The jealous are troublesome to others,  
but a torment to themselves.  
— William Penn (1693)

**Topic:** Single-Phase, Steady-State Flow in Porous Media

**Objectives:** (things you should know and/or be able to do)

- Be familiar with and be able to develop the single-phase, steady-state flow relations for the laminar flow of gases and compressible liquids in terms of pressure, pressure-squared, and pseudopressure, as appropriate.

- **Liquid Flow:** (Constant Fluid Properties)

  **Linear Flow**

  **Differential Form:**
  
  \[ q_{sc} = c_x \frac{kA}{B\mu} \frac{dp}{dx} \]

  **Radial Flow**

  \[ q_{sc} = c_r \frac{kh}{B\mu} \frac{dr}{dr} \]

  **Difference Form:** (at \( r_e \) at \( r_r \))
  
  \[ q_{sc} = c_x \frac{kA}{B\mu} \frac{p_r - p_w}{x_e} \]

  \[ q_{sc} = c_r \frac{kh}{B\mu} \ln\left(\frac{r_r}{r_w}\right) \]

  **Difference Form:** (any \( x \) or \( r \))
  
  \[ q_{sc} = c_x \frac{kA}{B\mu} \frac{p_r - p_w}{x} \]

  \[ q_{sc} = c_r \frac{kh}{B\mu} \ln\left(\frac{r_r}{r_w}\right) \]

  **Pressure Form:** (any \( x \) or \( r \))
  
  \[ p_x = p_w + \frac{1}{c_x} q_{sc} \frac{B\mu}{kA} x \]

  \[ p_r = p_w + \frac{1}{c_r} q_{sc} \frac{B\mu}{kh} \ln\left(\frac{r_r}{r_w}\right) \]

- **Gas Flow:** (Variable Fluid Properties)

  **Pseudopressure Relations**

  **Linear Flow**

  **Differential Form:**
  
  \[ q_{sc} = c_x \frac{kA}{Bg_i\mu_i} \frac{dp}{dx} \]

  **Radial Flow**

  \[ q_{sc} = c_r \frac{kh}{Bg_i\mu_i} \frac{dr}{dr} \]

  **Difference Form:**
  
  \[ q_{sc} = c_x \frac{kA}{Bg_i\mu_i} \frac{p_r(p_e) - p_p(p_w)}{x_e} \]

  \[ q_{sc} = c_r \frac{kh}{Bg_i\mu_i} \frac{p_r(p_e) - p_p(p_w)}{\ln\left(\frac{r_r}{r_w}\right)} \]

  **Pressure Form:** (Linear Flow)
  
  \[ p_r(p_x) = p_r(p_w) + \frac{1}{c_x} q_{sc} \frac{Bg_i\mu_i}{kA} x \]

  **Pressure Form:** (Radial Flow)
  
  \[ p_r(p_r) = p_r(p_w) + \frac{1}{c_r} q_{sc} \frac{Bg_i\mu_i}{kh} \ln\left(\frac{r_r}{r_w}\right) \]

- **Pressure-Squared Relations**

  **Rate Form:** (Linear Flow)
  
  \[ q_{sc} = c_x \frac{kA}{x_e} \left[ \frac{T_s c z_{sc}}{T P_{sc}} (\mu z)_c \right] \frac{1}{2} \left( p_e^2 - p_w^2 \right) \]

  **Rate Form:** (Radial Flow)
  
  \[ q_{sc} = c_r \frac{kh}{\ln\left(\frac{r_r}{r_w}\right)} \left[ \frac{T_s c z_{sc}}{T P_{sc}} (\mu z)_c \right] \frac{1}{2} \left( p_e^2 - p_w^2 \right) \]

  **Note:** For the pressure-squared case the \((\mu z)_c\) term is evaluated at some "average" pressure.

**Table of Units Conversions:** (for the equations given above)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Darcy Units</th>
<th>Field Units</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_x )</td>
<td>1</td>
<td>1.127x10^{-3}</td>
<td>8.527x10^{-5}</td>
</tr>
<tr>
<td>( c_r )</td>
<td>( 2\pi )</td>
<td>2\pi1.127x10^{-3}</td>
<td>2\pi8.527x10^{-5}</td>
</tr>
<tr>
<td></td>
<td>or</td>
<td>or</td>
<td>or</td>
</tr>
<tr>
<td></td>
<td>7.081x10^{-3}</td>
<td>5.358x10^{-4}</td>
<td></td>
</tr>
</tbody>
</table>
Petroleum Engineering 324
Well Performance
Single-Phase, Steady-State Flow in Porous Media

Objectives: (continued)

- Be familiar with and be able to derive the "skin factor" for a well where a near-well "skin zone" is used to model damage or stimulation in a radial flow system. Be familiar with the theoretical and practical limitations of this approach.
- Be able to derive the pressure-squared forms of the "gas deliverability" equations using steady-state, laminar flow theory. Also, be able to establish the conditions of validity for the pressure-squared forms of the gas deliverability equations.
- Be familiar with and be able to develop flow relations for gases and compressible liquids, in terms of pressure, pressure-squared, and pseudopressure using the non-laminar Forchheimer flow relation, which is quadratic in terms of velocity.

Lecture Outline:

- Linear and radial steady-state laminar flow relations for liquid flow
  - Horizontal radial flow
- Linear and radial steady-state laminar flow relations for horizontal gas flow
  - Development of "pressure" equations
  - Development of "pressure-squared" equations
  - Development of "pseudopressure" equations
  - Ranges of applicability and limitations
- Derivation of the pressure-squared forms of the "gas deliverability" equations
  - Empirical evidence--log-log plot of flow-after-flow test data
  - Derivation of the steady-state gas flow relations
  - Relation between empirical behavior and steady-state flow equations
  - Conditions for validity of the pressure-squared concept ($\mu$£=constant)
- Derivation of the "skin factor" for a well where a near-well "skin zone" is used to model damage or stimulation in a radial flow system.
  - Conceptual model
  - Physical representation of the "effective" wellbore radius concept
  - Limitations
- Developments using the Forchheimer, non-laminar flow relation.
  - Liquid flow relations
  - Gas flow relations
  - Plotting functions
- Various applications of the non-laminar flow equations (discussion)
  - Transient flow (isochronal tests)
  - Pseudosteady-state flow (flow-after-flow tests)

Reading Assignment:

- Review attached notes
  - Derivation of the Steady-State Flow Equations (Horizontal Linear and Radial Flow Cases).
  - Derivation of the Pressure-Squared ($p^2$) Forms of the "Gas Deliverability" Equations.
  - Derivation of the Skin Factor for a Zone of Altered Permeability Near the Well (Radial Flow Case).
  - Derivation of the Effective Wellbore Concept Concept for the Near Well Skin Factor.
Reading Assignment: (continued)

- Text Reading:

- Review attached papers—Generalized Flow Behavior in Porous Media

- Review attached papers—Non-Laminar/Non-Darcy Flow
Derivation of the Steady-State Flow Equations
(Horizontal Linear and Radial Flow Cases)

- Liquid Flow (Constant Properties)
- Gas Flow (Gas Properties = f(p))
  - Pseudopressure ($p_p(p)$) Formulation
  - Pressure-Squared ($p^2$) Formulation

(from Petroleum Engineering 412 Course Notes -- 1997)
Steady-State Flow / Fundamental Relations

Linear Flow System

Radial Flow System

Note that both systems assume that flow is convergent towards the well.

Governing Equation:

\[ q_{sc} = \frac{c_x KA}{8u} \frac{dp}{dx} \]  \( (1) \)

Table of Conversion Factors

<table>
<thead>
<tr>
<th>Factor</th>
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<th>SI Units</th>
</tr>
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<tbody>
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</tr>
<tr>
<td>( c_r )</td>
<td>( 2\pi )</td>
<td>( 2\pi \times 1.127 \times 10^{-3} )</td>
<td>( 2\pi \times 8.527 \times 10^{-5} ) or ( 7.081 \times 10^{-3} ) or ( 5.358 \times 10^{-4} )</td>
</tr>
</tbody>
</table>
Linear Flow System

Separating Eq. 1
\[ q_{sc} \, dx = c_x \, \frac{KA}{Bu} \, dp \quad (3) \]

Assuming liquid flow (i.e., \( Bu = \text{constant} \)), Eqs. 3 and 4 integrate as follows

\[ q_{sc} \int_{0}^{X_e} dx = c_x \, \frac{KA}{Bu} \int_{P_w}^{P_e} dp \]

Reducing
\[ q_{sc} (X_e - 0) = c_x \, \frac{KA}{Bu} (P_e - P_w) \]

Solving for \( q_{sc} \)
\[ q_{sc} = c_x \, \frac{KA}{Bu} \, \frac{(P_e - P_w)}{X_e} \quad (5) \]

Similarly, integrating Eqs from 0 to \( X_e \) gives:

\[ q_{sc} = c_x \, \frac{KA}{Bu} \, \frac{(P_e - P_w)}{X} \quad (6) \]

Solving Eq. 7 for \( P_e \) gives
\[ P_e = P_w + \frac{1}{c_x} \, q_{sc} \, \frac{Bu}{KA} \quad (7) \]

Radial Flow System

Separating Eq. 2
\[ q_{sc} \, \frac{1}{r} \, dr = c_r \, \frac{kh}{Bu} \, dp \quad (4) \]

Integrating Eqs. 4 and 4 from \( r_w \) to \( r \) gives:

\[ q_{sc} \int_{r_w}^{r} \frac{1}{r} \, dr = c_r \, \frac{kh}{Bu} \int_{P_w}^{P_e} dp \]

Reducing
\[ q_{sc} \ln(r/r_w) = c_r \, \frac{kh}{Bu} (P_e - P_w) \]

Solving for \( q_{sc} \)
\[ q_{sc} = c_r \, \frac{kh}{Bu} \, \frac{(P_e - P_w)}{\ln(r/r_w)} \quad (8) \]

Similarly, integrating Eq. 4 from \( r_w \) to \( r \) gives:

\[ q_{sc} = c_r \, \frac{kh}{Bu} \, \frac{(P_e - P_w)}{\ln(r/r_w)} \]

Solving Eq. 8 for \( r \) gives
\[ r = r_w + \frac{1}{c_r} \, q_{sc} \, \frac{Bu}{kh} \ln(r/r_w) \quad (9) \]

We will next develop the gas flow equations—which requires the definition of the gas formation volume factor, \( B_g \), and the definition of gas density. These are

\[ B_g = \frac{q_{sc}}{q_g} \quad (11) \]
\[ q_g = \frac{P_e}{Z} \, \frac{M}{RT} \quad (12) \]
\[ B_g = \frac{q_{sc}}{q_g} \, \frac{TP}{p} \, \frac{T_{sc}}{Z_{sc}} \quad (13) \]
Linear Flow System

Recalling Eq. 9 we have

$$P_x = P_w + \frac{1}{C_x} \frac{q_{sc}}{kA} x \quad (9)$$

which is of the form:

$$y = b + mx$$

where

$$y = P_x; \ x = x; \ m = \frac{1}{C_x} \frac{q_{sc}}{kA}$$

$$b = P_w$$

---

Radial Flow System

Recalling Eq. 10 we have

$$P = P_w + \frac{1}{Cr} \frac{q_{sc}}{kh} \ln(r/r_w) \quad (10)$$

which is of the form:

$$y = b + mx$$

where

$$y = P; \ x = \ln(r); \ m = \frac{1}{Cr} \frac{q_{sc}}{kh}$$

$$b = P_w$$

---

Cartesian Plot:

```
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{cartesian_plot}
\caption{Cartesian Plot for Linear Flow System}
\end{figure}
```

```
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{cartesian_plot}
\caption{Cartesian Plot for Radial Flow System}
\end{figure}
```

---

Semilog Plot:

```
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{semilog_plot}
\caption{Semilog Plot for Linear Flow System}
\end{figure}
```

```
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{semilog_plot}
\caption{Semilog Plot for Radial Flow System}
\end{figure}
```
Development of Gas Flow Equations

Linear Gas Flow System

Multiplying through Eq. 3 by \( n_i \) gives

\[
\frac{\partial \xi_i}{\partial t} + \frac{\partial \xi_i}{\partial x} = S_i \left( \frac{\partial \xi_i}{\partial x} \right) \frac{p_{\text{ii}}}{p_{\text{i}}} \quad (16)
\]

Substituting the definition of \( S_i \) (Eq. 13) into the right-hand-side of Eq. 14 gives

\[
\frac{\partial \xi_i}{\partial t} + \frac{\partial \xi_i}{\partial x} = c_i \left( \frac{\partial \xi_i}{\partial x} \right) \frac{p_{\text{ii}}}{p_{\text{i}}} \quad (17)
\]

Integrating Eq. 16 gives us

\[
\frac{\partial \xi_i}{\partial t} + \int_{x_0}^{x} \frac{\partial \xi_i}{\partial x} dp = c_i \int_{p_{\text{ii}}}^{p_i} \frac{p_{\text{ii}}}{p_{\text{i}}} dp \quad (18)
\]

\[
\text{Isolating the general integral term in both Eqs. 18 and 19 (i.e., the plus integral term), we have}
\]

\[
I = \frac{\xi_i}{p_{\text{i}}} \int_{p_{\text{ii}}}^{p_i} \frac{p_{\text{ii}}}{p_{\text{i}}} dp \quad (20)
\]

Expanding

\[
I = c_i \int_{p_{\text{ii}}}^{p_i} \frac{p_{\text{ii}}}{p_{\text{i}}} dp - c_i \int_{p_{\text{ii}}}^{p_{\text{base}}} \frac{p_{\text{ii}}}{p_{\text{i}}} dp \quad (21)
\]

or

\[
I = \frac{p_i(p_i)}{p_{\text{base}}} - \frac{p_i(p_{\text{base}})}{p_{\text{i}}} \quad (22)
\]

where \( \frac{p_i(p)}{p_{\text{i}}} = \xi_i \int_{p}^{p_i} \frac{p}{p_{\text{i}}} dp \quad (23) \)
Note that the function \( \eta(p) \) is called "pseudo-pressure" and by our definition \( \eta(p) \) has the units of pressure. Our definition of \( \eta(p) \) is important because using this formulation \( \eta(p) \) gives us liquid equivalent gas flow equations. Simply put, the gas and liquid flow relations are identical—provided that \( \eta(p) \) is used for \( p \) in the liquid flow equations.

Continuing...

Linear Gas Flow Relations
Completing the integration of Eq. 18 using the \( \eta(p) \) result gives

\[
\frac{q_{sc} \eta \mu_l}{x_e} = C_x \frac{K}{A} [\eta(p_e) - \eta(p_w)]
\]

Solving for \( q_{sc} \)

\[
q_{sc} = C_x \frac{K}{A} \frac{\eta \mu_l}{x_e} \eta(p_e) - \eta(p_w)
\]

Integrating Eq. 18 from \( 0 \) to \( x \), and solving for \( \eta(p) \) gives

\[
\eta(p) = \eta(p_w) + \frac{1}{C_x} \frac{q_{sc} \eta \mu_l}{K} x
\]

Recalling the liquid flow result, Eq. 9, we have

\[
x = x_w + \frac{1}{C_x} \frac{q_{sc} \eta \mu_l}{K} x
\]

Radial Gas Flow Relations
Completing the integration of Eq. 19 using the \( \eta(p) \) result gives

\[
\frac{q_{sc} \eta \mu_l}{x_e} \ln(r/\eta_kw) = C_r \frac{K}{h} [\eta(p_e) - \eta(p_w)]
\]

Solving for \( q_{sc} \)

\[
q_{sc} = C_r \frac{K}{h} \frac{\eta \mu_l}{x_e} \ln(r/\eta_kw)
\]

Integrating Eq. 19 from \( r_w \) to \( r \), and solving for \( \eta(p) \) gives

\[
\eta(p) = \eta(p_w) + \frac{1}{C_r} \frac{q_{sc} \eta \mu_l}{K} \ln(r/\eta_kw)
\]

Recalling the liquid flow result, Eq. 10, we have

\[
x = x_w + \frac{1}{C_r} \frac{q_{sc} \eta \mu_l}{K} \ln(r/\eta_kw)
\]
We can derive the pressure-squared ($p^2$) formulations for linear and radial directly from the pseudopressure ($P_p(p)$) relations. Recalling the definition of $P_p(p)$ in Eq. 23, we have:

$$P_p(p) = \frac{m_i \ddot{z}_i}{p_i} \int_{P_{base}}^{p} \frac{p}{m^2} dp$$  \hspace{1cm} (23)

Assuming $m^2$ = constant, Eq. 23 becomes:

$$P_p(p) = \frac{m_i \ddot{z}_i}{p_i} \frac{1}{(m^2)c} \int_{P_{base}}^{p} p dp$$  \hspace{1cm} (32)

Completing the integration in Eq. 32, we obtain:

$$P_p(p) = \frac{m_i \ddot{z}_i}{p_i} \frac{1}{(m^2)c} \frac{1}{Z} (p^2 - P_{base}^2)$$  \hspace{1cm} (33)

Dividing through Eq. 33 by $E_i/m_i$ gives:

$$\frac{1}{E_i/m_i} P_p(p) = \frac{1}{m_i} \left[ \frac{T_{sc}}{T} \frac{E_{sc}}{E_{sc}} \right] \frac{m_i \ddot{z}_i}{p_i} \frac{1}{(m^2)c} \frac{1}{Z} (p^2 - P_{base}^2)$$  \hspace{1cm} (34)

or

$$\frac{1}{E_i/m_i} P_p(p) = \left[ \frac{T_{sc}}{T} \frac{E_{sc}}{E_{sc}} \right] \frac{1}{(m^2)c} \frac{1}{Z} (p^2 - P_{base}^2)$$  \hspace{1cm} (34)

Writing a difference expression using Eq. 34, gives us:

$$\frac{1}{E_i/m_i} [P_p(p_2) - P_p(p_1)] = \left[ \frac{T_{sc}}{T} \frac{E_{sc}}{E_{sc}} \right] \frac{1}{(m^2)c} \frac{1}{Z} (p_2^2 - p_1^2)$$  \hspace{1cm} (35)

Substituting Eq. 35 into Eqs. 26 (linear flow) and 27 (radial flow) give the proper $p^2$ formulations.
Linear $p^2$ Gas Flow Equations

Substituting Eq. 35 into Eq. 26 gives

$$ q_{sc} = C_x \frac{KA}{Xe} \left[ \frac{T_{sc} \cdot Z_{sc}}{T \cdot Z_c} \right] \frac{1}{(uZ)_c} \frac{1}{Z} \left( \frac{1}{p^2 - p_{in}^2} \right) $$

(36)

Radial $p^2$ Gas Flow Equations

Substituting Eq. 35 into Eq. 27 gives

$$ q_{sc} = C_r \frac{k_h}{In(e/kw)} \left[ \frac{T_{sc} \cdot Z_{sc}}{T \cdot Z_c} \right] \frac{1}{(uZ)_c} \frac{1}{Z} \left( \frac{1}{p^2 - p_{in}^2} \right) $$

(37)

The $(uZ)_c$ term is evaluated at some "average" pressure, which is unfortunate— but a fact of life, this is because the $p^2$ approach is an approximation.

The validity of this approach can be established by observing plots of $uZ$ versus pressure for various cases of temperature and gas composition (see attached plots).
Comparison of $\mu z$ versus Pressure—Cartesian Format

Comparison of $\mu z$ versus Pressure for a Temperature of 100°F and Various Gas Compositions, Cartesian Format.

Comparison of $\mu z$ versus Pressure for a Temperature of 200°F and Various Gas Compositions, Cartesian Format.

Comparison of $\mu z$ versus Pressure for a Temperature of 300°F and Various Gas Compositions, Cartesian Format.
Comparison of $\mu z$ versus Pressure—Log-Log Format

Comparison of $\mu z$ versus Pressure for a Temperature of 100°F and Various Gas Compositions, Log-Log Format.

Comparison of $\mu z$ versus Pressure for a Temperature of 200°F and Various Gas Compositions, Log-Log Format.

Comparison of $\mu z$ versus Pressure for a Temperature of 300°F and Various Gas Compositions, Log-Log Format.
Comparison of \( p/\mu z \) versus Pressure—Cartesian Format

Comparison of \( p/\mu z \) versus Pressure for a Temperature of 100°F and Various Gas Compositions, Cartesian Format.

Comparison of \( p/\mu z \) versus Pressure for a Temperature of 200°F and Various Gas Compositions, Cartesian Format.

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Comparison of $p/\mu z$ versus Pressure for a Temperature of 300°F and Various Gas Compositions, Log-Log Format.
Derivation of the Pressure-Squared ($p^2$) Forms of the "Gas Deliverability" Equations
(from Petroleum Engineering 620 Course Notes -- 1995)
Multi-Point Testing Sequence / Conventional 4-Point Test

The figure shown above illustrates the multipoint well testing sequence for gas wells. This procedure has "evolved" as a mechanism for state regulatory agencies to monitor and regulate gas production (to guard against overproduction). The empirical analysis that is used to interpret these data is shown below.

\[
\log (p - p_{f1}) = a \log (q_{sc}) + b
\]

"log-log plot" of \( p - p_{f1} \) vs. \( q_{sc} \)

\( AOF = \text{Absolute Open Flow} \),

(Maximum Theoretical Flowrate)
From the empirical analysis of the data on the log-log plot we have
\[
\log \left( \frac{p^2 - p_f^2}{p_{sc}} \right) = a \log \left( \frac{q_{sc}}{c} \right) + b
\]  
(1)

At this point Eq. 1 is simply an empirical observation-stated without any theoretical basis. We will derive the theoretical basis for Eq. 1 but first we must reduce this expression to a more useful form.

Defining
\[
a = \frac{1}{n}
\]  
(2)
and
\[
b = a \log \left( \frac{1}{c} \right)
\]  
(3)

Substituting Eqs. 2 and 3 into Eq. 1 we have
\[
\log \left( \frac{p^2 - p_f^2}{p_{sc}} \right) = \frac{1}{n} \log \left( \frac{q_{sc}}{c} \right)
\]

Multiplying through by \( n \) gives
\[
\log \left( \frac{q_{sc}}{c} \right) = n \log \left( \frac{p^2 - p_f^2}{p_{sc}} \right) = \log \left[ \left( \frac{p^2 - p_f^2}{p_{sc}} \right)^n \right]
\]

Exponentiating gives
\[
\frac{q_{sc}}{c} = \left( \frac{p^2 - p_f^2}{p_{sc}} \right)^n
\]
or
\[
q_{sc} = c \left( \frac{p^2 - p_f^2}{p_{sc}} \right)^n
\]  
(4)

Recall that Eq. 4 (or its alternate form, Eq. 1) are both empirical results—without any theoretical justification. Our job is to establish a rigorous (i.e., theoretical) basis for these expressions using our knowledge of the relations for gas flow in porous media.

Since flow geometry may become an issue we will develop relations for both horizontal linear and horizontal radial flow.
Horizontal Linear Flow Case

Governing relation: Darcy's Law

\[ q_{sc} = -kA \ \frac{T_{sc} \ \varnothing_{sc}}{m^2} \ \frac{p}{d\varnothing} \]  

(5)

"Lumping" the constant terms

\[ q_{sc} = -C_L \ \frac{p}{m^2} \ \frac{d\varnothing}{d\varnothing} \]  

(6)

\[ C_L = kA \ \frac{T_{sc} \ \varnothing_{sc}}{m^2} \ ]  

(7)

separating

\[ q_{sc} \ d\varnothing = -C_L \ \frac{p}{m^2} \ d\varnothing \]  

(8)

The dilemma for both Eq. 8 and Eq. 12 is how do we integrate the \( \frac{p}{m^2} \) term? Do we assume something constant—and if so, how do we establish what is constant? We know that we want to end up with a pressure-squared formulation, so assuming \( m^2 = \) constant will permit direct integration of Eqs. 8 and 12. We will address whether or not \( m^2 = \) constant as a post-mortem exercise where we will consider the \( m^2 \) product under a variety of pressures, temperatures, and compositions (gas gravities). Proceeding with the integration gives:

Integrating \( \int [p, L], \varnothing \ [p, \varnothing] \)

\[ q_{sc} \ ln \ \frac{T_{sc}}{m^2} = -C_L \ \frac{1}{m^2} \ \int \frac{p}{\varnothing} \ d\varnothing \]

which yields

\[ q_{sc} \ L = -C_L \ \frac{1}{m^2} \ ln \ \frac{\varnothing^2}{p} \]  

(13)

reducing

\[ q_{sc} \ L = C_L \ \frac{1}{m^2} \ \left( \frac{p^2}{\varnothing^2} \right) \]  

(14)

Substituting Eq. 7 in Eq. 13 and solving for \( q_{sc} \)

\[ q_{sc} = \frac{kA \ \frac{T_{sc} \ \varnothing_{sc}}{m^2}}{L} \ \frac{1}{m^2} \ \left( \frac{p^2}{\varnothing^2} \right) \]

Horizontal Radial Flow Case

Governing relation: Darcy's Law

\[ q_{sc} = \frac{2\pi k h}{T \ \varnothing_{sc} \ \frac{r \ \varnothing}{m^2} \ dr} \]  

(9)

"Lumping" the constant

\[ q_{sc} = c_r \ \frac{p}{m^2} \ dr \]  

(10)

\[ c_r = 2\pi k h \ \frac{T_{sc} \ \varnothing_{sc}}{T \ \varnothing_{sc}} \]  

(11)

separating

\[ q_{sc} \ ln \ \frac{T_{sc}}{m^2} = c_r \ \frac{p}{m^2} \ dr \]  

(12)

Integrating \( \int [k_w, r], \varnothing \ [p, \varnothing] \)

\[ q_{sc} \ ln \ \frac{k_w}{m^2} = c_r \ \frac{p}{m^2} \]  

which gives

\[ q_{sc} \ ln \ \frac{k_w}{m^2} = c_r \ \frac{1}{m^2} \ \left( \frac{p^2}{\varnothing^2} \right) \]  

(15)

reducing

\[ q_{sc} \ ln \ \frac{k_w}{m^2} = c_r \ \frac{1}{m^2} \ \left( \frac{p^2}{\varnothing^2} \right) \]  

(16)

Substituting Eq. 11 into 15 and solving for \( q_{sc} \)

\[ q_{sc} = \frac{2\pi k h}{T} \ \frac{T_{sc} \ \varnothing_{sc}}{m^2} \ \frac{1}{m^2} \ \left( \frac{p^2}{\varnothing^2} \right) \]  

\[ \ln \left( \frac{k_w}{m^2} \right) \]  

(16)
Horizontal Linear Flow Case
Comparing Eqs. 4.14 we have
\[ q_{sc} = C \left( \frac{p_e - p_w}{p_e - p_i} \right)^n \]  (17)
\[ q_{sc} = C \left( \frac{p_e - p_i}{p_i} \right)^n \]  (20)

where
\[ C = \frac{kA}{L} \frac{\tau_{sc}}{T} \frac{z_{sc}}{M} \frac{1}{z} \]  (18)
and
\[ n = 1 \]  (19)

Horizontal Radial Flow Case
Comparing Eqs. 4.16 gives
\[ q_{sc} = C \left( \frac{p_e - p_w}{p_e - p_i} \right)^n \]  (21)
\[ q_{sc} = C \left( \frac{p_i}{p_e - p_i} \right)^n \]  (22)

where
\[ C = \frac{2nk L}{\ln \left( \frac{r_e}{r_w} \right)} \frac{T_{sc}}{T} \frac{\tau_{sc}}{M} \frac{1}{Z} \]  (21)
and
\[ n = 1 \]  (22)

There are some issues regarding the definitions of \( \rho \) and \( \eta \), which are constant with time and the analogy with the average reservoir pressure, \( \bar{p} \), which varies with time. The "simple solution" is to note that we have derived a steady-state solution (i.e., time invariant) for a problem that varies with time. If we assume a "snapshot" in time for \( \bar{p} \), then the concept should be applicable. The actual name for this time variant flow regime is pseudosteady-state flow, where pseudosteady-state conditions can be thought of as a succession of steady-state conditions with respect to time.

The other, more urgent question is whether or not we can assume that \( \mu \approx \mu_0 \) and if so, what are the conditions where this holds. The attached plots indicate that for \( p_e > 2000 \, \text{psi} \), \( \mu \approx \text{constant} \), which validates our empirical concept that \( \frac{p_e - p_w}{p_e - p_i} \) and \( q \) are linearly proportional, which in turn validates the gas well deliverability analysis approach.
Comparison of $\mu z$ versus Pressure for a Temperature of 200°F and Various Gas Compositions, Cartesian Format.

Comparison of $\mu z$ versus Pressure for a Temperature of 300°F and Various Gas Compositions, Cartesian Format.
Comparison of $\mu z$ versus Pressure for a Temperature of 100°F and Various Gas Compositions, Log-Log Format.

Comparison of $\mu z$ versus Pressure for a Temperature of 200°F and Various Gas Compositions, Log-Log Format.

Comparison of $\mu z$ versus Pressure for a Temperature of 300°F and Various Gas Compositions, Log-Log Format.
Comparison of $p/\mu z$ versus Pressure for a Temperature of 100°F and Various Gas Compositions, Cartesian Format.

Comparison of $p/\mu z$ versus Pressure for a Temperature of 200°F and Various Gas Compositions, Cartesian Format.

Comparison of $p/\mu z$ versus Pressure for a Temperature of 300°F and Various Gas Compositions, Cartesian Format.
Comparison of $p/\mu z$ versus Pressure for a Temperature of 100°F and Various Gas Compositions, Log-Log Format.

Comparison of $p/\mu z$ versus Pressure for a Temperature of 200°F and Various Gas Compositions, Log-Log Format.

Comparison of $p/\mu z$ versus Pressure for a Temperature of 300°F and Various Gas Compositions, Log-Log Format.