A Quadratic Cumulative Production Model for the Material Balance of Abnormally-Pressured Gas Reservoirs

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Executive Summary — "p/z-G_p^2" Relation (1/4)

The rigorous relation for the material balance of a dry gas reservoir system is given by Fetkovich, *et al.* as:

\[
\frac{p}{z} \left[ 1 - c_e(p)(p_i - p) \right] = \frac{p_i}{z_i} - \frac{p_i}{z_i} \frac{1}{G} \left[ G_p - G_{inj} + W_p R_{sw} + \frac{5.615}{B_g} (W_p B_w - W_{inj} B_w - W_e) \right]
\]

Eliminating the water influx, water production/injection, and gas injection terms; defining \( \omega G_p = c_e(p)(p_i - p) \) and assuming that \( \omega G_p < 1 \), then rearranging gives the following result:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \left( \frac{1}{G} - \omega \right) G_p - \frac{\omega}{G} G_p^2 \right]
\]
Simulated Dry Gas Reservoir Case — Abnormal Pressure:

- Volumetric, dry gas reservoir — with $c_f(p)$ (from Fetkovich).
- Note extrapolation to the "apparent" gas-in-place (previous approaches).
- Note comparison of data and the new "Quadratic Cumulative Production" model.
Executive Summary — "\( p/z - G_p^2 \)" Relation (3/4)

- Anderson L Reservoir Case — Abnormal Pressure:
  - South Texas (USA) gas reservoir with abnormal pressure.
  - Benchmark literature case.
  - Note performance of the new "Quadratic Cumulative Production" model.
Presentation Outline

- Executive Summary
- Objectives and Rationale
  - Rigorous technique for abnormal pressure analysis.
- Development of the $p/z-G_p^2$ model
  - Derivation from the rigorous material balance.
- Validation — Field Examples
  - Case 1 — Dry gas simulation ($c_f(p)$ from Fetkovich).
  - Case 3 — Anderson L (South Texas, USA).
- Demonstration (MS Excel — Anderson L case)
- Summary
- Recommendations for Future Work
Objectives and Rationale

- **Objectives:**
  - Develop a rigorous functional form (i.e., a model) for the $p/z$ vs. $G_p$ behavior demonstrated by a typical abnormally pressured gas reservoir.
  - Develop a sequence of plotting functions for the analysis of $p/z$—$G_p$ data (multiple plots).
  - Provide an exhaustive validation of this new model using field data.

- **Rationale:** The analysis of $p/z$—$G_p$ data for abnormally pressured gas reservoirs has evolved from empirical models and idealized assumptions (e.g., $c_f(p) =$ constant). We would like to establish a rigorous approach — one where any approximation is based on the observation of some characteristic behavior, not simply a mathematical/graphical convenience.
Development of the $p/z-G_p^2$ model

- **Concept:**
  - Use the rigorous material balance relation given by Fetkovich, et al. for the case of a reservoir where $c_f(p)$ is NOT presumed constant.
  - Use some observed limiting behavior to construct a semi-analytical relation for $p/z—G_p$ behavior.

- **Implementation:**
  - Develop and apply a series of data plotting functions which clearly exhibit unique behavior relative to the $p/z—G_p$ data.
  - Use a "multiplot" approach which is based on the dynamic updating of the model solution on each data plot.
  - Develop a "dimensionless" type curve approach that can be used to validate the model and estimate $G$. 
\(p/z\)-Cumulative Model:  

The rigorous relation for the material balance of a dry gas reservoir system is given by Fetkovich, *et al.* as:

\[
\frac{p}{z} \left[1 - \bar{c}_e(p)(p_i - p)\right] = \frac{p_i}{z_i} - \frac{p_i}{z_i} \frac{1}{G} \left[G_p - G_{inj} + W_p R_{sw} + \frac{5.615}{B_g} (W_p B_w - W_{inj} B_w - W_e)\right]
\]

Eliminating the water influx, water production/injection, and gas injection terms, then rearranging gives the following definition:

\[
p/z \equiv \frac{p_i/z_i}{(1 - \omega G_p)} \left[1 - \frac{G_p}{G}\right] \text{[where } \omega G_p \equiv \bar{c}_e(p)(p_i - p)]
\]
Considering the condition where:

$$\omega_D \equiv \omega G_p \leq 1$$

Then we can use a geometric series to represent the $$\omega_D$$ term in the governing material balance. The appropriate geometric series is given by:

$$1/[1 - x] \approx 1 + x + x^2 + x^3 + ... \quad (-1 < x < 1)$$

or, for our problem, we have:

$$\frac{1}{(1 - \omega G_p)} \approx 1 + \omega G_p \quad (-1 < \omega G_p < 1)$$

Substituting this result into the material balance relation, we obtain:

$$\frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \left( \frac{1}{G} - \omega \right) G_p - \frac{\omega}{G} G_p^2 \right]$$
A more convenient form of the \( p/z \)-cumulative model is:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} - \alpha G_p - \beta G^2_p
\]

\[
\alpha \equiv \left( \frac{1}{G} - \omega \right) \frac{p_i}{z_i} \quad \beta \equiv \frac{\omega}{G} \frac{p_i}{z_i}
\]

We note that these parameters presume that \( \omega \) is constant. Presuming that \( \omega \) is linear with \( G_p \), we can derive the following form:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} - \left( \frac{1}{G} - a \right) \frac{p_i}{z_i} G_p - \left( \frac{a}{G} + b \right) \frac{p_i}{z_i} G^2_p + \frac{b}{G} \frac{p_i}{z_i} G^3_p
\]

where \( \omega \equiv a - b G_p \)

Obviously, one of our objectives will be the study of the behavior of \( \omega \) vs. \( G_p \) (based on a prescribed value of \( G \)).
\( \omega-G_p \) Performance (Case 1) (1/2)

a. Case 1: Simulated Performance Case — Plot of \( \omega \) versus \( G_p \) (requires an estimate of gas-in-place). Note the apparent linear trend of the data. Recall that \( \omega G_p = c_\omega (p_p - p) \).

b. Case 1: Simulated Performance Case — Plot of \( p/z \) versus \( G_p \). The constant and linear \( \omega \) trends match well with the data — essentially a confirmation of both models.

- Simulated Dry Gas Reservoir Case — Abnormal Pressure:
  - A linear trend of \( \omega \) vs. \( G_p \) is reasonable and should yield an accurate model.
  - \( \omega \) is approximated by a constant value within the trend.
  - A physical definition of \( \omega \) is elusive — \( \omega G_p = c_\omega (p_p - p) \) implies that \( \omega \) has units of 1/volume, which suggests \( \omega \) is a scaling variable for \( G \).
a. Case 3: Anderson L Reservoir Case (South Texas, USA) — Plot of $\omega$ versus $G_p$ (requires an estimate of gas-in-place). Some data scatter exists, but a linear trend is evident (recall that $\omega G_p = c_e(p)(G_p)$).

b. Case 3: Anderson L Reservoir Case (South Texas, USA) — Plot of $p/z$ versus $G_p$. Both models are in strong agreement.

Anderson L Reservoir Case — Abnormal Pressure:

- Field data will exhibit some scatter, method is relatively tolerant of data scatter.
- Constant $\omega$ approximation is based on the "best fit" of several data functions.
- The linear approximation for $\omega$ is reasonable (should favor later data).
Validation of the $p/z-G_p^2$ model: Orientation

● Methodology:
  - All analyses are "dynamically" linked in a spreadsheet program (MS Excel). Therefore, all analyses are consistent — should note that some functions/plots perform better than others — but the model results are the same for every analysis plot.

● Validation: Illustrative Analyses
  - $p/z-G_p^2$ plotting functions — based on the proposed material balance model.
  - $\omega-G_p$ performance plots — used to calibrate analysis.
  - Gan analysis — presumes 2-straight line trends on a $p/z-G_p$ plot for an abnormally pressured reservoir.
  - $p_D-G_{pD}$ type curve approach — use $p/z-G_p^2$ material balance model to develop type curve solution — this approach is most useful for data validation.
Case 1 — Simulated Dry Gas Reservoir

Quadratic $G_p$ Material Balance Relation for Abnormally Pressured Gas Reservoirs

Presentation Plot: $\Delta(p/z) - G_p$ Format

\[ \Delta(p/z) = \left[ \frac{p_i - p}{z_i} \right] \] vs. $G_p$

\[ \int_0^{G_p} \Delta(p/z) \, dG_p \] vs. $G_p$

\[ \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) \, dG_p \] vs. $G_p$

\[ \frac{1}{G_p} \left[ \Delta(p/z) - \int_0^{G_p} \Delta(p/z) \, dG_p \right] \] vs. $G_p$

A New $p/z-G_p^2$ Material Balance for Abnormally-Pressured Reservoirs
$\omega - G_p$ Plotting Functions: Case 1


c. Case 1: Simulated Performance Case — Plot of $\omega$ versus $G_p$ (requires estimate of $G$).

Simulated Dry Gas Reservoir Case — Abnormal Pressure:

- Summary $p/z - G_p$ plot for $\omega =$constant and $\omega =$linear cases.
- Good comparison of trends, $\omega =$linear trend appears slightly conservative as it emerges from data trend — but both solutions appear to yield same $G$ estimate.
Gan-Blasingame Analysis (2001): Case 1 (4/5)

a. Case 1: Simulated Performance Case — Gan Plot 1 $c_e(p)(p_r-p)$ versus $(p/z)/(p_i/z_i)$ (requires est. of $G$).

b. Case 1: Simulated Performance Case — Gan Plot 2 $(p/z)/(p_i/z_i)$ versus $(G_p/G)$ (requires est. of $G$).

- Gan-Blasingame Analysis:
  - Approach considers the "match" of the $c_e(p)(p_r-p) = (p/z)/(p_i/z_i)$ data and "type curves."
  - Assumes that both abnormal and normal pressure $p/z$ trends exist.
  - Straight-line extrapolation of the "normal" $p/z$ trend used for $G$.

A New $p/z\cdot G_p^2$ Material Balance for Abnormally-Pressured Reservoirs
**$p_{D} - G_{pD}$ Type Curve Approach: Case 1**

The $p_{D} - G_{pD}$ type curve analysis is based on the $p/z-G_{p}^2$ model. $p_{D} = ([p/z] - (p/z)) / (p/z)$ and $G_{pD} = G_{p} / G$ — both $p_{D}$ and $p_{Di}$ functions are plotted.

**Case 1: Simulated Performance Case** — Type curve analysis of $(p/z)-G_{p}$ data, this case is "force matched" to the same results as all of the other plotting functions.

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**A New $p/z-G_{p}^2$ Material Balance for Abnormally-Pressured Reservoirs**
**A New p/z-\(G_p^2\) Material Balance for Abnormally-Pressured Reservoirs**

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**Case 3 — Anderson L (SPE 02938)**

**Quadratic \(G_p\) Material Balance Relation for Abnormally Pressured Gas Reservoirs**

**Presentation Plot:** \(\Delta(p/z) - G_p\) Format

**Plotting Fcns:**

- **a.** \(\Delta(p/z) = \left[ \frac{p_i - p}{z_i - z} \right] \) vs. \(G_p\)
- **b.** \(\frac{1}{G_p} \Delta(p/z) vs. G_p\)
- **c.** \(\frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p\) vs. \(G_p\)
- **d.** \(\int_0^{G_p} \Delta(p/z) dG_p\) vs. \(G_p\)
- **e.** \(\Delta(p/z) - \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p\) vs. \(G_p\)
- **f.** \(\frac{1}{G_p} \left[ \Delta(p/z) - \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p \right]\) vs. \(G_p\)
ω-\(G_p\) Plotting Functions: Case 3

a. Case 3: Anderson L (South Texas) — Plot of \(c_e(p)(p_f-p)\) versus \(G_p\) (requires estimate of \(G\)).

b. Case 3: Anderson L (South Texas) — Plot of \(1/c_e(p)(p_f-p)\) versus \(G_p\) (requires estimate of \(G\)).

c. Case 3: Anderson L (South Texas) — Plot of \(ω\) versus \(G_p\) (requires estimate of \(G\)).

d. Case 3: Anderson L (South Texas) — Plot of \(ω\) versus \(G_p/G\) (requires estimate of \(G\)).
Case 3 — Anderson L Reservoir (South Texas (USA))

- Summary $p/z - G_p$ plot for $\omega =$ constant and $\omega =$ linear cases.
- Good comparison of trends, $\omega =$ constant and $\omega =$ linear cases in good agreement.
- Data trend is very consistent.
a. Case 3: Anderson L Reservoir — Gan Plot 1 $c_e(p)(p_i-p)$ versus $(p/z)/(p_i/z_i)$ (requires est. of $G$).

b. Case 3: Anderson L Reservoir — Gan Plot 2 $(p/z)/(p_i/z_i)$ versus $(G_p/G)$ (requires est. of $G$).

c. Case 3: Anderson L Reservoir — Gan Plot 3 $(p/z)$ versus $G_p$ (results plot).

Gan-Blasingame Analysis:

- We note an excellent "match" of the $c_e(p)(p_i-p) - (p/z)/(p_i/z_i)$ data and the "type curves."
- Both the abnormal and normal pressure $p/z$ trends appear accurate and consistent.
- Straight-line extrapolation of the "normal" $p/z$ trend used for $G$. 

A New $p/z-G_p^2$ Material Balance for Abnormally-Pressured Reservoirs
Case 3 — Anderson L Reservoir (South Texas (USA))

- $p_D-G_{pD}$ type curve solution matched using field data.
- Note the "tail" in the $p_D$ trend for small values of $G_{pD}$ (common field data event).
- "Force matched" to the same results as each of the other plotting functions.
Example Analysis Using MS Excel: Case 3

- Case 3 — Anderson L (South Texas (USA))
  - Literature standard case.
  - A 3-well reservoir, delimited by faults.
  - Good quality data.
  - Evidence of overpressure from static pressure tests.

- Analysis: (Implemented using MS Excel)
  - $p/z-G_p^2$ plotting functions.
  - $\omega-G_p$ performance plots.
  - Gan analysis (2-straight line trends on a $p/z-G_p$ plot).
  - $p_D-G_{pD}$ type curve approach.
Summary: (1/3)

- Developed a new \( p/z-G_p^2 \) material balance model for the analysis of abnormally pressured gas reservoirs:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \left( \frac{1}{G} - \omega \right) G_p - \frac{\omega}{G} G_p^2 \right]
\]

where:

\[
\omega \equiv \frac{1}{G_p} \bar{c}_e(p)(p_i - p)
\]

The \( \omega \)-function is presumed (based on graphical comparisons) to be either constant, or approximately linear with \( G_p \). For the \( \omega=\text{constant} \) case, we have:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} - \alpha G_p - \beta G_p^2
\]

\[
\alpha \equiv \left( \frac{1}{G} - \omega \right) \frac{p_i}{z_i} \quad \beta \equiv \frac{\omega}{G} \frac{p_i}{z_i}
\]
Summary:

- Base relation: $p/z - G_p^2$ form of the gas material balance

\[
\frac{p}{z} \approx \frac{p_i}{z_i} - \alpha G_p - \beta G_p^2 \\
\alpha \equiv \left( \frac{1}{G} - \omega \right) \frac{p_i}{z_i} \\
\beta \equiv \frac{\omega}{G} \frac{p_i}{z_i}
\]

a. **Plotting Function 1:**
   (quadratic)

\[
\Delta(p/z) = \left[ \frac{p_i}{z_i} - \frac{p}{z} \right] \text{ vs. } G_p
\]

d. **Plotting Function 4:**
   (linear)

\[
\frac{1}{G_p^2} \int_0^{G_p} \Delta(p/z) dG_p \text{ vs. } G_p
\]

e. **Plotting Function 5:**
   (quadratic)

\[
\Delta(p/z) - \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p \text{ vs. } G_p
\]

b. **Plotting Function 2:**
   (linear)

\[
\frac{1}{G_p} \Delta(p/z) \text{ vs. } G_p
\]

f. **Plotting Function 6:**
   (linear)

\[
\frac{1}{G_p} \left[ \Delta(p/z) - \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p \right] \text{ vs. } G_p
\]
The plotting functions developed in this work have been validated as tools for the analysis reservoir performance data from abnormally pressured gas reservoirs. Although the straight-line functions ($PF_2$, $PF_4$, and $PF_6$) could be used independently, but we recommend a combined/simultaneous analysis.

The $\omega-G_p$ plots are useful for checking data consistency and for guiding the selection of the $\omega$-value. These plots represent a vivid and dynamic visual balance of all of the other analyses.

The Gan analysis sequence is also useful for orienting the overall analysis — particularly the $c_e(p)(p_i-p)$ versus $(p/z)/(p_i/z_i)$ plot.

The $p_D-G_{pD}$ type curve is useful for orientation — particularly for estimating the $\omega$ or ($\omega_D$) value.
Recommendations for Future Work:

- Consider the extension of this methodology for cases of external drive energy (e.g., water influx, gas injection, etc.).
- Continue the validation of this approach by applying the methodology to additional field cases.
- Implementation into a stand alone software.
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End of Presentation

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