simultaneously match the independent plotting functions (developed from Eq. 1), where we note that each function exhibits unique characteristics (e.g., a straight-line trend) which can be used as an extrapolation mechanism for estimating gas reserves.

In this work we utilize a "spreadsheet" approach in which the data and plotting functions are linked by "global" model parameters, thus ensuring consistent evaluations. We verify our methodology using a simulated case and we then illustrate the application of this method using a field case. Although theoretical considerations limit the rigorous use of the proposed methodology to reservoir pressures less than 6,000 psia, we have successfully applied of our method in practice for reservoir pressures as high as 10,000 psia.

Introduction

The classic works of Arps\(^\text{1}\) and Fetkovich\(^\text{2}\) illustrate the analysis of well performance data using empirically-derived exponential, harmonic, and hyperbolic functions. Although Arps' work is completely empirical (see derivation in Appendix B), this work does provide us with a family of rate-time and cumulative production-time relations that are valid (at least in a practical sense) for a variety of producing conditions.

In fact, the Arps' equations continue to enjoy widespread use in the upstream petroleum industry, particularly for production predictions and for estimating reserves from production decline behavior. However, the motivation for this work (i.e., our new reserves estimation methods for gas wells) is our observation that the Arps' relations can yield inconsistent results (unreliable matches, poor extrapolations, etc.). We do not conclude the hyperbolic relations have no utility, but rather, we find that the "quadratic cumulative" relations derived in this work tend to be more consistent, and provide better results for a wide variety of gas reservoir conditions.

For reference, the Arps' relations are summarized in Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>Rate Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential: ((b=0))</td>
<td>( q_g(t) = q_{gi} \exp(-D_i t) ) .........................(3)</td>
</tr>
<tr>
<td>Hyperbolic: ((0&lt;b&lt;1))</td>
<td>( q_g(t) = \frac{q_{gi}}{1 + b D_i t} ) ................................(4)</td>
</tr>
<tr>
<td>Harmonic: ((b=1))</td>
<td>( q_g(t) = \frac{q_{gi}}{D_i} ) ..................................(5)</td>
</tr>
</tbody>
</table>

Arps Cumulative Production Relations:

<table>
<thead>
<tr>
<th>Case</th>
<th>Cumulative Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential: ((b=0))</td>
<td>( G_p(t) = \frac{q_{gi}}{D_i} \left[ 1 - \exp(-D_i t) \right] ) ...................(6)</td>
</tr>
<tr>
<td>Hyperbolic: ((0&lt;b&lt;1))</td>
<td>( G_p(t) = \frac{q_{gi}}{D_i} \left[ 1 - (1 + b D_i t)^{1/b} \right] ) ......(7)</td>
</tr>
<tr>
<td>Harmonic: ((b=1))</td>
<td>( G_p(t) = \frac{q_{gi}}{D_i} \ln(1 + D_i t) ) .....................(8)</td>
</tr>
</tbody>
</table>

Regarding the work by Fetkovich\(^3\) and Fetkovich, et al\(^4\), we do note that their original work does discuss the use of the Arps' hyperbolic relations, and they do provide a semi-analytical...
result for gas flow behavior (which unfortunately, is never valid in practice). For reference, the "Fetkovich" gas rate-time relation is given as: \((\text{from ref. 2})\)

\[
\frac{q_g}{q_{gi}} = \frac{1}{1 + (2n-1) \left[ \frac{q_{gi}}{G} \right]^{2n-1}} \quad (z=1, \rho_{wf} = 0) \quad \text{.........(9)}
\]

Eq. 9 "looks" hyperbolic, but as noted above, this relation is a very poor approximation, and should only be considered as a concept approximation (and should never be used in practice). The Arps' hyperbolic relations were utilized as rate-time decline type curves and typically show a behavior where the actual gas rate (synthetic or observed) changes evaluation curves over time (i.e., changes from one hyperbolic model (or "stem") to another with time). These deviations have been attributed to changes in gas properties as a function of reservoir pressure (i.e., pressure-dependent properties).

To address the impact of pressure-dependent gas properties on the evaluation of gas production data, Carter\(^{2}\) presented a decline type curve for gas reservoir systems. Although the Carter approach is more rigorous than simply using the hyperbolic model, the Carter solution is not universal. The Carter solution suffers from the constant bottomhole flowing pressure assumption that the Arps' relations (and our present work) are limited to. In addition, both Fetkovich and Carter fail to address the analysis of the gas rate and cumulative gas production data functions.

Ansah, \textit{et al}\(^{5}\) proposed semi-analytical, direct solutions for determining average reservoir pressure, rate, and cumulative production for gas wells produced at a constant bottomhole flowing pressure. This work forms the basis for development of our new plotting functions and analysis methodology. In this paper, we extend the applicability of Ansah, \textit{et al}\(^{5}\) work to develop a systematic method for direct estimation of contacted gas-in-place and gas reserves using only production data.

The specific objectives of this paper are to:

- Develop a new technique and plotting functions to estimate gas-in-place and gas reserves directly from production data analysis; and
- Demonstrate the applicability of our new method using both field data and reservoir simulation cases.

**Summary of Historical Methods for Estimating Gas Reserves by Extrapolation**

While something of an aside, we also need to document the historical methods for estimating gas reserves by extrapolation. The Arps' relations given in Table 1 can be rearranged into extrapolation forms as shown in Table 2. We also include the new "quadratic" rate-cumulative model proposed in this work, and present a sequence example plots (Figs. 1 and 2) which illustrate the behavior of these techniques.

### Table 2 — Summary of Historical Methods for Estimating Gas Reserves by Extrapolation.

**Arps' Relations:** (derived from Ref. 1)

<table>
<thead>
<tr>
<th>Case</th>
<th>Rate-Cumulative Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential: ((\text{b}=0))</td>
<td>Plot: (q_0) versus (G_p)</td>
</tr>
<tr>
<td></td>
<td>(q_g = q_{gi} - D_i G_p) (G = \frac{q_{gi}}{D_i}) \text{.........(10)}</td>
</tr>
<tr>
<td>Hyperbolic: ((0&lt;\text{b}&lt;1))</td>
<td>Plot: (\log(q_{gi})) versus (\log[1-(G/G_p)])</td>
</tr>
<tr>
<td></td>
<td>(q_g = q_{gi} \left[ 1 - \frac{G_p}{G} \right]^b \quad G = \frac{q_{gi}}{(1-b)D_i} \text{.........(11a)} )</td>
</tr>
<tr>
<td></td>
<td>Alternative Plot: (\log(G-G_p)) versus (q_{gi})</td>
</tr>
<tr>
<td></td>
<td>((G-G_p) = \frac{q_{gi}^b}{(1-b)D_i} q_{gi}^{-1-b} \quad \text{.........(11b)} )</td>
</tr>
<tr>
<td>Harmonic: ((\text{b}=1))</td>
<td>Plot: (\log(q_{gi})) versus (G_p)</td>
</tr>
<tr>
<td></td>
<td>(q_g = q_{gi} \exp \left[ -\frac{D_i}{q_{gi}} G_p \right] ) \text{.........(12)}</td>
</tr>
</tbody>
</table>

* The Arps hyperbolic plots are confirmation plots, \textit{not} interpretation/extrapolation plots. If the \(q_{gi}, D_i\), and \(b\) parameters are known, then the straight-line model should fit the straight-line portion of the data trend.

**Ansah, \textit{et al}\"Quadratic Cumulative\" Relation:**

<table>
<thead>
<tr>
<th>Case</th>
<th>Rate-Cumulative Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Plot: (q_{gi}/G_p) versus (G_p)</td>
<td></td>
</tr>
<tr>
<td>(\overline{p} &lt; 6000 \text{ psia})</td>
<td>(q_g = q_{gi} - D_i G_p + \frac{1}{2} D_i G_p^2 ) \text{.........(13)}</td>
</tr>
</tbody>
</table>

The "historical" extrapolation relations (i.e., Eqs. 10-12), as well as the new "quadratic cumulative" extrapolation relation proposed by Ansah, \textit{et al}\ (i.e., Eq. 13) will be compared on appropriate plots in the examples provided later in this work.

**New Diagnostic Plotting Functions**

The sequence of development of the "quadratic cumulative" solution for gas well performance behavior is given below:

**Ansah, \textit{et al}\ "Quadratic Cumulative" Relation:**

<table>
<thead>
<tr>
<th>Author</th>
<th>Ref.</th>
<th>Year</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowles</td>
<td>6</td>
<td>1999</td>
<td>Original development of modified gas flow equation.</td>
</tr>
<tr>
<td>Buba</td>
<td>7</td>
<td>2003</td>
<td>Development/valid&amp; of &quot;quadratic cumulative&quot; plotting functions for gas reserves.</td>
</tr>
</tbody>
</table>

As noted, the semi-analytical plotting functions for our new method are derived in Appendix A (using ref. 7, we reproduce all relevant details). To demonstrate the function and utility of these plotting functions, we provide a series of schematic diagnostic plots generated using synthetic (simulated) performance data. We will not only illustrate the characteristic behavior of each function, but also provide guidance on how to analyze and interpret data on a given plot. The individual plotting functions are summarized in Table 3 — for reference, the "straight-line" plotting functions (used for reserves
cumulative gas production against cumulative gas production (i.e., \( q_{g,i,Gp} \) vs. \( G_p \)). Comparing Figs. 1 and 2, the \( q_{g,i,Gp} - G_p \) trend has many characteristics similar to the \( q_{g,Gp} - G_p \) trend — the data in both cases exhibit a quadratic trend where the minimum value of \( G_p \) defines the location of the maximum gas production (i.e., gas-in-place, \( G \)). For this case (i.e., the \( q_{g,i,Gp} - G_p \) comparison), the root is \( G_{p,\text{max}} = 3/2G \) Unlike the \( q_{g,Gp} - G_p \) trend, the \( q_{g,i,Gp} - G_p \) trend does not become negative.

The first extrapolation plotting function \( (PF_1) \) is defined by \( (q_{g} - q_{g})/G_p \) versus \( G_p \) and yields a linear trend which can be extrapolated to provide an estimate of gas-in-place (\( G \)). The \( (q_{g} - q_{g})/G_p \) versus \( G_p \) data functions are typically not significantly affected by long-term transient flow (e.g., for a low permeability gas reservoir). Consequently, the function is relatively easy to construct and is generally well-behaved.

In Fig. 3 we present a schematic plot of \( (q_{g} - q_{g})/G_p \) versus \( G_p \) which yields a straight line with an intercept equal to the dimensionless decline \( (D_i) \) and a slope equal to \( D_i/2G \). The extrapolation to \( (q_{g} - q_{g})/G_p = 0 \) gives \( G_{p,\text{max}} = 2G \).

The second extrapolation plotting function \( (PF_2) \) is defined by \( (q_{g} - q_{g})/G_p \) versus \( G_p \), where the \( (q_{g} - q_{g})/G_p \) function is defined as:

\[
(q_{g} - q_{g})/G_p = \frac{1}{G_p} \int_0^{G_p} q_g \, dG
\]  

Similarly to the \( PF_1 \) trend \( ((q_{g} - q_{g})/G_p \) versus \( G_p \)), the \( PF_2 \) trend \( ((q_{g} - q_{g})/G_p \) versus \( G_p \) also yields a straight-line trend during boundary-dominated flow conditions, where this linear trend can be extrapolated to provide an estimate of gas-in-place (\( G \)). We note that \( PF_2 \) is generally smoother than \( PF_1 \), since the \( G_p \) integral averaging process (i.e., Eq. 19) provides an "integral smoothing" of the production data. Unfortunately, the \( (q_{g} - q_{g})/G_p \) versus \( G_p \) is occasionally affected by extended transient flow behavior, but we note that this issue can be resolved by a simple subtractive correction to remove the effect of transient flow behavior from the \( (q_{g} - q_{g})/G_p \) function.

In Fig. 4, a we provide a schematic plot of \( (q_{g} - q_{g})/G_p \) versus \( G_p \), and we note that this function \( (PF_2) \) yields a straight line with an intercept equal to \( D_i/2 \) and slope equal to \( D_i/6G \). The extrapolation to \( (q_{g} - q_{g})/G_p = 0 \) yields \( G_{p,\text{max}} = 3G \).
using our new "quadratic cumulative" functions (Fig. 6) and the Arps’ hyperbolic functions (Fig. 7). These type curves are adapted from those cases derived and presented in ref. 8 for the hyperbolic rate decline cases — and are used in this work to orient the analyst, as well as to verify the analysis of the rate-cumulative production data.

Fig. 6 — Rate-cumulative production decline type curve — gas case (solution developed using the Ansah, et al “quadratic cumulative” solution (ref. 5))

Fig. 7 — Rate-cumulative production decline type curve — hyperbolic rate case (solution developed using the Arps’ hyperbolic rate decline relations (refs. 1,9))

Arps’ Rate-Cumulative Production Reserves Methodology (Arps’ Reserves Extrapolation Plots)
The Cartesian "rate-cumulative" plot has evolved from empirical roots to become the primary reserves mechanism for the petroleum industry. We present a schematic version of this plot and provide orientation for the application of this plot.

Fig. 4 — Schematic behavior: \[ (q_t G_p - q_G G_p) \] versus \( G_p \) (PF3).

The last extrapolation plotting function (PF3) is a combination of PF1 and PF2 — specifically, PF3 is defined as \( (q_t G_p - q_G G_p) \) versus \( G_p \). One advantage of this formulation is that we have eliminated the intercept rate term \( q_G \). However, this formulation suffers from some potentially serious issues — first, an erratic rate function \( q_t \) would distort PF3 more than PF1 and PF2. Second, an extended transient flow period could affect the \( (q_t G_p - q_G G_p) \) function to the point of making PF3 difficult to employ. We have not experienced serious limitations with PF3, but we believe that vigilance is necessary to ensure effective use of PF3 in practice.

As noted, use of both the \( q_t \) and \( (q_t G_p - q_G G_p) \) functions tends to amplify distortions not only in the data but also any behavior that may not be represented by the base model. In other words, PF3 is the most sensitive of the proposed extrapolation plotting functions. As we show in Fig. 5, a plot of \( (q_t G_p - q_G G_p) \) versus \( G_p \) yields a straight line with an intercept equal to \( D_t/2 \) and the slope equal to \( D_t/3G \). The extrapolation to \( (q_t G_p - q_G G_p) / G_p = 0 \) gives \( G_p, max = 3G/2 \).

As part of our integrated analysis technique, we also link the interpretation of the new plotting functions with new gas rate versus cumulative gas production decline type curves which were developed using the new quadratic functions (see Appendix C and ref. 7 for details regarding the construction of these "rate-cumulative" decline type curves for the gas case). These new gas decline type curves are also compared with Arps’ hyperbolic functions. The type curves shown in Figs. 6 and 7 are plots of dimensionless rate and dimensionless time-averaged rate versus dimensionless cumulative production.
As an attempt to establish a "straight-line" analysis tool for the Arps' hyperbolic rate and cumulative relations we have used Eq. 11a to create a "log-log" type curve solution that does yield the desired "straight-line" behavior.

\[
q_g = q_{gi} \left( 1 - \frac{G_p}{G} \right) \left( \frac{1}{1-b} \right) D_i
\]

\[
G = q_{gi} \left( \frac{1}{1-b} \right) D_i
\]

In Fig. 9 we present the "Arps hyperbolic type curve" using \( q_g/q_{gi} \) on the log(\( y \))-axis and \( 1-(G_p/G) \) on the log(\( x \))-axis, as required by Eq. 11a.

Unfortunately, Fig. 9 cannot be used to directly estimate the hyperbolic parameters \( q_{gi}, G \) (or \( D_i \), \( b \)) because of the structure of the of the \( x \)-axis function \( 1-(G_p/G) \), the gas-in-place \( G \) must be known in advance — or must be estimated simultaneously (e.g., in a spreadsheet implementation). However, we believe that Fig. 9 can (and should) be used as a "confirmation plot" for a given rate-cumulative analysis.

As part of our effort to "qualify" the hyperbolic decline behavior of the \( q_g-G_p \) trend, we have also constructed a "type curve" (though presented on Cartesian coordinates) using the \( q_g/q_{gi} \) versus \( G_p/G \) functions. This plot is shown in Fig. 10, and should also be used as a "confirmation" (or characteristic behavior) plot.

**Systematic Evaluation Methodology**

To evaluate gas-in-place using our new plotting functions, we employ an integrated solution technique where the model parameters are systematically modified to match the function and data. Moreover, our approach utilizes the model and data functions interactively — i.e., we use a spreadsheet format with a graphical display of the model and the data.

It is important to note that, in this spreadsheet approach, all of our analyses are integrated, and each functional relation is linked by a common (or "global") set of model parameters. Although we have attempted to incorporate an interactive analysis approach, we do not, however, use or advocate a completely automated analysis — the quadratic reservoir model is very sensitive, and may yield erroneous results if not applied systematically (this caveat also holds true for the use of the hyperbolic rate relations as well).

When using the plotting functions, the most difficult variable to assess is the initial gas flow rate. Consequently, we utilize a "high-correct-low" estimate of \( q_{gi} \), i.e., we attempt to bracket this parameter value. In our case, we use a 10 percent high-low spread (i.e., \( 0.9q_{gi}, q_{gi}, 1.1q_{gi} \)). This methodology is coupled with our "linked" analysis, and we provide calculated production data functions trends against which the model functions are matched. This procedure will be illustrated in detail with both simulated and field data. For reference, we utilize the \( 0.9q_{gi}, q_{gi}, 1.1q_{gi} \) values in our analysis procedure as the \( q_{gi} \) parameter with all plotting functions except \( PF_3 \).

Recall that \( PF_3 \) is the most sensitive of the plotting functions, and the \( q_{gi} \) parameter is not used in that function.

**Validation of New Analysis Methodology**

We first illustrate application of our new production-based analysis method using simulated gas well performance. Pert-
sequent production and reservoir input parameters for the simulated case are summarized below.

**Reservoir Properties:**
- Wellbore radius, $r_w = 0.0745$ ft
- Net pay thickness, $h = 30$ ft
- Formation permeability, $k = 100$ md
- Porosity, $\phi$ (fraction) = 0.30
- Nominal well spacing = 40 acres
- Initial reservoir pressure, $p_i$ = 1000 to 10,000 psia
- Initial gas-in-place, $G$ = 4.20 BSCF

**Fluid Properties:**
- Gas specific gravity, $\gamma_g = 0.6$ (air = 1.0)
- Reservoir temperature, $T = 200$ deg F

**Production Parameters:**
- Constant bottomhole pressure, $p_{wf} = 500$ or 1000 psia

Figures 11-13 are log-log plots of the simulated gas flow rate and cumulative gas production versus time, as indicated. While we do not use these plots directly in our analysis, these plots do provide a "time-dependent" data perspective.

Fig. 11 — Simulated Performance Case: $q_g$ versus $t$ fitted using the "quadratic cumulative" rate model ($p_i = 5000$ psia, $p_{wf} = 1000$ psia, $G_{input} = 4.20$ BSCF).

We do note that the data and plotting functions match quite well (as we would expect for this case, since it fits our requisite condition i.e., $p_i < 6000$ psia). Perhaps most importantly, there are no surprises — the boundary-dominated flow behavior is accurately represented by the Ansah, et al. model.

We next present plots of $q_g$ and $G_p$ (i.e., our base quadratic plotting functions) in Figs. 14 and 15, respectively. Our objective is to illustrate the behavior of these quadratic functions in a format where we can verify agreement between the data and model functions. Recall that, in these plots, we present each model function computed using the 0.9$q_g$, $q_g$, and 1.1$q_g$ values so that we may generate comparative trends and view the correct solution with slightly erroneous solutions nearby.

As illustrated in Figs. 14 and 15, the multiple model functions match the respective data functions quite well. We also note that each function exhibits the expected quadratic minimum (i.e., the point at which we can estimate the gas-in-place). This estimate of gas-in-place is used as initial input for subsequent analyses using the linear plotting functions $PF_1$, $PF_2$ and $PF_3$. 
Fig. 14 — Simulated Performance Case: \( q_g \) versus \( G_p \) (\( p_i = 5000 \text{ psia}, \ p_w = 1000 \text{ psia}, \ G_{\text{input}} = 4.20 \text{ BSCF} \)).

Fig. 15 — Simulated Performance Case: (\( q_g \) versus \( G_p \)) (\( p_i = 5000 \text{ psia}, \ p_w = 1000 \text{ psia}, \ G_{\text{input}} = 4.20 \text{ BSCF} \)).

Figures 16 and 17 illustrate the characteristic trends for plotting functions 1 and 2, respectively. As discussed earlier, we utilize a "high-correct-low" estimate of \( q_{gi} \) in an attempt to bracket the parameter using a 10 percent high-low spread (i.e., \( 0.9 \times q_{gi}, q_{gi}, 1.1 \times q_{gi} \)) for \( PF_1 \) and \( PF_2 \) only.

The model and data functions shown in Fig. 16 for \( PF_1 \) agree very well once boundary-dominated flow behavior is established. Although the data were simulated, the convergence of the \( 0.9q_{gi}, q_{gi}, 1.1q_{gi} \) trends is somewhat universal in our experience. Similarly, Fig. 17 illustrates the characteristic behavior of both the data and model functions for the extrapolation plotting function, \( PF_3 \). We should note that the linear trends exhibited by the model and data functions in Fig. 17 appear to be converging at a slower rate than similar data functions in Fig. 16, but this slower convergence rate of the integral-type functions does not affect the overall data analysis/interpretation (i.e., we obtain the same model match).

The match shown in Fig. 18 emphasizes the importance of a calculation mechanism through which all analyses are linked (i.e., a fully integrated analysis/interpretation approach). Specifically, we might be tempted to make adjustments to individual analyses in order to obtain a better "local" match of the data and model — however, such uncoordinated evaluations would invalidate not only the rigor of the analysis but also the "global" coordination of our data analysis. Failure to provide simultaneous analyses of all data/model functions will result in inconsistent results and, ultimately, a loss in the rigor and utility of this approach.

In Figs. 19 and 20 we present the quadratic and hyperbolic rate-cumulative decline type curve analyses, respectively. We note that in Fig. 19, the simulated rate (circle symbols) and cumulative-averaged rate data (square symbols) overlay a single quadratic type curve for the complete duration of boundary-dominated flow.
Fig. 19 — Simulated Performance Case: "Quadratic" Rate-Cumulative Decline Type Curve Analysis (G_{input}=4.20 BSCF, G_{quad}=4.20 BSCF).

However, when these simulated data are plotted on the "hyperbolic" type curve (Fig. 20) we note immediately that the data appear to follow more than one hyperbolic type curve (or "stem"). This observed behavior on the hyperbolic type curve makes a unique estimate of gas-in-place unlikely.

In Figs. 21 and 22 we provide the specialized plots developed in this work for the hyperbolic rate decline relation given by Arps (ref. 1). Our goal is to use these plots as confirmation/validation devices to support (or dispute) that the hyperbolic model is relevant for the interpretation (and extrapolation) of gas well performance data for a particular case.

From Figs. 21 and 22 we can conclude that the data for our simulated case does mimic the hyperbolic trends. However, we must also note that the data trends cross several model stems in both Figs. 21 and 22 — which is a common observation for the gas case, but this behavior warrants caution in any application of the Arps hyperbolic relations to interpret, analyze, or predict the performance behavior of a gas well.

To further validate the results of our new integrated plotting function approach, we also analyze the simulated production data using the material balance decline type curve approach (refs. 8-9).
It is important to note that these type curves rigorously incorporate both pressure- and time-dependent gas properties using the appropriate pseudopressure and pseudotime functions. As shown in Fig. 23, the decline type curve analysis yields essentially exact results for this case (i.e., the simulation input data are reproduced from the analysis and interpretation of the performance data).

**Application of Integrated Approach to Field Data**

In this section, we discuss the analysis and interpretation of field data using our new analysis approach and plotting functions. Although we have performed this analysis procedure on a wide variety of field data, due to space considerations, we have selected a single representative case from the literature as a demonstration example.

**Field Case No. 1 — West Virginia Well A.**

Fetkovich, et. al. evaluated this data using the Fetkovich decline type curves. This particular case has become a standard for the analysis of gas well performance data since the results compare quite well regardless of the analysis methodology employed. Further, the data appear to be remarkably consistent with theoretical methods that are derived assuming constant flowing bottomhole pressure conditions. The reservoir, fluid, and production parameters for this case are summarized below.

**Reservoir Properties:**
- Wellbore radius, $r_w = 0.354$ ft
- Net pay thickness, $h = 70$ ft
- Porosity, $\phi$ (fraction) = 0.06
- Irr. water saturation, $S_{wi}$ (fraction) = 0.35
- Initial reservoir pressure, $p_i$ = 4175 psia

**Fluid Properties:**
- Gas specific gravity, $\gamma_g = 0.57$ (air = 1.0)
- Reservoir temperature, $T = 160$ deg F

**Production Parameters:**
- Constant bottomhole pressure, $p_{wf} = 710$ psia

The gas production and flowing pressure histories are plotted in Figs. 24 and 25, respectively, while the cumulative gas production match is shown in Fig. 26.
In Figs. 27-31 we present the $q_g$ vs. $G_p$, and $(q_{gi}, G_p)$ vs. $G_p$ functions, as well as plotting functions $PF_1$, $PF_2$, and $PF_3$ (all respectively). As in previous cases, we again present multiple and model functions (for $PF_1$, and $PF_2$) — and we note excellent "convergence" of these functions and the subsequent extrapolations to estimate the gas-in-place, $G_p$.

Fig. 32 — West Virginia Well A: "Quadratic" rate-cumulative decline type curve analysis ($G_{quadr} = 3.29$ BSCF).

Fig. 33 — West Virginia Well A: "Hyperbolic" rate-cumulative decline type curve analysis ($G_{hyp} = 2.52$ BSCF).

We believe the performance of this particular data set is extraordinary relative to our plotting methods/analyses. Even the "high-correct-low" functions (i.e., the $0.9q_{gi}$, $q_{gi}$, and $1.1q_{gi}$ trends) plotted in Figs. 27-28 and 30-31 yield excellent comparison for all functions. This performance is remarkable, and we further note that we obtained a gas-in-place estimate of 3.29 BSCF using this approach, which is comparable to the material balance estimate of 3.36 BSCF given in ref. 3. In short, this example has demonstrated exceptional performance relative to our proposed analysis methodology.
The quadratic and hyperbolic rate-cumulative type curve analyses for the West Virginia Well A are presented in Figs 32 and 33, respectively. Although the data are sparse, the rate data (circle symbols) and the cumulative-averaged rate data (square symbols) shown on the quadratic rate-cumulative type curve (Fig. 32) are matched to a single type curve or stem.

In Figs. 34 and 35 we present the specialized "hyperbolic" plots for this example. We note that on Fig. 34 \( \frac{q_g}{q_{gi}} \) versus \( 1 - \frac{G_p}{G} \), the data function appears to overlay the hyperbolic model uniquely — with a \( b \)-value of approximately 0.45. Fig. 35 exhibits more "data scatter" — but the \( \frac{q_g}{q_{gi}} \) versus \( G_p/G \) data trend agrees with our estimated \( b \)-value of 0.45 (we actually used \( b=0.42 \) for this case). The performance of these plots arguably makes the case that this particular data is accurately represented by the hyperbolic rate decline model.

As with the simulated case, we have also performed rigorous material balance decline type curve analysis (refs. 8-9) on the production data for this case. The results of this analysis are shown in Fig. 36., and we note an excellent match for each of the data functions used in this analysis.

The results from the evaluation of West Virginia Well A using our new quadratic method are summarized below and compared to results from other researchers. We recognize that there are differences in these analyses — in particular, the gas-in-place estimate from the material balance decline type curve analysis is always conservative (most likely due to the "contacted" volume in communication at that time).

**Analysis Results: West Virginia Well A**

**New "quadratic analysis" methods:**
- Initial gas production rate, \( q_{gi} \) = 1920 MSCF/D
- Decline constant, \( D_i \) = 0.00133 1/D
- Dimensionless pressure, \( p_{wD} \) = 0.35
- Gas-in-place, \( G \) = 3.29 BSCF

**Material balance decline type curve analysis:**
- Gas-in-place, \( G \) = 2.79 BSCF
- Gas permeability, \( k_g \) (\( r_{cD}=18 \)) = 0.0524 md
- Near-well skin factor, \( s \) (\( r_{cD}=18 \)) = -5.19 (dim-less)

**Literature Results:** (references as noted)
- Fetkovich, et al.:\(^3\)
  - Gas-in-place, \( G \) = 3.36 BSCF
  - Gas permeability, \( k_g \) (\( r_{cD}=20 \)) = 0.0524 md
  - Near-well skin factor, \( s \) (\( r_{cD}=20 \)) = -5.17 (dim-less)
- Fraim, et al.:\(^10\)
  - Gas-in-place, \( G \) = 3.035 BSCF
  - Gas permeability, \( k_g \) (\( r_{cD}=24 \)) = 0.077 md
  - Near-well skin factor, \( s \) (\( r_{cD}=24 \)) = -5.08 (dim-less)
Gas-in-place, $G_{rD} = 2.62$ BSCF
Gas permeability, $k_g (r_{eD} = 20) = 0.054$ md
Near-well skin factor, $s (r_{eD} = 20) = -4.71$ (dim-less)

Summary and Conclusions
We have developed a new semi-analytical model for analyzing gas rate-time and cumulative gas production-time data. The basis of our method is the integration of independent rate-time/cumulative production-time plotting functions derived from a base model which is derived from a rigorous coupling of the gas material balance equation for a volumetric dry gas reservoir and the stabilized gas flow equation for wells producing at constant bottomhole pressure. The base model, was developed using solutions originally given by Ansah, et al., and Knowles, has been validated using numerous synthetic (simulated) and field data cases.

We provide integrated analysis technique in which we simultaneously match multiple independent plotting functions, where each function exhibits unique characteristics. We utilize a "spreadsheet" approach in which the data and plotting functions are linked by "global" model parameters, thus ensuring consistent evaluations. We believe that the proposed analyses, plotting functions, and procedures are robust — and should be used for the practical analysis of field data.

We acknowledge the primary theoretical limitations of our method, as discussed in Appendix A, are pressure constraints (i.e., constant bottomhole flowing pressure for the duration of production and an initial reservoir pressure less than 6,000 psia). Although we have successfully applied the methodology to synthetic and field data cases where $p_i > 6000$ psia, we still urge caution when analyzing such cases. Furthermore, we recognize that no well produces at a constant flowing bottomhole pressure for its entire producing life, but the proposed methodology appears to give reasonable answers in cases where the pressure history does not vary substantially.

Nomenclature

Field Variables
- $c_f = \text{formation compressibility, psi}^{-1}$
- $c_g = \text{gas compressibility, psi}^{-1}$
- $c_i = \text{total compressibility, psi}^{-1}$
- $D_i = \text{quadratic model decline constant, 1/D}$
- $G = \text{original gas-in-place, MSCF}$
- $G_p = \text{cumulative gas production, MSCF}$
- $G_{p,max} = \text{maximum gas production, MSCF}$
- $q_g = \text{gas production rate, MSCF/D}$
- $q_{gr} = \text{initial gas production rate, MSCF/D}$
- $(q_{gr})_{GP} = \text{cumulative averaged rate function, MSCF/D}$
- $h = \text{net pay thickness, ft}$
- $k = \text{average reservoir permeability, md}$
- $p_i = \text{initial reservoir pressure, psia}$
- $p_{wf} = \text{bottomhole flowing pressure, psia}$
- $r_e = \text{reservoir drainage radius, ft}$
- $r_w = \text{wellbore radius, ft}$
- $S_z = \text{skin factor, dimensionless}$
- $S_{wi} = \text{initial water saturation, fraction}$
- $t = \text{time, days}$
- $T = \text{reservoir temperature, °F}$
- $z = \text{gas compressibility factor}$
- $\phi = \text{effective porosity, fraction}$
- $\gamma_g = \text{reservoir gas specific gravity (air = 1)}$

Dimensionless Variables
- $p_{wD} = \text{dimensionless wellbore pressure}$
- $G_{rD} = \text{dimensionless cumulative production}$
- $q_{rD} = \text{dimensionless rate}$
- $t_{Dw} = \text{dimensionless time}$

References

Appendix A — Derivation of "Buba" Plotting Functions Using the "Quadratic Cumulative" Solution for a Gas Well Produced at a Constant Bottomhole Pressure

This Appendix is developed from the academic work given by Buba (ref. 7). Starting from the generalized dimensionless rate-time and pressure-time relations from Knowles, we have

$$p_{wD} = p_{wD} \left[ \frac{1 - p_{wD}}{1 + p_{wD}} \right] \exp\left(-p_{wD} t_{Dw} \right) \quad \text{...(A-1)}$$

$$q_{rD} = \frac{p_{wD}}{(1 - p_{wD})^2} \left[ \frac{1 - p_{wD}}{1 + p_{wD}} \right] \exp\left(-p_{wD} t_{Dw} \right) \quad \text{...(A-2)}$$

where the dimensionless pressure is defined as
\[ p_D = \frac{\bar{p}}{\bar{z}} \]

The dimensionless time function is defined as:

\[ t_{Dd} = \frac{2 q_i}{(1 - p_{wD})^2} \quad \text{Eq. A-4} \]

And, the definition of dimensionless rate is:

\[ q_{Dd} = \frac{q_i}{q_{gi}} \quad \text{Eq. A-5} \]

The initial rate is estimated using:

\[ q_{gi} = 0.7026 \frac{k_h}{T} \left( \frac{1}{\ln \left( \frac{r_e}{r_w} + \frac{3}{4} + S \right)} \right) \left[ \frac{z_i}{p_i} \mu_{gi} c_{gi} \right] \left[ \frac{b_i}{z_i} \right] \left[ \frac{p_{off}}{z_{off}} \right] \quad \text{Eq. A-6} \]

The material balance relation for a volumetric dry gas reservoir is given as

\[ \frac{\bar{p}}{\bar{z}} = \frac{p_i}{\bar{z}} G \]

We also define the dimensionless cumulative production as

\[ G_{pD} = \frac{G_p}{G} \quad \text{Eq. A-8} \]

Substituting the dimensionless pressure relation (Eq. A-3) into the material balance expression (Eq. A-7) and using the definition of dimensionless cumulative production (Eq. A-8), we have:

\[ G_{pD} = 1 - p_{PD} \quad \text{Eq. A-9} \]

Substituting Eq. A-1 into Eq. A-9, we have

\[ G_{PD} = 1 - p_{wD} \left[ \frac{1 + \frac{1 - p_{PD}}{1 + p_{wD}} \exp(-p_{PD} t_{Dd})}{1 - \frac{1 - p_{PD}}{1 + p_{wD}} \exp(-p_{PD} t_{Dd})} \right] \quad \text{Eq. A-10} \]

We note that Ansah, et al. have used numerical simulation to demonstrate the validity of Eqs. A-2 and A-10 specifically for reservoir pressures less than 6,000 psia. The semi-analytic proof given by Ansah, et al. is based on the pressure-dependent behavior of the viscosity-compressibility factor for dry gases, the required pressure condition was observed to be less than 6,000 psia.

We can simplify the dimensionless cumulative production (Eq. A-10) by defining a temporary parameter, \( \alpha \),

\[ \alpha = \frac{1 + \frac{1 - p_{PD}}{1 + p_{wD}} \exp(-p_{PD} t_{Dd})}{1 - \frac{1 - p_{PD}}{1 + p_{wD}} \exp(-p_{PD} t_{Dd})} \quad \text{Eq. A-11} \]

Substituting the definition of \( \alpha \) into Eq. A-10, and then solving for the \( \alpha \) parameter yields:

\[ \alpha = \frac{1}{p_{wD} (1 - G_{PD})} \quad \text{Eq. A-12} \]

Combining Eq. A-11 and Eq. A-2 gives

\[ q_{Dd} = \frac{p_{wD}^2}{(1 - p_{wD})} \left( \alpha^2 - 1 \right) \quad \text{Eq. A-13} \]

Squaring Eq. A-12 to yield the \( \alpha^2 \)-term, we have

\[ \alpha^2 = \frac{1}{p_{wD}^2} \left( 1 - 2 G_{PD} + G_{PD}^2 \right) \quad \text{Eq. A-14} \]

Substituting Eq. A-14 into Eq. A-13 and simplifying, the final form of the dimensionless rate-cumulative production quadratic expression is given by:

\[ q_{Dd} = \frac{p_{wD}^2}{(1 - p_{wD})^2} \left( \frac{1}{p_{wD}^2} \left( 1 - 2 G_{PD} + G_{PD}^2 \right) - 1 \right) \]

Further reduction in the equation gives the final form of the dimensionless rate-cumulative production as

\[ q_{Dd} = 1 - \frac{2}{(1 - p_{wD})^2} G_{PD} - \frac{1}{(1 - p_{wD})^2} G_{PD}^2 \quad \text{Eq. A-15} \]

Although initially developed for low- to moderately-pressured gas reservoirs (i.e., \( p \leq 6,000 \) psia), the quadratic rate-cumulative behavior defined by Eq. A-15 has not only been validated in a number of field cases where the reservoir pressures exceed 6,000 psia, but has also been validated using numerical simulation. This apparent discrepancy between applicable pressure limitations of Eq. A-15 and Eqs. A-2 and A-10 should not limit the practical application of these relations.

Substituting the definitions of \( q_{Dd} \) and \( G_{PD} \) (Eqs. A-5 and A-8, respectively) into Eq. A-15, we obtain:

\[ q_{gi} = q_{gi} - \frac{2 q_i}{(1 - p_{wD})^2} G_p + \frac{q_i}{(1 - p_{wD})^2} G_p^2 \quad \text{Eq. A-16} \]

Substituting the definition of \( p_{wD} \) into Eq. A-16 yields

\[ q_{gi} = q_{gi} - \frac{2 q_i}{(1 - p_{wD})^2} G_p + \frac{q_i}{(1 - p_{wD})^2} G_p^2 \quad \text{Eq. A-17} \]

For convenience, we define the decline constant, \( D_i \) for this particular case (i.e., Eq. A-17) as:

\[ D_i = \frac{2 q_i}{(1 - p_{wD})^2} \quad \text{Eq. A-18} \]

Substitution of the definition of the decline constant, \( D_i \) into Eq. A-17 yields:

\[ q_{gi} = q_{gi} - D_i G_p + \frac{1}{2 G} G_p^2 \quad \text{Eq. A-19} \]

The form given by Eq. A-19 suggests that the quadratic root from a plot of \( q_{gi} \) versus \( G_p \) will yield an estimate of gas-in-place.

Dividing both sides of Eq. A-19 by \( G_p \) and rearranging into an equation for a straight-line yields
\[ \frac{q_{gi} - q_g}{G_p} = D_i - \frac{1}{2} \frac{D_i}{G_p} \]  
\[ \text{(A-20)} \]

The form given by Eq. A-20 suggests that a plot of \((q_{gi} - q_g)/G_p\) versus \(G_p\) will yield a straight line whose properties provide estimates of both \(D_i\) and \(G\). We define this as Plotting Function 1 (i.e., \((q_{gi} - q_g)/G_p\) versus \(G_p\)). We note for reference that the extrapolation of the \((q_{gi} - q_g)/G_p\) versus \(G_p\) trend to \((q_{gi} - q_g)/G_p = 0\) yields \(G_{p,extr} = 2G\).

We next define the cumulative production averaged rate function, \((q_g)_i G_p\), as

\[ (q_g)_i G_p = \frac{1}{G_p} \int_0^G q_g \, dG \]  
\[ \text{(A-21)} \]

Integrating Eq. A-21 with respect to \(G_p\), then dividing through by \(G_p\) yields:

\[ \frac{(q_g)_i G_p - q_g}{G_p} = \frac{D_i}{2} G_p + \frac{1}{6} D_i G_p^2 \]  
\[ \text{(A-22)} \]

As with Eq. A-19, the form of Eq. A-22 suggests that the quadratic root from a plot of \((q_g)_i G_p\) versus \(G_p\) will yield an estimate of \(G\). While not derived in this Appendix, this particular root equals \(3G/2\).

Rearranging Eq. A-22, then dividing through by \(G_p\) yields:

\[ \frac{1}{G_p} \frac{(q_g)_i G_p - q_g}{G_p} = \frac{1}{2} D_i - \frac{1}{6} D_i G_p \]  
\[ \text{(A-23)} \]

The form of Eq. A-23 suggests a plot of \(\frac{(q_g)_i G_p - q_g}{G_p}\) versus \(G_p\) will yield a straight line that can be used to estimate both \(D_i\) and \(G\). We define this as Plotting Function 2 (or \(PF_2\)).

We immediately note the similarities in Eqs. A-20 and A-23 (\(PF_1\) and \(PF_2\), respectively), and comment that both are linear trends — and differ only by the slope and intercept terms. Specifically, the extrapolation of Eq. A-20 to \((q_{gi} - q_g)/G_p = 0\) yields \(G_{p,extr} = 2G\). Similarly, the extrapolation of Eq. A-23 to \((q_g)_i G_p - q_g = 0\) yields \(G_{p,extr} = 3G\).

We can also construct a final plotting function—Plotting Function 3 (\(PF_3\)) — which is defined using the \((q_g)_i G_p\) and the \(q_g\) data functions. The advantage of \(PF_3\) is that it does not require knowledge of \(q_{gi}\) — however, a disadvantage of this approach is that since the \(q_g\) data function is used, the \(PF_3\) trend mimics any erroneous behavior in the \(q_g\) data function. Subtracting Eq. A-20 from A-23 yields \(PF_3\):

\[ \frac{(q_g)_i G_p - q_g}{G_p} = \frac{1}{2} D_i - \frac{1}{3} \frac{D_i}{G_p} \]  
\[ \text{(A-24)} \]

Again, the form of Eq. A-24 suggests a plot of \(\frac{(q_g)_i G_p - q_g}{G_p}\) versus \(G_p\) will be a straight line, the slope and intercept of which yield estimates of both \(D_i\) and \(G\). The extrapolation of Eq. A-24 to \((q_g)_i G_p - q_g = 0\) yields \(G_{p,extr} = 3/2G\).

The "Buba" plotting functions are summarized in Table A-1.

### Appendix B — Derivation of Arps' Exponential and Hyperbolic Rate Relations (empirical basis)

The sole purpose of this Appendix is to provide a modern documentation of the "Arps" rate relations for the exponential \((b=0)\) and general hyperbolic \((0<b<1)\) cases (see ref. 1). While these relations were documented by Arps over 60 years ago (and were based on a various work by others as far back as the late 1910's), we believe that an informed analyst should review the basis (and hence, limitations) of the Arps' rate relations.

The starting point for the Arps' rate relations is given (in modern nomenclature) by the "loss ratio" and the "derivative of the loss ratio" given as:

\[ a = \frac{1}{D} = -\frac{q}{dq/dt} \]  
\[ \text{(B-1)} \]

Derivative of the Loss Ratio:

\[ b = \frac{d}{dt} \left[ \frac{1}{D} \right] = -\frac{d}{dt} \left[ \frac{q}{dq/dt} \right] \]  
\[ \text{(B-2)} \]

where:

- \(a\) = The original Arps' variable for the loss ratio
- \(D\) = The modern "decline" parameter (general)
- \(D_t\) = Constant "decline" parameter

**Exponential Rate Decline Case: \((b=0)\)**

Taking the reciprocal of Eq. B-1, and working only in terms of the \(D\)-parameter, we have:

\[ \frac{1}{D} \frac{dq}{dt} = -D \]  
\[ \text{(B-3)} \]

Presuming that the \(D\)-parameter is constant, we now employ the use of the variable, \(D_t\), where \(D_t\) is always constant. This yields:

\[ \frac{1}{D} \frac{dq}{dt} = -D_t \]  
\[ \text{(B-4)} \]
Separating the rate \( (q) \) and time \( (t) \) variables, we have:

\[
\frac{1}{q} \int q \, dq = -D_i \int \, dt \tag{B-5}
\]

Integrating Eq. B-5 gives us:

\[
\int_{q_i}^{q} \frac{1}{q} \, dq = -D_i \int_{0}^{t} \, dt
\]

Recalling the definition of the logarithm, we have:

\[
\ln(x) = \int_{1}^{x} \frac{1}{x} \, dx
\]

and the appropriate rule for this case is:

\[
\ln(a/b) = \int_{1}^{a} \frac{1}{x} \, dx - \int_{1}^{b} \frac{1}{x} \, dx
\]

Therefore, the rate integral becomes:

\[
\int_{q_i}^{q} \frac{1}{q} \, dq = \ln \left( \frac{q}{q_i} \right) = -D_i \int_{0}^{t} \, dt
\]

And the right-hand-side reduces to:

\[
\ln \left( \frac{q}{q_i} \right) = -D_i \, t \tag{B-6}
\]

Exponentiating both sides of Eq. B-6 and rearranging we obtain the familiar "exponential rate decline" relation:

\[
q = q_i \exp(-D_i \, t) \tag{B-7}
\]

As noted earlier in this Appendix, the purpose of this derivation is to provide a modern review of a very old (empirical) development. In fact, the form given by Eq. B-7 can be derived rigorously for the case of an oil well (compressible liquid) producing at a constant bottomhole pressure during boundary-dominated flow conditions.

This theory was not explicitly known to Arps at the time of his publication, but ironically, he did note that the "b=0" case typically occurred for oil wells producing above the bubble-point pressure (which coincides with the assumption of a slightly compressible liquid). Specifically, Eq. B-7 should not be considered applicable for the gas flow case as anything other than a "late transient" or "early boundary-dominated flow" approximation.

**General Hyperbolic Rate Decline Case: (0<b<1)**

Starting with Eq. B-2 (slightly rearranged), we have:

\[
\frac{d}{dt} \left[ \frac{q}{dq/dt} \right] = -b \tag{B-8}
\]

Separating the rate \( (q) \) and time \( (t) \) variables — then setting up an "indefinite integration" of Eq. B-8, we obtain:

\[
\int \frac{d}{dt} \left[ \frac{q}{dq/dt} \right] \, dt = -b \int dt \tag{B-9}
\]

Completing the integration on the right-hand-side of Eq. B-9, gives us:

\[
\frac{q}{dq/dt} = -bt + c \tag{B-10}
\]

Where the "c" parameter is a constant of integration resulting from the indefinite integration process. Recalling the definition of the "loss ratio" in terms of the parameter \( D \):

\[
\frac{1}{D} = -\frac{q}{dq/dt} \tag{B-11}
\]

Substitution of Eq. B-11 into Eq. B-10 yields:

\[
\frac{1}{D} = bt - c \tag{B-12}
\]

Evaluation of Eq. B-12 at zero time (i.e., \( t=0 \)) allows us to solve for the "initial" (or constant) value of the \( D \)-parameter (i.e., \( D_i \)). Specifically, we have:

\[
c = \frac{1}{D_i} \tag{B-13}
\]

Substitution of Eq. B-13 into Eq. B-10 yields:

\[
\frac{1}{D} \frac{dq}{dt} = -bt - \frac{1}{D_i} \tag{B-14}
\]

Taking the reciprocal of Eq. B-14 yields:

\[
\frac{1}{D} \frac{dq}{dt} = -bt - \frac{1}{D_i} \tag{B-15}
\]

Integrating Eq. B-15 gives us:

\[
\int_{q_i}^{q} \frac{1}{q} \, dq = \ln \left[ \frac{q}{q_i} \right] = -D_i \int_{0}^{t} \frac{1}{1+bD_i t} \, dt \tag{B-16}
\]

We need to define a variable of substitution as a mechanism to solve the integral on the right-hand-side. We will use the following variable of substitution:

\[
z = 1 + bD_i t \quad (at \, t = 0; \, z = 1 \text{ and at } \, t = t; \, z = 1 + bD_i t)
\]

Therefore:

\[
dz = 1 + bD_i \, dt \quad \text{or} \quad dt = \frac{1}{bD_i} \, dz
\]

Using this substitution, the right-hand-side (RHS) becomes:

\[
\text{RHS} = -D_i \int_{0}^{t} \frac{1}{1+bD_i t} \, dt
\]

\[
= -D_i \int_{0}^{1+bD_i t} \frac{1}{z} \frac{1}{bD_i} \, dz
\]

\[
= -\frac{1}{b} \int_{0}^{1+bD_i t} \frac{1}{z} \, dz
\]

\[
= -\frac{1}{b} \ln[1+bD_i t]
\]

\[
= \ln \left[ \frac{1}{1+bD_i t} \right] = \frac{1}{b} \]

\[
= \ln \left( \frac{1}{1+bD_i t} \right) \frac{1}{b}
\]
Substituting our solution for the right-hand-side of Eq. B-16, we obtain:

\[
\ln\left(\frac{q}{q_i}\right) = \ln\left(1 + \frac{1}{b}\right) \quad \text{.......................................................... (B-17)}
\]

Exponentiating both sides of Eq. B-17, we have:

\[
\frac{q}{q_i} = (1 + \frac{1}{b}) \quad \text{.......................................................... (B-18)}
\]

Solving Eq. B-18 for the rate term \(q\), we have the final form for this derivation:

\[
q = q_i \left(1 + \frac{1}{b}\right)^{-1} \quad \text{.................................................. (B-19)}
\]

It is important to note that the "hyperbolic" condition is defined specifically by Eq. B-2. Recalling Eq. B-2, we have:

\[
b = -\frac{d}{dt}\left[\frac{q}{dq/dt}\right] \quad \text{.................................................. (B-2)}
\]

Conceptually, it is difficult to imagine the relevance of the derivative condition stated by Eq. B-2. In fact, many studies have purported causes of the hyperbolic decline case (dry gas expansion, solution gas-drive (variation in relative permeability, compressibility, etc.), water influx, etc.). However, to date, there has been no rigorous derivation of Eq. B-19 from fundamental theory (although some theoretical work has generated reasonably approximate conditions for a hyperbolic rate trend to occur).

In addition, simple empirical experiments using the addition of say, three exponential trends of significant contrast do often yield "hyperbolic" behavior \(i.e.,\) the combined performance is represented very well by Eq. B-19.

Is all of this mere coincidence? Is there more to the hyperbolic case than is indicated by the condition given by Eq. B-2? Should we continue to consider other cases \(i.e.,\) allowing the \(b\) parameter to vary with time or even recasting the problem with another derivative condition? These questions are relevant for the following reasons.

- The hyperbolic rate decline relation (Eq. B-19) appears to have near universal appeal and application in the upstream petroleum industry.
- "Simple" models remain as an important component of reservoir engineering due to the quality of legacy data.
- What if — what if there is some reasonable and relevant theory governing "hyperbolic" rate performance?

None of these discussions have considered specific reservoir and/or fluid properties or producing conditions. In the end, the case of the hyperbolic rate decline model remains one for in-depth study — and analysts are warned against seeing too much in the hyperbolic relation — the hyperbolic rate decline model should be utilized for what it is — a very useful empirical relation.

**Appendix C — Rate-Cumulative Production Type Curve Derived Using the "Quadratic Cumulative" Gas Well Performance Relation**

In this Appendix we provide the governing equations for the rate-cumulative production type curve derived from the "quadratic cumulative" gas performance model. This type curve is useful for distinguishing the characteristic behavior of the "quadratic cumulative" model — and is used to check the other analyses obtained from the "quadratic cumulative" model \(i.e.,\) the extrapolation plotting functions.

The fundamental relation for the "quadratic cumulative" model is derived in Appendix A. The base result is given as:

\[
q_t = q_{gi} - \frac{2q_{gi}}{(1 - p_{wd})^2} G_p + \frac{q_{gi}}{(1 - p_{wd})^2} G_p^2 \quad \text{............................................... (C-1)}
\]

Where the dimensionless pressure parameter, \(p_{wd}\), is given as:

\[
p_{wd} = \frac{p_{wi} z_i^2}{p_{wi} z_i} \quad \text{.................................................. (C-2)}
\]

The decline constant, \((D)_q\), for the "quadratic cumulative" model is defined as: (this alternative nomenclature is necessary to avoid confusion with the hyperbolic case)

\[
(D)_q = \frac{2q_{gi}}{(1 - p_{wd})^2} G_p \quad \text{.................................................. (C-3)}
\]

Substitution of the definition of the decline constant, \((D)_q\), into Eq. C-1 yields:

\[
q_t = q_{gi} - (D)_q G_p + \frac{1}{2} (D)_q G_p^2 \quad \text{.......................................................... (C-4)}
\]

Dividing through Eq. C-4 by the initial rate, \(q_{gi}\), yields:

\[
\frac{q_t}{q_{gi}} = 1 - \frac{(D)_q}{q_{gi}} G_p + \frac{1}{2} \frac{(D)_q}{q_{gi}} G_p^2 \quad \text{.................................................. (C-5)}
\]

Solving Eq. C-3 for the reciprocal of the gas-in-place, \(1/G\), we have:

\[
\frac{1}{G} = \frac{(1 - p_{wd})^2 (D)_q}{2 q_{gi}} \quad \text{.................................................. (C-6)}
\]

Substituting Eq. C-6 into Eq. C-5, we obtain:

\[
\frac{q_t}{q_{gi}} = 1 - \frac{(D)_q}{q_{gi}} G_p + \frac{(1 - p_{wd})^2}{4} \left(\frac{(D)_q}{q_{gi}}\right)^2 G_p^2 \quad \text{.................................................. (C-7)}
\]

Defining the following specialized dimensionless "decline" variables:

\[
(D_{qd})_q = \frac{(D)_q}{q_{gi}} \quad \text{.................................................. (C-8)}
\]

\[
(G_{pdq})_q = \frac{(G_{pd})_q}{q_{gi} (D)_q} \quad \text{.................................................. (C-9)}
\]

Substitution of these specialized dimensionless "decline" variables \((q_{dd})_q\) and \((G_{dd})_q\) — Eqs. C-8 and C-9 — into Eq. C-7 gives us:

\[
(D_{qd})_q = 1 - (G_{pdq})_q + \frac{(1 - p_{wd})^2}{4} (G_{pdq})_q^2 \quad \text{.................................................. (C-10)}
\]

Eq. C-10 is the basis of the proposed "quadratic cumulative" rate-cumulative production type curve for gas well performance. Eq. C-10 represents the solution for "boundary-dominated" performance, and this relation is plotted in Fig. 6, along with the transient "stems" which were generated using a numerical simulator.
A Production-Based Method for Direct Estimation of Gas in Place and Reserves

SPE 98042

T.A. Blasingame                J. A. Rushing
Texas A&M University                 Anadarko Petroleum Corp.

2005 SPE ERM
Morgantown, WVA

September 15, 2005
Presentation Overview

• New method for direct estimation of gas-in-place and reserves using only production data
  – Basis of method is five independent plotting functions—each exhibiting unique characteristics
  – Plotting functions derived from new quadratic rate-time/rate-cumulative functions

• Integrated, Systematic Analysis Technique
  – Field data simultaneously matched to five plotting functions
  – Data and plotting functions automatically linked by global model parameters—thus ensuring consistent evaluation
Quadratic Model Overview

• Model Assumptions
  – Volumetric dry gas reservoir
  – Initial reservoir pressure $\leq 6,000$ psia
  – Pseudosteady-state flowing conditions
  – Production at constant bottomhole flowing pressure

• Notes and Observations
  – Quadratic model matches rate-time data better than empirical Arps hyperbolic relationship
  – New model incorporates pressure-dependent properties
  – Simulation studies suggest initial reservoir pressure limitation can be extended to higher pressures
  – Also valid for slowly changing bottomhole flowing pressures
Quadratic Rate-Time Function

Dimensionless rate-time relation for gas flow based on Ansah and Knowles model (papers SPE 35268, 66280)

\[ q_{gDd} = \frac{p_{wd}^2}{1 - p_{wd}^2} \left\{ \frac{1 + \left( \frac{1 - p_{wd}}{1 + p_{wd}} \right) \exp(-p_{wd}t_{Dd})}{1 - \left( \frac{1 - p_{wd}}{1 + p_{wd}} \right) \exp(-p_{wd}t_{Dd})} \right\}^2 - 1 \]

where

\[ p_{wd} = \left( \frac{p_{wf}/z_{wf}}{p_i/z_i} \right), \quad t_{Dd} = \left[ \frac{2q_{gi}}{1 - \left( \frac{p_{wf}/z_{wf}}{p_i/z_i} \right)^2} \right] t \]
Quadratic Rate-Cumulative Function

Dimensionless cumulative-rate relation for gas flow based on Ansah and Knowles model (papers SPE 35268, 66280)

\[ q_{Dd} = 1 - \frac{2}{(1 - p_{wD}^2)} G_{pD} + \frac{1}{(1 - p_{wD}^2)} G_{pD}^2 \]

where

\[ p_{wD} = \left( \frac{p_{wf}}{z_{wf}} \right) \frac{z_{wf}}{p_i} \frac{z_i}{z_i} \], \quad G_{pD} = 1 - p_{wD} \left[ \frac{1 + \left( \frac{1 - p_{wD}}{1 + p_{wD}} \right) \exp(-p_{wD} t_{Dd})}{1 - \left( \frac{1 - p_{wD}}{1 + p_{wD}} \right) \exp(-p_{wD} t_{Dd})} \right] \]
## Overview of Quadratic Plotting Functions

<table>
<thead>
<tr>
<th>Plotting Function</th>
<th>Function Type</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_g \text{ vs. } G_p$</td>
<td>Quadratic</td>
<td>Root = $G$</td>
</tr>
<tr>
<td>$(q_g)_{i,G_p} \text{ vs. } G_p$</td>
<td>Quadratic</td>
<td>Root = $1.5 \ G$</td>
</tr>
<tr>
<td>$\frac{q_{gi} - q_g}{G_p} \text{ vs. } G_p$</td>
<td>Linear</td>
<td>Intercept, $G_{pmax} = 2 \ G$</td>
</tr>
<tr>
<td>$(q_g)<em>{i,G_p} - q</em>{gi} \text{ vs. } G_p$</td>
<td>Linear</td>
<td>Intercept, $G_{pmax} = 3 \ G$</td>
</tr>
<tr>
<td>$(q_g)_{i,G_p} - q_g \text{ vs. } G_p$</td>
<td>Linear</td>
<td>Intercept, $G_{pmax} = 1.5 \ G$</td>
</tr>
</tbody>
</table>
**Flow Rate-Cumulative Production Analysis**

$q_g$ vs. $G_p$ Model

- Quadratic trend above zero line reflects “typical” $q_g$-$G_p$ decline
- Slope of tangent through $q_{gi}$ equals decline rate constant, $D_i$
- Intercept ($G_p$ at $q_g = 0$) is maximum gas that can be produced at initial conditions
- Minimum point on model trend denotes actual value of gas-in-place, $G_{p_{max}} = G$
**Flow Rate-Cumulative Production Analysis**

**$(q_g)_{i,Gp}$ vs. $G_p$ Model**

- Cumulative production averaged rate function defined by
  
  $$(q_g)_{i,Gp} = \frac{1}{G_p} \int_0^{G_p} q_g \, dG_p$$

- Similar quadratic behavior displayed by $q_g$-$G_p$ plot

- Minimum point has root equal to $G_{p_{max}} = 1.5G$
Special Plotting Function No. 1

- Quadratic Function:
  \[ q_g = q_g - D_i G_p + \frac{1}{2} \frac{D_i}{G_p} G_p^2 \]

- Plotting Function:
  \[ \frac{q_{g_i} - q_g}{G_p} \text{ vs. } G_p \]

- Straight Line Characteristics
  - Intercept = \( D_i \)
  - Slope = \( D_i/2G \)
  - Extrapolation, \( G_{p,\text{max}} = 2G \)

\[ (q_{g_i} - q_g)/G_p \text{ Versus Cumulative Gas Production (} G_p \text{)} \]

Quadratic Rate-Cumulative Production Analysis

\[ (q_{g_i} - q_g)/G_p = D_i - (D_i/2G)G_p \]

Linear Trend:

\[ \text{Slope} = (D_i/2G) \]

Extrapolation:

\[ G_{p,\text{max}} = 2G \]

\( PF_1 \) reflects character of gas flow rate data—which can be erratic (i.e., noisy)
Special Plotting Function No. 2

- **Quadratic Function:**
  \[(q_g)_{i,G_p} = q_{gi} - \frac{D_i}{2} G_p + \frac{1}{6} \frac{D_i}{G} G_p^2\]

- **Plotting Function:**
  \[\frac{(q_g)_{i,G_p} - q_{gi}}{G_p} \text{ vs. } G_p\]

- **Straight Line Characteristics**
  - Intercept = \(D_i / 2\)
  - Slope = \(D_i / 6G\)
  - Extrapolation, \(G_{p,max} = 3G\)

**PF_2 is smoother than underlying gas flow rate data since it is an integral function**
Special Plotting Function No. 3

- Combination of special plotting functions 1 and 2

- Plotting Function:
  
  \[ \frac{(q_g)_{i,G_p} - q_g}{G_p} \text{ vs. } G_p \]

- Straight Line Characteristics
  
  - Intercept = \( \frac{D_i}{2} \)
  - Slope = \( \frac{D_i}{3G} \)
  - Extrapolation, \( G_{p,\text{max}} = \frac{3G}{2} \)

\( PF_3 \) combines \( PF_1 \) and \( PF_2 \) functions—which yields consistent (but noisy) trend
Integrated Evaluation Methodology

• Integrated Evaluation Technique
  – Utilize iterative approach in which data and plotting functions are linked interactively by global model parameters
  – Match five independent rate-time and rate-cumulative production plotting functions simultaneously

• Assessment of Initial Gas Flow Rate, $q_{gi}$
  – Most difficult parameter to match
  – Utilize a “high-correct-low” technique to bracket correct value
  – Use a 10% high-low spread for matching process
Validation of Quadratic Plotting Functions

• Evaluated Simulated Production Data
  – Illustrate application of new plotting functions
  – Validate new quadratic model and plotting functions

• Example of Simulated Model Input Properties
  – Net pay, \( h = 30 \text{ ft} \)
  – Gas Permeability, \( k_g = 100 \text{ md} \)
  – Effective Porosity, \( \phi = 30\% \)
  – Initial reservoir pressure, \( p_i = 5,000 \text{ psia} \)
  – Original gas-in-place, OGIP = 4.2 Bcf (40-acre spacing)
  – Constant bottomhole flowing pressure, \( p_{wf} = 1,000 \text{ psia} \)
Gas Flow Rate vs. Cumulative Production: $q_g$ vs. $G_p$

Simulated Case: $p_i = 5000$ psia, $p_{wf} = 1000$ psia, $G_{input} = 4.20$ BSCF

- $q_{i, high} = 1.1 \cdot q_i$ (quadratic model)
- $q_{i, low} = 0.9 \cdot q_i$ (quadratic model)
- $q_{i, correct} = 1.0 \cdot q_i$ (quadratic model)

Quadratic Trend:

$$q_g = q_{gi} - D_i G_p - (D_i/2G)G_p^2$$

Quadratic Root:

$$G_{p, max} = G$$

Results: Quadratic

- $q_i = 495,000$ MSCF/D
- $D_i = 0.247$ 1/D
- $p_{WD} = 0.216$
- $G = 4.20$ BSCF
Cumulative Averaged Gas Rate vs. Cumulative Gas Production: \( (q)_{gi,Gp} \) vs. \( Gp \)

Simulated Case: \( p_i = 5000 \) psia, \( p_{wf} = 1000 \) psia, \( G_{input} = 4.20 \) BSCF

Gas Rate-Integral \( ((q_g)_{i,Gp}) \) Versus Cumulative Gas Production \( (G_p) \)

Quadratic Rate-Integral-Cumulative Production Analysis

Results: Quadratic
\( q_i = 495,000 \) MSCF/D
\( D_i = 0.247 \) 1/D
\( p_{WD} = 0.216 \)
\( G = 4.20 \) BSCF

Quadratic Trend:
\[ (q_g)_{i,Gp} = q_{gi} - (D_i/2)G_p - (D_i/6G)G_p^2 \]

Quadratic Root:
\( G_{p,max} = 3/2 \) \( G \)
Plotting Function No. 1: \( \frac{q_{gi} - q_g}{G_p} \) vs. \( G_p \)

Simulated Case: \( p_i = 5000 \) psia, \( p_{wf} = 1000 \) psia, \( G_{input} = 4.20 \) BSCF

\( \frac{q_{gi} - q_g}{G_p} \) Versus Cumulative Gas Production (\( G_p \))

Quadratic Rate-Cumulative Production Analysis

Linear Trend:

\[
\frac{q_{gi} - q_g}{G_p} = D_i - \frac{D_i}{2G} G_p
\]

Intercept = \( D_i \)

\( q_{i,high} = 1.1 \) \( q_i \)

(quadratic model)

\( q_{i,low} = 0.9 \) \( q_i \)

(quadratic model)

Extrapolation:

\( G_{p,max} = 2 \) \( G \)

Results: Quadratic

\( q_i = 495,000 \) MSCF/D

\( D_i = 0.247 \) 1/D

\( p_{WD} = 0.216 \)

\( G = 4.20 \) BSCF
Plotting Function No. 2: \( \frac{(q_{gi,Gp} - q_{gi})}{G_p} \) vs. \( G_p \)

Simulated Case: \( p_i = 5000 \) psia, \( p_{wf} = 1000 \) psia, \( G_{input} = 4.20 \) BSCF

\( \frac{(q_g)_{i,Gp} - q_{gi}}{G_p} \) Versus Cumulative Gas Production (\( G_p \))

Quadratic Rate-Integral-Cumulative Production Analysis

\[
\frac{(q_g)_{i,Gp} - q_{gi}}{G_p} = \left( \frac{D_i}{2} \right) - \left( \frac{D_i}{6G} \right) G_p
\]

Linear Trend:

Intercept = \( \left( \frac{D_i}{2} \right) \)

Extrapolation:

\( G_{p,\text{max}} = 3 \ G \)

Results:

<table>
<thead>
<tr>
<th>Quadratic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_i = 495,000 ) MSCF/D</td>
</tr>
<tr>
<td>( D_i = 0.247 ) 1/D</td>
</tr>
<tr>
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</tr>
<tr>
<td>( G = 4.20 ) BSCF</td>
</tr>
</tbody>
</table>

\( q_{i,high} = 1.1 \ q_i \) (quadratic model)

\( q_{i,low} = 0.9 \ q_i \) (quadratic model)

\( q_{i,correct} = 1.0 \ q_i \) (quadratic model)

Slope = \( \left( \frac{D_i}{6G} \right) \)
Plotting Function No. 3: \( (q_{gi,Gp} - q_g)/G_p \) vs. \( G_p \)

Simulated Case: \( p_i = 5000 \) psia, \( p_{wf} = 1000 \) psia, \( G_{input} = 4.20 \) BSCF

\( ((q_g)_i, G_p - q_g)/G_p \) Versus Cumulative Gas Production (\( G_p \))

Quadratic Rate-Integral-Cumulative Production Analysis

Linear Trend:

\[ ((q_g)_i, G_p - q_g)/G_p = \left( \frac{D_i}{2} \right) - \left( \frac{D_i}{3G} \right) G_p \]

Intercept = \( \left( \frac{D_i}{2} \right) \)

Slope = \( \left( \frac{D_i}{3G} \right) \)

Extrapolation:

\( G_{p,\text{max}} = \frac{3}{2} G \)

Results: Quadratic

\( q_i = 495,000 \) MSCF/D

\( D_i = 0.247 \) 1/D

\( p_{\text{wd}} = 0.216 \)

\( G = 4.20 \) BSCF
Application of Quadratic Functions to Field Data


Reservoir and Fluid Properties

- Net pay, \( h = 70 \text{ ft} \)
- Water Saturation, \( S_w = 35 \)
- Effective Porosity, \( \phi = 6\% \)
- Initial reservoir pressure, \( p_i = 4,175 \text{ psia} \)
- Reservoir temperature, \( T = 160^\circ \text{F} \)
- Reservoir gas specific gravity, \( \gamma_g = 0.57 \) (air = 1.0)
- Constant bottomhole flowing pressure, \( p_{wf} = 710 \text{ psia} \)
Gas Flow Rate vs. Cumulative Production: $q_g$ vs. $G_p$

Example: Fetkovich — West Virginia Well A
Gas Flowrate ($q_g$) Versus Cumulative Gas Production ($G_p$)

Quadratic Rate-Cumulative Production Analysis

Results: Quadratic
$q_i = 1920$ MSCF/D
$D_i = 0.00133$ 1/D
$p_{wD} = 0.35$
$G = 3.29$ BSCF

$q_{i,low} = 0.9 \, q_i$
(qquadratic model)

$q_{i,high} = 1.1 \, q_i$
(qquadratic model)

Quadratic Root:
$G_{p,max} = G$
Cumulative Averaged Gas Rate vs. Cumulative Gas Production: \((q)_{gi,Gp} vs. G_p\)

Example: Fetkovich — West Virginia Well A
Gas Rate-Integral \((q_{gi,Gp})\) Versus Cumulative Gas Production \((G_p)\)

Quadratic Rate-Integral-Cumulative Production Analysis

Results: Quadratic
\(q_i = 1920\) MSCF/D
\(D_i = 0.00133\) 1/D
\(p_{wD} = 0.35\)
\(G = 3.29\) BSCF

**Quadratic Root:**
\(G_{p,\text{max}} = \frac{3}{2} G\)
Plotting Function No. 1: \( \frac{q_{gi} - q_g}{G_p} \) vs. \( G_p \)

Example: Fetkovitch — West Virginia Well A

\( \frac{q_g - q_{gi}}{G_p} \) Versus Cumulative Gas Production (\( G_p \))

Quadratic Rate-Cumulative Production Analysis

Linear Trend:

\( \frac{q_{gi} - q_g}{G_p} = D_i - \left( \frac{D_i}{2G} \right) G_p \)

Results: Quadratic

- \( q_i = 1920 \) MSCF/D
- \( D_i = 0.00133 \) 1/D
- \( p_{wd} = 0.35 \)
- \( G = 3.29 \) BSCF

Intercept = \( D_i \)

Extrapolation:

- \( G_{p,max} = 2G \)
Plotting Function No. 2: \( \frac{(q_{gi,Gp} - q_{gi})}{G_p} \) vs. \( G_p \)

Example: Fetkovich — West Virginia Well A
\( ((q_g)_{i,Gp} - q_{gi})/G_p \) Versus Cumulative Gas Production (\( G_p \))
Quadratic Rate-Integral-Cumulative Production Analysis

Linear Trend:
\( ((q_g)_{i,Gp} - q_{gi})/G_p = (D_i/2) - (D_i/6G)G_p \)

Results: Quadratic
\( q_i = 1920 \) MSCF/D
\( D_i = 0.00133 \) 1/D
\( p_{WD} = 0.35 \)
\( G = 3.29 \) BSCF

Extrapolation:
\( G_{p,max} = 3G \)
Plotting Function No. 3: \( \frac{(q_{g_i,G_p} - q_g)}{G_p} \) vs. \( G_p \)

Example: Fetkovich — West Virginia Well A

\[
\frac{(q_{g_i,G_p} - q_g)}{G_p} = \frac{(D_i/2) - (D_i/3G)G_p}{G_p}
\]

Results: Quadratic

\( q_i = 1920 \) MSCF/D

\( D_i = 0.00133 \) 1/D

\( p_{WD} = 0.35 \)

\( G = 3.29 \) BSCF
## Comparison of Results: West Virginia Well A

<table>
<thead>
<tr>
<th>Method</th>
<th>Gas-in-Place (Bcf)</th>
<th>Effective Gas Permeability (md)</th>
<th>Skin Factor (dimensionless)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic Analysis Method</td>
<td>3.29</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Fetkovich, et al. (SPE Form. Eval., Dec. 1987)</td>
<td>3.36</td>
<td>0.053</td>
<td>-5.17</td>
</tr>
<tr>
<td>Fraim, et al. (SPE Form. Eval., Dec. 1987)</td>
<td>3.04</td>
<td>0.077</td>
<td>-5.08</td>
</tr>
</tbody>
</table>
Summary and Conclusions

- Developed new method for direct estimation of gas-in-place and reserves using only production data
  - Basis of method is five independent plotting functions—each exhibiting unique characteristics
  - Plotting functions derived from new quadratic rate-time/rate-cumulative functions

- Developed integrated, systematic analysis technique
  - Field data simultaneously matched to five plotting functions
  - Data and plotting functions automatically linked by global model parameters—thus ensuring consistent evaluation
Summary and Conclusions (continued)

• Primary Theoretical Limitations of Quadratic Model
  – Initial reservoir pressure ≤ 6,000 psia
  – Constant bottomhole flowing pressure

• New method has been successfully applied to simulated and field data at initial reservoir pressures > 6,000 psia

• New method has also been successfully applied to simulated and field cases where bottomhole flowing pressure is variable but is changing slowly