Application of a General Material Balance for High-Pressure Gas Reservoirs

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Summary

This paper presents the derivation of a general material balance that has particular application to high-pressure gas reservoirs (both normal pressured and overpressured [geopressured]). Its main application is to calculate original gas in place and assist in calculating remaining recoverable reserves from pressure-production data.

The form of the material-balance equation is 
\[
(\rho/p)(1 - \varepsilon_p(p/p_o - p)) = (\rho/k)(1 - G/(G_0)),
\]
which includes a pressure-dependent cumulative gas-in-place term \(\varepsilon_p(p)\) that is defined in terms of the following reservoir parameters: pore compressibility, water compressibility, gas solubility, and total water associated with the gas reservoir volume. "Associated" water includes connate water, water within interbedded shales and nonpore reservoir rock, and any limited aquifer volume. \(\varepsilon_p\) physically represents the cumulative change in hydrocarbon pore volume (PV) caused by compressibility effects and encroaching water.

High pressure gas reservoirs typically have concave downward \(p/\rho\) vs. \(G_p\) plots which may result in serious overestimation of original gas in place and remaining recoverable reserves. The proposed form of the gas material balance equation provides a method to linearize the \(p/\rho\) vs. \(G_p\) plot; and thereby predict the true original gas in place. A method is suggested to determine initial gas in place by analyzing the behavior of cumulative effective compressibility backcalculated from pressure-production data. The \(\varepsilon_p(p)\) function determined by this procedure, or estimated from logs and geological maps when sufficient production data is not available, is then used to forecast pressure/cumulative behavior. Two field examples are provided showing the application of the material balance equation to high pressure gas reservoirs.

Introduction

High pressure gas reservoirs experiencing depletion drive typically have downward curving \(p/\rho\) vs. \(G_p\) behavior. Incorrect extrapolation of early depletion data may result in serious overestimation of original gas in place and remaining reserves.

Brunt et al.¹¹ work in 1965 was a result of a field study conducted on a large, moderately overpressured gas reservoir in the Texas Gulf Coast area. Investments were made, and never needed, based on linear extrapolation of the early field \(p/\rho\) vs. \(G_p\) performance to an apparent original gas in place that is later found to be overstated by about 200 Bcfe. Fig. 3 in Ref. 1 (Run 20) shows the concave downward curvature typical for the pressure response of a limited external aquifier system that simulated the reservoir's response.

This type of "limited" aquifer behavior, where pressure in the reservoir and aquifer are virtually equal, led to the derivation of a general material balance for high pressure gas reservoirs (see Appendix, Ref. 2). The derivation includes pressure-dependent rock and water compressibility (with gas evolving from solution). All water and rock volumes associated with the reservoir and available for expansion, including a limited aquifer volume, were included in a cumulative effective compressibility term \(\varepsilon_p(p)\). Rock and water compressibilities were defined to account for cumulative changes in volume to be multiplied by the cumulative pressure drop \((\rho_o - \rho)(\rho_o - p)\).

Instantaneous compressibilities are not used at all. The final form of the material balance is similar to that published by Ramagorty and Farhashed,² except that they considered \(c_p\) as a constant. The general material balance as presented in this paper defines a cumulative effective compressibility \(\varepsilon(p)\) as a function of pressure.

Literature Review

Harville and Hawkinson ³ and Hammerling⁴ attribute the concave downward shape of \(p/\rho\) vs. \(G_p\) curves obtained in abnormally pressured gas reservoirs entirely to pore collapse and formation compaction. No definition of pore collapse is given in Ref. 4, but a plot of backcalculated PV change indicated a system compressibility change from \(28 \times 10^{-6}\) psi⁻¹ at initial pressure to about \(6 \times 10^{-6}\) psi⁻¹ at low pressures. This magnitude of PV change implies associated water volume. The decreasing "system" compressibility is expected for an overpressured reservoir with pressure-dependent PV compressibility, and used results presented in this paper pore collapse is not a necessary condition for such behavior.

The Anderson "L" reservoir performance presented by Duggan⁵ shows curved \(p/\rho\) vs. \(G_p\) behavior which was primarily attributed to shale water influx with no evidence of reservoir pore compaction. The water influx drive mechanism was supported by the fact that several wells watered out. Wallace⁶ also concluded that shale water influx is an important drive mechanism in abnormally pressured gas reservoirs. Bass² discounts shale water influx, and attributes curved \(p/\rho\) vs. \(G_p\) behavior to peripheral water influx from a limited aquifer and formation compaction treated with a constant PV compressibility \(c_p\). For a limited aquifer, Bass defines a term \(F_p\) as the ratio of peripheral water PV to the PV of gas-bearing rock.

Roche⁷ and Ramagot and Farhashed⁸ both use the term \(\rho/\rho_o - \varepsilon(p_o - p)\) for geopressed and abnormally pressured gas reservoirs. Both authors consider \(c_p\) a constant and they consider the Anderson "L" example.

Bernard⁹ does not accept the rock collapse theory as the cause for overpressured \(p/\rho\) vs. \(G_p\) behavior, concluding that water influx is the basic drive mechanism. He also uses \(\rho/\rho_o - \varepsilon(p_o - p)\) where \(c_p\) is a "catch-all" term for treating the effects of rock and water compressibility, a small steady-state acting aquifer, and steady state shale water influx. He further states that the term \(c_p\) is almost impossible to quantify in terms of reservoir properties.

Belgean and Whitehead,¹¹ Prasad and Rogers,¹² and Wang and Teasdale¹³ all present studies of overpressured gas reservoirs based on computer models. Refs. 11 and 12 treat \(c_o\) and \(c_p\) as functions of pressure, including the effect of solution gas in the water. External water sources are also included in Refs. 12 and 13. The differential forms of the material balance used in these references correctly apply instantaneous compressibility in a history-matching approach to determine initial gas in place. A direct plot of \(\rho/\rho_o - \varepsilon(p_o - p)\) vs. \(G_p\) was not made because the \(\varepsilon_p\) term had not been defined.

Poston and Chen¹⁴ analyzed several abnormally pressured gas reservoirs, and recognized that calculated values of \(c_p \geq 30 \times 10^{-6}\) psi⁻¹ required to linearize the material-balance plot reflected the influence of water influx.

Bourgoyne¹⁵ demonstrates that reasonable values of shale permeability and compressibilities treated as a function of pressure can be used to match abnormally pressured gas reservoir performance behavior. He points out, however, that determining \(c_o\) and \(c_p\) of the shale necessary for modeling this behavior is practically impossible.

Ambastha¹⁶ uses Bourgoyne's general material-balance equation to develop a graphical matching technique based on a constant effective compressibility \(c_p\). The example given in that paper shows a lack of uniqueness in determining initial gas in place.
The general form of the gas material balance is

$$P \left[ 1 - \bar{c}(p)(\rho - p) \right] = \frac{(p/c)}{G} .$$

$$\left[ G_s - G_m + W_0 R_s + \frac{5.615}{R_s} (W_0 R_s - W_m R_m - W_s) \right].$$

which reduces to

$$P \left[ 1 - \bar{c}(p)(\rho - p) \right] = \frac{(p/c)}{G} \left[ G_s \right].$$

(1)

when water terms and gas injection are neglected. The cumulative effective compressibility term \( \bar{c}(p) \) is pressure-dependent, consisting of connate water compressibility \( \bar{c}_w(p) \), cumulative total water compressibility \( \bar{c}_w(p) \), and the total pore and water volumes associated (i.e., in pressure communication) with the gas reservoir,

$$\bar{c}_w(p) = \frac{S_o c_m(p) + \bar{c}_w(p) + M c_m(p) + \bar{c}_w(p)}{1 - S_o}.$$  

(3)

The formation and total water compressibility terms \( \bar{c}_w \) and \( c_m \) account for cumulative changes in volume from initial pressure to the current pressure.

The interbedded nonpay volume and limited aquifer contributions to pressure support are quantified in terms of the M ratio,

$$M = \frac{V_{non}}{V_{aq}}.$$  

(4)

An important aspect of the material balance for high-pressure gas reservoirs is that the gas in solution in the connate and associated water provide both pressure support and additional gas available for production. The level of pressure support provided by the evolved solution gas depends on the level of depletion, and it is shown that this support is significant below about 1,000 psia. The solution gas available for production also depends on the level of depletion, i.e., how much of the original solution gas has evolved \( (R_m - R_m(p)) \) and the quantity of this gas that is mobile.

The term \( G \) is used for the initial free gas in place, and it is this quantity that will be determined from the material balance plot given by Eq. 2 when extrapolated to \( (\rho/c)(1 - \bar{c}_w(p) - p) = 0 \). This condition is reached as a pressure when \( 1 - \bar{c}_w(p)(\rho - p) = 0 \), and not when \( p = 0 \), i.e., additional gas may be produced after \( G \) reaches original free gas in place \( G \). At pressures where \( G \) exceeds \( G \), the corrected \( \rho/c \) term \( (\rho/c)(1 - \bar{c}_w(p) - p) \) becomes negative. If reservoir pressure could be brought to standard conditions \( (p = p_w) \), the total gas would be \( G \) plus the total solution gas in place \( G \).

The effect of connate water saturation \( S_o \) and \( M \) are important to the magnitude of \( \bar{c}_w \). With typical values of \( \bar{c}_w = 4 \times 10^{-4} \) psi" and \( c_m = 3 \times 10^{-4} \) psi" for a high-pressure gulf coast sandstone reservoir, the cumulative effective compressibility is initially \( \bar{c}_w = 7.5 \times 10^{-4} \) psi" for \( S_o = 35 \) and \( M = 0 \); and \( \bar{c}_w = 15 \times 10^{-4} \) psi" for \( S_o = 35 \) and \( M = 1 \). Fig. 1 shows the percentage of true original free gas in place that would be overestimated by extrapolating early \( p/c \) vs. \( G \) data, indicating that the overestimation is greater for larger initial pressure and higher \( \bar{c}_w \) values at initial conditions. For an initial pressure of 10,000 psi and a \( \bar{c}_w = 10 \times 10^{-4} \) psi" extrapolation of early data gives an estimate of \( G \) that is about 25% higher than the true original free gas in place. The sections below discuss the calculation of \( \bar{c}_w(p) \) and \( S_o(p) \).

Cumulative PV Compressibility \( \bar{c}_w \). The material balance presented in this paper uses a cumulative PV compressibility \( \bar{c}_w \) defined as

$$\bar{c}_w(p) = \frac{1}{V_{aq}} \frac{V_{aq} - V_{aq}(p)}{\rho - p}.$$  

(5)

The term in brackets is the slope of the curve from the initial condition \( (\rho, V_{aq}) \) to any lower pressure \( (\rho, V_{aq}) \), as shown in Fig. 2. This implies that \( \bar{c}_w(p) \) is a function of both pressure and the initial condition. The instantaneous PV compressibility \( c_w(p) \) is defined as

$$c_w(p) = \frac{1}{V_{aq}} \frac{dV_{aq}}{dp}.$$  

(6)

and is only a function of pressure. At initial pressure the two PV compressibilities are equal: \( \bar{c}_w(p) = c_w(p) \). The instantaneous compressibility function \( c_w(p) \) should be used in reservoir simulation and differential forms of the material balance, while the cumulative compressibility function \( \bar{c}_w(p) \) must be used with forms of the material balance that apply the cumulative pressure drop \( (\rho - p) \), i.e., \( p/c \) vs. \( G \) plots.

The pressure dependence of \( \bar{c}_w \) is best determined by special core analysis under appropriate reservoir conditions. Table 1 summarizes the calculation of \( \bar{c}_w \) as a function of pressure using laboratory data for a gulf coast sandstone. Fig. 3 shows how \( \bar{c}_w \) and \( c_w \) vary as a function of pressure for this overpressured reservoir rock.

In the absence of pore collapse \( \bar{c}_w \) is always greater than or equal to \( c_w \). The cumulative PV compressibility remains higher than the instantaneous compressibility because of an averaging effect that reduces the pressure dependence of \( \bar{c}_w \) compared with \( c_w \). An important consequence of this behavior is that a rock exhibiting large PV change because of a high level of overpressure will initially have and maintain a high cumulative compressibility \( \bar{c}_w \) as shown in Fig. 3.
TABLE 1—CALCULATION OF PORE VOLUME COMPRESSIBILITY FROM LABORATORY DATA

<table>
<thead>
<tr>
<th>$p_r$ (psia)</th>
<th>$V_r$ (cm$^3$)</th>
<th>$V_o$ (cm$^3$)</th>
<th>$\phi$ (%)</th>
<th>$c_r$</th>
<th>$c_r$</th>
<th>$p - p_r$</th>
<th>$V_o - V_r$</th>
<th>$c_r$</th>
<th>$c_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200.0</td>
<td>3.42</td>
<td>20.53</td>
<td>16.70</td>
<td>16.50</td>
<td></td>
<td>9,800</td>
<td>0</td>
<td>0.000</td>
<td>16.50</td>
</tr>
<tr>
<td>1,000.0</td>
<td>3.379</td>
<td>20.48</td>
<td>16.49</td>
<td>13.70</td>
<td></td>
<td>9,000</td>
<td>800</td>
<td>0.041</td>
<td>14.99</td>
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<tr>
<td>2,000.0</td>
<td>3.337</td>
<td>20.44</td>
<td>16.32</td>
<td>11.40</td>
<td></td>
<td>8,000</td>
<td>1,800</td>
<td>0.083</td>
<td>13.48</td>
</tr>
<tr>
<td>3,000.0</td>
<td>3.303</td>
<td>20.40</td>
<td>16.18</td>
<td>9.10</td>
<td></td>
<td>7,000</td>
<td>2,800</td>
<td>0.117</td>
<td>12.22</td>
</tr>
<tr>
<td>4,000.0</td>
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<td>20.36</td>
<td>16.07</td>
<td>6.90</td>
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<td>6,000</td>
<td>3,800</td>
<td>0.144</td>
<td>11.08</td>
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<td>5,000.0</td>
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<td>20.32</td>
<td>15.99</td>
<td>5.50</td>
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<td>5,000</td>
<td>4,800</td>
<td>0.183</td>
<td>9.93</td>
</tr>
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<td>6,000.0</td>
<td>3.243</td>
<td>20.29</td>
<td>15.93</td>
<td>3.80</td>
<td></td>
<td>4,000</td>
<td>5,800</td>
<td>0.177</td>
<td>9.22</td>
</tr>
<tr>
<td>7,000.0</td>
<td>3.229</td>
<td>20.24</td>
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<td>4.10</td>
<td></td>
<td>3,000</td>
<td>6,800</td>
<td>0.190</td>
<td>8.17</td>
</tr>
<tr>
<td>8,000.0</td>
<td>3.215</td>
<td>20.20</td>
<td>15.81</td>
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<td></td>
<td>2,000</td>
<td>7,800</td>
<td>0.227</td>
<td>7.76</td>
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<td>9,000.0</td>
<td>3.177</td>
<td>20.17</td>
<td>15.70</td>
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<td>1,000</td>
<td>8,800</td>
<td>0.243</td>
<td>8.07</td>
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<tr>
<td>9,500.0</td>
<td>3.144</td>
<td>20.24</td>
<td>15.50</td>
<td>2.50</td>
<td></td>
<td>600</td>
<td>9,300</td>
<td>0.276</td>
<td>8.68</td>
</tr>
</tbody>
</table>

All compressibilities in $10^{-6}$ psia$^{-1}$.

Pore collapse is defined as the condition when a rock's instantaneous PV compressibility starts to increase at decreasing reservoir pressure. Pore collapse provides greater pressure support when collapse occurs at a high pressure. However, pore collapse is not reflected by the $c_r(p)$ function and will not therefore be seen on the $p_r$ vs. $G_p$ plot as the pressure when pore collapse occurs. In fact, pore collapse may not be identifiable at all on the cumulative compressibility term. For example, the Gulf coast sandstone in Fig. 3 exhibits pore collapse at 4,000 psia (about 5,000 psig less than initial pressure $p_i$). Despite the increase in $c_r$ from 4 to 25 x $10^{-6}$ psia$^{-1}$ in the pressure range 4,000 to 1,000 psia, the change in $c_r$ over the same pressure range is almost insignificant. Fig. 4 shows a North Sea chalk sample from a reservoir with initial pressure of 7,000 psia exhibiting pore collapse at 6,000 psia. Here the effect of pore collapse is greater, causing $c_r$ to increase from 6 to 20 x $10^{-6}$ psia$^{-1}$ in the pressure range from 6,000 to 2,000 psia. In general, however, pore collapse is not of itself does not have a significant effect on the $p_r$ vs. $G_p$ plot.

In the absence of laboratory data, PV compressibilities can be estimated from correlations presented by Hall 11 and by Von Genten and Choudhary. 12 Hall's correlation (see Fig. 2) gives instantaneous PV compressibility as a function of porosity, i.e., there is no pressure dependence. Von Genten develops correlations for instantaneous PV compressibility $c_r$ as a function of net overburden pressure ($p_o$), where $p_o$ equals the overburden gradient times depth minus reservoir pressure.

Cumulative Total Water Compressibility $c_{w, c}$: The pressure support provided by water is made up of two components. First, the water expansion with decreasing pressure, and second, the release of solution gas and its expansion. The total or composite compressibility effect is expressed as

$$c_{w, c}(p) = \frac{1}{\rho_w} \left[ B_w(p) - B_w(p) \right] \rho_o - p$$

in terms of the total water formation volume factor $B_w$. $B_w(p) = B_w(p) + \frac{[R_{sat} - R_w(p)]p_o}{5.615}$

Fig. 5 shows typical behavior for $B_w$ and $B_w$ as a function of pressure; the figure also shows the behavior of $c_{w, c}(p)$ which is seen that little increase occurs before a pressure of about 1,500 psia, and that, at pressures below 1,000 psia, there is a significant increase in $c_{w, c}$ with a limiting relationship $c_{w, c} \approx 1/p$ at low pressures.

$$c_{w, c}(p \to 0) = \frac{1}{5.615} \frac{R_{sat} - R_w(p)}{\rho_o} \rho_o$$

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Associated Water Volume Ratio M. The total compressibility effect on the gas material balance depends on the magnitudes of rock and total water compressibilities and on the total pore and water volumes in pressure communication with the gas reservoir (including connate water and the PV within the net pay).

Associated water and PVs external to the net pay include nonnet pay (NNP) such as interbedded shales and dirty sands, plus external water volume found in limited aquifers. Including these water volumes in reservoir simulation is referred to as using a "gross" model. In the proposed material balance equations this associated volume is expressed as a ratio relative to the PV of the net pay reservoir,

$$M = M_{\text{NNP}} + M_{\text{AQ}}$$  \hspace{1cm} (12)

where

$$M_{\text{NNP}} = \frac{V_{\text{NNP}}}{V_{\text{NN}}^*}$$  \hspace{1cm} (13)

and

$$M_{\text{AQ}} = \frac{V_{\text{AQ}}}{V_{\text{AQ}}^*}$$  \hspace{1cm} (14)

In the simplest case when $M = 0$, there will be pressure support only from connate water and the net pay PV. This is equivalent in simulation to building a net model. The cumulative effective compressibility term $z_{w}$ will then be expected to have values ranging from 7 to $15 \times 10^{-8}$ psi$^{-1}$ for normal-pressure reservoirs, where the larger values will generally result from high connate water saturation.

Net pay compressibility effects alone can cause noticeable curvature in the $p$ vs. $G_{w}$ plot with potential overestimation of initial free gas in place ($G_{w}$) (see Fig. 1).

$M_{\text{NNP}}$. The nonnet pay water volume ratio $M_{\text{NNP}}$ comprises interbedded reservoir PV, including shales and poor quality rock, that are assumed to be completely filled with water. With this definition $M_{\text{NNP}}$ can be written in terms of the net to gross ratio $R_{\text{NG}}$ defined as

$$R_{\text{NG}} = \frac{h_{w}}{h_{w} + h_{\text{NNP}}} = \frac{h_{w}}{h_{c}}$$  \hspace{1cm} (15)

Accounting for different porosities in the net pay and nonnet pay $M_{\text{NNP}}$ is given by

$$M_{\text{NNP}} = \frac{\phi_{\text{NN}}}{\phi_{\text{NNP}}} \left(1 - R_{\text{NG}}\right)$$  \hspace{1cm} (16)

Properties and thicknesses of the net pay and nonnet pay are readily available from log analysis.

$M_{\text{AQ}}$. Aquifers with sufficient permeability and limited areal extent can be treated as part of the total cumulative compressibility term. The water volume ratio of the aquifer $M_{\text{AQ}}$ can be determined using geological maps and well control to define areal extent, and electric logs to define the gas/water contact. In general, $M_{\text{AQ}}$ is defined as

$$M_{\text{AQ}} = \frac{\phi_{\text{AQ}}}{\phi_{\text{AQ}}^*}$$  \hspace{1cm} (17)

and for a radial aquifer geometry quantified in terms of the aquifer to reservoir radius $r_{\text{AQ}}/r_{c}$, the aquifer volume ratio can be expressed

$$M_{\text{AQ}} = \left[\frac{\phi_{\text{AQ}}}{\phi_{\text{AQ}}^*}\left(\frac{r_{c}}{r_{\text{AQ}}}\right)^2 - 1\right]$$  \hspace{1cm} (18)

Bruns et al. show that limited aquifers with $r_{\text{AQ}}/r_{c}$ ratios up to 5 have the same $p_{w}$ vs. $G_{w}$ behavior for permeabilities 100 md and higher. This implies that the transient effects in the aquifer have

"TABLE 2—COMPARISON OF $c_p$ FOR NORMAL PRESSURE AND OVERPressURED CONDITIONS"

<table>
<thead>
<tr>
<th>Sample Type</th>
<th>Initial Porosity (%)</th>
<th>Normal Pressure $c_p$ (psu$^{-1}$)</th>
<th>Over-Pressed $c_p$ (psu$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gulf coast sandstones</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample 1</td>
<td>13</td>
<td>4.8</td>
<td>6.4</td>
</tr>
<tr>
<td>Sample 2</td>
<td>20</td>
<td>4.4</td>
<td>16.5</td>
</tr>
<tr>
<td>North Sea chalk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample 9 (pore collapse)</td>
<td>32</td>
<td>18.3</td>
<td>7.9</td>
</tr>
<tr>
<td>Sample 10 (pore collapse)</td>
<td>30</td>
<td>20.1</td>
<td>7.4</td>
</tr>
<tr>
<td>Von Gonten</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample 5A</td>
<td>11</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Sample 4A</td>
<td>22</td>
<td>4.6</td>
<td>9.2</td>
</tr>
<tr>
<td>Sample 7A</td>
<td>26</td>
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<td>7.2</td>
</tr>
<tr>
<td>Sample 3A</td>
<td>28</td>
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<td>10.8</td>
</tr>
<tr>
<td>Sample 6A</td>
<td>25</td>
<td>7.8</td>
<td>8.8</td>
</tr>
</tbody>
</table>

Normal Pressured is 0.5 psu in. Depth. Overpressed is 0.5 psu in. Depth. Used in 10,000 ft.
negligible effect on reservoir performance and the aquifer can be treated as part of the cumulative effective compressibility term. Values of $M_{AP}$ used in the definition of $\tilde{e}_p$ may be as high as 25, $[M_{AP} = (C_{sw}/C_{pw})^{-1}]$, in reservoirs with moderate permeability. With higher permeabilities, limited aquifers can include $C_{sw}/C_{pw}$ ratios greater than 5 and still be treated as part of the cumulative effective compressibility term.

When the aquifer is sufficiently large and requires treatment with either superposition or the Schilthuin infinite aquifer model, the $\tilde{e}_p$ term should still be used, but it will only contain the effect of net pay and nonnet pay volumes; i.e., $M = M_{AP}$.

**Cumulative Effective Compressibility $\tilde{e}_p$.** Total cumulative effective compressibility represents all available pressure support from rock and water. The equation for $\tilde{e}_p$ is

$$\tilde{e}_p(p) = \sum \tilde{e}_w(p) + \tilde{e}_l(p) = \frac{M \tilde{e}_w(p) + \tilde{e}_l(p)}{1 - \tilde{e}_l}$$

(19)

For a specific reservoir, a family of $\tilde{e}_p(p)$ curves can be generated for several $M$ values. These curves will have specific characteristics depending on the pressure dependence of rock and water compressibilities. The $\tilde{e}_w(p)$ curves are relatively constant at high pressure, increasing slightly as pressure decreases, then rising sharply at low pressure around 1,000 psi. Typically, a constant PV compressibility $\tilde{c}_p$ can be assumed and the $\tilde{e}_l(p)$ curves will then have the same character as the $\tilde{e}_w(p)$ curve. Fig. 6 illustrates an example of $\tilde{e}_w(p)$ curves at various $M$ ratios for a typical Gulf coast reservoir with $p_r = 9,000$ psi, $T = 200^\circ F$, $\gamma_s = 0.7$ (air = 1), and a constant $\tilde{c}_r = 3.2 \times 10^{-6}$ psi$^{-1}$.

For overpressured reservoirs exhibiting a pressure-dependent $\tilde{e}_r(p)$, the family of $\tilde{e}_r(p)$ curves at high pressures will tend to decrease with depletion. In the absence of pore collapse $\tilde{e}_l(p)$ decreases to a constant value at lower pressure and the $\tilde{e}_r(p)$ curves at lower pressure are dominated by the increasing $\tilde{e}_w(p)$ function. If pore collapse occurs, but not early in depletion, the pore collapse is almost insignificant because the $\tilde{e}_r(p)$ function starts increasing until low pressures because it represents a cumulative PV change, and, when the $\tilde{e}_r(p)$ function finally starts to increase it will be masked by the $\tilde{e}_w(p)$ function which is increasing as $1/p$.

Fig. 7 illustrates this point for a Gulf coast overpressured reservoir with $p_r = 9,000$ psi, $T = 300^\circ F$, and $\gamma_s = 0.71$ (air = 1). Although pore collapse occurs at approximately 3,500 psi (Fig. 3), $\tilde{e}_p$ does not start increasing until 2,000 psi. The increase is insignificant relative to the increase in $\tilde{e}_w(p)$ at lower pressures.

The next example is a North Sea chalk (Fig. 4) that shows pore collapse at a pressure only 1,000 psi below initial pressure of 7,000 psi. The $\tilde{e}_r(p)$ function increases almost simultaneously with instantaneous $\tilde{c}_p$ and the effect of $\tilde{e}_r(p)$ on $\tilde{e}_p(p)$ is shown in Fig. 8. Although $\tilde{e}_r(p)$ has an impact on $\tilde{e}_p(p)$ at moderate and high pressures for this example, the $\tilde{e}_w(p)$ function still dominates the behavior of $\tilde{e}_p(p)$ at pressures less than 1,500 psi.

**Estimating Gas-In-Place.** A method is proposed for estimating the initial (free) gas in place $G_i$ based on historical pressure/ cumulative data. The procedure also determines the water volume ratio $M$ and the $\tilde{e}_p(p)$ function. First, a plot of $p_r$ versus cumulative gas production $G_i$ should have the characteristic concave downward shape of a high-pressure reservoir influenced by associated water and PV compressibility. A range of values for $G_i$ should then be assumed, with the largest value based on an extrapolation of the early depletion data, and the lowest value being somewhat larger than the current $G_i$. For an assumed value of $G_i$, calculate for each measured $p_r$ and $G_i$, data the $\tilde{e}_p$ value from the rearranged material balance, Eq. 2.

$$\tilde{e}_p(p) = \left[1 - \frac{(p_r)^2}{(p_r - p)} \left(1 - \frac{G_i}{G_i}ight)\right] - \frac{1}{p_r - p}$$

(20)

At this point, a plot can be made of backcalculated $\tilde{e}_p$ as a function of pressure given the assumed $G_i$. Using reservoir rock and water properties, a family of $\tilde{e}_r(p)$ curve at various $M$ values can be generated independently to match against the backcalculated $\tilde{e}_p$ values. The data should honor the shape and magnitude of one $\tilde{e}_r(p)$ curve, where this match yields $G_i$, the $M$ value, and a $\tilde{e}_r(p)$ function that can be used to forecast future $p_r$ vs. $G_i$ behavior. This procedure gives a sound physical significance to the estimation of $G_i$ as opposed to a pure statistical best fit that may lead to unrealistic.
Characteristics of $p/z$ vs. $G_p$ Plots for High-Pressure Reservoirs

PV reduction, water expansion, and solution gas evolution, expressed in terms of $C_p(p)$ in the general material balance equation, provide pressure support for all reservoirs during depletion. The reservoir does not have to be overpressured or gaspressed. The term $C_p(p) = \frac{1}{\phi} \frac{d\phi}{dp}$ determines whether the conventional $p/z$ vs. $G_p$ plot yields a straight line. For most low-pressure reservoirs this term is small and is often neglected because a straight-line $p/z$ vs. $G_p$ plot is observed. Reservoirs undergoing depletion with initial pressure exceeding 1,000 psi are automatically candidates for being treated with the complete material balance equation.

Fig. 9 presents three generated $p/z$ vs. $G_p$ curves for a Gulf Coast overpressured sandstone reservoir using $M = 0$ (i.e., $C_p(p) = \frac{1}{\phi}$). Curve A accounts for PV reduction, including pore collapse at about 4,000 psi. Curve B uses the same $C_p(p)$ function as Curve A down to 4,000 psi (where pore collapse occurs) and thereafter uses a constant instantaneous compressibility of $4 \times 10^{-6}$ psi$^{-1}$. Curves C uses the initial instantaneous compressibility of $13 \times 10^{-6}$ psi$^{-1}$, which is equal to the constant throughout depletion. The difference between the two $p/z$ vs. $G_p$ plots. Curves A and C is a result of the actual decrease in PV compressibility. Including an external water volume that is 2 produces more curvature in the $p/z$ vs. $G_p$ plots, but the separation between curves with and without pore collapse is still very small (not shown).

Another example relates to a North Sea chalk reservoir where pore collapse occurs just below initial pressure. Fig. 10 presents generated $p/z$ vs. $G_p$ plots for $M = 0$ with pore collapse (Curve A) and with no pore collapse (Curve B). The effect of pore collapse is more significant than in the previous example because it occurs at a relatively high pressure.

Field Examples

Ellenburger Gas Reservoir. This field example is for a normal pressured (0.5 psi/ft) 1,800-ft-thick, dry gas reservoir with initial reservoir pressure of 6,675 psi at 200°F. Average porosity is about 5% with connate water saturation in the pay of about 35%. Permeability is high because of an extensive microfracture system that results in a high degree of interwell pressure communication and almost instantaneous pressure buildup to static conditions. Initial CO$_2$ concentration was about 28 mol%, and a gradual increase in CO$_2$ concentration to 31 mol% at the present time has been observed. The reservoir has produced about 3.1 Tcf, and currently has an average fieldwide bottomhole pressure of approximately 1,000 psi. The $p/z$ vs. $G_p$ plot shows a characteristic concave downward behavior, with an initial gas in place estimate of more than 4.4 Tcf using early data (Fig. 11). The $p/z$ vs. $G_p$ plot at low pressures has started flattening.

The procedure outlined earlier for determining initial free gas in place $G_f$ was used for this reservoir. Fig. 12 shows a plot of backcalculated $C_p$ vs. pressure for a range of $G_f$ from 3.0 Tcf to 3.6 Tcf. Another plot of $C_p(p)$ was generated independently from rock and fluid properties by use of an equation of state and by an estimated value of $M$ with $S_w = 0.35$, $S_o = 6.5 \times 10^{-6}$ psi$^{-1}$ (from Hall), and $C_{p0}(p)$. Fig. 13 shows the best-fit of data on the $C_p(p)$ curve for $M = 3.3$, corresponding to an initial free gas in place $G_f = 3.15$ Tcf.

The total water volume including connate and associated waters is given by

$$ W = \frac{GB_{p0}(S_w + M)}{5.615 \times (1 - S_w)^2} \quad (21) $$

which yields 8.45(10$^9$) STB. The initial solution gas in place $G_s$ is equal to $W$ times the initial solution gas/water ratio $R_{swi}$

$$ G_s = WR_{swi} \quad (22) $$

Because of the high CO$_2$ concentration in this reservoir, the solution gas/water ratio ($R_{swi}$) is about three times larger than for hydrocarbon systems. This yields a solution gas in place of $G_s = 0.55$ Tcf and a total initial gas in place of $G_t = 3.70$ Tcf. Fig. 11 shows the $p/z$ vs. $G_p$ forecast using the $M$ value determined from the match to calculate the $C_p(p)$ function from $S_w$, $M$, $S_o$, and $C_{p0}(p)$. Also shown on this figure is the plot of $p/z$ vs. $G_p$, sandstone. 

Fig. 9—Effect on $p/z$ vs. $G_p$ with and without pore collapse, sandstone.

Fig. 10—Effect on $p/z$ vs. $G_p$ with and without pore collapse, chalk.

Fig. 11—Pressure vs. cumulative production, Ellenburger gas reservoir.
Fig. 12—Backcalculated $\varepsilon_0$ vs. $p$ at various original gas in place (OGIP) values, Ellenburger gas reservoir.

Fig. 13—Matching backcalculated $\varepsilon_0$ to generated $\varepsilon_0$ curves, Ellenburger gas reservoir.

Fig. 14—$p/c$ vs. cumulative production, Anderson "L" reservoir.

Fig. 15—Backcalculated $\varepsilon_0$ vs. $p$ at various OGIP, Anderson "L" reservoir.

$\varepsilon_0(p)\{p = \text{G}_\text{e}\}$ vs. $\text{G}_\text{e}$ for historical performance data and for the forecast, where it is seen that the current cumulative gas produced equals the original free gas in place.

The associated water volume given by $M = 3.3$ consists of nonnet pay and an external limited aquifer. Log analysis indicates a net-to-gross ratio $R_{\text{ng}} = 0.5$, $\phi_s = 0.05$, and $\phi_{\text{sw}} = 0.03$, yielding $M_{\text{sw}} = 0.6$. External water is known to exist but has not been mapped because of lack of well control. The calculated aquifer water volume ratio $M_{\text{a}} = 2.7 (3.3 - 0.6)$, or an equivalent $R_{\text{sw}}/R_{\text{ng}} = 1.9$, seems reasonable for a limited aquifer.

Anderson "L". This reservoir has been studied by several authors and it is perhaps the best recognized example of a high-pressure gas reservoir with concave downward $p/c - G_e$ behavior (Fig. 14). The reservoir was abandoned after producing 55 Bscf, but pressure tests of public record were discontinued after 40 Bscf had been produced.

Different analyses by other authors have indicated original free gas in place between 65 to 75 Bscf. Fig. 15 shows backcalculated $\varepsilon_0$ vs. pressure for values of $G$ equal to 65, 72, and 90 Bscf. The 72 Bscf volume is chosen based on a best-fit match with the $\varepsilon_0(p)$ function calculated using $M = 2.25$, $\varepsilon_{\text{sw}} = 0.35$, $\varepsilon_{\text{sw}} = 3.2 \times 10^{-4}$ psi$^{-1}$, and a $\varepsilon_0(p)$ function from equation of state results. Although the first four data do not fall on the slightly increasing $\varepsilon_0(p)$ curve, data at pressures below this value do follow the trend down to the last pressure datum near 3,000 psi.

The 90 Bscf estimate produces unrealistically low $\varepsilon_0$ values, lower than would be calculated using the net reservoir PV and connate water compressibilities. The lowest estimate of 65 Bscf gives a slope for $\varepsilon_0(p)$ that cannot be accounted for using normal $\varepsilon_0(p)$ and $\varepsilon_{\text{sw}}(p)$ functions.

The forecasted $p/c$ vs. $G_e$ performance (Fig. 14) is calculated with the match determined above. Total gas in place is 76 Bscf, which includes 72 Bscf of original free gas plus 4 Bscf of solution gas.

Conclusions

1. A general form of the material balance equation for gas reservoirs has been presented. This equation has particular application to high-pressure reservoirs. A cumulative effective compressibility term $\varepsilon_0(p)$ has been defined in terms of pressure-dependent PV and total water cumulative compressibilities, $\varepsilon_{\text{sw}}(p)$ and $\varepsilon_{\text{sw}}(p)$, and the total volume of water associated with the net pay reservoir expressed as a ratio $M$.

2. The general material balance equation applies to all high-pressure reservoirs, both normal pressured and abnormally pressured (overpressured and geopressed).

3. The effect of a limited aquifer can be included as part of the $M$ term for most depletion-type reservoirs. Using the water volume ratio $M$ in the cumulative effective compressibility term, together with normal values of $\varepsilon_0$ and $\varepsilon_{\text{sw}}$, explains the "large" $\varepsilon_0$ values commonly reported for high-pressure gas reservoirs when linearizing the material balance equation. In fact, large values of $\varepsilon_0$ backcalculated from field performance data indicate that associated water influx is a dominant drive mechanism.

4. Only cumulative compressibilities ($\varepsilon_0$ and $\varepsilon_{\text{sw}}$) can be used in the general gas material balance equations because they are applied.
against the cumulative pressure drop ($p_1 - p$) in psf vs. $G_t$ plane. A method is given for calculating cumulative total water and PV compressibility $\frac{d}{dt}(p)$ and $\frac{d}{dp}(p)$.  
5. A method is proposed for estimating the original free gas in place from production data. This method uses backcalculated cumulative effective compressibility $\bar{c}_f$ which is plotted against pressure and compared with expected $\bar{c}_f(p)$ behavior calculated solely from rock and water properties for a range of values of the associated water volume ratio $M$.  
6. Pore collapse, in and of itself, does not contribute significantly to pressure support in overpressured gas reservoirs. In fact, pore collapse has little effect unless it occurs early in depletion at a relatively high pressure. The effect of pore collapse, if present, is a positive effect and tends to flatten the $p_1$ vs. $G_t$ curve, not bending the curve downward as has been implied by others.  
7. Gas found initially in solution in the connate and associated water is an important component of pressure support late in depletion (below 1500 psi) and may contribute additional producible volumes of gas. Typically, the solution gas in place $G_s$ represents 2 to 10% of the original free gas in place, the value depending primarily on total water volume ($M + S_w)(1 - S_o)$ and the initial solution gas/water ratio $R_{sw}$. Gas reservoirs with high CO$_2$ concentration (>20 mol%) can have even higher solution gas in place, $G_s$.  

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References
Appendix A—Derivation of General Gas Material Balance

The derivation that follows is based on the following assumptions:
1. Any pressure change caused by production or injection into the reservoir will be felt immediately throughout the total system including (a) net pay reservoir (R); (b) nonnet pay (NFP), including interbedded shales and poor quality rock assumed to be 100% water-saturated; and (c) limited aquifer (AQ), when present, also assumed to be water-saturated. The net pay and aquifer volumes are referred to as "associated" water volumes and both contribute to water influx during depletion.

2. Simple modifications to the material balance equations can be made to generalize for nonnet pay that has an initial free gas saturation.

3. All water in the system is initially saturated with solution gas.

Practically, the assumption of equal pressure throughout the system is reasonable, and any transient effects caused by a large aquifer may be treated by a conventional water influx term \( \left( \frac{W}{B_A} \right) \) as shown below.

For the sake of brevity we have chosen to omit explicit reference to pressure dependence—i.e., \( \varepsilon, \varepsilon_p, \eta, \sigma, \delta, \xi, \zeta \), and \( \eta_w \) should actually read \( \varepsilon_{\text{w}(p)}, \varepsilon_{\text{p}(p)}, \xi_{\text{w}(p)}, \eta_{\text{w}(p)}, \delta_{\text{w}(p)}, \xi_{\text{p}(p)}, \zeta_{\text{w}(p)} \), and \( \eta_{\text{w}(p)} \).

**Derivation.**

The volumetric balance at any pressure states that the total PV \( (V_p + V_w) \) equals the net reservoir PV occupied by gas and water \( (V_p + V_w) \) plus the associated (nonnet pay and aquifer) PV which also is occupied by gas and water \( (V_p + V_w) \):

\[
V_{ps} = (V_p + V_w) + (V_{ps} + V_{ws}). \tag{A-1}
\]

The net-pay reservoir PV \( V_{ps} \) is given by the initial volume \( V_{ps} \) less the change in PV \( \Delta V_{ps} \):

\[
V_{ps} = V_{ps} - \Delta V_{ps}. \tag{A-2}
\]

\[
V_{ps} = V_{ps} + V_{ws}. \tag{A-3}
\]

\[
V_{ps} = GB_p + GB_{ps} \tag{A-4}
\]

and

\[
\Delta V_{ps} = GB_p \frac{\varepsilon_p}{1 - \varepsilon_p} \sigma_p - GB_{ps} \frac{\varepsilon_p}{1 - \varepsilon_p} \sigma_p. \tag{A-5}
\]

PV of the associated rock is given by the initial PV less the change in PV, i.e.,

\[
V_{ps} = \frac{GB_p}{1 - \varepsilon_p} M - \frac{GB_{ps}}{1 - \varepsilon_p} M \varepsilon_p. \tag{A-6}
\]

The net reservoir gas volume is given by the sum of unproduced free gas, gas released from solution, and any injected gas,

\[
V_{ps} = (V_{ps} + \text{injection}) + (V_{ps} + \text{solution}) + (V_{ps} + \text{injection}). \tag{A-7}
\]

resulting in

\[
V_{ps} = (G - (G - W_p R_w))B_p + GB_p \frac{S}{1 - \varepsilon_p} R_w - \frac{B_p}{5.615} \tag{A-8}
\]

pressure/volume/temperature properties \( B_p \) and \( R_w \) are evaluated at current reservoir pressure. Value \( G \) for a gas condensate is the wet gas volume calculated by adding separator gas to liquid condensate converted to an equivalent surface gas volume. Also, the two-phase Z-factor must be used to calculated \( B_p \) for gas condensate reservoirs. Strictly speaking the cumulative water production term \( W_p \) represents "free" water production and not the water condensed out of solution from the produced gas wellstream.

The gas volume in the associated PV is a function of the amount of gas that has come out of solution,

\[
V_{ps} = \frac{GB_p}{1 - \varepsilon_p} M \frac{1}{B_p} (R_{ps} - R_w) + \frac{1}{5.615} \tag{A-9}
\]

The water volume in the net-pay reservoir equals the unproduced initial water plus injected water plus water encroachment from an external aquifer,

\[
V_{ws} = (V_{ws})_{\text{injection}} + (V_{ws})_{\text{solution}} + (V_{ws})_{\text{encroachment}} \tag{A-10}
\]

yielding

\[
V_{ws} = \left( GB_{ps} \frac{S}{1 - \varepsilon_p} B_w - \frac{W_p R_w}{5.615} \right) + 5.615 W_p B_w. \tag{A-11}
\]

The aquifer encroachment term \( W_p \) represents any external water volume that is not already included in the \( M \) term. Later in the derivation, we show the conditions required so that water encroachment (treated rigorously by the method of superposition) can be included as part of the \( M \) term used in the cumulative effective compressibility \( \varepsilon \).

The water volume in the associated PV is given by simple expansion,

\[
V_{ws} = \frac{GB_{ps}}{1 - \varepsilon_p} M B_w \tag{A-12}
\]

Inserting the appropriate equations above in Eq. A-1, rearranging, and grouping terms yields,

\[
(G - (R_{ps} - R_w) B_p) \frac{S}{1 - \varepsilon_p} B_w \tag{A-13}
\]

\[
(G - (R_{ps} - R_w) B_p) \frac{S}{1 - \varepsilon_p} B_w \tag{A-14}
\]

Defining the total water/gas formation volume factor \( B_w \),

\[
B_w = B_p - \frac{(R_{ps} - R_w) B_p}{5.615} \tag{A-15}
\]

Noting that \( B_w = B_{ps} \), and defining the cumulative total water/gas compressibility \( \varepsilon_w \),

\[
\varepsilon_w = \frac{(R_{ps} - R_w) B_p}{5.615} \tag{A-16}
\]

Now, defining a cumulative effective compressibility \( \varepsilon_w \),

\[
\varepsilon_w = \frac{S \varepsilon_{\text{p}(p)} + B_w + M \varepsilon_p}{1 - \varepsilon_p} \tag{A-17}
\]
gives
\[ G(B_p - B_w) + GB_p \tilde{e} \tilde{z}(p_t - p) \]
\[ = \frac{B_p}{R_p} \left[ G_s - G_m + W_p B_w + \frac{5.615}{B_p} (W_w B_w - W_m B_m - W_i) \right]. \]  

(A-17)

Dividing through by \( GB_p \) and expressing \( B_p = \frac{G_p}{T_p} \), gives the final form of the material balance
\[ \frac{p}{2} \left[ 1 - \tilde{e}_i(p_t - p) \right] = \left[ \frac{p}{2} \right] \left[ 1 - \frac{1}{G_p} \left( G_s - G_m + W_p B_w \right) \right] + \frac{5.615}{B_p} (W_w B_w - W_m B_m - W_i). \]  

(A-18)

The \( p/2 \) vs. cumulative plot, including all terms, would consider \( G_p \left[ 1 - \tilde{e}_i(p_t - p) \right] \) vs. the entire production/injection/encroachment term \( Q \)
\[ \frac{p}{2} \left[ 1 - \tilde{e}_i(p_t - p) \right] = \left[ \frac{p}{2} \right] \left[ 1 - \frac{G_s}{G_p} \right] Q. \]  

(A-19)

with
\[ Q = G_s - G_m + W_p B_w + \frac{5.615}{B_p} (W_w B_w - W_m B_m - W_i), \]  

(A-20)

where the intercept is given by \( p/2 \), and the slope equals \( (p/2)/G \).

Setting \( G_m = W_m = W_w = W_i = 0 \) gives the common form of the gas material balance,
\[ \frac{p}{2} \left[ 1 - \tilde{e}_i(p_t - p) \right] = \left[ \frac{p}{2} \right] \left[ 1 - \frac{G_s}{G_p} \right]. \]  

(A-21)

Treatling Limited Aquifers in \( \tilde{e}_i \) Term. The material balance thus far has considered any associated water volume expressed in terms of the \( M \) parameter. In fact \( M \) may include a limited aquifer with up to 25 times the reservoir PV for a system permeability greater than about 100 md, and even larger aquifer volumes for higher permeabilities. The condition that determines when a limited aquifer can be treated as part of the \( \tilde{e}_i \) term is outlined below. We start with the general material-balance equation including a water encroachment term \( W_w \) and a \( \tilde{e}_i \) term that considers only net pay,
\[ \frac{p}{2} \left[ 1 - \tilde{e}_i(p_t - p) \right] = \left[ \frac{p}{2} \right] \left[ 1 - \frac{G_s}{G_p} \right] + \frac{5.615}{B_p} W_w. \]  

(A-22)

and
\[ \tilde{e}_i = \frac{S_w \tilde{e}_w + \tilde{e}_v + (W_w \text{sat} V_{sat}) \tilde{e}_w}{1 - S_w}, \]  

(A-23)

The water encroachment term calculated by superposition is expressed,
\[ W_w = B \sum Q_w \Delta t \Delta p, \]  

(A-24)

where \( Q_w \Delta t \Delta p \) is the dimensionless cumulative influx given as a function of dimensionless time \( \tau \) and aquifer to reservoir radius \( r_d = r_{aq}/r_p \). Value \( \Delta t \) is given by \( p_t - p_{t-1} \) (in the limit for small time steps), and \( \Delta p \) to be \( p_t - p_{t-1} \). Assuming that permeability is reasonably high and the ratio \( r_d \) is not too large, \( Q_w \) for the smallest time step will approach the limiting value \( Q_w^* \) and the summation can be closely approximated by
\[ \sum Q_w \Delta t \Delta p = Q_w^* \Delta t, \]  

(A-25)

giving a simple expression for \( W_w \), which is dependent on time and only dependent on reservoir pressure,
\[ W_w = B Q_w^* \Delta t \]  

(A-26)

and
\[ B = \frac{2 \pi}{5.615} \frac{D_m}{G} \left[ \frac{D_m}{G} + \Delta p \right]. \]  

(A-27)

Expressing \( W_w \) in terms of aquifer PV \( V_{aq} \),
\[ W_w = \frac{G_p}{G} \Delta t \left[ \frac{D_m}{G} + \Delta p \right]. \]  

(A-28)

The material-balance equation can then be written
\[ \frac{5.615}{B_p} W_w = \left[ \frac{G}{G_p} \right] \left[ 1 - \frac{G_s}{G_p} \right] Q, \]  

(A-29)

and simplified in a form where the \( \tilde{e}_i \) term includes the aquifer contribution to pressure support,
\[ \frac{p}{2} \left[ 1 - \tilde{e}_i(p_t - p) \right] = \left[ \frac{p}{2} \right] \left[ 1 - \frac{G_s}{G_p} \right] \frac{Q_p}{G_p} + \frac{p}{2} \frac{V_{aq} \text{sat} V_{sat} \tilde{e}_w}{G_p \frac{1}{2}}, \]  

(A-30)

Rearranging, we arrive at the general form of the material balance (without water production and gas/water injection terms):
\[ \frac{p}{2} \left[ 1 - \tilde{e}_i(p_t - p) \right] = \left[ \frac{p}{2} \right] \left[ 1 - \frac{G_s}{G_p} \right] \frac{Q_p}{G_p} \]  

(A-31)

where
\[ \tilde{e}_w = \frac{S_w \tilde{e}_w + \tilde{e}_v + (V_{aq} \text{sat} V_{sat}) \tilde{e}_w}{1 - S_w}, \]  

(A-32)

and
\[ M = \frac{V_{aq} + V_{aq} \text{sat} V_{sat}}{V_{aq}}. \]  

(A-33)

\[ M = \frac{V_{aq} + V_{aq} \text{sat} V_{sat}}{V_{aq}}. \]  

(A-34)

**SI Metric Conversion Factors**

- \(^\circ\text{F}\) to \(^\circ\text{C}\): \( \frac{\text{C}}{9/5} - 32 \)
- m to cm: \( \times 10^2 \)
- m to ft: \( \times 3.281 \)
- psi to kPa: \( \times 6.895 \)

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