

Course Notes
for
Petroleum Engineering 311
Reservoir Petrophysics

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I. ROCK POROSITY

I) Definition

A measure of the pore space available for the storage of fluids in rock

In general form:

$$\text{Porosity} = \frac{V_p}{V_b} = \frac{V_b - V_m}{V_b}$$

where:

V_b is expressed in fraction

$$V_b = V_p + V_m$$

V_b = bulk volume of reservoir rock, (L^3)

V_p = pore volume, (L^3)

V_m = matrix volume, (L^3)

II) Classification

A. Primary (original) Porosity

Developed at time of deposition

B. Secondary Porosity

Developed as a result of geologic process occurring after deposition

C. Total Porosity

$$t = \frac{\text{total pore space}}{V_b} = \frac{V_b - V_m}{V_b}$$

D. Effective Porosity

$$e = \frac{\text{interconnected pore space}}{V_b}$$

1. Clean sandstones: $e = t$

2. Carbonate, cemented sandstones: $e < t$

III) Range of values of porosity

A. Maximum porosity value obtained from a cubic packing of uniform spheres:

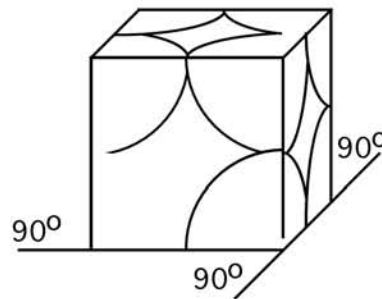
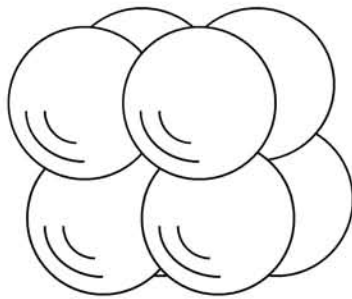
r = sand grain radius

$$V_b = (2r)^3 = 8r^3$$

$$V_m = 8(1/8 \text{ sphere}) = 1 \text{ sphere} = (4/3)\pi r^3$$

$$\phi = \frac{V_b - V_m}{V_b} = \frac{8r^3 - (4/3)\pi r^3}{8r^3}$$

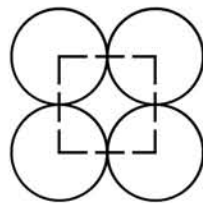
$$\phi = 1 - \pi/6 = 0.476 \text{ max. value}$$



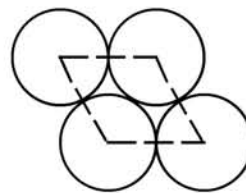
$$\phi = 0.476$$

B. Intermediate porosity values

For uniform spheres, porosity is a function of packing:



cubic
Porosity = 0.476



rhombohedral
Porosity = 0.259

C. Minimum Value:

Porosity = 0.0

VI) Factors affecting porosity

- A. Factors:
1. Particle shape
 2. Particle arrangement
 3. Particle size distribution
 4. Cementation
 5. Vugs and fractures

B. Particle shape

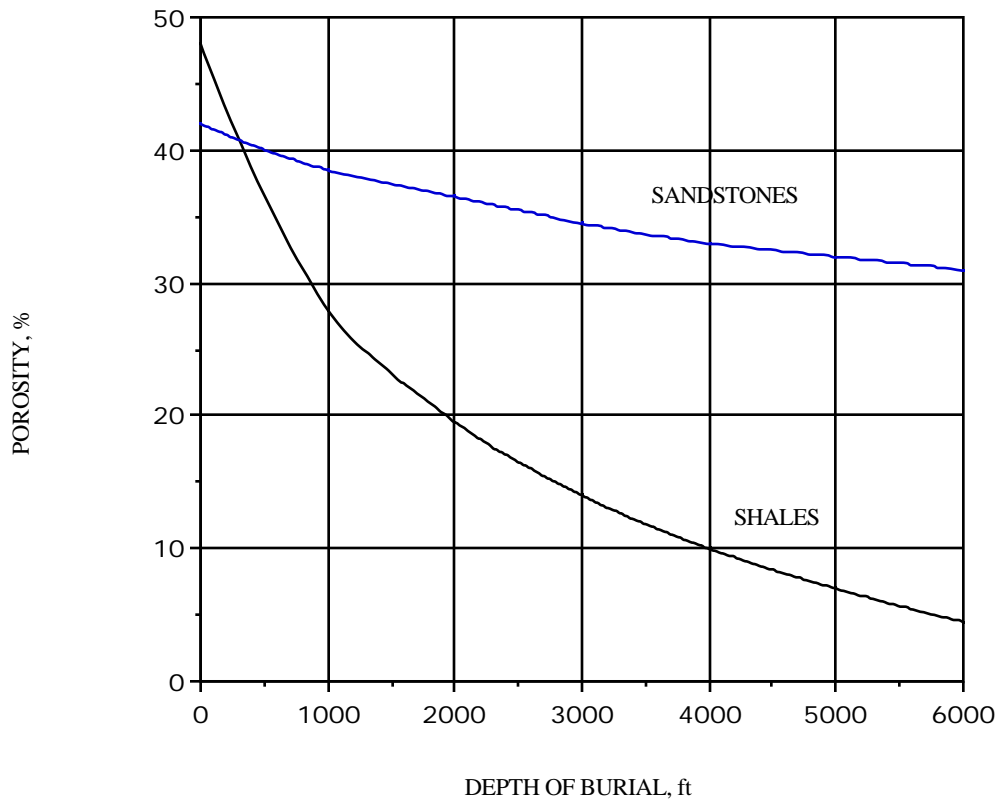
Porosity increases as particle uniformity decreases.

C. Packing Arrangement

Porosity decreases as compaction increases

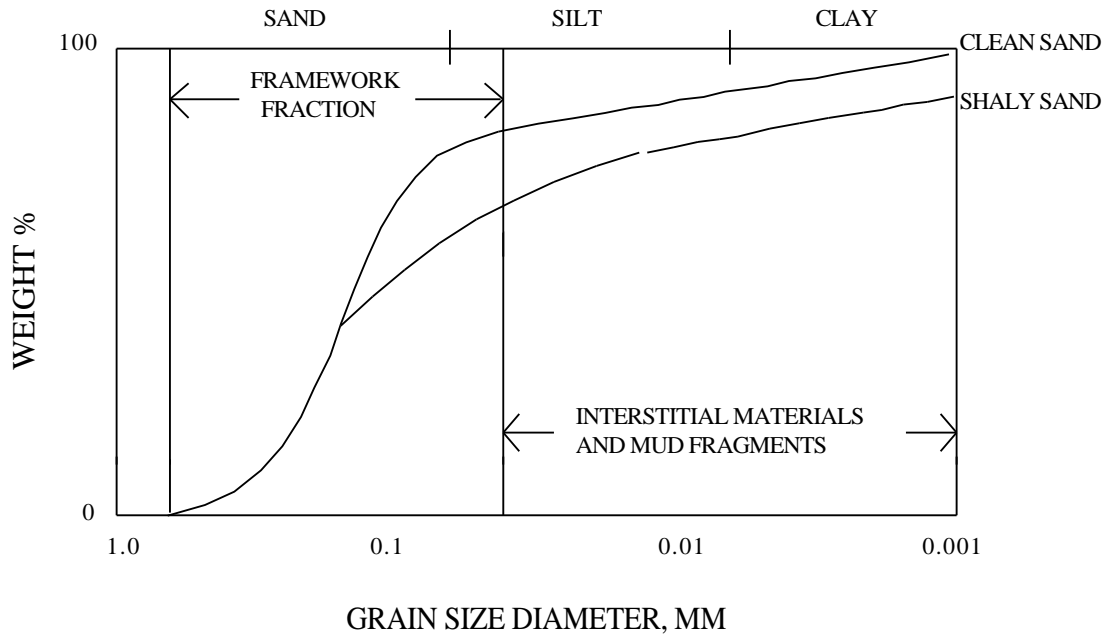
EFFECT OF NATURAL COMPACTION ON POROSITY

(FROM KRUMBEIN AND SLOSS.)



D. Particle Size Distribution

Porosity decreases as the range of particle size increases



E. Interstitial and Cementing Material

1. Porosity decreases as the amount of interstitial material increases
2. Porosity decreases as the amount of cementing material increases
3. Clean sand - little interstitial material
Shaly sand - has more interstitial material

F. Vugs, Fractures

1. Contribute substantially to the volume of pore spaces
2. Highly variable in size and distribution
3. There could be two or more systems of pore openings - extremely complex

V) Measurement of porosity

$$= \frac{V_b - V_m}{V_b} = \frac{V_p}{V_b}$$

Table of matrix densities

Lithology	m (g/cm ³)
Quartz	2.65
Limestone	2.71
Dolomite	2.87

A. Laboratory measurement

1. Conventional core analysis

a. measure any two

- 1) bulk volume, V_b
- 2) matrix volume, V_m
- 3) pore volume, V_p

b. bulk volume

- 1) calculate from dimensions
- 2) displacement method

a) volumetric (measure volume)

- (1) drop into liquid and observe volume change of liquid
- (2) must prevent test liquid from entering pores space of sample
 - (a) coat with paraffin
 - (b) presaturate sample with test liquid
 - (c) use mercury as test liquid

b) gravimetric (measure mass)

- (1) Change in weight of immersed sample- prevent test liquid from entering pore space
- (2) Change in weight of container and test fluid when sample is introduced

c. matrix volume

1) assume grain density

$$V_m = \frac{\text{dry weight}}{\text{matrix density}}$$

2) displacement method

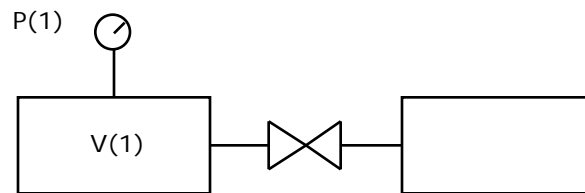
Reduce sample to particle size, then

a) volumetric

b) gravimetric

3) Boyle's Law: $P_1V_1 = P_2V_2$

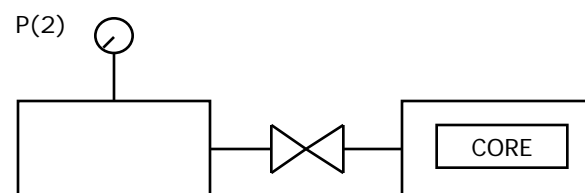
a)



VALVE CLOSED

b) Put core in second chamber, evacuate

c) Open valve



VALVE OPEN

V_2 = Volumetric of first chamber &
volume of second chamber-matrix
volume or core (calculated)

V_T = Volume of first chamber +
volume second chamber (known)

4) $V_m = V_T - V_2$

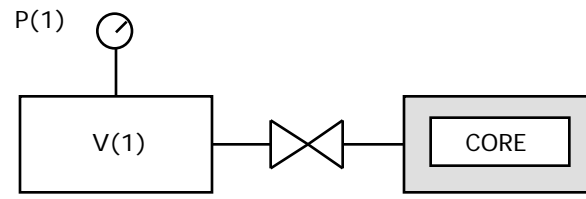
d. pore volume

1) gravimetric

$$V_p = \frac{\text{saturated weight} - \text{dry weight}}{\text{density of saturated fluid}}$$

2) Boyle's Law: $P_1V_1 = P_2V_2$

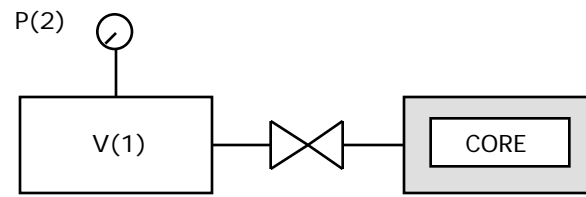
a)



VALVE CLOSED

b) Put core in Hassler sleeve, evacuate

c) Open valve



VALVE OPEN

V_2 = Volume of first chamber + pore volume of core (calculated)

3) $V_p = V_2 - V_1$

2. Application to reservoir rocks
 - a. intergranular porosity
(sandstone, some carbonates)
 - 1) use representative plugs from whole core in laboratory measurements
 - 2) don't use sidewall cores
 - b. secondary porosity
(most carbonates)
 - 1) use whole core in laboratory measurements
 - 2) calculate bulk volume from measurements
 - 3) determine matrix or pore volume from Boyle's Law procedure

Example I-1

A core sample coated with paraffin was immersed in a Russell tube. The dry sample weighed 20.0 gm. The dry sample coated with paraffin weighed 20.9 gm. The paraffin coated sample displaced 10.9 cc of liquid. Assume the density of solid paraffin is 0.9 gm/cc. What is the bulk volume of the sample?

Solution:

$$\text{Weight of paraffin coating} = 20.9 \text{ gm} - 20.0 \text{ gm} = 0.9 \text{ gm}$$

$$\text{Volume of paraffin coating} = 0.9 \text{ gm} / (0.9 \text{ gm/cc}) = 1.0 \text{ cc}$$

$$\text{Bulk volume of sample} = 10.9 \text{ cc} - 1.0 \text{ cc} = 9.9 \text{ cc}$$

Example I-2

The core sample of problem I-1 was stripped of the paraffin coat, crushed to grain size, and immersed in a Russell tube. The volume of the grains was 7.7 cc. What was the porosity of the sample? Is this effective or total porosity.

Solution:

$$\text{Bulk Volume} = 9.9 \text{ cc}$$

$$\text{Matrix Volume} = 7.7 \text{ cc}$$

$$= \frac{V_b - V_m}{V_b} = \frac{9.9 \text{ cc} - 7.7 \text{ cc}}{9.9 \text{ cc}} = 0.22$$

It is total porosity.

Example I-3

Calculate the porosity of a core sample when the following information is available:

Dry weight of sample = 427.3 gm

Weight of sample when saturated with water = 448.6 gm

Density of water = 1.0 gm/cm³

Weight of water saturated sample immersed in water = 269.6 gm

Solution:

$$V_p = \frac{\text{sat. core wt. in air} - \text{dry core wt.}}{\text{density of water}}$$

$$V_p = \frac{448.6 \text{ gm} - 427.3 \text{ gm}}{1 \text{ gm/cm}^3}$$

$$V_p = 21.3 \text{ cm}^3$$

$$V_b = \frac{\text{sat. core wt. in air} - \text{sat. core wt. in water}}{\text{density of water}}$$

$$V_b = \frac{448.6 \text{ gm} - 269.6 \text{ gm}}{1 \text{ gm/cm}^3}$$

$$V_b = 179.0 \text{ cm}^3$$

$$= \frac{V_p}{V_b} = \frac{21.3 \text{ cm}^3}{179.0 \text{ cm}^3} = .119$$

$$= 11.9\%$$

What is the lithology of the sample?

$$V_m = V_b - V_p$$

$$V_m = 179.0 \text{ cm}^3 - 21.3 \text{ cm}^3 = 157.7 \text{ cm}^3$$

$$m = \frac{\text{wt. of dry sample}}{\text{matrix vol.}} = \frac{427.3 \text{ gm}}{157.7 \text{ cm}^3} = 2.71 \text{ gm}/(\text{cm}^3)$$

The lithology is limestone.

Is the porosity effective or total? Why?

Effective, because fluid was forced into the pore space.

Example I-4

A carbonate whole core (3 inches by 6 inches, 695 cc) is placed in cell two of a Boyles Law device. Each of the cells has a volume of 1,000 cc. Cell one is pressured to 50.0 psig. Cell two is evacuated. The cells are connected and the resulting pressure is 28.1 psig. Calculate the porosity of the core.

Solution:

$$\begin{aligned}P_1 V_1 &= P_2 V_2 \\V_1 &= 1,000 \text{ cc} \\P_1 &= 50 \text{ psig} + 14.7 \text{ psia} = 64.7 \text{ psia} \\P_2 &= 28.1 \text{ psig} + 14.7 = 42.8 \text{ psia} \\V_2 &= (64.7 \text{ psia}) (1,000 \text{ cc}) / (42.8 \text{ psia}) \\V_2 &= 1,512 \text{ cc} \\V_m &= V_T - V_2 \\V_m &= 2,000 \text{ cc} - 1,512 \text{ cc} - 488 \text{ cc} \\&= \frac{V_T - V_m}{V_T} = \frac{695 \text{ cc} - 488 \text{ cc}}{695 \text{ cc}} = .298 = 29.8\%\end{aligned}$$

VI) Subsurface measurement of porosity

A. Types of logs from which porosity can be derived

1. Density log:

$$d = \frac{m - L}{m - f}$$

2. Sonic log:

$$s = \frac{t_L - t_m}{t_f - t_m}$$

3. Neutron log:

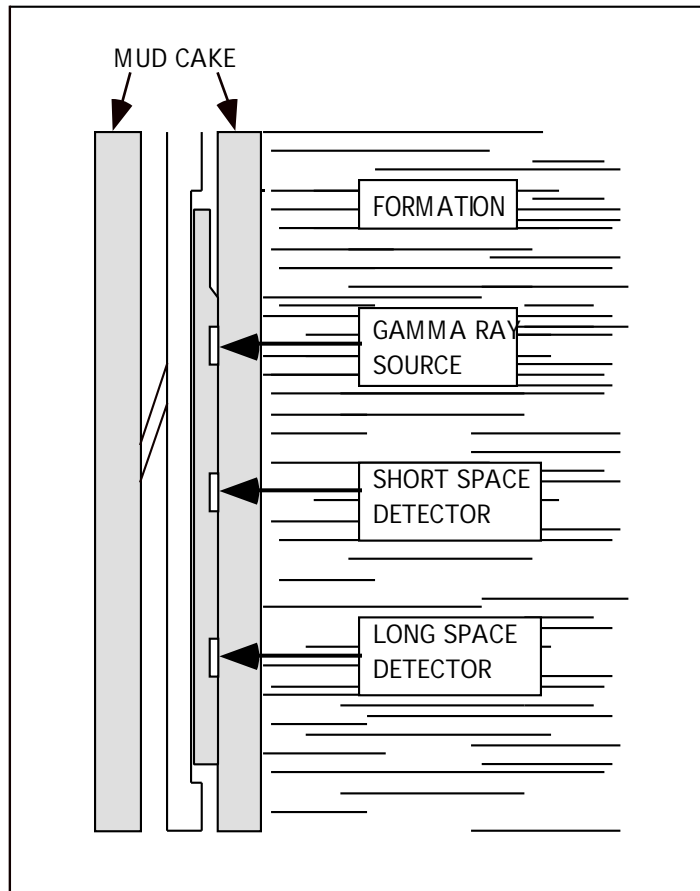
$$e^{-k} = CN_f$$

Table of Matrix Properties
(Schlumberger, Log Interpretation Principles, Volume I)

<u>Lithology</u>	<u>t_m(μsec/ft)</u>	<u>m(gm/cc)</u>
Sandstone	55.6	2.65
Limestone	47.5	2.71
Dolomite	43.5	2.87
Anhydrite	50.0	2.96
Salt	67.0	2.17
Water	189.0	1.00

B. Density Log

1. Measures bulk density of formation



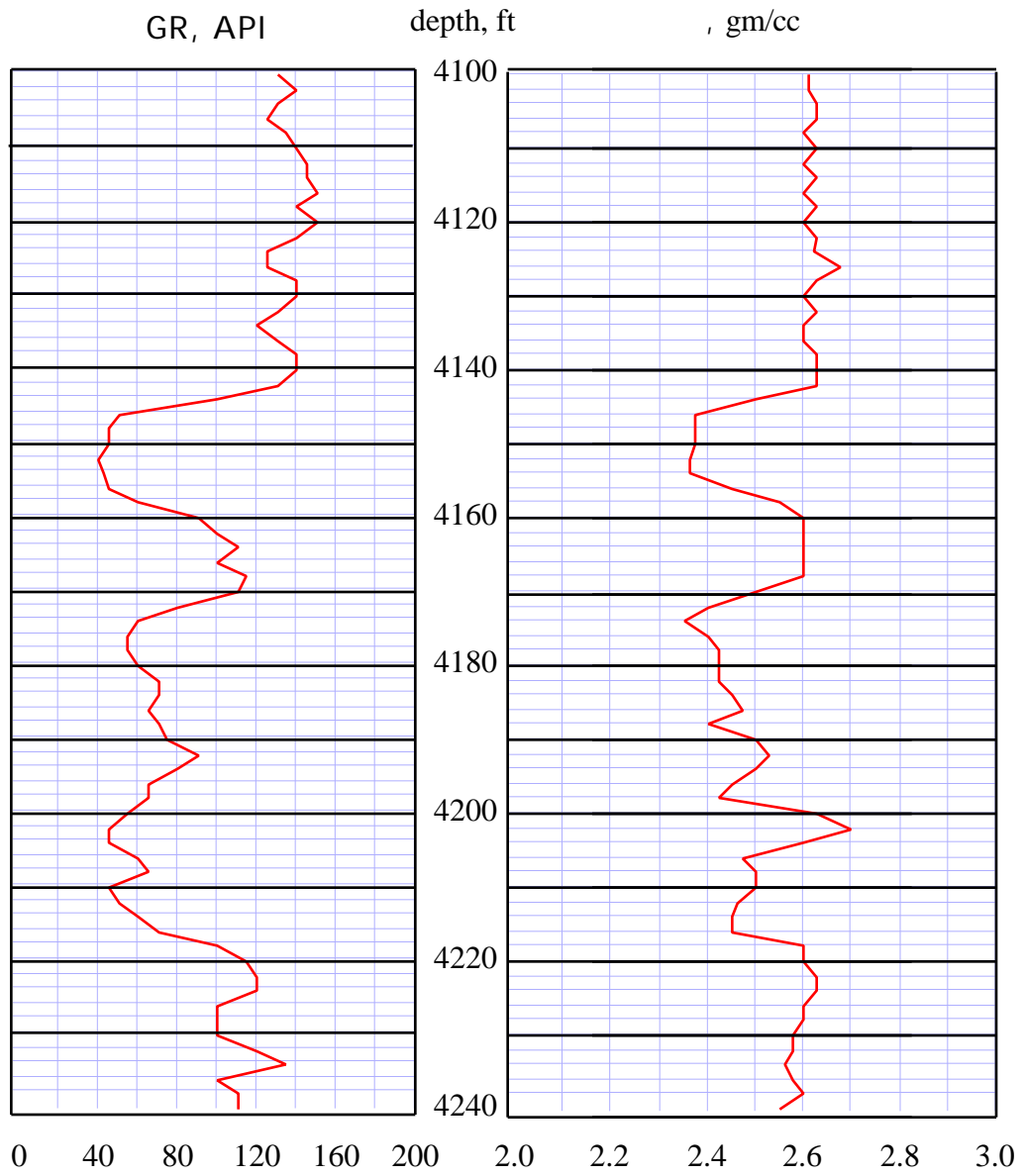
2. Gamma rays are stopped by electrons - the denser the rock the fewer gamma rays reach the detector

3. Equation

$$L = m(1 -) + f()$$

$$d = \frac{m - L}{m - f}$$

FORMATION DENSITY LOG



Example I-5

Use the density log to calculate the porosity for the following intervals assuming $\rho_{\text{matrix}} = 2.68$ gm/cc and $\rho_{\text{fluid}} = 1.0$ gm/cc.

Interval, ft	L, gm/cc	d, %
4143-4157	2.375	18
4170-4178	2.350	20
4178-4185	2.430	15
4185-4190	2.400	17
4197-4205	2.680	0
4210-4217	2.450	14

Example:

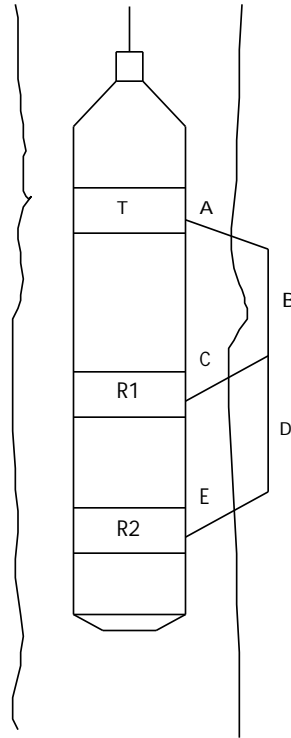
Interval 4,143 ft -4,157 ft :

$$L = 2.375 \text{ gm/cc}$$

$$d = \frac{\rho_{\text{matrix}} - L}{\rho_{\text{matrix}} - \rho_{\text{fluid}}} = \frac{2.68 \text{ gm/cc} - 2.375 \text{ gm/cc}}{2.68 \text{ gm/cc} - 1.0 \text{ gm/cc}} = 0.18$$

C. Sonic Log

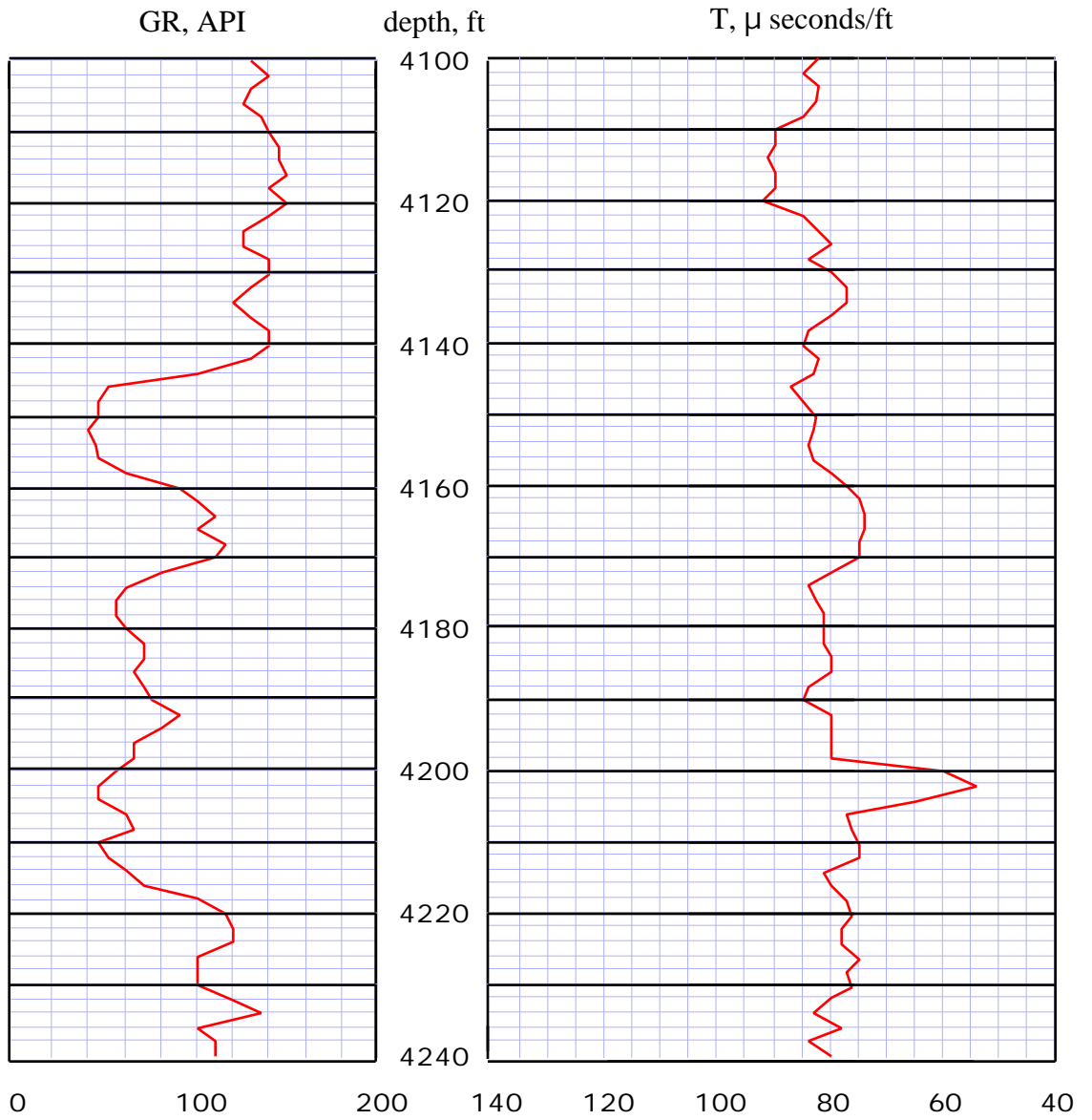
1. Measures time required for compressional sound waves to travel through one foot of formation



2. Sound travels more slowly in fluids than in solids. Pore space is filled with fluids. Travel time increases as porosity increases.
3. Equation

$$t_L = t_m(1 - \phi) + t_f(\phi) \quad (\text{Wylie Time Average Equation})$$

SONIC LOG



Example I-6

Use the Sonic log and assume sandstone lithology to calculate the porosity for the following intervals.

Interval (ft)	t_L μ second/ft	s, %
4,144-4,150	86.5	25
4,150-4,157	84.0	24
4,171-4,177	84.5	24
4,177-4,187	81.0	21
4,199-4,204	53.5	1
4,208-4,213	75.0	17

Example:

Interval 4144 ft - 4150 ft :

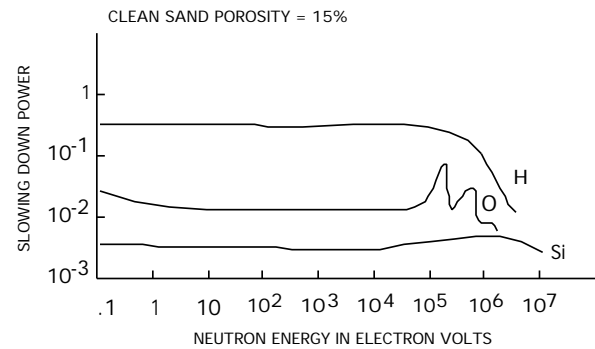
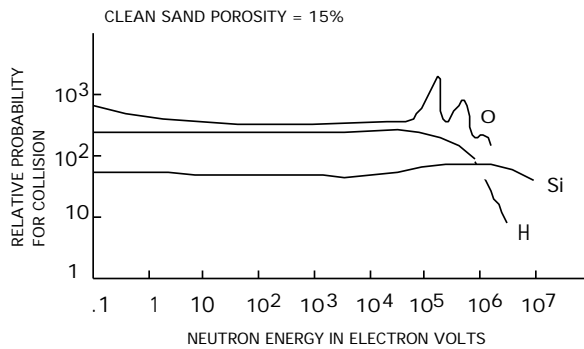
$$t_L = 86.5 \mu\text{-sec/ft}$$

$$s = \frac{t_L - t_m}{t_f - t_m} = \frac{86.5 \mu \text{ sec/ft} - 51.6 \mu \text{ sec/ft}}{189.0 \mu \text{ sec/ft} - 51.6 \mu \text{ sec/ft}} = 0.25$$

D. Neutron Log

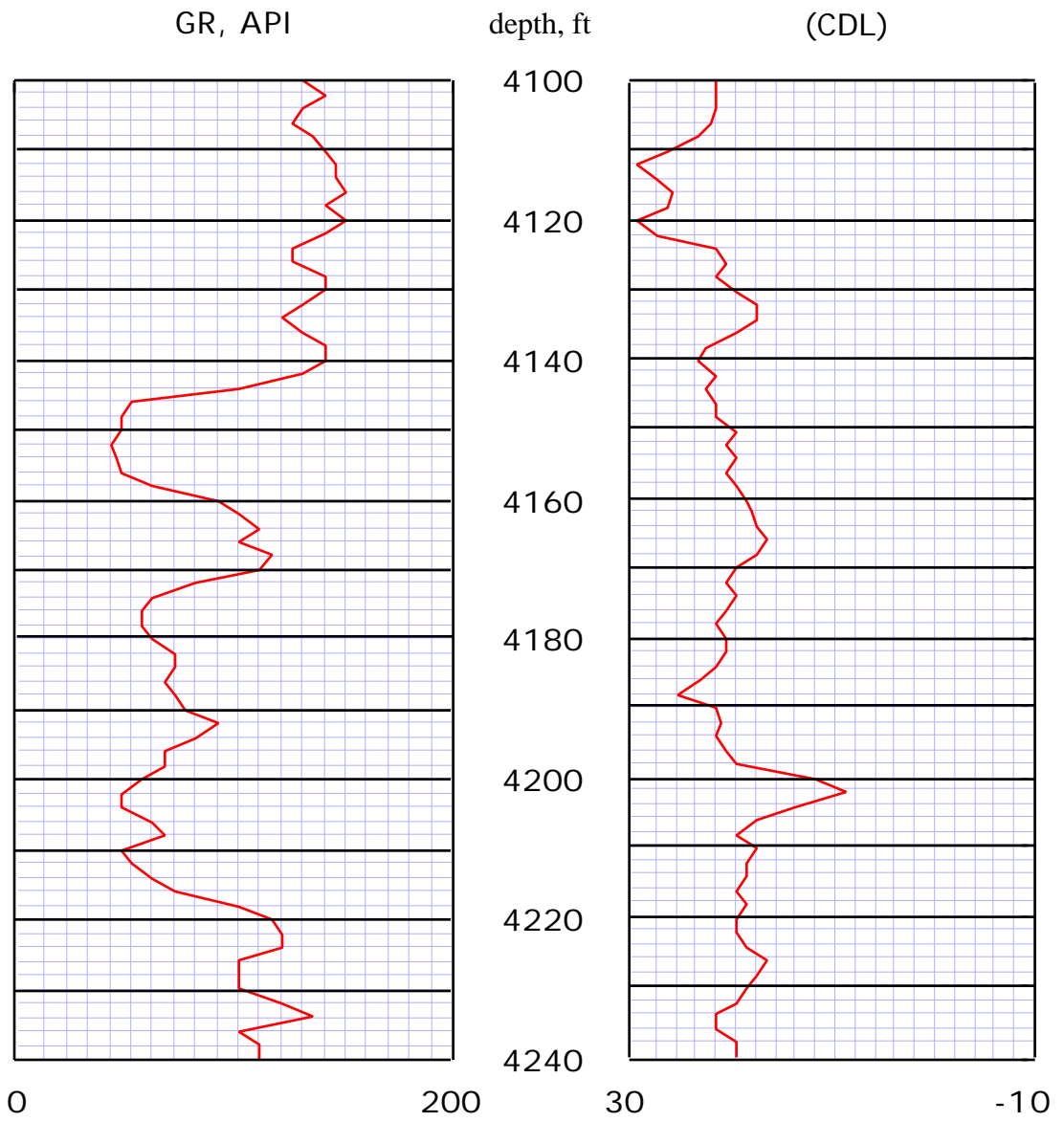
1. Measures the amount of hydrogen in the formation (hydrogen index)

Element	Average Number Collisions	Maximum Energy Loss/ Collision, %	Atomic Collision	Atomic Number
Calcium	371	8	40.1	20
Chlorine	316	10	35.5	17
Silicone	261	12	28.1	14
Oxygen	150	21	16.0	8
Carbon	115	28	12.0	6
Hydrogen	18	100	1.0	1



2. In clean, liquid filled formations, hydrogen index is directly proportional to porosity. Neutron log gives porosity directly.
3. If the log is not calibrated, it is not very reliable for determining porosity. Run density log to evaluate porosity, lithology, and gas content.

NEUTRON DENSITY LOG



Example I-7

Use the neutron log to determine porosity for the following intervals.

Solution:

<u>Interval (ft)</u>	<u>n (%)</u>
4,143-4,149	23
4,149-4,160	20
4,170-4,184	21
4,198-4,204	9
4,208-4,214	19

Example I-8

Calculate the porosity and lithology of the Polar No. 1 drilled in Lake Maracaibo. The depth of interest is 13,743 feet. A density log and a sonic log were run in the well in addition to the standard Induction Electric Survey (IES) survey.

The readings at 13,743 feet are:

$$\begin{aligned} \text{bulk density} &= 2.522 \text{ gm/cc} \\ \text{travel time} &= 62.73 \text{ } \mu\text{-sec/ft} \end{aligned}$$

Solution:

Assume fresh water in pores.

Assume sandstone:

$$m = 2.65 \text{ gm/cc}$$

$$t_m = 55.5 \text{ } \mu\text{-sec/ft}$$

$$d = \frac{m - L}{m - f} = \frac{2.65 \text{ gm/cc} - 2.522 \text{ gm/cc}}{2.65 \text{ gm/cc} - 1.0 \text{ gm/cc}} = 7.76\%$$

$$s = \frac{t_L - t_m}{t_f - t_m} = \frac{62.73 \text{ } \mu\text{ sec/ft} - 55.5 \text{ } \mu\text{ sec/ft}}{189.0 \text{ } \mu\text{ sec/ft} - 55.5 \text{ } \mu\text{ sec/ft}} = 5.42\%$$

Assume limestone:

$$m = 2.71 \text{ gm/cc}$$

$$t_m = 47.5 \text{ } \mu\text{-sec/ft}$$

$$d = \frac{m - L}{m - f} = \frac{2.71 \text{ gm/cc} - 2.522 \text{ gm/cc}}{2.71 \text{ gm/cc} - 1.0 \text{ gm/cc}} = 10.99\%$$

$$s = \frac{t_L - t_m}{t_f - t_m} = \frac{62.73 \text{ } \mu\text{ sec/ft} - 47.5 \text{ } \mu\text{ sec/ft}}{189.0 \text{ } \mu\text{ sec/ft} - 47.5 \text{ } \mu\text{ sec/ft}} = 10.76\%$$

Assume dolomite:

$$m = 2.87 \text{ gm/cc}$$

$$t_m = 43.5 \text{ } \mu\text{-sec/ft}$$

$$d = \frac{m - L}{m - f} = \frac{2.87 \text{ gm/cc} - 2.522 \text{ gm/cc}}{2.87 \text{ gm/cc} - 1.0 \text{ gm/cc}} = 18.619\%$$

$$s = \frac{t_L - t_m}{t_f - t_m} = \frac{62.73 \text{ } \mu\text{ sec/ft} - 43.5 \text{ } \mu\text{ sec/ft}}{189.0 \text{ } \mu\text{ sec/ft} - 43.5 \text{ } \mu\text{ sec/ft}} = 13.22\%$$

$$\text{limestone} = 11\%$$

Since both logs "read" nearly the same porosity when a limestone lithology was assumed then the hypothesis that the lithology is limestone is accepted.

Are the tools measuring total or effective porosity? Why?

The density log measures total compressibility because it "sees" the entire rock volume, including all pores. The sonic log tends to measure the velocity of compressional waves that travel through interconnected pore structures as well as the rock matrix. The general consensus is that the sonic log measures effective porosity when we use the Wyllie "time-average" equation.

It is expected that the effective porosity is always less than, or equal to, the total porosity.

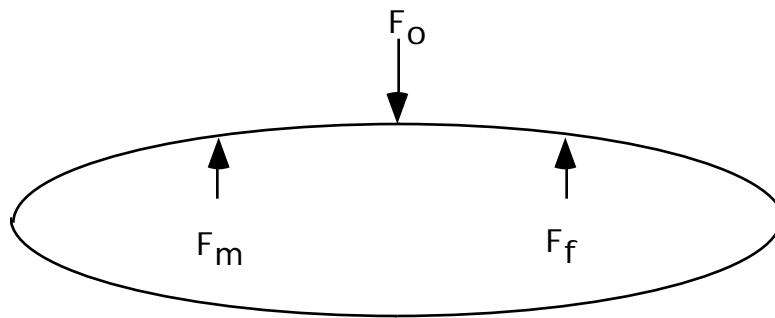
VII) Compressibility of porous rocks

Compressibility, c is the fractional change in volume per unit change in pressure:

$$c = -\frac{1}{V} \left(\frac{V}{P} \right)_T - \frac{\left(\frac{V}{V} \right)_T}{P}$$

A. Normally pressured reservoirs

1. Downward force by the overburden must be balanced by upward force of the matrix and the fluid



2. Thus,

$$F_o = F_m + F_f$$

it follows that

$$P_o = P_m + P_f$$

3. P_o 1.0 psi/ft

$$P_f$$
 0.465 psi/ft

4. As fluid is produced from a reservoir, the fluid pressure, P_f will usually decrease:
- the force on the matrix increases
 - causing a decrease in bulk volume
 - and a decrease in pore volume

B. Types of compressibility

1. Matrix Compressibility, c_m

$$c_m = 0$$

2. Bulk Compressibility c_b

used in subsidence studies

3. Formation Compressibility, c_f - also called pore volume compressibility

- a. important to reservoir engineers

- depletion of fluid from pore spaces
- internal rock stress changes
- change in stress results in change in V_p, V_m, V_b
- by definition

$$c_f = -\frac{1}{V_p} \left(\frac{dV_p}{dp_m} \right)$$

- b. since overburden pressure, P_o , is constant

$$dP_m = -dP_f$$

1) Thus,

$$c_f = - \frac{1}{V_p} \left(\frac{V_p}{P_m} \right)$$

2) where the subscript of f on c_f means "formation" and the subscript of f on P_f means "fluid"

3) procedure

(a) measure volume of liquid expelled as a function of "external" pressure

(b) "external" pressure may be taken to represent overburden pressure, P_o

(c) fluid pressure, p_f , is essentially constant, thus,

$$dP_o = dP_m$$

(d) expelled volume increases as pore volume, v_p , decreases, thus,

$$dV_p = - dV_{\text{expelled}}$$

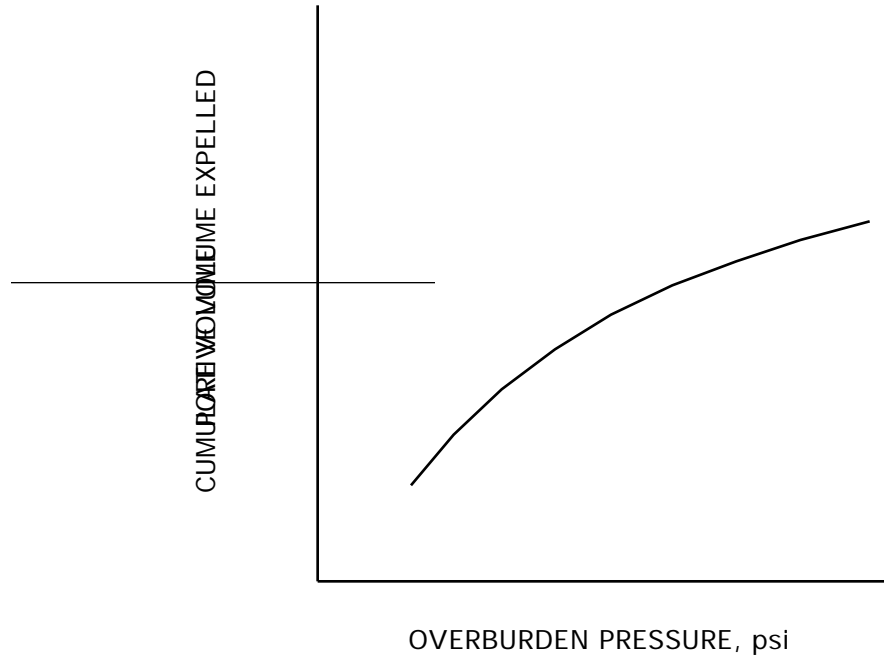
(e) from definition

$$c_f = - \frac{1}{V_p} \left(\frac{V_p}{P_m} \right)$$

it follows that

$$c_f = + \frac{1}{V_p} \left(\frac{(V_p)_{\text{expelled}}}{P_o} \right)$$

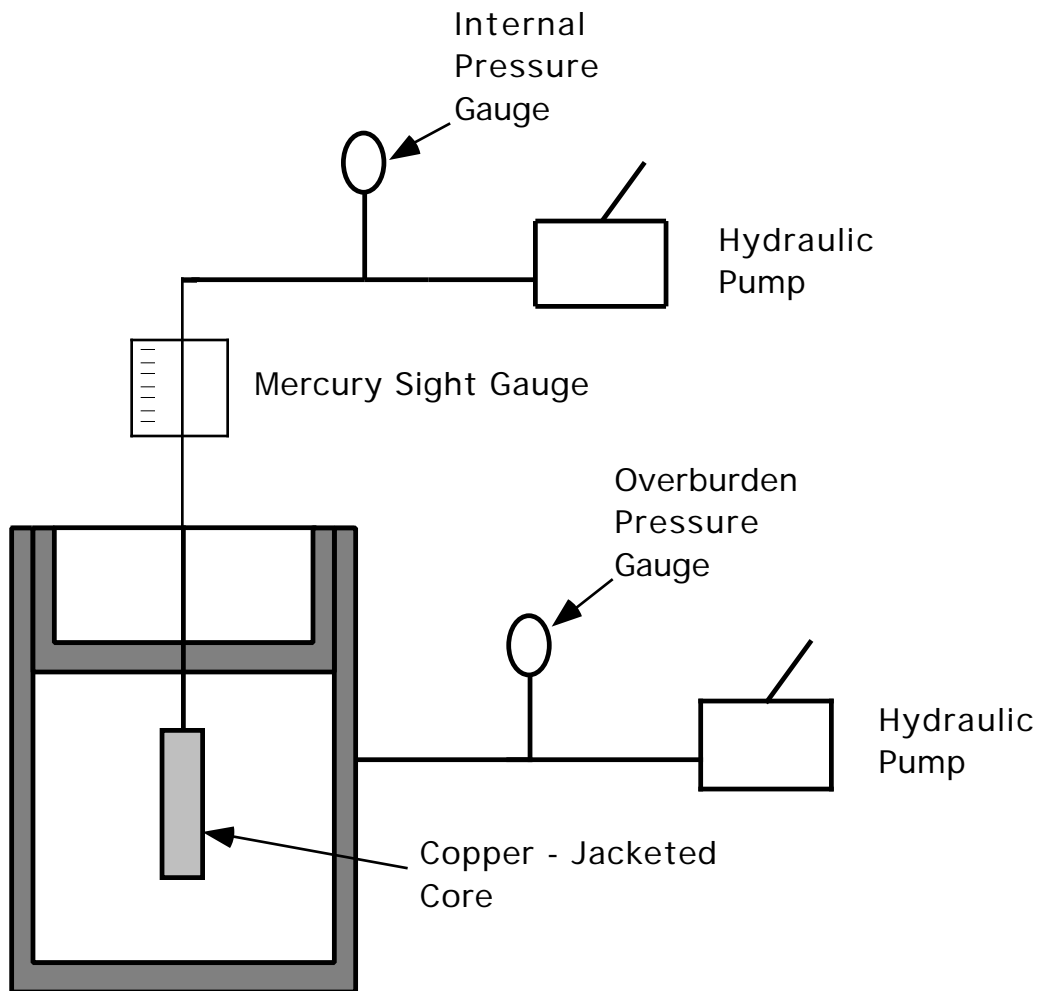
(f) plot



slope = c_f .

C. Measurement of compressibility

- 1) Laboratory core sample
 - a) apply variable internal and external pressures
 - b) internal rock volume changes
- 2) Equipment



Apparatus for measuring pore volume compressibility (hydrostatic)

Example I-9

Given the following lab data, calculate the pore volume compressibility for a sandstone sample at 4,000 and 6,000 psi.

pore volume = 50.0 cc

pressure, psi	vol. fluid expelled, cc
1000	0.244
2000	0.324
3000	0.392
4000	0.448
5000	0.500
6000	0.546
7000	0.596
8000	0.630

Solution:

from graph

@ 4,000 psi:

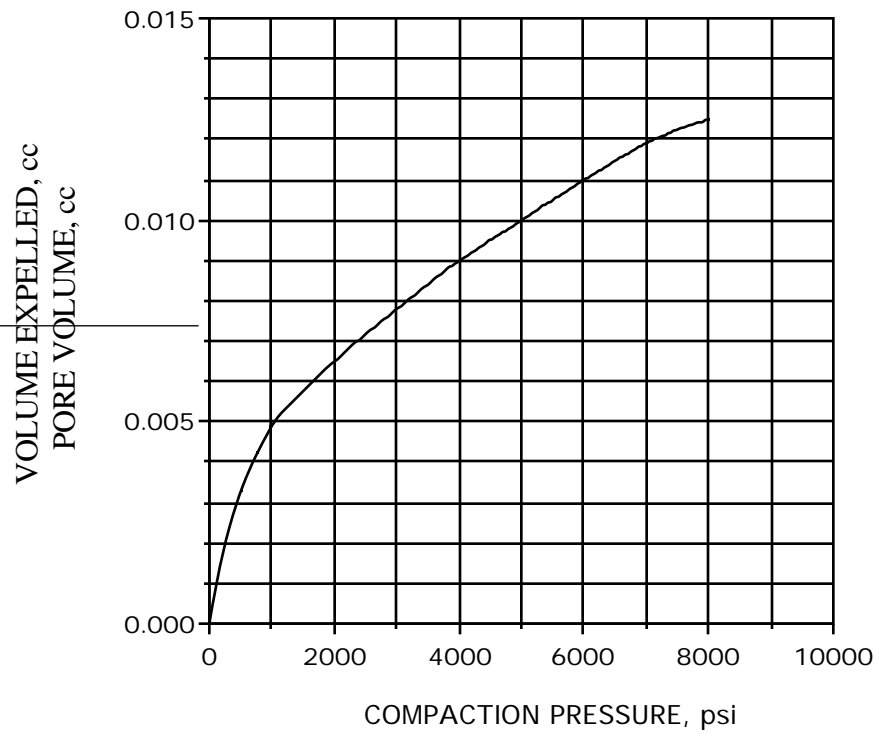
$$\text{Slope} = \left(\frac{0.009}{4000 \text{ psi}} \right)$$

$$c_f = \left(2.25 \times 10^{-6} \left(\frac{1}{\text{psi}} \right) \right)$$

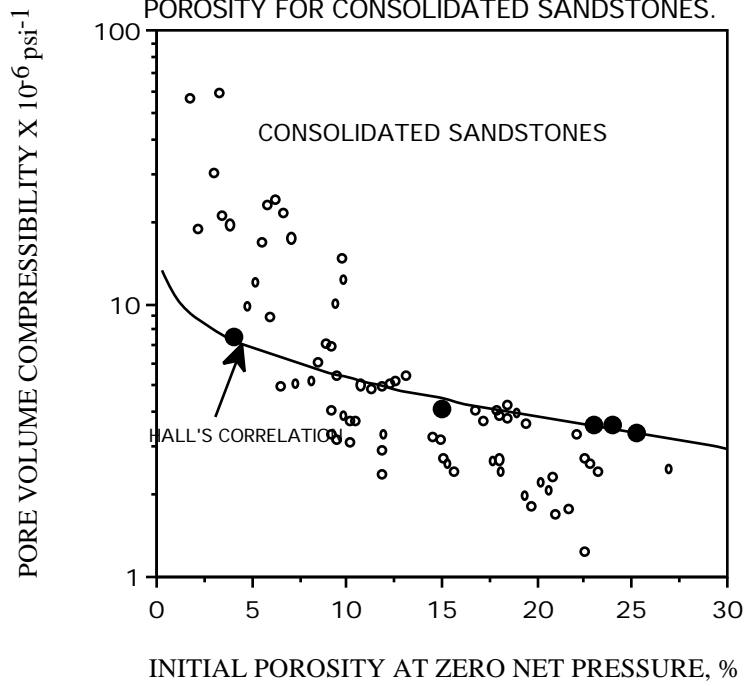
@ 6000 psi:

$$\text{Slope} = \left(\frac{0.011}{6000 \text{ psi}} \right)$$

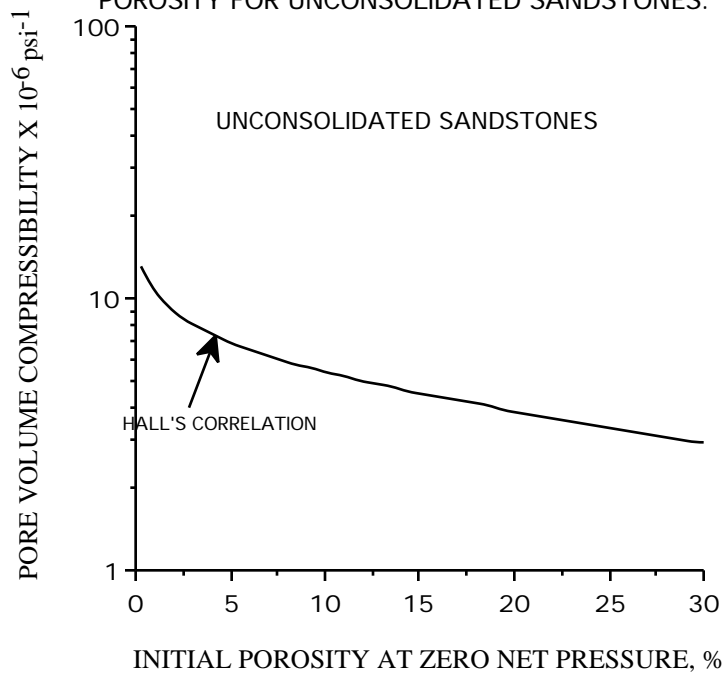
$$c_f = \left(1.83 \times 10^{-6} \left(\frac{1}{\text{psi}} \right) \right)$$



PORE-VOLUME COMPRESSIBILITY AT 75 % LITHOSTATIC PRESSURE VS INITIAL SAMPLE POROSITY FOR CONSOLIDATED SANDSTONES.

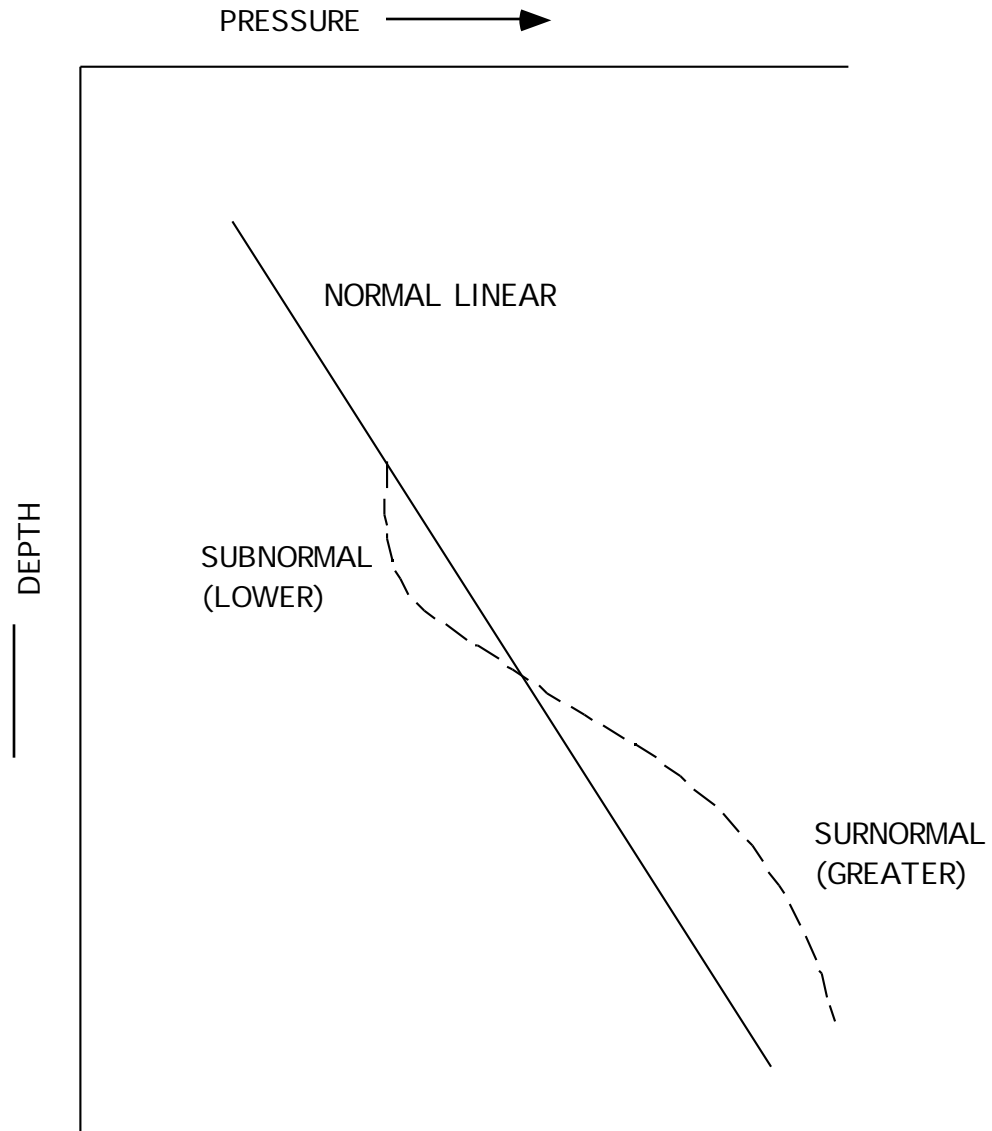


PORE-VOLUME COMPRESSIBILITY AT 75 % LITHOSTATIC PRESSURE VS INITIAL SAMPLE POROSITY FOR UNCONSOLIDATED SANDSTONES.



E. Abnormally pressured reservoirs

"abnormal pressure": fluid pressures greater than or less than the hydrostatic fluid pressure expected from an assumed linear pressure gradient



Compressibility/Porosity Problem No. 1

A limestone sample weighs 241.0 gm. The limestone sample coated with paraffin was found to weigh 249.5 gm. The coated sample when immersed in a partially filled graduated cylinder displaced 125.0 cc of water. The density of the paraffin is 0.90 gm/cc.

What is the porosity of the rock? Does the process measure total or effective porosity?

Solution:

$$V_m = \frac{\text{wt. dry}}{\rho_s} = \frac{241.0 \text{ gm}}{2.71 \text{ gm/cc}} = 88.9 \text{ cc}$$

$$V_{\text{paraffin}} = \frac{\text{wt. coated sample} - \text{wt. uncoated sample}}{\rho_{\text{paraffin}}}$$

$$V_{\text{paraffin}} = \frac{249.5 \text{ gm} - 241.0 \text{ gm}}{0.90 \text{ gm/cc}} = 9.4 \text{ cc}$$

$$V_b = 125 \text{ cc} - 9.4 \text{ cc} = 115.6 \text{ cc}$$

$$V_p = V_b - V_m$$

$$V_p = 115.6 \text{ cc} - 88.9 \text{ cc} = 26.7 \text{ cc}$$

$$= \frac{V_p}{V_b} = \frac{26.7 \text{ cc}}{115.6 \text{ cc}} = 0.231$$

$$= 23.1\% \quad (\text{total porosity})$$

Compressibility/Porosity Problem No. 2

You are furnished with the results of a sieve analysis of a core from Pete well #1. Previous laboratory work indicates there is a correlation between grain size and porosity displayed by those particular particles. The correlation is seen below:

gravel	-	25% porosity
coarse sand	-	38% porosity
fine sand	-	41% porosity

What would be the minimum porosity of the mixture?
 What basic assumption must be made in order to work the problem?

Solution:

Begin calculation with a volume of 1 cu. ft.

component	remaining pore volume (ft ³)	porosity (%)	remaining matrix volume (ft ³)
void space	1.000	100.0	0.000
gravel	0.250	25.0	0.750
coarse sand	0.095	9.5	0.905
fine sand	0.039	3.9	0.961

Final porosity - 3.9%

(Complete mixing of the grains)

Compressibility/Porosity Problem No. 3

A sandstone reservoir has an average thickness of 85 feet and a total volume of 7,650 acre-feet. Density log readings through the fresh water portion of the reservoir indicate a density of 2.40 gm/cc.

The Highgrade #1 Well was drilled and cored through the reservoir. A rock sample was sent to the laboratory and the following tests were run.

pressure (psig)	cum. pore vol. change (-cc)
1,000	0.122
2,000	0.162
3,000	0.196
4,000	0.224
5,000	0.250
6,000	0.273
7,000	0.298
8,000	0.315

The dry weight of the core sample was found to be 140 gm while the sample dimensions were 1.575 inches long and 1.960 inches in diameter.

Assuming the compressibility at 4,500 psi is the average compressibility in the reservoir, how much subsidence occurs when the reservoir pressure declines from 5,500 psi to 3,500 psi?

Calculate:

- A. Reservoir Porosity
- B. Sample Pore Volume
- C. Compressibility at 4,500 psi
- D. Amount of Ground Subsidence.

Solution:

- A. Reservoir Porosity

$$= \frac{m - L}{m - f} = \frac{2.65 - 2.40}{2.65 - 1.00} = 15.22\%$$

B. Sample Pore Volume

$$L = (1.575 \text{ in}) (2.54 \text{ cm/in}) = 4.0 \text{ cm}$$

$$D = (1.960 \text{ in}) (2.54 \text{ cm/in}) = 5.0 \text{ cm}$$

$$V_b = \text{bulk volume} = \frac{D^2 h}{4} = \frac{(3.14)(5.0)^2(4.0)}{4.0} = 78.5 \text{ cc}$$

$$V_m = \text{matrix volume} = (140 \text{ gm}) \left(\frac{\text{cc}}{2.65 \text{ gm}} \right) = 52.8 \text{ cc}$$

$$V_p = V_b - V_m = 78.5 \text{ cc} - 52.8 \text{ cc}$$

$$V_p = 25.7 \text{ cc}$$

C. Compressibility (see graph)

$$V_p = 25.7 \text{ cc}$$

D. Subsidence

$$H = (H)(c_p) \left(\frac{\Delta P}{P} \right)$$

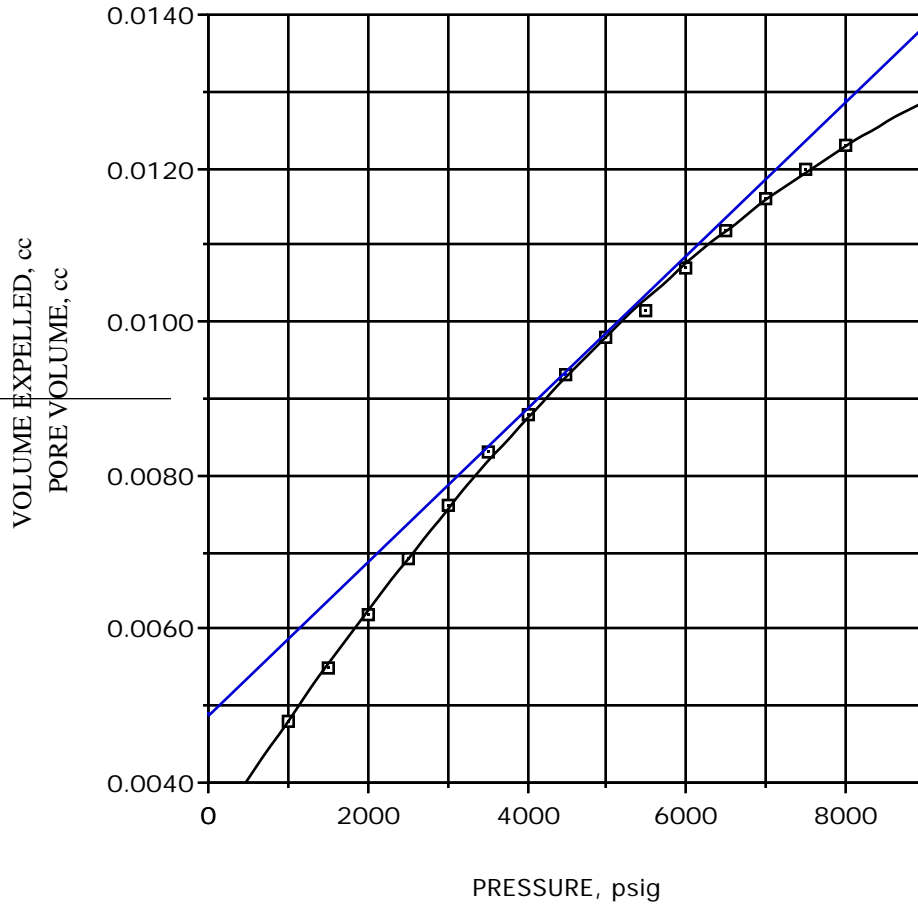
$$H = (85 \text{ ft}) (9.69 \times 10^{-7} \text{ psi}^{-1}) (0.152) (2,000 \text{ psi})$$

$$H = 0.026 \text{ ft}$$

$$H = 0.32 \text{ inches}$$

Note: the pore volume (formation) compressibility is somewhat smaller than usually encountered. An experienced engineer would be wary of this small number. Also it was assumed that the formation compressibility was exactly the same as the bulk volume compressibility. Experience shows that this is not the case.

POROSITY PROBLEM No. 3



$$\text{SLOPE} = \frac{.0118 - .0068}{7000 - 2000}$$

$$C_p = 9.96 \times 10^{-7} \text{ psi}^{-1}$$

Compressibility Problem

A 160-acre and 100 ft thick reservoir has a porosity of 11%. The pore compressibility is 5.0×10^{-6} (1/psi). If the pressure decreases 3,000 psi, what is the subsidence (ft)? Assume $C_f = C_b$

Solution:

$$A = 160 (43,560) = 6,969,600 \text{ ft}^2$$

$$V_b = 100 (6,969,600) = 696,960,000 \text{ ft}^3$$

$$V_p = V_b(f) = (696,960,000) (.11) = 76,665,600 \text{ ft}^3$$

$$C_p = -\frac{1}{V_p} \frac{dV_p}{dp}$$

$$5 \times 10^{-6} \text{ (1/psi)} = \frac{-1}{76,665,600 \text{ (ft}^3)} \frac{dV_p}{3,000 \text{ psi}}$$

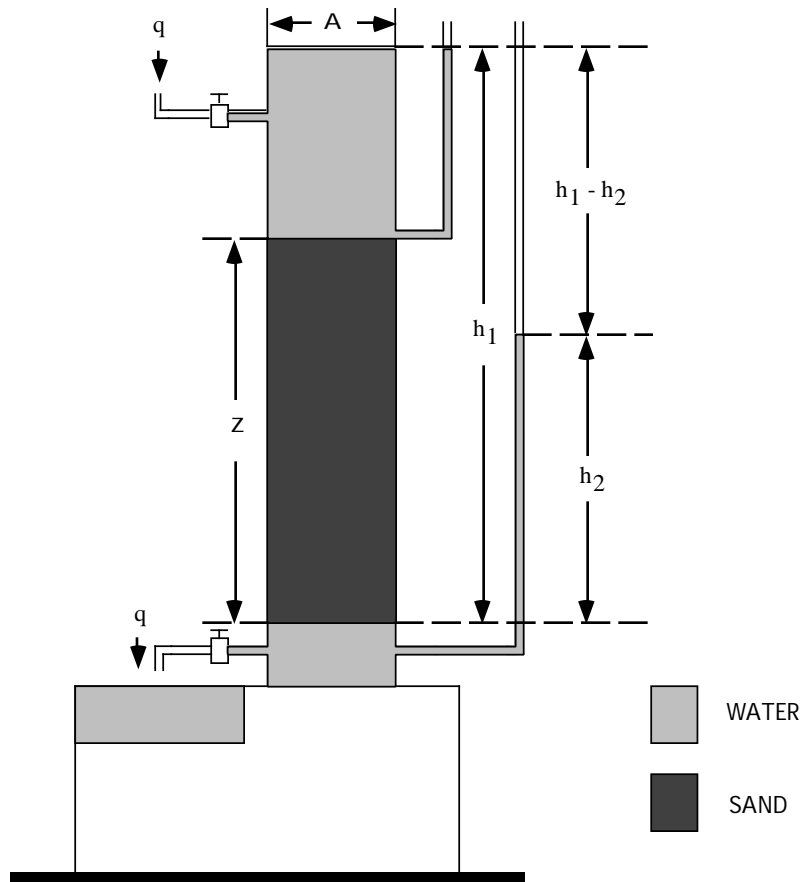
$$dV_p = 1.15 \times 10^6 \text{ ft}^3$$

$$H = 1.15 \times 10^6 \text{ (ft}^3) \times \frac{1}{6,969,600 \text{ (ft}^2)} = 0.165 \text{ ft}$$

II. SINGLE PHASE FLOW IN POROUS ROCK

I) Darcy's equation (1856)

A. Water flow through sand filters



DARCY'S FOUNTAIN.

$$q = \frac{kA(h_1 - h_2)}{\mu L}$$

Length of sand pack, $L = Z$

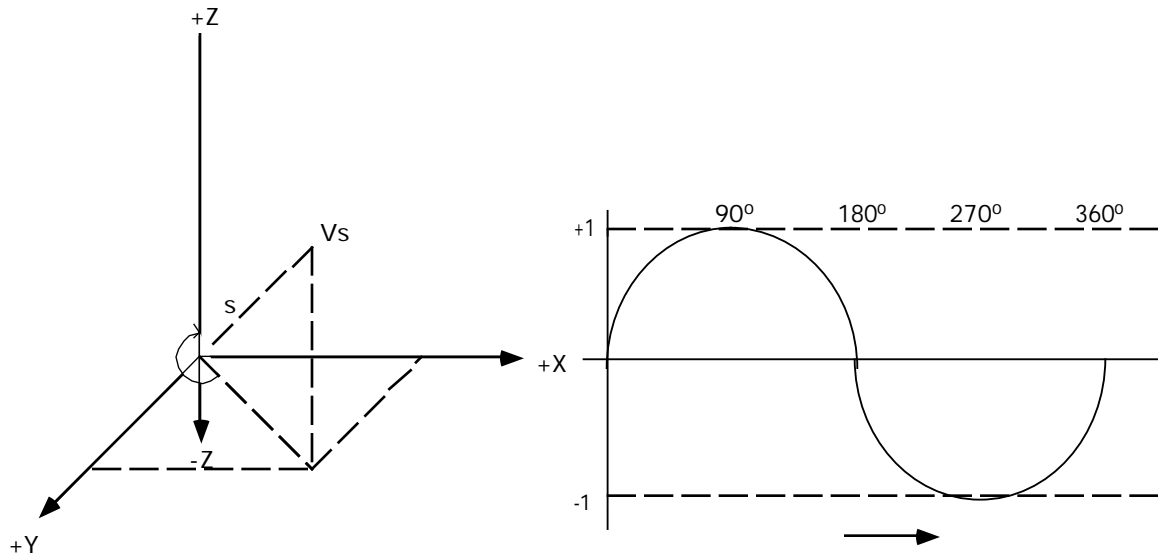
1. constant of proportionality, k , characteristics of particular sand pack, not sample size
2. Darcy's work confined to sand packs that were 100% saturated with water
3. equation extended to include other liquids using viscosity

$$q = \frac{kA(h_1 - h_2)}{\mu L}$$

B. Generalized form of Darcy's equation

1. Equation

$$v_s = \frac{-k}{\mu} \left[\frac{dP}{ds} - \frac{g}{1.0133 \times 10^6} \frac{dz}{ds} \right]$$



2. Nomenclature

v_s = superficial velocity (volume flux along path s) - cm/sec

v_s/ρ = interstitial velocity - cm/sec

ρ = density of flowing fluid - gm/cm³

g = acceleration of gravity - 980 cm/sec²

$\frac{dP}{ds}$ = pressure gradient along s - atm/cm

μ = viscosity - centipoise

k = permeability - darcies

3. Conversion factors

$$\text{dyne} = \text{gm cm/sec}^2 = \text{a unit of force}$$

$$\text{atm} = 1.0133 \times 10^6 \text{ dyne/cm}^2$$

$$\text{gh} = \text{dyne/cm}^2 = \text{a unit of pressure}$$

$$\text{poise} = \text{gm/cm sec} = \text{dyne sec/cm}^2$$

4. The dimensions of permeability

$$L = \text{length}$$

$$m = \text{mass}$$

$$t = \text{time}$$

$$v_s = L/t$$

$$\mu = m/Lt$$

$$= m/L^3$$

$$p = m/Lt^2$$

$$g = L/t^2$$

$$v_s = - \frac{k}{\mu} \left[\frac{dp}{ds} - \frac{g}{1.0133 \times 10^6} \frac{dz}{ds} \right]$$

$$\frac{L}{t} = - \frac{k}{m/Lt} \left[\frac{m/Lt^2}{L} - \frac{(m/L^3)(L/t^2)(L)}{L} \right]$$

$$k = L^2 = \text{cross-sectional area}$$

5. Definition of Darcy units
- a. conventional units would be:
 - 1) feet squared in the English system
 - 2) centimeter squared in the cgs system
 - b. both are too large for use in porous media
 - c. definition of darcy

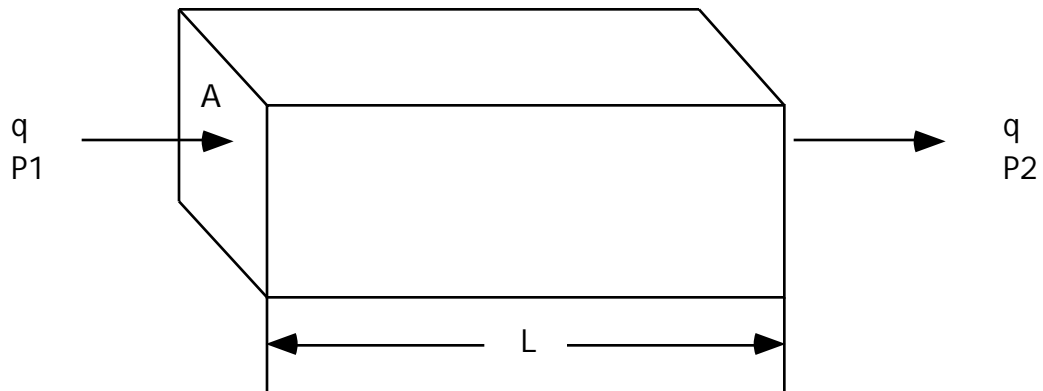
A porous medium has a permeability of one darcy when a single-phase fluid of one centipoise that completely fills the voids of the medium will flow through it under conditions of viscous flow at a rate of one cubic centimeter per second per square centimeter cross-sectional area under a pressure or equivalent hydraulic gradient of one atmosphere per centimeter.

$$q = \frac{k A (P_1 - P_2)}{\mu L}$$

II) Reservoir systems

A. Flow of incompressible liquid

1. Horizontal, linear flow system



a. Conditions

- 1) horizontal system, $\frac{dz}{ds} = 0$
- 2) linear system, $A = \text{constant}$
- 3) incompressible liquid, $q = \text{constant}$
- 4) laminar flow, can use Darcy's equation
- 5) non-reactive fluid, $k = \text{constant}$
- 6) 100% saturated with one fluid
- 7) constant temperature, μ, q

b. derivation of flow equation

$$v_s = - \frac{k}{\mu} \left[\frac{dP}{ds} - \frac{g}{1.0133 \times 10^6} \frac{dz}{ds} \right]$$

$$v_s = - \frac{k}{\mu} \frac{dP}{ds} = \frac{q}{A}$$

$$q \int_0^L ds = - \frac{kA}{\mu} \int_{P_1}^{P_2} dP$$

$$q(L - 0) = - \frac{kA}{\mu} (P_2 - P_1)$$

$$q = \frac{kA}{L\mu} (P_2 - P_1)$$

Note: P_1 acts at $L = 0$

P_2 acts at $L = L$

q is + if flow is from $L = 0$ to $L = L$

Example II-1

What is the flow rate of a horizontal rectangular system when the conditions are as follows:

$$\text{permeability} = k = 1 \text{ darcy}$$

$$\text{area} = A = 6 \text{ ft}^2$$

$$\text{viscosity} = \mu = 1.0 \text{ cp}$$

$$\text{length} = L = 6 \text{ ft}$$

$$\text{inlet pressure} = P_1 = 5.0 \text{ atm}$$

$$\text{outlet pressure} = P_2 = 2.0 \text{ atm}$$

Solution:

We must insure all the variables are in the correct units.

$$k = 1 \text{ darcy}$$

$$A = 6 \text{ ft}^2 (144 \text{ in}^2/1 \text{ ft}^2) (6.45 \text{ cm}^2/1 \text{ in}^2) = 5572.8 \text{ cm}^2$$

$$L = 6 \text{ ft} (12 \text{ in}/1 \text{ ft}) (2.54 \text{ cm}/1 \text{ in}) = 182.88 \text{ cm}$$

$$P_1 = 5.0 \text{ atm}$$

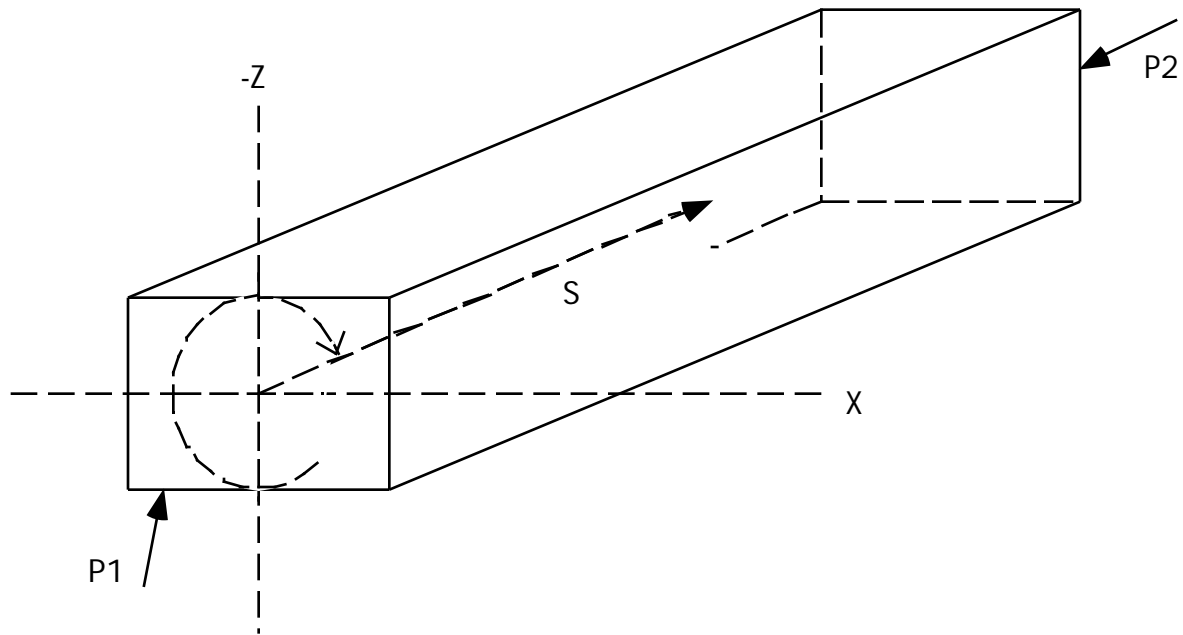
$$P_2 = 2.0 \text{ atm}$$

$$q = \frac{kA}{L\mu} (P_2 - P_1)$$

$$q = \frac{(1)(5,572.8)}{(1)(182.88)} (5.0 - 2.0)$$

$$q = 91.42 \text{ cm}^3/\text{sec}$$

2. Non-horizontal, linear system



a. Conditions

- 1) non-horizontal system, $\frac{dz}{ds} = \sin \theta = \text{constant}$
- 2) linear system, $A = \text{constant}$
- 3) incompressible liquid, $q = \text{constant}$
- 4) laminar flow, use Darcy equation
- 5) non-reactive fluid, $k = \text{constant}$
- 6) 100% saturated with one fluid
- 7) constant temperature μ, q

b. derivation of equation

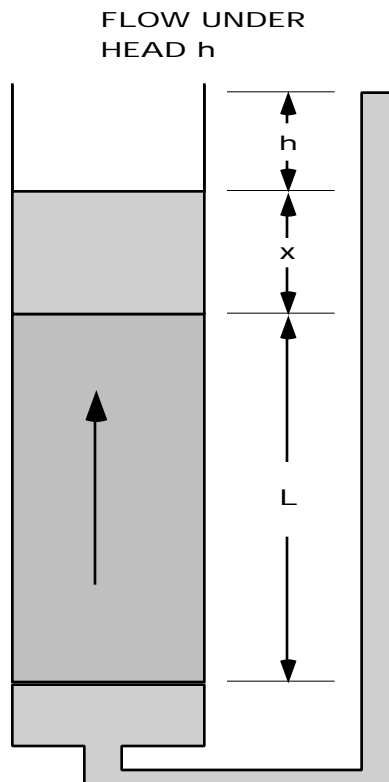
$$v_s = - \frac{k}{\mu} \left[\frac{dP}{ds} - \frac{g}{1.0133 \times 10^6} \frac{dz}{ds} \right]$$

$$v_s = - \frac{q}{A} = - \frac{k}{\mu} \frac{dP}{ds} + \frac{k g \sin}{\mu 1.0133 \times 10^6}$$

$$q \int_0^L ds = - \frac{kA}{\mu} \int_{P_1}^{P_2} dp + \frac{kA g \sin}{\mu 1.0133 \times 10^6} \int_0^L ds$$

$$q = - \frac{kA}{\mu L} \left[P_1 - P_2 + \frac{gL \sin}{1.0133 \times 10^6} \right]$$

3. Vertical, upward flow, linear system



a. Conditions

- 1) vertical system, $\frac{dz}{ds} = \sin \theta = \text{constant}$
- 2) upward flow, $q = 270^\circ$, $\sin \theta = -1$
- 3) linear system, $A = \text{constant}$
- 4) incompressible liquid, $q = \text{constant}$
- 5) laminar flow, use Darcy equation
- 6) non-reactive fluid, $k = \text{constant}$
- 7) 100% saturated with one fluid
- 8) constant temperature, μ

b. derivation of flow equation

$$v_s = \frac{k}{\mu} \left[\frac{dP}{ds} - \frac{g}{1.0133 \times 10^6} \frac{dz}{ds} \right]$$

$$v_s = \frac{q}{A} = - \frac{k}{\mu} \left[\frac{dP}{ds} + \frac{g}{1.0133 \times 10^6} \right]$$

$$q = \frac{kA}{\mu} \left[\frac{P_1 - P_2}{L} - \frac{g}{1.0133 \times 10^6} \right]$$

$$P_1 = - \frac{g (h + x + L)}{1.0133 \times 10^6}$$

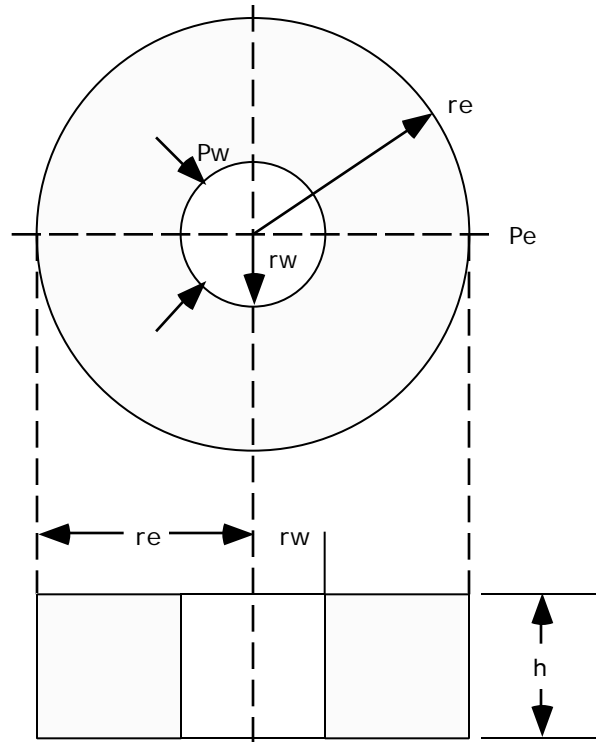
$$P_2 = \frac{g x}{1.0133 \times 10^6}$$

$$\frac{P_1}{L} - \frac{P_2}{L} = \frac{g h}{1.0133 \times 10^6 L} + \frac{g}{1.0133 \times 10^6}$$

$$q = \frac{kA}{\mu} \left[\frac{g h}{1.0133 \times 10^6 L} + \frac{g}{1.0133 \times 10^6} - \frac{g}{1.0133 \times 10^6} \right]$$

$$q = \frac{kA}{\mu L} \left[\frac{g h}{1.0133 \times 10^6} \right]$$

4. Horizontal, radial flow system



a. Conditions

- 1) horizontal system, $\frac{dz}{ds} = 0$
- 2) radial system, $A = 2 rh$, $ds = - dr$, flow is inward
- 3) constant thickness, $h = \text{constant}$
- 4) incompressible liquid, $q = \text{constant}$
- 5) laminar flow, use Darcy equation
- 6) non-reactive fluid, $k = \text{constant}$
- 7) 100% saturated with liquid,
- 8) constant temperature, μ, q

b. Derivation of flow Equation

$$v_s = - \frac{k}{\mu} \left[\frac{dP}{ds} - \frac{g}{1.0133 \times 10^6} \frac{dz}{ds} \right]$$

$$v_s = + \frac{k}{\mu} \frac{dP}{dr} = \frac{q}{A} = \frac{q}{2 rh}$$

$$\frac{q}{2 h} \int_{r_w}^{r_e} \frac{dr}{r} = \frac{k}{\mu} \int_{P_w}^{P_e} dp$$

$$\frac{q}{2 h} (1n(r_e) - 1n(r_w)) = \frac{k}{\mu} (P_e - P_w)$$

$$q = \frac{2 hk}{\mu 1n(r_e/r_w)} (P_e - P_w)$$

Note: if q is + , flow is from r_e to r_w

B. Flow of gas (compressible fluid)

1. horizontal, linear flow system



a. Conditions

- 1) horizontal system, $\frac{dz}{ds} = 0$
- 2) linear system, $A = \text{constant}$
- 3) compressible gas flow, $q = f(p)$
- 4) laminar flow, use Darcy equation
- 5) non-reactive fluid, $k = \text{constant}$
- 6) 100% saturated with one fluid
- 7) constant temperature

b. Assumptions

- 1) $\mu, Z = \text{constant}$
- 2) $Z(\text{and } \mu)$ can be determined at mean pressure

c. Derivation of equation for q_{sc}

$$v_s = - \frac{k}{\mu} \left[\frac{dP}{ds} - \frac{g}{1.0133 \times 10^6} \frac{dz}{ds} \right]$$

$$v_s = - \frac{k}{\mu} \frac{dP}{ds} = \frac{q}{A_{ds}}$$

but

$$q = \frac{P_{sc} q_{sc} Z T}{P T_{sc}}$$

thus

$$\frac{P_{sc} T q_{sc}}{T_{sc} A} \int_0^L ds = -k \int_{p_1}^{p_2} \frac{PdP}{\mu z}$$

$$\frac{P_{sc} T q_{sc}}{T_{sc} A} (L - 0) = - \frac{k}{\mu z} \left(\frac{P_2^2}{2} - \frac{P_1^2}{2} \right)$$

$$q_{sc} = \frac{kA}{\mu L} \frac{T_{sc}}{T z P_{sc}} \frac{(P_1^2 - P_2^2)}{2}$$

Note: real gas equation of state

$$Pq = Z n R T$$

where $q = \text{volumetric flow/time}$
 $n = \text{mass flow/time}$

thus,

$$\frac{Pq}{P_{sc} q_{sc}} = \frac{Z n R T}{n R T_{sc}}$$

$$q = \frac{P_{sc} q_{sc} Z T}{T_{sc}} \frac{1}{P}$$

where q_{sc} is constant

Z is determined at P, T

d. Derivation of equation for \bar{q}

$$q_{sc} = \frac{kA}{\mu L} \frac{T_{sc}}{Tz P_{sc}} \frac{(P_1^2 - P_2^2)}{2}$$

but

$$q_{sc} = \frac{\bar{P} \bar{q} T_{sc}}{\bar{z} P_{sc} T} = \frac{kA}{\mu L} \frac{T_{sc}}{Tz P_{sc}} \frac{(P_1^2 - P_2^2)}{2}$$

$$\bar{q} = \frac{kA}{\mu L} \frac{1}{\bar{P}} \frac{(P_1^2 - P_2^2)}{2}$$

$$\bar{q} = \frac{kA}{\mu L} \frac{2}{P_1 + P_2} \frac{(P_1^2 - P_2^2)}{2}$$

$$\bar{q} = \frac{kA}{\mu L} (P_1 - P_2)$$

This equation is identical to the equation for horizontal, linear flow of incompressible liquid

thus

if gas flow rate is determined at mean pressure, \bar{P} , the equation for incompressible liquid can be used for compressible gas!

Note: real gas equation of state

$$Pq = Z n R T$$

thus

$$\frac{P_{sc} q_{sc}}{\bar{P} \bar{q}} = \frac{n R T_{sc}}{\bar{z} n R T}$$

where

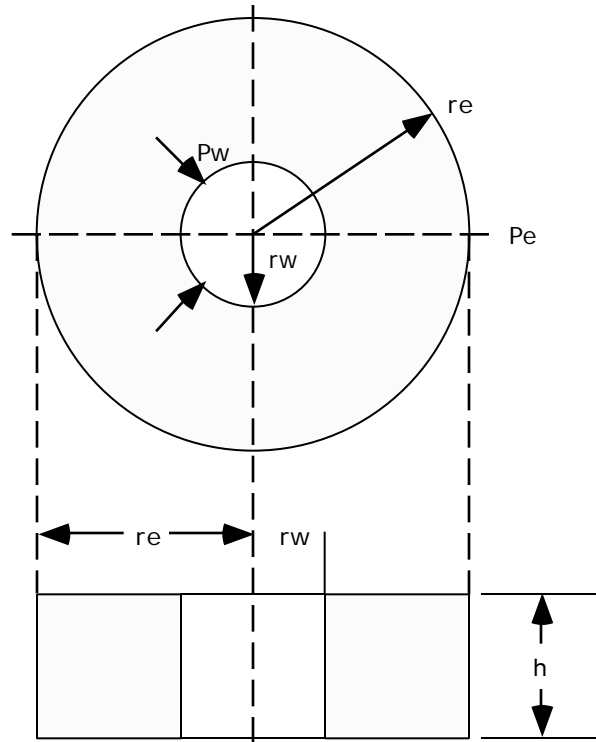
$$\bar{P} = \frac{P_1 + P_2}{2}$$

$$\bar{P} = \text{volumetric flow rate at } P, T$$

\bar{z} is determined at P, T

$$q_{sc} = \frac{\bar{P} \bar{q} T_{sc}}{\bar{z} P_{sc} T}$$

2. Horizontal, radial flow system



a. Conditions

- 1) horizontal system $\frac{dz}{ds} = 0$
- 2) radial system, $A = 2 rL$, $ds = - dr$, inward flow
- 3) constant thickness, $h = \text{constant}$
- 4) compressible gas flow, $q = f(P)$
- 5) laminar flow, use Darcy equation
- 6) non-reactive fluid, $k = \text{constant}$
- 7) 100% saturated with one fluid
- 8) constant temperature

b. Assumptions

$$\mu z = \text{constant}$$

z (and μ) can be determined at mean pressure

c. derivation of equation for q_{sc}

$$v_s = -\frac{k}{\mu} \left[\frac{dP}{ds} - \frac{g}{1.0133 \times 10^6} \frac{dz}{ds} \right]$$

$$v_s = -\frac{k}{\mu} \frac{dP}{ds} = \frac{q}{A}$$

but

$$q = \frac{P_{sc} q_{sc} z T}{P T_{sc}}$$

and

$$A = 2 r h \text{ and } ds = -dr$$

thus

$$\frac{P_{sc} T q_{sc}}{2 T_{sc} h} \int_{r_w}^{r_e} \frac{dr}{r} = k \int_{P_w}^{P_e} \frac{dP}{\mu z}$$

$$\frac{P_{sc} T q_{sc}}{2 T_{sc} h} \ln \frac{r_e}{r_w} = \frac{k}{\mu z} \left[\frac{P_e^2 - P_w^2}{2} \right]$$

$$q_{sc} = \frac{2 h k}{\mu \ln r_e/r_w} \frac{T_{sc}}{P_{sc} z T} \left[\frac{P_e^2 - P_w^2}{2} \right]$$

d. derivation of equation for \bar{q}

$$q_{sc} = \frac{2}{\mu} \frac{h k}{\ln r_e/r_w} \frac{T_{sc}}{P_{sc} z T} \left[\frac{P_e^2 - P_w^2}{2} \right]$$

but

$$\bar{q} = \frac{\bar{P} \bar{q} T_{sc}}{z P_{sc} T}$$

thus

$$\frac{\bar{P} \bar{q} T_{sc}}{z P_{sc} T} = \frac{2}{\mu} \frac{h k}{\ln r_e/r_w} \frac{T_{sc}}{P_{sc} z T} \left[\frac{(P_e^2 - P_w^2)}{2} \right]$$

$$\bar{q} = \frac{2}{\mu} \frac{h k}{\ln r_e/r_w} \frac{1}{\bar{P}} \left[\frac{(P_e^2 - P_w^2)}{2} \right]$$

$$\bar{q} = \frac{2}{\mu} \frac{h k}{\ln r_e/r_w} \frac{2}{(P_e + P_w)} \left[\frac{(P_e^2 - P_w^2)}{2} \right]$$

$$\bar{q} = \frac{2}{\mu} \frac{h k}{\ln r_e/r_w} (P_e - P_w)$$

Note: Equation for real gas is identical to equation for incompressible liquid when volumetric flow rate of gas, \bar{q} , is measured at mean pressure.

C. Conversion to Oilfield Units

Symbol	Darcy units	Oil field
q	cc/sec	bbbl/d or cu ft/d
k	darcy	md
A	sq cm	sq ft
h	cm	ft
P	atm	psia
L	cm	ft
μ	cp	cp
r	gm/cc	lb/cu ft

Example:

$$q = \frac{hkA(P_1 - P_2)}{\mu L} \quad \text{in Darcy's units}$$

$$q \frac{\text{cc}}{\text{sec}} = q \frac{\text{bbbl}}{\text{d}} \left(\frac{5.615 \text{ cu ft}}{\text{bbbl}} \right) \left(\frac{1,728 \text{ cu in}}{\text{cu ft}} \right) \left(\frac{16.39 \text{ cc}}{\text{cu in}} \right) \left(\frac{\text{d}}{24\text{hr}} \right) \left(\frac{\text{hr}}{3,600 \text{ sec}} \right)$$

$$q \frac{\text{cc}}{\text{sec}} = 1.841 q \frac{\text{bbbl}}{\text{d}}$$

$$k \text{ darcy} = k \text{ md} \left(\frac{\text{darcy}}{1,000\text{md}} \right)$$

$$k \text{ darcy} = 0.001 k \text{ md}$$

$$A \text{ sq cm} = \left(\frac{929.0 \text{ sq cm}}{\text{sq ft}} \right) A \text{ sq ft}$$

$$A \text{ sq cm} = 929.0 A \text{ sq ft}$$

$$(P_1 - P_2) \text{ atm} = (P_1 - P_2) \text{ psia} \left(\frac{\text{atm}}{14.696 \text{ psia}} \right)$$

$$(P_1 - P_2) \text{ atm} = 0.06805 (P_1 - P_2) \text{ psia}$$

$$L \text{ cm} = L \text{ ft} \left(\frac{30.48 \text{ cm}}{\text{ft}} \right)$$

$$\text{meter} = 100 \text{ cm}$$

$$1.841 q = \frac{(0.001 k)(929.0 A)(.06805 (P_1 - P_2))}{\mu (30.48 L)}$$

$$q = \frac{0.01127 k A (P_1 - P_2)}{\mu L} \quad \text{in oilfield units}$$

D. Table of Equations

1. Darcy Units

System	Fluid	Equation
Horizontal, Linear	Incompressible Liquid	$q = \frac{kA}{\mu L} (P_1 - P_2)$
Dipping, Linear	Incompressible Liquid	$q = \frac{kA}{\mu L} \left[(P_1 - P_2) + \frac{g L \sin}{1.0133 \times 10^6} \right]$
Horizontal, Radial	Incompressible Liquid	$q = \frac{2 k h}{\mu \ln (r_e/r_w)} (P_e - P_w)$
Horizontal, Linear	Real Gas	$q_{sc} = \frac{kA}{\mu L} \frac{T_{sc}}{T_z P_{sc}} \frac{(P_1^2 - P_2^2)}{2}$ $\bar{q} = \frac{kA}{\mu L} (P_1 - P_2)$
Horizontal, Radial	Real Gas	$q_{sc} = \frac{k h}{\mu \ln (r_e/r_w)} \frac{T_{sc}}{T_z P_{sc}} (P_e^2 - P_w^2)$ $\bar{q} = \frac{2 k h}{\mu \ln (r_e/r_w)} (P_e - P_w)$

2. Oilfield Units

System	Fluid	Equation
Horizontal, Linear	Incompressible Liquid	$q = 0.001127 \frac{kA}{\mu L} (P_1 - P_2)$ $q = \text{res bbl/d}$
Dipping, Linear	Incompressible Liquid	$q = 0.001127 \frac{kA}{\mu L} \left[(P_1 - P_2) + \frac{g L \sin}{1.0133 \times 10^6} \right]$
Horizontal, Radial	Incompressible Liquid	$q = .007082 \frac{kh}{\mu \ln(r_e/r_w)} (P_e - P_w)$
Horizontal, Linear	Real Gas	$q_{sc} = .1118 \frac{kA}{\mu L z T} (P_1^2 - P_2^2)$ $q_{sc} = \text{scf/d}$ $\bar{q} = .001127 \frac{kA}{\mu L} (P_1 - P_2)$ $\bar{q} = \text{res bbl/d}$
Horizontal, Radial	Real Gas	$q_{sc} = .7032 \frac{kh}{\mu \ln(r_e/r_w) Tz} (P_e^2 - P_w^2)$ $\bar{q} = .007082 \frac{kh}{\mu \ln(r_e/r_w)} (P_e - P_w)$

Example II-2

What is the flow rate of a horizontal rectangular system when the conditions are as follows:

$$\text{permeability} = k = 1 \text{ darcy}$$

$$\text{area} = A = 6 \text{ ft}^2$$

$$\text{viscosity} = \mu = 1.0 \text{ cp}$$

$$\text{length} = L = 6 \text{ ft}$$

$$\text{inlet pressure} = P_1 = 5.0 \text{ atm.}$$

$$\text{outlet pressure} = P_2 = 2.0 \text{ atm.}$$

Solutions:

We must insure that all the variables are in the correct units.

$$k = 1 \text{ darcy} = 1,000 \text{ md}$$

$$A = 6 \text{ ft}^2$$

$$L = 6 \text{ ft}$$

$$P_1 = (5.0 \text{ atm}) (14.7 \text{ psi/atm}) = 73.5 \text{ psi}$$

$$P_2 = (2.0 \text{ atm}) (14.7 \text{ psi/atm}) = 29.4 \text{ psi}$$

$$q = 1.1271 \times 10^{-3} \frac{kA}{\mu L} (P_1 - P_2)$$

$$q = 1.1271 \times 10^{-3} \frac{(1,000)(6)}{(1)(6)} (73.5 - 29.4)$$

$$q = 49.7 \text{ bbl / day}$$

Example II-3

Determine the oil flow rate in a radial system with the following set of conditions:

K	=	300 md	r _e	=	330 ft
h	=	20 ft	r _w	=	0.5 ft
P _e	=	2,500 psia	r _e /r _w	=	660
P _w	=	1,740 psia	ln (r _e /r _w)	=	6.492
μ	=	1.3 cp			

Solution:

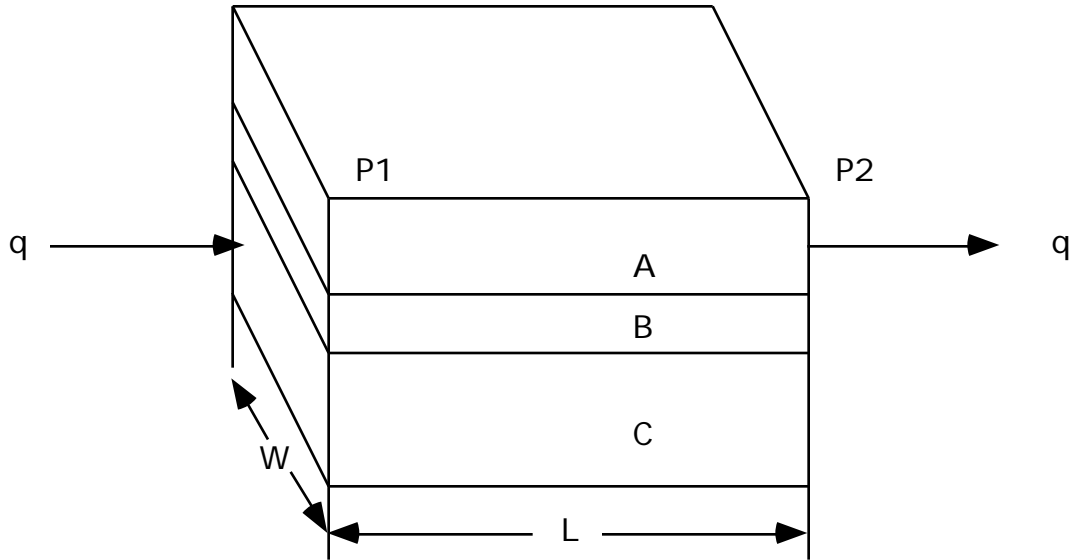
$$q = \frac{7.082 \times 10^{-3} kH (P_e - P_w)}{\mu \ln (R_e / R_w)}$$

$$q = \frac{7.082 \times 10^{-3} (300)(20)(2,500 - 1,740)}{1.3 (6.492)}$$

$$q = 3,826 \text{ res bbl/d}$$

E. Layered Systems

1. Horizontal, linear flow parallel to bedding



$$q_t = q_A + q_B + q_C$$

$$h = h_A + h_B + h_C$$

let \bar{k} be "average" permeability,

then

$$q_t = \frac{\bar{k} w h (P_1 - P_2)}{\mu L}$$

and

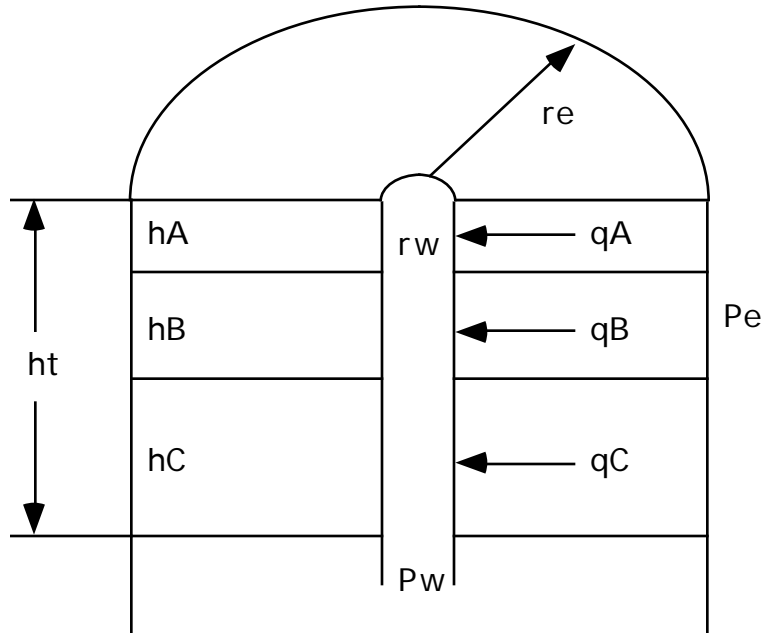
$$q_t = \frac{k_A w h_A}{\mu L} (P_1 - P_2) + \frac{k_B w h_B}{\mu L} (P_1 - P_2) + \frac{k_C w h_C}{\mu L} (P_1 - P_2)$$

then

$$\bar{k} h = k_A h_A + k_B h_B + k_C h_C$$

$$\bar{k} = \frac{1}{h} \sum_{j=1}^n k_j h_j$$

2. Horizontal, radial flow parallel to bedding



again

$$q_t = q_A + q_B + q_C$$

$$h = h_A + h_B + h_C$$

$$q_t = \frac{2 \bar{k} h}{\mu \ln(r_e/r_w)} (P_e - P_w)$$

and

$$q_t = \frac{2 k_A h_A}{\mu \ln(r_e/r_w)} (P_e - P_w) + \frac{2 k_B h_B}{\mu \ln(r_e/r_w)} (P_e - P_w) + \frac{2 k_C h_C}{\mu \ln(r_e/r_w)} (P_e - P_w)$$

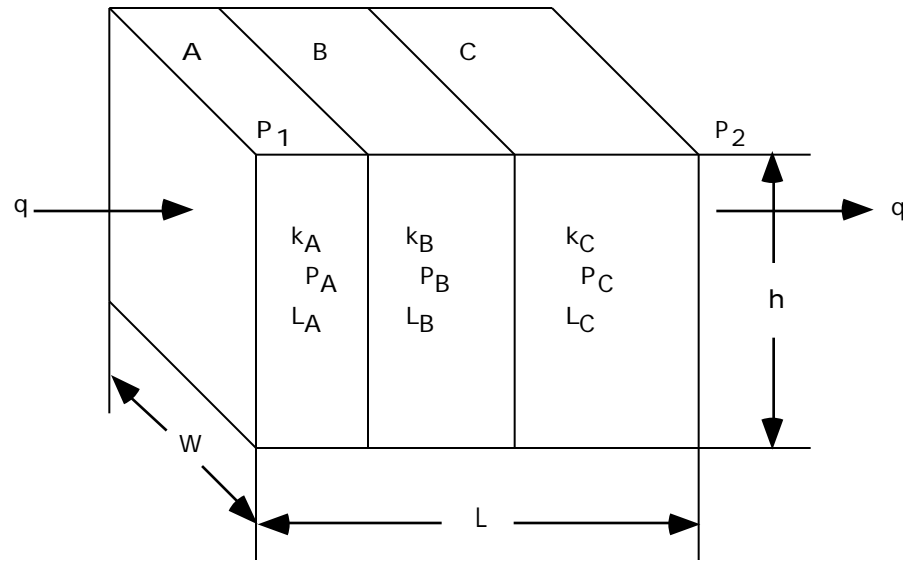
then

$$\bar{k} h = k_A h_A + k_B h_B + k_C h_C$$

and again

$$\bar{k} = \frac{n}{j=1} \frac{k_j h_j}{h}$$

3. Horizontal, linear flow perpendicular to bedding



$$q_t = q_A = q_B = q_C$$

$$P_1 - P_2 = P_A + P_B + P_C$$

$$L = L_A + L_B + L_C$$

$$q_t = \frac{\bar{k} wh (P_1 - P_2)}{\mu L}$$

and since $P_1 - P_2 = P_A + P_B + P_C$

$$P_1 - P_2 = \frac{q_t \mu L}{\bar{k} wh} = \frac{q_A \mu L_A}{k_A wh} + \frac{q_B \mu L_B}{k_B wh} + \frac{q_C \mu L_C}{k_C wh}$$

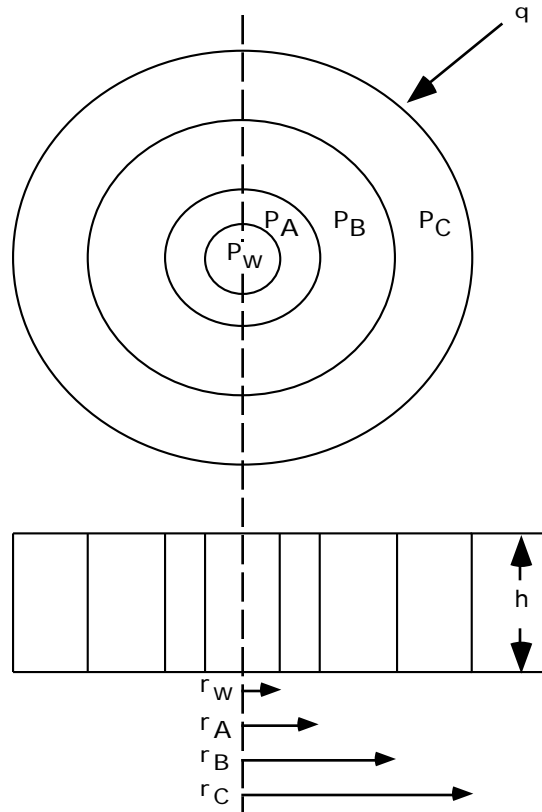
since $q_t = q_A = q_B = q_C$

$$\frac{L}{\bar{k}} = \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}$$

thus

$$\bar{K} = \frac{L}{n \sum_{j=1}^n \frac{L_j}{k_j}}$$

4. Horizontal, radial, flow perpendicular to bedding



$$q_t = q_A = q_B = q_C$$

$$P_e - P_w = P_A + P_B + P_C$$

$$q = \frac{2 \bar{k} h (P_e - P_w)}{\mu \ln(r_e/r_w)}$$

$$P_e - P_w = \frac{q_t \mu \ln(r_e/r_w)}{2 \bar{k} h} = \frac{q_A \mu \ln(r_A/r_w)}{2 k_A h} + \frac{q_B \mu \ln(r_B/r_A)}{2 k_B h} + \frac{q_C \mu \ln(r_C/r_e)}{2 k_C h}$$

then

$$\bar{k} = \frac{\ln r_e/r_w}{n \sum_{j=1} \frac{\ln(r_j/r_{j-1})}{k_j}}$$

Example II-4

Damaged zone near wellbore

$$k_1 = 10 \text{ md}$$
$$k_2 = 200 \text{ md}$$

$$r_1 = 2 \text{ ft}$$
$$r_2 = 300 \text{ ft}$$
$$r_w = 0.25 \text{ ft}$$

Solution:

$$\bar{k} = \frac{\ln(r_e/r_w)}{n \sum_{j=1}^n \frac{\ln(r_j/r_{j-1})}{k_j}}$$

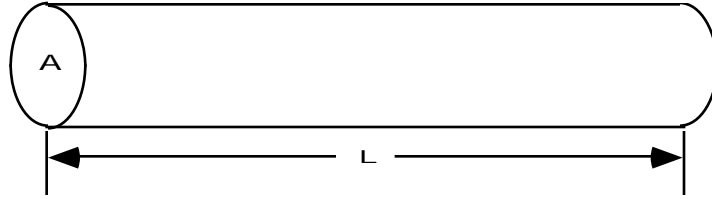
$$\bar{k} = \frac{\ln\left(\frac{300}{0.25}\right)}{\frac{\ln(2/0.25)}{10} + \frac{\ln(300/2)}{200}}$$

$$\bar{k} = 30.4 \text{ md}$$

The permeability of the damaged zone near the wellbore influences the average permeability more than the permeability of the undamaged formation.

F. Flow through channels and fractures

1. Flow through constant diameter channel



a. Poiseuille's Equation for viscous flow through capillary tubes

$$q = \frac{r^4}{8 \mu L} (P_1 - P_2)$$

$$A = r^2, \text{ therefore}$$

$$q = \frac{Ar^2}{8 \mu L} (P_1 - P_2)$$

b. Darcy's law for linear flow of liquids

$$q = \frac{kA}{\mu L} (P_1 - P_2)$$

assuming these flow equations have consistent units

$$\frac{Ar^2}{8 \mu L} (P_1 - P_2) = \frac{kA}{\mu L} (P_1 - P_2)$$

thus

$$k = \frac{r^2}{8} = \frac{d^2}{32}$$

where $d = \text{inches}$, $k = 20 \times 10^9 d^2 \text{ md}$

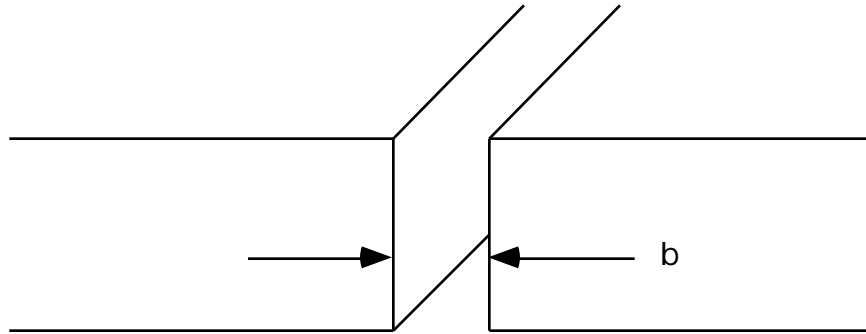
Example II-4

- A. Determine the permeability of a rock composed of closely packed capillaries 0.0001 inch in diameter.
- B. If only 25 percent of the rock is pore channels ($f = 0.25$), what will the permeability be?

Solution:

- A. $k = 20 \times 10^9 d^2$
 $k = 20 \times 10^9 (0.0001 \text{ in})^2$
 $k = 200 \text{ md}$
- B. $k = 0.25 (200 \text{ md})$
 $k = 50 \text{ md}$

2. Flow through fractures



$$v = \frac{q}{A} = \frac{h^2}{12 \mu L} (P_1 - P_2)$$

$$q = \frac{b^2 A}{12 \mu L} (P_1 - P_2)$$

setting this flow equation equal to Darcy's flow equation,

$$\frac{b^2 A}{12 \mu L} (P_1 - P_2) = \frac{kA}{\mu L} (P_1 - P_2)$$

solve for permeability of a fracture:

$$k = \frac{b^2}{12} \text{ in darcy units, or}$$

$$k = 54 \times 10^9 b^2$$

where b = inches
 k = md

Example II-6

Consider a rock of very low matrix permeability, 0.01 md, which contains on the average a fracture 0.005 inches wide and one foot in lateral extent per square foot of rock.

Assuming the fracture is in the direction of flow, determine the average permeability using the equation for parallel flow.

Solution:

$$\bar{k} = \frac{k_j A_j}{A}, \text{ similar to horizontal, linear flow parallel to fracture}$$

$$\bar{k} = \frac{(\text{matrix } k)(\text{matrix area}) + (\text{fracture } k)(\text{fracture area})}{\text{total area}}$$

$$\bar{k} = \frac{(0.01) ((12 \text{ in})^2 + (12 \text{ in})(0.005 \text{ in}))}{144 \text{ in}^2} + \frac{(54 \times 10^9 \times (0.005)^2)(12 \text{ in} \times (0.005 \text{ in}))}{144 \text{ in}^2}$$

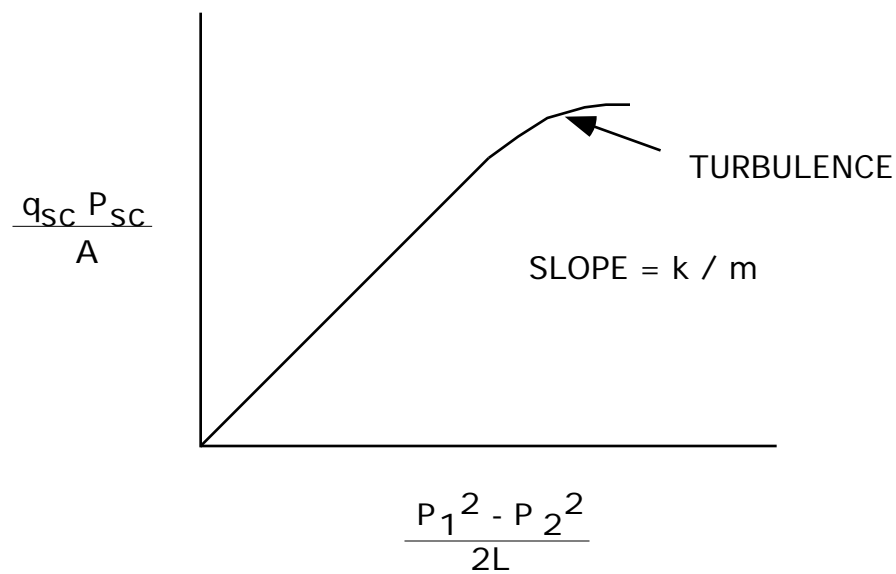
$$\bar{k} = \frac{1.439 + 81,000}{144}$$

$$\bar{k} = 563 \text{ md}$$

III) Laboratory measurement of permeability

A. Procedure

1. Perm plug method
 - a. cut small, individual samples (perm plugs) from larger core
 - b. extract hydrocarbons in extractor
 - c. dry core in oven
 - d. flow fluid through core at several rates



$$q_{sc} = \frac{kA (P_1^2 - P_2^2)}{2 \mu L P_{sc}} \quad \text{horizontal, linear, real gas flow with}$$

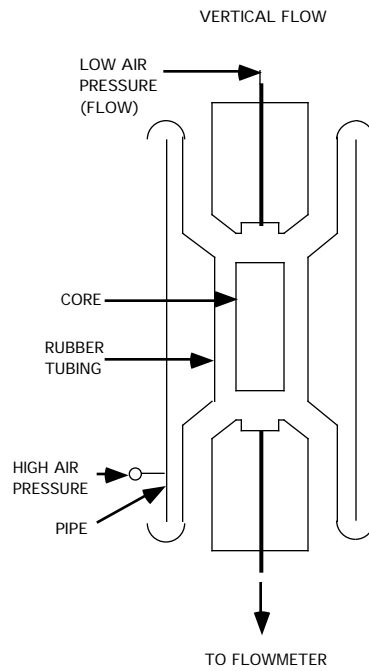
$$T = T_{sc} \text{ and } Z = 1.0$$

$$\frac{q_{sc} P_{sc}}{A} = \frac{k}{\mu} \left[\frac{P_1^2 - P_2^2}{2L} \right]$$

$$k = (\text{slope}) m$$

2. Whole core method

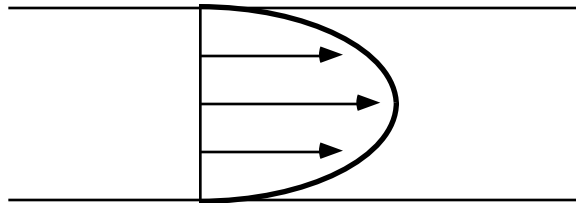
- a. prepare whole core in same manner as perm plugs
- b. mount core in special holders and flow fluid through core as in perm plug method



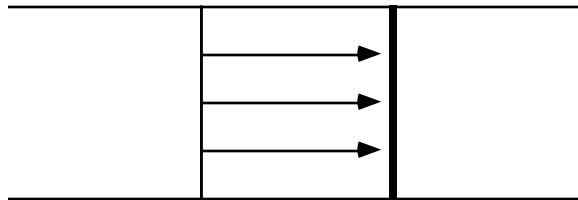
- c. the horizontal flow data must be adjusted due to complex flow path
- d. whole core method gives better results for limestones

B. Factors which affect permeability measurement

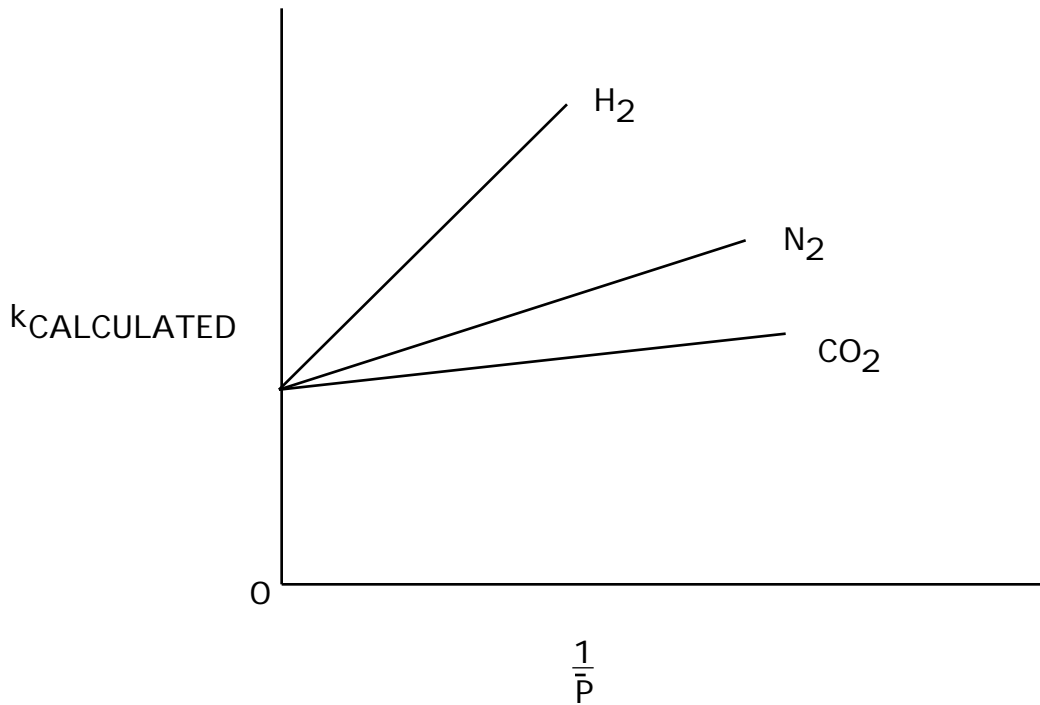
1. Fractures - rocks which contain fractures in situ frequently separate along the planes of natural weakness when cored. Thus laboratory measurements give "matrix" permeability which is lower than in situ permeability because typically only the unfractured parts of the sample are analyzed for permeability.
2. Gas slippage
 - a. gas molecules "slip" along the grain surfaces
 - b. occurs when diameter of the capillary openings approaches the mean free path of the gas molecules
 - c. Darcy's equation assumes laminar flow



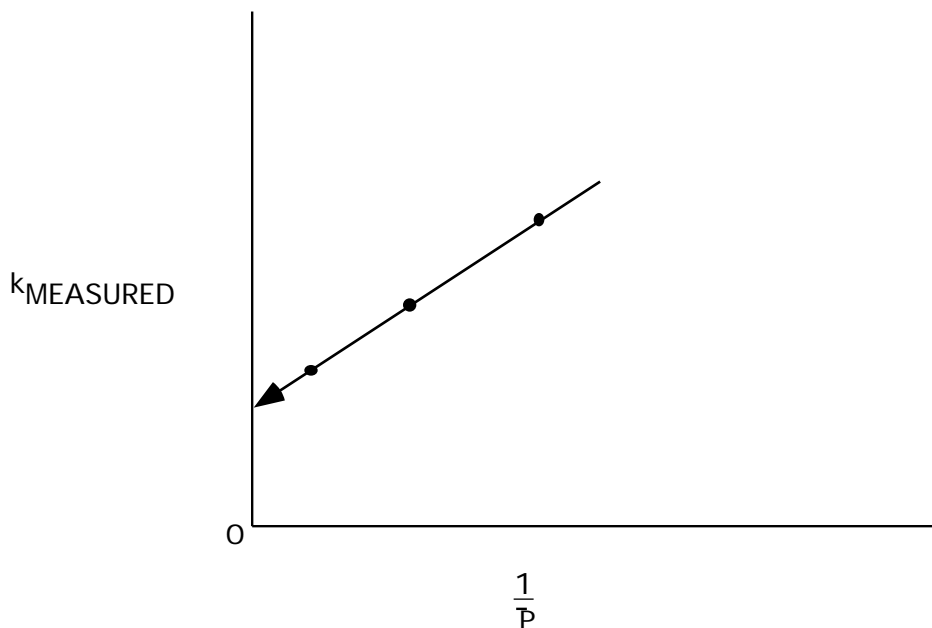
- d. gas flow path with slippage



- e. called Klinkenberg effect
- f. mean free path is function of size of molecule thus permeability measurements are a function of type of gas used in laboratory measurement.



- g. mean free path is a function of pressure, thus Klinkenberg effect is greater for measurements at low pressures - negligible at high pressures.
- h. permeability is a function of size of capillary opening, thus Klinkenberg effect is greater for low permeability rocks.
- i. effect of gas slippage can be eliminated by making measurements at several different mean pressures and extrapolating to high pressure ($1/p \Rightarrow 0$)



Example II-7

Another core taken at 8815 feet from the Brazos County well was found to be very shaly. There was some question about what the true liquid permeability was, since nitrogen was used in the permeameter.

Calculate the equivalent liquid permeability from the following data.

Mean Pressure (atm)	Measured Permeability (md)
1.192	3.76
2.517	3.04
4.571	2.76
9.484	2.54

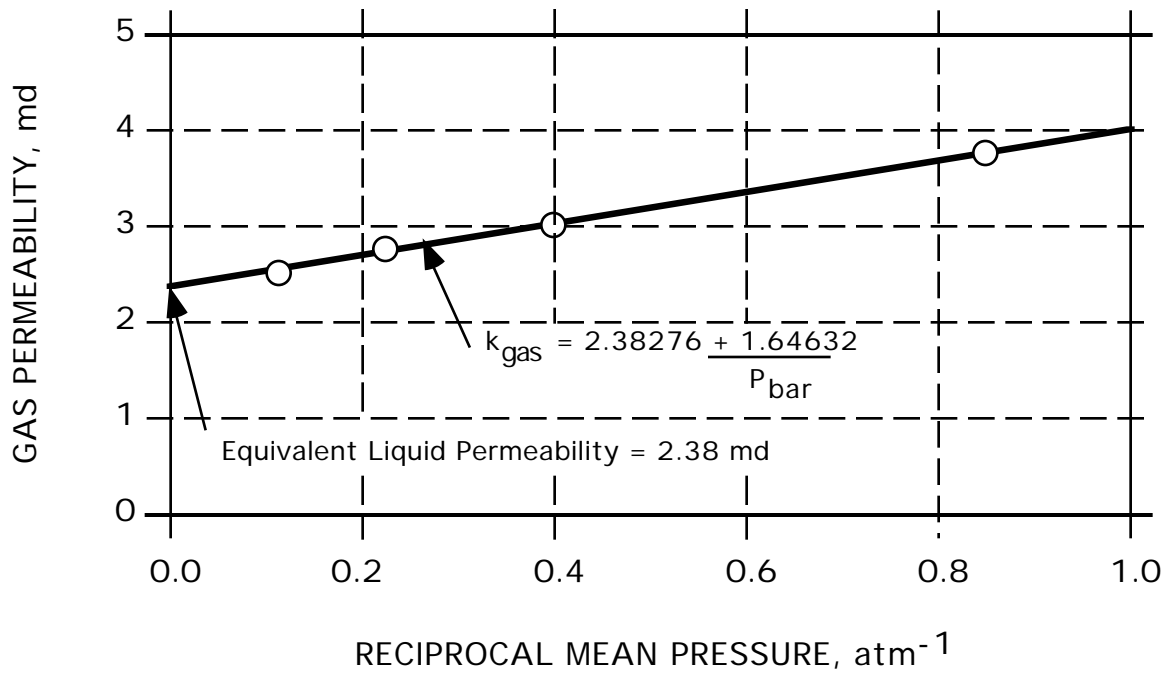
Solution:

Plot k_{measured} vs. $1/\text{pressure}$

Intercept is equivalent to liquid permeability

From graph:

$$k_{\text{liq}} = 2.38 \text{ md}$$

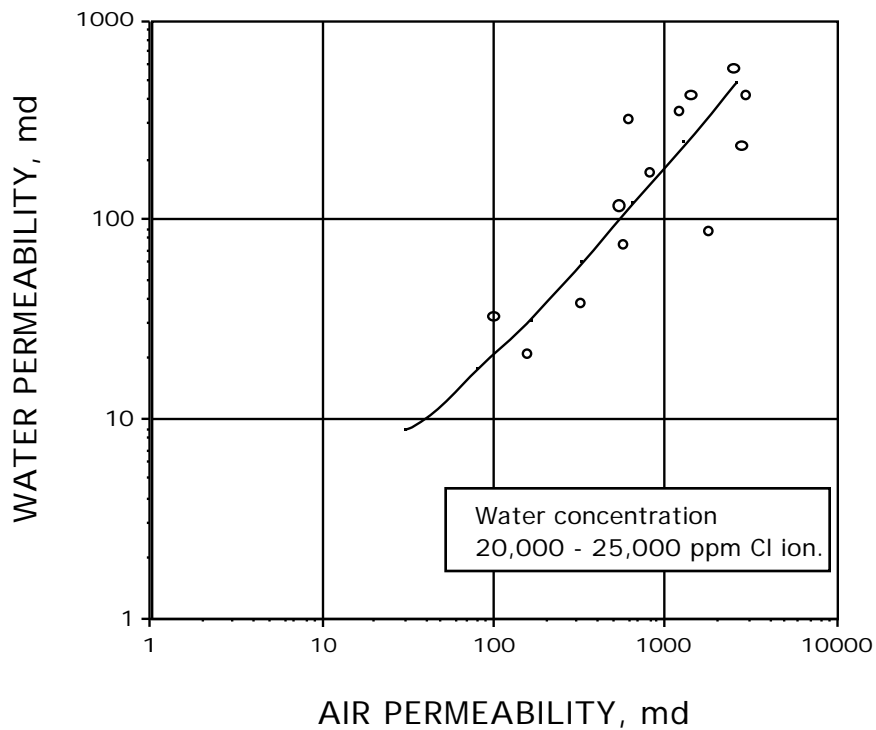


3. Reactive fluids

a. Formation water reacts with clays

- 1) lowers permeability to liquid
- 2) actual permeability to formation water is lower than lab permeability to gas

RELATIONSHIP OF PERMEABILITIES MEASURED WITH AIR TO THOSE MEASURED WITH WATER



b. Injection water may, if its salinity is less than that of the formation water, reduce the permeability due to clay swelling.

Effect of Water Salinity on Permeability of Natural Cores
(Grains per gallon of chloride ion as shown).

Field	Zone	K _a	K1000	K500	K300	K200	K100	K _w
S	34	4080	1445	1380	1290	1190	885	17.2
S	34	24800	11800	10600	10000	9000	7400	147.0
S	34	40100	23000	18600	15300	13800	8200	270.0
S	34	4850	1910	1430	925	736	326	5.0
S	34	22800	13600	6150	4010	3490	1970	19.5
S	34	34800	23600	7800	5460	5220	3860	9.9
S	34	13600	5160	4640	4200	4150	2790	197.0
S	34	7640	1788	1840	2010	2540	2020	119.0
T	36	2630	2180	2140	2080	2150	2010	1960.0
T	36	3340	2820	2730	2700	2690	2490	2460.0
T	36	2640	2040	1920	1860	1860	1860	1550.0
T	36	3360	2500	2400	2340	2340	2280	2060.0

K_a means permeability to air; K500 means permeability to 500 grains per gallon chloride solution; K_w means permeability to fresh water

4. Change in pore pressure

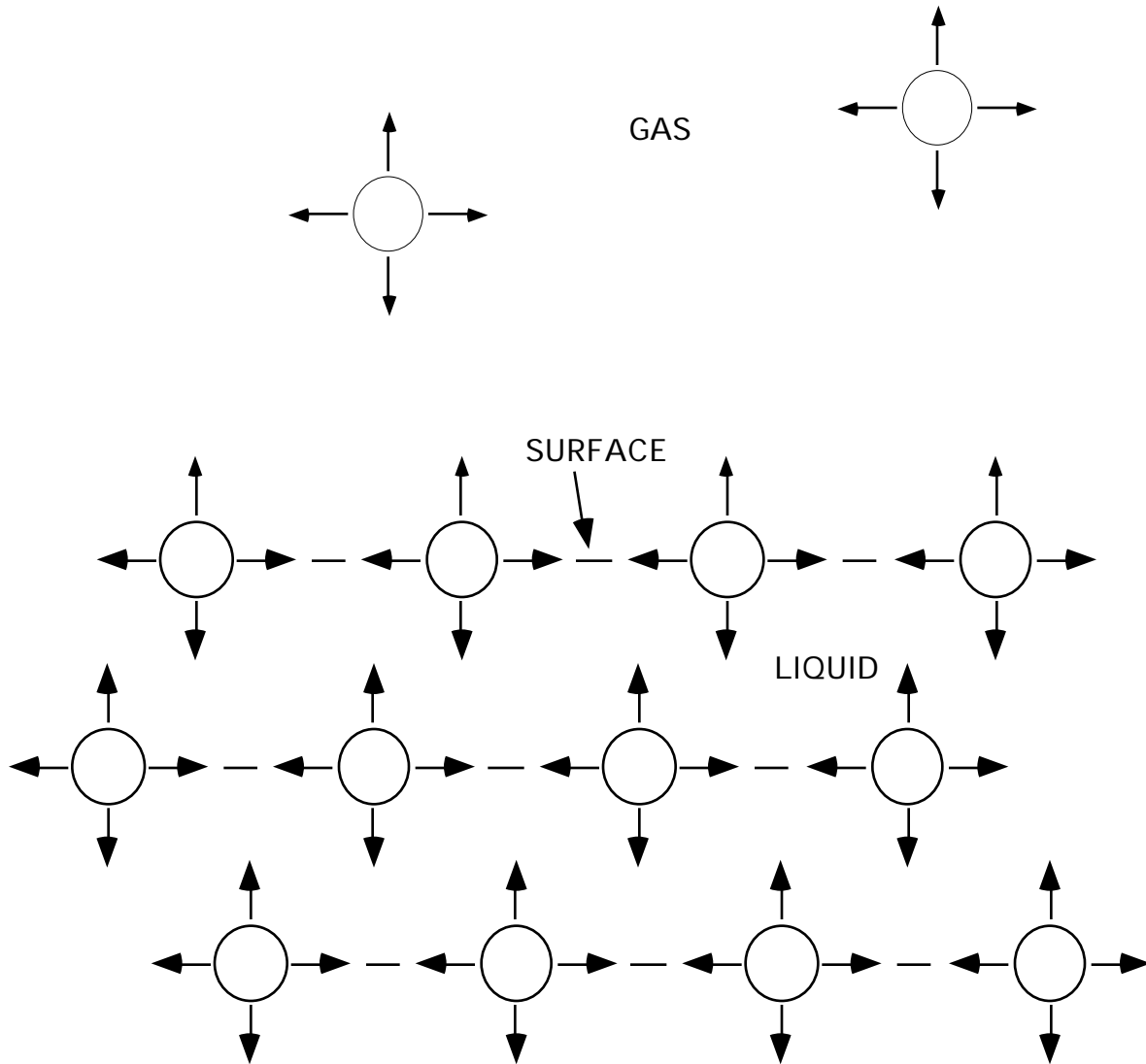
a. The removal of the core from the formation will likely result in a change in pore volume. This is likely to result in a change in permeability (+ or -).

b. The production of fluids, especially around the well, will result in a decrease in pore pressure and a reduction of in-situ permeability.

III. BOUNDARY TENSION AND CAPILLARY PRESSURE

I) Boundary tension,

- A. at the boundary between two phases there is an imbalance of molecular forces
- B. the result is to contract the boundary to a minimum size



- C. the average molecule in the liquid is uniformly attracted in all directions
- D. molecules at the surface attracted more strongly from below
- E. creates concave or convex surface depending on force balance
- F. creation of this surface requires work
 - 1. work in ergs required to create 1 cm^2 of surface (ergs/cm^2) is termed "boundary energy"
 - 2. also can be thought of as force in dynes acting along length of 1 cm required to prevent destruction of surface (dynes/cm) - this is called "boundary tension"
 - 3. Boundary Energy = Boundary Tension x Length
- G. Surface Tension - Boundary tension between gas and liquid is called "surface tension"
- H. Interfacial Tension - Boundary tension between two immiscible liquids or between a fluid and a solid is called "interfacial tension"

- gw = surface tension between gas and water
- go = surface tension between gas and oil
- wo = interfacial tension between water and oil
- ws = interfacial tension between water and solid
- os = interfacial tension between oil and solid
- gs = interfacial tension between gas and solid

- I. Forces creating boundary tension
 - 1. Forces
 - a. Law of Universal Gravitation applied between molecules
 - b. physical attraction (repulsion) between molecules
 - 2. Liquid-Gas Boundary

attraction between molecules is directly proportional to their masses and inversely proportional to the square of the distance between them
 - 3. Solid-Liquid Boundary

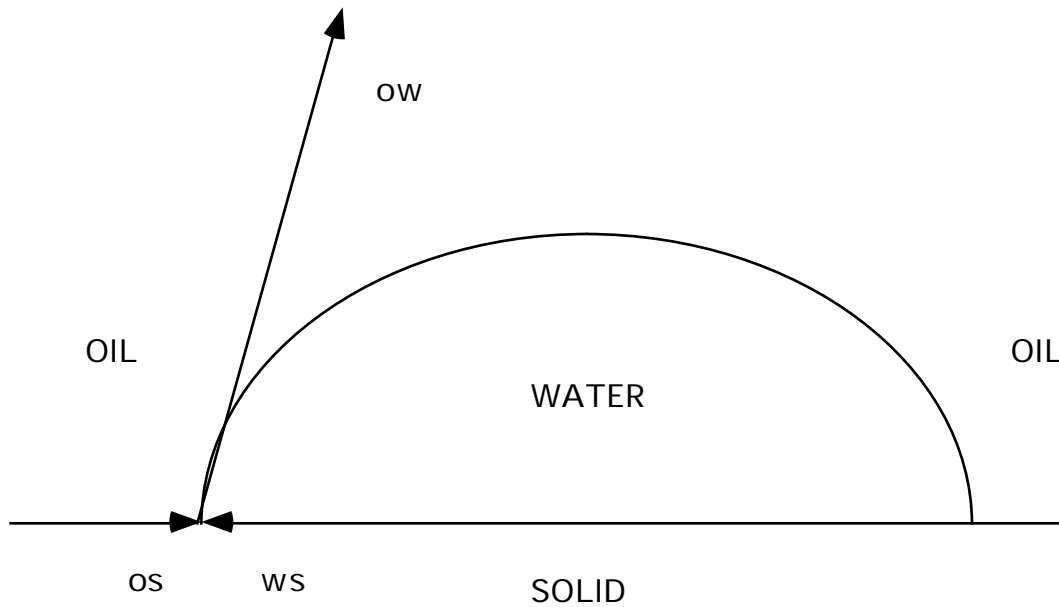
physical attraction between molecules of liquid and solid surface

4. Liquid-Liquid Boundary

some of each

II) Wettability

A. forces at boundary of two liquids and a solid (or gas-liquid-solid)



$$w_s = o_s + o_w \cos \theta$$

B. Adhesion Tension, A_T

$$A_T = w_s - o_s = o_w \cos \theta$$

C. if the solid is "water-wet"

$$w_s > o_s$$

$$A_T = +$$

$$\cos \theta = +$$

$$0^\circ < \theta < 90^\circ$$

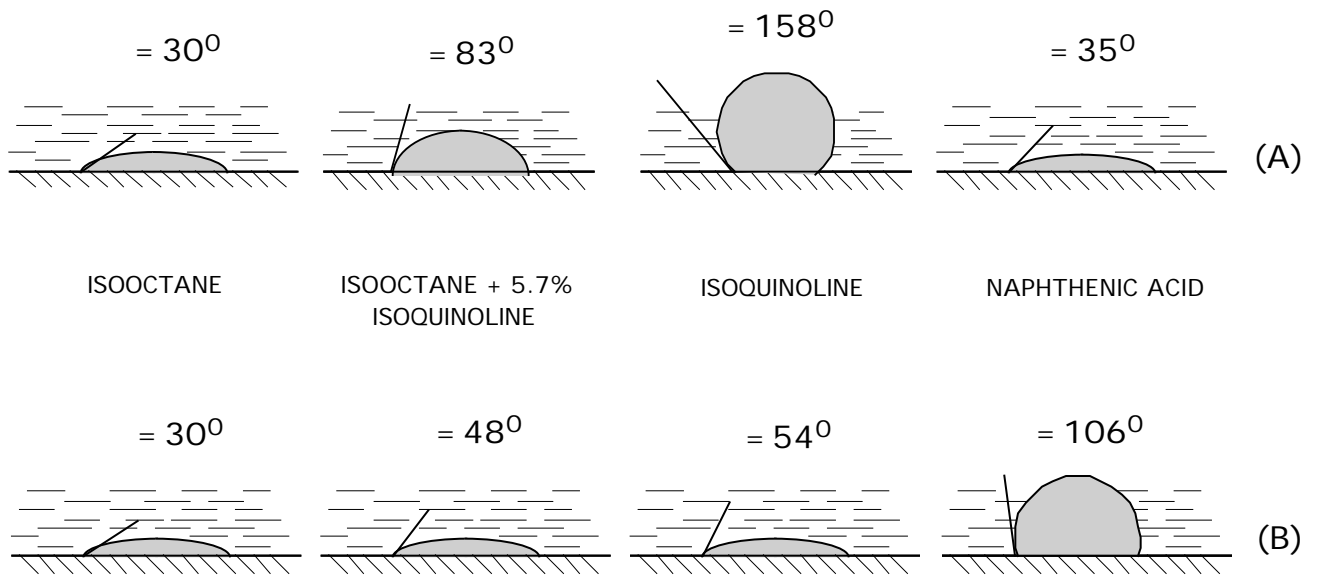
if $\theta = 0^\circ$ - strongly water-wet

D. if the solid is "oil-wet"

$$\cos \theta = \frac{\sigma_{os} - \sigma_{ws}}{\sigma_{ow}}$$

$$\cos 90^\circ = \frac{\sigma_{os} - \sigma_{ws}}{\sigma_{ow}}$$

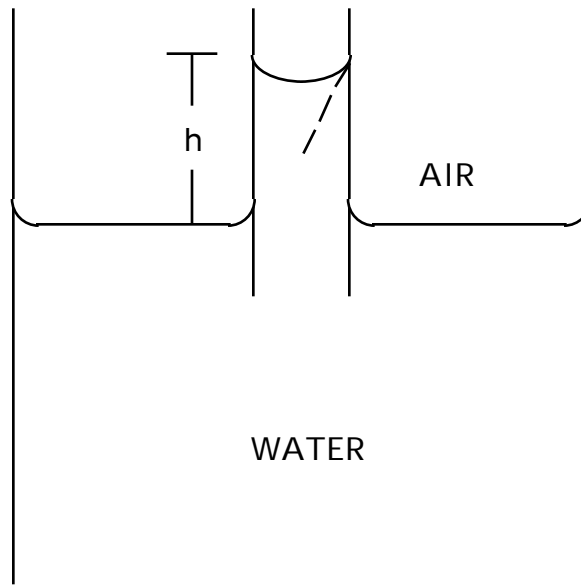
if $\theta = 180^\circ$ - strongly oil-wet



Interfacial contact angles. (A) Silica surface; (B) calcite surface

III) Capillary pressure

A. capillary pressure between air and water



1. liquid will rise in the tube until total force up equals total force down

a. total force up equals adhesion tension acting along the circumference of the water-air-solid interface

$$= 2 r A_T$$

b. total force down equals the weight of the column of water converted to force

$$= r^2 h g_w$$

c. thus when column of water comes to equilibrium

$$2 r A_T = r^2 h g_w$$

d. units

$$\text{cm} \frac{\text{dyne}}{\text{cm}} = \text{cm}^2 \text{cm} \frac{\text{cm}}{\text{sec}^2} \frac{\text{gm}}{\text{cm}^3}$$

$$\text{dyne} = \frac{\text{gm cm}}{\text{sec}^2}$$

dyne = force unit

e. adhesion tension

$$A_T = \frac{1}{2} r h g_w \left(\frac{\text{dyne}}{\text{cm}} \right)$$

2. liquid will rise in the tube until the vertical component of surface tension equals the total force down

a. vertical component of surface tension is the surface tension between air and water multiplied by the cosine of the contact angle acting along the water-air-solid interface

$$= 2 r \sigma_{aw} \cos \theta$$

b. total force down

$$= r^2 h \rho_w g$$

c. thus when the column of water comes to equilibrium

$$2 r \sigma_{aw} \cos \theta = r^2 h \rho_w g$$

d. units

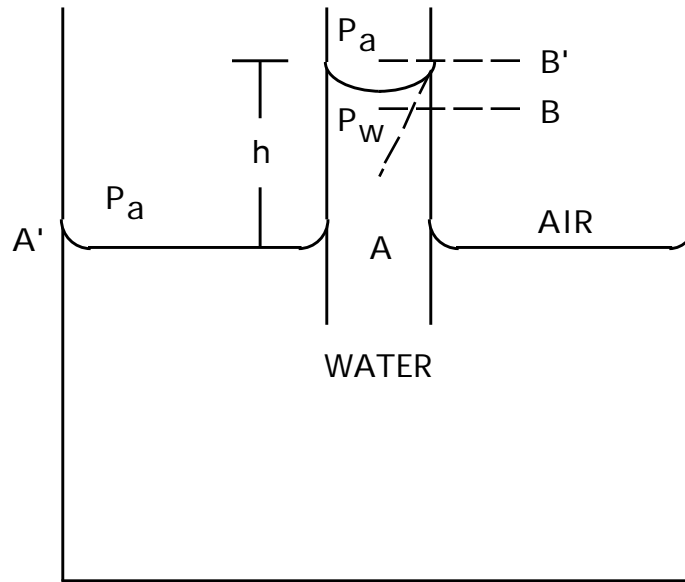
$$\text{cm} \frac{\text{dyne}}{\text{cm}} = \text{cm}^2 \text{cm} \frac{\text{cm}}{\text{sec}^2} \frac{\text{gm}}{\text{cm}^3}$$

$$\text{cm} \frac{\text{dyne}}{\text{cm}} = \text{cm} \frac{\text{gm}}{\text{sec}^2}$$

3. since $A_T = \sigma_{aw} \cos \theta$, 1 and 2 above both result in

$$h = \frac{2 \sigma_{aw} \cos \theta}{r \rho_w g}$$

4. capillary pressure (air-water system)



pressure relations in capillary tubes

- a. pressure at A' is equal to pressure at A

$$P_{a'} = P_a$$

- b. pressure at B is equal to the pressure at A minus the head of water between A & B

$$P_w = P_a - wgh$$

$$\text{units: } \frac{\text{dyne}}{\text{cm}^2} = \frac{\text{dyne}}{\text{cm}^2} - \frac{\text{gm}}{\text{cm}^3} \frac{\text{cm}}{\text{sec}^2} \text{ cm}$$

- c. thus between B' and B there is a pressure difference

$$P_a - P_w = P_a - (P_a - wgh)$$

$$P_a - P_w = wgh$$

- d. call this pressure difference between B' and B "capillary pressure"

$$P_c = P_a - P_w = wgh$$

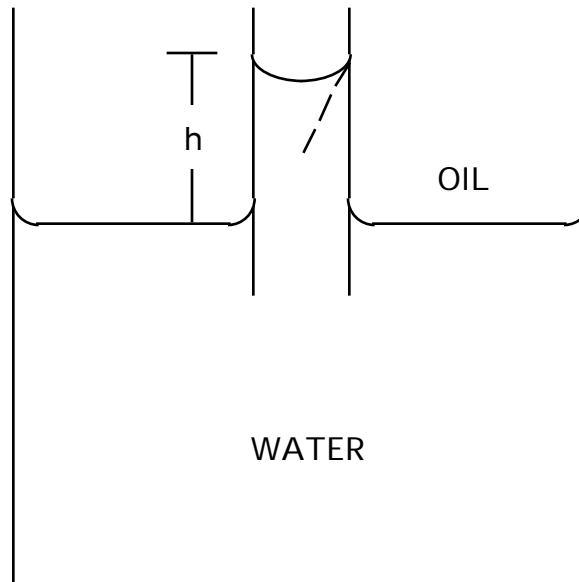
- e. remember

$$h = \frac{2}{rg} \frac{g_w \cos}{w}$$

f. thus

$$P_c = \frac{2 \gamma_{ow} \cos \theta}{r}$$

B. capillary pressure between oil and water



1. liquid will rise in the tube until the vertical component of surface tension equals the total force down
 - a. vertical component of surface tension equals the surface tension between oil and water multiplied by the cosine of the contact angle acting along the circumference of the water-oil-solid interface

$$= 2 r \gamma_{ow} \cos \theta$$

- b. the downward force caused by the weight of the column of water is partially offset (bouyed) by the weight of the column of oil outside the capillary

- c. thus, total force down equals the weight of the column of water minus the weight of an equivalent column of oil converted to force

- 1) weight per unit area of water

$$= \gamma_w h$$

- 2) weight per unit area of oil

$$= \gamma_o h$$

3) net weight per unit area acting to pull surface down

$$= w h - o h = h (w - o)$$

4) total force down

$$= r^2 g h (w - o)$$

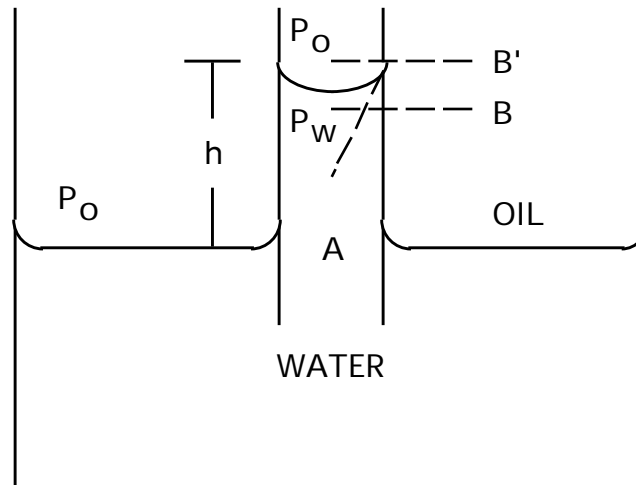
d. thus when the column of water comes to equilibrium

$$2 r o w \cos = r^2 g h (w - o)$$

2. thus the equilibrium for the height of the column of water

$$h = \frac{2 o w \cos}{r g (w - o)}$$

3. capillary pressure (oil-water system)



a. pressure at A' equals pressure at A

$$P_{oa} = P_{wa}$$

b. pressure at B is equal to the pressure at A minus the head of water between A and B

$$P_{wb} = P_{wa} - w g h$$

c. pressure at B' equal to the pressure at A' minus the head of oil between A' and B'

$$P_{ob} = P_{oa} - \rho gh$$

d. thus capillary pressure, the difference between pressure at B' and the pressure at B is

$$P_c = P_{ob} - P_{wb}$$

$$P_c = (P_{oa} - \rho gh) - (P_{wa} - \rho gh)$$

$$\text{since } P_{oa} = P_{wa}$$

$$P_c = (\rho_w - \rho_o)gh$$

e. remember

$$h = \frac{2 \rho_w \cos \theta}{r(\rho_w - \rho_o)}$$

f. thus

$$P_c = \frac{2 \rho_w \cos \theta}{r}$$

4. same expression as for the air-solid system except for the boundary tension term

$$P_c = \frac{2 \cos \theta}{r}$$

C. remember adhesion tension is defined as

$$A_T = \rho_w \cos \theta,$$

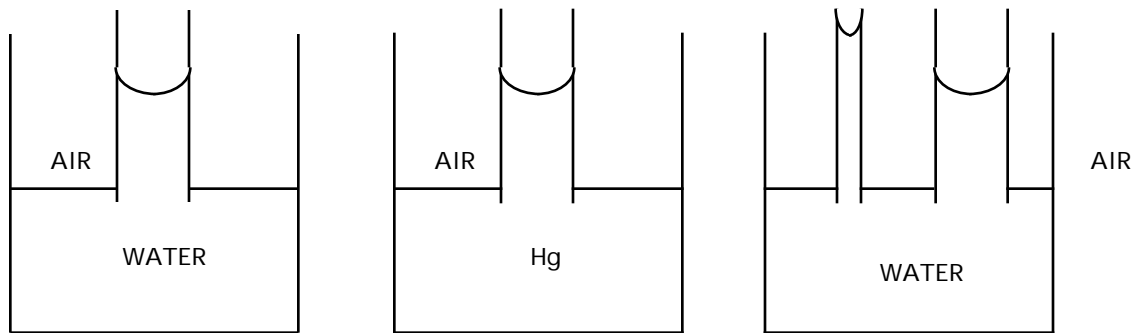
and

$$P_c = \frac{2 \rho_w \cos \theta}{r}$$

thus

$$P_c = f(\text{adhesion tension, } 1/\text{radius of tube})$$

ADHESION TENSION



1/radius of tube

D. an important result to remember

1. $P_{wb} < P_{ob}$
2. thus, the pressure on the concave side of a curved surface is greater than the pressure on the convex side
3. or, pressure is greater in the non-wetting phase

E. capillary pressure-unconsolidated sand

1. the straight capillary previously discussed is useful for explaining basic concepts - but it is a simple and ideal system
2. packing of uniform spheres

$$P_c = \left[\frac{1}{R_1} + \frac{1}{R_2} \right]$$

R_1 and R_2 are the principal radii of curvature for a liquid adhering to two spheres in contact with each other.

3. by analogy to capillary tube

$$\left[\frac{1}{R_1} + \frac{1}{R_2} \right] = \frac{2 \cos}{r}$$

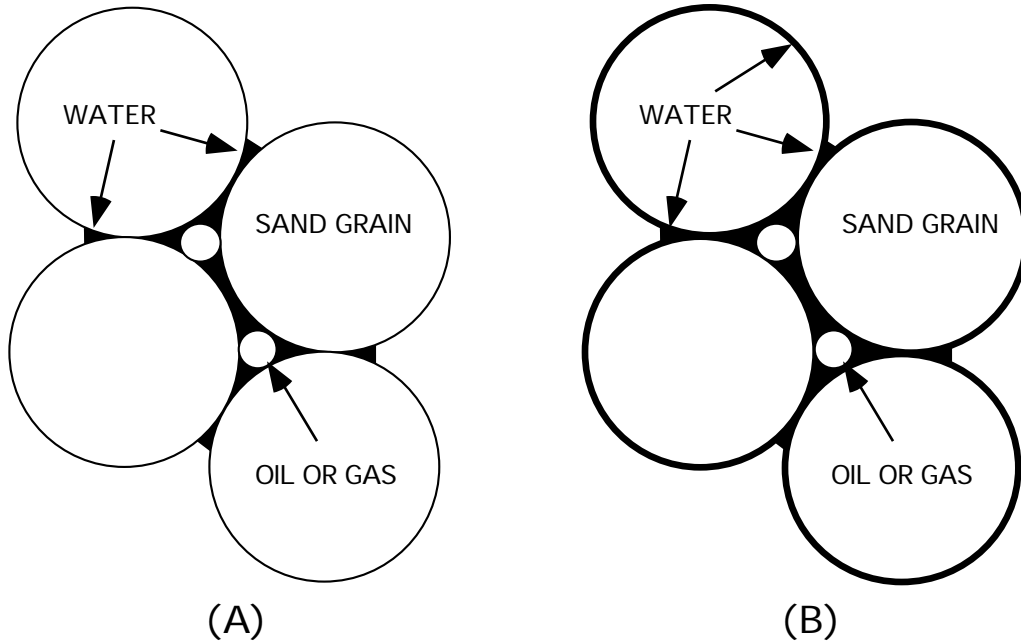
$$\text{where } P_c = \frac{2 \cos}{r}$$

call it R_m (mean radius), i.e.

$$\frac{1}{R_m} = \frac{2 \cos}{r_m} = ()gh$$

F. wettability-consolidated sand

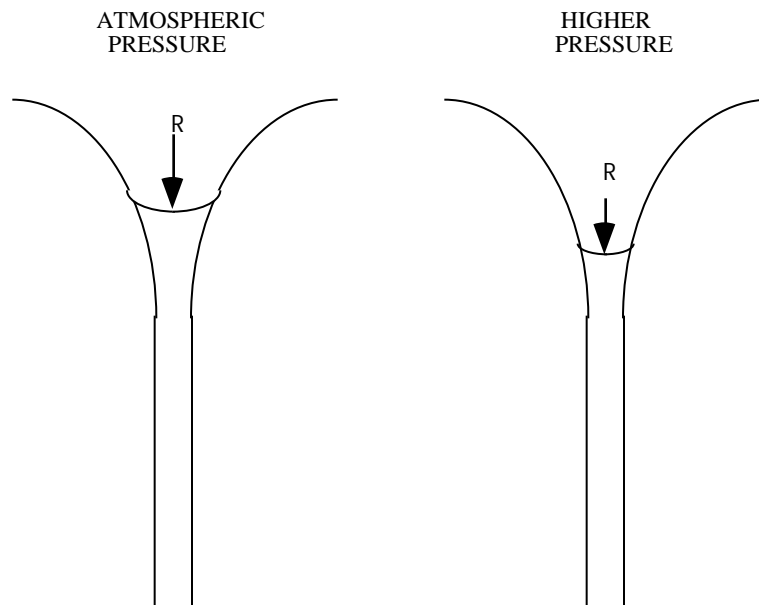
1. Pendular-ring distribution-wetting phase is not continuous, occupies the small interstices-non-wetting phase is in contact with some of the solid
2. Funicular distribution - wetting phase is continuous, completely covering surface of solid



Idealized representation of distribution of wetting and nonwetting fluid phase about intergrain contacts of spheres. (a) Pendular-ring distributions; (b) funicular distribution

IV) Relationship between capillary pressure and saturation

- A. remember that the height a liquid will rise in a tube depends on
1. adhesion
 2. fluid density
 3. variation of tube diameter with height
- B. consider an experiment in which liquid is allowed to rise in a tube of varying diameter under atmospheric pressure. Pressure in the gas phase is increased forcing the interface to a new equilibrium position.

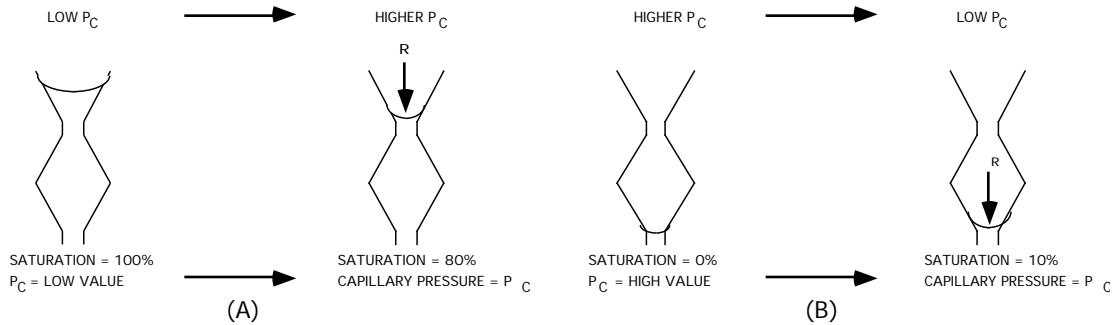


DEPENDENCE OF INTERFACIAL CURVATURE ON FLUID SATURATION IN A NON-UNIFORM PORE

1. Capillary pressure is defined as the pressure difference across the interface.
2. This illustrates:
 - a. Capillary pressure is greater for small radius of curvature than for large radius of curvature
 - b. An inverse relationship between capillary pressure and wetting-phase saturation
 - c. Lower wetting-phase saturation results in smaller radius of curvature which means that the wetting phase will occupy smaller pores in reservoir rock

V) Relationship between capillary pressure and saturation history

- A. consider an experiment using a non-uniform tube (pore in reservoir rock)
1. tube is filled with a wetting fluid and allowed to drain until the interface between wetting fluid and non-wetting fluid reaches equilibrium (drainage)
 2. tube is filled with non-wetting fluid and immersed in wetting fluid allowing wetting fluid to imbibe until the interface reaches equilibrium (imbibition)

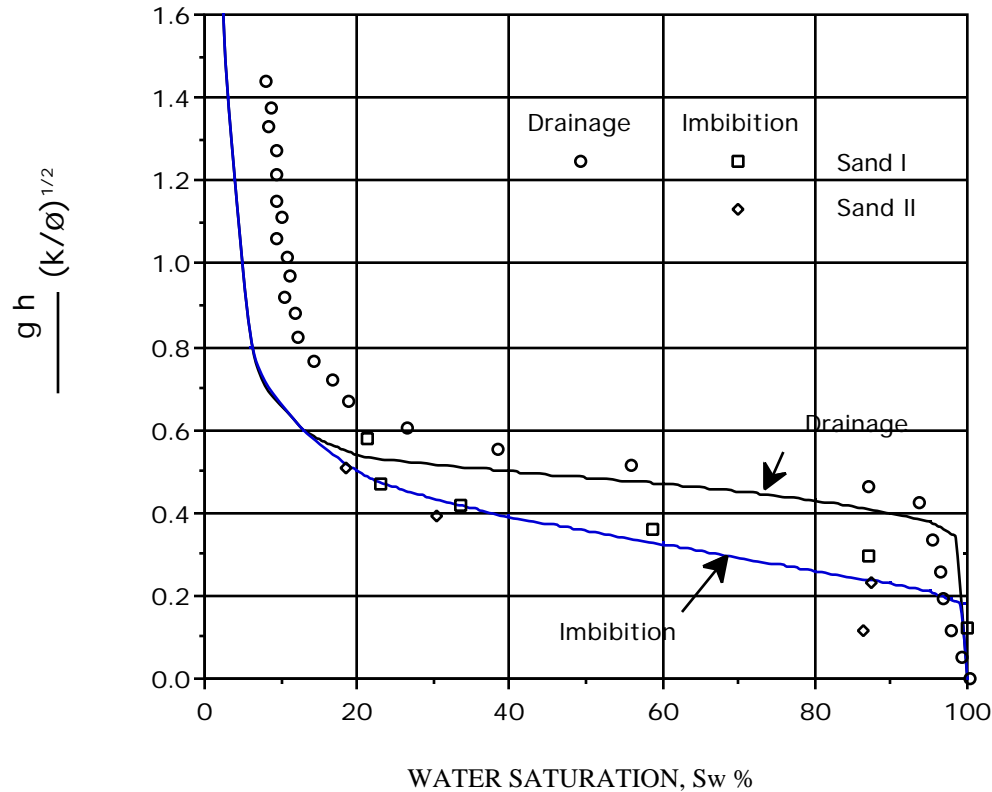


Dependence of equilibrium fluid saturation upon the saturation history in a nonuniform pore. (a) Fluid drains; (b) fluid imbibe. Same pore, same contact angle, same capillary pressure, different saturation history

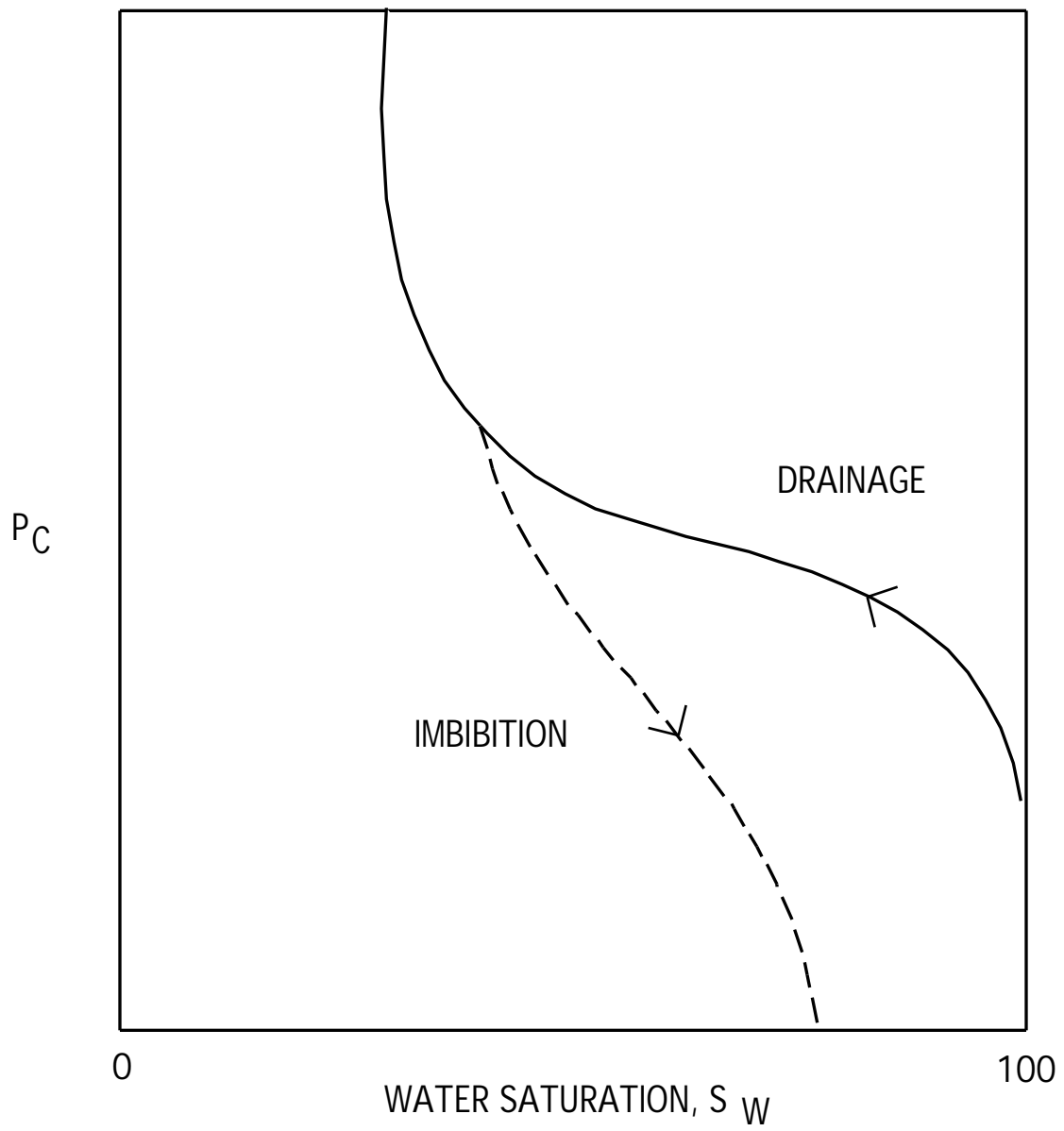
3. This is an oversimplified example, however it illustrates that the relationship between wetting-phase saturation and capillary pressure is dependent on the saturation process (saturation history)
 - a. for given capillary pressure a higher value of wetting-phase saturation will be obtained from drainage than from imbibition

B. Leverett conducted a similar experiment with tubes filled with sand.

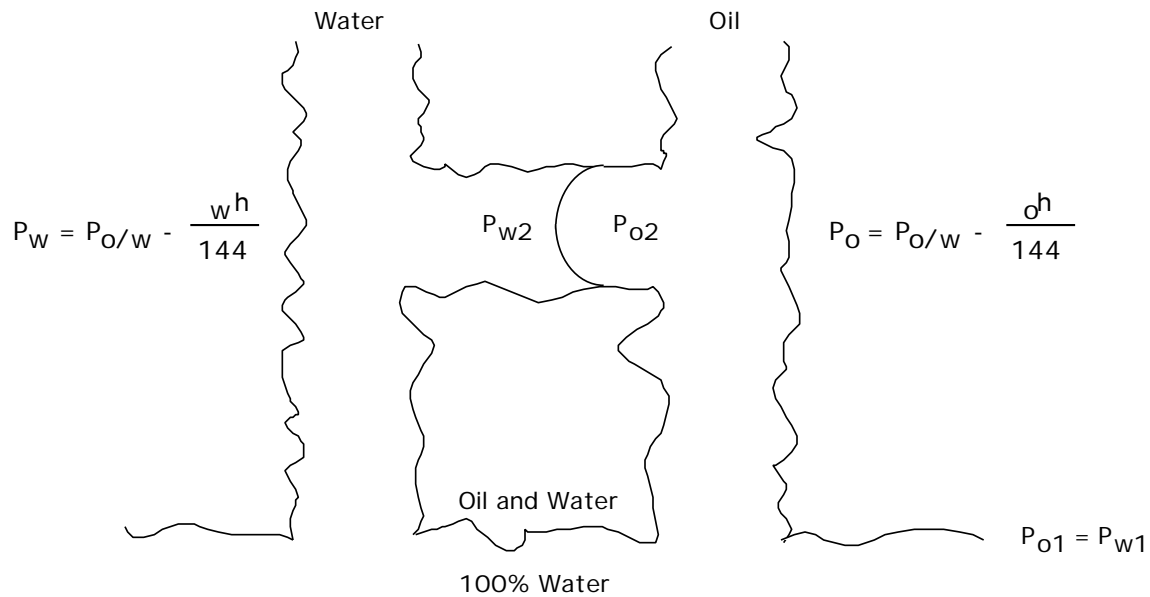
DATA FROM HEIGHT-SATURATION EXPERIMENTS
ON CLEAN SANDS. (FROM LEVERETT)



1. capillary pressure is expressed in terms of a non-dimensional correlating function (remember $P_c = (gh)$)
2. in general terms,
 - a. drainage means replacing a wetting fluid with a non-wetting fluid
 - b. imbibition means replacing a non-wetting fluid with a wetting fluid



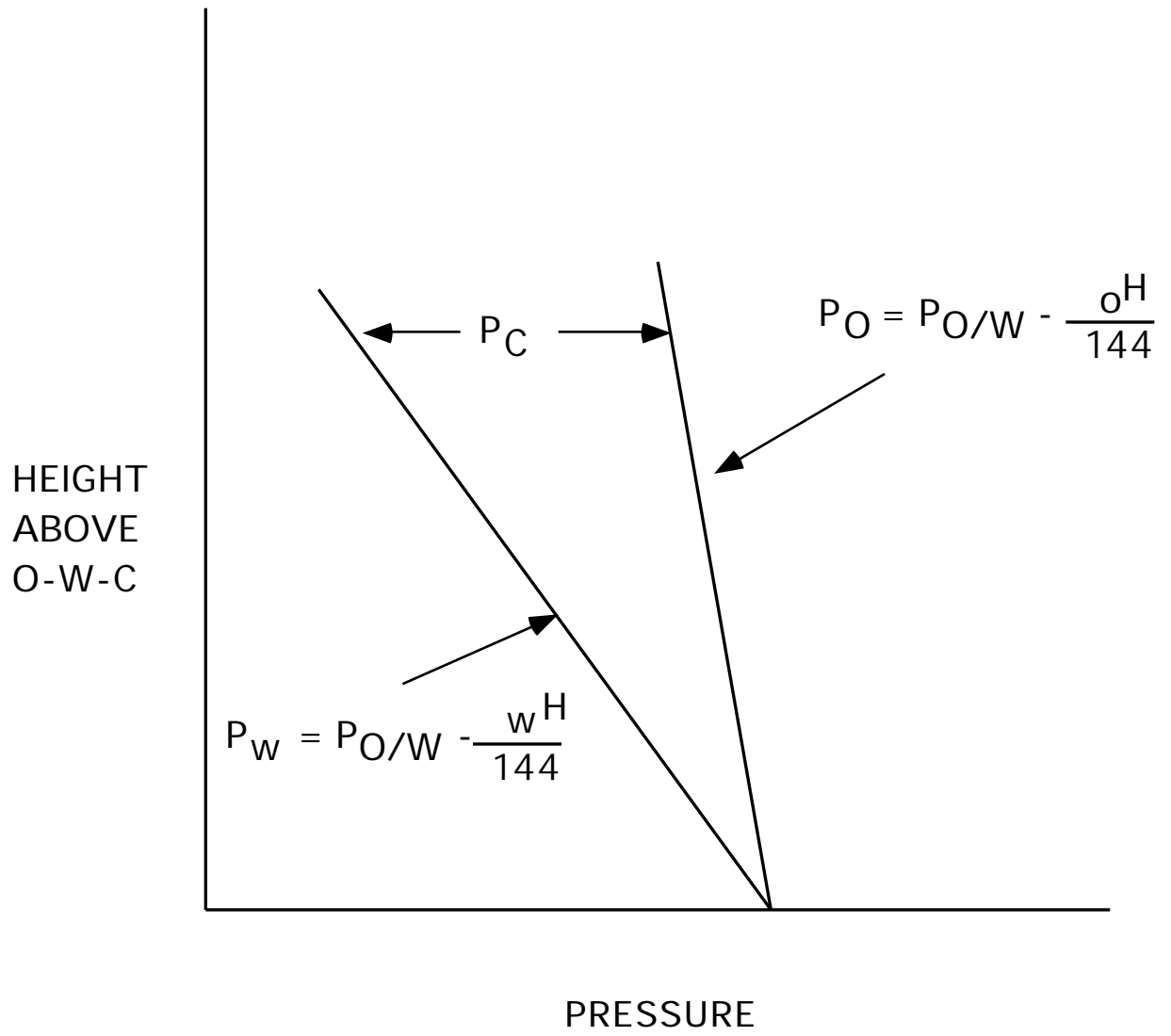
VI) Capillary pressure in reservoir rock



$$P_c = P_o - P_w = \frac{h}{144} (w - o)$$

- Where:
- P_o = pressure in oil phase, psia
 - P_w = pressure in water phase, psia
 - h = distance above 100% water level, ft
 - $P_{O/w}$ = pressure at oil-water contact, psia
 - w = density of water, lb/cf
 - o = density of oil, lb/cf

At any point above the oil-water contact, $p_o > p_w$



VII) Laboratory measurement of capillary pressure

A. Methods

1. porous diaphragm
2. mercury injection
3. centrifuge
4. dynamic method

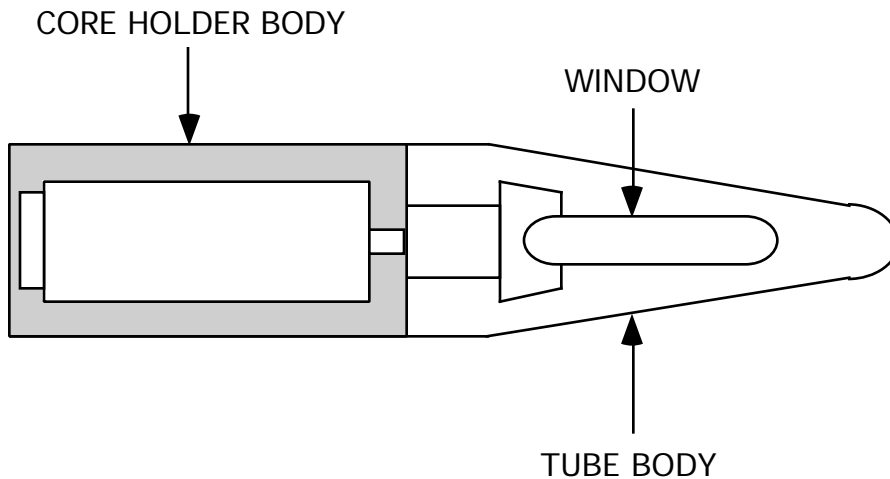
B. Porous diaphragm

1. Start with core saturated with wetting fluid.
2. Use pressure to force non-wetting fluid into core-displacing wetting fluid through the porous disk.
3. The pressure difference between the pressure in the non-wetting fluid and the pressure in the wetting fluid is equal to P_c .
4. Repeat at successively higher pressures until no more wetting fluid will come out.
5. Measure S_w periodically.
6. Results
7. Advantages
 - a. very accurate
 - b. can use reservoir fluids
8. Disadvantages
 - a. very slow - up to 40 days for one core
 - b. pressure is limited by "displacement pressure" of porous disk

C. Mercury Injection Method

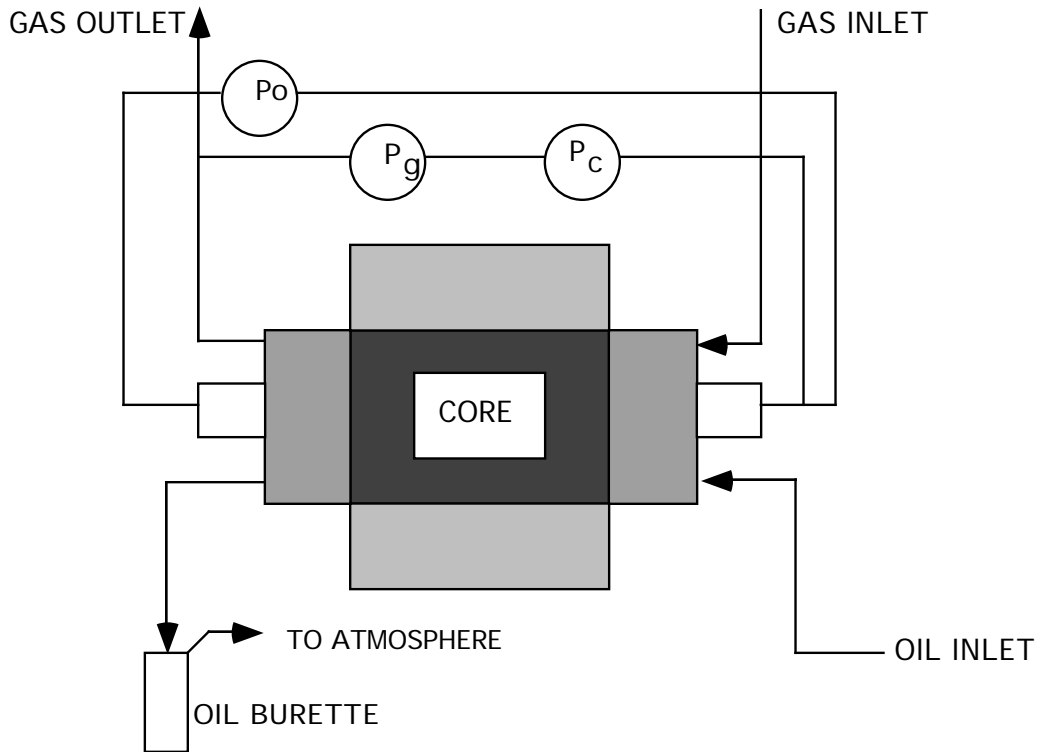
1. Force mercury into core - mercury is non-wetting phase - air (usually under vacuum) is wetting phase
2. Measure pressure
3. Calculate mercury saturation
4. Advantages
 - a. fast-minutes
 - b. reasonably accurate
5. Disadvantages
 - a. ruins core
 - b. difficult to relate data to oil-water systems

D. Centrifuge Method



1. Similar to porous disk method except centrifugal force (rather than pressure) is applied to the fluids in the core
2. Pressure (force/unit area) is computed from centrifugal force (which is related to rotational speed)
3. Saturation is computed from fluid removed (as shown in window)
4. Advantages
 - a. fast
 - b. reasonably accurate
 - c. use reservoir fluids

E. Dynamic Method



DYNAMIC CAPILLARY - PRESSURE APPARATUS
(HASSLER'S PRINCIPLE)

1. establish simultaneous steady-state flow of two fluids through core
2. measure pressures of the two fluids in core (special wetted disks) - difference is capillary pressure
3. saturation varied by regulating quantity of each fluid entering core
4. advantages
 - a. seems to simulate reservoir conditions
 - b. reservoir fluids can be used
5. Disadvantages
 - a. very tedious

F. Comparison of methods

1. diaphragm method (restored state) is considered to be most accurate, thus used as standard against which all other methods are compared
2. comparison of mercury injection data against diaphragm data
 - a. simple theory shows that capillary pressure by mercury injection should be five times greater than capillary pressure of air-water system by diaphragm method
 - b. capillary pressure scale for curves determined by mercury injection is five times greater than scale for diaphragm air-water data
 - c. these comparisons plus more complex theory indicate that the ratio between mercury injection data and diaphragm data is about 6.9 (other data indicate value between 5.8 and 7.5)

Example VIII-1

Comparison of Mercury Injection Capillary Pressure Data with Porous Diaphragm Data

- A. Calculate capillary pressure ratio,

$$\frac{P_{cAH_g}}{P_{cAW}}, \text{ for the following data}$$

$$AH_g = 480 \text{ Dynes/cm}$$

$$AW = 72 \text{ Dynes/cm}$$

$$A_{H_g} = 140^\circ$$

$$AW = 0^\circ$$

- B. Pore geometry is very complex. The curvature of the interface and pore radius are not necessarily functions of contact angles. Calculate the ratio using the relationship.

$$\frac{P_{cAH_g}}{P_{cAW}} = \frac{AH_g}{AW}$$

Solution:

$$(A) \quad \frac{P_{cAH_g}}{P_{cAW}} = \frac{AH_g \cos A_{H_g}}{AW \cos AW} = \frac{480 \cos(140^\circ)}{72 \cos(0^\circ)}$$

$$\frac{P_{cAH_g}}{P_{cAW}} = 5.1$$

$$(B) \quad \frac{P_{cAH_g}}{P_{cAW}} @ \frac{AH_g}{AW} = \frac{480}{70}$$

$$\frac{P_{cAH_g}}{P_{cAW}} = 6.9$$

Discussion:

- A. Best way to determine the relationship between mercury and air-water data is to generate capillary pressure curves for each set of data and compare directly.

Mercury Injection and Porous Diaphragm Methods

- B. For this given set of conditions, mercury injection method requires a higher displacement pressure, must adjust ratio between scales until match is obtained.
- C. Minimum irreducible wetting phase saturations are the same.
- D. Reduction in permeability results in a higher minimum irreducible wetting phase saturation. For both cases, mercury system still has higher required displacement pressure.

VIII) Converting laboratory data to reservoir conditions

$$P_{cL} = \frac{2 L \cos \theta_L}{r}$$

$$P_{cR} = \frac{2 R \cos \theta_R}{r}$$

setting $r = r$

$$r = \frac{2 L \cos \theta_L}{P_{cL}} = \frac{2 R \cos \theta_R}{P_{cR}}$$

$$P_{cR} = \left(\frac{\cos \theta_R}{\cos \theta_L} \right) \left(\frac{R}{L} \right) (P_{cL})$$

where

P_{cR} = reservoir capillary pressure, psi

P_{cL} = capillary pressure measured in laboratory, psi

L = interfacial tension measured in laboratory, dynes/cm

R = reservoir interfacial tension, dynes/cm

θ_R = reservoir contact angle, degrees

θ_L = laboratory contact angle, degrees

Example III-2

Converting Laboratory Data to Reservoir Conditions

Express reservoir capillary pressure by using laboratory data.

lab data: $A_W = 72$ dynes

$$A_W = 0^\circ$$

reservoir data: $O_W = 24$ dynes/cm

$$O_W = 20^\circ$$

Solution:

$$P_{cR} = \frac{(\cos \theta)_R}{(\cos \theta)_L} (P_{cL})$$

$$P_{cR} = \frac{24(\cos 20^\circ)}{72(\cos 0^\circ)} (P_{cL})$$

$$P_{cR} = 0.333 P_{cL}$$

IX) Determining water saturation in reservoir from capillary pressure data

- A. convert laboratory capillary pressure data to reservoir conditions
- B. calculate capillary pressure in reservoir for various heights above height at which capillary pressure is zero

$$P_c = \frac{(\quad)gh}{144 g_c}$$

in English units

$$= \rho_w - \rho_o, \text{ lb/cu ft}$$

$$g = 32 \text{ ft/sec}^2$$

$$g_c = 32 \frac{\text{lbm ft}}{\text{lbf sec}^2}$$

$$h = \text{ft}$$

$$144 = (\text{sq in})/(\text{sq ft.})$$

thus

$$P_c = \text{lbf}/(\text{sq in}), \text{ psi}$$

Example III-3

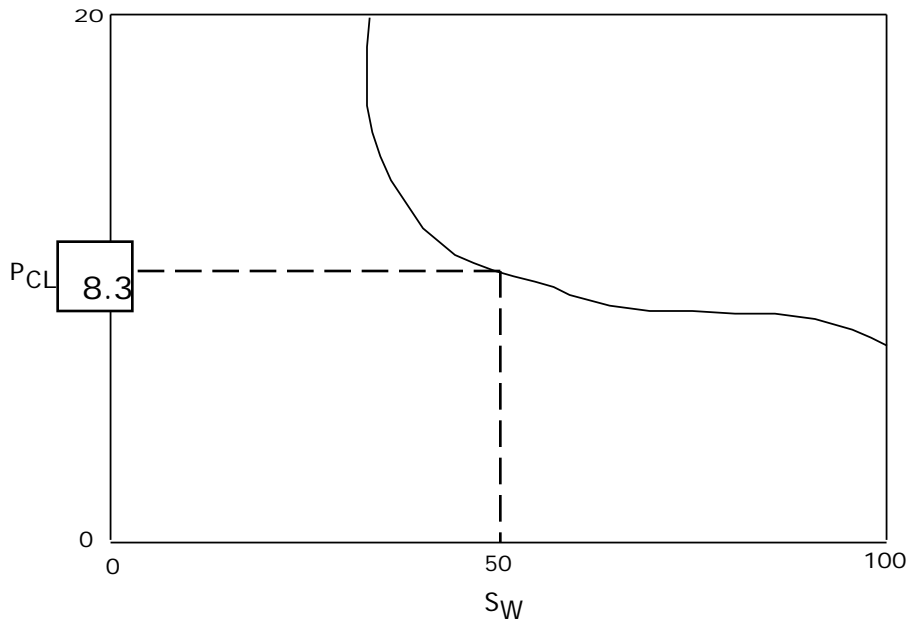
Determining Water Saturation From Capillary Pressure Curve

Given the relationship,

$P_{cR} = 0.313 P_{cL}$, use the laboratory capillary pressure curve to calculate the water saturation in the reservoir at a height of 40 ft. above the oil-water contact.

$$\rho_o = 0.85 \text{ gm/cm}^3$$

$$\rho_w = 1.0 \text{ gm/cm}^3$$



Solution:

$$P_{cR} = \frac{(\rho_w - \rho_o) h}{144}$$

$$P_{cR} = \frac{(1.0 - 0.85) \left(62.4 \frac{\text{lb}}{\text{ft}^3} \right) (40)}{144} = 2.6 \text{ psi}$$

$$P_{cL} = \frac{P_{cR}}{0.313}$$

$$P_{cL} = \frac{2.6}{0.313} = 8.3 \text{ psi}$$

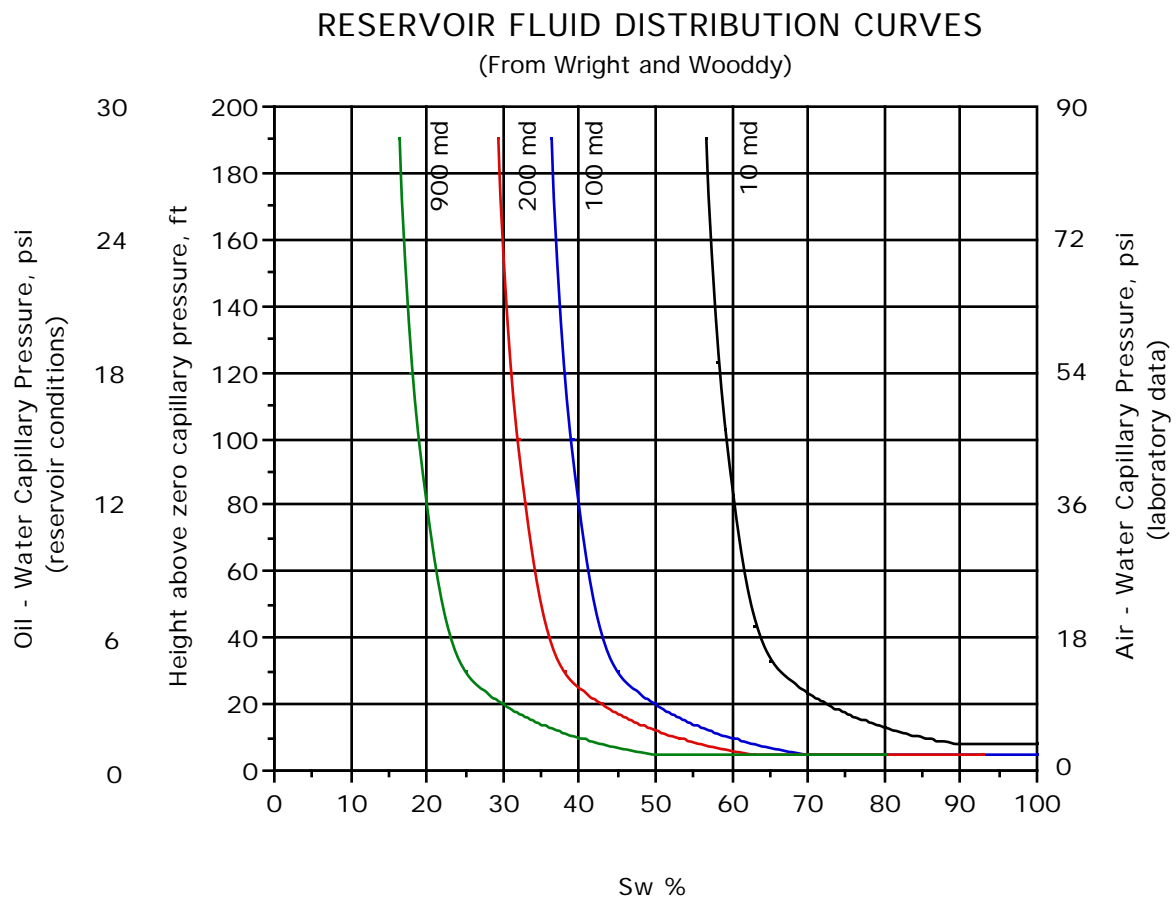
move to the right horizontally from $P_{cL} = 8.3$ psi to the capillary pressure curve. Drop vertically to the x-axis, read S_w .

$$S_w = 50\%$$

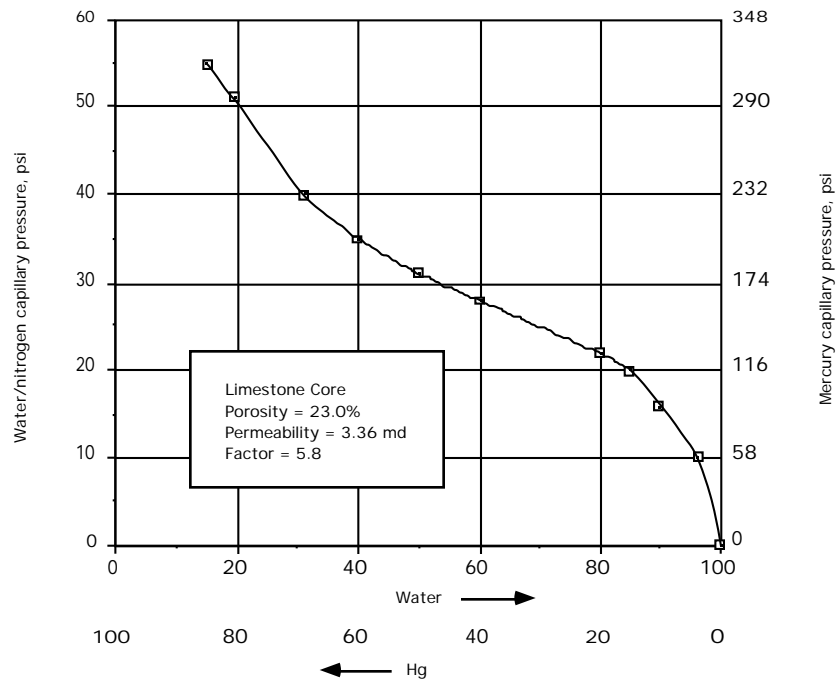
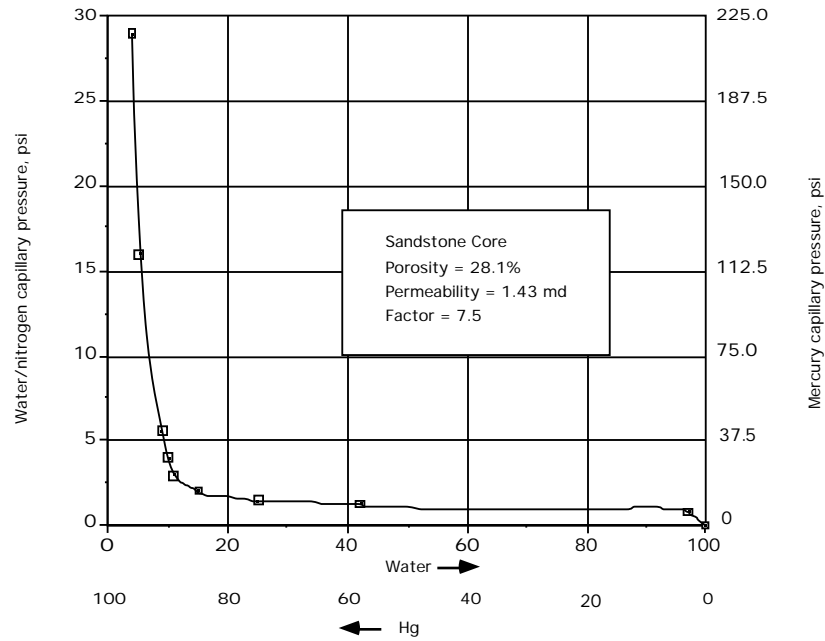
X) Capillary pressure variation

A. effect of permeability

1. displacement pressure increases as permeability decreases
2. minimum interstitial water saturation increases as permeability decreases



B. Effect of grain size distribution



1. majority of grains same size, so most pores are same size - curve (a) (well sorted)
2. large range in grain and pore sizes - curve (b) (poorly sorted)

XI) Averaging capillary pressure data

J-function

$$J(S_w) = \frac{P_c}{\cos} \left(\frac{k}{\mu} \right)^{1/2}$$

attempt to convert all capillary pressure data to a universal curve

universal curve impossible to generate due to wide range of differences existing in reservoirs

concept useful for given rock type from given reservoir

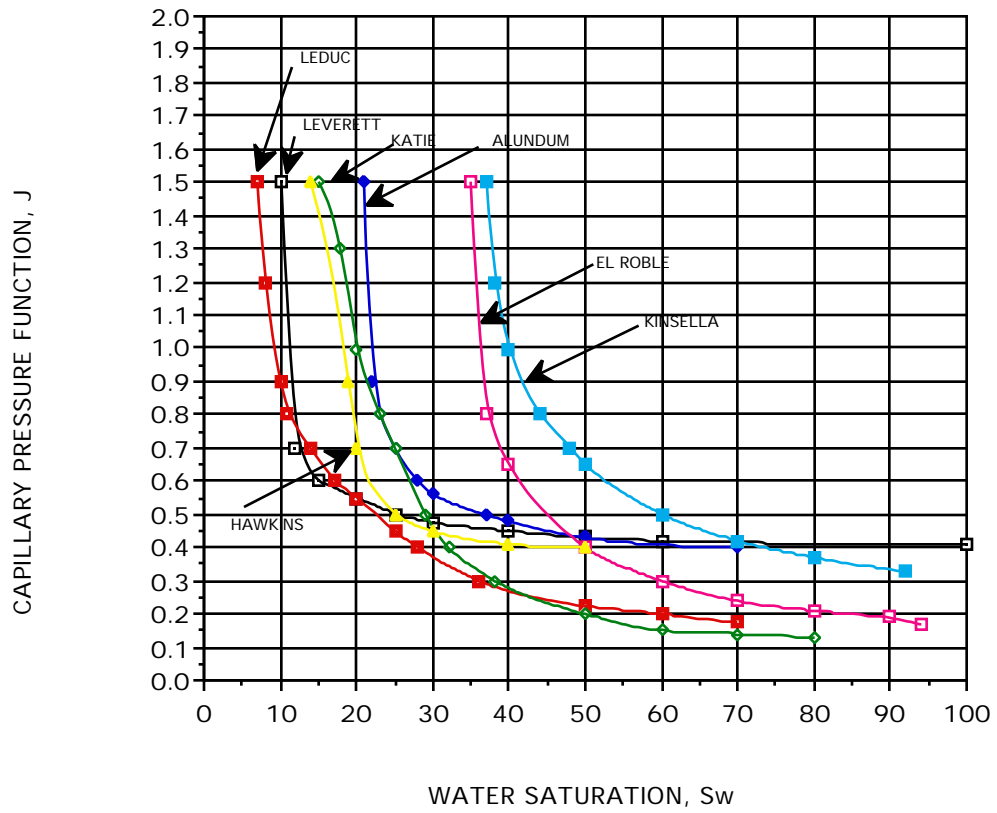
where

$$\begin{aligned} P_c &= \text{dyne/(sq cm)} \\ &= \text{dyne/cm} \\ k &= \text{(sq cm)} \\ &= \text{fraction} \end{aligned}$$

or can use any units as long as you are consistent

CAPILLARY RETENTION CURVES.

(From Rose and Bruce.)



Reservoir	Formation
Hawkins	Woodbine
El Roble	Moreno
Kinsella	Viking
Katie	Deese
Leduc	Devonian
Alundum	(consolidated)
Leverett	(unconsolidated)

Capillary Pressure Problem 1

1. A glass tube is placed vertically in a beaker of water. The interfacial tension between the air and water is 72 dynes/cm and the contact angle is 0 degree.

Calculate:

- a. the capillary rise of water in the tube if the radius of the tube is 0.01 centimeters.
 - b. what is the difference in pressure in psi across the air-water interface in the tube.
2. The displacement pressure for a water saturated porcelain plate is 55 psi of air. What is the diameter in inches of the largest pore in the porcelain plate? Assume 72 dynes/cm and 0 degrees.

Solution:

(1) $AW = 72 \text{ dynes/cm}$

$W = 1 \text{ gm/cm}^3$

$g = 980 \text{ dynes/gm}$

$= 0^\circ$

- (a) capillary rise of water if radius is .01 cm

$$h = \frac{2 AW \cos}{r g} = \frac{2(72) \cos 0^\circ}{(.01)(1.0)(980)}$$

$h = 14.69 \text{ cm}$

- (b) pressure drop in psi across interface

$P_c = P_a - P_w = wgh = (1.0)(980)(14.69)$

$P_c = 0.0142 \text{ atm} \left(\frac{14.696 \text{ psi}}{\text{atm}} \right)$

$P_c = 0.209 \text{ psi}$

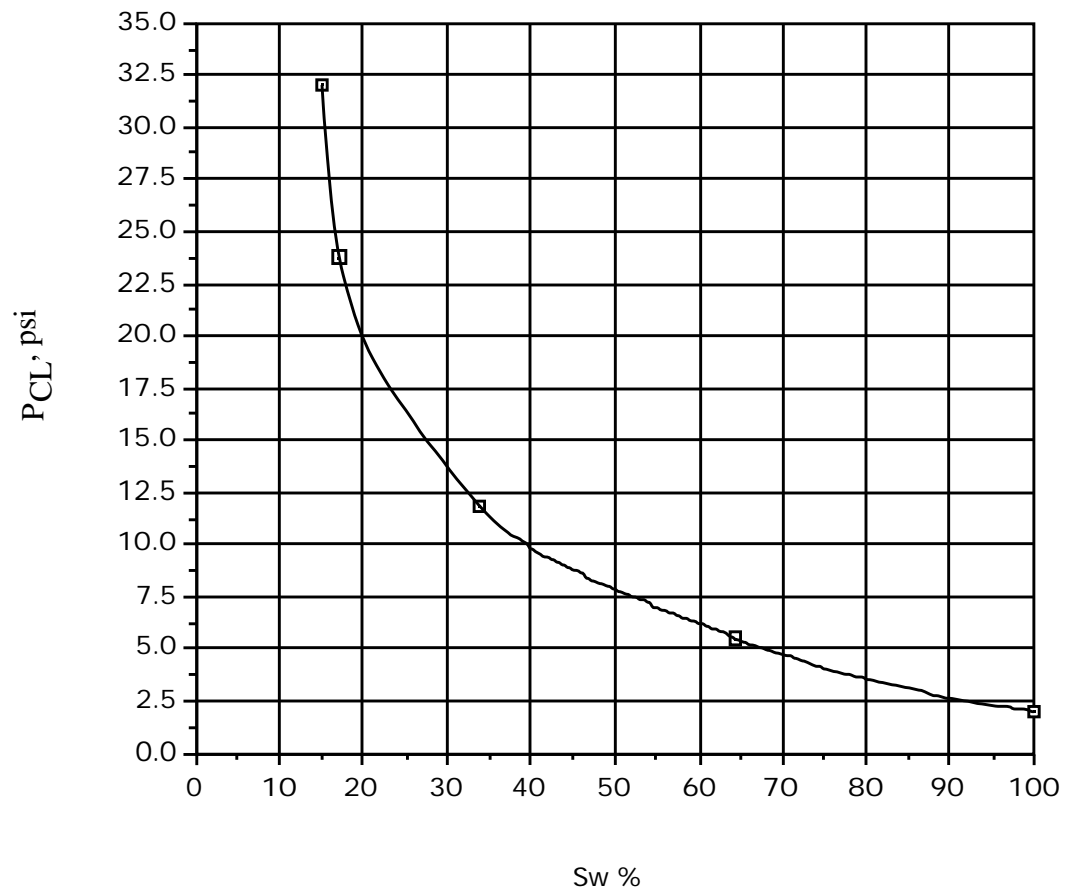
$$\begin{aligned}
(2) \quad P_c &= \frac{2 AW \cos}{r} \\
P_c &= 55 \text{ psi} \\
P_c &= 55 \text{ psi} \left(\frac{\text{atm}}{14.696 \text{ psi}} \right) \left(\frac{1.0133 \times 10^6 \text{ dynes/cm}^2}{\text{atm}} \right) \\
&= 3.792 \times 10^6 \text{ dynes/cm}^2 \\
r &= \frac{2 AW \cos}{P_c} \\
r &= \frac{2(72) \cos 0^\circ}{3.792 \times 10^6} = 3.797 \times 10^{-5} \text{ cm} \left(\frac{\text{in}}{2.54 \text{ cm}} \right) \\
r &= 1.495 \times 10^{-5} \text{ in} \\
d &= 2.99 \times 10^{-5} \text{ in}
\end{aligned}$$

Capillary Pressure Problem 2

Given the information below and graph of P_{cL} vs. wetting phase saturation (S_w), construct the curves for P_{cR} , h in reservoir, and J-function vs. S_w . Water is the wetting phase in both the laboratory and the reservoir.

fluids	<u>lab</u> air-water	<u>res</u> oil-water
	0°	25°
	60 dyne/cm	20 dyne/cm
wet	1.0 gm/cm^3	1.1 gm/cm^3
non-wet	0 gm/cm^3	0.863 gm/cm^3
k	37 md	variable
	16%	variable

$$J = \frac{P_c (k/h)^{1/2}}{\cos \theta}$$



Solution:

$$\begin{aligned}
 (1) \quad P_{cR} &= \frac{R \cos R}{L \cos L} P_{cL} \\
 &= \frac{(20) \cos 25}{(60) \cos 0} P_{cL} \\
 P_{cR} &= 0.302 P_{cL}
 \end{aligned}$$

$$(2) P_{cR} = \frac{h_R (w - o)}{144}$$

$$= \frac{h_R (1.1 - .863)(62.4)}{144}$$

$$P_{cR} = .103 h_R$$

$$h_R = 9.74 P_{cR}$$

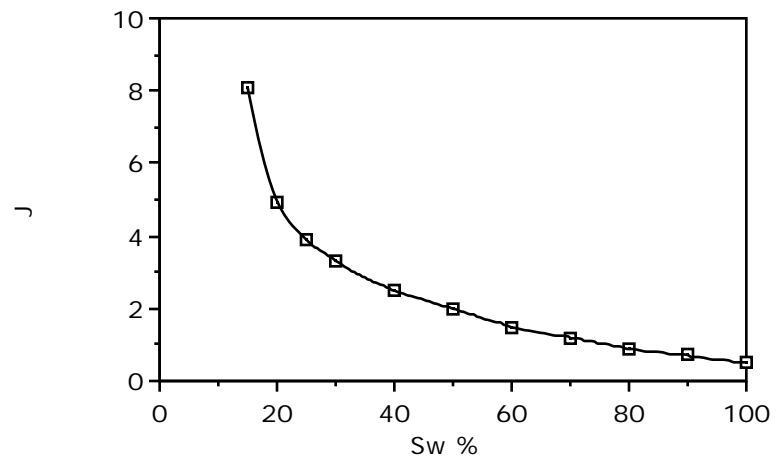
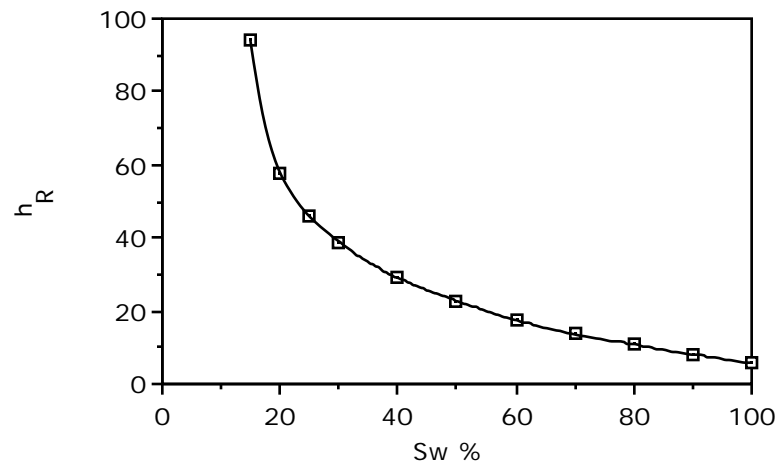
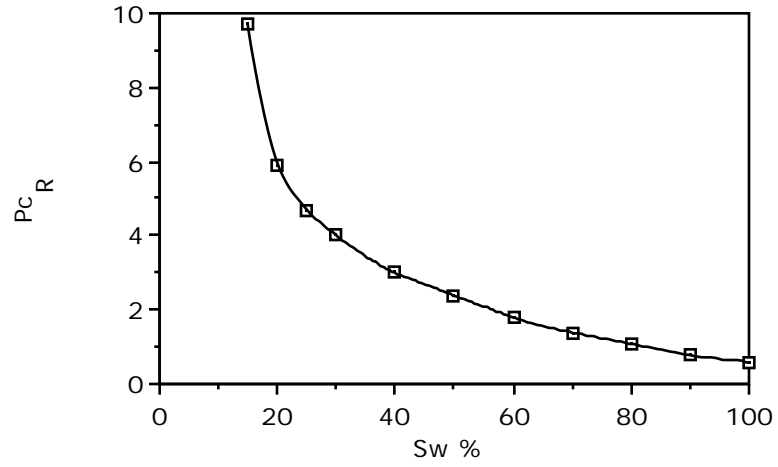
$$(3) J = \frac{P_c}{\cos \left(\frac{k}{L} \right)^{1/2}}$$

$$= \frac{P_{cL}}{AW \cos L \left(\frac{k}{L} \right)_L^{1/2}}$$

$$= \frac{P_{cL}}{(60) \cos 0^\circ \left(\frac{37}{.16} \right)^{1/2}}$$

$$J = .253 P_{cL}$$

<u>S_w</u> <u>%</u>	<u>P_{cL}</u> <u>psi</u>	<u>P_{cR}</u> <u>psi</u>	<u>h_R</u> <u>ft</u>	<u>J</u> <u>assorted</u>
15	32	9.7	94.1	8.1
20	19.5	5.9	57.4	4.9
25	15.6	4.7	45.9	3.9
30	13.2	4.0	38.8	3.3
40	9.9	3.0	29.1	2.5
50	7.8	2.4	22.9	2.0
60	6.0	1.8	17.6	1.5
70	4.7	1.4	13.8	1.2
80	3.7	1.1	10.9	0.9
90	2.8	0.8	8.2	0.7
100	2	0.6	5.9	0.5



IV. FLUID SATURATIONS

- I) Basic concepts of hydrocarbon accumulation
 - A. Initially, water filled 100% of pore space
 - B. Hydrocarbons migrate up dip into traps
 - C. Hydrocarbons distributed by capillary forces and gravity
 - D. Connate water saturation remains in hydrocarbon zone

- II) Methods for determining fluid saturations
 - A. Core analysis (direct method)
 - 1. factors affecting fluid saturations
 - a. flushing by mud filtrate
 - 1) differential pressure forces mud filtrate into formation
$$P_h > P_{res}$$
 - 2) for water base mud, filtrate displaces formation water and oil from the area around the well (saturations likely change)
 - 3) for oil base mud, filtrate will be oil; saturations may or may not change.

Example: Effects of flushing by mud filtrates

Coring with water base mud

Oil zone at minimum interstitial water saturation:

	<u>flushing by bit trip to surface</u>	<u>sat at surface</u> <u>compared to res</u>
S_w		? probably
S_o		
S_g	-	

Gas zone at minimum interstitial water saturation:

	<u>flushing by bit trip to surface</u>	<u>sat at surface</u> <u>compared to res</u>
S_w		?
S_o	-	-
S_g		?

Water zone:

	<u>flushing by bit trip to surface</u>	<u>sat at surface</u> <u>compared to res</u>
S_w	-	
S_o	-	-
S_g	-	

Coring With Oil Base Mud

Oil zone at minimum interstitial water saturation:

	<u>flushing by bit trip to surface</u>		<u>sat at surface compared to res</u>
S_w	-	-	-
S_o	-		
S_g	-		

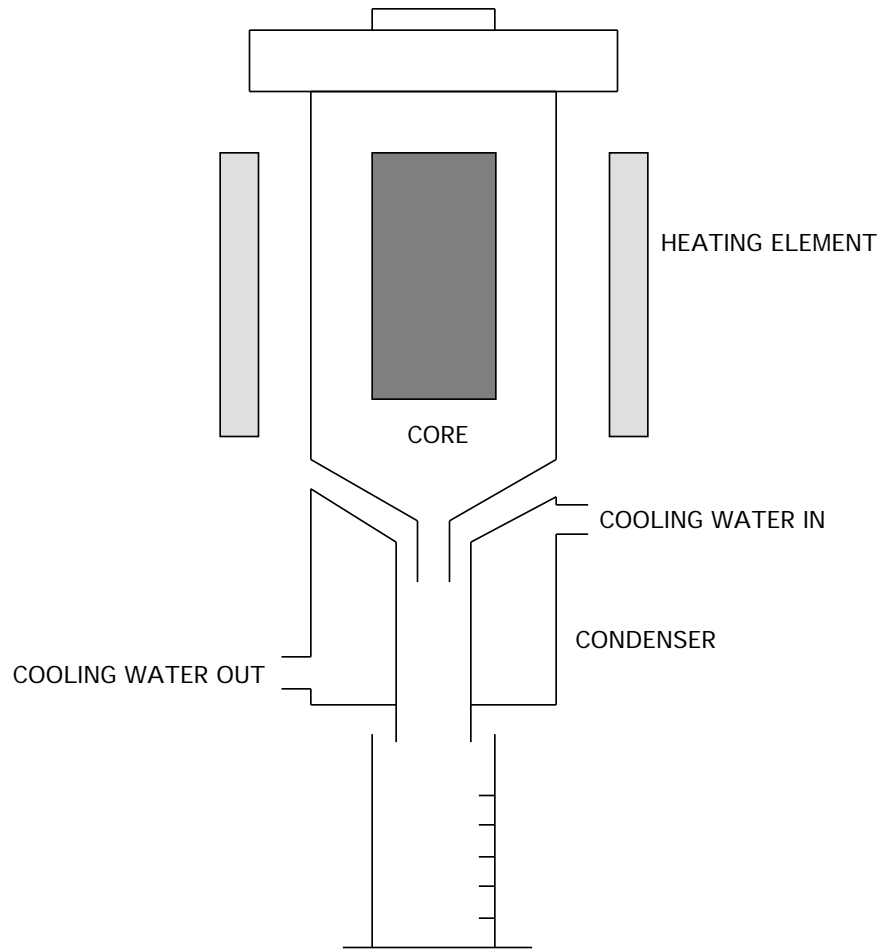
Gas zone at minimum interstitial water saturation:

	<u>flushing by bit trip to surface</u>		<u>sat at surface compared to res</u>
S_w	-	-	-
S_o			
S_g			

Water zone:

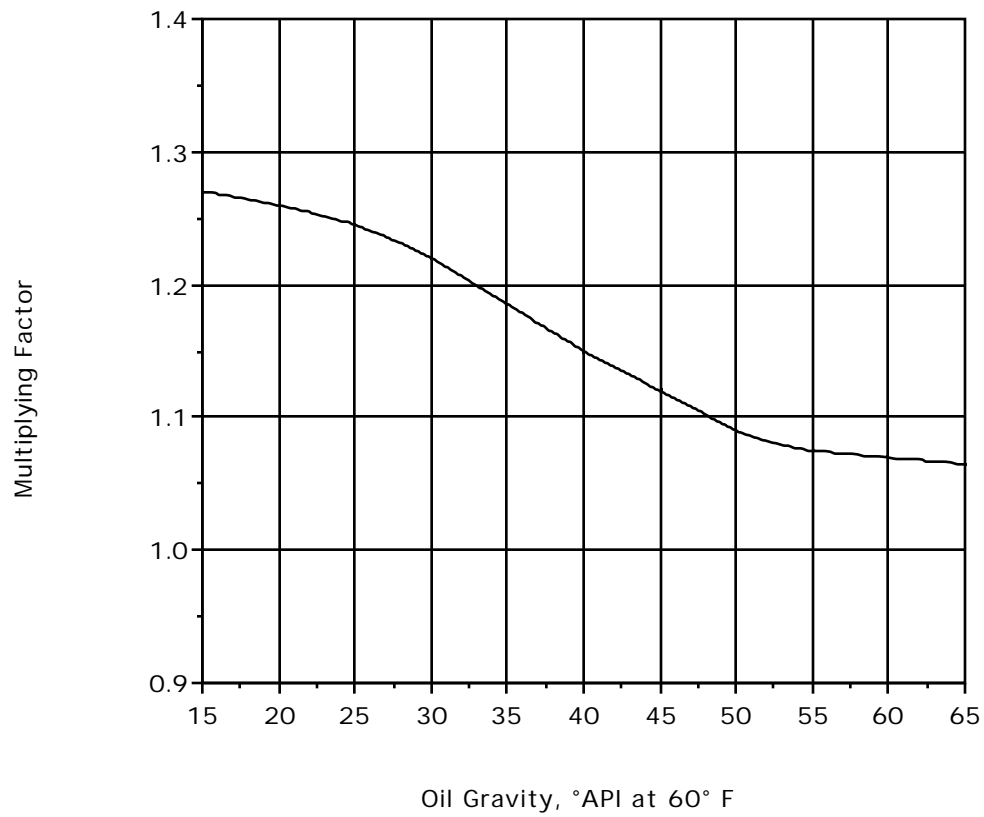
	<u>flushing by bit trip to surface</u>		<u>sat at surface compared to res</u>
S_w			
S_o			
S_g	-		

- b. bringing core to surface
 - 1) reduction in hydrostatic pressure causes gas to come out of solution
 - 2) gas displaces oil and water causing saturations to change
- 2. laboratory methods
 - a. evaporation using retort distillation apparatus



- 1) process
 - a) heat small sample of rock
 - b) oil and water vaporize, then condense in graduated cylinder
 - c) record volumes of oil and water
 - d) correct quantity of oil

For converting distilled oil volume to oil volume originally in a sample, multiply oil volume recovered by factor corresponding to gravity of oil in core



e) determine saturations

$$S_w = \frac{V_w}{V_p} \quad S_o = \frac{V_o}{V_p}$$

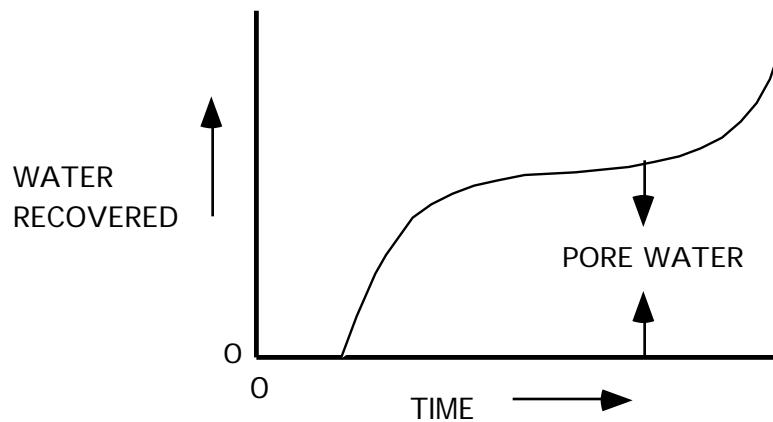
$$S_g = 1 - S_o - S_w$$

where

- S_w = water saturation, fraction
- S_o = oil saturation, fraction
- S_g = gas saturation, fraction
- V_p = pore volume, cc
- V_w = volume of water collected, cc
- V_o = volume of oil collected, cc

2) disadvantages of retort process

a) must obtain temperature of 1000-1100°F to vaporize oil, water of crystallization from clays also vaporizes causing increase in water recovery

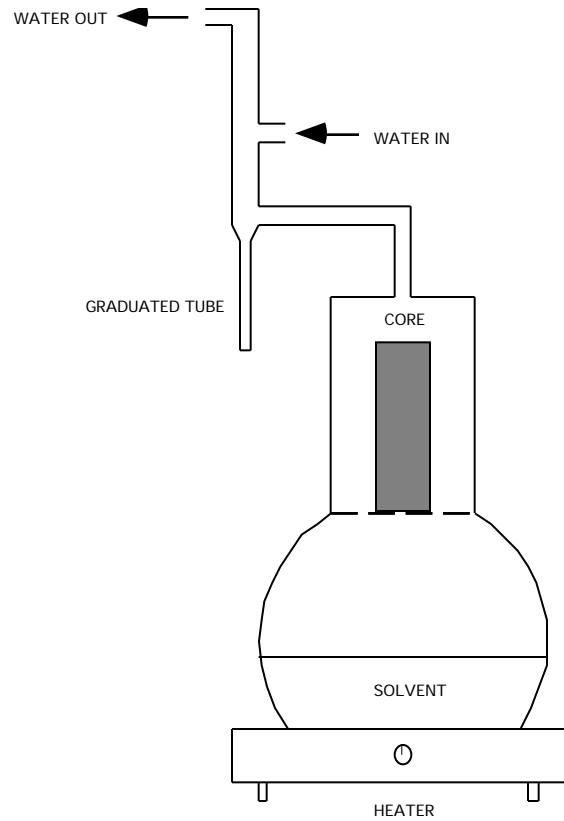


b) at high temperatures, oil will crack and coke. (change in hydrocarbon molecules) amount of recoverable liquid decreases.

c) core sample ruined

- 3) advantages of retort process
 - a) short testing time required
 - b) acceptable results obtained

b. leaching using solvent extraction apparatus



- 1) process
 - a) weigh sample to be extracted
 - b) heat applied to system causes water from core to vaporize
 - c) solvent leaches hydrocarbons from core

- d) water condenses, collects in trap. Record final water volume
- e) reweigh core sample
- f) determine volume of oil in sample

$$V_o = \frac{(W_i - W_{dry}) - (V_w) w}{o}$$

where:

W_i = weight of core sample after leaching

W_{dry} = weight of core sample after leaching

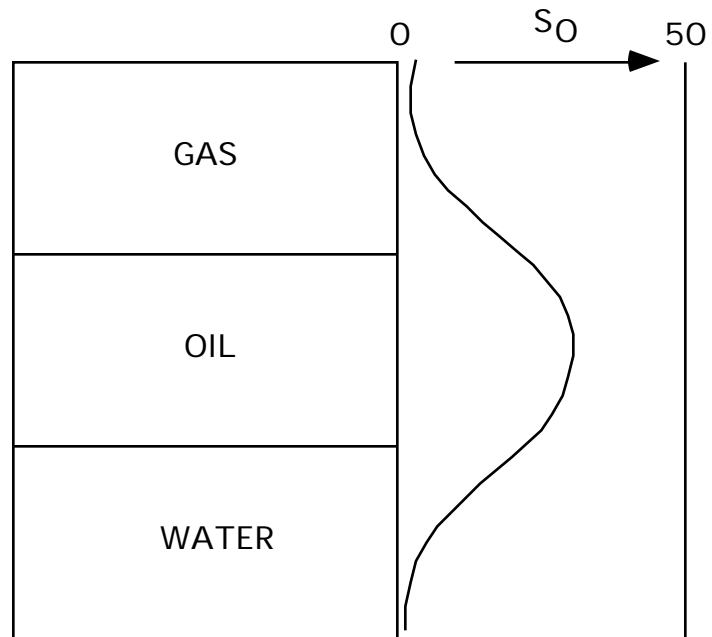
$$S_w = \frac{V_w}{V_p} \qquad S_o = \frac{V_o}{V_p}$$

- 2) disadvantages of leaching
 - a) process is slow
 - b) volume of oil must be calculated
- 3) advantages of leaching
 - a) very accurate water saturation value obtained
 - b) heating does not remove water of crystallization
 - c) sample can be used for future analysis

3. uses of core determined fluid saturation

a. cores cut with water base mud

- 1) presence of oil in formation
- 2) determination of oil/water contact
- 3) determination of gas/oil contact



$S_O = 0$ in gas zone

$S_O = 15\%$ in oil zone

$0 < S_O < S_{OR}$ in water zone

S_{OR} = residual oil saturation

- b. cores cut with oil base mud ("natural state" cores)
 - 1) minimum interstitial water saturation
 - 2) hydrocarbon saturation
 - 3) oil/water contact

- B. Capillary pressure measurements (discussed in Chapter VIII)

- C. Electric logs

Example IV-1

You want to analyze a core sample containing oil, water and gas.

$$V_b \text{ bulk volume} = 95 \text{ cm}^3$$

$$W_t \text{ initial} = 216.7 \text{ gm}$$

the sample was evacuated and the gas space was saturated with water ($\rho_w = 1 \text{ gm/cm}^3$)

$$W_t \text{ new} = 219.7 \text{ gm}$$

the water with in the sample is removed and collected

$$V_w \text{ removed} = 13.0 \text{ cm}^3$$

the oil is extracted and the sample is dried

$$W_t \text{ dry} = 199.5 \text{ gm}$$

calculate:

- (1) porosity
- (2) water saturation
- (3) oil saturation assuming 35° API
- (4) gas saturation
- (5) matrix density
- (6) lithology

Solution:

$$\text{gas vol.} = 219.7 - 216.7 ; \quad V_g = 3 \text{ cc}$$

$$\text{water vol.} = 13 - 3 ; \quad V_w = 10 \text{ cc}$$

$$W_t \text{ fluids} = 219.7 - 199.5 = 20.2 \text{ gm}$$

$$W_t \text{ oil} = 20.2 - 10 - 3 = 7.2 \text{ gm}$$

$$\rho = \frac{141.5}{131.5 + 35^\circ \text{API}} = 0.85 \text{ gm/cc}$$

$$V_o = 7.2/0.85 = 8.49 \text{ cc}$$

$$V_p = 8.49 + 3 + 10 = 21.47 \text{ cc}$$

$$= 21.47/95 = 22.6\%$$

$$S_w = 10/21.47 = 46.57\%$$

$$S_o = 8.49/21.47 = 39.46\%$$

$$S_g = 3/21.47 = 13.97\%$$

$$m = 199.5/(95-21.47) = 2.71 \text{ gm/cc}$$

lithology = limestone

Example IV-2

A core sample was brought into the laboratory for analysis. 70 gm of the core sample were placed in a mercury pump and found to have 0.71 cc of gas volume. 80 gm of the core sample was placed in a retort and found to contain 4.5 cc of oil and 2.8 cc of water. A piece of the original sample weighing 105 gm was placed in a pycnometer and found to have a bulk volume of 45.7 cc.

(Assume $w = 1.0$ gm/cc and 35° API oil)

calculate:

- (1) porosity
- (2) water saturation
- (3) oil saturation
- (4) gas saturation
- (5) lithology

Solution:

$$V_g = \frac{.71 \text{ cc}}{70 \text{ gm}} (100 \text{ gm}) = 1.014 \text{ cc}$$

$$V_o = \frac{4.5 \text{ cc}}{80 \text{ gm}} (100 \text{ gm}) = 5.63 \text{ cc}$$

$$V_w = \frac{2.8 \text{ cc}}{80 \text{ gm}} (100 \text{ gm}) = 3.50 \text{ cc}$$

$$V_b = \frac{45.7 \text{ cc}}{105 \text{ gm}} (100 \text{ gm}) = 43.52 \text{ cc}$$

$$W_t \text{ matrix} = 100 - 5.63(.85) - 3.5(1.0) = 91.71 \text{ gm}$$

$$V_m = 43.52 - 1.014 - 5.63 - 3.50 = 33.37 \text{ cc}$$

$$V_p = 1.014 + 5.63 + 3.50 = 10.14 \text{ cc}$$

$$= \frac{10.14}{43.52} = 23.31\%$$

$$\begin{aligned} S_w &= 3.50/10.14 = 34.5\% \\ S_o &= 5.63/10.14 = 55.5\% \\ S_g &= 1.014/10.14 = 10\% \\ m &= (91.71/33.38) = 2.75 \end{aligned}$$

Fluid Saturation Problem 1

Calculate porosity, water, oil, and gas saturations, and lithology from the following core analysis data.

How should the calculated saturations compare with the fluid saturations in the reservoir?

Oil well core with water base mud

initial weight of saturated core = 86.4 gm

after gas space was saturated with water, weight of core = 87.95 gm

weight of core immersed in water = 48.95 gm

core was extracted with water recovery being 7.12 cc

after drying core in oven, core weighed 79.17 gm

assume $\rho_w = 1.0 \text{ gm/cc}$

oil gravity = 40° API

Solution:

$$\rho_o = \frac{141.5}{131.5 + \text{°API}}$$

$$\rho_o = \frac{141.5}{131.5 + 40^\circ} = 0.825$$

$$\rho_o = 0.825 \text{ gm/cc}$$

$$(1) \quad \rho_p = \frac{V_p}{V_b}$$

$$V_p = V_w + V_o + V_g$$

$$W_o = W_{\text{sat}} - V_w \rho_w - W_{\text{dry}}$$

$$= 87.95 - 7.12(1.0) - 79.17$$

$$W_o = 1.66 \text{ gm}$$

$$V_o = \frac{W_o}{\rho}$$

$$V_o = \frac{1.66 \text{ gm}}{0.825 \text{ gm/cc}} = 2.01 \text{ cc}$$

$$V_w = V_{w_{\text{rec}}} - (W_{\text{sat}} - W_j) / (\rho_w)$$

$$= 7.12 - (87.95 - 86.4) / (1.0)$$

$$V_w = 5.57 \text{ cc}$$

$$V_g = 1.55 \text{ cc}$$

$$V_p = 5.57 + 2.01 + 1.55$$

$$V_p = 9.13 \text{ cc}$$

$$V_b = \frac{W_{\text{sat}} - W_{\text{imm}}}{\rho}$$

$$V_b = \frac{(87.95 - 48.95) \text{ gm}}{1 \text{ gm/cc}} = 39.0 \text{ cc}$$

$$= \frac{9.13}{39.0} = 23.4\%$$

$$(2) \quad S_w = \frac{V_w}{V_p}$$

$$S_w = \frac{5.57 \text{ cc}}{9.13 \text{ cc}} = 61.0\%$$

$$S_o = \frac{V_o}{V_p}$$

$$S_o = \frac{2.01 \text{ cc}}{9.13 \text{ cc}} = 22.0\%$$

$$S_g = \frac{V_g}{V_p}$$

$$S_g = \frac{1.55 \text{ cc}}{9.13 \text{ cc}} = 17.0\%$$

$$(3) \quad V_m = V_b - V_p$$

$$V_m = 39 - 9.13 = 29.87 \text{ cc}$$

$$m = \frac{W_{\text{dry}}}{V_m}$$

$$m = 79.17 \text{ gm} / 29.87 \text{ cc} = 2.65 \frac{\text{gm}}{\text{cc}}$$

.. lithology is sandstone

- (4) water saturation at surface will probably be greater than reservoir water saturation

oil saturation at surface will be less than reservoir oil saturation

gas saturation at surface will be greater than reservoir gas saturation

Fluid Saturation Problem 2

Calculate porosity, water saturation, oil saturation, gas saturation, and lithology from the following core analysis data.

How should the saturations you have calculated compare with the fluid saturations in the reservoir?

Oil well core cut with an oil base mud

Sample 1 weighed 130 gm and was found to have a bulk volume of 51.72 cc

Sample 2 weighed 86.71 gm, and from the retort method was found to contain 1.90 cc of water and 0.87 cc of oil

Sample 3 weighed 50 gm and contained 0.40 cc of gas space

assume $\rho_w = 1.0 \text{ gm/cc}$

oil gravity = 40° API

Solution: $\rho_o = \frac{141.5}{131.5 + \text{°API}}$

$$\rho_o = \frac{141.5}{131.5 + 40^\circ} = 0.825$$

$$\rho_o = 0.825 \text{ gm/cc}$$

$$(1) \quad V_p = \frac{V_p}{V_b}$$

$$V_p = V_o + V_w + V_g$$

$$V_o = \frac{0.87 \text{ cc}}{86.71 \text{ gm}} \times 100 = \frac{1.00 \text{ cc}}{100 \text{ gm}}$$

$$V_w = \frac{1.90 \text{ cc}}{86.71 \text{ gm}} \times 100 = \frac{2.19 \text{ cc}}{100 \text{ gm}}$$

$$V_g = \frac{0.40 \text{ cc}}{50 \text{ gm}} \times 100 = \frac{0.80 \text{ cc}}{100 \text{ gm}}$$

$$V_p = (1.00 + 2.19 + 0.80) \text{ cc}/100 \text{ gm}$$

$$V_p = 3.99 \text{ cc/100 gm}$$

$$V_b = \frac{51.72 \text{ cc}}{130 \text{ gm}} \times 100 = \frac{39.78 \text{ cc}}{100 \text{ gm}}$$

$$= \frac{3.99/100}{39.78/100} \times 100 = 10\%$$

$$(2) \quad S_w = \frac{V_w}{V_p} \times 100$$

$$= \frac{2.19 \text{ cc}}{3.99 \text{ cc}} \times 100$$

$$S_w = 54.8\%$$

$$(3) \quad S_o = \frac{V_o}{V_p} \times 100$$

$$= \frac{1.0 \text{ cc}}{3.99 \text{ cc}} \times 100$$

$$S_o = 25.1\%$$

$$(4) \quad S_g = \frac{V_g}{V_p} \times 100$$

$$= \frac{0.80 \text{ cc}}{3.99 \text{ cc}} \times 100$$

$$S_g = 20.1\%$$

$$(5) \quad V_m = V_b - V_p$$

$$= 39.78 - 3.99$$

$$V_m = 35.79 \text{ cc/100 gm}$$

$$W_m = (W_{\text{sat}} - oV_o - wV_w)/W_{\text{sat}}$$

$$= \frac{(86.71 - 0.825(0.87) - 1.0(1.90))}{86.71}$$

$$W_m = 97 \text{ gm/100 gm}$$

$$m = \frac{W_m}{V_m}$$

$$m = \frac{97 \text{ gm}/100 \text{ gm}}{35.79 \text{ cc}/100 \text{ gm}}$$

$$m = 2.71 \text{ gm/cc}$$

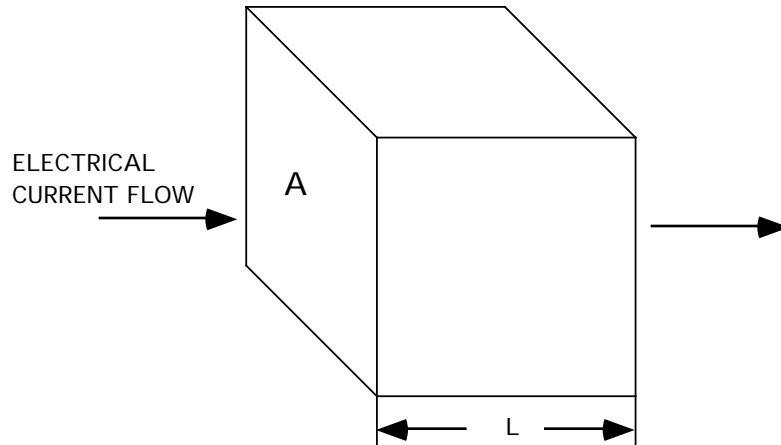
. . lithology - limestone

- (6) water saturations should be fairly close in value
oil saturation will be less than reservoir oil saturation
gas saturation will be greater than reservoir gas saturation

V. ELECTRICAL PROPERTIES OF ROCK-FLUID SYSTEMS

I) Electrical conductivity of fluid saturated rock

A. Definition of resistivity



given a box of length (L) and cross-sectional area (A) completely filled with brine of resistivity (R_w)

the resistance of the brine in the box to current flow may be expressed as

$$r = R_w \frac{L}{A}$$

r = resistance - ohm

R_w = resistivity - ohm meters

L = length - meters

A = area - (meters)²

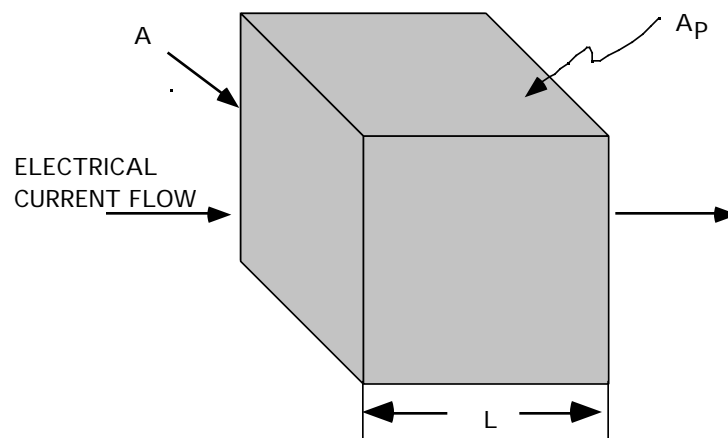
B. Nonconductors of electricity

1. oil
2. gas
3. pure water
4. minerals
5. rock fragments

C. Conductors of electricity

1. water with dissolved salts conducts electricity (low resistance)
2. clay

D. Development of saturation equation (ignore clay)



1. the electrical current flows through the water (brine)
 - a. the area available for current flow is the cross-sectional area of the pores.

$$A_p < A$$

- b. the path through the pores is L_p .

$$L_p > L$$

2. resistance to electrical flow through the porous media is equal to the resistance of a container of area A_p and length L_p filled with water (brine)

$$r = \frac{R_w L_p}{A_p}, \text{ water filled cube}$$

$$r = \frac{R_o L}{A}, \text{ porous media}$$

thus

$$R_o = \frac{R_w A L_p}{A_p L}$$

where r = resistance of rock cube with pores filled with brine, ohm

R_w = formation brine resistivity, ohm-m
(from water sample or SP log)

R_o = resistivity of formation 100% saturated
with brine of resistivity, R_w , ohm-m

A_p = cross-sectional area available for current
flow, m^2

L_p = actual path length ion (current) must
travel through rock, m

A = cross-sectional area of porous media,
 m^2

L = length of porous media, m

3. Since

$$\frac{A_p}{A} \text{ porosity,}$$

and

$$\frac{L_p}{L} \text{ tortuosity, a measure of rock cementation.}$$

then

$$R_o = \frac{R_w A L_p}{A_p L}$$

becomes

$$R_o = f(R_w, \text{ , tortuosity})$$

E. Electrical formation resistivity factor, F

1. the equation for resistivity of a formation 100% saturated with a brine of resistivity of R_w

$$R_o = f(R_w, \text{ , tortuosity})$$

2. can be written as

$$R_o = F R_w$$

where F is the electrical formation resistivity factor

$$F = \frac{R_o}{R_w}$$

3. cementation factor, m

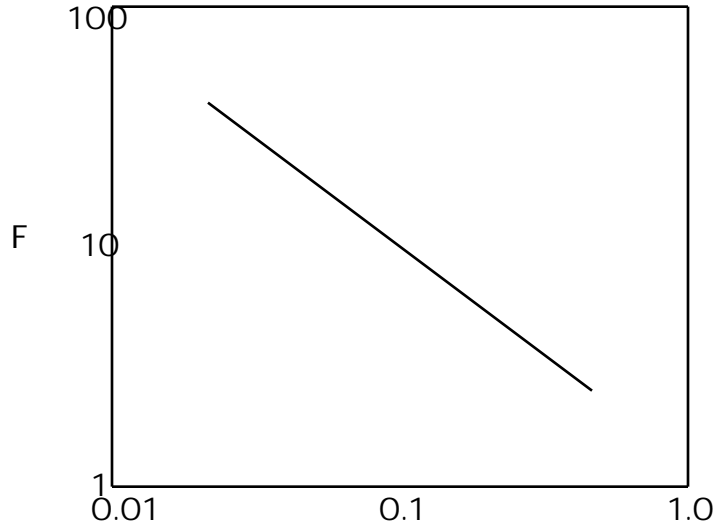
- a. it has been found experimentally that the equation for F takes the form

$$F = C^{-m}$$

where C is a constant
m is the cementation factor

- b. thus

$$\log F = \log C - m \log$$



when intercept = C

slope = -m, the cementation factor

4. commonly used equation for electrical formation resistivity factor

a. Archie's Equation

$$F = \frac{C}{\phi^m}$$

b. Humble Equation

$$F = 0.62 \phi^{-2.15}$$

(best suited for sandstones)

Cementation Factor (m) and Lithology

Lithology	m values
Unconsolidated rocks (loose sands, oolitic limestones)	1.3
Very slightly cemented (Gulf Coast type of sand, except Wilcox)	1.4-1.5
Slightly cemented (most sands with 20% porosity or more)	1.6-1.7
Moderately cemented (highly consolidated sands of 15% porosity or less)	1.8-1.9
Highly cemented (low-porosity sands, quartzite, limestone, dolomite)	2.0-2.2

Example V-1

Determine the porosity for a sandstone using Archie's and Humble equation .
The formation water's resistivity was 0.5 ohm-meters. The formation rock 100% saturated with this water was 21.05 ohm-meters.

Which of the two equations will give the most reasonable answer?

Solution:

$$F = 21.05 / 0.5 = 42.1$$

Archie's: $F = \frac{1}{m}$

$m = 2.0$ for sandstone

$$\frac{1}{2} = 1/F$$
$$= \sqrt{\frac{1}{42.1}}$$
$$= 15.41\%$$

Humble: $F = 0.62 / \frac{1}{2.15}$

$$2.15 = 0.62/F$$
$$= \frac{2.15 \sqrt{0.62}}{\sqrt{42.1}}$$
$$= 14.06\%$$

The Humble equation was developed for sandstone.

F. Resistivity Index, I, and Saturation Exponent, n

1. definition of resistivity index

$$I = \frac{R_t}{R_o}$$

where R_o = resistivity of formation 100% saturated with water (brine) of resistivity R_w , ohm-m

R_t = resistivity of formation with water (brine) saturation less than 100%, ohm-m

2. it has been found experimentally that

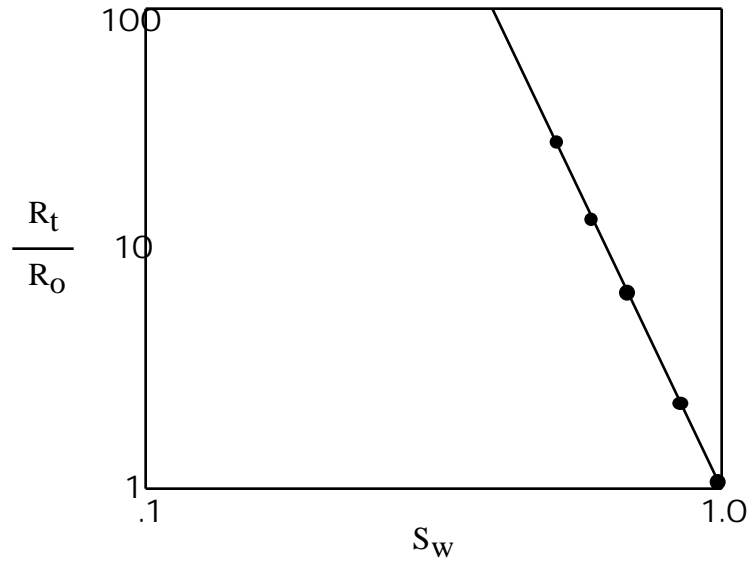
$$S_w = I^{-\frac{1}{n}} = \left[\frac{R_t}{R_o} \right]^{-\frac{1}{n}}$$

where n is the saturation exponent 2.0

3. rearrange

$$S_w^{-n} = \frac{R_t}{R_o}$$

$$-n \log S_w = \log \frac{R_t}{R_o}$$



slope = -n, when n is the saturation exponent

NOTE:

$$\text{slope} = \frac{\log y_1 - \log y_2}{\log x_1 - \log x_2}$$

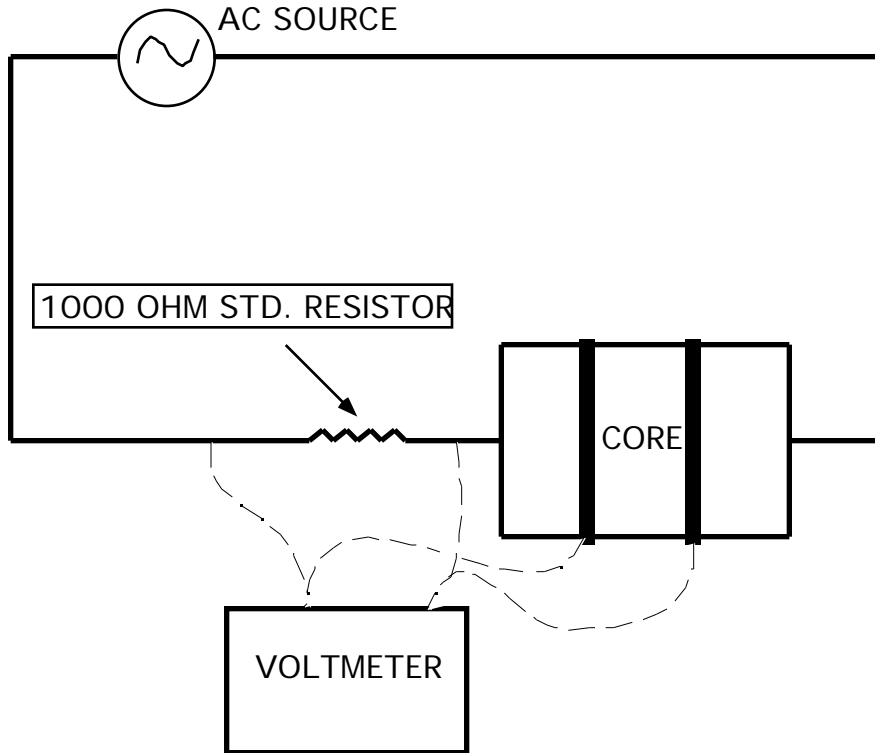
II) Use of Electrical Formation Resistivity Factor, Cementation Factor, and Saturation Exponent

- A. obtain porosity, ϕ , from electric log or core analysis
- B. $F = C^{-m}$ (usually use Archie or Humble equation)
- C. obtain water resistivity, R_w , from water sample or electric log
- D. $R_o = F R_w$
- E. convert R_t from electric log to water saturation

$$S_w = \left[\frac{R_o}{R_t} \right]^{\frac{1}{n}}$$

III) Laboratory measurement of electrical properties of rock

A. Apparatus



B. Calculations

1. resistance of core

$$E = Ir$$

where: E = voltage, volts
I = current, amperes
r = resistance, ohms

$$r_{\text{core}} = \frac{E}{I}$$

2. resistivity of core

$$R_{\text{core}} = \frac{r_{\text{core}}A}{L}$$

substituting $r = \frac{E}{I}$ into the equation

$$R_{\text{core}} = \frac{EA}{IL}$$

C. Procedure

1. determine resistance of core
 - a. set desired current from AC source, low current preferred so core does not heat up.
 - b. record voltage from voltmeter
2. determine resistivity of core
 - a. for the first test completely saturate core with brine $S_w = 100\%$, $R_{\text{core}} = R_o$
 - b. for next test, desaturate core by 15-20%, until $S_w < 100\%$
 $R_{\text{core}} = R_t$
 - c. repeat tests until $S_w = S_{\text{wir}}$

where

S_{wir} = minimum interstitial brine saturation (irreducible), fraction

R_o = resistivity of core 100% saturated with brine, ohm-m

R_t = resistivity of core less than 100% saturated with brine of R_w , ohm-m

$\frac{R_t}{R_o}$ = resistivity index = I

D. Determine saturation exponent, n

1. rearrange saturation equation

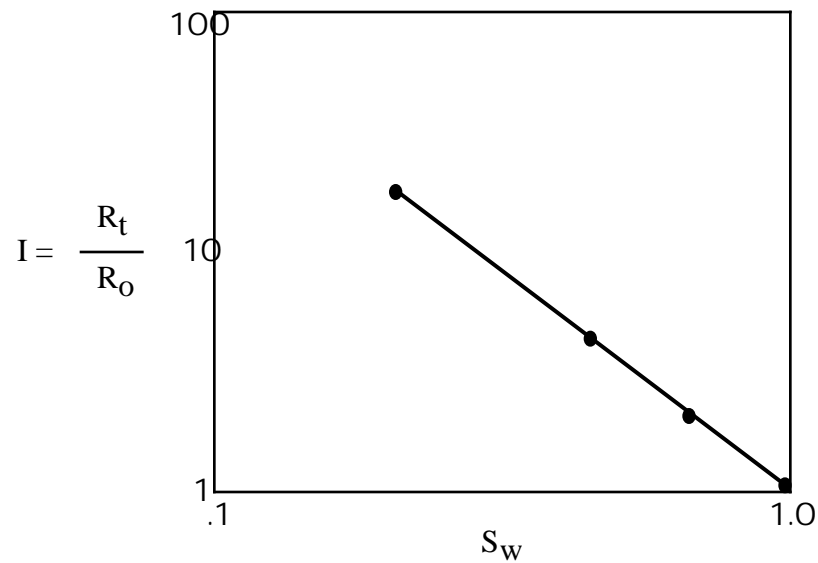
$$S_w = \left[\frac{R_o}{R_t} \right]^{1/n}$$

$$S_w^n = \frac{R_o}{R_t}$$

$$\frac{R_o}{R_t} = S_w^{-n}$$

$$\log \frac{R_t}{R_o} = -n \log S_w$$

2. Plot $\log \left(\frac{R_t}{R_o} \right)$ vs $\log S_w$ or $\log I$ vs $\log S_w$



3. the slope of the plot is -n, where n is the saturation exponent

Example V-2

Given the following data, calculate the electrical formation resistivity factor and saturation exponent of the core.

$$R_w = 55 \text{ ohm-cm}$$

$$I = 0.01 \text{ amp}$$

$$D = 2.54 \text{ cm}$$

$$L = 3.2 \text{ cm}$$

Sw Water Saturation, %	E Voltage across Core, volts
100.0	7.64
86.0	10.50
74.0	14.34
63.0	20.16
54.0	27.52
49.0 = S _{wir}	34.67

Solution:

- (1) electrical formation resistivity factor

$$F = \frac{R_o}{R_w}$$

$$r_o = \frac{E}{I} = \frac{7.64}{.01} = 764 \text{ ohm}$$

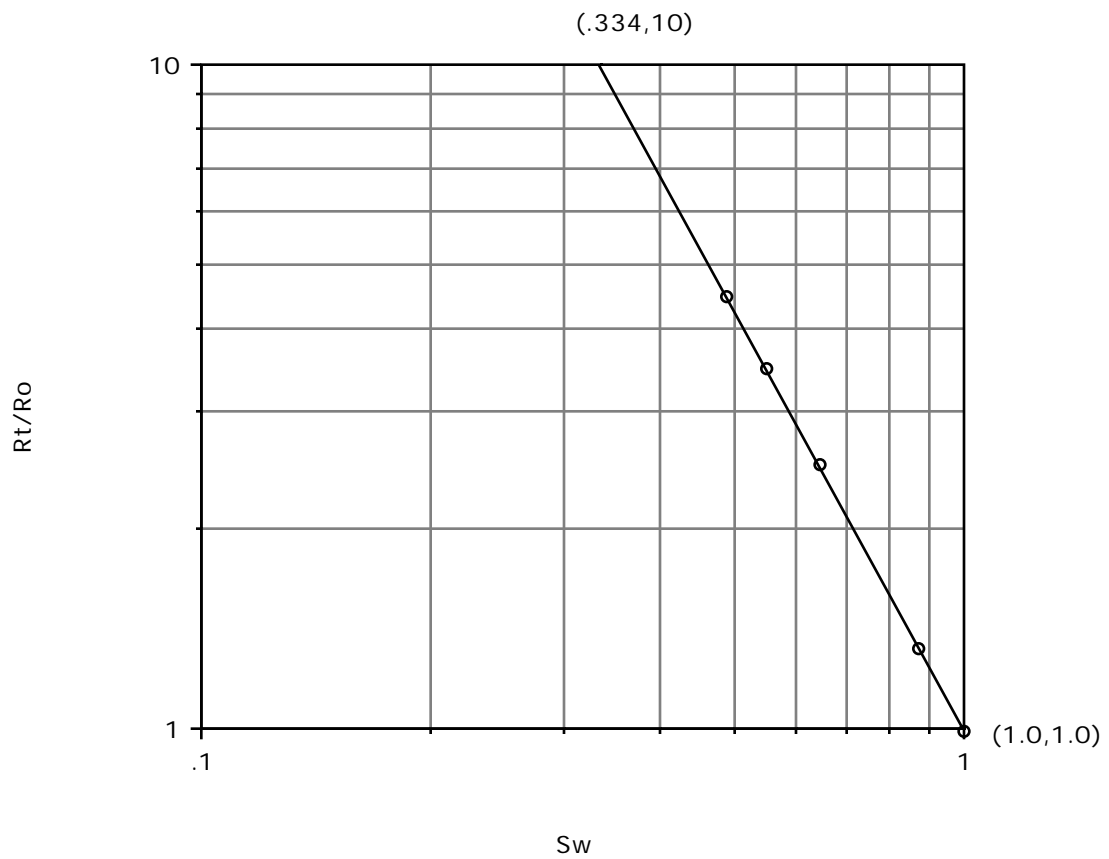
$$R_o = \frac{r_o A}{L} = \frac{(764)(2.54^2 / 4)}{3.2} = 1210 \text{ ohm cm}$$

$$F = \frac{R_o}{R_w} = \frac{1210}{55} = 22$$

saturation exponent

$$-n \log S_w = \log \frac{R_t}{R_o}$$

S_w %	$r_t = \frac{E}{I}$ (ohm)	$R_t = \frac{r_t A}{L}$ (ohm-cm)	$\frac{R_t}{R_o}$
1.00			1.000
.86	1050	1663	1.374
.74	1434	2271	1.877
.63	2016	3192	2.638
.54	2452	4358	3.601
.49	3467	5490	4.537



$$\begin{aligned}
 -n &= \text{slope} = \frac{\log 10 - \log 1}{\log .334 - \log 1} \\
 -n &= \frac{1 - 0}{-.4763 - 0} \\
 n &= 2.10 = \text{saturation exponent}
 \end{aligned}$$

NOTE:

$$\frac{R_t}{R_o} = \frac{E_t}{E_o}$$

so $\frac{R_t}{R_o}$ could have been calculated as the ratio of voltage at S_w divided by the voltage at $S_w = 1.0$

E. Determine cementation factor, m , and constant C for electrical formation resistivity factor equation

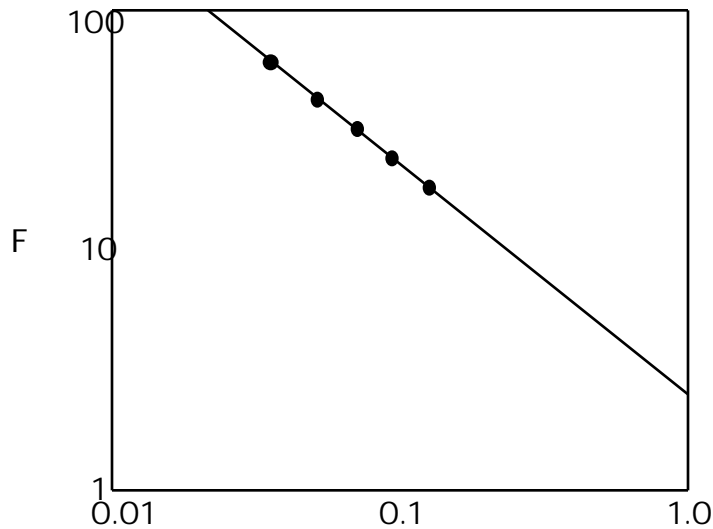
1. test several core samples from reservoir with formation brine
 - a. determine R_O and f for each sample
 - b. determine R_w for formation brine

c.
$$F = \frac{R_O}{R_w}$$

2. plot data according to form of equation for electrical formation resistivity factor

$$F = C \cdot r^{-m}$$

$$\log F = \log C - m \log r$$



slope = $-m$, m = cementation factor

intercept = C (intercept found at $r = 1.0$)

Example V-3

The laboratory test of Example IV-2 has been repeated for several core samples from the reservoir. Data is given below. Calculate the cementation factor and intercept for the formation resistivity factor equation.

Porosity Formation Resistivity Factor

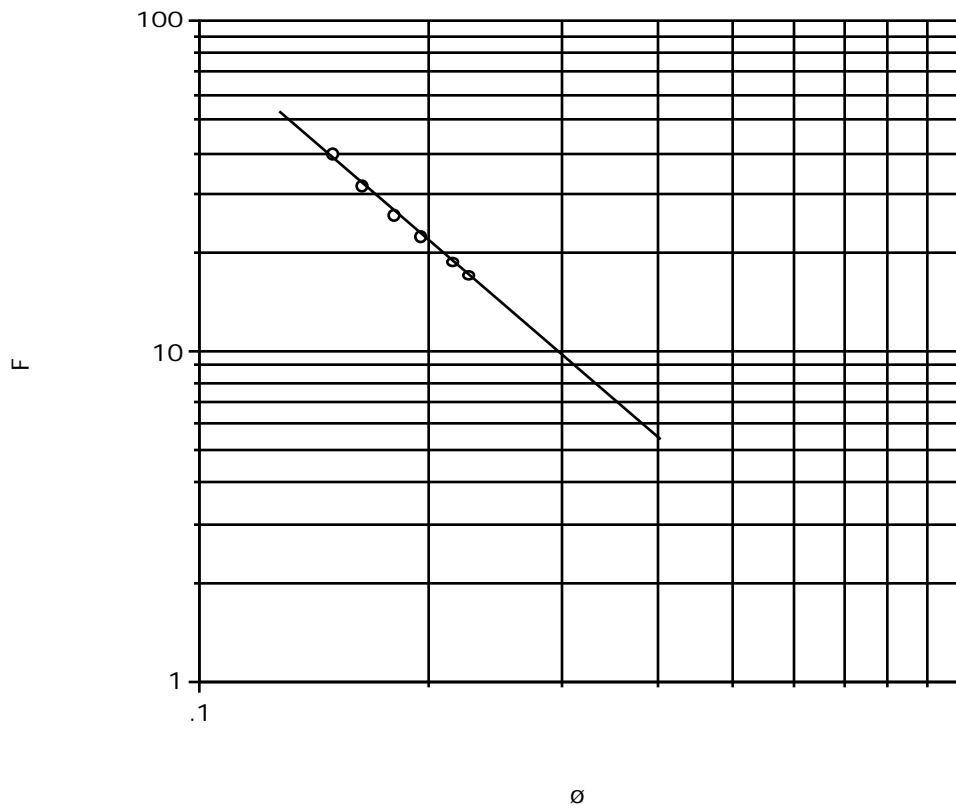
	F
0.152	40
0.168	32
0.184	26
0.199	22
0.213	19
0.224	17

Solution:

$$F = C \phi^{-m}$$

$$\log F = \log C - m \log \phi$$

plot $\log F$ vs $\log \phi$



$$\text{slope} = \frac{\log 50 - \log 10}{\log 0.137 - \log 0.284} = -2.21$$

$$-m = \text{slope} = -2.21$$

$$m = 2.21 = \text{cementation factor}$$

intercept

$$\log F = \log C - m \log f$$

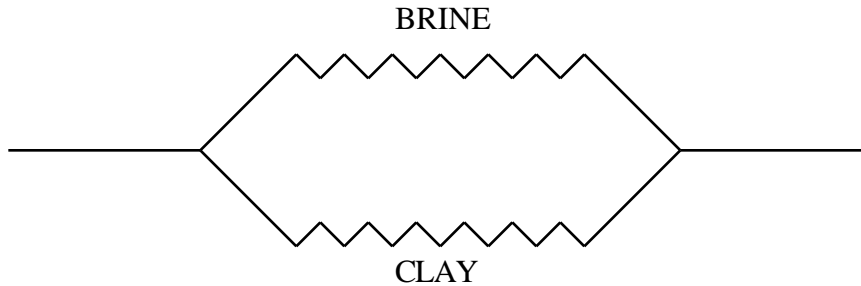
$$\log 10 = \log C - 2.21 \log 0.284$$

$$\log C = -.2082$$

$$C = 0.62 = \text{intercept}$$

IV) Effect of clay on resistivity

- A. ideally, only water conducts a current in rock
- B. if clay is present, portion of current conducted through the clay



$$\frac{1}{R_{OA}} = \frac{1}{R_{clay}} + \frac{1}{R_O}$$

where R_{OA} = resistivity measured on sample of reservoir rock with clay, 100% saturated with brine of resistivity R_W , ohm-m

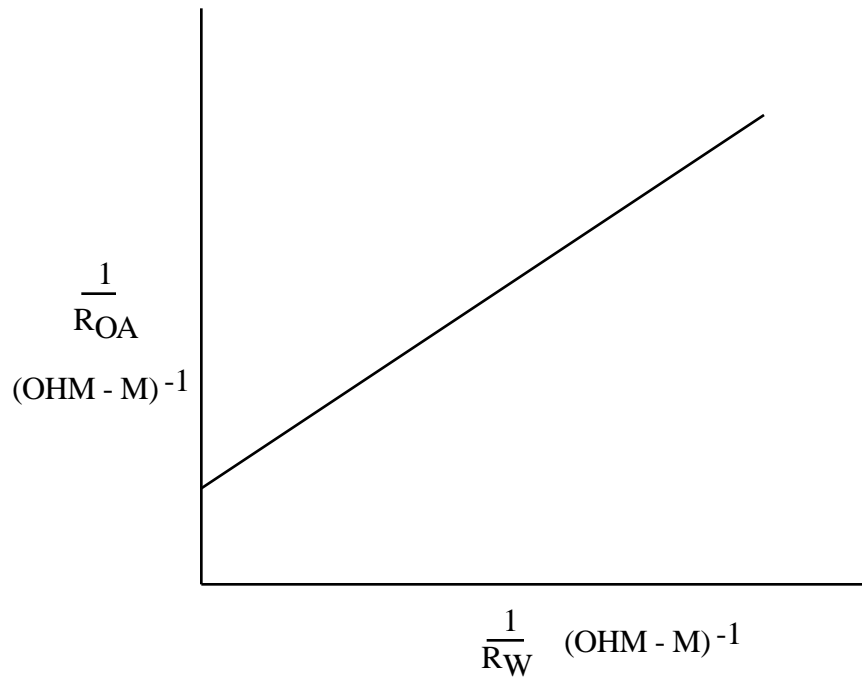
R_{clay} = component of measured resistivity due to clay, ohm-m

R_O = component of measured resistivity due to brine, ohm-m

$$\frac{1}{R_{OA}} = \frac{1}{R_{clay}} + \frac{1}{F R_W}$$

- C. to determine electrical formation resistivity factor
 - 1. measure resistivity of core sample (containing clay) in usual manner, this will be R_{OA}
 - 2. measure resistivity of brine, R_W , in usual manner

3. plot



$$\frac{1}{R_{OA}} = \frac{1}{R_{clay}} + \left[\frac{1}{F}\right] \left[\frac{1}{R_W}\right]$$

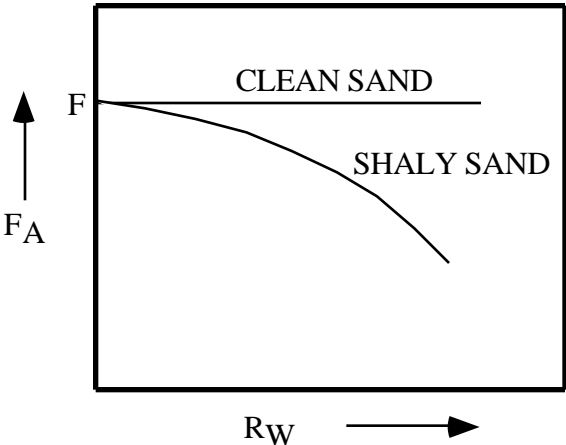
where

$$\frac{1}{R_{clay}} = \text{intercept}$$

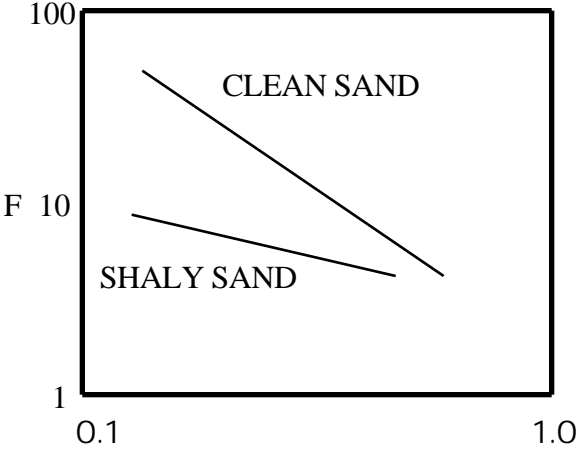
$$\frac{1}{F} = \text{slope}$$

D. effect of clay

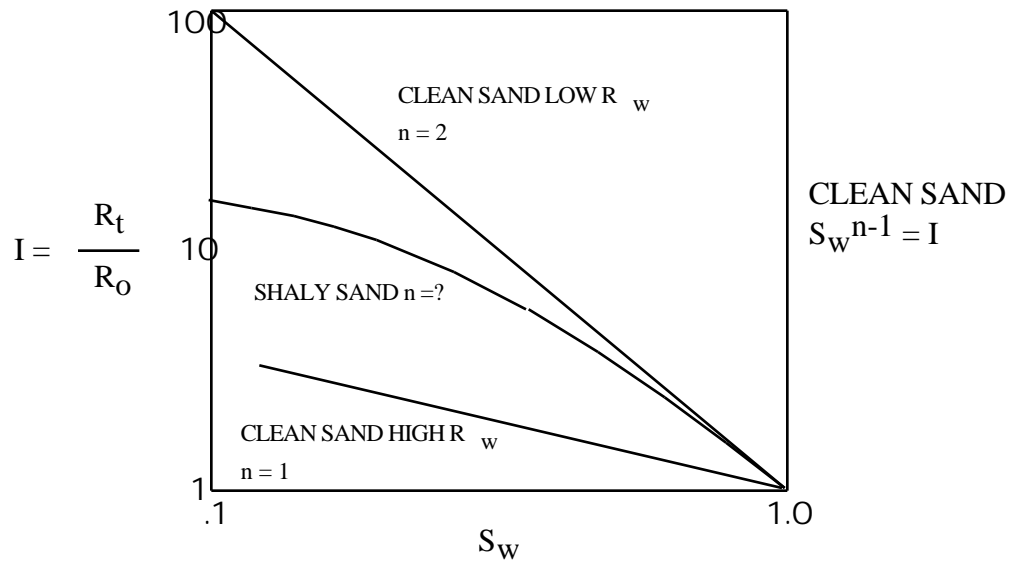
1. define $F_A = \frac{R_oA}{R_w}$, clays reduced the apparent formation resistivity factor



2. formation resistivity factor decreases more gradually when clay is present in the formation



3. saturation exponent n is not constant when clay is present in formation.



VI. MULTIPHASE FLOW IN POROUS ROCK

I) Effective permeability

- A. Permeability, k , previously discussed applies only to flow when pores are 100% saturated with one fluid - sometimes called absolute permeability

$$q = \frac{kA}{\mu L}$$

- B. When pore space contains more than one fluid, the above equation becomes

$$q_o = \frac{k_o A P}{\mu_o L}$$

$$q_w = \frac{k_w A P_w}{\mu_w L}$$

$$q_g = \frac{k_g A P_g}{\mu_g L}$$

where q_o = flow rate of oil, volume/time

q_w = flow rate of water, volume/time

q_g = flow rate of gas, volume/time

and k_o = effective permeability to oil, md

k_w = effective permeability to water, md

k_g = effective permeability to gas, md

- C. Effective permeability is a measure of the fluid conductance capacity of porous media to a particular fluid when the porous media is saturated with more than one fluid

- D. Effective permeability is a function of:

1. geometry of the pores of the rock
2. rock wetting characteristics
3. fluid saturations

E. Darcy equation for multiple fluids in linear flow, in oilfield units

$$q_o = 1.1271 \times 10^{-3} \frac{k_o A (P_1 - P_2)}{\mu_o L}$$

$$q_w = 1.1271 \times 10^{-3} \frac{k_w A (P_1 - P_2)_w}{\mu_w L}$$

$$q_g = 1.1271 \times 10^{-3} \frac{k_g A (P_1 - P_2)_g}{\mu_g L}$$

when k = md
 A = ft²
 P = psia
 L = ft
 q = res bbl/day

II) Relative permeability

A. Defined as the ratio of the effective permeability to a fluid at a given saturation to the effective permeability to that fluid at 100% saturated (absolute permeability)

$$k_{r_o} = \frac{k_o}{k}$$

$$k_{r_w} = \frac{k_w}{k}$$

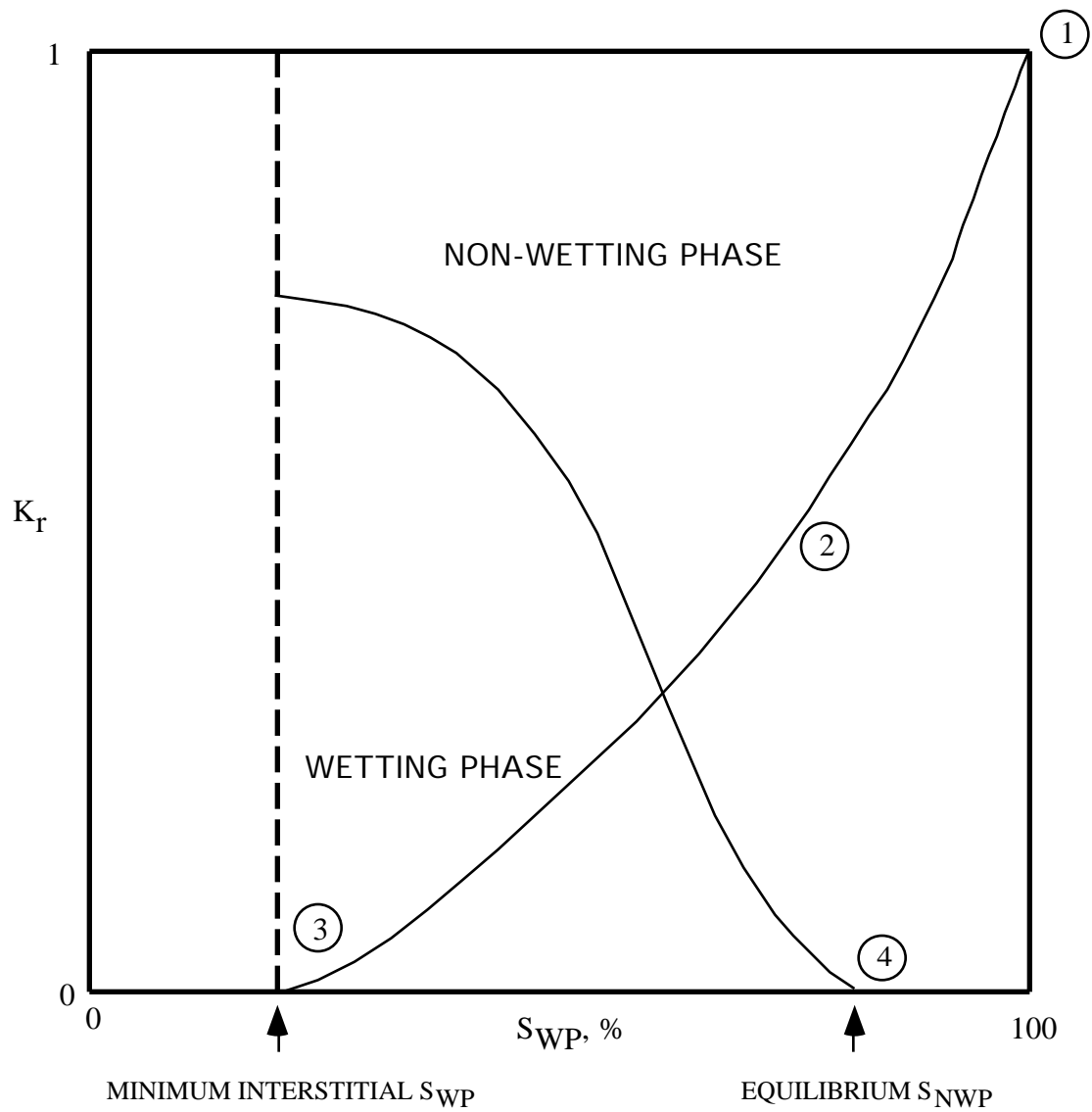
$$k_{r_g} = \frac{k_g}{k}$$

B. It is normally assumed that the effective permeability at 100% saturation is the same for all fluid in a particular rock. (not necessarily true in shaly sand)

III) Typical relative permeability curves

A. Use subscript wp to represent the "wetting phase"

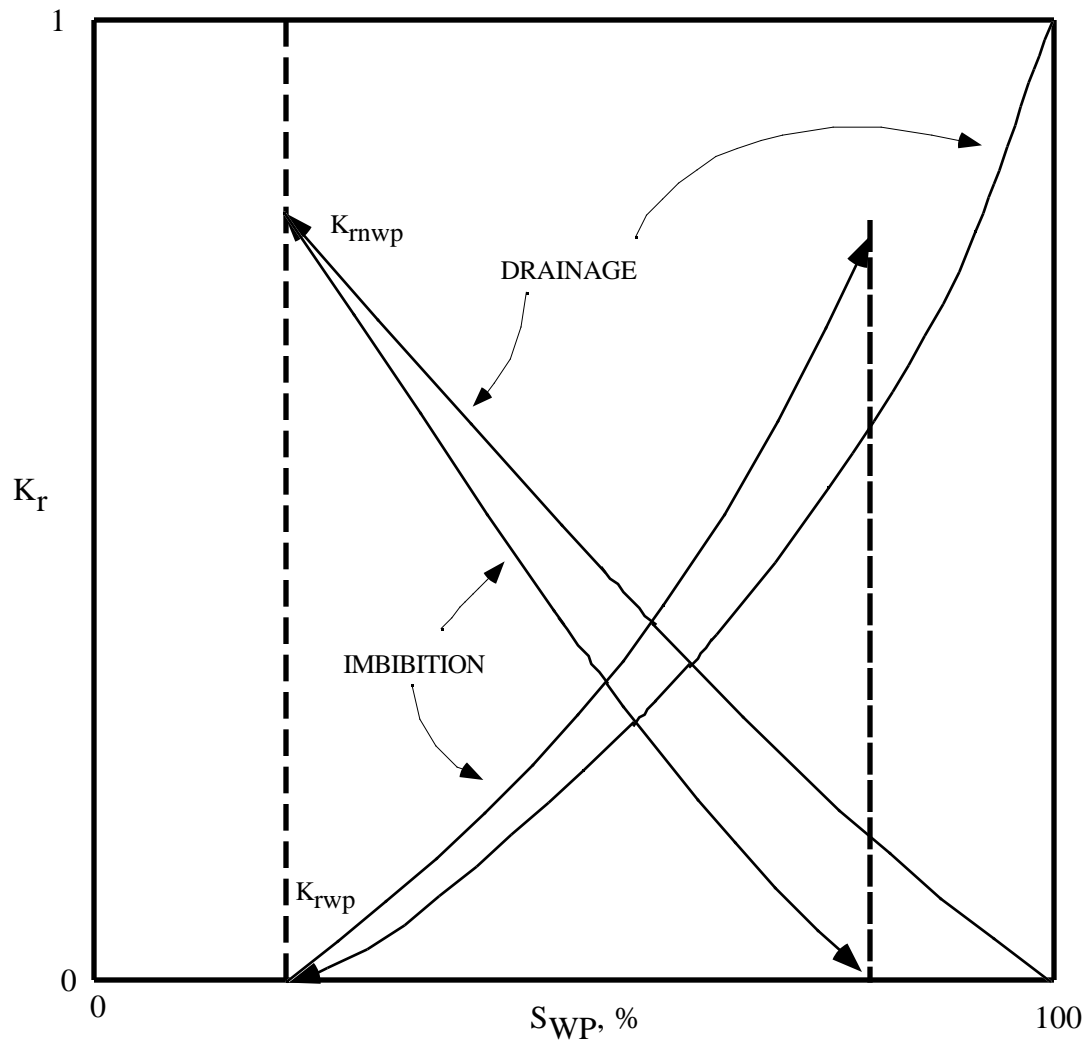
Use subscript nwp to represent the "non-wetting phase"



1. $k_{rwp} = 1$, only at $S_{wp} = 100\%$
2. rapid decrease in k_{rwp} as S_{wp} decreases
3. $k_{rwp} = 0$, at minimum interstitial S_{wp}
4. $k_{rnwp} = 0$, at equilibrium S_{nwp}

Note that $k_{rwp} + k_{rnwp} < 1.0$

- B. Effect of saturation history
1. two types of relative permeability curves
 - a. drainage curve - wetting phase is displaced by non-wetting phase, i.e., wetting phase saturation is decreasing
 - b. imbibition curve - non-wetting phase is displaced by wetting phase, i.e., wetting phase saturation is increasing
 2. the typical relative permeability curve shown below represents a process in which
 - a. process begins with porous rock 100% saturated with wetting phase ($S_{wp} = 100\%$)
 - b. wetting phase is displaced with non-wetting phase (drainage) until wetting phase ceases to flow ($S_{wp} =$ minimum interstitial wetting phase saturation)
 - c. then non-wetting phase is displaced with wetting phase (imbibition) until non-wetting phase ceases to flow ($S_{wp} =$ equilibrium or residual non-wetting phase saturation)



minimum interstitial
wetting phase saturation

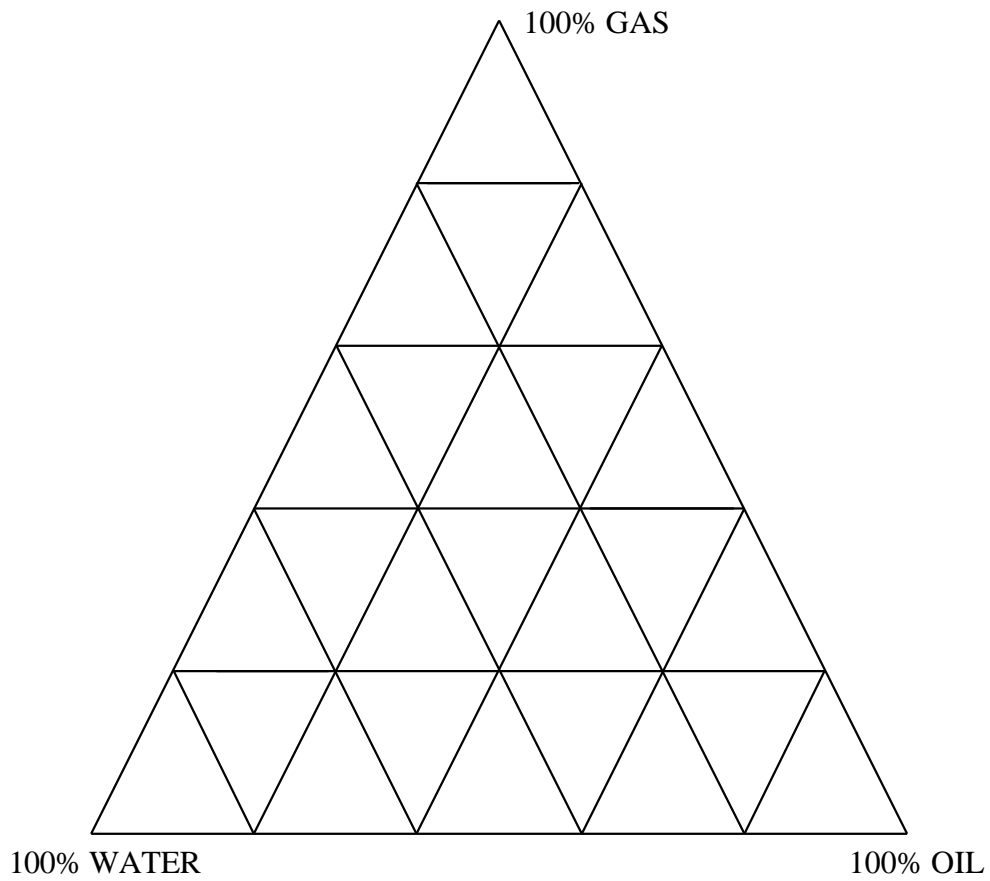
residual non-wetting
phase saturation

3. the word "hysteresis" describes the process in which the results (k_r) are different when measurements are made in different directions
4. the procedure (drainage or imbibition) used to obtain k_r data in laboratory must correspond to the process in the reservoir
 - a. initial distribution of fluids in reservoir was by drainage
 - b. at and behind a water front (flood or encroachment) the process is imbibition
5. wetting preference for reservoir rocks is usually water first, then oil, finally gas

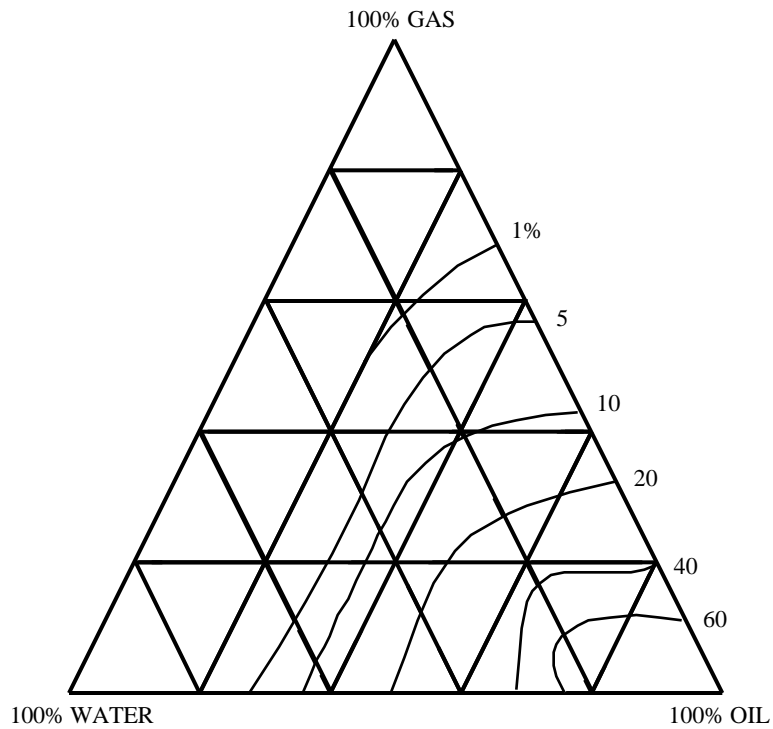
Fluids Present	Wetting Phase
Water & Oil	Water
Water & Gas	Water
Oil & Gas	Oil

C. Three phase relative permeability

1. often three phases are present in petroleum reservoirs
2. tertiary (triangular) diagram is used to represent a three-phase system



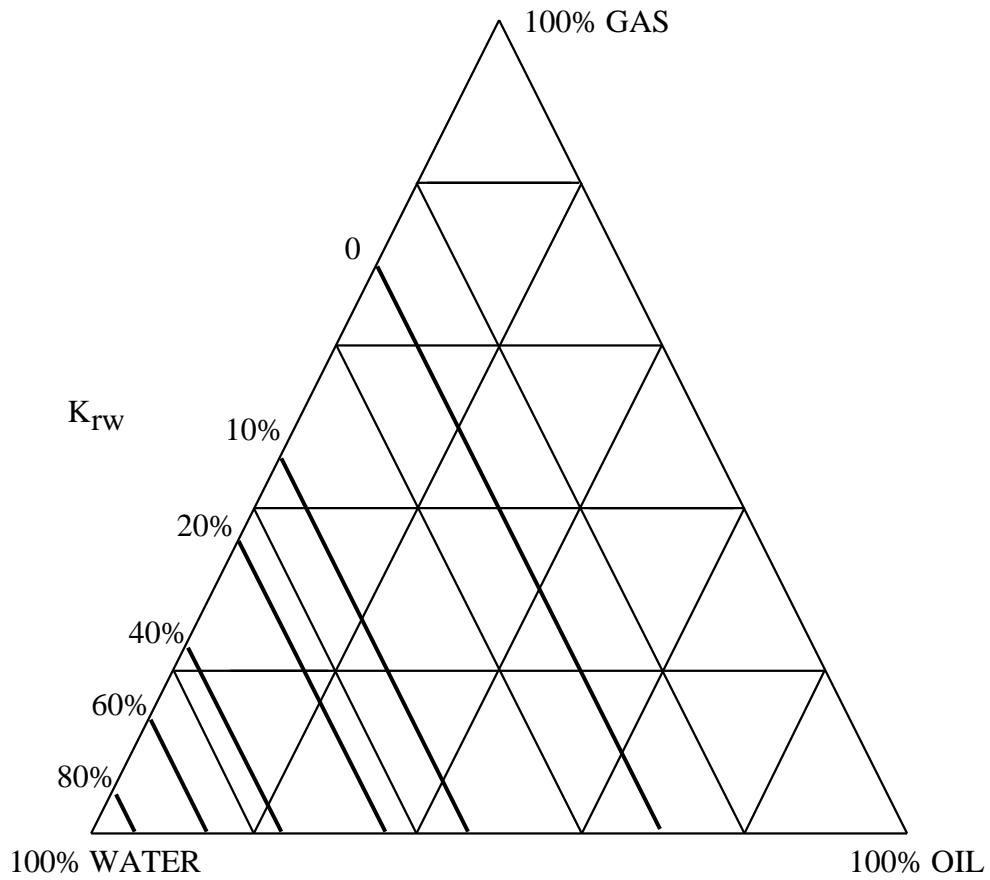
3. relative permeability to oil in a three phase system



Note, k_{rO} is shown in %

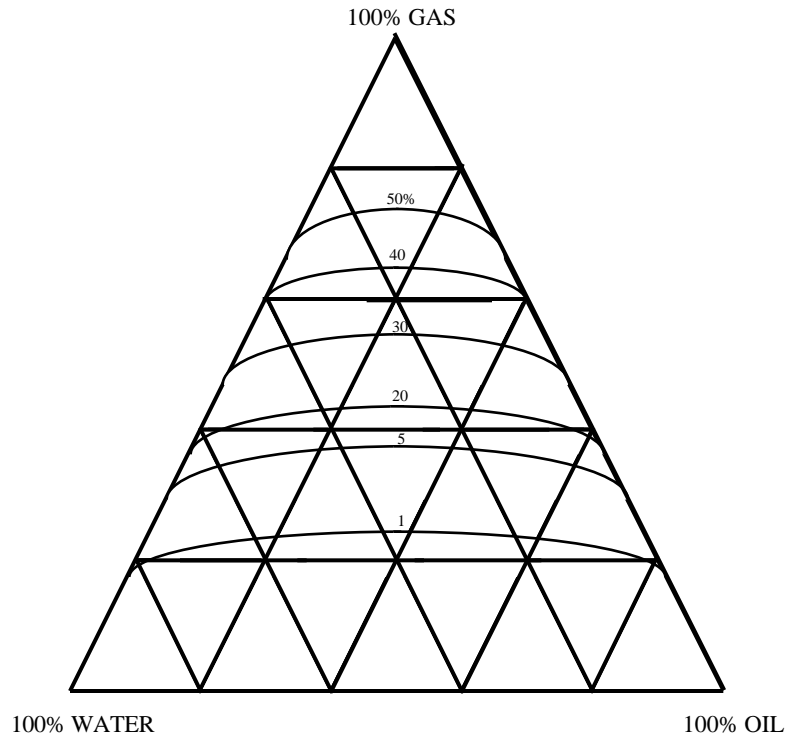
- a. dependence of relative permeability to oil on saturations of other phases is established as follows:
- 1) oil phase has a greater tendency than gas to wet the solid
 - 2) interfacial tension between water and oil is less than that between water and gas
 - 3) oil occupies portions of pore adjacent to water
 - 4) at lower water saturations the oil occupies more of the smaller pores. The extended flow path length accounts for the change in relative permeability to oil at constant oil saturation and varying water saturation

4. Relative permeability to water in a three-phase system

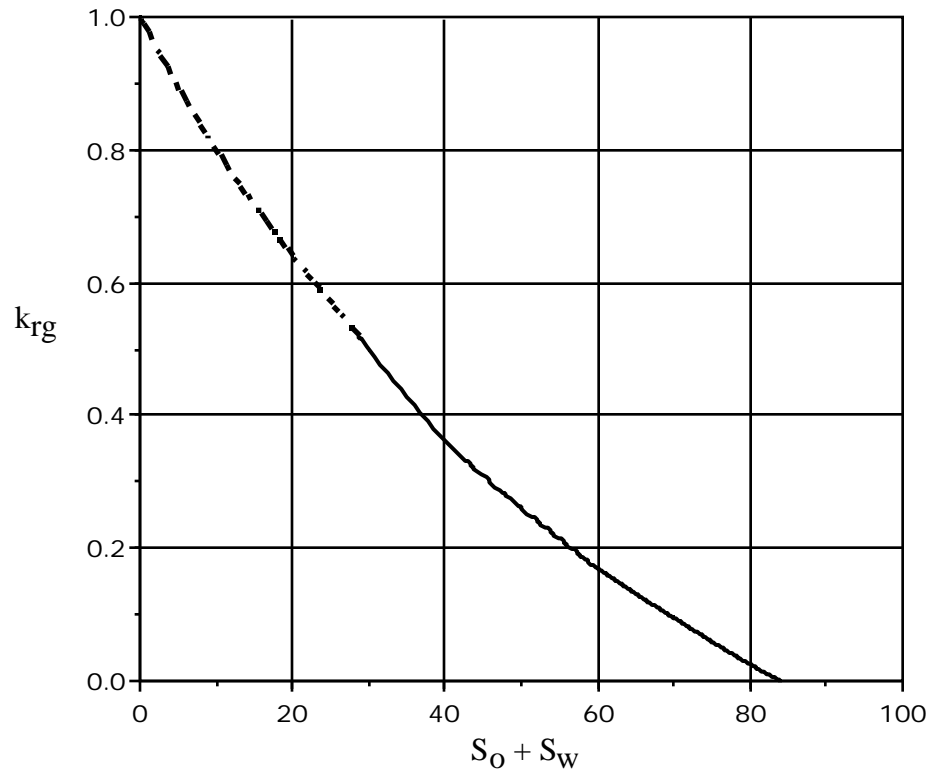


- a. straight lines indicate relative permeability to water is a function of water saturation only
- b. thus, k_{rw} can be plotted on cartesian coordinates against S_w .

5. Relative permeability to gas in a three-phase system



- a. curves above indicate that k_{rg} is a function of saturations of other phases present.
- b. other research shows that k_{rg} is a unique function of gas saturation
- c. the other phases, oil and water, occupy the smaller pore openings and wet the surface of the rock
- d. therefore, k_{rg} should be dependent only on the total saturation of the other two phases (i.e. $1-S_g$) and independent of how much of that total is composed of either phase
- e. thus k_{rg} can be plotted on Cartesian coordinates against $S_o + S_w$



6. Bottom line - for three-phase system in water wetted rock

a. water

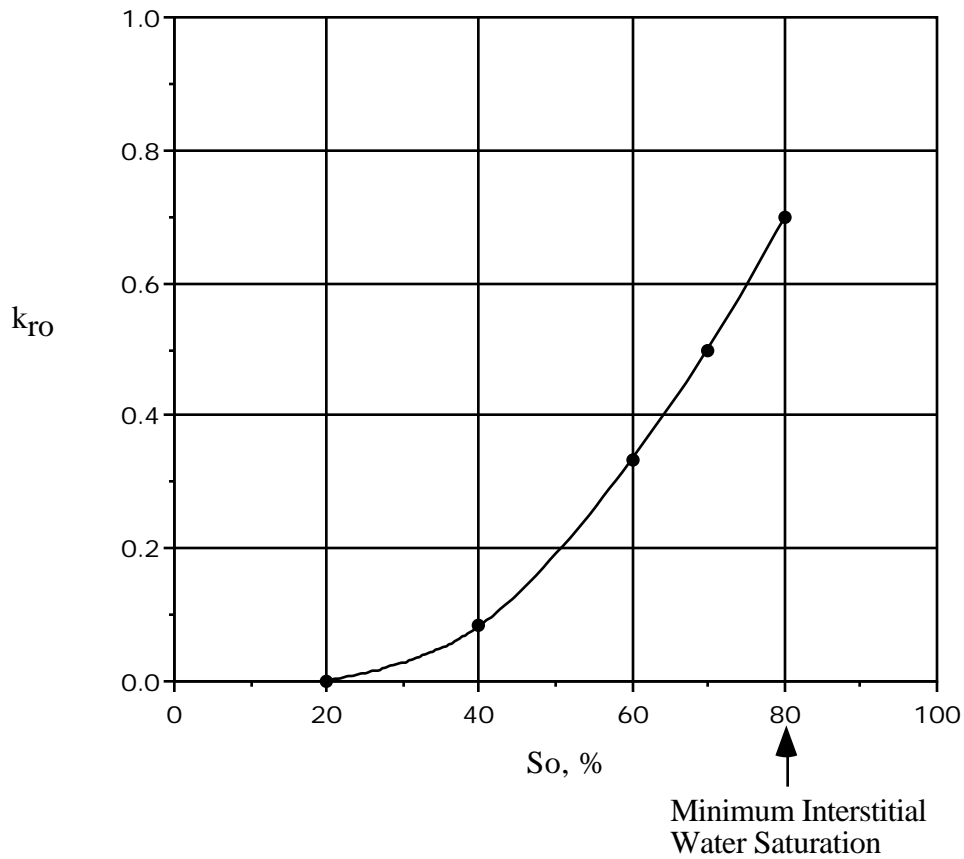
- 1) is located in smaller pore spaces and along sand grains
- 2) therefore k_{rw} is a function of S_w only
- 3) thus plot k_{rw} against S_w on rectangular coordinates

b. gas

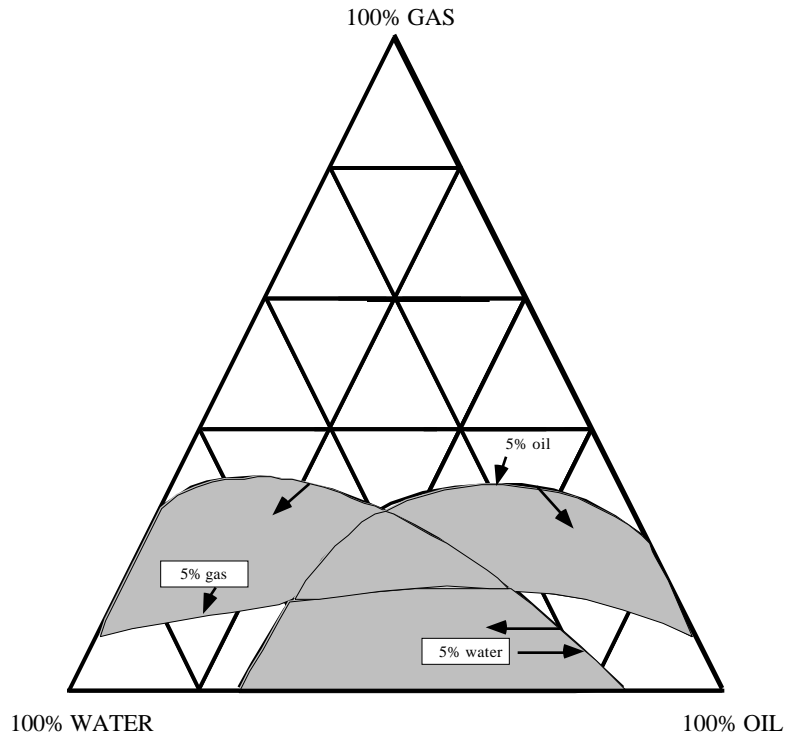
- 1) is located in center of larger pores
- 2) therefore k_{rg} is a function of S_g only
- 3) thus plot k_{rg} against S_g (or $S_o + S_w$) on rectangular coordinate

c. oil

- 1) is located between water and gas in the pores and to a certain extent in the smaller pore spaces
- 2) therefore k_{rO} is a function of S_O , S_W , and S_g
- 3) thus plot k_{rO} against S_O , S_W , S_g on a triangular diagram
- 4) if S_W can be considered to be constant (minimum interstitial) k_{rO} can be plotted against S_O on a rectangular diagram



7. Flow in three-phase system



Arrows point to increasing fraction of respective components in stream

Region of three-phase flow in reservoir centers around 20% gas, 30% oil, 50% water

IV) Permeability ratio (relative permeability ratio)

A. Definitions

1. When the permeability to water is zero (as at minimum interstitial water saturation) it is sometimes convenient to use permeability ratio to represent the flow conductance of the rock to gas and oil as a ratio.

$$\text{permeability ratio} = \frac{k_g}{k_o} = \frac{k_{rg}}{k_{ro}}$$

2. When the permeability to gas is zero (no gas or gas below "critical gas saturation") it is sometimes convenient to use permeability ratio to represent the flow conductance of the rock to oil and water as a ratio

$$\text{permeability ratio} = \frac{k_o}{k_w} = \frac{k_{ro}}{k_{rw}}$$

V) Measurement of relative permeability

A. Methods

1. Laboratory - steady-state flow process
2. Laboratory - displacement (unsteady-state process)
3. Calculation from capillary pressure data (not covered here)
4. Calculation from field performance data

B. Laboratory Methods

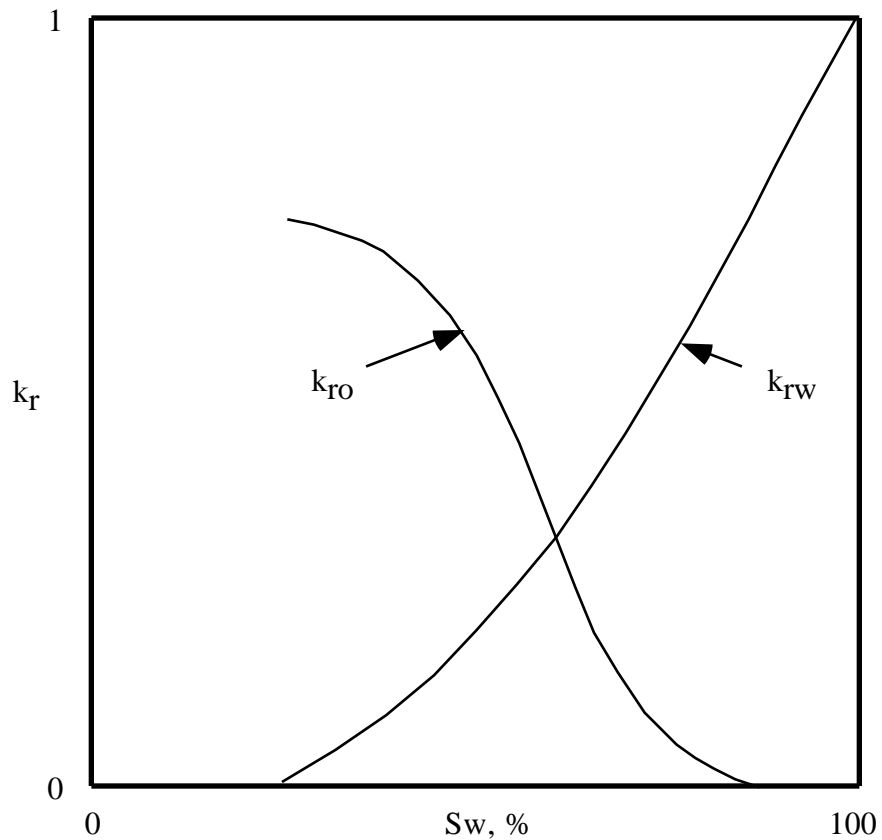
1. Steady-state flow process
 - a. saturate core with wetting-phase fluid
 - b. inject wetting-phase fluid through core (this will determine absolute permeability)

- c. inject a mix of wetting-phase and non-wetting phase (start with small fraction of non-wetting phase)
- d. when inflow and outflow rates and portion of non-wetting phase equalize, record inlet pressure, outlet pressure and flow rates of each phase
- e. measure fluid saturation in core (see below)
- f. calculate relative permeability

$$k_o = \frac{q_o \mu_o L}{A p}$$

$$k_w = \frac{q_w \mu_w L}{A p}$$

- g. repeat b through f with injection mixtures containing relatively more non-wetting phase until irreducible wetting-phase saturation is reached



h. determination of fluid saturations

1) resistivity

$$S_w = \left(\frac{R_o}{R_t}\right)^{\frac{1}{n}} = \left(\frac{E_o}{E_t}\right)^{\frac{1}{n}}$$

where: R_o = resistivity of core 100% saturated with wetting-phase, ohm-m

R_t = resistivity of core with saturation of wetting phase less than 100%, ohm-m

E_o = voltage across core 100%, saturated with wetting phase, volts

E_t = voltage across core with saturation of wetting phase less than 100%, volts

2) volumetric balance

3) gravimetric method - remove core and weigh it

$$W_f = W_t - W_d$$

where: W_f = weight of fluid in core, gm

W_t = weight of saturated core, gm

W_d = weight of dry core, gm

$$W_f = \rho_o V_o + \rho_w V_w$$

and

$$V_f = V_o + V_w$$

where: ρ = density, gm/cc

V = volume, cc

$$S_w = V_w / V_f$$

where: S_w = saturation of wetting phase

thus

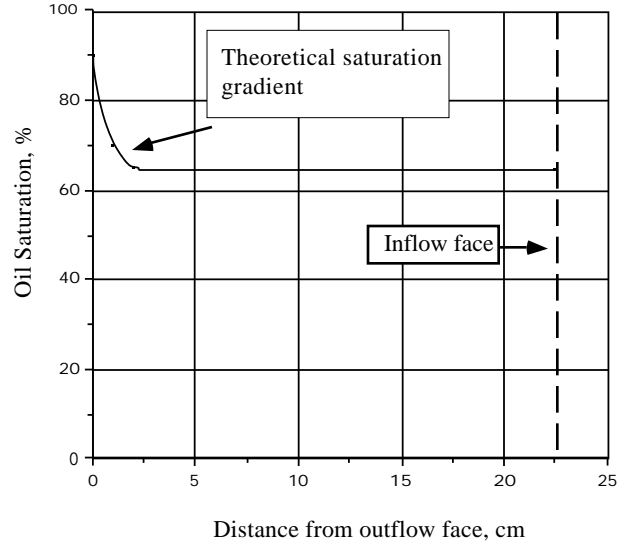
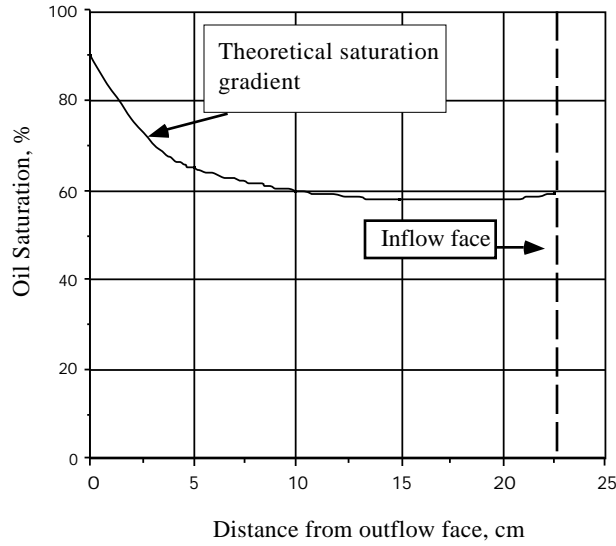
$$S_w = \frac{W_f/V_f - o}{w - o}$$

- i. same procedure can be used starting with 100% saturation of non-wetting phase
- 1) injection ratio start with high ratio of non-wetting phase
 - 2) procedure ends at residual non-wetting phase saturation
 - 3) then is a hysteresis effect of same type as discussed with capillary pressure measurements
 - 4) choice of starting saturation depends on reservoir process which is being simulated
- j. end effects
- 1) causes of end effects
 - a) in the bulk of the core there is a wetting-phase saturation and a non-wetting phase saturation, therefore there is a finite value of capillary pressure
 - b) thus there is a difference in pressure between the wetting-phase and non-wetting phase

$$P_{cap} = P_{nwp} - P_{wp}$$
 - c) at the face of the core the pressures in the wetting-phase and the non-wetting phase are essentially equal

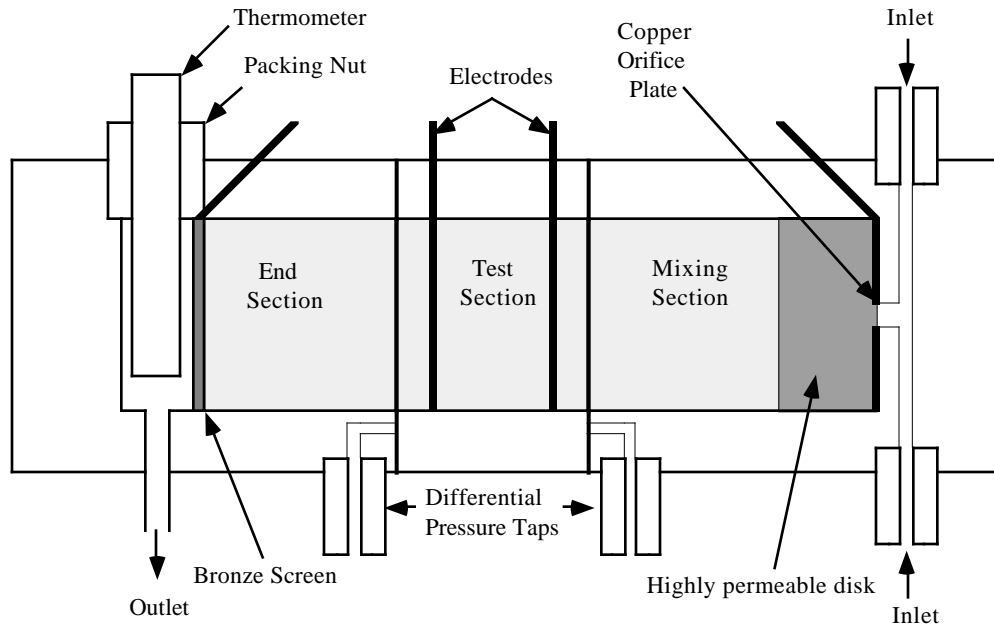
$$P_{nwp} = P_{wp}$$

thus capillary pressure is essentially zero
 - d) if capillary pressure is zero, the saturation of the wetting phase must be 100% (see capillary pressure curve)
 - e) there must be a saturation gradient from essential 100% wetting phase at the "end" to some value of S_{wp} less than 100% in the bulk of the core



2) elimination of end effects

- a) install end pieces to contain end effects
- b) flow at rapid rates to make end effect negligible (pressure gradient ≥ 2 psi/inch)



PENN STATE RELATIVE-PERMEABILITY APPARATUS

Example VI-1

The relative permeability apparatus shown above was used in a steady-state flow process to obtain the data given below at a temperature of 70°F. See figure on previous page.

The Core

sandstone
 length = 2.30 cm
 diameter = 1.85 cm
 area = 2.688 cm²
 porosity = 25.5%

The Fluids

brine, 60,000 ppm
 oil, 40°API
 $\mu_w = 1.07$ cp
 $\mu_o = 5.50$ cp

Oil Flow cc/sec	Water Flow cc/sec	Inlet Pressure psig	Outlet Pressure psig	Voltage Drop volts	Electrical Current amps
0.0000	1.1003	38.4	7.7	1.20	0.01
0.0105	0.8898	67.5	13.5	2.10	0.01
0.0354	0.7650	88.1	17.6	2.80	0.01
0.0794	0.3206	78.2	15.6	4.56	0.01
0.1771	0.1227	85.6	17.1	8.67	0.01
0.2998	0.0000	78.4	15.7	30.00	0.01

Draw the relative permeability curve

Solution:

1. Calculate absolute permeability using data with core 100% saturated with water

$$k = \frac{q_w \mu_w L}{A p}$$

$$k = \frac{(1.1003)(1.07)(2.30)}{(2.688)(38.4 - 7.7)(14.696)}$$

$$k = 0.482 \text{ darcy}$$

2. Calculate effective permeabilities to oil and water

$$k_o = \frac{q_o \mu_o L}{A P}$$

$$k_o = \frac{(0.0105)(5.50)(2.30)}{(2.688)(67.5 - 13.5)(14.696)}$$

$$k_o = 0.0134 \text{ darcy}$$

$$k_w = \frac{q_w \mu_w L}{A P}$$

$$k_w = \frac{(0.8898)(1.07)(2.30)}{(2.688)(67.5 - 13.5)(14.696)}$$

$$k_w = 0.2217 \text{ darcy}$$

3. Calculate relative permeabilities

$$k_{r_o} = \frac{k_o}{k} = \frac{.0134}{.482} = 0.028$$

$$k_{r_w} = \frac{k_w}{k} = \frac{.2217}{.482} = 0.460$$

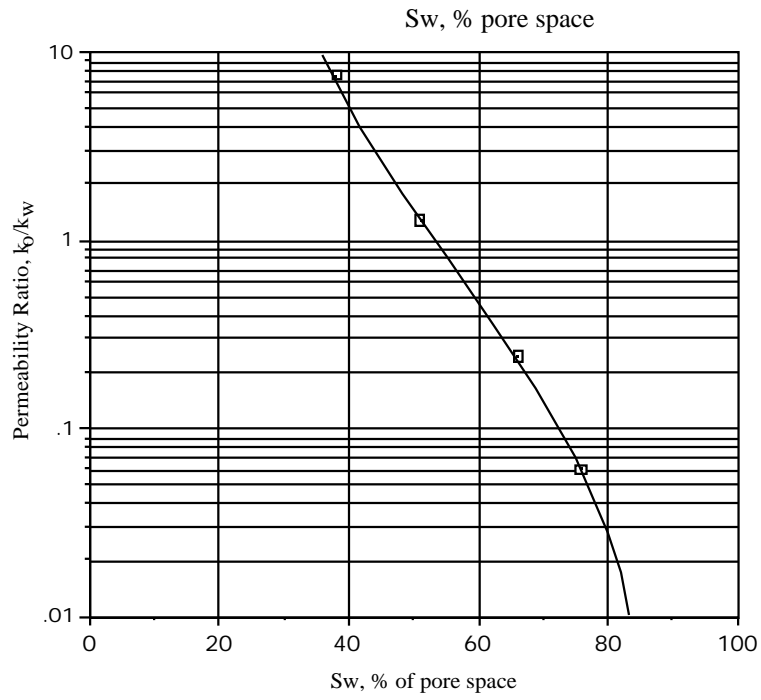
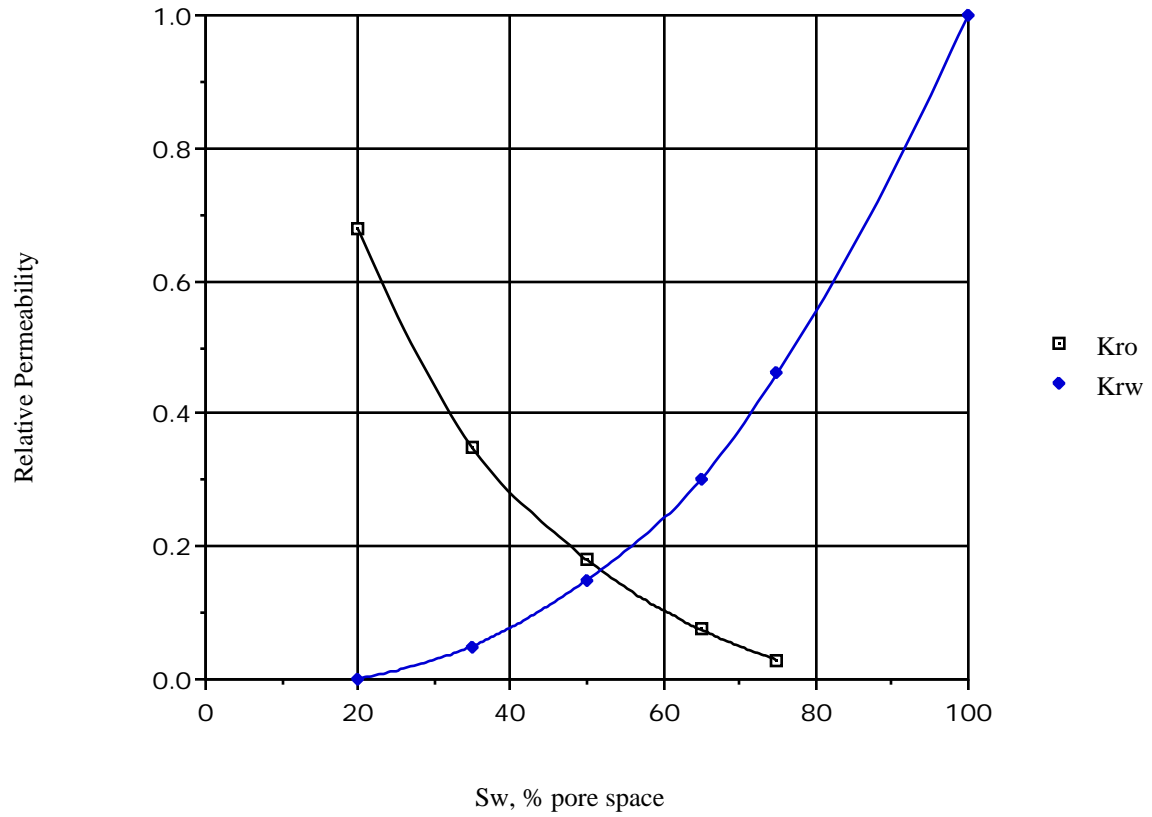
4. Calculate water saturations

$$S_w = \left(\frac{E_o}{E_t} \right)^{1/2}$$

$$S_w = \left(\frac{1.20}{2.10} \right)^{1/2} = .756$$

5. Results

Water Saturation S_w	Relative Permeability to oil k_{r_o}	Relative Permeability to water k_{r_w}	k_o/k_w
1.000	0.000	1.000	0.000
0.756	0.028	0.460	0.061
0.655	0.072	0.303	0.238
0.513	0.182	0.143	1.273
0.372	0.371	0.050	7.419
0.200	0.686	0.000	-----



6. The data permit certain checks to be made

$$F = 0.62^{-2.15}$$

$$F = \frac{R_o}{R_w}$$

$$R_w = 12 \text{ ohm-m for } 60,000 \text{ ppm brine}$$

$$R_o = \frac{EA}{IL} = \frac{(1.20)(2.688)}{(.01)(2.3)} = 140 \text{ ohm-m}$$

$$F = \frac{140}{12} = 11.7$$

$$= \left(\frac{.62}{F}\right)^{2.15} = \left(\frac{.62}{11.7}\right)^{2.15}$$

$$= .255$$

2. Displacement (unsteady-state)(Welge)
- a. does not result in relative permeability only give permeability ratio
 - b. procedure
 - 1) mount core in holder
 - 2) saturate with wetting phase (usually oil)
 - 3) inject non-wetting phase (usually gas) at constant inlet and outlet pressures
 - 4) measure
 - a) cumulative gas injected as a function of time
 - b) cumulative oil produced as a function of time
 - c. conditions
 - 1) pressure drop across core high enough to make end effects negligible, but not enough to cause turbulent (non-darcy) flow.
 - 2) gas saturation can be described at mean pressure
 - d. calculations
 - 1) convert gas injected into pore volumes

$$P_m = \frac{P_i + P_o}{2}$$

- 3) flow is horizontal and core is short so that effects of gravity can be neglected

$$G_{ipv} = \frac{G_i p_i}{LA p_m}$$

where G_i = cumulative gas injected (measured at pressure p_i), cc

G_{ipv} = cumulative gas injected in pore volume

p_i = inlet pressure, psi

$$P_m = \frac{P_i + P_o}{2}, \text{ psi}$$

LA = pore volume, cc

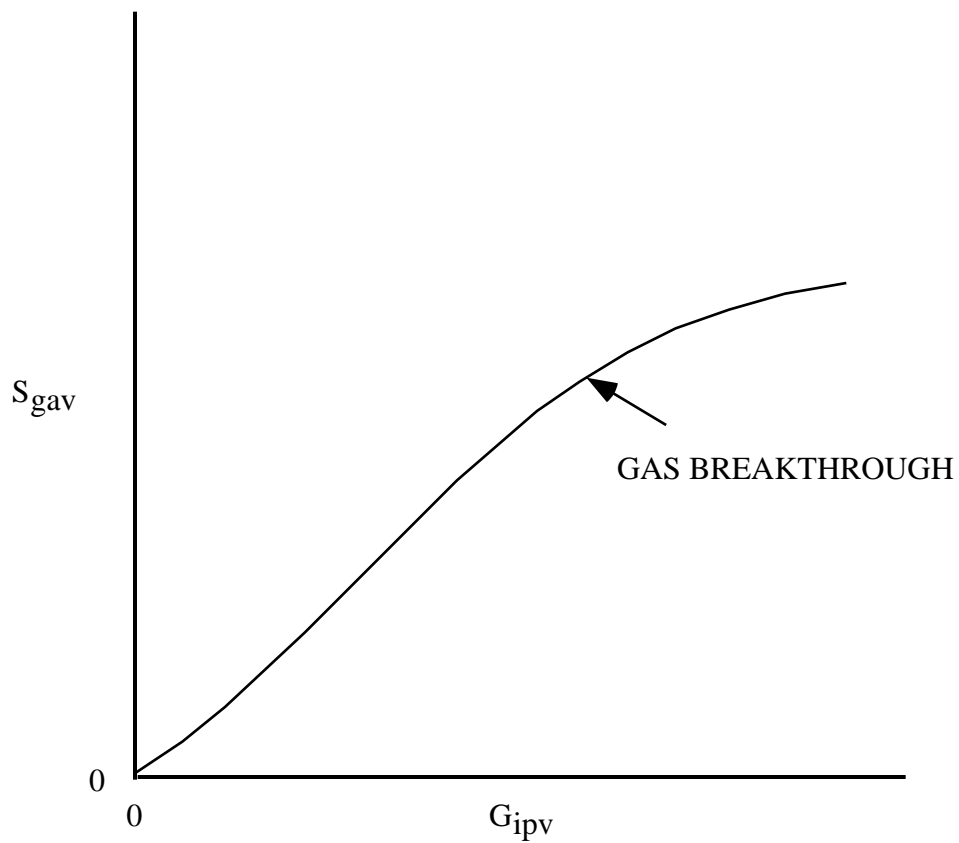
2) calculate average gas saturation, S_{gav}

$$S_{gav} = \frac{N_p}{LA}$$

where N_p = cumulative oil produced, cc

LA = pore volume, cc

3) plot S_{gav} vs G_{ipv}



- 4) determine fractional flow of oil, f_o

$$f_o = \frac{d(S_{g_{av}})}{d(G_{i_{pv}})}$$

f_o = slope of plot of $S_{g_{av}}$ vs $qG_{i_{pv}}$

- 5) calculate permeability ratio, k_g/k_o

$$f_o = \frac{\frac{k_o A p}{\mu_o L}}{\frac{k_o A p}{\mu_o L} + \frac{k_g A p}{\mu_g L}}$$

$$f_o = \frac{k_o/\mu_o}{k_o/\mu_o + k_g/\mu_g}$$

$$\frac{k_g}{k_o} = \frac{1 - f_o}{f_o(\mu_o/\mu_g)}$$

where $\frac{k_g}{k_o}$ = permeability ratio of gas to oil

f_o = fractional flow of oil

- 6) Permeability ratio, k_g/k_o , calculated above applies only at the gas saturation of the outflow face, thus must calculate S_{go}

$$S_{go} = S_{g_{av}} - G_{i_{pv}} f_o$$

where S_{go} = gas saturation at outlet face of core

$G_{i_{pv}}$ = cumulative gas injected, pore volumes

f_o = fractional flow of oil at outlet face of core

- e. advantages
 - 1) minimum amount of equipment
 - 2) rapid
- f. disadvantages
 - 1) results in k_g/k_o , not k_{ro} and k_{rg}
 - 2) equations don't apply until gas breaks through, thus initial value of gas saturation may be high, resulting in incomplete k_g/k_o vs S_{go} curve.

Example VI-2

The data from an unsteady-state displacement of oil by gas in a 2 inch diameter by 5 5/8 inch long sandstone core are given below.

Cumulative Gas Injection, G_i , cc	Cumulative Oil Produced, N_p , cc
14.0	14.6
50.2	19.5
112.6	22.5
202.3	25.5
401.4	28.6
546.9	30.4
769.9	32.2
1226.5	33.4
3068.9	35.3
5946.6	35.9

Other data

$$T = 70^\circ\text{F}, \quad \mu_o = 2.25 \text{ cp}, \quad \mu_g = .0185 \text{ cp}$$

$$= .210, \quad p_{\text{inlet}} = 5.0 \text{ psig}, \quad p_{\text{out}} = 0.0 \text{ psig}$$

$$L = 5 \frac{5}{8} \times 2.54 = 14.3 \text{ cm} \quad A = \pi (2.54)^2 = 20.27 \text{ cm}^2$$

Prepare to determine k_g/k_o by calculating S_{gav} and G_{ipv} .

Solution:

1. Calculate S_{gav}

$$S_{gav} = \frac{N_p}{L A}$$

$$S_{gav} = \frac{14.6 \text{ cc}}{(14.3 \text{ cm})(20.27 \text{ cm}^2)(.210)}$$

$$S_{gav} = 0.24$$

2. Calculate G_{ipv}

$$G_{ipv} = \frac{G_i p_i}{LA \quad p_m}$$

$$G_{ipv} = \frac{(14.0 \text{ cc})(19.7 \text{ psia})}{(14.3 \text{ cm})(20.27 \text{ cm}^2)(.210)(19.7 \text{ psia} + 14.7 \text{ psia})/2}$$

$$G_{ipv} = 0.264 \text{ pv}$$

3. Results

S_{gav}	$G_{ipv} \text{ pv}$
0.24	0.264
0.32	0.945
0.37	2.12
0.42	3.81
0.47	7.56
0.50	10.3
0.53	14.5
0.55	23.1
0.58	57.8
0.59	112.0

Example VI-3

A core sample initially saturated with oil is flooded with gas. The following data was obtained:

S_{gav}	$G_{ipv} \text{ pv}$
0.24	0.264
0.32	0.945
0.37	2.12
0.42	3.81
0.47	7.56
0.50	10.3
0.53	14.5
0.55	23.1
0.58	57.8
0.59	112.0

$$\mu_o = 2.25 \text{ cp}$$

$$\mu_g = 0.0185 \text{ cp}$$

Calculate and construct a f_g versus S_{go} plot. Convert S_{gav} to S_{go} . Determine k_g/k_o for each of the given saturations. Construct a graph of k_g/k_o versus S_{go} .

Solution:

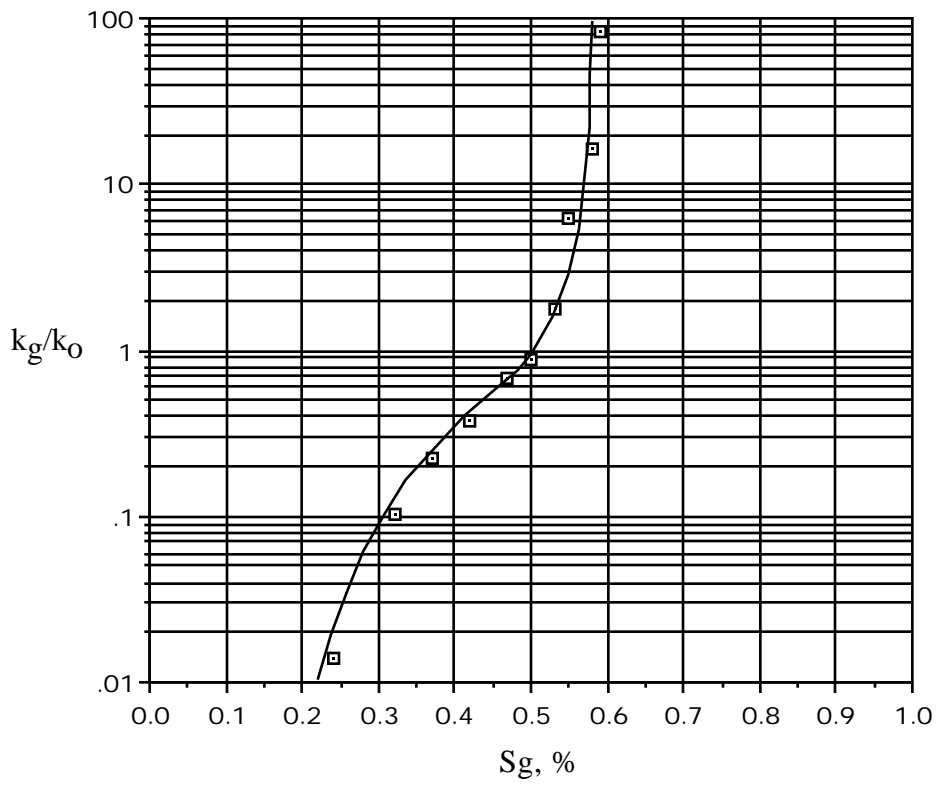
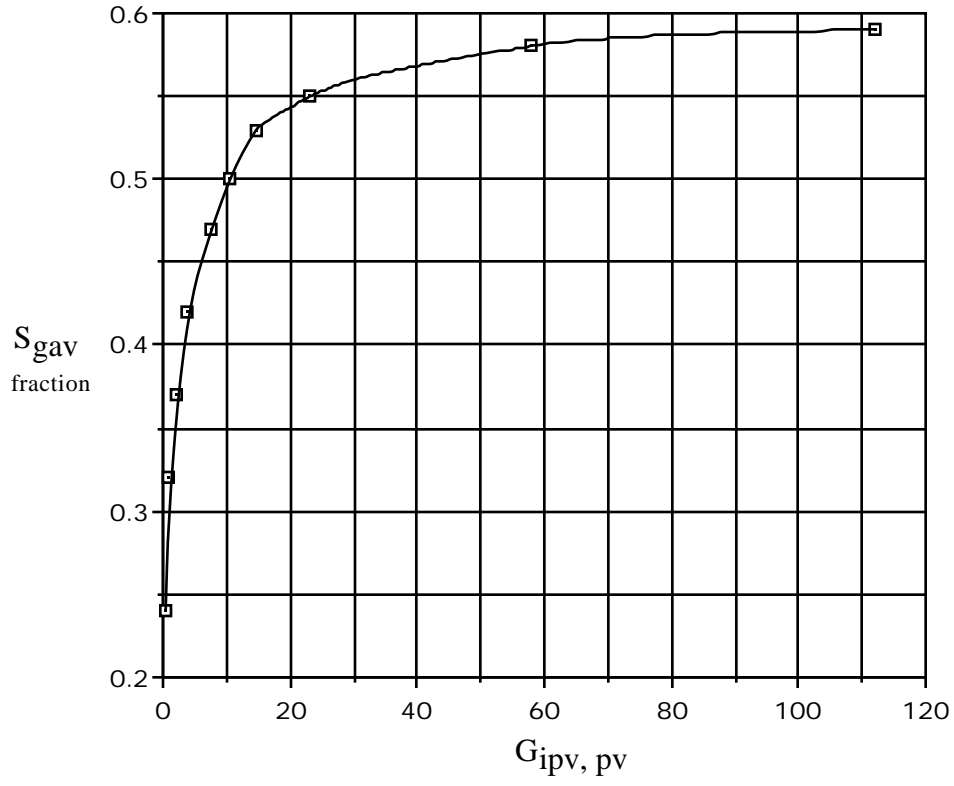
Plot S_{gav} vs. G_{ipv}

The slope from this plot is f_o .

$$S_{go} = S_{gav} - f_o G_{ipv}$$

$$k_g/k_o = \frac{1 - f_o}{f_o} \frac{\mu_o}{\mu_g}$$

S_{gav}	$G_{ipv} \text{ pv}$	f_o	S_{go}	k_g/k_o
0.24	.264	.375	.141	.0137
0.32	.94	.075	.249	.101
0.37	2.17	.0357	.294	.222
0.42	3.81	.0214	.338	.376
0.47	7.56	.0118	.381	.689
0.50	10.3	.0092	.405	.886
0.53	14.5	.0046	.463	1.78
0.55	23.1	.0013	.521	6.32
0.58	57.8	.0005	.550	16.4
0.59	112.0	.0001	.581	82.2



C. Field determination of permeability ratios

1. equations

$$\frac{q_g}{q_o} = \frac{\frac{k_g A p}{\mu_g L}}{\frac{k_o A p}{\mu_o L}}$$

where q_g = gas flow rate measured at reservoir conditions,
vol/time

q_o = oil flow rate measured at reservoir conditions,
vol/time

thus,

$$\frac{k_g}{k_o} = \frac{q_g \mu_g}{q_o \mu_o}$$

replace q_g/q_o with

$$\frac{q_g}{q_o} = \frac{B_g(R_p - R_s)}{5.615 B_o}$$

where B_g = formation volume factor of gas, res cu ft/scf

B_o = formation volume factor of oil, res bbl/STB

R_p = producing gas-oil ratio, scf/STB must include both
separator gas and stock tank gas)

thus

$$\frac{k_g}{k_o} = \frac{\mu_g B_g (R_p - R_s)}{\mu_o (5.615) B_o}$$

2. procedure

a. producing gas-oil ratio, R_p , and physical properties, B_g ,
 B_o , R_s , μ_g , μ_o must be determined at some known
reservoir pressure

b. saturations in reservoir, S_g or S_o , must be calculated from
production data and material balance calculations

Example VI-4

Discovery pressure for your well was 4250 psia, temperature is 200^oF, and initial producing gas-oil ratio was 740 SCF/STB. Stock tank oil gravity is 30^oAPI and surface gas gravity is 0.7. Production history and correlations indicate the bubble point at 3500 psia. Reservoir pressure is now 3000 psia. Producing gas-oil ratio is 18,100 SCF/STB. What is k_g/k_o in the reservoir at this time.

Solution:

Correlations covered in the fluid properties portion of this course yield the following value of the physical properties of the gas and oil at 3000 psia and 200^o F.

$$R_s = 560 \text{ SCF/STB}$$

$$B_o = 1.314 \text{ res. BBL/STB}$$

$$T_{pc} \text{ of gas} = 390 \text{ } ^\circ\text{R}$$

$$P_{pc} \text{ of gas} = 665 \text{ psia}$$

$$z = 0.86$$

$$B_g = 0.0282 z T/p = 5.34 \times 10^3 \text{ res cu ft/SCF}$$

$$\mu_g = 0.0192 \text{ cp}$$

$$\mu_o = 0.75 \text{ cp}$$

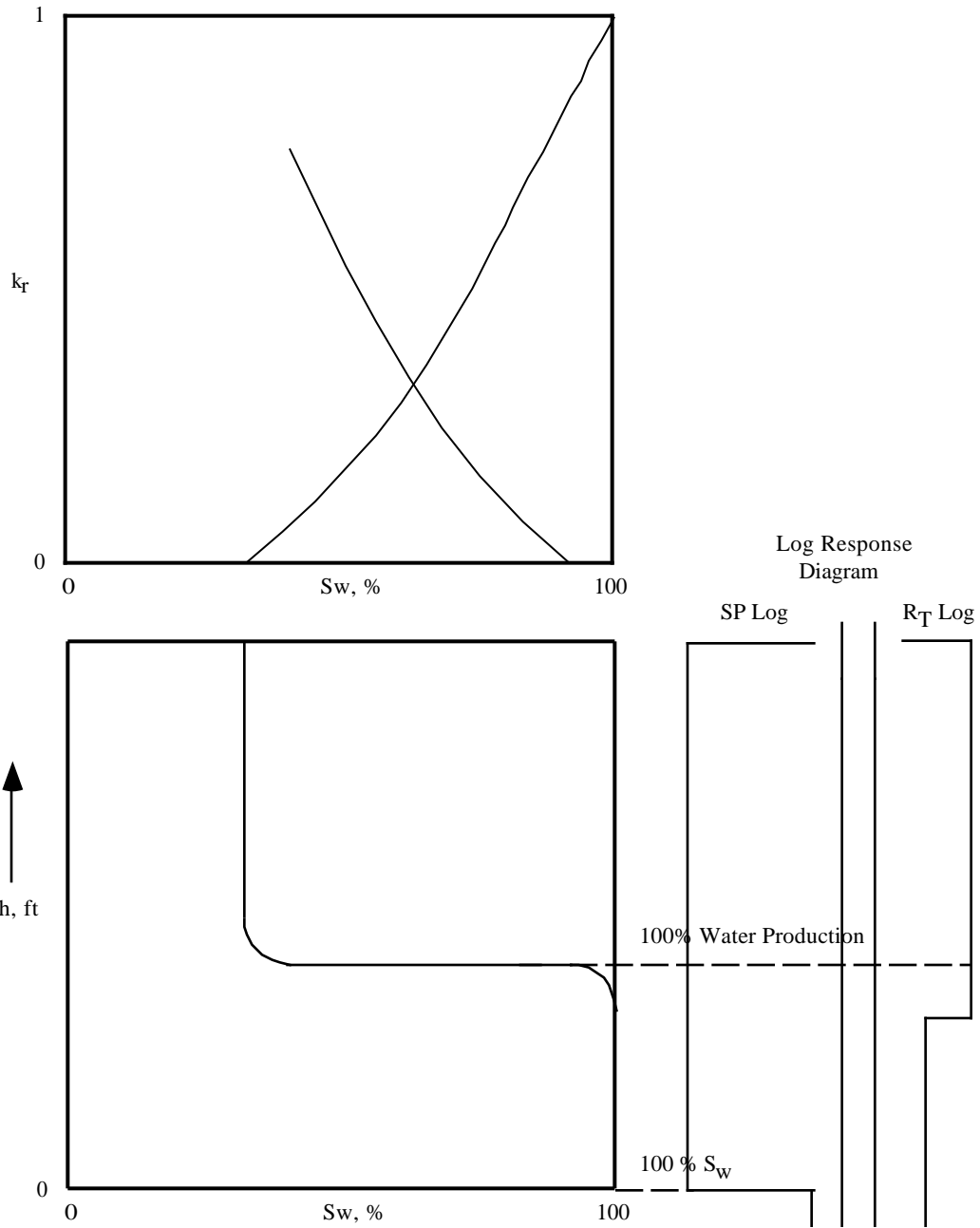
$$\frac{k_g}{k_o} = \frac{\mu_g B_g (R_p - R_s)}{\mu_o (5.615) B_o}$$

$$\frac{k_g}{k_o} = \frac{(0.0192)(5.34 \times 10^3)(18100 - 560)}{(0.75)(5.615)(1.314)}$$

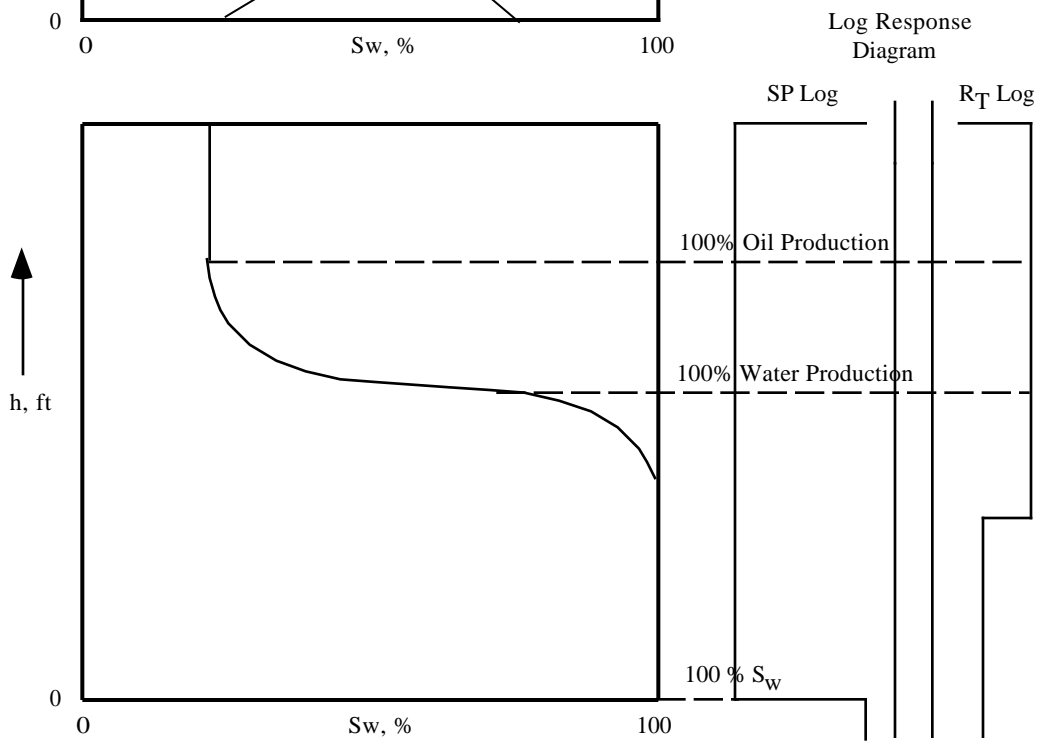
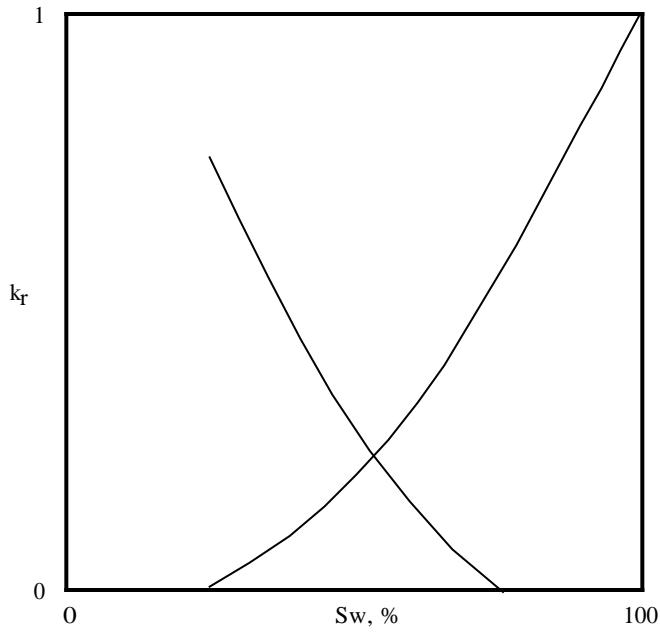
$$\frac{k_g}{k_o} = 0.325$$

VI) Uses of relative permeability data

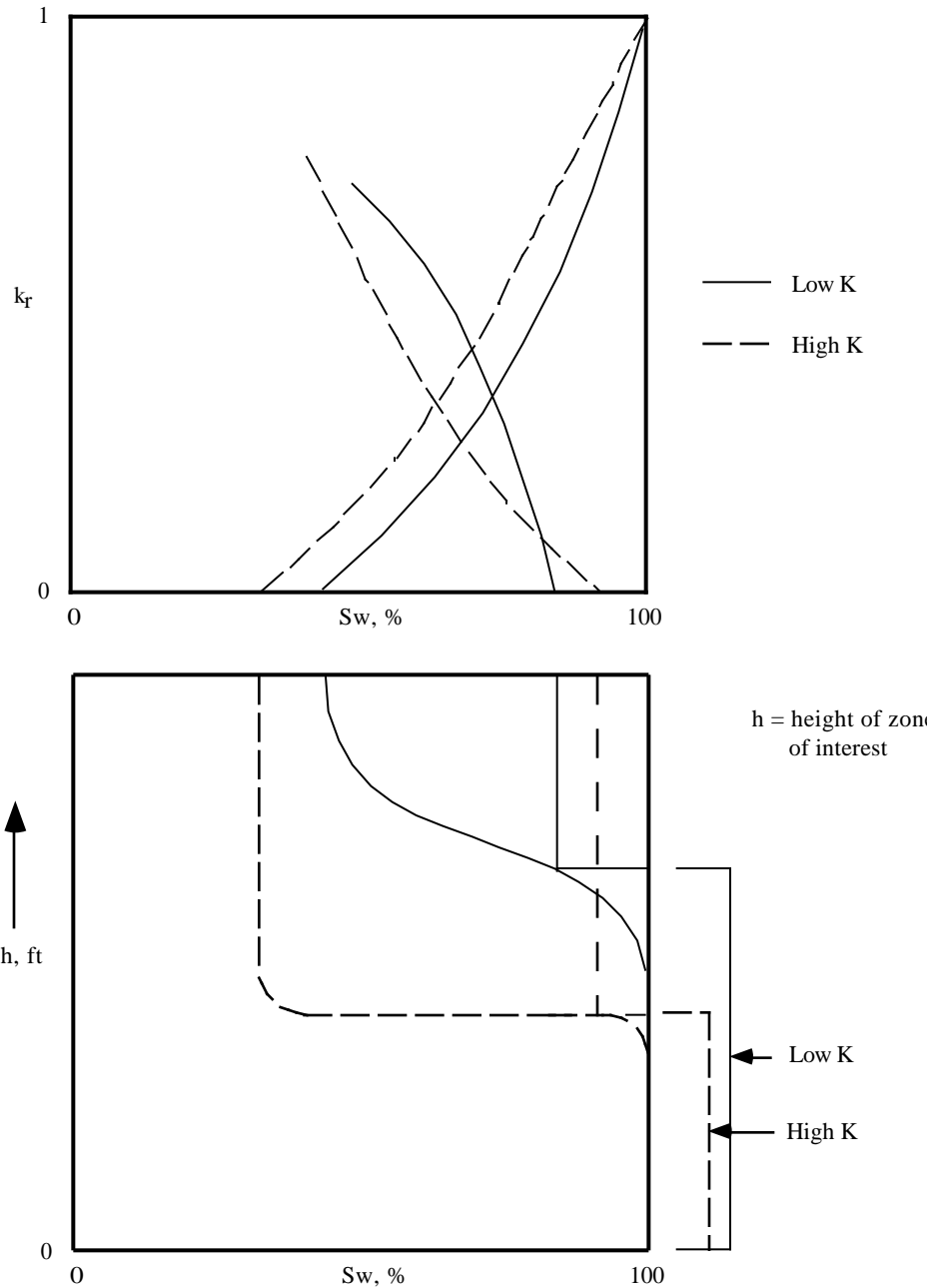
A. Determination of free water surface in reservoir (100% water production)



B. Determination of height of 100% oil production

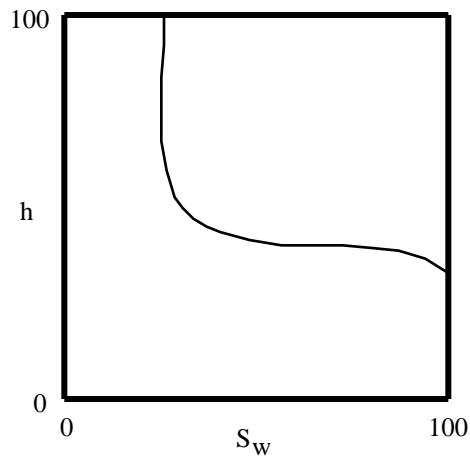
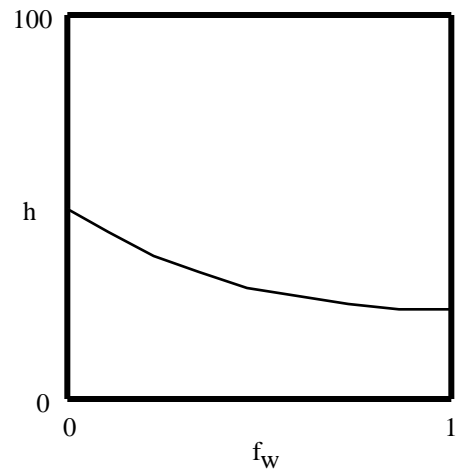
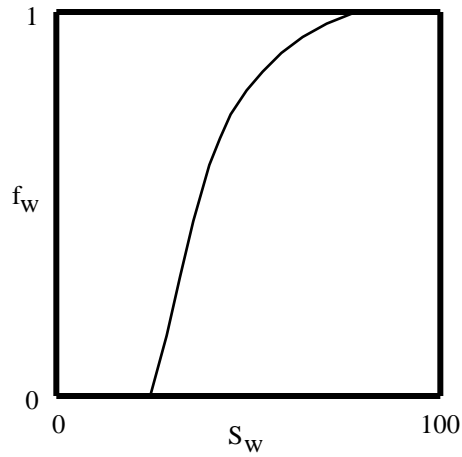


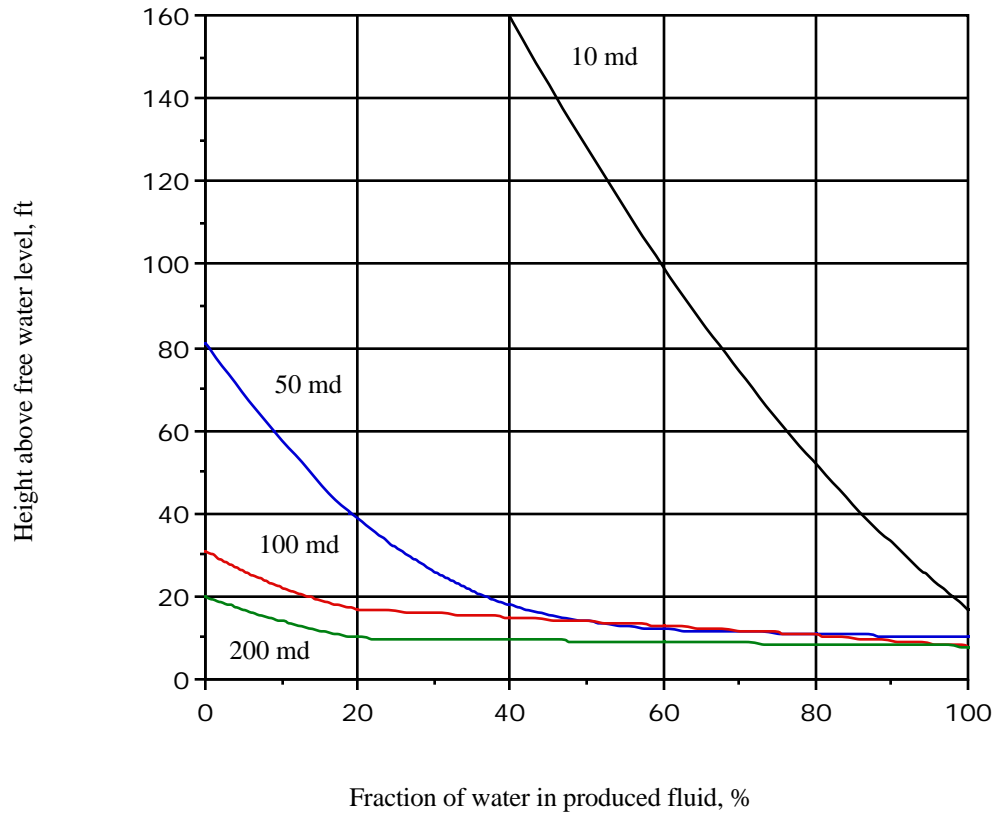
C. Effect of permeability on thickness of transition zone



D. Fractional flow of water as a function of height

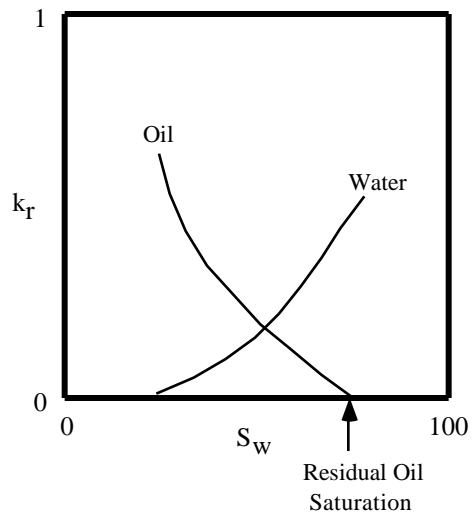
$$f_w = \frac{q_w}{q_{tot}} = \frac{q_w}{q_o + q_w} = \frac{\frac{k_w A P}{\mu_w L}}{\frac{k_o A P}{\mu_o L} + \frac{k_w A P}{\mu_o L}} = \frac{1}{1 + \frac{k_o \mu_w}{k_w \mu_o}}$$





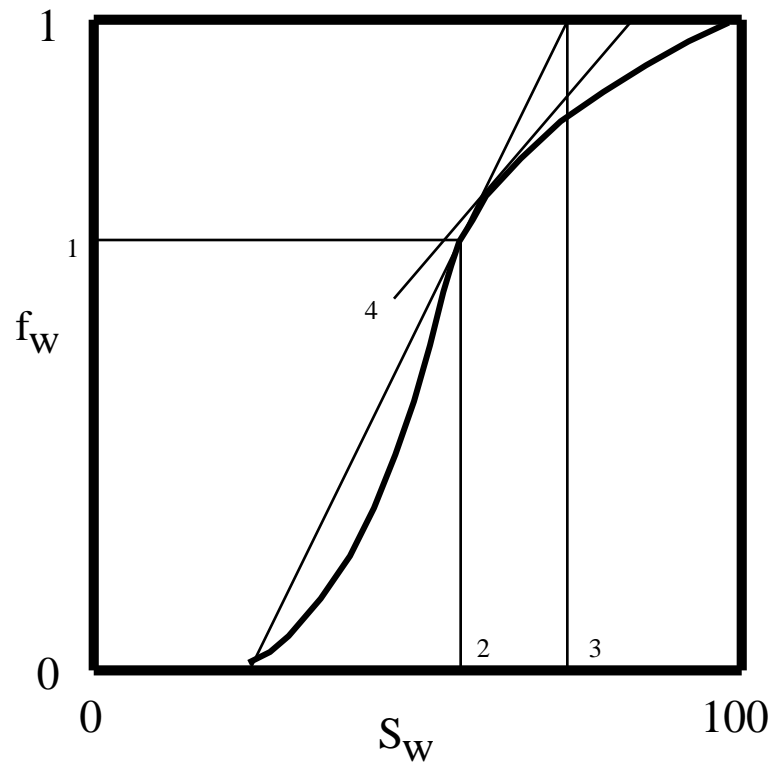
This figure indicates that lower permeabilities result in longer transition zones

E. Determination of residual fluid saturations

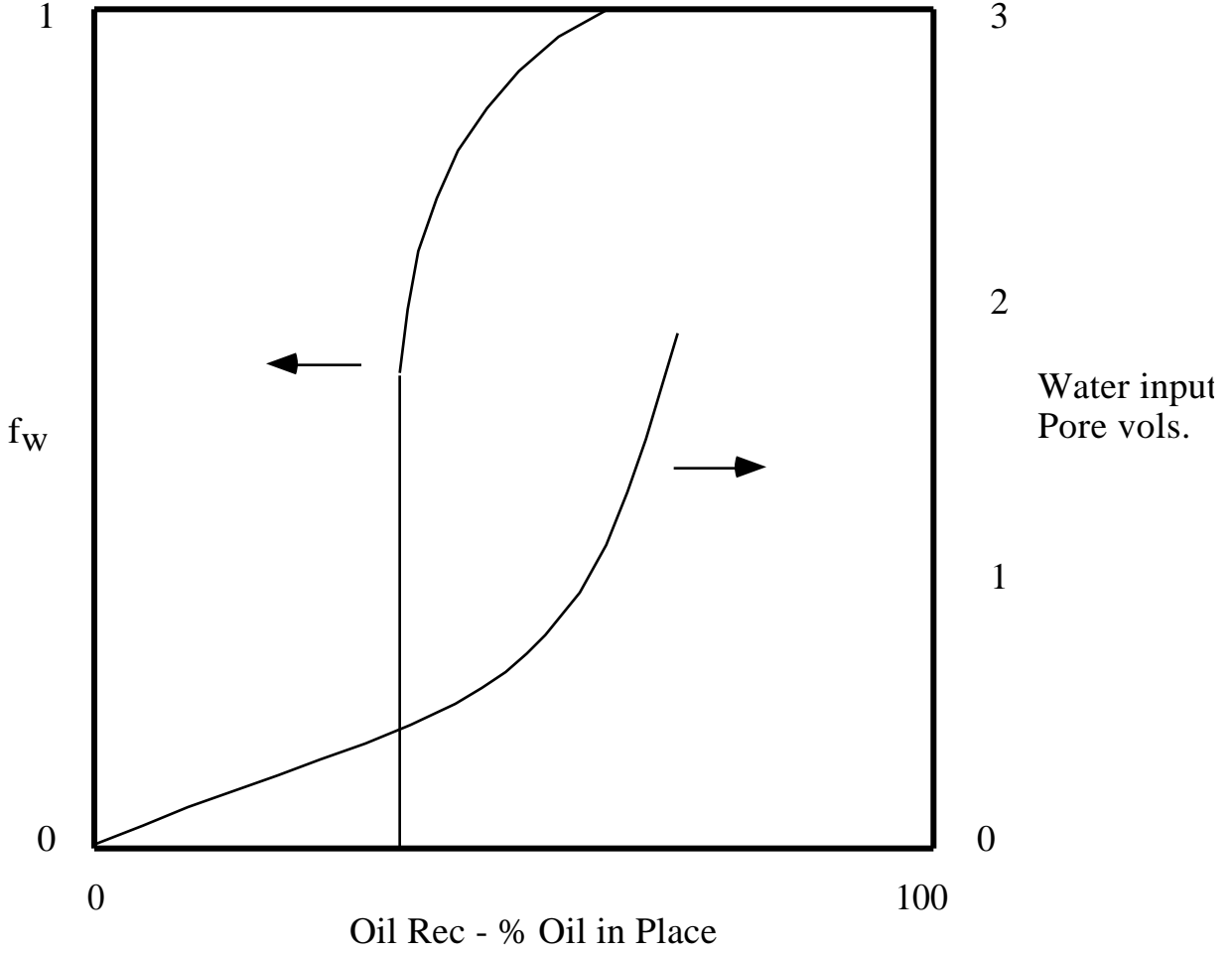


1. Imbibition curve used in water flood calculations
2. Maximum oil recovery = area (acre) x h(ft) x f x 7758 BBL/acre ft x S_w

F. Interpretation of fractional flow curve



1. f_w at water breakthrough
2. S_w at well at water breakthrough
3. $S_{w_{av}}$ in reservoir between wells at water breakthrough
4. $\frac{1}{\text{slope}} = \text{pore volume of water injected}$



VII. STATISTICAL MEASURES

D) Introduction

Usually we can not examine an entire "population" (i.e. we can not dig up an entire reservoir, cut it into plugs, and measure the porosity of every plug). We can only "sample" the population and use the properties of the sample to represent the properties of the population. Often we seek a single number (porosity or permeability) to represent the population (reservoir) for use in reservoir engineering calculations.

If the sample is representative of the population, we have a statistical basis for estimating properties of the population.

The sample data is said to be unclassified or classified depending on whether it is arranged or grouped in a particular order. Unclassified data is randomly arranged. The classification of data for a large number of samples will often provide additional information to help describe the physical properties of the population.

II) Frequency Distributions

It is often useful to distribute data into classes. The number of individuals belonging to each class is called the class frequency. A tabular arrangement of these data according to class is called a frequency distribution or frequency table. Sometimes classified data is called grouped data.

The division of unclassified data into classified data is accomplished by allocating all data to respective class intervals. The midpoint of each class interval is called the class mark.

Rules for forming frequency distributions

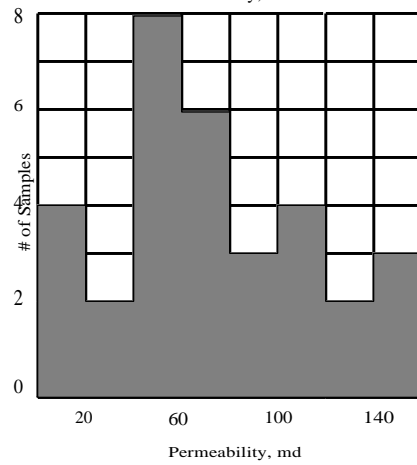
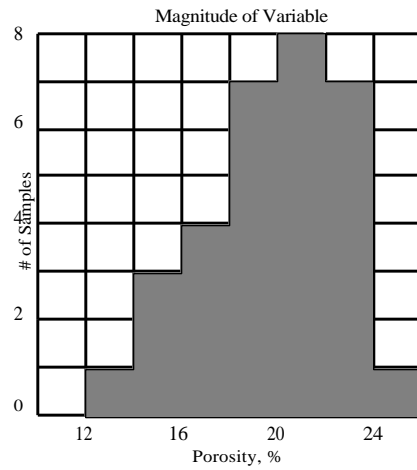
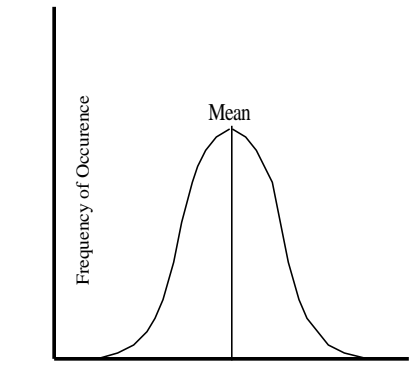
- A. Determine largest and smallest numbers in the raw data.
- B. Divide the range of numbers into a convenient number of equal sized class intervals. The number of class intervals depends on the data but is usually taken between 5 and 20 in number.
- C. The number of observations for each class interval is the class frequency.
- D. The relative frequency of a class is the frequency of the class divided by the total frequency of all the classes.

III) Histogram

A histogram is a graphical representation of a frequency distribution.

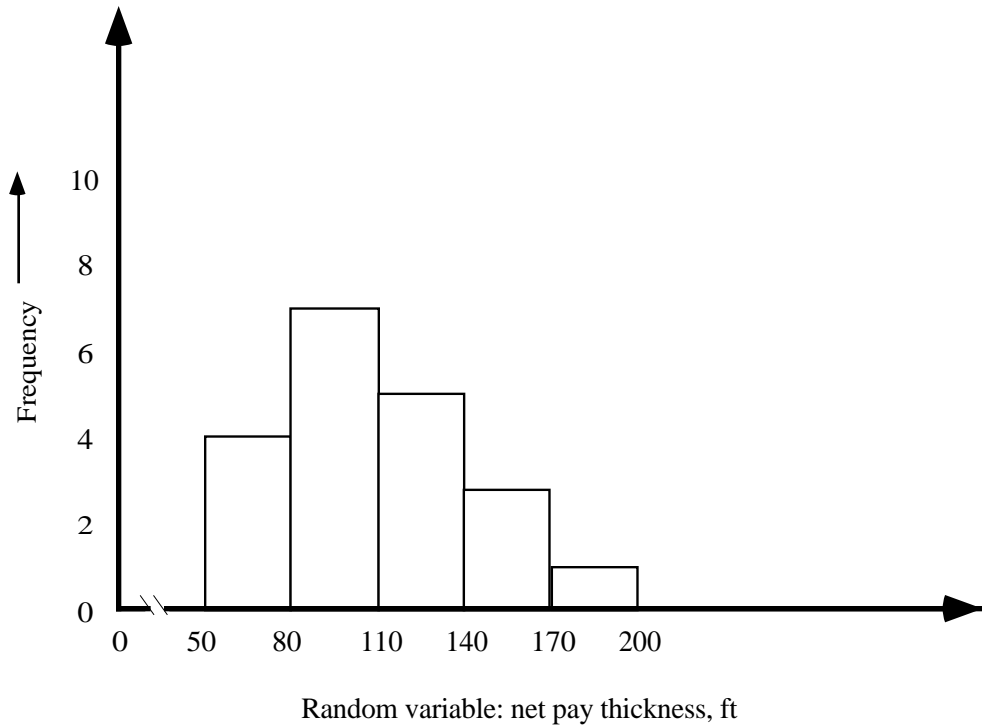
The vertical scale is the number of data points - the class frequency - in each class.

The width of the rectangle corresponds to the class interval.

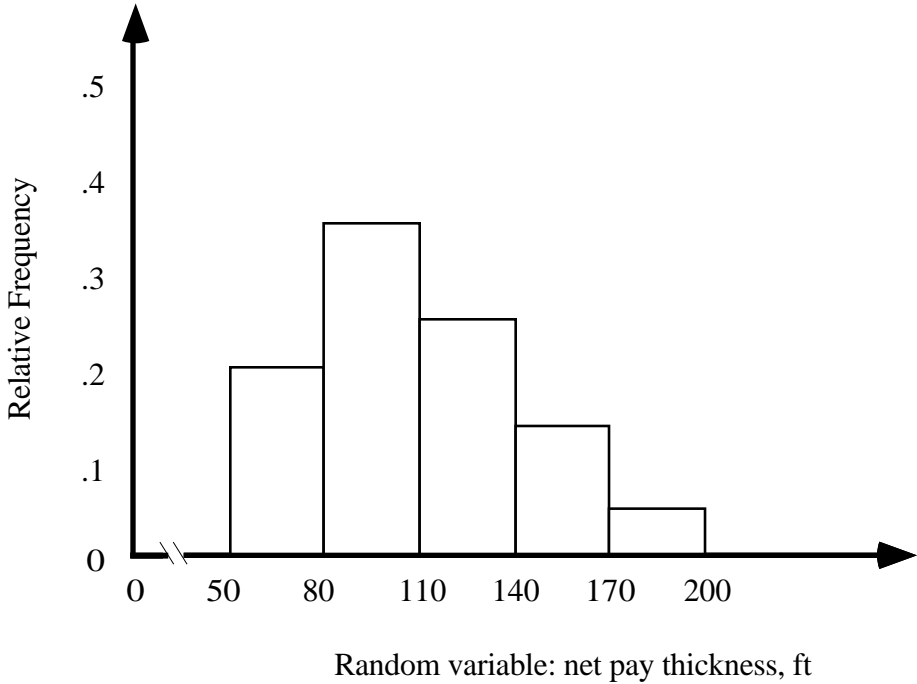


Net pay thickness data from 20 wells summarized as relative frequency data

Range of thickness, ft.	Frequency (No. of wells having thickness values in the range)	Relative Frequency (No. of wells having thickness values in each range, fraction of total wells)	Relative Frequency as percentage.
50-80	4	0.20	20%
81-110	7	0.35	35%
111-140	5	0.25	25%
141-170	3	0.15	15%
171-200	1	0.05	5%
	20	1.00	100%

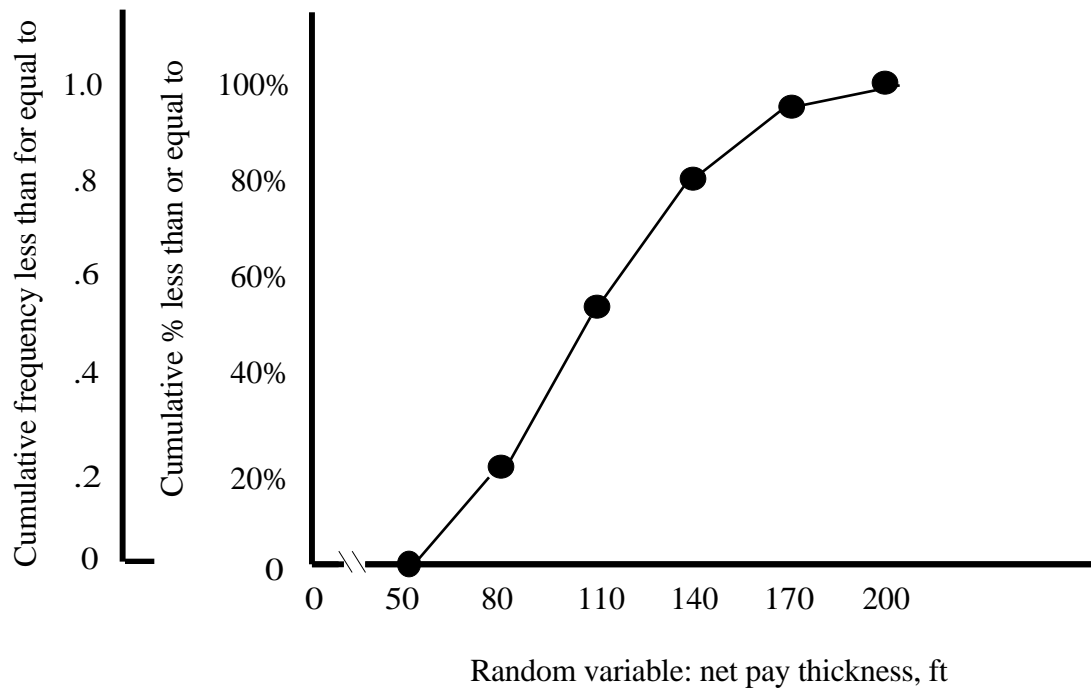


Sometimes the relative frequency is plotted on a histogram

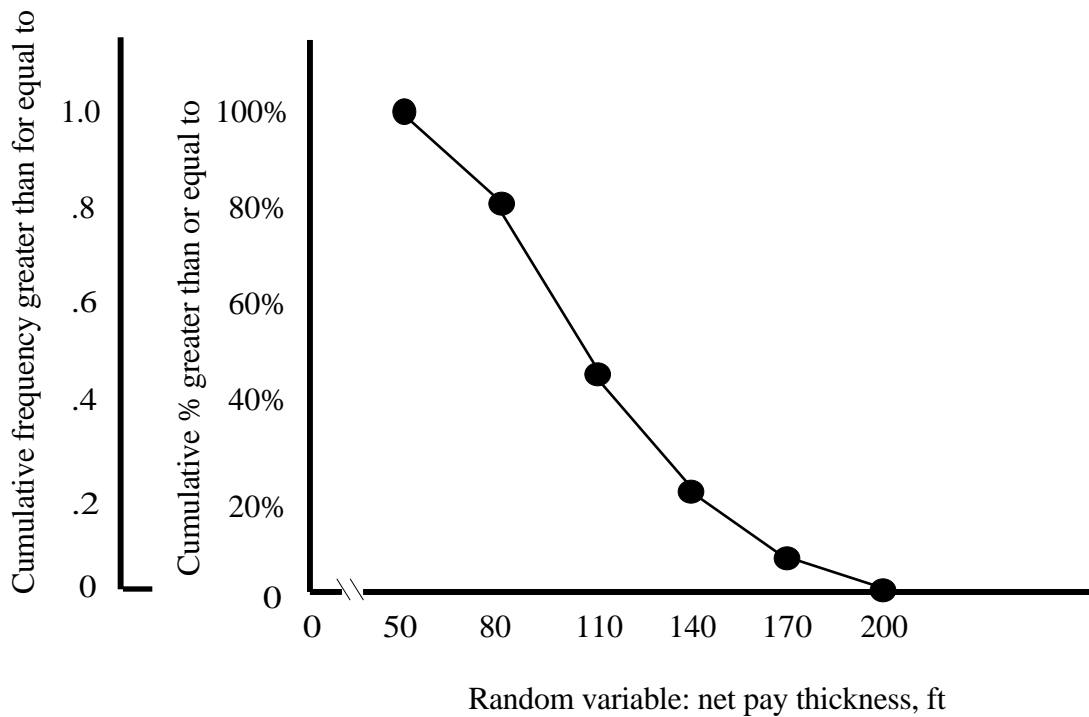


IV) Cumulative Frequency Distributions

Relative frequencies are summed and plotted at the higher ends of the class intervals to create a "cumulative frequency less than or equal to" distribution



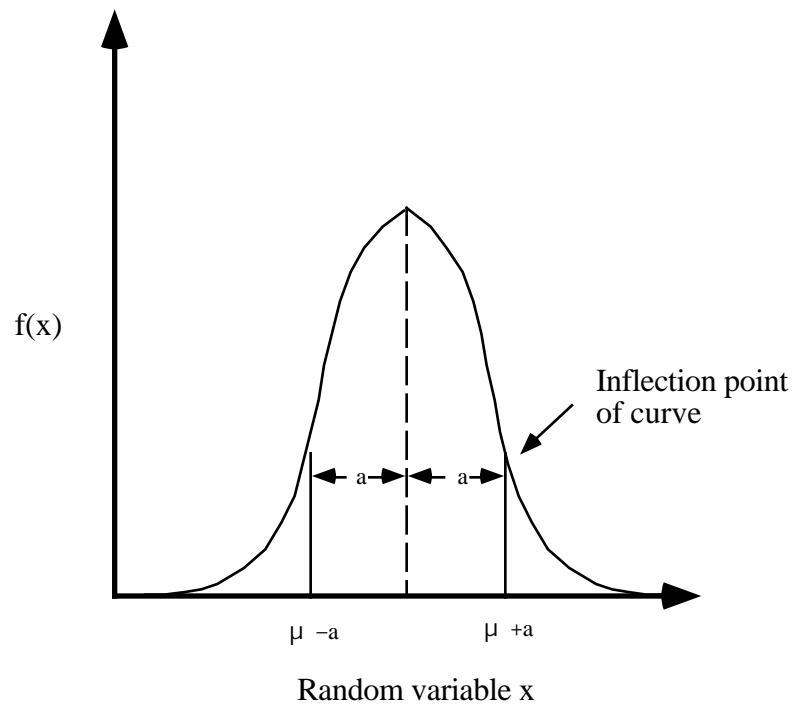
Occasionally a "cumulative frequency greater than or equal to" distribution is plotted. Relative frequencies are summed from the highest class interval and plotted at the lower ends of the intervals



Probability graph paper has been constructed so that data from certain probability distributions plot as a straight line. Different probability paper is used for data with different distributions

V) Normal Distribution

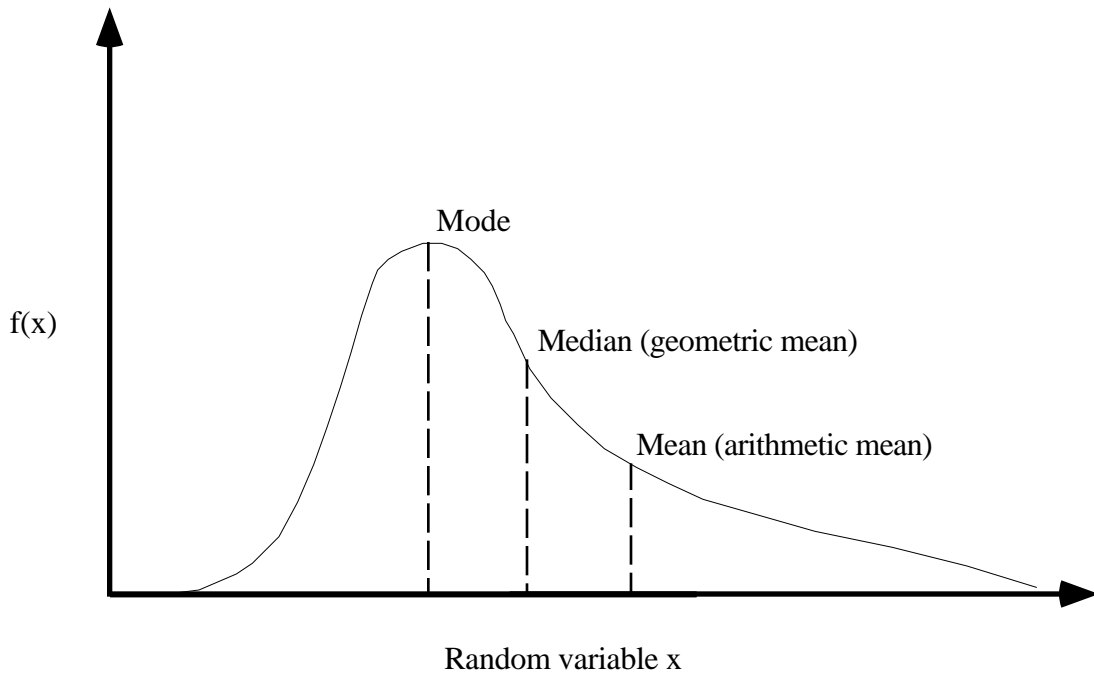
The normal distribution is continuous probability distribution having a symmetrical shape similar to a bell, sometimes called a Gaussian distribution.



This distribution is completely and uniquely defined by two values - the mean, m , and standard deviation, σ .

VI) Log Normal Distribution

The log normal distribution is a continuous probability distribution that appears similar to a normal distribution except that it is skewed to one side. It is also called an exponential distribution.



This distribution can also be completely and uniquely defined by the mean, m , and the standard deviation, σ .

If random variable x_i are log normally distributed then the variables $\log x_i$ are normally distributed.

VII) Measures of Central Tendency

An average is a value which is typical or representative of a set of data. When a set of data is arranged according to magnitude the average value tends to lie in the center of these data. These averages are called measure of central tendency.

mean - the arithmetic average value of the samples

$$\mu = \frac{\sum_{i=1}^n x_i}{n}$$

where x_i = values of the variable of interest for each sample

n = number of samples

median - the value equalled or exceeded by exactly one-half of the samples.

mode - the value which occurs with the greatest frequency

geometric mean - the n th root of the product of n numbers

$$\mu_g = (x_1 \cdot x_2 \cdot x_3 \cdot \dots \cdot x_n)^{1/n}$$

$$\mu_g = \left(\prod_{i=1}^n x_i \right)^{1/n}$$

where μ_g = the geometric mean

VIII) Measures of Variability (dispersion)

A measure of central tendency is the "average" or expected value of a set of variables, however it does not show the spread or variability of the variables on either side of the central tendency.

- A. Standard deviation - The square root of the mean of the squared deviations about μ , where deviation is defined as the distance of the variable from μ .

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \mu)^2}{n-1}$$

where σ^2 is the variance

σ is the standard deviation

- B. Mean deviation - another measure of the dispersion about the central tendency

$$MD = \frac{\sum_{i=1}^n |x_i - \mu|}{n}$$

For classified data

$$\sigma^2 = \frac{\sum_j f_j (x_j - \mu)^2}{\sum_j f_j}$$

where f_j = frequency for each class

x_j = class mark

or

$$\sigma^2 = \sum_j f_{rj} (x_j - \mu)^2$$

where f_{rj} = relative frequency for each class

IX) Normal Distribution

Porosity data is usually assumed to have a normal distribution.

For the normal distribution the mean, median and mode have the same numerical values. They are identical measures of central tendency.

Thus, for unclassified data

$$\mu = \frac{\sum_{i=1}^n x_i}{n}$$

where i refers to each individual data point and, for classified data

$$\mu = \frac{\sum_j f_j x_j}{\sum_j f_j}$$

where j refers to each class interval

f_j is the frequency of the class

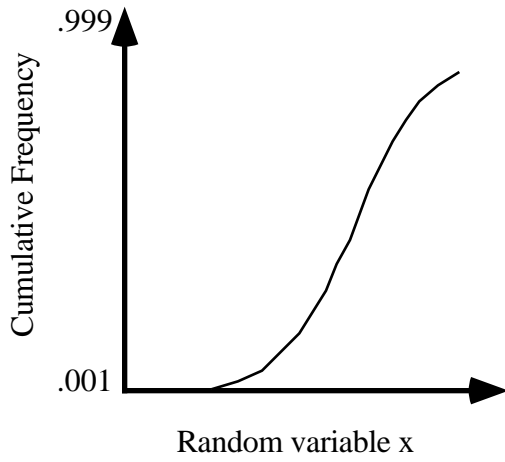
x_j is the class mark

or

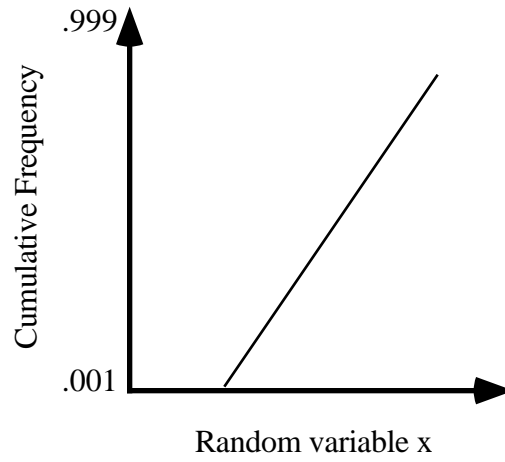
$$\mu = \sum_j f_{rj} x_j$$

where f_{rj} is the relative frequency of the class.

x_j is the class mark

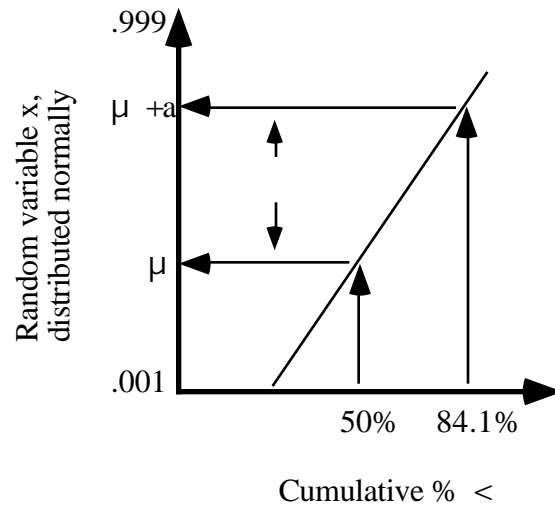
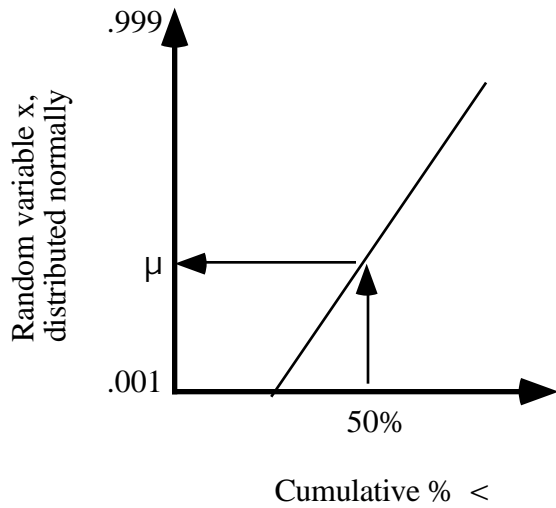


Cumulative frequency plotted on coordinate graph paper



Cumulative frequency plotted on normal probability paper

Normal probability graph paper



Porosity and permeability data from a well in the Denver-Julesburg Basin

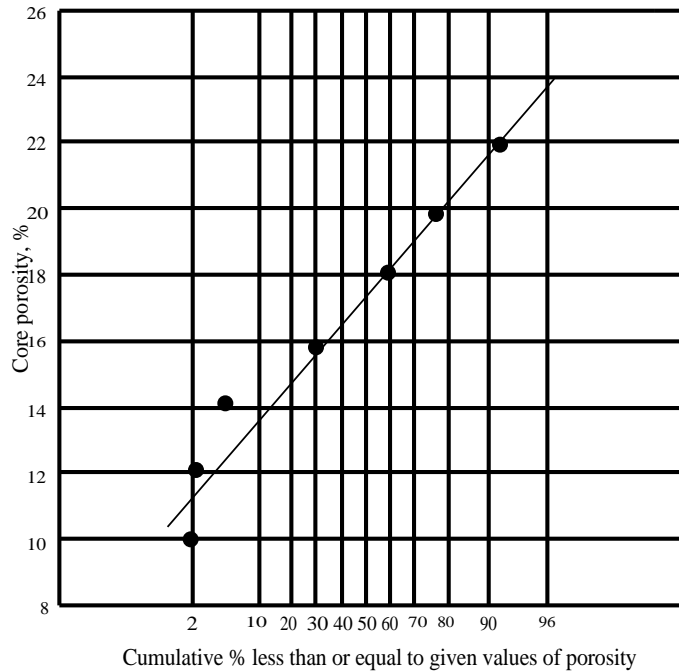
i	Porosity Interval Percent	Frequency f_i	Interval Midpoint x_i	$f_i x_i$	$(x_i - \mu)$	$(x_i - \mu)^2$	$f_i(x_i - \mu)^2$
1	7.0<x<10.0	1	8.5	8.5	-9.2	84.64	84.64
2	10.0<x<12.0	0	11.0	0.0	-6.7	44.89	0.00
3	12.0<x<14.0	1	13.0	13.0	-4.7	22.09	22.09
4	14.0<x<16.0	10	15.0	150.0	-2.7	7.29	72.90
5	16.0<x<18.0	12	17.0	204.0	-0.7	0.49	5.88
6	18.0<x<20.0	8	19.0	152.0	+1.3	1.69	13.52
7	20.0<x<22.0	7	21.0	147.0	+3.3	10.89	76.23
8	22.0<x<25.0	3	23.5	70.5	+5.8	33.64	100.92
		42		745.0			376.18

$$\mu = \frac{\sum f_i x_i}{\sum f_i} = \frac{745.0}{42} = 17.7\%$$

$$\sigma^2 = \frac{\sum f_i (x_i - \mu)^2}{\sum f_i} = \frac{376.18}{42} = 8.96$$

$$= \sqrt{8.96} = 2.99\%$$

Porosity Interval, %	Frequency	Cumulative Frequency Than or Equal to Upper Limit of Interval	Cumulative Frequency Expressed as Percentage
7.0<x<10.0	1	1	2.4%
10.0<x<12.0	0	1	2.4%
12.0<x<14.0	1	2	4.8%
14.0<x<16.0	10	12	28.6%
16.0<x<18.0	12	24	57.1%
18.0<x<20.0	8	32	76.2%
20.0<x<22.0	7	39	92.9%
22.0<x<25.0	3	42	100.0%
	42		



at 50th percentile

$$f = 17.7\%$$

at 84th percentile

$$f + = 20.7\%$$

$$= 20.7 - 17.7 = 3\%$$

X) Log Normal Distribution

Permeability data is usually assumed to have a log normal distribution.

For log normal distribution the median, mode, and mean have different numerical values. The median has been chosen as the value of central tendency which best represents the data.

The median of a log normal distribution is equal to the geometric mean.

Thus, for unclassified data

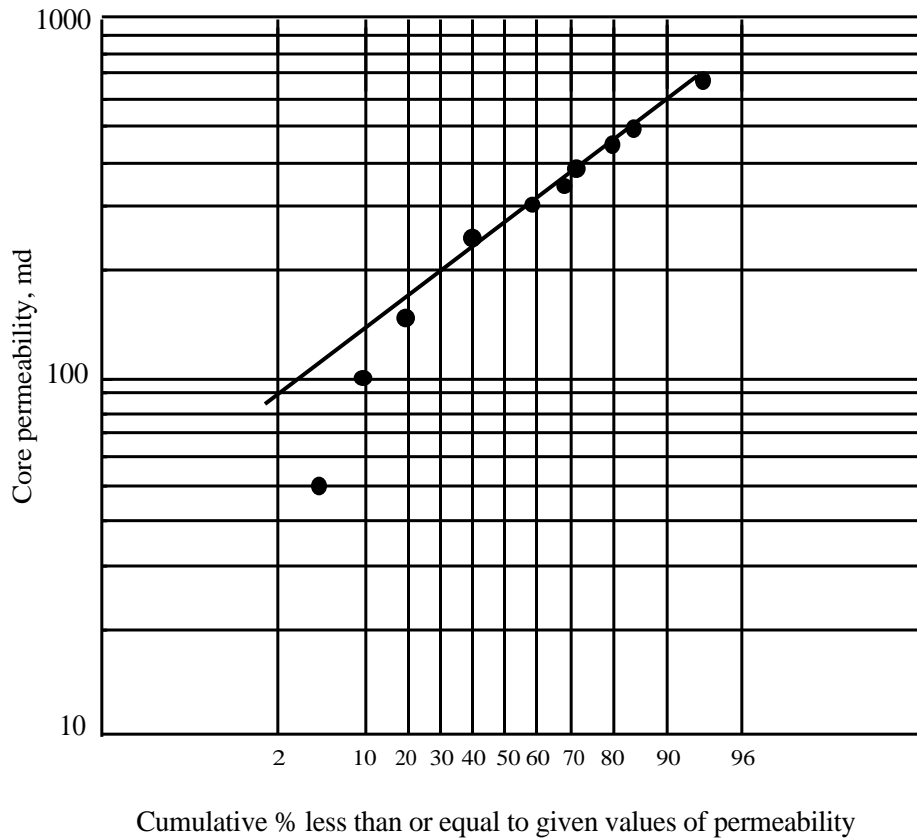
$$\mu = \left(\prod_{i=1}^n x_i \right)^{1/n}$$

or

$$\log(m) = \frac{1}{n} \sum_{i=1}^n \log(x_i)$$

Permeability Interval (millidarcies)	Frequency	Cumulative Frequency Less Than or Equal to Upper Limit of Interval	Cumulative Frequency Expressed As Percentage
0-50	2	2	4.8%
51-100	2	4	9.5%
101-150	4	8	19.0%
151-200	4	12	28.6%
201-250	4	16	38.1%
251-300	8	24	57.1%
301-350	4	28	66.7%
351-400	2	30	71.4%
401-450	4	34	81.0%
451-500	1	35	83.3%
501-700	5	40	95.2%
701-1000	2	42	100.0%
	42		

Lognormal probability graph paper



PETE 306 HANDOUT 3/5/92

Calculation of Permeability using Capillary Pressure Data

Purcell Approach
(ABW: pages 167-172)

Three basic considerations:

1. Capillary pressure in a capillary,

$$P_c = \frac{2 \sigma \cos \theta}{r}$$

2. Capillary flow: Poiseuille's law

$$q_i = \frac{r_i^4 p}{8 \mu L}$$

3. Darcy's equation,

$$q_t = \frac{kA p}{\mu L}$$

$$P_c = \frac{\text{dynes}}{\text{cm}^2}$$

$$k = \text{cm}^2$$

$$= \frac{\text{dynes}}{\text{cm}}$$

$$A = \text{cm}^2$$

$$r = \text{cm}$$

$$L = \text{cm}$$

$$q = \frac{\text{cm}^3}{\text{sec}}$$

$$\mu = \text{poise} = \frac{\text{dynes-sec}}{\text{cm}^2}$$

$$p = \frac{\text{dyne}}{\text{cm}^2}$$

Let $V_i = r_i^2 L$,

then the flow rate in a capillary is

$$q_i = \frac{V r_i^2 p}{8 \mu L^2}, \quad V = \text{cm}^3$$

Since

$$r_i = \frac{2 \cos}{P_{ci}}$$

$$q_i = \frac{\left(\frac{\cos}{P_{ci}}\right)^2 \frac{V_i}{2 \mu L^2} p}{}$$

For a bundle of n capillary tubes,

$$q_t = \frac{i=n}{i=1} \left[\frac{\left(\frac{\cos}{P_{ci}}\right)^2 p}{2 \mu L^2} \right] \left[\frac{i=n}{i=1} \frac{V_i}{(P_{ci})^2} \right]$$

Since,

$$q_t = \frac{k A p}{\mu L}$$

$$k = \frac{\left(\frac{\cos}{P_{ci}}\right)^2 \frac{i=n}{i=1} V_i}{2 A L (P_{ci})^2}$$

Define the fractional volume of ith capillary

$$S_i = \frac{V_i}{V_T}, \quad S_i = \text{fraction}$$

and

$$= \frac{V_T}{AL}, \quad = \text{fraction}$$

$$k = \frac{(\cos)^2}{2} \sum_{i=1}^{i=n} \frac{S_i}{(P_{ci})^2}$$

Introducing a lithology factor for deviation of the actual pore space,

$$k = \frac{(\cos)^2}{2} \sum_{i=1}^{i=n} \frac{S_i}{(P_{ci})^2}$$

In integral form,

$$k = \frac{(\cos)^2}{2} \int_{S=0}^{S=1} \frac{dS}{P_c^2}$$

PETE 306 HANDOUT 4/16/92

Calculation of Relative Permeabilities using Capillary Pressure Data

Purcell and Burdine Approach (ABW: pages 196-199)

Purcell approach:

The absolute permeability may be expressed as

$$k = \frac{(\cos \theta)^2}{2} \int_{S=0}^{S=1} \frac{dS}{P_c^2}$$

The effective permeability of the wetting phase may be expressed as

$$k_{wt} = \frac{(\cos \theta)^2}{2} \int_{S=0}^{S=S_{wt}} \frac{dS}{P_c^2}$$

The relative permeability of the wetting phase is the ratio of the wetting phase effective permeability to the absolute permeability

$$k_{rwt} = \frac{k_{wt}}{k} = \frac{\int_{S=0}^{S=S_{wt}} \frac{dS}{P_c^2}}{\int_{S=0}^{S=1} \frac{dS}{P_c^2}}$$

Similarly, the effective permeability of the nonwetting phase may be expressed as

$$k_{nwt} = \frac{(\cos \theta)^2}{2} \int_{S=S_{wt}}^{S=1} \frac{dS}{P_c^2}$$

The relative permeability of the nonwetting phase is the ratio of the nonwetting phase effective permeability to the absolute permeability

$$k_{rnwt} = \frac{k_{nwt}}{k} = \frac{\int_{S=S_{wt}}^{S=1} \frac{dS}{P_c^2}}{\int_{S=0}^{S=1} \frac{dS}{P_c^2}}$$

Burdine Approach:

Burdine considered the tortuosity factors for one-phase and multiphase systems and modified the Purcell equations for the effective permeabilities.

$$r_{wti} = \frac{i}{wti}$$

The relative permeability of the wetting phase is the ratio of the wetting phase effective permeability to the absolute permeability

$$k_{rwt} = \frac{k_{wt}}{k} = (r_{wt})^2 \frac{\int_{S=0}^{S=S_{wt}} \frac{dS}{P_c^2}}{\int_{S=0}^{S=1} \frac{dS}{P_c^2}}$$

The tortuosity ratio is related to the minimum wetting-phase saturation S_m , as

$$r_{wt} = \frac{S_{wt} - S_m}{1 - S_m}$$

Similarly, the relative permeability of the nonwetting phase is the ratio of the nonwetting phase effective permeability to the absolute permeability

$$k_{rnwt} = \frac{k_{nwt}}{k} = (r_{nwt})^2 \frac{\int_{S=S_{wt}}^{S=1} \frac{dS}{P_c^2}}{\int_{S=0}^{S=1} \frac{dS}{P_c^2}}$$

The tortuosity ratio for the nonwetting phase is related to the minimum wetting-phase saturation, S_m , and the equilibrium saturation to the nonwetting phase, S_e , as

$$r_{nwt} = \frac{S_{nwt} - S_e}{(1 - S_m) - S_e}$$