Permeability-Porosity Relationships in Sedimentary Rocks


Abstract: In many consolidated sandstone and carbonate formations, plots of core data show that the logarithm of permeability \( k \) is often linearly proportional to porosity \( \phi \). The slope, intercept, and degree of scatter of these \( \log(k) - \phi \) trends vary from formation to formation, and these variations are attributed to differences in initial grain size and sorting, diagenetic history, and compaction history. In unconsolidated sands, better sorting systematically increases both permeability and porosity. In sands and sandstones, an increase in gravel and coarse grain size content causes \( k \) to increase even while decreasing \( \phi \). Diagenetic minerals in the pore space of sandstones, such as cement and some clay types, tend to decrease \( \log(k) \) proportionately as \( \phi \) decreases.

Models to predict permeability from porosity and other measurable rock parameters fall into three classes based on either grain, surface area, or pore dimension considerations. (Models that directly incorporate well log measurements but have no particular theoretical underpinnings form a fourth class.) Grain-based models show permeability proportional to the square of grain size times porosity raised to (roughly) the fifth power, with grain sorting as an additional parameter. Surface-area models show permeability proportional to the inverse square of pore surface area times porosity raised to (roughly) the fourth power; measures of surface area include irreducible water saturation and nuclear magnetic resonance. Pore-dimension models show permeability proportional to the square of a pore dimension times porosity raised to a power of (roughly) two and produce curves of constant pore size that transgress the linear data trends on a \( \log(k) - \phi \) plot. The pore dimension is obtained from mercury injection measurements and is interpreted as the pore opening size of some interconnected fraction of the pore system. The linear \( \log(k) - \phi \) data trends cut the curves of constant pore size from the pore-dimension models, which shows that porosity reduction is always accompanied by a reduction in characteristic pore size. The high powers of porosity of the grain-based and surface-area models are required to compensate for the inclusion of the small end of the pore size spectrum.

INTRODUCTION

An objective of applied petrophysics is to provide detailed and accurate estimates of permeability \( k \) in wells where no core measurements are available. This objective is difficult to achieve because no log has yet been developed that directly measures permeability. Consequently, permeability must be estimated by indirect methods based on either core or logs or on some combination of the two. In this review, permeability-porosity data and algorithms relating them are presented and compared.

Data published in the open literature are difficult to compare because different authors use different scales for porosity and permeability. Moreover, predictive algorithms are often published without an indication of how the adjustable parameters affect the permeability prediction. In this paper this lack of compatibility has been overcome by presenting all data and algorithms on a uniform scale in which one decade of permeability change is equivalent in scale to a change of 0.1 porosity (10 porosity units).

In Part 1 of this paper, data from the open literature are compiled on \( \log(k) - \phi \) plots of uniform scales. The tendency of \( \log(k) - \phi \) data from individual formations to form linear trends of comparable slopes and differing offsets was pointed out by Archie (1950). Although the petroleum industry has continued to measure the permeability and porosity of core samples during the decades since Archie’s paper, there is not a great deal of published data in which other relevant properties of the rock samples are described. Some authors note the influence of grain size, grain shape, cement, and clays upon flow properties, and their comments are mentioned in this review.

After examining the dependence of permeability on porosity and other petrological controls, predictive models relating permeability to various rock and pore parameters are reviewed in Part 2. Because the concepts based on the capillary tube model (Kozeny-Carman equation) permeate so many models developed by many workers over the years, the forms of the Kozeny-Carman equation are briefly reviewed. Ideas concerning the dependence of permeability on porosity, pore throat size, and surface area stem from the Kozeny-Carman equation.

Predictive models are presented under four headings to classify their underpinnings and applications. The first category is for models incorporating grain size and sorting. A theoretical model derived from consideration of pore space geometries and a strongly empirical model arrive at very similar formulations incorporating grain diameter and a single sorting parameter. These models are attractive whenever petrological data are available and are consistent with published permeability-porosity data.

Next, the emphasis turns to models incorporating measures of pore surface area. We discuss how Timur’s equation evolved from the older Kozeny-Carman equation and show that the common petrophysical charts are in turn based on Timur’s equation. The best of the log-based predictors rely on good estimates of irreducible water saturation. Labora-
Permeability-Porosity Relationships in Sedimentary Rocks

Figure 1: Permeability-porosity plot showing data fields for subsequent figures.

Throughout this paper permeability values are presented in darcies, millidarcies, and microdarcies, which originate in a system of mixed units. The SI unit for permeability is square meter, and 1 darcy = 9.869 \times 10^{-12} m^2. Most authors do not state measurement conditions or methods. Unless otherwise stated, it can be assumed that permeability was measured with an air permeameter at low confining pressures, without Klinkenberg correction.

PART 1 – PETROLOGICAL CONTROLS

Permeability values of rocks range over many orders of magnitude; the log(\(k\))–\(\phi\) plot of Figure 1 illustrates several fields of values that are of interest to the petrophysicist. Values commonly encountered in petroleum reservoirs range from a fraction of a millidarcy to several darcies. The fields of Figure 1 will reappear on subsequent log(\(k\))–\(\phi\) plots; the scales will always be the same but the axis values will shift to suit the data. All log(\(k\))–\(\phi\) plots are scaled so that a factor of 10 change in \(k\) is the same length as a change of 0.1 in fractional porosity.

Unconsolidated Sand Packs

Using sand from two Texas rivers, Beard and Weyl (1973) sieved 48 sand samples into 8 size classes and 6 sorting classes. Each data point shown in Figure 2 represents the permeability and porosity of a sample with a unique grain size and sorting. Median grain size ranges from 0.840 mm for the coarse sample to 0.074 mm for the very fine sample (Table 1). The authors present photomicrographs of thin section comparators for each of the 48 samples to enable the reader to see the wide range of size and sorting represented by the sample suite.

Sand samples were wet-packed in vials 1.8 cm in diameter and 5 cm in length. Permeability was determined by flowing a measured volume of 0.5 N brine through the sample for a fixed time while measuring the pressure drop across the sample. Except for a few very fine, poorly sorted samples, the values of \(k\) exceed 1 darcy (Figure 2). The maximum value for a well-compacted, unconsolidated sand pack is about 500 darcies. Porosity ranges from 23.4% to 43.5%. Note the general increase in permeability as grain size increases from very fine to coarse and the increase in both porosity and permeability as sorting progresses from very poor to well sorted.

Consider the extremely well-sorted samples represented by the open-circle data points along the right edge of Figure 2. For these extremely well-sorted samples, porosity is independent of grain size, as it should be for a packing of uniform spheres. However, for those samples that are not well sorted, an increase in the coarse grain content results in somewhat decreased \(\phi\) as \(k\) increases. We will see this pattern preserved in some consolidated samples.
Table 1: Range and median grain size of sieved unconsolidated samples by Beard and Weyl (1973).

<table>
<thead>
<tr>
<th>Size range</th>
<th>Size range (μm)</th>
<th>Median (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse, upper</td>
<td>1,000-710</td>
<td>840</td>
</tr>
<tr>
<td>Coarse, lower</td>
<td>710-500</td>
<td>600</td>
</tr>
<tr>
<td>Medium, upper</td>
<td>500-350</td>
<td>420</td>
</tr>
<tr>
<td>Medium, lower</td>
<td>350-250</td>
<td>300</td>
</tr>
<tr>
<td>Fine, upper</td>
<td>250-177</td>
<td>210</td>
</tr>
<tr>
<td>Fine, lower</td>
<td>177-125</td>
<td>148</td>
</tr>
<tr>
<td>Very fine, upper</td>
<td>125-88</td>
<td>105</td>
</tr>
<tr>
<td>Very fine, lower</td>
<td>88-63</td>
<td>74</td>
</tr>
</tbody>
</table>

Continuing to inspect the extremely well-sorted samples of Figure 2, we observe that log(k) increases in equal increments as grain size increases. The samples were sized so that the mean grain diameter of each adjacent size interval increases by the square root of two; inspection of Figure 2 shows that permeability increases by a factor of 2 for each increment of grain size. Thus, Beard and Weyl's data show that permeability is proportional to the square of grain size. Because theoretical models show that flow is proportional to the square of the radius of a pore opening, it can be said that Beard and Weyl's data demonstrate that pore size is proportional to grain size.

Consolidated Clays

Ground dry clays were mixed with a synthetic sea water to form a slurry, which was then placed in a consolidometer (Thompson and Callanan, 1981). Sample load was increased in steps from 0 to 10,000 psi and the porosity and permeability were determined at each step. Assuming a lithostatic stress gradient of 1 psi/ft, the 10,000 psi maximum pressure represents a depth of 10,000 ft. The data in Figure 3 represent three types of clay with a power law fit to each one.

The data reflect the remarkable sealing power of compacted clays (note the superior sealing capability of bentonite, a primary ingredient of drilling mud). Even though
the porosity of compacted clays remains quite high, the permeability values are 10 microdarcies or less. These results cannot be directly extended to clay-bearing rocks, but it is well known that clay minerals impair permeability. The gamma-ray log is usually a good indicator of clay content, and it often correlates negatively with permeability.

Unconsolidated Clays and Silts

Bryant et al. (1974) applied consolidometer techniques to determine the permeability of samples taken at shallow depths in the Gulf of Mexico. The particular data set shown in Figure 4 represents silts and clays having a sand fraction greater than 5%; unfortunately, no grain size information or clay description is given. The data illustrate that low permeabilities (less than a millidarcy) can be established in young, shallow, high-porosity, very fine-grained sediments.

Effect of Clays and Cement

Having looked at the extremes in permeability afforded by sands and clays, let us now look at the behavior of sedimentary rocks. Figures 5, 6, and 7 exhibit a linear relationship between \( \log(k) \) and \( \phi \) as determined in “relatively pure” quartz sandstones. Such linear trends are often seen, but they are not understood. They are significant because \( \log(k) \) is so well predicted by \( \phi \), having the form \( \log(k) = a + b\phi \).

Thomson (1978) describes continental sandstones from the Lower Cretaceous Hosston Formation in Mississippi (Figure 5): “Secondary quartz cement and compaction through pressure solution of grains are the principal causes of porosity reduction. The early introduction of large amounts of dolomite has inhibited compaction of framework grains. Kaolinite ranges from 5% to 15% of total rock volume. All samples contain a little illite. The permeability-porosity plots indicate a progressive and uniform loss of permeability as porosity is reduced, which suggests the
sandstones underwent a simple diageneric history, uncomplicated by such late processes as leaching, development of authigenic clay minerals, and so forth." Thomson also suggests that the introduction of hydrocarbons caused a cessation in diagenesis in the lower part of the reservoir.

In the case of the Upper Carboniferous samples (Figure 6), the author states that the progressive decrease in $k$ is due to the high clay content. In the case of the Tertiary Bausteinlachen Sandstone (Figure 7), calcite cement, increasing clay content, and decreasing grain size are all possible contributors to the decreasing log($k$)--$\phi$ trend.

Bos (1982) describes results from an exploration well that encountered 1) clean sandstone; 2) sandstone with pores filled with kaolinite; 3) laminated sandstone, part clean and part filled with kaolinite (indicated as "laminated" in Figure 8); and 4) shale. Scanning electron microscope (SEM) photographs document the extent to which kaolinite fills the pores, thereby reducing $k$ as shown in Figure 8. Here again we see the linear relationship between log($k$) and $\phi$, with pore-occluding clays reducing both $k$ and $\phi$ in a fairly systematic fashion.

The term "tight gas sands" is sometimes used to refer to gas-bearing sandstones of unusually low porosity and permeability. Data obtained by Luffel et al. (1991) from the Lower Cretaceous Travis Peak formation in east Texas are shown in Figure 9. Permeability values were measured with samples confined at net overburden pressure and were either corrected for Klinkenberg effect or else were conducted at high pore pressure. Porosity values range from 0.02 to 0.12 while permeability ranges from 0.1 md to 100 md, maintaining a remarkably well defined and unusually steep trend on the log($k$)--$\phi$ plot. Luffel et al. also present data for siltstones and mudstones from the same wells; the slope for the siltstones and mudstones is very similar to that of the channel sandstones, but most of the permeability values are less than 0.1 md.

Soeder and Chowdiah (1990) examined thin sections cut from three of the wells examined by Luffel et al. To examine the pore geometry, the thin sections were impregnated
Permeability-Porosity Relationships in Sedimentary Rocks

Figure 8: Permeability-porosity data from a Lower Cretaceous sandstone from the North Sea by Bos (1982).

Figure 9: Permeability-porosity data from channel sandstones of the Lower Cretaceous Travis Peak formation, east Texas. Data are measured at net overburden pressure and corrected for Klinkenberg effect. From Figure 17 of Luffel et al. (1991).

with fluorescent-dyed epoxy. Soeder and Chowdiah report that "the most common tight-sand pore geometry consists of highly altered primary porosity, extensively occluded with authigenic quartz overgrowths, coupled with significant secondary pore development. Primary porosity is reduced to narrow, flat 'slot' pores along the boundaries of adjoining quartz overgrowths, which are propped open by irregular bumps or asperities on the overgrowth faces. Although most of the porosity occurs in the secondary pores, the narrow slots often provide the only flow paths and create a bottleneck for movement of pore fluids, making the rock tight." Recognition of the flat slotlike pores between quartz overgrowths suggests that the pore space geometry of these tight sandstones differs substantially from the more equidimensional pores of most consolidated rocks.

The unusual log(k)–φ character, that is, the extreme sensitivity of log(k) to small changes in φ shown in Figure 9, is attributed to the presence of the slotlike pores. In addition, the presence of secondary porosity may explain the increasing scatter of data with increasing permeability in Figure 9.

At low permeability and porosity, there probably is little secondary porosity; scatter increases at higher porosity because the secondary porosity is poorly correlated with the slot pores that furnish the permeable pore paths. Interestingly, in examining the behavior of the Klinkenberg correction factor, Luffel et al. note that their samples appear to contain pores that are slotlike as well as pores controlled by spherical grains.
Effect of Grain Size

Data from the Jurassic Dogger Beta sandstone (Figure 10) exhibit more scatter than the previous data sets. Fuchtbauer (1967) attributes the vertical variation in permeability to variation in grain size. Porosity decreases are attributed to denser packing because of greater depth of burial and to carbonate, silicate, anhydrite, or pyrite cementation. Unfortunately, no subsidiary data are presented to support these contentions.

The Lower Cretaceous Bentheimer sandstone (Figure 11) is uncemented and consequently displays high $k$ values. The pattern of the coarser grained samples (open circles) is somewhat similar to the sand-pack data of Figure 2. It appears that interconnected large pores are preserved in these coarse-grained samples, thereby preserving high $k$. While sorting variations produce porosity variations that are independent of $k$. Clearly, $k$ cannot be predicted from $\phi$ in such an environment. The finer-grained samples (solid circles) contain clay that increases from 5% to 30% as both $k$ and $\phi$ decrease.

Data from an eolian environment (Figure 12) show that dune sandstones possess higher porosity and permeability than interdune and extradune deposits. Lupe and Ahlbrandt (1979) define interdune as the area between dunes of a dune complex, originally deposited by the same wind activity that built the dune, and extradune as the area extending beyond the dune field, probably deposited by processes independent of the dune-building process. Both interdune and extradune deposits would be expected to have poorer sorting and be less porous and less permeable than dune deposits. The progression shown in Figure 12 reflects this understanding of dune deposits.

Shenhav (1971) classified samples by grain size; his results are shown in Figure 13. The sandstones are cemented by calcite and dolomite. Note the progressive increase in the log($k$)–$\phi$ slopes and intercepts as grain size increases. The trends are not as linear as observed in other data sets. Shenhav also plotted log($k$)–$\phi$ according to depositional facies, but the plots (not shown here) do not exhibit any dependence on depositional facies.
A composite of data from the Prudhoe Bay field (Figure 14) shows that the character of the permeability-porosity plot varies from zone to zone. Zone 4B, the uppermost zone, and Zones 1B/1A, the lowermost zones, exhibit quite linear trends similar to those discussed above. We infer that these trends are mainly controlled by the cement and clay content. However, Zones 3 and 2C exhibit a great deal more scatter, attributed to the presence of conglomerates and coarse-grain sandstones in these zones. Except for Zone 2A, each of the plots shows a crisp edge on the lower right below which no data appear. By comparison with the data by Shenhav, this crisp edge implies a lack of fine- to very fine-grained rocks in the Prudhoe Bay reservoir.

Clay Coatings on Grains

Diederix (1982) found anomalous electrical properties in Rotliegend sandstone, which he attributed to the presence of illite and kaolinite on sand grain surfaces. Where the saturation exponent \( \eta \) was low, SEM photographs showed the presence of clays. Where it was high, the grains were relatively free of clay minerals. Figure 15 shows the log(\( k \))–\( \phi \) plot for two wells. The data with low \( \eta \), asserted to contain clay coatings on sand grains, have the lower \( k \) at any given \( \phi \).

Wilson (1992) contends that many clay coatings, particularly on eolian sandstones, formed on the framework grains prior to deposition. Their presence actually preserves porosity because quartz overgrowths cannot readily form. According to Wilson, many of the largest petroleum reservoirs (North Sea, north slope of Alaska) have retained good porosity because of detrital clay coatings. Samples in which kaolinite and illite occur as clay coatings fall within the boundaries of the three upper fields in Figure 16.

Fibrous Illite

In the same Rotliegend sandstones, fibrous illite can form within the pore space, reducing the permeability one to two orders of magnitude compared with rocks in which clay occurs as grain coatings (lower two fields in Figure 16).
Nonvuggy Carbonate Rocks

There is not much information in the open literature on the permeability-porosity relationships of carbonate rocks; only two cases are reproduced here.

Based on SEM photography, Turner (1983) describes a poorly sorted oolitic limestone for which data are shown in Figure 17: "The overall rock fabric consists of a poorly sorted mixture of large and small oolites and coated particles ranging from about 25 to 275 microns in diameter. Generally, the grain contacts do not show much evidence of pressure solution... Overall, the particles have a very rough surface texture in some ways resembling clay-coated sandstone grains."

The data in Figure 17 fall on a reasonable log(k)-Φ trend that is attributed to the oolitic grain rock fabric. The slope is somewhat steeper than in most clastics (a steep slope mitigates against successful permeability prediction), which we attribute to the apparent wide range of grain size. The surface roughness alluded to by the author is of much smaller textural scale than the grains themselves, which may be why surface roughness does not reduce k in this case.

Lucia (1983) plotted carbonate data selected from compilations by Shell (Figure 18). He claims that the plot can be used to estimate permeability of a nonvuggy carbonate.

Figure 14: Permeability-porosity data from the seven productive zones at Prudhoe Bay field, Alaska. From Wendt et al. (1986).
rock if the porosity and particle size are known. He points out that the effect of vugs is to increase the porosity but not alter the permeability much. In Figure 18 we can observe the quasi-linear log($k$)--$\phi$ relationship and the decline in slope (and $k$) with decreasing grain size. It appears that the fundamental controls observed in the sandstones are also present in these selected carbonates.

Summary of Petrological Controls

What has been gained by this review of laboratory data? Figure 19 illustrates the effect of different textural and mineralogical controls, based on the preceding log($k$)--$\phi$ plots. Better sorting, as seen on the Beard and Weyl plot, increases both $k$ and $\phi$. Gravel and coarse grain size increase $k$ even though decreasing $\phi$. Diagenetic effects, in the form of cement and some clay types filling the pore space, tend to decrease log($k$) proportionately as $\phi$ is decreased. Very fine grains of silt and clay produce low permeability at high porosity.

Figure 16: Permeability-porosity data fields from wells penetrating the Permian Rotliegend sandstones of the southern North Sea, from Wilson (1992).

PART 2 — PETROPHYSICAL MODELS

Kozeny-Carman Equation

The problem of predicting permeability is one of selecting a model expressing $k$ in terms of other measurable rock properties. Historically, the first approaches were based on a tube-like model of rock pore space known as the Kozeny-Carman relationship (Carman, 1956; Amyx et al., 1960; Timur, 1968; Hearst and Nelson, 1985). The derivation of this "equivalent channel model" has been reworked by Paterson (1983) and Walsh and Brace (1983). The model assumes that flow through a porous medium can be represented by flow through a bundle of tubes of different radii. Within each tube the flow rate is low enough so that flow is laminar rather than turbulent. A tube is assigned a shape factor, $f$, a dimensionless number between 1.7 and 3, and length, $L_w$, which is greater than the sample length $L$. The assumption is that each flow path comprises a twisted, toruous yet independent route from one end of the rock to the other. The tortuosity is defined as $\tau = (L_w/L)^2$. From considerations of flow through tubes, the resulting equation is
Figure 17: Permeability-porosity data from a Jurassic carbonate, Smackover formation, by Turner (1983).

Figure 18: Permeability-porosity data from selected nonvuggy carbonates, by Lucia (1983).

\[
k = \frac{\phi R_a^2}{f \Sigma_p} = \frac{\phi}{f \Sigma_p}. \quad (1a)
\]

where the hydraulic radius \( R_a \) is defined as the reciprocal of \( \Sigma_p \), the ratio of pore surface area to pore volume. The pore surface area normalized by a volume is often called the specific surface area. The form of Equation (1a) depends on which volume is used to normalize the pore surface area. If specific surface area is instead expressed as \( \Sigma_p \), the ratio of pore surface area to rock volume, then Equation (1a) becomes

\[
k = \frac{\phi^3}{f \Sigma_p^2}. \quad (1b)
\]

Whereas if specific surface area is defined as the ratio of pore surface area to grain volume, \( \Sigma_g \), the expression is

\[
k = \frac{\phi^3}{f \Sigma_g^2 (1 - \phi)^2}. \quad (1c)
\]

Thus, the functional dependence of \( k \) on \( \phi \), which differs among Equations (1a), (1b), and (1c), depends on the definition of specific surface area.

Both Paterson (1983) and Walsh and Brace (1983) establish a relationship between electrical properties and tortuosity, determining that formation factor \( F = (L_d/L)^2 \phi / \tau = \tau / \phi \). They note that this expression differs from earlier incorrect formulations. Using it, tortuosity can be eliminated from Equation (1a) to obtain

\[
k = \frac{R_a^2}{f F} = \frac{1}{f F \Sigma_p^2}. \quad (1d)
\]

Most models that relate \( k \) to a pore dimension \( R \) are derived, either in spirit or in rigor, from the Kozeny-Carman relationship, which recognizes explicitly the dependence of \( k \) on \( R^2 \).
MODELS BASED ON GRAIN SIZE AND MINERALOGY

Krumbein and Monk’s Equation

Using experimental procedures that were later adopted by Beard and Weyl (1973), Krumbein and Monk (1943) measured permeability in sand packs of constant 40% porosity for specified size and sorting ranges. Analysis of their data, coupled with dimensional analysis of the definition of permeability, led to

\[ k(\text{darcy}) = 760D_g^2 \exp(-1.3 \sigma_D) \]  

(2)

where \( D_g \) is the geometric mean diameter in millimeters, \( \sigma_D \) is the standard deviation of grain diameter in phi units, where \( \phi = -\log_2[D \text{ (mm)}] \). Although the Krumbein and Monk equation is based on sand packs of 40% porosity and does not include porosity as a parameter, Beard and Weyl showed that Equation (2) fits their own data fairly well even though porosity of the Beard and Weyl samples ranges from 23% to 43%. In fact, because of difficulties in obtaining homogeneous sand packs, Beard and Weyl chose to use computed \( k \) values from Equation (1) rather than their measured data in tabulating values for fine and very fine samples with poor or very poor sorting. If Equation (2) can predict \( k \) for a varying \( \phi \) in unconsolidated sand packs, then the exponential dependence on sorting must be adequate to describe all the effects associated with porosity reduction. In other words, both \( k \) and \( \phi \) reduction pictured in Figure 2 are primarily due to a decrease in sorting.

The laboratory studies of Krumbein and Monk (1943) and Beard and Weyl (1973) dealt with sieved sands from a common source so that grain properties such as angularity, sphericity, and surface texture did not vary much. Moreover, sorting was purposely controlled to be log normal. In situations where these ideal conditions are not met, other techniques must be invoked to predict permeability in unconsolidated sands. A disproportionate amount of fines can drastically reduce \( k \) in unconsolidated sands. Morrow et al.
Table 2: Relation of sorting coefficient $C$ to spread of dominant grain diameter, $D_d$, from Van Baaren (1979).

<table>
<thead>
<tr>
<th>Sorting</th>
<th>$C$</th>
<th>$D_\text{max}/D_\text{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extremely well to very well sorted</td>
<td>0.70</td>
<td>2.5</td>
</tr>
<tr>
<td>Very well to well</td>
<td>0.77</td>
<td>—</td>
</tr>
<tr>
<td>Well</td>
<td>0.84</td>
<td>3.5</td>
</tr>
<tr>
<td>Well to moderately</td>
<td>0.87</td>
<td>—</td>
</tr>
<tr>
<td>Moderately</td>
<td>0.91</td>
<td>8.0</td>
</tr>
<tr>
<td>Moderately to poorly</td>
<td>0.95</td>
<td>—</td>
</tr>
<tr>
<td>Poorly</td>
<td>1.00</td>
<td>—</td>
</tr>
</tbody>
</table>

(1969), using statistical techniques on data from Gulf Coast sands, found that permeability correlated best with the log of grain size times sorting if the fine fraction, taken to be of size less than 44 $\mu m$, was accounted for.

Berg's Model

An interesting model linking petrological variables—grain size, shape, and sorting—to permeability is that of Berg (1970). Berg considers "rectilinear pores," defined as those pores that penetrate the solid without change in shape or direction, in various packings of spheres. Simple expressions for $k$ are derived from each packing and form a linear trend when $\log(k)$ is plotted against $\log(\phi)$. From these geometrical considerations comes an expression relating $k$ to $\phi$ raised to a power and to the square of the grain diameter,

$$k(\text{darcy}) = 5.1 \times 10^{-6} \phi^{5.1} D^2 e^{-1.385 p},$$  \hspace{1cm} (3a)

where $D$ (mm) is the median grain diameter, $\phi$ is porosity in percent, and $p$, a sorting term, is explained below. If permeability is expressed in millidarcies, $D$ in micrometers, and $\phi$ as fractional porosity, this expression becomes

$$k(\text{md}) = 80.8 \phi^{5.1} D^2 e^{-1.385 p}.$$ \hspace{1cm} (3b)

To account for a range of grain sizes, Berg considers two mixtures of spheres and assumes that $k$ will be controlled primarily by the smaller grains. This introduces a sorting term $p = P_{90} - P_{10}$, called the percentile deviation, to account for the spread in grain size. The $p$ term is expressed in phi units, where $\phi = -\log_2 D$ (mm). For a sample with a median grain diameter of 0.177 mm, a value of 1 for $p$ implies that 10% of the grains are larger than 0.25 mm and 10% are smaller than 0.125 mm.

Berg's expression Equation (3b) is illustrated in Figure 20 for $p = 1$ and varying $D$. Permeability increases rapidly with increasing porosity, depending on $\phi$ to the fifth power, and the curves migrate downward and to the right with decreasing grain size. By overlaying Figure 20 on the other charts that have some data on grain size, one can verify that it is remarkably concordant with the Lower Cretaceous Bentheimer sandstone, with Shenav's data on a Lower Cretaceous sandstone, and with Lucia's data on carbonates. Thus, Berg's theoretical model is consistent with three independent data sets. Berg's model appears to be a usable means of estimating permeability in unconsolidated sands and in relatively clean consolidated quartzose rocks. This is true even though Berg did not expect his model to be applicable for porosity values less than 30%.

Van Baaren's Model

Proceeding along more empirical lines, Van Baaren (1979) obtains a result nearly identical to that of Berg. Van Baaren begins with the Kozeny-Carman expression of form 1b and substitutes $F \phi = \tau, \Sigma_\phi = 4\phi/d$, and $F = \phi^{-m}$ to obtain

$$k = c_2 d^2 \phi^m,$$ \hspace{1cm} (4)

where $c_2$ is a constant. This substitution appears to be in error because $4\phi/d$ is the ratio of surface area to volume of a cylindrical tube of diameter $d$ (equivalent to $\Sigma_\phi$ in Equation 1a) rather than the tube surface area normalized by rock volume as required by Equation (1b). Nevertheless, van Baaren proceeds empirically, first using a set of experimental measurements of $k, \phi$, and mercury injection data to obtain

$$k = 10d^2 \phi^m,$$ \hspace{1cm} (5)

where $d_{70}$ is the effective pore diameter ($\mu m$) at 70% wetting saturation. We will see in the section on "models based upon pore dimension" that Equation (5) is quite similar to equations obtained by other workers. However, van Baaren uses Equation (5) only as an intermediate step in deriving a permeability predictor based on petrological variables. Using data from sandstones and carbonates, an empirical relationship between $\phi$ and $d_{70}/D$ is established,

$$\phi = C \left( \frac{d_{70}}{D_d} \right)^{0.55},$$ \hspace{1cm} (6)

where $D_d (\mu m)$ is the dominant grain size from petrological observation, and $C$ is a sorting index given in Table 2. Substitution of Equation (6) into Equation (5) gives

$$k = 10D_d^2 \phi^{3.64+m} C^{-3.64}.$$ \hspace{1cm} (7a)

Van Baaren also gives a table relating formation factor $m$ to the state of consolidation for sands and sandstones. Consequently, Equation (7a) can be used to estimate $k$ from petrological observations (consolidation state, dominant
Permeability-Porosity Relationships in Sedimentary Rocks

Figure 21: Empirical model relating permeability to porosity with mineralogy as a parameter, after Herron et al. (1987).

Figure 22: Empirical model relating permeability to porosity with critical interstitial water $S_{crw}$ as a parameter, after Granberry and Keelan (1977).

The grain diameter $D_g$ and sorting index $C$ and a porosity estimate obtained from either core or logs.

Assuming that the dominant grain size $D_g$ is equivalent to Berg's median grain diameter $D$, then Equation (7a) is very close to Equation (3). For example, a sorting parameter $p = 1$ in Equation (3b) results in

$$k (md) = 20.2 \phi^{5.1} D^2,$$

where, for a well-sorted sandstone, $C = 0.84$ (Table 2) and Equation (7a) becomes

$$k = 18.8 D^2 \phi^{3.64 + m}.$$  

Van Baaren's Equation (7a) is so close to Berg's Equation (3b) that a separate log($k$)--$\phi$ plot is not warranted here.

Van Baaren's expression is probably easier to use as the parameters are directly related to practical petrological variables. Note that both models are compatible with the data of Beard and Weyl on unconsolidated sands in that $k$ increases with the square of grain size.

A Mineralogical Model

Herron (1987) uses Equation (1c) as a starting point for a model using mineralogical abundances in place of specific surface area. He obtains

$$k = A_f \left[ \frac{\phi^3}{(1-\phi)^2} \right] \exp(\Sigma B_i M_i),$$

where $M_i$ is the weight fraction of each mineral component in the solid rock and $B_i$ is a constant for each mineral (values of $B_i$ are given in Figure 21). Mineral abundances are obtained by performing an element-to-mineral transform on data from a logging tool that measures chemical elemental concentrations by means of neutron-induced gamma-ray spectroscopy. The coefficient $A_f = 4.9 + 2 F_{max}$ where $F_{max}$ is the maximum feldspar content over an interval of interest. $A_f$ represents the textural maturity of the sediment; that is, if feldspars are present, the sediment is likely to have a higher permeability than a rock where feldspars have been

May-June 1994

The Log Analyst
altered to clay minerals. Herron notes that criteria other than feldspar content could be used to assess $A_F$.

The logarithmic form of Equation (8) is shown in Figure 21, along with hypothetical examples for a pure quartz rock and five examples for a 90% quartz rock with 10% allotted to five other minerals. The effect of the $B_i$ and $A_F$ coefficients is to translate the permeability curve upwards or downwards. Note that the presence of feldspar (Curve 1 in Figure 21) increases permeability with respect to the pure quartz example (Curve 2), while smectite, with the most negative coefficient, depresses permeability the most (Curve 6).

The dependence of permeability on grain size and sorting is left implicit in Equation (8). The characteristics of grain size and sorting are partially compensated by stipulating mineral type and abundance. Herron uses data from three wells in different geographic areas to test the model.

Figure 23: Data from three US oil fields, by Timur (1968).

Figure 24: Permeability data from three US oil fields as a function of porosity and residual water saturation, after Timur (1968).

MODELS BASED ON SURFACE AREA AND WATER SATURATION

Two ideas inherent in Equation (1) are important for later developments: the dependence of $k$ on a power of porosity and on the inverse square of surface area. Equation (1) has been used as a starting point for predicting permeability from well log data by assuming that residual water saturation $S_{rw}$ is proportional to specific surface area $\Sigma$.

Granberry and Keelan’s Chart

Granberry and Keelan (1977) published a set of curves relating permeability, porosity, and “critical water” $S_{crw}$ for Gulf Coast Tertiary sands that frequently are poorly consolidated. Their chart, originally presented with $S_{crw}$ as a function of permeability with porosity as a parameter, is transposed into $\log(k) - \phi$ coordinates in Figure 22. The $S_{crw}$ parameter is taken from the “knee” of a capillary pressure curve; it is said that if the water saturation in the formation is less than this critical value, the well will produce water free. Because $S_{crw}$ is taken from the capillary pressure curve, it is a measure of the size of interconnected pores.

Timur’s Model

Timur (1968) used a data base of 155 sandstone samples from three oil fields (Figure 23). The three sandstones
Figure 25: Interpretation chart incorporating irreducible water saturation as a parameter, from Schlumberger (1985).

Figure 26: Interpretation chart incorporating irreducible and bound water as parameters.

Exhibit varying degrees of sorting, consolidation, and ranges of porosity. Timur measured $S_{wi}$ using a centrifuge and then held $k$ proportional to $S_{wi}^{-2}$ in the general power-law relationship,

$$ k = \frac{ab}{S_{wi}}. \quad (9) $$

Coefficients $a$ and $b$ were determined statistically. Timur’s statistical results show that the exponent $b$ can range between 3 and 5 and still give reasonable results. Results for $b = 4.4$ produced a fit somewhat better than other values; it was obtained by taking the logarithm of both sides of Equation (2) and testing the correlation coefficient with respect to $\phi/S_{wi}$. There is no theoretical basis for the substitution of $S_{wi}$ for specific surface area $\Sigma$, so although the form of Equation (9) is similar to Equation (1), it is strictly an empirical relationship. The effectiveness of Equation (9) as a predictor of permeability is shown in Figure 24. The popularity of Timur’s equation is such that versions of it can be found in commercial chart books (Figure 25).

It is not easy to apply Equation (9), which is based totally on core data, to an oil reservoir. The $S_{wi}$ core data used to establish Equation (9) were obtained for a fixed value of capillary pressure $P_c$. In a reservoir $P_c$ varies with height, and because $S_{wi}$ varies with $P_c$ it is necessary to assume a functional dependence of $S_{wi}$ on $P_c$ (Raymer, 1981). There are also some practical difficulties in establishing the coefficients $a$ and $b$ in a reservoir in which the oil-water contact cuts across lithologies because of regional dip or structure. In particular, within the transition zone only part of the water is irreducible ($S_{wi}$); the remainder is movable. Thus, a log-based estimate of saturation immediately above the oil-water contact will overestimate $S_{wi}$.

**Dual Water Model**

An algorithm discussed by Schlumberger (1988) and Ahmed et al. (1989) is attributed to Coates. An extension of
Equation (9) and Figure 25, it assures that permeability declines to zero as $S_{wi}$ increases to fill the entire pore space

$$k = \left[ \frac{100\phi_w^2 (1 - S_{wa})}{S_{wi}} \right]^2. \tag{10}$$

A further refinement incorporates the presence of clay minerals and is based on the dual water model. It requires log-based estimates of the total porosity $\phi_t$, effective porosity $\phi_e$, and bound water saturation $S_{bw}$. The volume of bound water, $V_{bw} = S_{bw} \phi_e$, is computed, and an estimate of a parameter $V_{br} = S_{wi} \phi_e$, called the bulk volume irreducible water in clean wet rock, must also be provided. Then computed as a function of depth is the total immovable water

$$V_{wi} = V_{br} (1 - S_{bw}) + V_{bw} \tag{11}$$

and the permeability

$$k = \left[ \frac{100\phi_w^2 (\phi_t - V_{wi})}{V_{wi}} \right]^2 \tag{12}$$

The algorithm of Equations (11) and (12) uses a pair of parameters $V_{br}$ and $V_{bw}$, which in effect sweep out a broad region of the log($k$)-$\phi$ crossplot (Figure 26). For the solid curves $V_{bw}$ has been set to zero as if the rock were entirely clay free. As irreducible water $V_{br}$ increases, the curves shift downward and to the right into the regime populated by fine-grained rock. The dashed curve is drawn for $V_{bw}$ and $V_{br}$ (each equal to 0.05), thereby representing one of a second family of curves for a fine-grained dirty sandstone. Note how $S_{bw}$ increases with decreasing $\phi$.

This algorithm produces reasonable results in sandstones if $V_{br}$ is chosen judiciously. One difficulty is choosing a value for $V_{br}$ in coarse-grained and gravel-bearing sandstones.
Permeability-Porosity Relationships in Sedimentary Rocks

“Tight” Sandstones

The prediction of permeability becomes much more difficult in formations with small grain size and an abundance of clay minerals. Such rocks are called “tight gas sands” or “submillidarcy reservoirs.” Kukal and Simons (1986) show that the Timur equation produces $k$ values too high in such formations and establish some predictive equations that decrease the porosity by multiplying $\phi$ by $1 - V_{cl}$, where $V_{cl}$ is the clay fraction. They show that the water saturation term $S_{sw}$ is not so important in these rocks with high clay content. Although their predictive equation is a welcome improvement, the scatter shows the difficulty in dealing with such low-porosity systems.

Nuclear Magnetic Resonance and Other Measures of Surface Area

Equation (1a) indicates that other measures of specific surface area could be correlated with permeability. A study by Sen et al. (1990) provides laboratory data on 100 sandstone samples on exchange cation molarity, $Q_e$, nuclear magnetic resonance decay time, $T_1$, and pore surface area-to-pore volume ratio $\Sigma_n$ from the gas adsorption method. Borgia et al. (1992) provide data on $\Sigma_n$ and $T_1$ on 32 samples. Both studies include measurements of $k$, $\phi$, and formation factor $F$ on their samples. Both sample suites are comprised of samples from different formations so the $\log(k)$–$\phi$ plots exhibit scatter, as shown by Figure 27.

Both groups of experimenters found that $k$ correlated best with measures of specific surface when it formed a product with $\phi^n$ or $\phi^2$. For example, Sen et al. found $k$ to be strongly correlated ($R$ around 0.9) with $(\phi^n/\Sigma_n)^{12.08}$, with $(\phi^n/T_1)^{2.15}$, and with $(\phi^n/Q_e)^{2.11}$. Two of these correlations are shown as insets in Figure 27. Borgia et al. (1992) did not incorporate $m$ into their regression equations, but found $k$ to be best correlated with $(\phi^n/\Sigma_n)^{0.76}$ and with $(\phi^n/T_1)^{0.72}$. As an example of these statistical fits, the expression from Sen et al.,

$$k(\text{md}) = 0.794(\phi^n T_1)^{2.15},$$

where $T_1$ is in milliseconds and $\phi$ is fractional porosity, is graphed in Figure 28. Because the porosity exponent is so close to that established by Timur (Equation (9)), the curves in Figure 28 are quite similar to those in Figure 25. Because the nuclear magnetic resonance method can be instrumented for borehole logging, Equation (13) could be useful for both laboratory and borehole applications.

Summary

Timur’s equation and its corresponding chart offer a viable method of permeability estimation where porosity and irreducible water saturation can be estimated. Difficulties arise if there is uncertainty in $S_{sw}$ as there is within an extensive transition zone. The dual water predictor is an interesting embellishment that can include a clay content parameter.

WELL LOG MODELS

In fields where log and core data are plentiful, approaches based on grain size and water saturation give way to reliance on statistical techniques. A widely used statistical approach is multiple linear regression (Allen, 1979; Wendt et al., 1986). Linear regression techniques have become very popular for establishing predictors of geological variables because the methods are effective at predicting mean values, are fast computationally, are available in statistical software packages, and provide a means of assessing errors.

Predictors with One or Two Input Variables

The simplest regression model takes porosity to be the single independent variable,

$$\log(k) = a + b\phi. \quad (14)$$

When a straight-line relationship between $\log(k)$ and $\phi$ exists, the computation of a predictor by Equation (14) is straightforward and merits little discussion. Curvature in the $\log(k)$–$\phi$ relationship is treated by adopting a polynomial in $\phi$. Increased accuracy is also afforded by dividing the field area tally or vertically and computing regression coefficients for each area. In one area the curvature in the statistical predictor may be rather pronounced; in another, no curvature may be present.

Predictors with Several Input Variables

The quality of the predictor can often be enhanced by adding a variable such as gamma-ray response or depth normalized to top of formation. As variables are added to Equation (14), families of curves are required to present graphically the effect of combinations of variables. By varying one or two parameters, the curves sweep out a large area on the $\log(k)$–$\phi$ plot. Predictive power can be increased by adding other parameters. Predictive accuracy does not increase indefinitely as parameters are added, but instead usually reaches a limit after several (anywhere from two to six) parameters are included in the regression. Refer to Figure 17 of Wendt et al. (1986) for an example.

Predictors Using Computed Parameters

Computed logs such as shale volume and differences between porosities from different logs can be included as independent variables. In this way, petrological information
can also be incorporated into the predictive relationships. A
petrological parameter (cement or gravel) is first “pre-
dicted” from well logs using core observations as “ground
truth.” The predicted petrological parameters can then be
included in a relationship to estimate permeability.

As the complexity of the log($k$)–$\phi$ plot increases (that is,
as the data deviate from a linear trend), more variables must
be incorporated into the predictive model in order to main-
tain predictive accuracy, although instability can result
from having too many variables. The better the under-
standing of petrological controls on permeability, the more
effective will be the predictor and its application. Other
complications with regression methods, including the un-
derestimation of high-permeability zones, are mentioned by
Allen (1979), Wendt et al. (1986), and Nicolaysen and

A database approach equivalent to an $n$-dimensional
lookup table can also be used for predicting permeability
within a field or common geology (Nicolaysen and Svend-
sen, 1991). In this approach, the user must first select the
logs or log-derived variables that offer sufficient discrimi-
nating power for permeability. One must also choose a
suitable bin size for each variable, based on its resolution.
Then, a database is constructed from the core permeability
values and associated log values. Each $n$-dimensional bin or
volume is bounded by incremental log values and contains
mean and standard deviation values of permeability plus the

number of samples. In application, permeability estimate:
are extracted from a bin addressed by the log values. An
interpolation scheme is used to extract an estimate from an
empty bin. Like the regression method, the database ap-
proach can only be used where adequate core data are
available to build the model and results generally cannot be
transferred to other areas.

MODELS BASED ON PORE DIMENSION

Capillary Pressure and Pore Size

It is, of course, the dimension of connected pores that
determines permeability, not grain size and sorting, nor
porosity. Thus, all the methods of estimating permeability
discussed up to this point are indirect methods. A viable
direct method requires both adequate theoretical underpin-
ings relating pore dimension to permeability and exper-
imental determination of the critical pore dimension
parameters. Many workers have made use of the capil-
ary pressure curve, obtained experimentally by injecting mer-
cury into a dried sample. As mercury pressure is increased,
more mercury is forced into progressively smaller pores in
the rock and the resident pore fluid (water or air) is ex-
pelled. A length, $R$, usually referred to as the pore throat
radius, is related to the injection pressure by the Washburr
equation,

$$P_c = 2\sigma \cos \theta \frac{R}{R},$$

(15)

where $\sigma$ the interfacial tension and $\theta$ is the wetting
angle. The injection process can be visualized by examining
the idealized capillary pressure curve of Figure 29. A finite
pressure is required to inject mercury into a 100% water-saturated sample (right side of Figure 29). At the first
inflection point (entry pressure), mercury occupies only a
small fraction of the pore volume containing the largest
pores. Next, much of the pore space becomes filled with
mercury with a comparatively slight increase in pressure
(progressing from the circle labelled Katz and Thompson to
the circle labelled Swanson in Figure 29). Finally, large
pressure increases are required to force more mercury into
the smallest pores (steep curve to left of Swanson circle,
Figure 29).

Many authors have linked capillary pressure curves to
permeability. Purcell (1949; summarized by Amyx et al.,
1960, p. 167-172) derived an expression relating $k$ to an
integral of $P_c^2$ over the entire saturation span and achieved
a good match with core data. The relationships established
by Timur (1968) and Granberry and Keelan (1977), dis-
cussed in a previous section, are symbolized at low water
saturation in Figure 29. Contributions by Swanson (1981),
Swanson's Equation

Swanson (1981) provides a method of determining air and brine permeabilities from a single point on the capillary pressure curve. His regression relationships are based on permeabilities and capillary pressure data on 203 sandstone samples from 41 formations and 116 carbonates from 33 formations. His method picks the maximum ratio of mercury saturation to pressure, \((S_b/P_c)_{\text{max}}\), from the capillary pressure curve, arguing that at this point all the connected space is filled with mercury and "this capillary pressure corresponds to pore sizes effectively interconnecting the total major pore system and, thus, those that dominate fluid flow." From linear regression, Swanson obtains simple equations of the form

\[
k = a \left( \frac{S_b}{P_c} \right)^c,
\]

(16)

where the constants \(a\) and \(c\) depend on rock type (carbonate versus sandstone) and on fluid type (air or brine). For carbonates and sandstones combined, \(c\) is equal to 2.005. Because \(S_b\) is defined as the mercury saturation as percent of bulk volume, it must be proportional to \(\phi(1-S_w)\); through Equation (15), \(P_c\) can be linked with a pore throat radius \(R_{\text{pore}}\). Thus, Swanson's result shows that \(k\) is proportional to \(\phi(1-S_w)R_{\text{pore}}^2\), again demonstrating the dependence of \(k\) on the square of a pore throat size.

Winland's Equation and Pittman's Results

An empirical equation relating permeability, porosity, and a capillary pressure parameter is referred to as Winland's equation (Kolodzie, 1980; Pittman, 1992). Based on laboratory measurements on 312 samples, Winland's regression equation is

\[
\log R_{35} = 0.732 + 0.588 \log k - 0.864 \log \phi,
\]

(17)

where \(R_{35}\) is the pore throat radius at 35% mercury saturation, \(k\) is air permeability, and \(\phi\) is porosity in percent. A log(\(k\))–\(\phi\) plot based on Equation (17) and showing five characteristic lines for pore throat radius is graphed in Figure 30. Note that at a given porosity, permeability increases roughly as the square of the pore throat radius. However, for a given throat size, the dependence of permeability on porosity is not great (slightly less than \(\phi^2\)). Kolodzie indicates that pore throat size was used as a cutoff for reserve determinations, in preference to the use of \(k\) or \(\phi\). Hartmann and Coalson (1990) also present Winland's equation in the same format as Figure 30. They state that \(R_{35}\) is a function of both entry size and pore-throat sorting and
is a good measure of the largest connected pore throats in a rock with intergranular porosity.

Pittman (1992) sheds additional light on Winland’s equation, linking it to Swanson’s results. Pittman used a set of 202 sandstone samples from 14 formations on which $k$, $\phi$, and mercury injection data had been obtained. Using Equation (15), he associated a pore size $R_{apec}$ with the capillary pressure $P_c$ determined by Swanson’s method and found that the mean value of $R_{apec}$ has a mercury saturation of 36%. That is, on a statistical basis, the points denoted by circles labelled “Swanson” and “Winland” in Figure 29 are practically identical, and the two methods are sampling the same fraction of the pore space.

Pittman (1992) also established regression equations for pore aperture sizes ranging from 10% to 75% mercury saturation. His expressions have been rearranged and displayed in Table 3 to show the exponents of $R$ and $\phi$ required to predict $k$. (Because $R$ was used as the dependent variable in Pittman’s regressions, the coefficients in Table 3 differ somewhat from what would be obtained if $k$ were the dependent variable; however, the changes would not invali-

FIGURE 31: Permeability equation with critical pore size radius $R_c$ as a parameter, from Katz and Thompson (1987).
Perteeability-Porosity Relationships in Sedimentary Rocks

Table 4: Eight equations relating permeability $k$ (md) to fractional porosity $\phi$. Other parameters are defined in the text, referenced by equation number given in # column. Dimensions of grain diameters $D$ and pore throat radii $R$ are in micrometers.

<table>
<thead>
<tr>
<th>Class</th>
<th>Author</th>
<th>#</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand pack</td>
<td>Krumbein &amp; Monk</td>
<td>2</td>
<td>$k = 0.76D_{g}^2 e^{-1.3a}$</td>
</tr>
<tr>
<td>Grain-based</td>
<td>Berg</td>
<td>3a</td>
<td>$k = 80.8D_{g}^2 \phi^{1.1} e^{-1.25a}$</td>
</tr>
<tr>
<td>models</td>
<td>Van Baaren</td>
<td>7a</td>
<td>$k = 10D_{g}^2 \phi^{1.64+6C -3.64}$</td>
</tr>
<tr>
<td>Surface area</td>
<td>Timur</td>
<td>9</td>
<td>$k = 0.136S_{ss}^{0.4} \phi^{1.5}$</td>
</tr>
<tr>
<td>models</td>
<td>Sen et al.</td>
<td>13</td>
<td>$k = 0.794 R_{s}^{2.13} \phi^{1.25}$</td>
</tr>
<tr>
<td>Pore size</td>
<td>Kozeny-Carman</td>
<td>1d</td>
<td>$k = 400 R_{s}^{2} \phi^{1.5}$</td>
</tr>
<tr>
<td>models</td>
<td>Winland</td>
<td>17</td>
<td>$k = 49.4 R_{s}^{2} \phi^{1.47}$</td>
</tr>
<tr>
<td>Pore size</td>
<td>Katz &amp; Thompson</td>
<td>18</td>
<td>$k = 17.9 R_{s}^{2} \phi^{1.6}$</td>
</tr>
</tbody>
</table>

The percolation concepts used to derive Equation (18b’s) are quite different from the geometrical arguments used to derive the Kozeny-Carman expression. The coefficient in Equation (1d), which is around 0.4, is considerably greater than that of 0.0177 in Equation (18b’s). Consequently, the characteristic radius $R_{c}$ is about 4.7 times greater than the hydraulic radius, $R_{h}$. Although $R_{h}$ is defined as the ratio of core volume to pore surface area, it can be determined in a variety of ways, including the use of mercury injection. Conceptually, then, the Kozeny-Carman equation could also be represented by an extended horizontal line across Figure 29; that is, as a method that samples a broad spectrum of pore sizes.

It is interesting to compare the Katz and Thompson model (Figure 31) with Winland’s empirical equation (Figure 30). The shape of the curves is comparable; the models agree on the approximate $\phi^2$ dependence. The pore radii given by the Winland equation are smaller than comparable radii in the Katz and Thompson model. This is expected because the Winland equation requires a saturation of 35%, a criterion of greater injection pressure than that of Katz and Thompson. What is noteworthy is the general agreement between the two models regarding the form of the $\log(k)$-$\phi$ relationship. They demonstrate that the models invoking higher powers of $\phi$, which we have shown in previous graphs, are not well grounded physically; the higher powers of $\phi$ are required to compensate for lack of knowledge regarding the critical pore dimension. It does seem, however, that the empirical data that often show a “straightline” $\log(k)$-$\phi$ relationship contain some fundamental information regarding how the critical pore dimension relates to porosity.

PRACTICAL APPLICATIONS

The problem of predicting permeability has been reviewed by compiling data and predictive algorithms from the literature. Which approach should be used to estimate permeability from core and well log data? As a practical matter, it depends on what data are available from a given well or field:

1. In cases where no core logs or core data are available, one can proceed by analogy using data developed in formations with geological properties similar to the one under study. Figures 5 through 19 can give some general guidance in such cases.

2. Where porosity and grain size estimates are available, refer to Figure 20. This chart appears to give good estimates for many consolidated rocks. Exceptions will exist, such as rocks containing illite in pore space and low-permeability formations, such as those shown in Figures 9, 15, and 16.

3. In situations where porosity and water saturation can be estimated, permeability can be estimated from Timur’s relationship [Equation (9) and Figure 25]. In clay-bearing rocks, the dual water relationship for permeability [Equations (11) and (12)] is an interesting enhancement, but the user is required to provide estimates of both interstitial and bound water.

4. In field developments where core data are abundant, it is worthwhile constructing a petrophysical predictor that can be used to compute a permeability log based on openhole logs. Such a statistically based algorithm will be unique to the field or formation for which it is developed.

5. The most recent research relates permeability to a pore dimension of a selected subset of the pore population [Figures 30 and 31 and Equations (16–18)]. The critical pore dimension is determined from capillary pressure by mercury injection. These advances offer the opportunity to determine the permeability of small or fragmented samples.

CONCLUSIONS

Expressions relating permeability $k$ to porosity $\phi$ are summarized in Table 4. In Table 4, permeability is in millidarcies and grain sizes $D$ and pore sizes $R$ are expressed in micrometers, so the coefficients may differ from the originating equation in the text. From these representative equations, it can be seen that 1) the predictive equations are simple in form, 2) that $k$ is related to a power of $\phi$ (with the exception of the Krumbein and Monk equation), and 3) that $k$ is related to the square of either a characteristic length or a measure of surface area.

We have seen that models relying on estimates of surface area, whether that estimate comes from irreducible water saturation, nuclear magnetic resonance, gas adsorption, or cation exchange data, require porosity raised to a power of roughly 4. How can surface area models requiring a porosity power of 4 (Figures 25 and 28) be reconciled with pore
dimension models requiring a power of 2 (Figures 30 and 31)? The answer lies in the pore size distribution. Because most of the surface area is contributed by the smallest grains ( pores), measures of surface area emphasize the small end of the pore size spectrum. Yet, the small pores contribute least to permeability. The high (∼4) power of porosity serves to unweight the contribution of the small pores. In the surface area models, porosity serves a dual role, first as a measure of tortuosity and second as a measure of the pore size distribution function.

A similar question arises with the grain size models. Both models by Berg (Figure 20) and van Baaren require a porosity power of around 5, multiplied by the square of a dominant grain size. Why is the porosity power so high? Probably because the dominant grain size becomes a progressively poorer measure of dominant pore size as the spread in grain size increases and small grains ( pores) become more abundant. Again, porosity serves both as a measure of tortuosity and as a weighting factor to compensate for the presence of small pores at lower porosities. Moreover, the retention of a sorting term in Equations (3a) and (7a) is inadequate compensation for small pores, even though a sorting term is all that is needed in sized samples [Equation (2)].

Models incorporating an estimator of pore size [Equations (1d), (17), and (18) in Table 4] include porosity raised to a power of m (=2). Estimates of dominant or characteristic pore size are more effective at predicting k than estimates of grain size or surface area, so the higher exponent of porosity to compensate for the low end of the porosity spectrum is not required. Given a measure of R and φ, the more information that R contains regarding the large through-going pores, the lower the dependence on φ. Indeed, the findings of Beard and Weyl (1973), Swanson (1981), Katz and Thompson (1986), and Pittman (1992) all show that φ is not so important as a predictor of k as long as the dominant R is well specified. Conversely, using Pittman’s findings of Table 3, as R decreases below Raper, R becomes a progressively poorer estimator of the dominant R, and a higher exponent of φ is required to compensate for the inclusion of pore throats that do not contribute to flow.

The preceding considerations hold for predicting k on individual samples from a wide range of rock formations, whereas Part I of this paper shows that φ can be a good predictor of k for samples from a given rock formation. Why is this? The pore size models produce curves of constant pore size that transgress the steeper log(k)-φ data trends. The cutting of the log(k)-φ trends by the curves of constant pore size shows that porosity reduction is always accompanied by a reduction in characteristic pore size. As rocks from a common source are compacted and undergo diagenesis, pore space is reduced and permeable pathways are progressively occluded in a systematic way that maintains a consistent relationship between φ and R. Samples from different formations that have undergone different diagenetic processes follow different evolutionary paths in log(k)-φ-R space and thus produce different trends on a log(k)-φ plot.

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May-June 1994 The Log Analyst 61
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