Practical Use of Modern Well Test Analysis

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ABSTRACT

Analysis of short time well test data by log-log type curve was introduced in 1970. The first type curve concerned the wellbore storage effect with damaged permeability in an annular shaped region adjacent to the sand face. Later type curves for vertical and horizontal fractures were added. Practical experience with these curves now indicates a great deal of useful information may be extracted from complete matching of both log-log and conventional semi-log curves. Examples include almost certain detection of the correct semi-log straight line, as well as detection and correction of errors in data. On the other hand, some early claims for log-log type curves appear to be incorrect. The purpose of this paper is to summarize useful findings to date with field case illustrations.

INTRODUCTION

Based upon fundamental studies of transient flow of fluids through porous media by Hurst in 1934,1 van Everdingen and Hurst in 1949,2 Muskat,3 the two pressure buildup studies by Horner4 in 1950, and by Miller, Dyes and References and illustrations at end of paper.

Hutchinson5 in 1951, there began an intensive study of pressure transient (well test) analysis which continues vigorously to this day. Both Refs. 4 and 5 demonstrated that it was possible to determine the effective permeability to the flowing fluid phase by means of an appropriate graph of pressures measured vs. time after shut-in of a well previously produced at a constant rate. However, both studies contained ideas sufficiently different to cause a great deal of confusion.

Figure 1 is a schematic of a Horner pressure buildup graph which demonstrates some of the problems. Horner taught that a graph of buildup pressures vs. a logarithm of a time ratio involving the sum of the producing and shut-in time to the shut-in time should yield a straight line whose slope was inversely proportional to the permeability. The correct Horner straight line for permeability determination is represented by the straight segment on Fig. 1 indicated by the letter B.

Horner also showed that for wells in closed boundary drainage systems, eventually the pressure would stop rising, become static and provide a measure of a mean pressure within the drainage boundary of the well. It
was not necessary to close the well in long enough to reach the static pressure. It was possible to compute the eventual static pressure by means of a pressure correction function first presented by Horner. An example of a buildup curve going to a static pressure may be seen by following the semi-log straight line $B$ up to the portion of the curve labeled $G$, which shows the pressures leveling off and becoming static.

Horner also showed that if a well were close to a linear barrier such as a sealing fault, buildup pressures could bend upwards, forming another straight line of slope twice the initial semi-log straight line. An example of this sort of effect can be seen on Fig. 1 by comparing the semi-log straight line labeled $B$ with the portion of the buildup graph labeled $D$. Unfortunately, within a few years other papers were published which indicated that a bend upwards in buildup pressure could also be caused by the effect of two zones commingled within a single wellbore or by wells being off-center within an asymmetric drainage shape.

The Miller-Dyes-Hutchinson study presented information on the effect of either a closed outer boundary or a constant pressure outer boundary. However, it also indicated that buildup pressures should be graphed against the logarithm of the shut-in time only, as long as the well had been produced a long time before shut-in. Either a closed outer boundary or a constant pressure outer boundary (for water drive) could lead to pressures bending over and becoming static.

Miller-Dyes-Hutchinson also investigated the effect of near-well permeability changes and wellbore storage phenomena. They showed that if the well were stimulated, buildup pressures should approach the correct semi-log straight line from above. This is indicated by the early line labeled $C$ on Fig. 1. Figure 1 is a Horner, rather than a Miller-Dyes-Hutchinson graph. Although they are not the same in detail, the general characteristics of the two buildup graphs are similar. Miller-Dyes-Hutchinson also showed that if a well were damaged; that is, if the permeability were reduced in the immediate vicinity of the wellbore, the shut-in pressures would approach the semilog straight line from below, as indicated by the curve labeled $A$ on Fig. 1. They also investigated the effect of the phenomena called "after flow."

After flow phenomena results because shutting in a well at the surface does not cause the flow rate to go to zero at the sandface immediately. Fluid continues to enter the wellbore and pressurize the wellbore until the formation pressure becomes static. The flow through the sand face into the well after shut in was thus termed "afterflow." This phenomena is precisely the same as the phenomena termed "annulus unloading" by van Everdingen and Hurst in 1949 for the case where a well was opened at the surface and produced. They observed that if a well were produced at a constant rate starting at time zero by opening the valve at the surface, the first production from the well would be complicated by the fact that portions would come from the lowering of the liquid level in the annulus, thus the term annulus unloading. Both afterflow and annulus unloading involve storage of fluids in the wellbore. Both phenomena are now termed wellbore storage effects.

Van Everdingen and Hurst presented quantitative information for the annulus unloading case in the form of log-log graphs of dimensionless pressure vs. dimensionless time for various dimensionless wellbore storage constants. This 1949 graph was the first wellbore storage log-log type curve. (It appears that the earliest attempt to use the wellbore unloading effect quantitatively was made by Moore, Schilthuis and Hurst in 1933.)

Miller, Dyes, and Hutchinson found that the afterflow effect on a pressure buildup curve looked very much like wellbore damage. The curve labeled $A$ on Fig. 1 was typical of wellbore storage or afterflow as well as a damaged wellbore. This raised the obvious question as to whether a well that was physically stimulated (by hydraulic fracturing, acidizing, etc.) could have such a large afterflow effect that the well would appear to be damaged.

In 1955, Perrine prepared a review of pressure buildup literature which summarized the findings of Miller, Dyes, and Hutchinson and Horner. Perrine referred to the Horner style of pressure buildup graphing as the "van Everdingen-Hurst," or "VEH" method. He discussed the difference between the Horner style of pressure buildup graph and the Miller, Dyes, Hutchinson style of graph. Because Horner introduced his graph by means of mathematical expressions correct for a well in an infinitely large system, Perrine concluded that the Horner (VEH) pressure buildup graph was only correct for new wells in very large reservoirs. On the other hand, the Miller-Dyes-Hutchinson study had been developed under the assumption that the wells had been produced either to steady or to pseudo-steady state prior to shut-in. For this reason, Perrine concluded that the Miller-Dyes-Hutchinson graph was most appropriate for old wells in fully-developed reservoirs. Although logical, these conclusions proved to be incor-
rect. Ramey and Cobb\(^9\) showed on purely empirical bases that the Horner style of graphing usually produces much longer semi-log straight lines than the Miller-Dyes-Hutchinson graph for any duration producing period. Furthermore, the Miller-Dyes-Hutchinson graph can lead to serious problems in identification of the correct straight line if the producing time prior to shut-in is not very long. Nothing further will be said concerning the difference between the Miller-Dyes-Hutchinson and Horner style of graphing in this paper. As a historical note, it is worth mentioning that the Horner type of pressure buildup graph was actually first suggested by C. V. Theis\(^10\) in 1935.

Perrine did make a substantial contribution to pressure buildup theory, however, in a discussion of the effect of multiphase flow on pressure buildup. He pointed out that the multiphase flow method proposed by Miller, Dyes, and Hutchinson appeared to be incorrect, and that an alternate method which he had developed intuitively appeared to work better. Later, Martin\(^11\) provided a theoretical basis for Perrine's multiphase flow method. Perrine's method is still the most widely used method for interpreting multiphase flow pressure buildup tests.

Miller, Dyes, and Hutchinson observed that it was possible to get into trouble with a pressure buildup test for a damaged well or a well with large afterflow if the test were not run long enough. Consider, for example, the case of a pressure buildup test which followed the curve A on Fig. 1 and ended at the point E. It would be possible to place a straight line on the pressure buildup graph in the portion of the curve labeled A, and to conclude erroneously that the bend down toward the point E was an indication that the well was going toward a static pressure, or an indication of increased permeability distant from the wellbore. Miller, Dyes, and Hutchinson showed that it was possible to test for the correct straight line in this case because the point E could be established in terms of a dimensionless pressure buildup time for either a closed outer boundary or a constant pressure outer boundary. Matthews\(^12\) later discussed the Miller-Dyes-Hutchinson straight line check criterion, and extended it. The basis for this straight line check was that the slope of the early portion of the buildup curve, like curve A, would be far too steep. This would yield a very low value of permeability, and thus a very short dimensionless buildup time. The Miller-Dyes-Hutchinson time check, however, was based upon a time equivalent to the top of the semi-log straight line, rather than the start of the semi-log straight line.

In the case of stimulated wells, such as curve C on Fig. 1, the Miller-Dyes-Hutchinson test fails. For stimulated wells, it is often possible to draw one straight line of low slope corresponding to curve C and then a later bend upward to another straight line indicated by curve B on Fig. 1. In this case, both straight lines will give permeabilities equal to or greater than the actual effective permeability, and the Miller-Dyes-Hutchinson time check will be satisfied for either straight line.

As can be seen on Fig. 1, it is possible to have a combination of wellbore and outer boundary effects which can lead to curves with perhaps 3 to 5 apparent semi-log straight lines. An example might be the curve A-E-B-F-D on Fig. 1. Some method of detecting the start of the correct semi-log straight line would be highly desirable.

Another problem is that if such a time check were available, it is entirely possible that one would find that his pressure buildup test had not been run long enough. Examples would be pressure buildup curves for either stimulated or damaged cases on Fig. 1 that had not been shut in beyond the point E. A correct semi-log straight line would not exist for either case. A moot point is whether some interpretation techniques might be devised to handle determination of permeability and eventual static pressure for pressure buildup tests which had not been carried long enough to establish the slope of the correct semi-log straight line.

The log-log type curve analysis method has been found to provide answers to many problems suggested by Fig. 1. The subject of this paper, then, is to illustrate the use of log-log type curves as important diagnostic devices and to demonstrate practical interpretation and corrective methods that can be used with log-log type curves to permit a proper conventional buildup analysis by means of a Horner graph. All of the methods may also be applied to interpretation of Miller-Dyes-Hutchinson or Muskat\(^9\) buildup graphs, however, this will not be shown in detail in the following.

**SHORT-TIME ANALYSIS**

The first attempt to use the data obtained during pressure buildup controlled by afterflow was provided by Gladfelter, Tracy, and Wilsey.\(^13\) They suggested that the pressure rise after shut-in divided by the instantaneous sand face flow rate could be graphed vs. the logarithm of the shut-in time to provide a synthetic Miller-Dyes-Hutchinson buildup.
graph. The method was established empirically, and did not appear to receive wide usage. In 1965, it was shown that there was a fundamental foundation for the Gladfelter, et al., method for sandface flow rates which varied exponentially, and that the method could be extended to pressure drawdown analysis. The next year, Winestock and Colpitts established the technique for other types of production rate variation.

Aron and Scott also examined the effect of rate variation on water well test analysis in 1965. The basic method is the Gladfelter, et al., graph of pressure difference divided by instantaneous rate, for either drawdown or buildup testing. This procedure is not just a short-time method, and is widely used in waterwell testing. It is particularly useful for drawdown testing as it is very difficult to hold a constant producing rate. This method performs an approximate superposition for the rate changes. It is related to short-time testing with wellbore storage only because wellbore storage causes the sandface flow rate to change.

A different type of short-time well test analysis was presented by Brons and Miller. They pointed out that it was possible to obtain at least a minimum estimate of the eventual static pressure from a short pressure buildup analysis by passing a line of computed straight line slope through the highest buildup pressure measured. For example, if a pressure buildup test had been conducted which followed line A and terminated at point E on Fig. 1, and there was a previous or subsequent test which permitted an estimation of the permeability-thickness product for the well, then it would be possible to compute the correct straight line slope by means of the known permeability-thickness and the appropriate producing rate prior to shut-in. The straight line then could be passed through the last measured pressure buildup point, extrapolated to a Horner time ratio of unity, and corrected in the normal manner to the static pressure, p. The method is correct and is valuable for finding static pressures for material balance work.

In 1961, Carter presented a study of gas well behavior in which he speculated that strange field test behavior was a result of afterflow or annulus unloading during well testing. An investigation of the Carter speculation was begun, leading to a publication in 1965, which proved that wellbore storage effects could be significant for both gas and oil wells, and in some cases could persist for many hours after shut-in of a well.

The first important clue in unraveling the importance of wellbore storage was available in the graph of the logarithms of dimensionless pressure drawdown vs. the logarithm of dimensionless producing time, including wellbore storage, presented by van Everdingen and Hurst in 1949. This was the first published log-log type curve for the wellbore storage problem. This significant graph was reproduced with an extended range of wellbore storage constants (C = C_D) by Chataas in 1953, and used by Ramsey in 1965 to prepare the first equation relating the start of the correct semi-log straight line to the dimensionless wellbore storage constant C_D. The equation was:

\[ t_D = 60 \frac{C_D}{C} \]  

where: \[ t_D = \frac{0.000264 \times k}{\phi \mu t_w^2} \]  

\[ C = C_D = \frac{C}{2\pi \phi \mu t_w^2} \]  

In 1967, Papadopulos and Cooper independently presented a groundwater version of the wellbore storage type curve, and also derived a time of start of the semi-log straight line similar to Eq. 1, but about four times as long. It appears they used an unreasonably high precision in determination of their time criteria. However, Papadopulos and Cooper did first show the importance of the use of the wellbore storage type curve. It is amazing that the recognition of the use of the graph first published by van Everdingen and Hurst in 1949 took so long in coming.

In 1970, three papers presented log-log graphs of dimensionless pressure drop vs. dimensionless time with the wellbore damage represented by the van Everdingen-Hurst skin effect. Figure 2 presents the wellbore storage and skin effect type curve shown in Refs. 20 and 21. A similar graph is presented in Ref. 22, which also includes the radius of a finite thickness damaged region skin effect. The dimensionless pressure and skin effect are:

\[ P_D = \frac{kh(p_i - p_{wf})}{141.2 \phi \mu} \]  

\[ s = \frac{kh \Delta p_s}{141.2 \phi \mu} \]  

The Wattenberger study indicated that the damaged annular region about the wellbore would have to have a radius of more than 100 wellbore radii to be evident other than in the skin effect during a pressure buildup test.
for large storage constants. That is, a semi-log straight line is likely to the permeability of the damage zone would not appear unless the radius of the damage region were 100 or more times the wellbore region. If the wellbore storage effect was small, two distinct semi-log straight lines could appear, the first having a slope inversely proportional to the permeability of the damaged zone, and the second having a slope inversely proportional to the permeability of the reservoir proper. Let's consider Fig. 2.

The shallow curves across the top of Fig. 2 labeled \( C_D = 0 \) represent semi-log straight lines of negligible wellbore storage effect. This can be seen by graphing data points from the log-log curves onto semi-log graphs. The unit slope lines moving up toward the right on Fig. 2 represent wellbore storage. A perfect unit slope indicates that all of the fluid being produced is coming from the wellbore annulus on production, or that the sand face production rate has not diminished following shut-in during pressure buildup. The intersection of the curves starting with unit slopes with the shallow semi-log straight lines indicate the start of the correct semi-log straight lines. Such an intersection is indicated on Fig. 2 by a dimensionless time of about 10^5 for a wellbore constant, \( C_D \), of 1000, and a skin effect of 20, with zero wellbore storage. This point is indicated by the large dot on Fig. 2. On Fig. 2 it can be seen that the start of the correct semi-log straight line depends upon both the wellbore storage constant and the skin effect. Equation 1 presents the start of the correct semi-log straight line for a zero skin effect. Later, Standing and Ramey, et al., developed an equation for the start of the correct semi-log straight line for a damaged well:

\[
\text{t}_D = C_D \left( 60 + 3.58 \right) \quad (5)
\]

This equation is correct and useful for design purposes, but it is rarely necessary for well test interpretation purposes. This can be illustrated by means of the following example.

Fig. 3-A presents a Miller-Dyes-Hutchinson pressure buildup graph for well A from Russell. As can be seen on Fig. 3-A, there are two apparent semi-log straight lines. Several interpretations are possible. One is that the first straight line, labeled A, is indicative of the correct formation permeability, and the second line, labeled B, indicates the bend down caused by the outer boundary of the drainage area. Another interpretation is that line B is the correct straight line indicative of formation permeability, and curve A is simply an afterflow effect. The answer to this problem may be found by graphing the same data on a log-log type curve. In this case, the logarithm of the static pressure minus the logarithm of the flowing pressure prior to shut-in is graphed vs. logarithm of shut-in time. This graph is appropriate as long as producing time was large compared to the largest shut-in time shown on Fig. 3-B. As can be seen, the early pressure points do form a unit slope. This indicates all of the pressure shut-in data through 0.7 hours are dominated by afterflow, or wellbore storage. An approximate type-curve match of these data with the dimensionless type curve shown on Fig. 2 at any position on the graph would indicate the approximate start of the correct semi-log straight line as indicated by the arrow at 20 hours on Fig. 3-B.

An alternate approach is available. Inspection of Fig. 2 indicates that the top of the unit slope straight line on a log-log graph is about 1-1/2 log cycles prior to the start of the correct semi-log straight line. This can be seen on Fig. 2 by comparing the cross for the top of the unit slope straight line with a dot for the start of the semi-log straight line. This is termed the "one and one-half log cycle rule."

Although Eq. 5 and Fig. 2 both indicate the important concept of finding the start of the correct semi-log straight line, it is not obvious that real field data would actually match the simple solution graphed on Fig. 2. Figure 2 represents an analytic solution for an ideal well produced at constant rate from an ideal formation of constant properties. It is assumed that the damage effect on the well is caused by the van Everdingen-Hurst "infinitesimal" skin effect. The well is assumed to be in an infinitely large reservoir. This combination of ideal assumptions would lead one to question the application of the information to real field test data. Furthermore, at the time this solution was prepared, there was a general feeling that short-time well test data were dominated by physical phenomena too complex for practical mathematical description. For example, it was believed that on shutting in a well the initial effects were caused by frictional effects due to flow through the liner perforations, and to strange effects resulting from pressure waves rebounding from the top and bottom of the formation. After these effects, then the fluid would be produced from the annulus for some time. To the author's surprise, all of the field test examples cited in the well test monograph by Matthews and Russell and examples by
Russell" and many other field cases collected by the author all exhibited perfect unit slope straight lines on graphs of the log of the pressure rise vs. the log of shut in time. Furthermore, the start of the semi-log straight line usually was evident on the log-log graph. Almost complete success with this method led the author to publish the 1970 study advocating the use of the log-log type curve for determination of permeability and skin effect.

After a few years of further experience it was found that in all successful cases of log-log type curve analysis, there was a sufficient portion of the semi-log straight line evident to permit conventional analysis. If only the unit slope straight line and a small portion of the transition toward the semi-log straight line were available, it was possible to find a match between field data and Fig. 2 in almost any position on the graph. It now appears that the most important use of Fig. 2 is as a diagnostic device to determine the start of the semi-log straight line. Type curve matching should be done in emergency or as a checking device.

One interesting aspect of Fig. 2 is that it can be shown that as long as pressure data fall on a 45° unit slope line on a log-log graph, pressure data are completely controlled by wellbore storage. This can be shown by assuming that the sand face has such a high skin effect that it is completely closed to flow from the formation. In this case, the dimensionless pressure vs. dimensionless time can be computed simply, and a 45° straight line results.

This led to the speculation that as long as pressure buildup data lies on a 45° unit slope straight line on a log-log type curve, it was not possible to determine formation permeability or skin effect. R. M. McKinley pointed out that this was not necessarily the case. As long as field data formed a 45° slope straight line, it was possible to make a minimum estimate of the permeability by means of type curve matching. The next year, McKinley presented a different log-log type curve for analyzing pressure buildup data, shown in Fig. 4. Later, Earlougher and Kirsch presented yet a third log-log type curve, Fig. 5, and discussed the type curves shown on Figs. 2 and 4. These type curves will be discussed in the next section of this paper.

In the 1970 study which introduced the type curve shown in Fig. 2, a type curve for a well with a vertical fracture was also presented. Fig. 6 is a recent version of that curve. Fig. 6 presents the logarithm of the dimensionless producing pressures vs. the logarithm of a dimensionless time based on the producing area for a well in the center of a closed square of drainage area A.

$$t_{DA} = \frac{0.000264k}{\phi u c L A} = t_D(r_w^2/A)$$  \hspace{1cm} (6)

The data graphed on Fig. 6 were taken from ref. 32 for a vertical fracture of infinite fracture permeability. The parameter ($x_f/x_e$) shown on Fig. 6 is the ratio of the half-fracture length (well axis to fracture tip distance divided by the half length of the side of the drainage square). Dimensionless times for vertical fracture cases are also often defined based on the half-fracture length distance, $x_f$, also often given the symbol $L$.

$$t_{De} = \frac{0.000264k}{\phi u c x_f^2} = \frac{0.000264k}{\phi u c L^2}$$  \hspace{1cm} (7)

Ref. 21 stressed that Fig. 2 was appropriate for a damaged well with wellbore storage, or a well which had been stimulated by acidization to improve the permeability in an annular region about the wellbore. If the well had been fractured such as to cause a single planar vertical fracture passing through the vertical axis of the wellbore, then a different type curve was required, even though the van Everdingen-Hurst skin effect was perhaps -5, as shown on Fig. 2.

The original version of Fig. 6 was constructed from the classic fracture study by Russell and Truitt. The advantage of the fracture type curve was that it appeared possible to determine formation permeability, the effective length of the fracture, and perhaps even the size of the drainage system from a well test. A later regraphing of the Russell and Truitt data indicated that there were some inconsistencies in the short-time solutions. For this reason, the problem was resolved by Gringarten, et al., and the result is shown as Fig. 6. A comparison of Figs. 2 and 6 will indicate drastically different shapes for the two sets of curves. The curves for storage and skin effect on Fig. 2 begin with the unit slope, while the curves on Fig. 6 for a vertical fracture begin with a slope of one-half. This slope of one-half is caused by linear flow into the planar surface of the fracture at very short times. The end of the half-slope period is indicated on Fig. 6 at a dimensionless time of about $1.5 \times 10^{-2}$. The start of the semi-log straight line was found by regraphing the data from Fig. 6 to a semi-log graph. This was demonstrated in ref. 32. The start of the semi-log straight line is also shown. Thus an important fact for the start...
of the correct semi-log straight line for a
vertically fractured well was found. 35

The pressure response of a vertically frac-
tured well can be displayed in a different
type curve as $p_D$ vs $t_{DF}$ as shown on Fig. 7 32-33
Figs. 6 and 7 are different displays of the same information. Fig. 6 is convenient for
long-time tests which clearly show the bend upwards caused by the drainage boundary of the system. Fig. 7 is more convenient to empha-
size the short time results of a test. Refs. 32-33 also presented similar information for a constant flux (as opposed to infinite
fracture permeability) fracture. These re-
sults give the appearance of a high, but somewhat lower fracture permeability than the infinite fracture permeability case.

Two important features of log-log type curves were evident: short time pressure data might indicate either a unit slope (storage) or a half-slope (fracture). Not surprisingly, ref. 20 presented a set of field data which appeared to indicate a unit slope grading abruptly into a half-slope: a fractured well with very high wellbore (plus fracture) storage. Recently, data for a vertically-fractured well with large storage component were pre-
sented. 34 Tabulated data and both log-log and semi-log graphs were given in ref. 34. However, the log-log type curve is shown as Fig. 8. Although both the unit and half-
slope do occur, the unit slope takes place at very short times, and there is a long trans-
ition from the unit slope to a half slope. It appears that an abrupt transition from a unit slope to a half-slope just should not take place.

In a previous study of the behavior of verti-
cally fractured gas wells, Wattenbarger 35 had noticed that the dimensionless pressure at the start of the semi-log straight line was approximately twice the dimensionless pressure at the top of the one-half slope line. This rule is more correct for a constant flux frac-
ture than for the infinite conductivity frac-
ture shown on Fig. 7. Nevertheless, this rule of thumb is useful for analyzing the behavior of fractured wells. All of the storage and skin lines shown on Fig. 2 pass through a slope of one-half in the transition between the unit slope storage control line and the conventional semi-log straight line. If the first portion of the buildup is missed and the unit slope straight line is not evident, the first portion of a buildup test may appear to have a half-slope. In this event, applica-
tion of the Wattenbarger rule will lead to a pressure difference far greater than any possible for the start of the straight line if the well is actually a storage case rather than a fracture case. The Wattenbarger rule may be called the "double delta p" rule.

One final type curve seems needed at this juncture. What about horizontal fractures? Are there differences between horizontal and vertical fractures evident in type curves? Fig. 9 presents a type curve for a horizontal fracture taken from ref. 33. As can be seen, there is an initial half-slope line indicating vertical flow into the surface of the fracture. Otherwise, there are large differences from the vertical fracture case. The author has seen only a few cases of apparent horizontal fractures to date. Nothing else will be said of this case.

Although the preceding log-log type curves for the storage and skin case and the vertical fracture case may be used to find the start of a conventional straight line on a Horner buildup graph, many anomalies occur. In the following, we examine the use of log-log type curves as diagnostic aids to conventional pressure build-up analysis, and compare the relationships between different methods.

TYPE CURVES FOR WELLBORE STORAGE CASES

In regard to wellbore storage cases, several distinctly different appearing type curves are in the literature. First was the wellbore storage and skin type curve 29, second was the afterflow type curve of ref. 29 (McKinley type curve), and the third was the type curve of ref. 30 (Earlougher-Kersch). There appear to be essential differences between these three type curves. Results obtained from them can be sufficiently different to indicate sub-
stantial differences. The Earlougher-Kersch study considered differences between these three techniques and was substantially cor-
rect; however, a few points are worth further discussion.

The type curves of Figs. 4 and 5 employ real units in a convenient manner, but it is use-
ful to consider the relationships between these graphs by means of dimensionless graphs. Figs. 10 and 11 present generalized versions of the McKinley and Earlougher-Kersch type curves.

Fig. 10 presents a generalized McKinley type curve. The ordinate is the ratio $(t_D/C_D)$ and the abscissa is $p_D$. The parameter on the graph is the dimensionless storage constant, $C_D$. It is clear that Figs. 4 and 10 have exactly the same appearance. Lines for other dimen-
sionless storage constants may be graphed on Fig. 10 to complete the resemblance. Addi-
tional information may be shown on Fig. 10, however. The horizontal dashed lines shown in the upper portion of the figure represent the start of the semi-log straight line for
a conventional buildup graph, depending on the value of the van Everdingen-Hurst skin effect.

Another important fact is that in reality, wellbore storage effects are not constant. It is not unusual for a well test to begin with one value of the storage constant, \( C_s \), and change gradually or abruptly to another value either smaller or larger. Fig. 10 was obtained by graphing data tabulated in ref. 21.

Although Figs. 4 and 10 appear to be similar, there do appear to be some fundamental differences. Fig. 10 was developed for pressure drawdown, and may be applied to pressure buildup under the assumption that the largest buildup time is short compared to the production time before shut-in \( t_{sh} \), (say, less than 10% of the flow time). However, Fig. 4 appears to have been developed for pressure buildup, although nothing has been said of the conditions of application. Comments in ref. 29 indicate that some approximate adjustment was made to the shut-in pressure curves to make them approach static values. A single value of hydraulic diffusivity was also used. It is assumed that \( k/(\phi \mu C_s r_w^2) = 9.728 \times 10^6 \text{ md-psi/(cp-ft}^2) \). It would be necessary to assume a long producing time to be able to make a single buildup graph such as Fig. 4. By direct comparison between the early portions of Figs. 4 and 10, the same answers result. But Fig. 4 seems to approach static pressure during a time period that should correspond to the semi-log straight line.

It is possible to develop log-log type curves specific to pressure buildup interpretation. In order to do this from the pressure drawdown data indicated in either Figs. 2 or 7, it would be necessary to apply superposition for a variety of producing times, and then make an appropriate graph or graphs. It is beyond the scope of this paper to pursue this point further.

Either type curve Fig. 2 or the type curve for Fig. 4 requires a graph of the log of pressure difference vs log of time. Fig. 2 requires a graph of the log of pressure difference vs the log of time, while Fig. 4 requires a graph of the log of time vs the log of pressure difference, a simple rotation by 90°. The field graphs are exactly the same for either type curve. There is little fundamental difference between type curve matching on either of the graphs for times before the conventional semi-log straight line.

Several comparisons of results utilizing the two type curves have been published. The most thorough is ref. 44, an MS study by C.A. Solano C. The most important conclu-
The classic case of a pressure buildup test complicated by wellbore storage is given by Fig. 3A. There are several semi-log straight lines possible on the conventional pressure buildup graph. It is clear from Fig. 3B that the correct semi-log straight line must start at about 20 hours of shut-in time. This eliminates the long initial apparent straight line in Fig. 3A. Nothing more will be said about this test (see ref. 20), except to emphasize that it is now strongly believed that the best results may be obtained from a Horner pressure buildup graph by analyzing the straight line identified from a log-log type curve such as that indicated in Fig. 3B. Fig. 3B can serve an additional useful purpose in emphasizing that all of the pressure buildup data immediately after closing the valve on the well do match an analytical solution. Although many examples similar to that shown in Fig. 3 will be found in practice, the following may explain the anomalies which also occur sometimes.

Fig. 12 presents a log-log graph of pressure rise vs shut-in time for an oil well taken from information published by Cable. The circular data points indicate the information published by Cable and show an initial unit slope line followed by a half-slope straight line. This indicates a vertically fractured well with a large wellbore storage component. The same information is shown in Fig. 13 as a Horner pressure buildup graph. The circular data points in both Figs. 12 and 13 are the same. A comparison of Figs. 12 and 13 indicates that the long semi-log straight line of slope $m=212$ psi/cycle on Fig. 13 corresponds roughly to the half-slope information shown on Fig. 12. Cable stated that this slope corresponded to a permeability of 50 millidarcy-feet although core analysis indicated a value of only 7.2 millidarcy-feet. This indicates that the straight line shown on Fig. 13 is not steep enough. In order to obtain a value of permeability-thickness on the order of 7 millidarcy-feet, it would be necessary to have a straight line on Fig. 13 at a much steeper slope. An example would be line B on Fig. 13. Line B has a slope ten times the value of line A. Consequently, permeability-thickness corresponding to line B would be only 5 millidarcy-feet. On the other hand, line C has a slope one-tenth that of line A. For line C, the permeability-thickness product would be 500 millidarcy-feet. It does not seem likely that the buildup pressures would bend and follow a line as shallow as line C.

On the other hand, the type curve of Fig. 12 which shows a unit slope line trending toward a half-slope line suggests the possibility that the well is influenced by wellbore storage and a following fracture flow transition towards a semi-log straight line. If this were the case, then the correct slope on a Horner buildup graph such as Fig. 13 would have to be a line like that indicated by the label C. We've already concluded that this is highly unlikely.

An additional piece of information is available by virtue of Fig. 8, the figure for a vertically fractured well with a large wellbore storage effect. As was mentioned previously, the transition from a unit storage line to the one-half slope fracture line follows a long transitional period. Curves like those shown in the type curve of Fig. 12 do not appear on Fig. 8. This information was not available to Cable in his original interpretation of the data reported in ref. 38.

Consideration of the fundamental nature of log-log type curves and the Horner pressure buildup graph indicates that the flowing pressure prior to shut-in plays an important role in a log-log type curve. Although the flowing pressure is indicated on Fig. 13, it plays no role in the positioning of the pressure buildup points shown on the graph. On the other hand, Fig. 12 presents a graph of the difference between the static buildup pressures and the formation flowing pressure prior to shut-in. An error in the flowing pressure would be evident in all of the graphed points shown on Fig. 12. Because it is known that a unit-slope line cannot change abruptly into a half-slope line on a log-log-type curve for a fractured well with storage, the data shown on Fig. 12 are highly unlikely. In the case of flow into a fractured well, it is known that pressure should be a function of the square root of the shut-in time. Because the flowing pressure is suspect, then, we regraph Cable's information as the shut-in static pressure vs the square root of the shut-in time in minutes. This information is shown on Fig. 14, and tabulated in Table 1.

As can be seen from Fig. 14, all of the shut-in static pressures follow a straight line indicating a flowing pressure of 710 psi, as compared to the value of 740 psi used by Cable in the graphing of the solid line type curve shown on Fig. 12. If a flowing pressure of 710 psi, rather than 740 psi, is employed with the data of Table 1, the data points shown by the triangles on Fig. 12 result. All of the data points from one and one-half minutes after shut-in follow the dashed line on Fig. 12 representing a half-slope. The last four data points appear to fall below the dashed line, indicating the start of the transition from the half-slope linear flow control to pseudo-radial flow. If the data shown on Fig. 12 is matched with Fig. 7, a permeability of 5.3 millidarcy-feet
results. This appears a reasonable check with core data value of 7.2 millidarcy-feet reported by Cable. It is also possible to calculate an apparent fracture length for this case. The result is 215 feet. One of the clues to the preceding analysis was Cable's comment that the well was producing with an intermittent setting of four hours on and four hours off. This would make determination of the flowing pressure prior to shut-in difficult.

**FRACTURED WELL ANOMALIES**

The vertical fracture type curves such as Fig. 7 have been found to work very well in a majority of the field cases tested to date. However, a significant number of strange effects have been seen. A common anomaly is that all of the well test data matches the curve for a vertically-fractured well in an infinite system for the entire test: perhaps from five minutes to more than 100 hours of test time. The problem is that effective fracture lengths computed from the dimensionless time are orders of magnitude smaller than the design fracture length. In some field cases, apparent fracture lengths on the order of 10 feet have been obtained when the desired fracture length was 1000 feet or more. At the moment, the source or sources of this kind of anomaly have not been definitely established. Holditch and Morse have found that inertial flow effects within and near the surface of the fracture could be one source of this problem. Other causes might be limited fracture permeability due to ineffective propping, limited radial growth of the fracture due to excessive vertical growth, and many other factors which could cause the appearance of limited fracture conductivity.

Another strange effect that has been observed many times is that after a substantial fracture treatment on a well, a pressure test is found to appear like the wellbore storage and skin effect type curve presented in Fig. 2, rather than the vertical fracture type curve presented in Fig. 7. Generally, the skin effects are negative but a unit slope storage line is definitely present. The well has been stimulated, but it does not appear that a single planar vertical fracture was generated by the fracture treatment. The first field example of such a case was noticed and identified by Strobel. On checking the wellbore volume indicated by the unit slope storage line after the fracture treatment, Strobel found that the apparent wellbore volume was about twice the known wellbore volume. He theorized that the fracture treatment had resulted in propping open existing interlocking fractures in the region surrounding the well, rather than generating a single planar fracture. This being the case, it would be expected that the additional apparent wellbore volume would be a quantitative measure of the pore volume within the propped fracture. In this event, it is possible to estimate the volume of proppant used during the test by estimation of the porosity in the propped volume. Reasonably good checks were found in this case with the known volume of sand displaced during the fracture treatment. It seems reasonable that attempts to fracture in formations which are already generally fractured or whose main porosity is fracture porosity may result in simply propping open a part of the fracture porosity in the immediate vicinity of the well.

There is a tendency to think that a fracture treatment was unsuccessful if a planar fracture is not clearly evident in the pressure transient data taken following a treatment. This is not necessarily the case. Performance can be satisfactory with a propped volume as well as with a single planar fracture. The appearance of the storage result can reveal important information about the nature of the formation. It would be expected that general fracturing with cross jointing would be present in the event that a unit slope fracture line was found following a fracture treatment.

During the first year of application of the vertical fracture type curves, no examples were found which appeared to show the bend up toward outer boundary control indicated by the family of lines with the parameter $x/\bar{x}$ of Fig. 7. However, in the next few years' several cases were found which did appear to indicate drainage areas that were reasonable. The fact that drainage boundary effects do not often appear on fracture type curves appears to indicate that effective fracture lengths are not often as long as we would like to believe.

One strange type curve was found wherein the first data point on the fracture type curve appeared at a dimensionless time greater than about 10. The very first data point was already well up on the semi-log straight line and an outer boundary effect did appear in this case. This sort of situation can happen with either fracture or storage type curves. If the hydraulic diffusivity is such that five minutes of real time is equivalent to a large dimensionless time beyond the transition to the semi-log straight line, it may be difficult to obtain the early time data which are useful to prove the existence of the correct semi-log straight line. We turn now to some problems experienced with the storage and skin type curves.
STORAGE AND SKIN TYPE CURVE PROBLEMS

Fig. 15 presents a field graph in which the data points clearly show an early half slope line at a shut-in time near one hour. Also shown on Fig. 15 is application of the double delta p rule to find the start of the correct semi-log straight line. As can be seen, the top of the half-slope line occurs at a time of 2.2 hours. The pressure rise at 2.2 hrs is 180 psi. Doubling 180 psi yields 360 psi, and the dashed line across the top of Fig. 15 shows the result. This line will not intersect the buildup pressures. It falls well above the suit. This is a strange type curve which would be higher and the data points clearly show an early half slope pressure curve. As can be seen, the top of the half-slope line is lost because it occurred well before 0.9 hrs of buildup time. Perhaps if early data had been taken or if a pressure chart exists which can be read at times before 0.9 hrs, it would be possible to graph data points which show the unit slope straight line. We proceed without additional early data for Fig. 15, however. In this case, we cannot apply the one and one-half cycle rule which was used in Fig. 3 to find the start of the semi-log straight line. It is possible to find the start of the semi-log straight line by rough matching on Fig. 2 at almost any position on the graph. This is shown by Fig. 16. Fig. 16 shows Fig. 15 placed on the storage and skin type curve, Fig. 2, in two locations. The position in the upper left shows an approximate match for a skin effect of +20 and a storage constant, Cps, of 100. As can be seen on the figure, the data points appear to reach the semi-log straight line starting at a shut-in time of about 6 hours.

The same field data graph is also shown on the lower right hand portion of Fig. 16, and is matched at a skin effect of zero and a storage constant, Cps, of 10. A reasonable match occurs in this position also, and the start of the correct semi-log straight line again appears at a time of about 6 hrs. Even if the unit slope straight line is not available, it is still possible to find the start of the correct semi-log straight line with crude type curve matching procedures on Fig. 2. This shows one of the most powerful advantages of the use of type curves. Assuming that pressure measurements were accurate, we know the correct start of the semi-log straight line in this case, even though the early data are completely missing. Furthermore, we are absolutely certain that this is not a fractured well case, but is a storage and skin case.

From Figs. 12, 13, and 14 for the Cable field example of a fractured well, it is clear that the assumption that pressures were measured accurately can be doubtful. Even if pressures are measured with complete accuracy, there are operational procedures involved in running pressure bombs into wells prior to the pressure buildup test that lead to changes in pressure which affect the character of a type curve. For example, consider Fig. 17. The circular data points on Fig. 17 show a straight-forward case of wellbore storage for a damaged well. If the flowing pressure is taken to be 15 psi lower than the correct value, then the pressure difference will be higher and the data points shown as squares on Fig. 17 result. This is a strange type curve which could be taken to have a fracture slope of one-half in the early portion and bending upwards. This shape appears on some of the horizontal fracture solutions (see Fig. 9). If the producing pressure were taken even lower, the result could be a type curve that is almost completely flat across the top of Fig. 17. For example, if an error on the order of -300 psi in the flowing pressure were made, it is clear that the data points would be raised 300 psi above the correct line shown by the circular data points.

The square data points on Fig. 17 indicate the case where the flowing pressure is measured 15 psi high. In this event the pressure differences will be too low, and the early part of the type curve may have a very steep slope. On Fig. 17, the early square data points have a slope near two.

As time increases and the pressure difference rises, small errors such as 15 psi become a negligible fraction of pressure differences of several hundred psi, and the data points tend to approach the correct line.

In the case of the fractured well test reported by Cable, it was shown that the correct flowing pressure could be found by making a graph of shut-in pressures vs the square root of time. A similar procedure is possible in the case of the storage and skin type curve. During wellbore storage controlled by a constant value of the storage constant, Cps, static pressure should be a linear function of the shut-in time. It would be possible in the case of the data points shown by either the triangles or the squares on Fig. 17 to plot the shut-in pressure, p, vs the shut-in time, Δt, on Cartesian coordinates, pass a straight line through the data points, and extrapolate back to a shut-in time of zero. The value so obtained would be a more correct flowing pressure and upon replacement with the new value, a correct type curve will result.

It is not necessary that mistakes in measurement be made for the flowing pressure to be erroneous. The flowing pressure may not even
be available. In certain cases, wells are not equipped to lower the pressure bomb while flowing. It is necessary to shut in the well and then lower the bomb in place. Even in this event, an estimate of the flowing pressure can be made if the wellbore storage effect has a duration long enough that some measurements may be made before storage control disappears. It should be emphasized that the correct unit slope line on Fig. 17 is actually a measure of the flow rate and wellbore volume. In the case that the wellbore volume and storage effects are known, a data point on the unit slope line may be a more reliable measure of the production rate prior to shut-in than the rate actually recorded. The reason for this fact is that a well often is "stabilized" prior to a buildup test. This means that the flow rate is adjusted at the surface, although the flow through the liner slots may continue at a rate established by the pressure gradient in the formation prior to rate stabilization. This sort of an effect can be explained by means of Fig. 18, a schematic for a gas well during production and shut-in.

Wellbore storage appears complicated for the case of gas production. Assuming there is no liquid present in a gas well bore, the volume of gas which is stored is proportional to the compressibility of gas, which is not constant. The storage type curve, Fig. 2, is only approximately correct for a gas well in this case. A question often asked is whether the pressure, pressure squared, or the real gas potential is more correct in this case. The answer is none of them. The correct way to handle a volumetric gas material balance is through a p/z vs cumulative production graph. In the event the producing rate is constant, time is directly proportional to cumulative production. Thus a graph like Fig. 18 will be very close to a p/z - gas produced graph during periods of constant rate production. Fig. 18 is arranged to show events which frequently happen during production and shut-in of a gas well. The factors also affect the behavior of an oil well. The ratio p/z can be replaced by pressure. Now consider Fig. 18. The gas well begins to produce at rate q₁ at time zero. This causes the linear portion of the line from time zero shown by the solid line with the dashed extension on the left hand part of Fig. 18. If the line is extrapolated to p = 0, and the time increment, Δt, measured; then Δt times the rate q₁ should be a measure of the volume of gas initially stored within the wellbore. This is indicated in the equation near the left top part of Fig. 18.

After a time, the production rate of the well declines, and the p/z line will follow the solid line toward the right. Later, while the well is producing at rate q₂, it is decided to shut in the well and run a pressure buildup test. In order to run the bomb in the well, the well is shut in, causing the pressure rise shown, and then the well is stabilized at the new rate, q₃, and then eventually shut in at time t. The solid line pressure trace shown on Fig. 18 during the production phase does represent correct pressures. However, it also represents operational procedures involved in simply getting a bomb into the well and stabilizing the producing rate. Because there is a substantial wellbore storage effect for this well, the sand face producing rate continues at the rate q₄ during the time that the bomb is run and the surface rate is stabilized at rate q₃. This results because the fluid flow into the bottom of the well is controlled by the pressure gradient set up in the formation prior to the shut-in and adjustment of the rate. If nothing had happened to the well and the rate had continued at q₄, the pressures would have followed the dashed line shown to the "true" flowing pressure, p₂. Instead, pressures follow the solid line, causing the "apparent" flowing pressure. After shut-in, pressures rise, as shown by the solid line, and eventually approach a static pressure corresponding to the depletion of the system caused by producing for a time period t. Although the early shut-in data is controlled by wellbore storage, a type curve constructed from the solid line shown during the shut-in period on Fig. 18 will not produce a unit slope. The "apparent" flowing pressure is too high. This will lead to a steep type curve with perhaps an initial straight line of slope much greater than unity. The situation can be corrected, however. A graph of p/z vs shut-in time for the early shut-in data will yield a straight line which will extrapolate to the true flowing pressure at zero shut-in time. Correction of the flowing pressure will then lead to a proper type curve which will have the correct slope.

If the volume of the wellbore was measured during the drawdown portion of the test, it would even be possible to determine q₄ from the slope of the p/z vs time graph as is indicated on Fig. 18. This led Moser to propose in an unpublished Forum Note that the wellbore storage effect could be used to prorate gas well production when a number of wells are metered collectively by a single separator. Moser recommended measuring the pressure rise for 5 minutes shut-in on each of the wells entering the separator and allocating the total production rate in proportion to the pressure rise for each well. This is a sound procedure for gas wells controlled by wellbore storage.
DISCUSSION

The early claims made for the wellbore storage (afterflow) type curve now appear rather optimistic. Inadvertent errors in the flowing pressure prior to shut-in can cause misleading shapes on the type curves and, in the extreme, can make a fractured well test look like a wellbore storage case, and a storage-dominated case appear to be a fracture case. On the other hand, the conventional Horner pressure buildup graph does not depend on the correct value of the flowing pressure prior to shut-in. It is advisable to use the type curves mainly as diagnostic devices to identify and correct errors, and to select the proper case (mathematical model) and the start of the correct semi-log straight line. Once conventional analysis has been performed, however, it is usually possible to place all of the data back on an appropriate type curve to show that the entire test does fit the mathematical model selected.

The time for the start of the semi-log straight line on either fracture or storage type curves actually represents the minimum time for shut-in for a proper buildup test. If one log cycle of semi-log straight line is to be obtained, then the well would have to be shut in at least ten times the time for the start of the straight line. It is not unusual to compromise by using about one-third to one-half log cycle so that the shut-in time is usually three times the time of start of the semi-log straight line.

In certain cases, it is absolutely necessary to analyze data for which there is no correct straight line. An example would be a buildup test that was run just long enough to reach the start of the correct semi-log straight line. In the event that the well is a storage case, recourse can be made to any of the three type curves available: the storage and skin type curve (Fig. 2), the McKinley type curve (Fig. 4), or the Earlougher-Kersch type curve (Fig. 5). In the event Fig. 2 is used, the field data curve may be positioned on any one of the family of lines for a given storage constant and simply moved up and down along that family of lines until the best visual match is obtained. The permeability may then be calculated from the pressure match, and the skin effect approximated by reading the parameter on the family of lines on the right hand portion of Fig. 2.

In the case of inadequate data for a fracture case, a more satisfactory result may be achieved. The data can be matched with the fracture type curve shown on Fig. 7, and both permeability and fracture length determined from data before the correct semi-log straight line is observed. It should be obvious that the storage type curves, Figs. 2, 4, and 5, should not be used for fracture cases. In both fracture cases and storage cases, it should be remembered that the conventional Horner graph and the log-log type curve must agree. Log-log type curves are powerful diagnostic tools which will often permit unraveling of complex pressure buildup data. But log-log type curves are not an end unto themselves. They should be used in conjunction with proper conventional well test methods. The combination is a powerful tool that will often permit an analyst to be certain of his interpretation. The long-sought short-time pressure buildup method is not yet available.

DETERMINATION OF POROSITY

In the preprint, SPE 2416, by R. M. McKinley, which was later published as ref. 29, the comment is made that "...hydraulic diffusivity is not measured by a single well test." Furthermore, in 1971, it was emphasized that the Muskat pressure buildup graph was unique in that it appeared to offer the opportunity of obtaining porosity from a pressure buildup test. These two references make clear the general feeling that it was not possible to obtain porosity from a single well test, such as a pressure buildup test. This impression is correct for pressure drawdown, but it is not correct for pressure buildup testing. The explanation of this statement may be seen in consideration of Fig. 19. Fig. 19 presents a graph of dimensionless pressures vs dimensionless time for a single well in the center of a closed drainage area of size A/ρ = 4x10. The well is considered to have a skin effect of +20, and the case of significant wellbore storage is also considered by the dashed line shown on Fig. 19.

Let us first consider the case where the well has no storage effect. During production, pressures may be represented by the van Everdingen-Hurst dimensionless pressure function, p. If there is a skin effect, then the producing pressures become (p + S). This is shown by the solid line on Fig. 19. The van Everdingen-Hurst infinitesimal skin results in a pressure discontinuity at the start of production. The pressure drops immediately by an amount equal to the pressure drop across the skin effect, and then decreases monotonically until the time of shut-in. During pressure buildup, we superimpose pressures to cause the effect of a zero rate. This can be seen by the following equations. First, for pressure drawdown, we write:

\[
\frac{kh}{141.2q_{ew}} (p_{1} - p_{w}) = p_{D} (t_{D} A/\tau_{w}^2) + S \quad (9)
\]
Then for pressure buildup, we write:

\[
\frac{kh}{141.2qBu} (p_1 - p_{ws}) = p_D(t + \Delta t) - p_D(\Delta t) \quad (10)
\]

As a result, buildup pressures appear to be independent of the magnitude of the skin effect and the size of the system. This can be seen on Fig. 19 because at the end of the producing time, t, on shut-in the pressure jumps immediately by an amount equal to the pressure drop across the skin, and then from that point is independent of the skin effect and proceeds to the static pressure corresponding to the depletion of the system. If there had not been a skin effect during the producing phase, the pressures would have dropped smoothly from a dimensionless pressure of zero to the large dot shown at the top of the skin effect arrow at shut-in. This means that there is a whole family of producing curves, depending upon the magnitude of the skin effect, but there is a single shut-in curve that is independent of the skin effect. There is only one shut-in curve. To emphasize, the drawdown curve is a function of ratio of the drainage area to the well radius squared, while the pressure buildup curve is not. The pressure buildup curve is uniquely different from the infinite number of possible pressure drawdown curves.

Pressure buildup curves are independent of the skin effect, yet skin effects are determined in pressure buildup testing. In principle, it should not be possible to determine a skin effect by pressure buildup alone. In fact, this is not done. The flowing pressure just before shut-in is the source of the skin effect. On Fig. 19, it is indicated that the skin effect causes a discontinuous jump from the producing pressure, shown by the lower dot, to the first shut-in pressure, shown by the upper dot. It is necessary to have a measure of the flowing pressure to be able to compute the skin effect.

In reality, there is not a discontinuous jump in pressures on production or shut-in of a well. The wellbore storage effect and the finite thickness of the damaged region near a well leads to more realistic solutions. The dashed lines shown on Fig. 19 indicate the inclusion of a finite wellbore storage effect. Dimensionless pressures for this case were taken from ref. 21. As can be seen, the pressures immediately after shut-in do depend upon the skin effect and the storage constant. However, the essence of the previous comments still hold. By the time the storage case reaches the semi-log straight line, buildup pressures are still independent of both the skin effect and the storage constant. In the case of the drawdown data, however, a family of curves would result depending upon both the magnitude of the skin effect and the storage constant. In essence, then, it should be possible to determine both drainage pore volume and permeability through pressure buildup data. This can be done by a variety of methods. Probably the most useful is a direct type curve matching between a field semi-log Horner graph and dimensionless semi-log Horner graph, such as are presented in refs. 9 and 26. The Horner time ratio is dimensionless and is always known for field data. The dimensionless build-up pressures may also be known if the slope of the correct semi-log straight line and the initial pressure prior to production are known. This results because the dimensionless static pressure can be represented as follows:

\[
\frac{kh}{141.2qBu} (p_1 - p_{ws}) = \frac{(p_1 - p_{ws})}{0.87 m} \quad (11)
\]

where

\[
m = \frac{162.6qBu}{kh} \quad (12)
\]

Not only is it possible to determine drainage pore volume from identification of the dimensionless producing time, but it is also possible to identify drainage area effects such as asymmetric positioning of a well within a closed drainage shape, and the effect of water drive along the boundaries of the drainage region by means of examples such as those shown in ref. 26. These techniques are new, and will be the subject of much exploration in the coming years.

**CONCLUDING REMARKS**

The availability of new tools (pressure measuring devices) and new technology is a fortuitous combination that indicates rapid advances in the state of art of pressure transient analysis. The use of type curve procedures makes understanding of pressure transient data within the grasp of the average engineer. Almost magical determination of errors in data and estimation of missing pieces of information such as flowing pressures and production rate before shut-in are now frequently possible. This has removed pressure transient analysis from the mystique of the solution of partial differential equations and placed it on an operational basis with other petroleum engineering production tools. The new pressure transient analysis offers the possibility of obtaining much better information for the production and reservoir engineer for input to reservoir simulation tools now being developed. Computer tail pressure transient analysis systems will be available in the near future, and fully automatic pressure transient analysis will arrive in the not too far distant future.
### Nomenclature

- **k** = effective permeability to flowing phase, md
- **h** = net formation thickness, ft
- **p** = pressure, psi
- **q** = production rate, STB/d
- **B** = formation volume factor, reservoir volumes/std volume
- **μ** = viscosity of flowing fluid, centipoise
- **Z** = real gas law deviation factor \((pv = ZnRT)\)
- **n** = lb moles
- **R** = 10.73 \((\text{ft}^3 - \text{psi})/\text{lb mole - OR})\)
- **T** = absolute formation temperature, °F
- **PE** = pressure, psi
- **P** = volumetric average pressure within drainage region resulting from constant-rate production for a time \(t\)
- **R** = dimensionless radius, \(r/r_w\)
- **t** = time, hrs
- **m** = slope of semi-log graph, psi/log cycle for liquid
- **s** = skin effect, dimensionless
- **Ap** = pressure drop across skin effect, psi
- **C_D, C** = dimensionless storage constant
- **C_w** = wellbore storage, cu ft/psi
- **C_Df** = wellbore storage for a fractured well
- **x_e** = half length of side of square enclosing a vertically-fractured well, ft
- **x_f** = vertical fracture length from center of well to tip of fracture, ft
- **r_f** = horizontal fracture radius, ft
- **r_D** = dimensionless radius, \(r/r_w\)
- **P** = pressure, psi
- **r_w** = well radius, ft
- **A_w** = drainage area, sq ft
- **t_D** = dimensionless time
- **φ** = porosity, fraction of bulk volume
- **c_t** = total system effective isothermal compressibility, psi
- **r** = radial distance from a constant rate well, ft
- **ϕ_w** = skin effect, dimensionless
- **R_w** = well radius, ft
- **A_w** = drainage area, sq ft
- **t_d** = dimensionless time
- **ϕ_w** = skin effect, dimensionless
- **P** = volumetric average pressure within drainage region resulting from constant-rate production for a time \(t\)
- **r_D** = dimensionless radius, \(r/r_w\)
- **D** = dimensionless storage constant

### Subscripts

- **w** = bottom hole, well
- **t** = surface
- **s** = static (zero surface production rate)
- **f** = flowing
- **r** = radial dimension
- **z** = vertical dimension
- **D** = dimensionless
- **f** = fracture
- **A** = based on drainage area \(A\)
- **i** = initial

### References

| 38. | Cable, E.D.: "Inexpensive Well Testing to Increase Production," Oil & Gas J. (Feb. 11, 1974) 64. |


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**TABLE 1 - PRESSURE BUILDUP DATA FOR A FRACTURED OIL WELL (FROM CABLE 38)**

<table>
<thead>
<tr>
<th>Shut-in time in min.</th>
<th>Dimensionless time $\sqrt{t + \Delta t}$</th>
<th>Casing pressure at top of fluid, psia</th>
<th>Fluid level, joints</th>
<th>Fluid due to pressure, psig</th>
<th>Pressure difference, $P_{ws} - P_{wf}$ psi for $P_{wf} = 740$</th>
<th>$P_{wf} = 710$</th>
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COMMINGLED ZONES, OR ASYMMETRIC DRAINAGE SHAPE, OR?

SOUNDARY EFFECT?

PO, Psi

DAMAGED AFTERFLOW, OR BOTH? OR STIMULATED WITH LARGE AFTERFLOW?

FIG. 1 - CHARACTERISTICS OF HORNOR PRESSURE BUILDUP GRAPHS.

FIG. 2 - $P_D$ VS $t_D$ FOR WELL WITH STORAGE AND SKIN EFFECT.

FIG. 3-A - PRESSURE BUILDUP CURVE, WELL A (AFTER RUSSELL).

P$_{WF}$ = 1590 PSIG

FIG. 3-B - TYPE-CURVE MATCHING FOR WELL A.

$P_D (S, E,t_d)$
Fig. 4 - McKinley Type Curve. 29

Fig. 5 - Earlougher-Kersch Type Curve. 30

Fig. 6 - $P_D$ vs $T_D$ for a vertical fracture with infinite fracture conductivity.

Fig. 7 - $P_D$ vs $T_D$ for a constant rate well with an infinite conductivity vertical fracture (refs. 37, 33).
FIG. 8 - $P_0$ vs $T_D$ for an infinite conductivity vertical fracture with storage (Ref. 34).

FIG. 9 - Horizontal fracture type curve (Ref. 35).

FIG. 10 - Generalized McKinley type curve.

FIG. 11 - Generalized Earlougher-Kersch type curve.

FIG. 12 - Pressure rise vs shut-in time for an oil well (After Ref. 38).

FIG. 13 - Horner build-up graph for an oil well (After Ref. 38).
Fig. 14 - Static pressure vs square root of shut-in time for an oil well.

Fig. 15 - Type curve for apparent fracture case.

Fig. 16 - Rough type curve matching to find start of semi-log straight line.
Fig. 17 - Storage and Skin Type Curve Showing Effect of Flowing Pressure.

Fig. 18 - P/z vs Time for a Flow Test and Buildup for a Gas Well with Wellbore Storage.

Fig. 19 - Dimensionless Pressure Drop vs Dimensionless Time for a Damaged (s = 20) Well Centered in a Closed Square.