1. Natural Logarithm Function: \((\ln(1)=0, \ln(\exp(x))=x, \ln(\infty)=\infty)\)

<table>
<thead>
<tr>
<th>Integral Definition</th>
<th>Derivative Definition</th>
<th>Evaluate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ln(x) = \int \frac{1}{x} , dx)</td>
<td>(\frac{d}{dx}[\ln(x)] = \frac{1}{x})</td>
<td>(I = \int_{a}^{b} \frac{1}{x} , dx)</td>
</tr>
</tbody>
</table>

Ans. \(I = \int_{a}^{b} \frac{1}{x} \, dx = \) ____________________________ (Show all work)

2. Exponential Function: \((\exp(0)=1, \exp(1)=2.71828 182284 ..., \exp(\infty)=\infty, \exp(-\infty)=0)\)

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<td>(\int \exp(ax) , dx = \frac{1}{a} \exp(ax) + C)</td>
<td>(\frac{d}{dx}[\exp(ax)] = a \exp(ax))</td>
<td>(\int_{0}^{x} \exp(-ax) , dx = ?)</td>
</tr>
</tbody>
</table>

Ans. \(\int_{0}^{x} \exp(-ax) \, dx = \) ____________________________ (Show all work)
3. Solve the following:

\[ \int x^n \ln(x) \, dx = ? \]

a. For \( n \neq -1 \)
b. For \( n = -1 \)

*(Hint: Check the next page.)*
Integration by parts.

Let $u$ and $v$ be differentiable functions. According to the product rule for differentials

$$d(uv) = u dv + v du$$

Upon taking the antiderivative of both sides of the equation, we obtain

$$uv = \int u dv + \int v du$$

This is the formula for integration by parts when written in the form

$$\int u dv = uv - \int v du \quad \text{or} \quad \int f(x)g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx$$

where $u = f(x)$ and $v = g(x)$. The corresponding result for definite integrals over the interval $[a, b]$ is certainly valid if $f(x)$ and $g(x)$ are continuous and have continuous derivatives in $[a, b]$. See Problems 5.17 to 5.19.