An Analysis of High-Velocity Gas Flow Through Porous Media

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Introduction

Much research has been conducted to understand flow through porous media. Regarding high velocity, suitable correlations and nomenclature are the subject of controversy because of different views on the mechanism causing pressure drop.1-14

This study tries to improve the understanding of high-velocity flow through porous media. Using available data and this research, the best correlation is sought to permit the calculation of high-velocity flow based on permeability, porosity, and character of the rock. We hope that a nomenclature can be suggested that is acceptable to both reservoir engineers and fluid mechanics research scientists. The results should be useful for improving correlations of gas-well flow data and for predicting well flow from core data, fluid properties, and specified conditions.

Early Work on High-Velocity Flow

Fancher et al.1 measured pressure drop during flow through a large number of unconsolidated and consolidated porous media. They correlated the data by using the friction factor and Reynolds number, with grain diameter as a characteristic length. They showed that for flow through porous media, an increased pressure drop at high velocity is beyond that proportional to velocity. Data taken at the USBM2 were correlated using a quadratic equation of pressure drop with the second term of velocity to the nth power. Green and Duwez4 increased the understanding of high-velocity gas flow data when studying sintered metals. They adapted the equation with a velocity term squared that Forchheimer15 had developed:

\[- \frac{dp}{dL} = \frac{\nu v}{k} + \beta \nu v^2. \]  

Cornell and Katz5,20 measured the porosity, permeability, and \( \beta \) factors for cores, resulting in a correlation of \( \beta \) with permeability.16 The term \( \beta \) was called a "turbulence factor," but the expression was unacceptable to several researchers.14

Language Used in Literature

Space limitations prohibit quoting completely the language used when describing the mechanism that consumes energy at more than a linear rate with velocity. The term used in flow equations (generally the \( \beta \) in a quadratic flow equation) also has been given various names according to the author's view of the flow mechanism.

Table I assembles selected typical titles and statements quoted from published works. After the flow mechanism has been reviewed, a more appropriate language will be submitted to agree with the concepts developed here.

Mechanism for Consumption of Pressure Drop Energy

Here, we try to understand the mechanism by which

Darcy's law is inadequate for representing high-velocity gas flow in porous media, such as near the wellbore. An analysis of pressure lost during flow through conduits of alternating cross sections suggests more appropriate words for describing the mechanism for energy loss and terms in the flow equation. Updated correlations are presented for the coefficient of the velocity-squared term.

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increased velocity results in a pressure drop greater than that proportional to velocity increase. The mechanism of consuming pressure drop energy in pipe flow is well understood. In Fig. 1, the cylinders of fluid are flowing at different velocities. Work energy overcomes the longitudinal shear stresses between the cylinders flowing at different velocities. In the case of unidirectional flow for a constant cross section, there is a single term in the resistance equation that includes the velocity of the fluid. The increased energy consumed at higher velocities is

directly proportional to the velocity increase for streamline flow, as shown by Poiseuille's law.

Now consider the flow in the interstices of a porous solid, as idealized in Fig. 2. In flow through pores, there are two variations from the horizontal cylindrical flow. First, the cross section of the flow channel is increasing and decreasing alternately. Then, there is the displacement from a straight line when moving through a network of pores. Each deviation from the direction of fluid movement now has two components of the viscous resistance: (1) longitudinal shear $e_f$ in the direction of flow and (2) longitudinal tension or a normal stress component $f_g$ during the expansion and, correspondingly, $i_j$ and $h_i$ during contractions, commonly written as Eqs. 2 and 3.  

$$
\tau_{ef} = \mu \left( \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)
$$

longitudinal shear stress, \hspace{1cm} (2)

$$
\sigma_{ef} = \mu \left( \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right)
$$

longitudinal tension stress. \hspace{1cm} (3)

Now consider the effect of velocity on the resistances. The flowlines with increased velocity are no longer constant in length and are believed to increase the shear and tension areas with increased velocity (Figs. 3a and 3b). At still higher velocities, separation or reversed flow occurs in the enlarged cross sections to increase the viscous resistance (Fig. 3c). Here, the recirculating portion may be considered laminar. All these transverse effects are characteristics of irregular alternating cross-section flow paths and are not present in cylinders. Porous media (such as sandstones or carbonates with only matrix porosity and free from irregular solution processes) no doubt are always in the regimes in Figs. 3a and 3b. In some porous solids (such as vugular carbonates, reeds, and conglomerates), there are interstices large enough to allow the regimes in Fig. 3c and even in Fig. 3d to be described as turbulence. A student working with Katz observed that this turbulence occurred in pipes when an ink filament was used in a liquid flowing at increasing velocities through a bed of glass beads with about $\frac{3}{16}$ in. diameter. Regarding porous solids characterized by Figs. 3a and 3b, the term turbulence is unacceptable. However, flow through such

**TABLE 1—LANGUAGE USED IN LITERATURE FOR HIGH VELOCITY GAS FLOW**

<table>
<thead>
<tr>
<th>Reference</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fancher et al.¹ (1933)</td>
<td>&quot;... the flow of fluids through these porous materials closely resembles that through pipes; that there is a condition of flow in porous systems which resembles viscous flow, another which corresponds to turbulence.&quot;</td>
</tr>
<tr>
<td>Elenbaas and Katz² (1948)</td>
<td>&quot;A Radial Turbulent Flow Formula&quot;</td>
</tr>
<tr>
<td>Green and Duwez⁴ (1951)</td>
<td>&quot;The inertial coefficient $\beta$ may be interpreted as a measure of the tortuosity of the flow channels, perhaps as an average curvature of the streamline determining the acceleration experienced by the fluid.&quot;</td>
</tr>
<tr>
<td>Hubbert⁷ (1956)</td>
<td>&quot;... we have seen that the cause of the failure of Darcy's Law is the distortion that results in the flow lines when the velocity is great enough that the inertial force becomes significant.&quot;</td>
</tr>
<tr>
<td>Tek⁶ (1957)</td>
<td>&quot;The generalized Darcy equation may be referred to as the 'non-Darcy flow' regime. The transition from Darcy to non-Darcy flow is a gradual one.&quot;</td>
</tr>
<tr>
<td>Katz et al.⁸ (1959)</td>
<td>&quot;If one includes extra motion of the fluid to consume the extra pressure drop, then the term 'turbulent flow' here is justified.&quot;</td>
</tr>
<tr>
<td>Houpeurt⁷ (1959)</td>
<td>&quot;... we do not think, the flow can be really turbulent ... , we consider the kinetic energy losses are responsible for the deviation from Darcy's Law.&quot;</td>
</tr>
<tr>
<td>Tek et al.²³ (1962)</td>
<td>&quot;The Effect of Turbulence on Flow of Natural Gas,&quot;</td>
</tr>
<tr>
<td>Swift and Kiel⁸ (1962)</td>
<td>&quot;Prediction of Gas-Well Performance Including the Effect of Non-Darcy Flow&quot;</td>
</tr>
<tr>
<td>Wright⁹ (1968)</td>
<td>&quot;Four regimes of flow for water in an unconsolidated bed. 1) a laminar regime 2) a steady inertia regime 3) a turbulent transition regime 4) a fully turbulent regime.&quot;</td>
</tr>
<tr>
<td>Gewers and Nichol¹² (1969)</td>
<td>&quot;Gas Turbulence Factor in a Micravigular Carbonate.&quot;</td>
</tr>
<tr>
<td>Geertsma¹⁴ (1974)</td>
<td>&quot;Coefficient of Inertial Resistance&quot;</td>
</tr>
</tbody>
</table>

**Fig. 1—Flow in a cylindrical conduit.**

**Fig. 2—Flow in an idealized pore.**

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solids has a continuous function between pressure drop and flow rate since there is no change in mechanism between low and high flow rates.

Mathematical Representation of High-Velocity Flow

When correlating the data for the high-velocity water flow through porous media, Forchheimer found that a relationship of the type presented by Eq. 1 described his data best. In some cases another velocity term seemed better:

\[
-\frac{dp}{dL} = \frac{\mu \nu}{k} + \beta \nu^2 + \gamma \nu^3. \quad (4)
\]

To use Eq. 1 for gases, it should be changed to the following form:

\[
\frac{p_1^2 - p_2^2}{2zRTGL} = \frac{1}{k} + \frac{\beta \mu}{\mu}. \quad (5)
\]

A plot of \((p_1^2 - p_2^2)/2zRTGL\) vs \(\mu/\mu\) results in a straight line for many cores. The slope of the straight line is \(\beta\); the inverse of the intercept with the \(y\) axis is the permeability, \(k\). In many cases, Eq. 5 deviates from a straight line. This deviation is attributed to two factors. One factor, which appears predominantly in the lower portion of the line for flow of gases, results from the slip effect. The slip could be interpreted as the bouncing of the gas molecules on the wall at low pressures when the mean free path of the molecules becomes the same order of magnitude as the pore diameter. In low-velocity gas flow, where the Darcy equation describes the flow behavior, Klinkenberg demonstrated that the slip effect can be taken into the Darcy equation by Eq. 6:

\[
k = k \left(1 + \frac{b}{\rho}\right) \quad (6)
\]

The other factor causing deviation from straight-line behavior results from the inadequacy of Eq. 1 to represent high-velocity gas flow. In other words, in addition to the \(\nu^2\) quadratic term, a third term of the form \(\gamma \nu^3\) is necessary, as noted by Forchheimer, for high-velocity-water flow. One might also represent this type of deviation by substituting an exponent \(n\) instead of 2 for \(\nu\) in the second term, as was done by Johnson and Taliaferro.

Permeability Limit at Zero or Low Velocity

From the analysis of the mechanism of energy consumption for fluid flowing in alternating size conduits and from Eq. 1, one would expect that the high-velocity term, \(\beta \nu^2\), should apply over the entire range of velocity. To illustrate that the data do not deny this statement, the data are plotted to demonstrate that a constant value of permeability in reality is a limiting value.

Fig. 4 shows data on a consolidated sandstone with nitrogen flowing at high pressure at both outlet and inlet, as plotted by Iffly according to an equation similar to Eq. 5. The straight line represents the prior concept of Darcy’s law and the curve represents measured values. Accepting that the curve becomes tangent to the line at zero flow rate demonstrates that the constant value of \(k\) applies at the limit of zero velocity. Forchheimer’s data on water describe a similar relationship. This behavior would be expected when the velocity is the variable that changes the flowline length (Figs. 2 and 3).

In normal measurements of core permeability (in millidarcies), the results are valid as the limiting value since \(\beta \nu^2\) is less than the experimental error when measuring the flow rate or pressure drop. When gases near atmospheric pressure are used to measure permeability, the slip may err more than when neglecting the velocity effect.

Nomenclature

The introduction of the \(\beta\) factor by Cornell and Katz as the turbulence factor, although defined as "extra fluid motion consuming extra energy," was unacceptable to many researchers. This concept overlooked the stages of progress for the growth of shear and tension components in laminar mode before random movement that is typical of turbulence. Equally valid objections are made now to the terms "non-Darcy" and "inertial" flow.

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**Fig. 3**—Idealized flow through alternating cross sections.

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**Fig. 4**—Pressure drop and flow rate data of Iffly illustrating that Darcy's law applies as a limit. Nitrogen flow through a sandstone of 21.4% porosity is 73.3 md.
Since there is a continuous function between pressure drop and flow rate for porous media with alternating cross sections in the flow path, the term "non-Darcy flow" is out of place because it implies two different flow mechanisms. The same can be said for using "Darcy flow" at low velocities and "inertia regime" for higher velocities; the inertia effects are always present. There is a spectrum of velocities in the various flow channels at a given flow rate. Any implication that there is one type of flow mechanism at low velocities and another regime at high velocities (short of true turbulence) disregards the nature of the mechanism by which the pressure-loss energy is consumed in porous media.

A simple concept is to characterize the flows as "low velocity" and "high velocity" to distinguish conditions for which $\beta \nu^2$ may be ignored and where it should be used. High velocity becomes the velocity for which neglecting $\beta \nu^2$ gives an error in computed pressure drop more than the unreliability of the permeability measurement.

Continued use of the term "Darcy flow" to characterize low velocity is recommended. This concept is ingrained in our past and implies that permeability is constant with a range of pressure drops.

For flow rates where neglecting $\beta \nu^2$ calculates significantly less pressure drop than would occur, the term "high-velocity flow" would be used to describe the condition.

**What to Call $\beta$?**

The term $\beta$ is used in flow equations for gas wells and even for high flow-rate oil wells. Objections to the turbulence factor and inertial coefficient are clear; they denote changes in flow regimes.
TABLE 3—ADDED FLOW DATA ON SANDSTONE CORES

<table>
<thead>
<tr>
<th>Rock</th>
<th>$k$ (md)</th>
<th>$\phi$ (fraction)</th>
<th>$\beta$ (1 ft)</th>
<th>$k\phi$</th>
<th>Fig. 6</th>
<th>Fig. 5 (k only)</th>
</tr>
</thead>
<tbody>
<tr>
<td>St. Peter</td>
<td>2,087</td>
<td>0.267</td>
<td>$3.17 \times 10^6$</td>
<td>557.</td>
<td>$2 \times 10^6$</td>
<td>$2.3 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>1,158</td>
<td>0.266</td>
<td>$7.3 \times 10^6$</td>
<td>306.</td>
<td>$4 \times 10^6$</td>
<td>$5 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>614</td>
<td>0.24</td>
<td>$1.59 \times 10^7$</td>
<td>147.</td>
<td>$9 \times 10^6$</td>
<td>$9 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>178</td>
<td>0.201</td>
<td>$4.94 \times 10^7$</td>
<td>36.</td>
<td>$4 \times 10^7$</td>
<td>$5 \times 10^7$</td>
</tr>
<tr>
<td>Jordan</td>
<td>248</td>
<td>0.20</td>
<td>$3.94 \times 10^6$</td>
<td>50.*</td>
<td>$3 \times 10^7$</td>
<td>$3 \times 10^7$</td>
</tr>
<tr>
<td></td>
<td>811</td>
<td>0.233</td>
<td>$7.33 \times 10^7$</td>
<td>189.*</td>
<td>$7 \times 10^6$</td>
<td>$7 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>1,232</td>
<td>0.214</td>
<td>$4.46 \times 10^7$</td>
<td>264.</td>
<td>$5 \times 10^6$</td>
<td>$5 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>63</td>
<td>0.217</td>
<td>$1.05 \times 10^8$</td>
<td>13.7</td>
<td>$1.2 \times 10^6$</td>
<td>$1.5 \times 10^6$</td>
</tr>
<tr>
<td>Lower</td>
<td>16</td>
<td>0.183</td>
<td>$2.24 \times 10^8$</td>
<td>2.9</td>
<td>$5.5 \times 10^6$</td>
<td>$8 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>1,049</td>
<td>0.255</td>
<td>$3.46 \times 10^8$</td>
<td>268.</td>
<td>$5 \times 10^5$</td>
<td>$5 \times 10^5$</td>
</tr>
<tr>
<td></td>
<td>117</td>
<td>0.211</td>
<td>$1.17 \times 10^8$</td>
<td>24.7</td>
<td>$6 \times 10^7$</td>
<td>$8 \times 10^7$</td>
</tr>
<tr>
<td>Franconia</td>
<td>40</td>
<td>0.142</td>
<td>$3.58 \times 10^8$</td>
<td>5.7</td>
<td>$3 \times 10^6$</td>
<td>$2.7 \times 10^6$</td>
</tr>
<tr>
<td>Mt. Simon</td>
<td>240</td>
<td>0.153</td>
<td>$9.38 \times 10^7$</td>
<td>37.</td>
<td>$3 \times 10^7$</td>
<td>$3 \times 10^7$</td>
</tr>
<tr>
<td></td>
<td>665</td>
<td>0.255</td>
<td>$1.24 \times 10^7$</td>
<td>170.</td>
<td>$8 \times 10^6$</td>
<td>$9 \times 10^6$</td>
</tr>
</tbody>
</table>

*Deviation from correlation larger than by previous data.

Data taken by Core Laboratories Inc. and transmitted to authors by Dan Slagle of Northern Natural Gas Co.

5, the $k$ vs $\beta$ chart. The correlation lines with porosity as the parameter definitely have slopes quite different from the consolidated data. The correlating curve in Fig. 5 is similar to that presented earlier without the porosity parameter. If porosity is to be a parameter, Fig. 6 is preferred over the porosity lines of Janick and Katz.16

Casse and Ramey note the absence of temperature effect on $\beta$ for a Berea sandstone. Wong took data on limestone cores and found curvature using Eq. 5; the cores contained connate liquid. Johnson and Taliaferro (Fig. 7) also obtained curvature for data on a dry limestone core. To represent these data, a second term is needed, such as $\gamma^2\beta^2$ shown in Eq. 4, to represent pressure drop. Then, $\beta$ could be the first velocity coefficient and $\gamma$ the second velocity coefficient.

New Data

Since this paper was written, more data were found in the files for 14 cores (Table 3). Twelve of the points scattered in Figs. 5 and 6 were not unlike previous data. Two points were beyond the normal band of point scatter. The data included six to eight data points each, were plotted in the manner of Cornell, and reported $k$ corrected for Klinkenberg effect.

Complexity of Reservoir Flow

The influence of reservoir heterogeneity, fractures, infill at the wellbore, and presence of liquids all discount the value of simple flow calculations with Eq. 1. However, such calculations are valuable when designing aquifer gas storage reservoirs. Also, well flow data may be used to obtain in-situ $\beta$ values. Such data provide some measure of reservoir deviations from the model using core data alone and allow evaluation of the reservoir condition.

Conclusions

A greater variety of high-velocity flow data on oil and gas reservoir rocks would be helpful. The range of predictions could be evaluated better and could be related to well flow characteristics.

We hope the nomenclature used here will be acceptable to both practicing engineers and fluid flow specialists and will help establish language and terms that will eliminate future misunderstandings.

The correlation of $\beta$ used during the past 20 years has been generally representative of the character of reservoir rocks. The correlations given here demonstrate that more data show only modest improvement in the correlation curve. Additional data show that half the data may fit the correlation, while individual cores may deviate tenfold.

![Fig. 7—Plot according to Eq. 7 of USBM data for a limestone core.](image-url)
Nomenclature

- \( a \) = a constant
- \( A \) = cross-sectional area, \( L^2 \)
- \( b \) = slip coefficient, \( m/Lt^2 \)
- \( g_e \) = unit conversion factor
- \( G \) = gas gravity, \( M/29 \)
- \( k \) = absolute permeability, \( L^2 \)
- \( k_p \) = apparent permeability, \( L^2 \)
- \( L \) = length, \( L \)
- \( m \) = a correlating constant
- \( M \) = molecular weight
- \( p \) = pressure, \( m/Lt^2 \)
- \( p_i \) = inlet pressure, \( m/Lt^2 \)
- \( p_o \) = outlet pressure, \( m/Lt^2 \)
- \( \bar{p} \) = arithmetic average pressure, \( m/Lt^2 \)
- \( R \) = universal gas constant
- \( T \) = absolute temperature
- \( u \) = mass velocity, \( m/Lt \)
- \( v_x \) = velocity in the \( x \) direction, \( L/t \)
- \( v_y \) = velocity in the \( y \) direction, \( L/t \)
- \( x \) = coordinate
- \( y \) = coordinate
- \( \varepsilon \) = compressibility factor
- \( \beta \) = first velocity coefficient, \( 1/L \)
- \( \gamma \) = second velocity coefficient, \( L/tm \)
- \( \rho \) = density, \( m/L^3 \)
- \( \tau \) = longitudinal shear stress, \( m/Lt^2 \)
- \( \sigma \) = longitudinal tension stress, \( m/Lt^2 \)
- \( \phi \) = porosity

Acknowledgments

The assistance of C. S. Yih, U. of Michigan Mechanics Dept., in clarifying and understanding pressure losses during flow through porous media is appreciated. The correlations of \( \beta \) in Table 3 were performed by J. Azuonye Ironkwe.

References
