Improvements to Reservoir Material-Balance Methods

J.L. Pletcher,* SPE, Marathon Oil Co.

Summary
Experience with material-balance data sets from the field and from simulation has revealed some procedures that can be used to improve analysis of both oil and gas reservoirs:

- Failure to account for a weak waterdrive can result in significant material-balance errors.
- The assertion of previous authors that weak waterdrive exhibits a negative slope on the Cole (gas) and Campbell (oil) plots has been confirmed. A weak waterdrive is much more unambiguous on these plots than on commonly used plots, such as the p/z plot for gas.
- A modified version of the Roach plot is proposed to account for formation compressibility.
- The reservoir drive indices are a useful tool for determining the correctness of the material-balance solution because they must sum to unity. The drive indices should never be normalized to sum to unity because this obscures their usefulness and leads to a false sense of security.
- A modified version of the Roach plot (for gas) is proposed that improves interpretation in some waterdrive situations.
- Material balance has not been replaced by reservoir simulation; rather, it is complementary to simulation and can provide valuable insights to reservoir performance that cannot be obtained by simulation.

Introduction
Classical material balance is one of the fundamental tools of reservoir engineering. Many authors have addressed the difficult problem of solving the material balance in the presence of a waterdrive (Refs. 1 through 5 are just a few of the more significant ones). The emphasis in the literature has been on strong and moderate waterdrives. In this paper, examples of weak waterdrives are shown in which the effects on the material balance are significant. All aquifers studied here are of the “pot aquifer” type, which is time-independent. In gas reservoirs, the plot of p/z vs. cumulative gas production, Gp, is a widely accepted method for solving the gas material balance under depletion-drive conditions. Extrapolation of the plot to atmospheric pressure provides a reliable estimate of original gas in place (OGIP). If a waterdrive is present, the plot often appears to be linear, but the extrapolation will give an erroneously high value for OGIP. Many authors have addressed this problem (including those in Refs. 2 and 5 through 8), especially in cases of strong or moderate waterdrives. The p/z plot is actually more ambiguous in weak waterdrives than in strong or moderate ones.

The Cole plot has proven to be a valuable diagnostic tool for distinguishing between depletion-drive gas reservoirs and those that are producing under a waterdrive. The analogous plot for oil reservoirs is the Campbell plot. The literature has emphasized strong and moderate waterdrives, the signature shapes of which are a positive slope and a hump-shaped curve, respectively, on these plots. Previous authors have recognized that weak waterdrives can produce negative slopes on these two diagnostic plots, but this author is not aware of any example plots in the literature. This paper shows examples, using simulation and actual field data, wherein a negative slope clearly reveals a weak waterdrive. These plots are much more diagnostic than the p/z plot. Once a weak waterdrive has been diagnosed, the appropriate steps can be taken in the material-balance equations to yield more accurate results.

The Cole plot assumes that formation compressibility can be neglected, which is frequently the case with gas. However, in those reservoirs in which formation compressibility is significant, a modification to the Cole plot is presented that incorporates formation compressibility and gives more accurate results.

The reservoir drive indices have been used to quantify the relative magnitude of the various energy sources active in a reservoir. It is shown here that the drive indices are also a useful diagnostic tool for determining the correctness of a material-balance solution because they must sum to unity. If they do not sum to unity, a correct solution has not been obtained. In some commercial material-balance software, the drive indices are automatically normalized to sum to unity, which not only obscures their usefulness but also leads to the false impression of having achieved a correct solution.

The Roach plot has been presented as a tool for solving the gas material balance when formation compressibility is unknown, with or without the presence of a waterdrive. This paper shows that for waterdrives that fit the small pot aquifer model, incorporating cumulative water production into the x-axis plotting term improves the linearity of the Roach plot and gives more accurate values for OGIP.

Finally, it is argued that even in those reservoirs for which a simulation study is performed, classical material-balance evaluation should be performed on a stand-alone basis. Simulation should not be viewed as a replacement for material balance because the latter can yield valuable insights that can be obscured during simulation. Performing a separate material-balance study usually will improve overall reservoir understanding and enhance any subsequent simulation study. Material balance should be viewed as a complement to simulation, not as a competing approach.

In this paper, formation compressibility, εf, is assumed to be constant and unchanging over the reservoir life under investigation. References are given for recommended methods to be used in those cases in which εf is variable.

Gas Reservoirs
Cole Plot. The Cole plot is a useful tool for distinguishing between waterdrive and depletion-drive gas reservoirs. The plot is derived from the general material-balance equation for gas reservoirs:

\[ F = G(E_g + E_{fw}) + W_e, \]  
(1)

where \( F \) = cumulative reservoir voidage,

\[ F = G_p B_g + W_p B_w; \]  
(2)

\( E_g = \) cumulative gas expansion,

\[ E_g = B_g - B_{gi}; \]  
(3)

and \( E_{fw} = \) cumulative formation and water expansion,

\[ E_{fw} = B_g \frac{S_{ot} C_{wi} + \epsilon f (p_i - p)}{1 - S_{wi}}. \]  
(4)

In Eq. 1, \( G = \) OGIP, and \( W_e = \) cumulative water influx. Often in gas reservoirs, \( E_{fw} \) is negligible compared to \( E_g \) and can therefore
be ignored. Then, by substitution and rearranging, Eq. 1 can be expressed as

\[ \frac{G_t B_{gi}}{B_g - B_{gi}} = G + \frac{W_r - W_p B_{gi}}{B_g - B_{gi}}. \] .............................. (5)

Cole proposed plotting the left side of Eq. 5, \( G_t B_{gi} / (B_g - B_{gi}) \), on the y-axis vs. cumulative gas production on the x-axis. If the reservoir is depletion drive (i.e., no water influx), the term on the far right side of Eq. 5 goes to zero and the points plot in a horizontal line with the y-intercept equal to \( G \), the OGIP. If a waterdrive is present, the far right-side term is not zero, and the points will plot above the depletion-drive line with some type of slope. In other words, the existence of a sloping line vs. a horizontal line is a valuable diagnostic tool for distinguishing between depletion drive and waterdrive.

Cole and others have suggested that the sloping waterdrive line can be extrapolated back to the y-intercept to obtain the OGIP. However, the slope usually changes with each plotted point; thus, the correct slope for extrapolation is very difficult, if not impossible, to establish, so this method for estimating OGIP is not recommended. This does not, however, detract from the plot’s qualitative value in establishing that the reservoir is under waterdrive, as opposed to depletion drive.

Dake\(^\text{7}\) showed two types of curving Cole plots in his Fig. 6.6, a strong waterdrive curve and a moderate waterdrive curve, depicted here in Fig. 1. (Actually, Dake’s plot is a slightly modified version of Cole’s plot because Dake incorporated water production into the y-axis plotting term; that is, he in effect moved the water-production term to the left side of Eq. 5. The net effect on the curve shapes is negligible.)

Wang and Teasdale\(^\text{12}\) stated that in the presence of a weak waterdrive, the far right-side term in Eq. 5, \( W_r - W_p B_{gi} / (B_g - B_{gi}) \), would decrease with time because the denominator (gas expansion) would increase faster than the numerator (net water influx). Therefore, the plotted points will exhibit a negative slope, as shown in Fig. 1; indeed, this has been observed in practice, as will be shown later in this paper. As reservoir depletion progresses, the points migrate down and to the right toward the true OGIP; the smaller the aquifer, the closer the plot will approach the true OGIP.

Note that the negative slope of the weak waterdrive curve represents an unexpected anomaly. The y-axis plotting term gives the apparent OGIP that would be calculated, assuming no waterdrive is present. Therefore, under a weak waterdrive, the apparent OGIP decreases with time, contrary to that for a strong or moderate waterdrive.

Actually, before developing the signature negative slope, the weak waterdrive curve begins with a positive slope in the very early stages of reservoir depletion, as shown in Fig. 1. The very early points are difficult to use for determining OGIP, however, because they frequently exhibit a great deal of scatter that is introduced by even small errors in pressure measurement early in the reservoir life. Technically, then, the curve is hump-shaped like Dake’s moderate waterdrive curve in Fig. 1, except that the positive-slope portion of the hump is over with very early and in practice will not show up at all unless frequent and accurate very early time data are obtained.

**Modified Cole Plot.** In some gas reservoirs, formation compressibility is not negligible, in which case \( E_{fr} \) should not be ignored and Eq. 5 should be written:

\[ \frac{F}{E_t} = G + \frac{W_r - W_p B_{gi}}{B_g - B_{gi}}, \] .............................. (6)

where \( E_t \) = total reservoir expansion,

\[ E_t = E_g + E_{fr}. \] .............................. (7)

The left side of Eq. 6, \( F / E_t \), now incorporates in the denominator the energy contribution from formation (and water) compressibility, as well as from gas expansion. The modified Cole plot consists of plotting \( F / E_t \) on the y-axis vs. \( G_p \) on the x-axis. Curve shapes will be the same as in Fig. 1. Vertically, the points will lie closer to the true value of OGIP than the original Cole plot.

In reservoirs in which formation compressibility (\( c_f \)) is a significant contributor to reservoir energy, such as abnormally pressured reservoirs, the original Cole plot will exhibit a negative slope, even if no waterdrive is present. The modified Cole plot, however, will plot in a horizontal line, assuming the correct value of \( c_f \) is used in calculating the \( F / E_t \) term. Thus, constructing both the original and modified Cole plots will distinguish between those reservoirs that are subject to both a weak aquifer and significant formation compressibility and those reservoirs in which formation compressibility is significant but there is no aquifer attached; for the former, both plots will have a negative slope, and for the latter, the original Cole plot will have a negative slope while the modified plot will be horizontal. This assumes, of course, that formation compressibility is known with certainty, which is often problematical.

Actually, negative slopes on the original and modified Cole plots can result from any unaccounted-for source of energy that is decreasing with time relative to gas expansion. This could include, for example, communication with other depleting reservoirs.

**Drive Indices.** Drive indices have been defined for oil reservoirs\(^\text{13}\) to indicate the relative magnitude of the various energy forces contributing to the reservoir. Similarly, drive indices can be defined for gas reservoirs as follows.

**Gas drive index:**

\[ I_{Gd} = \frac{G E_t}{G_p B_g}. \] .............................. (8)

**Formation and connate water compressibility drive index:**

\[ I_{Gd} = \frac{G E_{fr}}{G_p B_g}. \] .............................. (9)

**Waterdrive index:**

\[ I_{Wd} = \frac{W_r - W_p B_{gi}}{G_p B_g}. \] .............................. (10)

The numerators of these three dimensionless fractions represent the cumulative gas expansion, cumulative rock and connate water expansion, and cumulative net water influx, respectively, all at reservoir conditions. The common denominator is the cumulative hydrocarbon voidage at reservoir conditions. If the material bal-
ance has been solved correctly, the sum of these three fractions equals unity.

\[ I_{GD} + I_{CD} + I_{KD} = 1 \]  \hspace{1cm} (11)

If the drive indices do not sum to unity, a correct material-balance solution has not been obtained.

In practice, drive indices calculated from actual field data rarely sum exactly to unity because the data are not perfect. The summed drive indices typically vary between values somewhat larger than or less than unity, or show a consistent increasing or decreasing trend, this is an indication that a correct solution to the material balance has not been obtained.

**Gas-Simulation Model.** A simple two-cell gas model was constructed with the Eclipse reservoir simulator to study the effects of weak water influx on gas reservoir material balance. One cell contained gas at irreducible water saturation (i.e., a “tank” model ideally suited to material-balance analysis), and the other cell contained an equal pore volume containing 100% water saturation. OGIP was approximately 101 Bcf. A single well was produced at a rate of 15 MMscf/D for 10 years, recovering a little more than one half (54.3%) of the OGIP. Other properties of the model are found in Table 1.

The simulator output at 1-year intervals was used to perform a material-balance evaluation of the reservoir. Production and pressure histories used in the material balance are given in Table 2, and pressure/volume/temperature (PVT) properties are given in Table 3. The p/z plot is shown in Fig. 2, where each point represents year-end conditions for Years 1 through 10. Because formation compressibility in this case is significant, the p/z plotting term was modified to account for the energy contribution from rock compressibility with a method equivalent to that of Ramagost and Farshad.

The plotted p/z points in Fig. 2 appear to lie on an almost perfect straight line (\( R^2 = 0.9998 \) after 10 years), giving the impression that an extrapolation to OGIP could be made with confidence. However, an extrapolation of the points made after 2 years, when 11% of the true OGIP had been produced, would yield a value for OGIP of 109 Bcf, or 8.2% too high. After 5 years, the error would be +6.5%. Even after 10 years and recovery of 54% of the OGIP, the error would still be +4.0%. Errors of this magnitude are not insignificant, even though this aquifer is very small.

Existence of a waterdrive would be practically impossible to detect from well performance because even after 10 years, the well made only 1.5 STB of water per MMscf of gas. In the simulation, the well produced water only because the encroached water is dispersed uniformly throughout the single cell. In an actual reservoir, the well would likely produce less water because of saturation gradients, depending on the proximity of the well with regard to the original gas/water contact.

**Cole and Modified Cole Plots.** The Cole plot (Fig. 3) for this weak aquifer data set exhibits a negative slope. The plot corroborates Wang and Teasdale’s contention that the Cole plot clearly indicates the presence of even a weak waterdrive, whereas the p/z...
plot is completely ambiguous. The negative slope distinguishes the weak waterdrive system from the strong waterdrive (positive slope), moderate waterdrive (hump-shaped), and depletion-drive (horizontal line) systems (Fig. 1).

Note that the ordinate values plotted in Fig. 3 appear to be migrating toward the true OGIP value, 101 Bcf, as reservoir depletion proceeds. Thus, the most recent plotted point on the Cole plot could be taken as the maximum possible value of OGIP, approximately 107 Bcf after 10 years ($G_0 = 54\%$ of OGIP).

Because formation compressibility is significant in this example, the modified Cole plot should be used. As expected, the points lie closer to the true value of OGIP than the original Cole plot, Fig. 3. The ordinate value after 10 years is 104.4 Bcf, more nearly approaching the true OGIP than the original Cole plot.

Included in Fig. 3 are values from the first year of production at 1-month intervals, plotted with smaller symbols. The early time points exhibit a steep positive slope. The negative slope develops after approximately 10 months, when 4.5\% of the true OGIP has been produced. To obtain this early-time portion of the plot in an actual reservoir, it would be necessary to obtain frequent and very accurate pressure measurements. Even then, the points do not plot with a constant slope, rendering impractical the extrapolation back to $G_0 = 0$ for the purpose of obtaining OGIP.

**Pot Aquifer Plot.** If the aquifer is relatively small and in good communication with the hydrocarbon reservoir, and permeabilities are sufficiently high, the aquifer can be represented with the pot aquifer model, and original hydrocarbons in place (OOGIP) can be obtained from the pot aquifer plot.\(^4\) This type of aquifer should apply in high-permeability reservoirs having a hydrocarbon/water contact where the “water leg” is isolated from large regional aquifers by permeability pinch-out or faulting. Examples would be found in the U.S. midcontinent, where high-permeability sands typically are broken up into relatively small reservoirs by faulting. An example from the U.S. midcontinent is shown later in the paper.

For the pot aquifer model, any drop in reservoir pressure is instantaneously transmitted throughout the entire aquifer. Mathematically,

$$W_e = (c_w + c_f)W(p_i - p), \quad \text{........................................... (12)}$$

where $W_e =$ aquifer original water in place (OWIP), res bbl.

Substituting Eq. 12 for $W_e$ and Eq. 4 for $E_{pi}$ in Eq. 1 and then rearranging yields an equation of a straight line:

$$F \frac{E}{E_t} = G + \frac{p_i - p}{E_t} \left[ \frac{G B_{p} (S_{u} c_w + c_f)}{1 - S_{ui}} + (c_w + c_f)W \right]. \quad \text{........................................... (13)}$$

Plotting $F \frac{E}{E_t}$ on the y-axis vs. $\frac{p_i - p}{E_t}$ on the x-axis yields a straight line with the y-intercept equal to $G$. This is the pot aquifer plot. The value of this plot is that it permits determination of OGIP without any prior knowledge of aquifer size, rock or water compressibility, or even initial water saturation. The sequence of plotted points will be from right to left.

The slope of the pot aquifer plot is given by the term in brackets in Eq. 13. The water in place in the aquifer, $W$, can be calculated from the slope if, in fact, $c_w$ is known with some degree of confidence. Rearranging the slope term,

$$A = \frac{G B_{p} (S_{u} c_w + c_f)}{1 - S_{ui}} + (c_w + c_f)W \quad \text{........................................... (14)}$$

where $G$ and the slope are obtained from the least-square fit straight line.

**Fig. 4** shows the pot aquifer plot for the two-cell gas-simulation example. As before, each plotted point (large symbols) represents conditions at the end of each year. Also shown are straight lines fitted to the data using the least-squares method, assuming that analyses had been performed at several times during the reservoir’s history (after 2, 5, and 10 years). Values of OGIP are obtained from extrapolation of those straight lines to the y-intercept. Typically with this plot, the early-time points fall below the true straight line that eventually develops, and such is the case with this data. After 2 years of performance, an analysis would consist only of points from Years 1 and 2, and the true straight line would not yet be apparent, giving a value for OGIP approximately 4\% too high. Analyses conducted after 5 and 10 years would likely have excluded the Year 1 data from the least-square fit. In all cases, the OGIP values are significantly closer to the actual value of 101 Bcf than the corresponding values obtained from the p/z plot (Fig. 2).

In Fig. 4, data points during the first year are plotted at 1-month intervals with the smaller symbols. These points have a negative slope and do not start “turning over” toward the correct positive slope until approximately three-quarters of the way through the year. This is typical of the pot aquifer plot; therefore, the plot may not be usable in the very early life of the reservoir.

**Table 4** summarizes the OGIP values obtained using the three evaluation methods (modified p/z, modified Cole, and pot aquifer), as well as the percent errors. Even the modified Cole plot solution is closer to the true OGIP than the p/z plot. The reason the modified Cole plot is so near the true OGIP is that the aquifer is so small for this example. For larger aquifers, neither the original nor the

<table>
<thead>
<tr>
<th>% of OGIP Produced</th>
<th>p/z Modified for $c_w$</th>
<th>Modified Cole Plot</th>
<th>Pot Aquifer Plot</th>
</tr>
</thead>
</table>
| \begin{tabular}{c|c|c|c|c}
11 & 109.0 & 8.2 & <108.9 & 8.0 & 105.3 & 4.5 \\
27 & 107.3 & 6.5 & <107.2 & 6.3 & 101.6 & 0.8 \\
54 & 104.8 & 4.0 & <104.4 & 3.6 & 101.0 & 0.2 \\
\end{tabular} |
modified Cole plot will give a value so close to the true OGIP as in this example.

The slope of the solution line in Fig. 4 after 10 years is 1,103 RB/psi, giving a calculated W of 69.1 million res bbl using Eq. 14, some 7% low compared to the true value of 74.5 million res bbl. Cumulative water influx can be calculated from Eq. 12 as 2,346,000 res bbl after 10 years, approximately 6% less than the 2,494,000 res bbl from the simulation. Accuracy of the calculated W and Wc would be improved by excluding from the least-squares fit additional early data points after Years 2 and 3 that deviate slightly from the true straight-line trend. However, when analyzing actual field data, such subtle deviations are difficult to detect owing to normal data scatter.

Note that if in fact there is no aquifer, the pot aquifer plot still applies. In this case, W goes to zero in Eq. 13. The formation compressibility can then be calculated from the slope:

\[ c_f = A \frac{1 - Sw \omega}{GB \rho} = Sw_c c_w \]  

(15)

If for this data set it had been assumed that no aquifer were present, a cf of 14.3×10^{-6} psi^{-1} would have been calculated from Eq. 15, significantly larger than the “known” value of 6×10^{-6} psi^{-1}. In a real-world setting, this would be another indication that an unaccounted-for energy source is present. Case 1 of Wang and Teasdale12 shows an application of this method to an actual reservoir believed to have no waterdrive.

**Drive Indices.** Drive indices were calculated for the two-cell simulation model, assuming that the OGIP obtained from the modified p/z method (modified to include cf effects) would, on the surface, give every indication that a correct material-balance solution had been obtained for depletion drive. Yet OGIP would be erroneously high, with the error ranging from approximately 4 to 8%, depending on the stage of reservoir depletion considered. Constructing the modified Cole plot or calculating drive indices would signal that the solution was, in fact, not correct. The Cole plot, original or modified, indicates unambiguously that a weak waterdrive exists, in which case the pot aquifer plot should be used to calculate the most accurate value of OGIP.

**Oklahoma Morrow Gas Reservoir.** Production history and other data for an Oklahoma Morrow sand gas reservoir are given in Table 7. The lack of water production, together with the decline in reservoir pressure, suggested that no aquifer was present. The p/z plot, Fig. 5, also gives no hint of aquifer support. The modified p/z extrapolation gives G = 6.02 Bcf. (Note that even though cf is only 3×10^{-6} psi^{-1}, extrapolation of the conventional p/z that ignores cf gives G = 6.32 Bcf, some 5% greater.)

The Cole and modified Cole plots are shown in Fig. 6 and exhibit the characteristic negative slope of a weak waterdrive system. (Note that the maximum possible value of OGIP from the modified Cole plot is slightly less than OGIP from the modified p/z.) Therefore, the pot aquifer plot was used to determine OGIP and aquifer size (Fig. 7). OGIP of 5.44 Bcf results from the extrapolation of a line fit to the three data points using the least-squares method (R^2 = 0.934). Thus the p/z extrapolation gave a value nearly 11% too high, even after being modified to account for formation compressibility.

The slope of Fig. 7, 58 RB/psi, was used with Eq. 15 to calculate a value for cf of 12×10^{-6} psi^{-1}, much greater than the estimated value of 3×10^{-6} psi^{-1} and too high for “hard rock country.” Therefore, the estimated cf = 3×10^{-6} psi^{-1} was used with Eq. 14 to greater than unity and sometimes less than unity, as opposed to a consistently increasing trend.

To summarize, evaluation of this reservoir taking the common approach of considering only the p/z method (modified to include cf effects) would, on the surface, give every indication that a correct material-balance solution had been obtained for depletion drive. Yet OGIP would be erroneously high, with the error ranging from approximately 4 to 8%, depending on the stage of reservoir depletion considered. Constructing the modified Cole plot or calculating drive indices would signal that the solution was, in fact, not correct. The Cole plot, original or modified, indicates unambiguously that a weak waterdrive exists, in which case the pot aquifer plot should be used to calculate the most accurate value of OGIP.
calculate \( W \) of 6.74 million res bbl. Aquifer size can be compared with reservoir size by first calculating the original pore volume of the hydrocarbon reservoir from:

\[
P_V = \frac{G_P}{1 - S_w} = \frac{5,440,000 \text{ Mcf} \times 0.5770 \text{ RB/Mcf}}{1 - 0.3} = 4.84 \text{ million res bbl.} \quad \text{(16)}
\]

Then, the aquifer is 6.74 million res bbl/4.48 million res bbl = 1.5 times as large as the gas reservoir. Cumulative water influx of 99,800 res bbl after 332 days is calculated with Eq. 12. This equates to only 3% of the original hydrocarbon pore volume (HCPV) of approximately 3,139,000 res bbl, yet it represents approximately 10% of the cumulative hydrocarbon voidage \( (G_p \times B_g = 1,350,000 \text{ Mcf} \times 0.7353 \text{ RB/Mcf} = 992,700 \text{ res bbl}) \).

Drive indices are shown in Table 8 for the modified \( p/z \) solution and the pot aquifer solution. Drive-index sums based on the OGIP obtained from \( p/z \) show a trend from too low at early time to near unity at late time. Had the drive indices been normalized to sum to unity, the fact that a problem existed with the \( p/z \) solution would have been obscured. Drive indices based on the OGIP obtained from the pot aquifer solution fluctuate around unity, exhibiting scatter typical of field data.

**Roach Plot.** Roach\textsuperscript{17} rearranged the \( p/z \) relationship to solve for the correct OGIP when formation compressibility is significant but of unknown magnitude, and no waterdrive is present. Poston and coworkers\textsuperscript{11} expanded Roach’s solution to incorporate water influx. (Poston and coworkers described this approach as the “solution plot” method, but the “Roach plot” terminology is retained in this paper as a more distinctive title.) Equation 6.10 from Ref. 11 can be expressed in modified form as

\[
\frac{(p/z)/(p/z) - 1}{p_i - p} = \frac{1}{G_p} \frac{(p/z)/(p/z) \cdot G_p}{p_i - p} - \left[ \frac{S_w + c_i}{1 - S_i} \cdot \frac{W - W_B}{1 - (p_i - p)G_P} \right] \quad \text{(17)}
\]

The Roach plot consists of plotting the left side of Eq. 17, \( (p/z)/(p/z) - 1 \), on the y-axis vs. \( (p/z)/(p/z) \cdot G_p \) on the x-axis. The slope of this plot is 1/G, so G is equal to the reciprocal of the slope. The y-intercept is the term in brackets on the right side of Eq. 17 and incorporates formation and water compressibility, as well as water influx and water production.

The difficulty in interpreting the plot in the presence of a waterdrive is that the y-intercept is not constant because the water-influx and water-production terms in brackets do not remain constant. Thus, the correct slope is difficult to ascertain, and significant errors in OGIP can easily result.

**Modified Roach Plot.** The problem can be solved, provided that the aquifer is of the pot aquifer type. Eq. 12 is substituted for \( W \) in Eq. 17, which is then rearranged to move the water-production term into the x-axis plotting term, resulting in:

\[
\frac{(p/z)/(p/z) - 1}{p_i - p} = \frac{1}{G_p} \frac{(p/z)/(p/z) \cdot G_p}{p_i - p} - \left[ \frac{S_w + c_i}{1 - S_i} \cdot \frac{W_B}{1 - (p_i - p)G_P} \right] \quad \text{(18)}
\]

**Fig. 5**—\( p/z \) plot, Oklahoma Morrow gas reservoir.

**Fig. 6**—Original and modified Cole plots, Oklahoma Morrow gas reservoir.

**Fig. 7**—Pot aquifer plot, Oklahoma Morrow gas reservoir.

**Table 8**—Drive Indices, Oklahoma Morrow Gas Reservoir

<table>
<thead>
<tr>
<th>Days</th>
<th>Modified ( p/z ) Solution</th>
<th>Pot Aquifer Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_d )</td>
<td>( l_d )</td>
<td>Total</td>
</tr>
<tr>
<td>72</td>
<td>0.849</td>
<td>0.090</td>
</tr>
<tr>
<td>237</td>
<td>0.886</td>
<td>0.080</td>
</tr>
<tr>
<td>332</td>
<td>0.959</td>
<td>0.048</td>
</tr>
</tbody>
</table>

February 2002 SPE Reservoir Evaluation & Engineering
The x-axis plotting term is now \( \frac{(p/z)/p_z}{\rho_i - \rho} \cdot \frac{W_p B_w}{B_w} \), and the y-axis term is the same as before. The y-intercept, the term in brackets, is now constant; thus, the plotted points will have a constant and correct slope. The method is demonstrated with a simulation example.

**Simulation Model.** The gas model described previously was modified to give a much stronger aquifer. The water-filled cell was removed, and aquifer strength was provided by attaching a Fetkovich-type aquifer \(^{18}\) to the single-cell gas reservoir. Aquifer OWIP was 633 million res bbl, or 10 times the HCPV. Aquifer productivity index (PI) was set to a high value, 485 RB/D/psi, and aquifer compressibility (sum of \( c_w \) and \( c_c \)) was set to 9x10\(^{-6}\) psi\(^{-1}\). The model was run for 10 years, as before. Simulation results and PVT compressibility are shown in Table 9, and the conventional and modified Roach plots are shown in Fig. 8. Plotted points migrate from left to right with time.

Examining Fig. 8, the conventional plot appears to be linear. In reality, however, the points are deviating slightly to the left with increasing time because water production causes the y-intercept (in brackets in Eq. 17) to migrate upward with time. The “slope” of the conventional plot is 1.042x10\(^{-5}\) MMscf/psi, giving the deviation in the late-time points on the conventional plot becomes more visible; for \( W = 1 \) billion res bbl, it is clearly noticeable, and the late-time points are excluded from the least-square fit. For \( W = 5 \)xHCPV, the conventional plot gives essentially the correct \( G \) because water production is not too great. In application, both the conventional and modified plots could be constructed as in Fig. 8 and compared to determine the amount of deviation. If only one plot is to be constructed, it should be the modified plot, to be on the safe side.

A word of caution: the modified Roach plot has not been verified with actual field data because suitable field data have not become available. Two questions come to mind when considering field cases: first, whether an actual aquifer could be as large as that used in the simulations and still perform like a pot aquifer, and second, whether water volumes sufficiently large to cause the original Roach plot to deviate from a straight line can realistically be produced before the wells load up. The method might at least find application in enhanced recovery projects in which gas reservoirs are aggressively dewatered.

**Oil Reservoirs**

**Campbell Plot.** For oil reservoirs, the Campbell plot \(^{10}\) is the counterpart to the modified Cole plot for gas. It is based on an equation analogous to Eq. 5 for gas:

\[
\frac{F}{E} = \frac{W_i}{E_i} \quad \text{or} \quad \frac{W_i}{E_i} = \frac{F}{E}
\]

where \( F = N \times (B_t + B_g (R_p - R_s)) \) is the cumulative reservoir voidage, and

\[
E_i = \frac{F}{E} = \frac{W_i}{E_i} = \frac{F}{E} = \frac{W_i}{E_i}
\]

The gas model described previously was run for 10 years, as before. Simulation results and PVT compressibility are shown in Table 9, and the conventional and modified Roach plots are shown in Fig. 8. Plotted points migrate from left to right with time. The modified Roach plot has been tested for varying Fetkovich aquifer volumes with this model, using values of \( \alpha \) and \( \beta \) that are known. A value of 629 million res bbl are known. A value of 629 million res bbl, almost 5% low to the true OGIP

The gas model described previously was run for 10 years, as before. Simulation results and PVT compressibility are shown in Table 9, and the conventional and modified Roach plots are shown in Fig. 8. Plotted points migrate from left to right with time. The modified Roach plot has been tested for varying Fetkovich aquifer volumes with this model, using values of \( \alpha \) and \( \beta \) that are known. A value of 629 million res bbl, almost 5% low to the true OGIP

<table>
<thead>
<tr>
<th>Year</th>
<th>Pressure (psia)</th>
<th>Cumulative Gas Produced (Scf)</th>
<th>Cumulative Water Produced (STB)</th>
<th>Gas Deviation Factor, z</th>
<th>( B_t ) (RB/Mscf)</th>
<th>( B_o ) (RB/STB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6,411</td>
<td>0.000</td>
<td>0</td>
<td>1.1192</td>
<td>0.6279</td>
<td>1.0452</td>
</tr>
<tr>
<td>1</td>
<td>6,130</td>
<td>5.475</td>
<td>2.163</td>
<td>1.1008</td>
<td>0.6459</td>
<td>1.0460</td>
</tr>
<tr>
<td>2</td>
<td>5,849</td>
<td>10.950</td>
<td>9.293</td>
<td>1.0828</td>
<td>0.6659</td>
<td>1.0470</td>
</tr>
<tr>
<td>3</td>
<td>5,565</td>
<td>16.425</td>
<td>22.286</td>
<td>1.0652</td>
<td>0.6885</td>
<td>1.0478</td>
</tr>
<tr>
<td>4</td>
<td>5,280</td>
<td>21.900</td>
<td>43.807</td>
<td>1.0482</td>
<td>0.7141</td>
<td>1.0488</td>
</tr>
<tr>
<td>5</td>
<td>4,992</td>
<td>27.375</td>
<td>78.152</td>
<td>1.0316</td>
<td>0.7434</td>
<td>1.0496</td>
</tr>
<tr>
<td>6</td>
<td>4,700</td>
<td>32.850</td>
<td>132.011</td>
<td>1.0158</td>
<td>0.7774</td>
<td>1.0505</td>
</tr>
<tr>
<td>7</td>
<td>4,403</td>
<td>38.325</td>
<td>219.211</td>
<td>1.0005</td>
<td>0.8174</td>
<td>1.0515</td>
</tr>
<tr>
<td>8</td>
<td>4,101</td>
<td>43.800</td>
<td>358.536</td>
<td>0.9865</td>
<td>0.8653</td>
<td>1.0524</td>
</tr>
<tr>
<td>9</td>
<td>3,787</td>
<td>49.275</td>
<td>607.252</td>
<td>0.9731</td>
<td>0.9243</td>
<td>1.0534</td>
</tr>
<tr>
<td>10</td>
<td>3,459</td>
<td>54.750</td>
<td>1,034,275</td>
<td>0.9610</td>
<td>0.9994</td>
<td>1.0544</td>
</tr>
</tbody>
</table>
The decline in reservoir pressure and lack of significant water production for 8 years could lead to the interpretation that no aquifer is present. The recommended method\textsuperscript{16,19} for solving the material balance for an undersaturated oil reservoir without a water drive is the plot of $F$ vs. $E$, which should be a straight line with OOIP equal to the slope. Fig. 10 is the plot for these data. Least-square straight lines were fit to the data, assuming that evaluations were performed at various stages in the life of the reservoir, after 3, 7, and 20\% of the true OOIP had been produced (after 700, 1,285, and 3,595 days, respectively). Calculated values of $N$ (shown in the legend of Fig. 10) are in error by +160\%, +90\%, and +50\%, respectively. For this perfect data set, it is obvious that the points do not lie in a straight line, but for real field data, the curvature could be obscured easily within normal data scatter, leading to the false conclusion that no aquifer is present.

The Campbell plot for these data, Fig. 11, clearly shows the signature negative slope of a weak waterdrive, even after just the first two or three data points (700 and 1,285 days, respectively). As with the modified Cole plot for gas reservoirs, the points migrate toward the true OOIP with time. Because a weak waterdrive is present, the correct material-balance solution for this case is obtained from the pot aquifer plot that has been derived for oil\textsuperscript{16} similar to that for gas. Because the oil is undersaturated, $F$ is plotted on the y-axis vs. $\frac{dP}{dE}$ on the x-axis (see Ref. 16 for derivation). The y-intercept gives the OOIP.

Several solutions were obtained from the pot aquifer plot at the same point in the reservoir's life as before. The initial data point at 305 days lies below the correct straight-line trend that has become apparent after 1,285 days (third plotted point) and so is excluded from that least-square fit. The solution at 1,285 days gives a value of $N$ of 21.7 million STB, within 10\% of the true value. Sometime after 1,285 days (that is, after the third plotted point), it becomes apparent that the second data point at 700 days is off trend as well. Therefore, the second point is excluded from subsequent fits, giving increasingly accurate answers.

Aquifer OWIP is calculated from the slope of the pot aquifer plot, using the oil version of Eq. 14 (i.e., $N$ replaces $G$, and $B_w$ replaces $B_{p, w}$). Formation compressibility is known in this simulation example). After 3,595 days, the slope is 3,090 RB/psi, from which $W$ of approximately 79 million res bbl is calculated, very close to the known value of approximately 80 million res bbl. Oil reservoir pore volume is approximately 35.7 million res bbl, so the aquifer is about 2.2 times as large as the reservoir.

**Drive Indices.** Drive indices for oil reservoirs as defined in Ref. 13 are presented here in modified form.

Depletion-drive index:

$$I_{DD} = \frac{NE_o}{F - W_o B_w} \quad \text{(26)}$$
The calculated values of OOIP would be obtained using the depletion-drive solution (compare the calculated values of $N$ in Fig. 10 with those in Fig. 12).

Carlson\'s pointed out that even when material-balance results are ambiguous or do not provide very accurate quantitative answers, valuable qualitative insights may still be obtained. For this oil-simulation case, the pot aquifer material-balance solution after 700 days is considerably in error, and even after 1,285 days, it is not particularly accurate (Fig. 12). However, the negative slope of the Campbell plot (Fig. 11) clearly shows the presence of a weak waterdrive even after only 700 days (first two data points), a valuable piece of information obtained early in the life of the reservoir.

**Other Considerations**

In this paper, water compressibility, $c_w$, considers only the liquid phase. That is, the energy contribution from gas dissolved in the water, coming out of solution as reservoir pressure declines, is ignored. Fetkovich et al.\(^{21}\) examined this problem for high-pressure gas reservoirs and concluded that the energy contribution from gas dissolved in the water is usually important only late in the reservoir life (below approximately 1,500 psia). To account for this additional energy, they defined water total compression volume factor, $B_{tw}$, analogous to oil total compression volume factor:

$$B_{tw} = B_u + B_g (R_{sat} - R_{sw})$$

They also defined water total compressibility, $c_{tw}$:

$$c_{tw} = \frac{B_{tw} - B_{sat}}{B_{sat} (\rho_o - \rho_w)}$$

The energy contribution from gas dissolved in the water can be incorporated in the equations presented in this paper by substituting $B_{tw}$ (Eq. 31) for $B_u$, and $c_{tw}$ (Eq. 32) for $c_w$. The Campbell and modified Cole plots would be affected, but not until later in the reservoir life when pressure has declined. The pot aquifer plot no longer applies because the slope is no longer constant, and the Roach plot no longer applies because the $y$-intercept is no longer

### Table 12—PVT Properties of Oil-Simulation Model with Pot Aquifer

<table>
<thead>
<tr>
<th>Days</th>
<th>Pressure (psia)</th>
<th>$B_u$ (RB/STB)</th>
<th>$R_o$ (Mscf/STB)</th>
<th>$B_w$ (RB/Mscf)</th>
<th>$B_g$ (RB/STB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2,855</td>
<td>1.2665</td>
<td>0.5010</td>
<td>0.9201</td>
<td>1.2665</td>
</tr>
<tr>
<td>305</td>
<td>2,779</td>
<td>1.2677</td>
<td>0.5010</td>
<td>0.9637</td>
<td>1.2677</td>
</tr>
<tr>
<td>700</td>
<td>2,627</td>
<td>1.2681</td>
<td>0.4973</td>
<td>1.0502</td>
<td>1.2720</td>
</tr>
<tr>
<td>1,285</td>
<td>2,547</td>
<td>1.2554</td>
<td>0.4671</td>
<td>1.0977</td>
<td>1.2926</td>
</tr>
<tr>
<td>1,465</td>
<td>2,402</td>
<td>1.2512</td>
<td>0.4574</td>
<td>1.1146</td>
<td>1.2998</td>
</tr>
<tr>
<td>2,005</td>
<td>2,223</td>
<td>1.2383</td>
<td>0.4269</td>
<td>1.2010</td>
<td>1.3273</td>
</tr>
<tr>
<td>2,365</td>
<td>2,090</td>
<td>1.2278</td>
<td>0.4024</td>
<td>1.2825</td>
<td>1.3543</td>
</tr>
<tr>
<td>2,905</td>
<td>1,833</td>
<td>1.2074</td>
<td>0.3579</td>
<td>1.4584</td>
<td>1.4161</td>
</tr>
<tr>
<td>3,235</td>
<td>1,665</td>
<td>1.1949</td>
<td>0.3277</td>
<td>1.6112</td>
<td>1.4741</td>
</tr>
<tr>
<td>3,595</td>
<td>1,460</td>
<td>1.1802</td>
<td>0.2908</td>
<td>1.8526</td>
<td>1.5696</td>
</tr>
</tbody>
</table>

### Figure 10—Solution plot for oil-simulation case, assuming no waterdrive.

![Solution plot for oil-simulation case, assuming no waterdrive.](image)

### Figure 11—Campbell plot for oil-simulation case.

![Campbell plot for oil-simulation case.](image)
constant. Fetkovich et al. presented a method for evaluating gas reservoirs under these conditions.

In this paper, formation compressibility, \(c_f\), is assumed to be constant and unchanged over the reservoir life being investigated. Fetkovich et al. presented a method to account for changing formation compressibility in gas reservoirs, and Yale et al.\(^\text{22}\) presented a method for oil reservoirs.

Various workers have investigated the effect of errors in measured reservoir pressure on material-balance results. However, to this author’s knowledge, such an evaluation has not been performed on weak aquifer material balance such as presented in this paper. Further study is needed of the sensitivity of these relationships to errors in pressure.

**Material Balance and Reservoir Simulation**

The perception exists among some that classical material-balance methods have been rendered obsolete by reservoir simulation. Because simulation incorporates material balance on a cell-by-cell basis, it may be argued that stand-alone material balance is superfluous and therefore serves no utility on those reservoirs that are subject to a simulation study.

In response, it is argued that material balance and simulation are complementary rather than competing tools. Material balance can provide valuable insights into reservoir mechanisms and processes that may be obscured by the multitude of parameters that go into simulation.

Consider the cases shown in this paper in which weak waterdrives are not apparent from performance data. Simulations performed on these reservoirs without benefit of a prior material-balance study might well have resulted in rock and fluid parameters being adjusted to achieve matches on the wrong values of OHIP. If the waterdrive is of the pot aquifer type, as in this paper, material balance can solve for OHIP and aquifer size simultaneously and unambiguously, without resorting to trial and error (provided that sufficient reservoir history is available). Even in cases in which the material-balance solution is more ambiguous, the analysis often yields qualitative insights that are as valuable as quantitative results.

Material balance should be performed before a simulation study to help narrow the range of the many parameters that can be adjusted during simulation as well as the magnitude of adjustments that are considered reasonable. And, of course, it is impractical to perform a simulation study on every reservoir.

Dake provided an especially cogent discussion of this issue in Ref. 7. He summarized the situation appropriately: “...numerical simulation and material balance must not be regarded as competitive techniques: we have too few tools in reservoir engineering to discard any of them.”

**Conclusions**

1. The Cole plot (gas) and Campbell plot (oil) diagnose the presence of a weak waterdrive unambiguously. Depletion-drive plots, such as the p/t, are ambiguous in the presence of a weak waterdrive and can give OHIP values that are erroneously high by a significant amount. As suggested by previous authors, the weak waterdrive signature on the Cole and Campbell plots is shown to be a negative slope.

2. The negative slope of the Cole and Campbell plots amounts to the counterintuitive characteristic of decreasing apparent OHIP with time.

3. The modified version of the Cole plot should be used in cases in which formation compressibility is not negligible compared to gas compressibility, such as abnormally pressured reservoirs.

4. If a correct solution to the material balance has been obtained, the drive indices will sum to unity (allowing for normal scatter). If the drive indices do not sum to unity, a correct solution has not been obtained. The drive indices should never be normalized to sum to unity because this obscures their usefulness as a criterion for determining the validity of the solution and gives a false sense of security. Only the raw calculated values should be reported.

5. The Roach plot can be modified to improve gas reservoir interpretation in the presence of a pot aquifer by incorporating cumulative water production in the \(x\)-axis plotting term. This procedure has not been tested on field data, however.

6. Reservoir simulation does not eliminate the need for classical material-balance analysis. Material balance can reveal insights into reservoir performance that cannot be obtained from simulation, such as the presence of a weak aquifer that is not otherwise obvious, as in examples presented in this paper. Material balance is complementary to, not competitive with, reservoir simulation.

**Nomenclature**

\(A\) = slope

\(B_g\) = gas formation volume factor, \(L^3/L^3\), RB/Mscf

\(B_o\) = oil formation volume factor, \(L^3/L^3\), RB/STB

\(B_t\) = total or two-phase oil formation volume factor (Eq. 25), \(L^3/L^3\), RB/STB

\(B_w\) = water formation volume factor, \(L^3/L^3\), RB/STB

\(B_{tw}\) = total or two-phase water formation volume factor (Eq. 31), \(L^3/L^3\), RB/STB

\(c_f\) = formation compressibility, \(L^3/L^3/(m/L^2)\), vol/vol/psi

\(c_w\) = water compressibility, \(L^3/L^3/(m/L^2)\), vol/vol/psi

\(c_{tw}\) = total or two-phase water compressibility (Eq. 32), \(L^3/L^3/(m/L^2)\), vol/vol/psi

\(E_g\) = cumulative gas expansion, \(L^3/L^3\), RB/STB in oil reservoirs, RB/Mscf in gas reservoirs

\(E_{fuc}\) = cumulative formation and water expansion, \(L^3/L^3\), RB/STB in oil reservoirs, RB/Mscf in gas reservoirs

\(E_{cu}\) = cumulative oil expansion, including original complement of solution gas, \(L^3/L^3\), RB/STB

\(E_{tu}\) = cumulative total expansion, \(L^3/L^3\), RB/STB in oil reservoirs, RB/Mscf in gas reservoirs

\(F\) = cumulative reservoir voidage, \(L^3\), res bbl

---

**Table 13—Drive Indices After 3,595 Days, Oil-Simulation Model with Pot Aquifer**

<table>
<thead>
<tr>
<th>Days</th>
<th>(I_{OIP})</th>
<th>(I_{AH})</th>
<th>Total</th>
<th>(I_{OIP})</th>
<th>(I_{AH})</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>305</td>
<td>0.151</td>
<td>0.405</td>
<td>0.556</td>
<td>0.100</td>
<td>0.700</td>
<td>0.809</td>
</tr>
<tr>
<td>700</td>
<td>0.209</td>
<td>0.368</td>
<td>0.577</td>
<td>0.139</td>
<td>0.636</td>
<td>0.775</td>
</tr>
<tr>
<td>1,285</td>
<td>0.454</td>
<td>0.233</td>
<td>0.747</td>
<td>0.301</td>
<td>0.506</td>
<td>0.807</td>
</tr>
<tr>
<td>1,465</td>
<td>0.489</td>
<td>0.282</td>
<td>0.771</td>
<td>0.325</td>
<td>0.486</td>
<td>0.811</td>
</tr>
<tr>
<td>2,005</td>
<td>0.582</td>
<td>0.256</td>
<td>0.838</td>
<td>0.386</td>
<td>0.442</td>
<td>0.828</td>
</tr>
<tr>
<td>2,365</td>
<td>0.643</td>
<td>0.240</td>
<td>0.883</td>
<td>0.427</td>
<td>0.415</td>
<td>0.842</td>
</tr>
<tr>
<td>2,905</td>
<td>0.739</td>
<td>0.214</td>
<td>0.953</td>
<td>0.491</td>
<td>0.369</td>
<td>0.860</td>
</tr>
<tr>
<td>3,235</td>
<td>0.806</td>
<td>0.196</td>
<td>1.001</td>
<td>0.535</td>
<td>0.336</td>
<td>0.871</td>
</tr>
<tr>
<td>3,595</td>
<td>0.892</td>
<td>0.174</td>
<td>1.065</td>
<td>0.592</td>
<td>0.290</td>
<td>0.882</td>
</tr>
</tbody>
</table>

*Excluded from least-square fit.*
Acknowledgments

I thank Marathon Oil Co. for permission to publish this paper following my retirement, particularly Jim Gilman for his special efforts. Teresa Schaller ran the oil-simulation case presented in the paper. Stuart Cox provided the Morrow Gas data and consulted on the interpretation. Lois Fitzpatrick provided valuable help formatting this paper.

References


SI Metric Conversion Factors

- acre × 4,046 873  E–01 = ha
- bbl × 1,589 873  E–01 = m³
- ft × 3.048*  E–01 = m
- ft³ × 2.831 685  E–02 = m³
- °F (°F–32)/1.8 = °C
- psi × 6.894 757  E+00 = kPa
- psi× 1.450 377  E–01 = kPa–1
- scf/bbl × 1.801 175  E+01 = m³ (st)

*Conversion factor is exact.

Jeff Pletcher retired in 1999 from Marathon Oil Co.’s Petroleum Technology Center in Littleton, Colorado. He was an advanced senior engineer working in the areas of reservoir evaluation and reservoir engineering training. His career spanned more than 30 years, all with Marathon. Previous assignments were in production and reservoir engineering in Illinois, Texas, Louisiana, and the general office in Findlay, Ohio. Pletcher holds a BS degree in petroleum engineering from Marietta College.