A Production Mechanism Diagnosis Approach to the Gas Material Balance
O. Morán-O. and F. Samaniego-V., PEMEX and UNAM.

Abstract
This paper presents the derivation of a more general gas material balance equation than that currently available in the literature, that considers a variable effective compressibility, water influx, an associated shale pore volume, that the reservoir could be naturally fractured, and abnormal pressure conditions. The diagnosis of the reservoir production mechanisms problem is thoroughly addressed. With presently available conventional material balance methods and limited diagnosis techniques, under conditions of a partial reservoir characterization, it is not possible to positively differentiate between an abnormal volumetric reservoir and one connected to a shale pore volume. This work discusses new diagnosis methods such as \[ \partial(p/z)\partial G_p \text{ vs } G_p \text{ or } (p_i - p) \text{ and } -\partial(p/z)\partial p \text{ vs } \partial G_p/\partial p. \]

The new methods are illustrated through its application to the most discussed gas field behavior examples published in the literature, among them the McEwen’s water influx case, the volumetric behavior, Beggs’s case, and the Anderson L.

Derivation of a General Material Balance Equation.
We follow basically the derivation of Fetkovich et al. \(^5\). The main difference is that in the present study the possibility of a naturally fractured formation is considered. The details of this general gas material balance equation (GGMBE) are shown in reference 6. The general form of this GGMBE is

\[ G(B_w - B_g) + GB_p \tilde{c}_e(p) dp = \left( G_p - W_p R_w - G_w \right) B_w + \left( W_p - W_{wi} - W_e \right) B_w, \]

(1)

where the cumulative effective compressibility term \( c^-e(p) \) is pressure-dependent, consisting of a cumulative pore volume \((PV)\) compressibility \( c_p(p) \) for a double porosity system\(^4\), cumulative total water compressibility \( c_w(p) \), and the total pore and water volumes associates (i.e., in pressure communication) with the gas reservoir,

\[ c_e(p) = \frac{\phi_{ma} c_{ma} + \phi_{mc} c_{mc} + \phi_{ma} S_w c_{mw} + M (\phi_{ma} c_{ma} + \phi_{mc} c_{mc} + \phi_{ma} c_{mw})}{\phi_{ma} (1 - S_w)} + \phi_{fm} \]

(2)

and \( \Delta p \) is the average reservoir pressure drop \((= p_i - p)\).
The first two terms of the numerator of Eq. 2 represent the effective total cumulative compressibility for a double porosity reservoir

\[ \bar{c}_f(p) = \phi_{ma} \bar{c}_{ma}(p) + \phi_{fi} \bar{c}_{fi}(p). \]  

Eq. 1 can be written in terms of pressure; first dividing through by GB and expressing \( B_g = \frac{\rho_{sc}}{T_{sc}} \beta T / p \) gives an alternate GGMBE:

\[ \frac{p_i(1 - \bar{c}_e(p) \Delta p)}{z_i} = \frac{p_i}{z_i} \left[ 1 - \frac{1}{G_p} \left( W_p - W_{w} \right) \left( B_w - B_g \right) \right] \]

\[ \bar{c}_e(p) = \frac{S_{wi} \bar{c}_{iw} + \bar{c}_f(p) + M \left( \bar{c}_w(p) + \bar{c}_f(p) \right)}{1 - S_{wi}}. \]

The interbedded nonpay volume \( V_{PNNP} \) and limited aquifer contributions \( V_{PAQ} \) support are quantified in terms of the M ratio,

\[ M = \frac{V_{PNNP} + V_{PAQ}}{V_{pr}}. \]

The cumulative effective compressibility \( \bar{c}_e(p) \) accounts for cumulative changes in volume from initial pressure to the current pressure. Fetkovich et al.\(^5\) clearly explain (their Fig. 2) the meaning of cumulative effective compressibility. For instance, for the case of a single porosity reservoir (Eqs. 4 and 5), the cumulative PV compressibility \( \bar{c}_f(p) \) has been defined\(^4\) as

\[ \bar{c}_f(p) = \frac{1}{V_{pf}} \left[ \frac{p_f - V_{pf}(p)}{p_i - p} \right]. \]

The term in brackets is the slope of the chord from the initial condition \( (p_i, V_p) \) to any lower pressure \( (p, V_p) \); this implies that \( \bar{c}_f(p) \) is a function of both pressure and the initial conditions. This cumulative compressibility function \( \bar{c}_f(p) \) should be used with material balance equations that consider the cumulative pressure drop \( (p_i - p) \), i.e., \( p \) vs. \( G_p \) plots. This parameter can be best estimated by special core analysis under in-situ reservoir conditions\(^7\).

**Cumulative Total Water Compressibility.** \( \bar{c}_w(p) \). The pressure support provided by water is made up of two components. The reservoir pressure decrease causes the water expansion and the release of gas and its expansion. The cumulative total compressibility is expressed as

\[ \bar{c}_w = \frac{1}{B_w(p)} \left[ B_w(p) \frac{B_w(p) - B_w(p_i)}{p_i - p} \right]. \]

where the total water formation volume factor \( B_w \) is defined as

\[ B_w(p) = B_w(p) + [R_{sw} - R_{sw}(p)] B_g(p). \]

To calculate \( B_w(p) \), values of \( B_w(p) \), \( R_{sw}(p) \) and \( z(p) \) can be obtained from correlations when necessary, but laboratory data measured on representative samples taken from the reservoir are always superior to general correlations, and should be used whenever possible\(^8\).

**Associated Water Volume Ratio M.** The total compressibility effect on the gas material balance depends for a naturally fractured system, on the magnitudes of matrix, fractures and total water compressibilities and on the total pore and water volumes in pressure communication with the gas reservoir (this includes connate water and the PV within the net pay).

Associated water \( (AW) \) and \( PVs \) external to the net pay include nonnet pay \( (NNP) \), such as interbedded shales and dirty sands, plus external water volume found in limited aquifers. Following the developments of Fetkovich et al.\(^4\), the associated volume (water and \( PVs \) ) is expressed (Eq. 6) as a ratio relative to the \( PV \) of the net pay reservoir \( V_{pr} \),

\[ M = M_{NNP} + M_{AQ}. \]

where

\[ M_{NNP} = \frac{V_{PNNP}}{V_{pr}}, \]

and

\[ M_{AQ} = \frac{V_{PAQ}}{V_{pr}}. \]

For the simplest case when \( M = 0 \) there will be pressure support only from connate water and the net \( PV \).

**MNNP.** Let's start assuming that properties and thickness of the net pay and nonnet pay are readily available from well log analysis. The nonnet pay water volume ratio \( M_{NNP} \)}
comprises interbedded reservoir PV, including shales and poor quality rock, assumed to be fully water saturated. Based on this definition, \( M_{\text{SNP}} \) can be expressed in terms of the net to gross ratio \( R_{NG} \) defined as

\[
R_{NG} = \frac{h_r}{h_r + h_{\text{SNP}}} = \frac{h_r}{h_i}.
\]  

(13)

Considering different porosities in the net pay and nonnet pay, \( M_{\text{SNP}} \) is given by

\[
M_{\text{SNP}} = \frac{(\phi h_A)_{\text{SNP}}}{(\phi h_A)_r} = \frac{\phi_{\text{SNP}}}{\phi_r} \left( \frac{1 - R_{NG}}{R_{NG}} \right).
\]  

(14)

\( M_{AQ} \): Associated aquifers of sufficient permeability (greater than 100 md²) and limited areal extent can be treated as part of the total cumulative compressibility. \( M_{AQ} \) is defined as

\[
M_{AQ} = \frac{(\phi h_A)_{AQ}}{(\phi h_A)_r}.
\]  

(15)

As expected, for large aquifers the required treatment will be the superposition methods originally discussed by van Everdingen and Hurst. For these cases the \( \bar{c}_e(p) \) term can still be used, but it should only contain the effect of net pay and nonnet pay volumes; i.e., \( M = M_{\text{SNP}} \) (from Eq. 10).

Diagnosis of the Production mechanism.

The diagnosis of the production mechanism of a gas reservoir is an important information regarding the implementation of the optimum exploitation conditions of these systems. A review of the literature indicates that the energy plot discussed by Beggs, where the reservoir behavior is plotted in a log-log graph in terms of \( 1 - z_i G_p / p_i z_i G_p \), is the most used for these purposes. This graph can be regarded as the \( \Delta p \) vs. \( t \) Ramey's first log-log method for the flow diagnosis in well test analysis. This problem is emphasized under limited reservoir characterization conditions.

The following discussion will present methods for the diagnosis of the production mechanism based on cartesian graphs.

Methods Based on the Derivative \( \bar{c}(p/z) / \partial G_p \).

The following discussion will present methods for the diagnosis of the production mechanism based on cartesian graphs.

Case 1: Volumetric reservoir.

Deriving the \( GGMBE \) given by Eq. 4 with respect to pressure

\[
\frac{\partial (p/z)}{\partial G_p} = \frac{p_i}{z_i G_p}.
\]  

(16)

This expression indicates that for the producing conditions of a volumetric (closed) reservoir, a graph of \( \bar{c} (p/z) / \partial G_p \) vs. \( G_p \) should result in a constant value, equal to \( p_i / z_i G_p \), which can be used to estimate the original gas in place, G. Fig. 1 shows this type of graph. Continuing with the discussion between the well test analysis flow diagnosis problem and the production mechanism diagnosis for gas reservoirs, this graph is similar to the derivative methods of Tiab and Kumar and Bourdet et al. A graph as previously described can be very useful as complement for analyzing the behavior of gas reservoirs.

Case 2: Over pressured Naturally Fractured Volumetric Reservoir (ONFR).

As previously stated regarding Eq. 2, the \( c_e(p) \) total effective compressibility of the system is considered as variable, or pressure dependent. Deriving Eq. 4 with respect to the cumulative gas production \( G_p \),

\[
-\bar{c}(p/z) = \frac{p_i}{z_i G_p} \frac{\partial (c_e(p) \Delta p)}{\partial G_p} \frac{1}{z} \frac{1}{(1 - \bar{c}_e \Delta p)} \ldots \ldots \ldots \ldots \ldots \ldots (17)
\]

Based on an analogy analysis between Eqs. 16 and 17, due to the second negative term in the latter equation, a graph of \( -\bar{c} (p/z) / \Delta p \) vs. \( G_p \) will result at early production times in smaller ordinate values than those corresponding to a volumetric reservoir (case 1), Fig. 2; later after the inflection point (•), either a straight line or concave behaviors can be shown. The shape of the late reservoir behavior, is strictly related with the values of the \( c_e(p) \) parameter as the reservoir pressure decreases. For the case of constant \( c_e(p) \) Eq. 17 indicates that a straight-line behavior would result (curve b, Fig.2), and with the information about the intercept to the origin an estimate of the original gas in place G can be obtained. For conditions of variable \( c_e(p) \) an upward concave behavior will be shown, curve c (Fig. 2).

Case 3: Conventional (Single Porosity) reservoir-aquifer system, considering solution gas in water.
Starting from the GGMBE (Eq. 4), for a reservoir with an associated aquifer, considering a constant $c_e$, deriving with respect to the cumulative gas production $G_p$, 

$$-\partial \left( \frac{p(z)}{z} \right) = \frac{p_i}{G} \left( 1 - \frac{\partial}{\partial G_p} \left( W_e - \frac{W_p B_w}{B_g} \right) + W_p R_{sw} \right) \cdots (18)$$

Fig. 3 shows a graph of the reservoir behavior for the conditions already stated; it can be observed from Eq. 18 that this straight line equation has a slope and an intercept of equal value, $p_i/zG$, which can be used to estimate the original gas in place $G$. The direction of production time increase is also indicated in this schematic figure.

Based on a comparison of the behavior of a volumetric reservoir (Eq. 16, Fig. 1) and the reservoir behavior presented on Fig. 3, and through Eq. 18 that includes a negative derivative of the net water influx and the solution gas released from water, can be concluded that a graph of $-\partial \left( \frac{p(z)}{z} \right)/\partial G_p$ vs. $G_p$ or $(p_i - p)$ should follow a straight line with negative slope, as shown in Fig. 4. The value of the slope of this reservoir behavior graph is strictly related to the two physical factors previously stated, the net water influx and the solution gas released from the produced water. For instant, as the activity of the aquifer (dimension) increases, the slope will also increase.

**Methods Based on the Derivative $-\partial \left( \frac{p(z)}{z} \right)/\partial p$ vs. $\partial G_p/\partial p$.**

**Case 4: Volumetric reservoir.** Considering the physical conditions of these reservoirs, the derivative of the GGMBE given by Eq. 4 gives 

$$\frac{\partial}{\partial p} \left( \frac{p(z)}{z} \right) = -\frac{p_i}{zG} \frac{\partial G_p}{\partial p} \cdots (19)$$

This is the basic starting equation for the methods to be discussed in this section.

For volumetric gas reservoirs the change (derivative) of the cumulative production with respect to pressure decreases with the increase in production time. This is due to a higher rate decline at early exploitation times. In addition, the change of $p/z$ with respect to pressure decreases as pressure decreases, because the $z$ compressibility factor increases at low pressure levels. This discussion can be better understood as follows. Deriving the left term of Eq. 19:

$$\frac{\partial}{\partial p} \left( \frac{p(z)}{z} \right) = -\frac{1}{z} \frac{\partial z}{\partial p} \cdots (20)$$

Thus, considering the physical behavior of the $z$ parameter at low pressures, $\partial \left( \frac{p(z)}{z} \right)/\partial p$ will show a decreasing behavior. It also can be concluded that for the (theoretical) limiting or minimum atmospheric reservoir pressure ($p = 0$ psi), the value of $\partial \left( \frac{p(z)}{z} \right)/\partial p \mid_{p=0}$ will be 1 ($z=1$ at this pressure), as shown in Fig. 5. In conclusion, Eq. 19 that describes the derivative behavior of a volumetric gas reservoir, is a straight line with zero interception and positive slope $p_i/(zG)$. Based on this slope, the original gas in place $G$ can be estimated. A further check for the estimation of $G$ can be as follows. For conditions of the minimum value of $\partial \left( \frac{p(z)}{z} \right)/\partial p \mid_{p=0}$, the value for the abscissa $-\partial G_p/\partial p \mid_{p=0}$ could be applied in Eq. 19 to estimate the original gas in place $G$:

$$G = \frac{p_i}{z} \left[ \frac{-\partial G_p}{\partial p} \bigg|_{p=0} \right] \cdots (21)$$

**Case 5: Overpressured naturally fractured reservoir (ONFR).** Deriving the GGMBE with respect to the reservoir pressure, assuming a constant cumulative effective compressibility $c_e(p)$, the derivative $\partial \left( \frac{p(z)}{z} \right)/\partial p$ is given by

$$\frac{\partial}{\partial p} \left( \frac{p(z)}{z} \right) = -\frac{p_i}{zG} \frac{\partial G_p}{\partial p} - c_e \frac{p}{z} \frac{\partial G_p}{\partial p} \cdots (22)$$

Considering a variable cumulative effective compressibility $c_e(p)$, the derivative can be expressed as

$$\frac{\partial}{\partial p} \left( \frac{p(z)}{z} \right) = -\frac{p_i}{zG} \frac{\partial G_p}{\partial p} - c_e(p) \frac{p}{z} \frac{\partial G_p}{\partial p} \cdot \left[ 1 - c_e(p)(p_i - p) \right] \cdots (23)$$

An analysis of the parameters of Eq. 23 will allow the conclusion that for an over pressured naturally fractured reservoir, the derivative $\partial \left( \frac{p(z)}{z} \right)/\partial G_p$ presents increasing positive values with respect to the exploitation time. The easiest way to analyze this problem is for the simplest case of a constant effective cumulative compressibility $c_e$ considered in Eq. 22, where as the exploitation time increases its numerator also increases due to the decrease in pressure, and its denominator decreases because the reservoir pressure drop $(p_i - p)$ increases. This behavior can be observed schematically in Fig. 6, part (a).

It can be stated that the behavior of an ONFR will show several straight lines in a graph of $\partial \left( \frac{p(z)}{z} \right)/\partial p$ vs. $\partial G_p/\partial p$, which are strictly related to the variability of the effective
cumulative compressibility $c_e(p)$, as shown in Fig. 6. As was previously stated in the last paragraph, while the effective cumulative compressibility $c_e(p)$ has a high value or keeps increasing during the transient reservoir behavior period, the derivative $\frac{\partial (p/z)}{\partial p}$ will also increase up to a maximum value at the end of part (b) of Fig. 6, then it will decrease, as indicated in part (c) of this figure. In addition, it has been discussed that the $z$ gas compressibility factor decreases in a pressure range starting from the initial pressure and ending in an intermediate pressure (see for instance the Standing and Katz correlation, Fig. 1.3, page 21 of Craft and Hawkins' 19).

For the case of a closed (limited) reservoir ($M=0$, Eqs. 2 and 6), the cumulative effective compressibility $c_e(p)$ will only be a function of the cumulative PV compressibility and of the cumulative total water compressibility $c_w(p)$, showing as the reservoir pressure declines a faster decrease with exploitation time than that for the general case, where the interbedded nonpay volume $V_{NP}$ and aquifer contribution are considered. Thus, in accordance to Eq. 23, the relation between the derivatives $\frac{\partial (p/z)}{\partial p}$ vs $-\frac{\partial G_p}{\partial p}$ will decrease as the exploitation time increases, as shown in Fig. 6 part (c).

A further explanation of the reservoir behavior shown in Fig. 6, part (b) will be as follows. When the reservoir fluids are produced, the reservoir pressure decreases, causing an increase of the effective stress (equal to the overburden stress minus the reservoir pressure), resulting in a deformation of the formation. This compressive effect provides an additional energy source for the gas production. This effect will show in this graph through decreasing values of the derivative $-\frac{\partial G_p}{\partial p}$ due to a decrease in the cumulative $PV$ effective compressibility caused by the formation deformation (pore collapse) already discussed.

Summarizing, based on Eqs. 22 and 23 and the previously presented discussion for this Case 5, the possible general behavior of an ONFR is as shown in Fig. 6.

**Case 6: Conventional (single porosity) reservoir-aquifer system, neglecting solution gas in water ($R_{w} = 0$) and injection of fluids ($G_{inj} = W_{w,j} = 0$).**

For these conditions already discussed in case 3, Eq. 4 can be expressed as

$$\frac{p}{z} = \frac{p_i}{z_i} \left[ G_p + (W_p B_w - W_e) \frac{1}{B_g} \right]. \quad \text{(24)}$$

Deriving this equation with respect to pressure

$$\frac{\partial (p/z)}{\partial p} = -\frac{p_i}{z_i} G_p + \frac{\partial G_p}{\partial p} - \frac{\partial \left( W_e - W_p B_w \right) - \frac{1}{B_g} \frac{\partial B_g}{\partial p}}{B_g}. \quad \text{(25)}$$

It can be observed from this Eq. 24 that for these conditions the derivative $\frac{\partial G_p}{\partial p}$ is not proportional to the derivative $\frac{\partial (p/z)}{\partial p}$, as it was for case 3 previously discussed, Eq. 18; the difference in this last Eq. 25 is the negative net water influx derivative term, which results in a decreasing $\frac{\partial (p/z)}{\partial p}$ derivative with the exploitation time.

Writing Eq. 25 in terms of the net water influx $W_{en} = W_e - W_p B_w$.

$$\frac{\partial (p/z)}{\partial p} = \frac{p_i}{z_i} G_p \left[ \frac{\partial W_{en}}{\partial p} - \frac{\partial G_p}{\partial p} \right]. \quad \text{(26)}$$

If we consider an increasing aquifer activity, the change of the cumulative gas production with respect to pressure will be bigger than those of the net water influx, resulting in decreasing values for the $\frac{\partial (p/z)}{\partial p}$ derivative.

This previous discussion for the present case can be continued considering the first version for the GGMBE written in terms of the gas volume factor $B_g$: for the conditions of these reservoirs, Eq. 1 can be simplified as

$$G_p B_g = G (B_g - B_g) + (W_e - W_p B_w). \quad \text{(27)}$$

Deriving this equation

$$\frac{\partial G_p}{\partial p} = G \frac{\partial}{\partial p} \left( \frac{B_g - B_g}{B_g} \right) + \frac{\partial \left( W_e - W_p B_w \right)}{B_g}. \quad \text{(28)}$$

Writing this equation in terms of the net water influx $W_{en}$,

$$\frac{\partial W_{en}}{\partial p} - \frac{\partial G_p}{\partial p} = -G \left( \frac{B_g}{B_g} \frac{\partial B_g}{\partial p} \right). \quad \text{(29)}$$

Inserting this equation in Eq. 25

$$\frac{\partial (p/z)}{\partial p} = -\frac{p_i}{z_i} \frac{B_g}{B_g} \frac{\partial B_g}{\partial p}. \quad \text{(30)}$$

The derivative of the gas formation volume factor $\frac{\partial B_g}{\partial p}$ presents negative values, that increase in absolute value as the reservoir pressure decreases. The absolute value of this derivative decreases with the activity (size) of the associated aquifer. In the limit for conditions of a very strong (large) aquifer where the reservoir pressure drop is small, the derivative $\frac{\partial B_g}{\partial p}$ presents a small negative value, resulting in value of $\frac{\partial (p/z)}{\partial p}$ close or equal to zero.
In summary, from the previous discussion based on Eq. 30, the behavior of a gas reservoir with an associated aquifer, presented in a graph of $\partial (p/z)/\partial p$ vs $-\partial G_p/\partial p$, Fig. 7, will show a negative slope straight line, that tends to reach a zero value at long exploitation times. For a very strong aquifer, the derivative $\partial (p/z)/\partial p$ reaches a constant positive, close to zero, value, following a horizontal straight line, and the derivative $-\partial G_p/\partial p$ continues to increase, as shown in Fig. 7.

Field Examples

McEwen's Gas Reservoir with an Associated Aquifer\(^\text{16}\).

A Gulf Coast gas reservoir was analyzed by the author. A conventional $p/z$ vs. $G_p$ graph shows the typical concave upward curve. Fig. 8 shows a diagnostic graph of the reservoir behavior in terms of $\partial (p/z)/\partial p$ vs. $G_p$. From the results shown in this graph three different reservoir behaviors corresponding to different water influx effects can be observed; the last part starting in point e and ending in f indicates a more intensive influx than the first two affecting this reservoir.

Estimates from other diagnostic mechanism methods, for instance one based on the derivative of the GGMBE with respect to the exploitation time $t$, resulted in values for the original gas in place $G$ of 195.35 Bscf\(^\text{16}\), which compares well with the 180 Bscf calculated by McEwen.

Begg's Volumetric Gas Reservoir\(^\text{1}\).

Fig. 9 shows a typical behavior for a volumetric reservoir. A Comparison is presented between two methods for the estimation of this derivative, mainly through the derivation of an interpolation polynomial of second degree and using a centered difference approximation\(^\text{17}\).

The estimation of the original gas in place $G$ by means of the extrapolation to the minimum point, results in a value of 46.4 MMScf, which can be compared with the value calculated by Begg of 48.348 MMScf.

The Anderson L Gas Condensate Reservoir.

This reservoir has been studied by several authors and it is usually recognized as the best example of a high-pressure gas condensate reservoir, with the characteristic concave downward $p/z$ vs. $G_p$ behavior. The reservoir was abandoned after producing 55 Bscf, but pressure tests of public record were discontinued after 40 Bscf had been produced.

The behavior of this reservoir is shown in Fig. 10 in terms of $\partial (p/z)/\partial p$ vs $-\partial G_p/\partial p$. The first finding is that the behavior of this reservoir is highly complex. The abnormal reservoir pressure causes an increase of the cumulative effective compressibility due to the inelastic compression of the formation, shown in Fig. 10 by the positive slope straight lines ab, de and fg, and also results in a decrease of the compressibility generated by the pore volume collapse, indicated by the negative slope straight line behavior gh. Starting in point b, a decrease on the y axis values of $d (p/z)/dp$ occurs, ending at point c, which indicates a possible water influx coming, form shales in contact with the reservoir, which ceases after a certain time. On a different basis, this was also stated by Duggan\(^\text{18}\).

Conclusions.

The main aim of this paper has been to present a more general gas material balance equation (GGMBE) than that currently available in the literature, that considers as an improvement that the reservoir could be naturally fractured. The diagnosis of the production mechanism is thoroughly addressed, presenting new methods of analysis.

From the results of this study the following conclusions are pertinent.

1. Applying the GGMBE derived in this work, new and more robust methods for the diagnosis of the production mechanisms of gas reservoirs have been derived.

2. Through the implementation of the new diagnostic methods, new alternate procedures were derived for the estimation of the original gas in place $G$, the cumulative effective compressibility $\tilde{c}_e(p)$, and the volume ratio $M$.

3. The new diagnostic methods result in a better reservoir characterization, because having accurate information regarding the production mechanisms and the original gas in place, will allow a better forecasting of the reservoir behavior.

Nomenclature

$B_g$ = gas $FVF$.

$B_w$ = water $FVF$, RB/STB.

$\tilde{c}_e$ = effective cumulative compressibility, psia\(^{-1}\).

$\tilde{c}_f$ = in-situ formation compressibility, psia\(^{-1}\).

$\tilde{c}_{fs}$ = in-situ formation compressibility of fractures, caverns and vugs, psia\(^{-1}\).

$\tilde{c}_{rw}$ = water compressibility, psia\(^{-1}\).

$G$ = original gas in place, Mscf.

$G_{og}$ = cumulative gas injection, Mscf.

$G_p$ = cumulative gas production, Mscf.

$M$ = volume ratio, dimensionless.

$p$ = reservoir pressure, psia.

$p_i$ = initial reservoir pressure, psia.

$p_{ce}$ = pressure at standard conditions, psia.

$R_{sw}$ = Solution gas water ratio, scf/res cf.

$S_w$ = connate water saturation, fraction.

$t$ = time, days.

$V_{ig}$ = associated (non-net pay and aquifer) pore volume.
occupied by gas, ft$^3$.

$V_{gr}$ = net reservoir pore volume occupied by gas, ft$^3$.

$V_{r}$ = pore volume of the associated rock, ft$^3$.

$V_{ph}$ = net reservoir pore volume occupied by gas and water, ft$^3$.

$V_{wa}$ = water volume in the associated pore volume, ft$^3$.

$V_{wr}$ = net reservoir pore volume occupied by water, ft$^3$.

$W_e$ = cumulative water influx, res bbl.

$W_{cw}$ = net water influx, $W_e - W_p B_{wr}$, Eq. 28, res bbl.

$W_{inv}$ = cumulative water injection, STB.

$W_p$ = cumulative water production, STB.

$z$ = gas compressibility factor, dimensionless.

$\phi_f$ = porosity of fractures, caverns and vugs, fraction.

$\phi_{ma}$ = matrix porosity.

References

8. Earlougher, R.C., Jr.: Advances in Well Test Analysis, SPE Monograph Series No. 5, Richardson, Texas (1967).

Fig. 1. Diagnosis of the production mechanism of a volumetric reservoir based on the derivative $\frac{\partial (p/z)}{\partial G_p}$. 

Fig. 2. Diagnosis of the production mechanism of an abnormally pressurized naturally fractured gas reservoir, based on the derivative \(-\partial(p/z)/\partial G_p\).

(a) Volumetric;
(b) Constant effective compressibility;
(c) Variable effective compressibility \(c_e(p)\).

Fig. 4. Diagnosis of the production mechanism for a water drive gas reservoir, based on the derivative \(-\partial(p/z)/\partial G_p\) vs. \(\partial G_p/\partial p\).

\[ b = p_i / (z_i G) \]

\[ m = p_i / (z_i G) \]

\[ t \]

Slope

\[ -\partial((W_e - W_p B_w) B_g + W_p R_{sw}) / \partial G_p \]

Fig. 3. Diagnosis of the production mechanism for a water drive gas reservoir, based on the derivative \(-\partial(p/z)/\partial G_p\) vs. \(\partial (W_e - W_p B_w) B_g + W_p R_{sw} / \partial G_p\).

Fig. 5. Diagnosis of the production mechanism for a volumetric gas reservoir, based on a graph of \(\partial(p/z)/\partial p\) vs. \(\partial G_p/\partial p\).
Fig. 6. Diagnosis of the production mechanism for over-pressured or naturally fractured gas reservoirs, based on a graph of $\frac{\partial (p/z)}{\partial p}$ vs. $-\frac{\partial G_p}{\partial p}$, for various effective compressibility conditions: (a) high, (b) increasing and (c) small and decreasing.

Fig. 8. Diagnosis of the production mechanism for the water drive gas reservoir of McEwen\textsuperscript{12}, based on a graph of $\frac{\partial (p/z)}{\partial G_p}$ vs. $-G_p$.

Fig. 7. Diagnosis of the production mechanism for a water drive gas reservoir, based on a graph of $\frac{\partial (p/z)}{\partial p}$ vs. $-\frac{\partial G_p}{\partial p}$.

Fig. 9. Diagnosis of the production mechanism based on a graph of $\frac{\partial (p/z)}{\partial p}$ vs. $-\frac{\partial G_p}{\partial p}$ for the gas reservoir data presented by Beggs.
Fig. 10. Diagnosis of the production mechanism for the Anderson L gas reservoir, based on a graph of $\frac{\partial (p/z)}{\partial p}$ vs. $-\frac{\partial G_p}{\partial p}$. 

$C_t$ increasing

NNP water influx

$C_t$ decreasing