Material Balance Orientation

Material Balance: Issues
- Oil MBE (must know all data, also $c_f(p)$).
- Gas MBE (abnormal pressure, water drive).

Material Balance: Topics
- "Accounting" Concept of Material Balance:
  - Require all inflows/outflows/generations.
  - (Average) reservoir pressure profile is REQUIRED.
  - Require rock, fluid, and rock-fluid properties (at some scale).
- Oil Material Balance:
  - Less common than gas material balance (pressure required).
- Gas Material Balance:
  - Volumetric dry gas reservoir ($p/z$ versus $G_p$ (straight-line)).
  - Abnormally-pressured gas reservoirs (various techniques).
  - Waterdrive/water influx cases (always problematic) Material Balance yields RESERVOIR VOLUME!
Reservoir Engineering (Material Balance)
Material Balance of a Petroleum Reservoir

● General Concept of Material Balance...

- Gas zone
- Connate water
- Oil zone
- Gas

a. Initial reservoir conditions.

- Gas zone
- Connate water
- Oil zone
- Gas

b. Conditions after producing $N_p$ STB of oil, and $G_p$ SCF of gas, and $W_p$ STB of water.


● Material Balance: Key Issues
  - Must have accurate production measurements (oil, water, gas).
  - Estimates of average reservoir pressure (from pressure tests).
  - Suites of PVT data (oil, gas, water).
  - Reservoir properties: saturations, formation compressibility, etc.
Reservoir Engineering (Material Balance)
Average Reservoir Pressure for Material Balance

- **Average Reservoir Pressure**

- **Average Reservoir Pressure: Key Issues**
  - Must "average" pressures over volume or area (approximation).
  - Pressure tests must be representative ($p_{avg}$ extrapolation valid).
  - Can average using cumulative production (surrogate for volume).

- *From: Engineering Features of the Schuler Field and Unit Operation — Kaveler (SPE-AIME, 1944).*
Black Oil Material Balance Case: (Example)
- Note that all fluid functions are given: \( N_p \), \( W_p \), and \( GOR \) (for \( G_p \)).
- Average reservoir pressure is presumed correct.
- Authors cite "partial waterdrive" — remains a contentious issue.
Oil Material Balance Relations:

"Black Oil" Material Balance: \((p>p_b)\)

\[
\bar{p} = p_i - \frac{1}{N_{C_t}} \frac{B_o}{B_{oi}} N_p
\]

"Solution Gas Drive" (Oil) Material Balance: \((\text{all } p)\)

\[
N_p \left[ B_o + (R_p - R_s)B_g \right] + W_p B_w = (\text{Withdrawal (RB)})
\]

\[
N \left[ (B_o - B_{oi}) + (R_{si} - R_s)B_g \right]
+ m N B_{oi} \left[ \frac{B_g}{B_{gi}} - 1 \right]
+ (1 + m) N B_{oi} \frac{c_w S_{wi} + c_f}{1 - S_{wi}} (p_i - \bar{p})
+ W_e B_w = (\text{Oil Expansion (RB)})
\]

\[
(\text{Gas Cap Expansion (RB)})
\]

\[
(\text{Water Exp./Pore Vol. Comp. (RB)})
\]

\[
(\text{Water Influx (RB)})
\]
Material Balance Notes
(from Department of Petroleum Engineering Course Notes -- 1984)

- Black Oil Cases
  - Undersaturated Oil
  - Solution Gas Drive
- Dry Gas Case
VOLUMETRIC OIL RESERVOIRS

I. Undersaturated Reservoir

Objective: To derive a material balance equation for an undersaturated reservoir

A. Assumptions

1. \( P > P_B \)
2. No original or final gas cap
3. No water influx or production

B. Derivation of Material Balance Equation
BY VOLUMETRIC BALANCE

ORIGINAL VOLUME = FINAL VOLUME

ORIGINAL VOLUME = \( N B_{01} \)

FINAL VOLUME = \((N-N_P)B_0 + \) VOLUME OCCUPIED BY WATER AND ROCK EXPANSION AS PRESSURE DECLINES

- ROCK AND WATER EXPANSION IMPORTANT IN UNDERSATURATED RESERVOIRS

FROM DEFINITION OF COMPRESSIBILITY

\[ c_w = -\frac{1}{V_w} \left( \frac{dV_w}{dP} \right) = -\frac{1}{V_{wi}} \frac{\Delta V_w}{\Delta P} \]

THUS, CHANGE IN RESERVOIR WATER VOLUME DUE TO PRESSURE CHANGE:

\[ \Delta V_w = -c_w V_{wi} \Delta P \]

AS PRESSURE DECREASES, MATRIX SUPPORTING STRUCTURE COLLAPSES INTO PORE SPACE

\[ c_f = -\frac{1}{V_P} \left( \frac{dV_P}{dP} \right) = -\frac{1}{V_{pi}} \frac{\Delta V_P}{\Delta P} \]
Thus, change in pore volume due to pressure change:

\[ \Delta V_p = -c_F V_{PI} \Delta P \]

Total change in water volume and pore volume:

\[ \Delta V_w + \Delta V_p = - \left[ c_w V_{WI} + c_F V_{PI} \right] \Delta P \]

\[ = \Delta V_{TOTAL} \]

Note that

\[ V_w = S_w V_p \]

\[ V_{WI} = S_{WI} V_{PI} \]

Thus

\[ \Delta V_{TOTAL} = - \left[ c_w S_{WI} + c_F \right] V_{PI} \Delta P \]

Also

\[ V_{PI} = \frac{NB_{OI}}{1 - S_{WI}} \]

Thus

\[ \Delta V_{TOTAL} = \frac{NB_{OI}}{1 - S_{WI}} \left[ c_w S_{WI} + c_F \right] \Delta P \]
THE VOLUMETRIC BALANCE BECOMES:

\[ NB_{OI} = (N-N_p)B_O - \frac{NB_{OI}}{1-S_{WI}} \left[ c_w S_{WI} + c_F \right] \Delta P \]

SOLVING FOR N:

\[ N = \frac{N_p B_O}{B_O + B_{OI} \left\{ \frac{c_w S_{WI} + c_F}{1-S_{WI}} \right\} (P-I-P) - B_{OI}} \] \hspace{1cm} (V-1)

TO SIMPLIFY, NOTE:

\[ c_O = - \frac{1}{V} \left( \frac{dV}{dP} \right) = - \frac{1}{V} \left( \frac{\Delta V}{\Delta P} \right) \]

IF \( V_{SC} \) IS VOLUME OF OIL IN STOCK TANK (STANDARD CONDITIONS)

\[ - \frac{1}{V} \left( \frac{\Delta V}{\Delta P} \right) = - \frac{1}{V_{SC} \sqrt{r}} \left( V/V_{SC} - V_{I}/V_{SC} \right) \]

\[ = \frac{1}{V_{I}/V_{SC}} \frac{(B_O-B_{OI})}{(P-I-P)} \]

THEN

\[ B_O-B_{OI} = c_O B_{OI} (P-I-P) \]
SUBSTITUTING INTO EQUATION V-1

\[ N = \frac{\frac{N_p B_0}{B_{OI}}} {c_o + \frac{c_w S_{WI}}{1-S_{WI}} + c_f} (p_1 - p) \]

DEFINE

\[ c_e = c_o + \frac{c_w S_{WI} + c_f}{1-S_{WI}} = \frac{c_o S_{OI} + c_w S_{WI} + c_f}{(1-S_{WI})} \]

THUS

\[ N = \frac{N_p B_0}{B_{OI} c_e (p_1 - p)} \] (V-2)

C. CONSIDERATIONS

1. Eqs. V-1 OR V-2 SHOULD BE USED FOR ESTIMATING OOIP ABOVE BUBBLE POINT WHERE ROCK AND WATER EXPANSION NOT NEGLECTIBLE

2. DIFFICULTY IN MEASURING \( c_f \) AND \( c_w \) MAY LIMIT ACCURACY
D. Example - Use of Material Balance to Determine Original Oil in Place in Under-saturated Reservoir

**Problem**

Determine the original oil in place for the undersaturated reservoir for which data are summarized below.

\[
\begin{align*}
N_p &= 1.4 \times 10^6 \text{ STB} \\
B_o &= 1.46 \text{ RB/STB} \\
B_{oi} &= 1.39 \text{ RB/STB} \\
c_w &= 3.71 \times 10^{-6} \text{ PSI}^{-1} \\
c_f &= 3.52 \times 10^{-6} \text{ PSI}^{-1} \\
S_{wi} &= 32\%
\end{align*}
\]

The reservoir was discovered at an initial pressure of 4300 PSI. Pressure has declined to 2450 PSI.

**Solution**

From equation V-1

\[
N = \frac{N_p B_o}{B_o + B_{oi} \left( \frac{c_w S_{wi} + c_f}{1 - S_{wi}} \right) (P_i - P) - B_{oi}}
\]
\[ N = \frac{1.4 \times 10^6(1.46)}{1.46 + 1.39 \left( \frac{3.71 \times 10^{-6}(0.32) + 3.52 \times 10^{-6}}{1 - 0.32} \right)(4300 - 2450) - 1.39} \]

\[ = 2.33 \times 10^7 \text{ STB} \]

**NOTE:**

**IF C_F IS ASSUMED TO BE 0:**

\[ N = \frac{1.4 \times 10^6(1.46)}{1.46 + 1.39 \left( \frac{3.71 \times 10^{-6}(0.32) + 0}{1 - 0.32} \right)(4300 - 2450) - 1.39} \]

\[ = 2.74 \times 10^7 \text{ STB} \]

**CALCULATED VALUE OF N IS SIGNIFICANTLY INCREASED IF C_F NEGLECTED**
II. Saturated Reservoir - Solution Gas Drive

OBJECTIVE: TO DERIVE A MATERIAL BALANCE EQUATION FOR A SOLUTION GAS DRIVE RESERVOIR AND TO APPLY IT TO ESTIMATE ORIGINAL OIL IN PLACE (OOIP)

A. Assumptions

1. $P \leq P_B$

2. No original gas cap

3. No water influx or production

4. Negligible rock and water expansion

B. Derivation of Material Balance Equation

\[ \text{OIL VOLUME } \frac{N_P}{N_{BOI}} \]

\[ \text{GAS VOLUME } \frac{G_P}{(N-N_P)B_O} \]

Original conditions by volumetric balance

Later conditions

Original volume = final volume
ORIGINAL OIL VOLUME = \( N_{BOI} \)
ORIGINAL FREE GAS VOLUME = 0
FINAL OIL VOLUME = \( (N-N_P)B_O \)

- DETERMINE FINAL FREE GAS VOLUME BY PERFORMING A GAS BALANCE

ORIGINAL DISSOLVED GAS = \( NR_{SI} \)
FINAL DISSOLVED GAS = \( (N-N_P)R_S \)
GAS PRODUCED = \( G_P \)

THEREFORE,

FINAL FREE GAS = \( NR_{SI} - (N-N_P)R_S - G_P \)

- CONVERT TO RESERVOIR CONDITIONS

FINAL FREE GAS = \( (NR_{SI} - (N-N_P)R_S - G_P)B_g/5.61 \)

THE VOLUMETRIC BALANCE BECOMES:

\[ NB_{OI} = (N-N_P)B_o + (NR_{SI} - (N-N_P)R_S - G_P)B_g/5.61 \]

SOLVING FOR \( N \)

\[
N = \frac{N_PB_O - (N_PR_S - G_P)B_G/5.61}{B_O + (R_{SI} - R_S)B_G/5.61 - B_{OI}} \quad (V-3)
\]
TO SIMPLIFY, NOTE THAT

\[ B_T = B_0 + (R_{SI} - R_S)B_G/5.61 \]

\[ R_p = G_p/N_p \]

ALSO,

\[ B_{OI} = B_{TI} \quad \text{(no gas evolved at } P_B) \]

SUBSTITUTING INTO EQUATION V-3

\[ N = \frac{N_p \{B_T + (R_p - R_{SI})B_G/5.61\}}{B_T - B_{TI}} \quad \text{(V-4)} \]

SUBSTITUTING FOR \( B_O \) IN THE NUMERATOR ONLY, AN
ALTERNATIVE FORM IS

\[ N = \frac{N_p \{B_O + (R_p - R_S)B_G/5.61\}}{B_T - B_{TI}} \quad \text{(V-5)} \]
III. Derivation of Material Balance Equation

- From Volumetric Balance

**INITIAL VOLUME = FINAL VOLUME**

\[
\frac{G(B_{G1})}{5.61} = (G-G_P)(B_G/5.61) + (W_E-W_P)(B_W)
\]

Where \( B_G \) = gas formation volume factor, \( \text{RCF/SCF} \)

\[
G(B_{G1})/5.61 = (G-G_P)(B_G/5.61) - W_P(B_W) + W_E(B_W)
\]

**THIS CAN BE REARRANGED TO BE**

\[
G(B_G-B_{G1})/5.61 = (G_P(B_G)/5.61) + W_P(B_W) - W_E(B_W)
\]
THE GENERALIZED FORM OF THE MATERIAL BALANCE EQUATION

\[
\frac{G_p(B_g)/5.61 + W_p(B_w)}{(B_g-B_{gI})/5.61} = G + \frac{W_e(B_w)}{(B_g-B_{gI})/5.61}
\]

(X-6)

A. PRESSURE DEPLETION CASE - NO WATER INFLUX

\( W_E = W_P = 0 \)

Therefore:

\[ G(B_g-B_{gI})/5.61 = G_p(B_g)/5.61 \]

or

\[ G_p = G(1-B_{gI}/B_g) \]  

(X-7)
SINCE \[ B_G = \frac{V_{R,C}}{V_{S,C}} = \frac{P_{S,C}}{P_{R,C}} \frac{T_{R,C}}{s,c} Z_{R,C}. \]

\[ B_G/B_{G1} = (P_1/z_1)(z/P) = \text{constant} \]

THEN \[ G_p = G(1 - (P/z)(z_1/P_1)) \]

THIS CAN BE REARRANGED TO BE

\[ (P_1/z_1)(1-G_p/G) = P/z \quad (X-8) \]

THIS SUGGESTS A PLOT OF \((P/z)\) vs. \(G_p\)

WHEN \((P/z) = 0\), NOTE THAT

\[ P_1/z_1 \ (1-G_p/G) = 0 \]

OR

\[ G_p = G \]

ALSO, THE PLOT SHOULD BE LINEAR AND THUS READILY EXTRAPOLATED TO \(P/z = 0\).
B. Example Problem - Determination of OGIP and Drive Mechanism Using a P/z Plot

**Problem**

An isopach map of the "Zapata Sand" in the Woodford Field in Atascosa County, Texas indicated an original gas in place of 44 mmscf. Production from the field has resulted in the following:

<table>
<thead>
<tr>
<th>Reservoir Pressure (psia)</th>
<th>Gp (mmscf)</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>0.00</td>
<td>0.80</td>
</tr>
<tr>
<td>3500</td>
<td>2.46</td>
<td>0.73</td>
</tr>
<tr>
<td>3000</td>
<td>4.92</td>
<td>0.66</td>
</tr>
<tr>
<td>2500</td>
<td>7.88</td>
<td>0.60</td>
</tr>
<tr>
<td>2000</td>
<td>11.20</td>
<td>0.55</td>
</tr>
<tr>
<td>200</td>
<td>---</td>
<td>0.94</td>
</tr>
</tbody>
</table>

The Z factors were derived from fluid analysis data. A volumetric type depletion is suspected.

Perform a P/z plot to confirm original gas in place estimates and the suspected drive mechanisms.
SOLUTION

\[
\begin{array}{cc}
G_p & P/z \\
(MMSCF) & (PSIA) \\
0.00 & 5000 \\
2.46 & 4795 \\
4.92 & 4545 \\
7.88 & 4167 \\
11.20 & 3636 \\
\end{array}
\]

(See Graph)

The characteristics of the best straight line fit of the data set are:

A) \( G = 43.6 \) MMSCF (by extrapolation)

B) The curve gives some indication of nonlinearity but still fits the expected gas-in-place estimate

C) The straight line and good agreement with original gas-in-place estimate confirm the volumetric depletion characteristics
Reservoir Engineering (Material Balance)  
Gas Material Balance Case (1/6)

- **Gas Material Balance Relations:**

**General Gas Material Balance:**

\[
\frac{\bar{p}}{z} \left[1 - \bar{c}_e(\bar{p})(p_i - \bar{p})\right] = \\
\frac{p_i}{z_i} - \frac{p_i}{z_i} \frac{1}{G} \left[ G_p - G_{inj} + W_p R_{sw} + 5.615 \frac{1}{B_g} \left( W_p - W_{inj} \right) B_w - W_e \right]
\]

"Dry Gas" Material Balance: *(no reservoir liquids)*

\[
\frac{\bar{p}}{z} = \frac{p_i}{z_i} \left[ 1 - \frac{1}{G} G_p \right]
\]

"Abnormal Pressure" Material Balance: *(c_f=f(p))*

\[
\frac{\bar{p}}{z} = \frac{p_i}{z_i} \frac{1}{1 - \bar{c}_e(\bar{p})(p_i - \bar{p})} \left[ 1 - \frac{G_p}{G} \right]
\]

\[
\bar{c}_e(\bar{p}) = \frac{1}{(1 - S_{wi})} \left[ S_{wi} \bar{c}_w + \bar{c}_f + \left[ \frac{V_{pNNP}}{V_p R} \right] + \left[ \frac{V_{pAQ}}{V_p R} \right] \right] (\bar{c}_w + \bar{c}_f)
\]

---

**Notes:**

- General Gas Material Balance
- "Dry Gas" Material Balance
- "Abnormal Pressure" Material Balance
Gas Material Balance: Abnormally Pressured Reservoir Example

- Normal pressure production sequence (volumetric depletion, $G_{app}$).
- Abnormal pressure production sequence (OGIP, $G$).
Reservoir Engineering (Material Balance)
Gas Material Balance Case (3/6)

(a) Gas Material Balance Plot: p/z vs. G_p — Illustration of the abnormal pressure trend starting at an inflection point defined by the hydrostatic (normal) pressure.

(b) Gas Material Balance Plot: p/z vs. G_p — Illustration of the abnormal pressure trend starting at an inflection point defined by the hydrostatic (normal) pressure.

Gas Material Balance: Abnormally Pressured Reservoir Example
- Normal pressure production sequence (volumetric depletion, G_{app}).
- Abnormal pressure production sequence (OGIP, G).
"Dry Gas" Material Balance: Normally Pressured Reservoir Example

- Volumetric reservoir — no external energy (gas expansion only).
- $p/z$ versus $G_p$ yields unique straight-line trend.
- Linear extrapolation yield gas-in-place ($G$).

Base Simulation — Volumetric Reservoir

Quadric $G_p$ Material Balance Relation for Abnormally Pressured Gas Reservoirs

Presentation Plot: $p/z$ — $G_p$ Format

Model Trend:

$$p/z = 7255.42 - 10.5703G_p - 0G_p^2$$

Extrapolation to $p/z = 0$:

$$G_{p,max} = G$$

($G = 686.397$ BSCF (input))
"Dry Gas" Material Balance: Abnormally Pressured Reservoir Example

- Volumetric reservoir — no water influx or leakage.
- \( p/z \) versus \( G_p \) yields unique quadratic trend (from approximated MBE).
- Quadratic extrapolation yield gas-in-place (\( G \)).
Reservoir Engineering (Material Balance)
Gas Material Balance Case (6/6)


b. Gas Material Balance Plot: \( p/z \) vs. \( G_p \) — simulated performance. Note effect of displacement efficiency \( (E_p) \).

Gas Material Balance: Water Drive Gas Reservoir

- Pressure (hence \( p/z \)) is maintained during production via communication with an unsteady-state aquifer (this study).