Diffusivity Equations
(Governing Flow Relations)

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Diffusivity Equations
(Governing Flow Relations)

- **Diffusivity Equations:**
  - "Black Oil" \((p > p_b)\)
  - "Solution-Gas Drive" (valid for all \(p\), referenced for \(p < p_b\))
  - "Dry Gas" \((p > p_d)\)
  - Multiphase Flow
Diffusivity Equations
Black Oil \((p>p_b)\)

**Diffusivity Equations for a Black Oil:**

- **Slightly Compressible Liquid: (General Form)**

\[
c(\nabla p)^2 + \nabla^2 p = \frac{\phi \mu c_t}{k} \frac{\partial p}{\partial t}
\]

- **Slightly Compressible Liquid: (Small \(\nabla p\) and \(c\) form)**

\[
\nabla^2 p = \frac{\phi \mu c_t}{k} \frac{\partial p}{\partial t}
\]
Behavior of the $\mu_o$ and $B_o$ variables as functions of pressure for an example black oil case. Note behavior for $p > p_b$ — both variables should be considered to be "approximately constant" for the sake of developing flow relations. Such an assumption (i.e., $\mu_o$ and $B_o$ constant) is not an absolute requirement, but this assumption is fundamental for the development of "liquid" flow solutions in reservoir engineering.
Behavior of the $c_o$ variable as a function of pressure — example black oil case. Note the "jump" at $p=p_b$, this behavior is due to the gas expansion at the bubblepoint.
Diffusivity Equations
Solution-Gas Drive ($p<p_b$)

Diffusivity Equations for Solution-Gas Drive: ($p<p_b$)

- **Oil Pseudopressure Form**: (Accounts for $\mu_o$ and $B_o$)

  \[ \nabla^2 p_{po} = \frac{\phi \mu c_t}{k} \frac{\partial p_{po}}{\partial t} \nabla^2 \]

- **Oil Pseudopressure Definition**: ($p_n$ is any reference pressure)

  \[ p_{po} = [\mu_o B_o]_{p_n} \int_{p_{base}}^{p} \frac{1}{\mu_o B_o} dp \]
Diffusivity Equation

Solution-Gas Drive — \(1/(\mu_oB_o)\) vs. \(p\) (\(p_b=5000\) psia, \(T=175\) Deg F)

\[
p_{po} = \left[\mu_oB_o\right]_p n \int_{p_{base}}^p \frac{1}{\mu_oB_o} \, dp
\]

"Solution-Gas Drive" Pseudopressure Condition: \(1/(\mu_oB_o)\) vs. \(p\)

- Concept: IF \(1/(\mu_oB_o) \cong \text{constant}\), THEN oil pseudopressure NOT required.
- \(1/(\mu_oB_o)\) is NEVER "constant" — but does not vary significantly with \(p\).
- Oil pseudopressure calculation straightforward, but probably not necessary.
Diffusivity Equations
Solution-Gas Drive — \((\mu_o c_o)\) vs. \(p\) \((p_b=5000\ \text{psia, } T=175\ \text{Deg F})\)

\[ t_{a,o} = \left[\mu_o c_t\right]_n \int_0^t \frac{1}{\mu_o(p)c_t(p)} \, dt \]

**"Solution-Gas Drive" Pseudopressure Condition:** \((\mu_o c_o)\) vs. \(p\)

- Concept: IF \((\mu_o c_o)\) \(\cong\) constant, THEN oil pseudotime NOT required.
- \((\mu_o c_o)\) is NEVER "constant" — BUT, oil pseudotime would be very difficult.
- Other evidence suggests that ignoring \((\mu_o c_o)\) variance is acceptable.
**Diffusivity Equations**

Solution-Gas Drive — Mobility/Compressibility (Camacho)

- **"Solution-Gas Drive" Behavior:** \((c_t/\lambda_t)\) vs. *time*
  - Observation: \((c_t/\lambda_t)\) constant for \(p>p_b\) and later, for \(p<p_b\).
  - \(p_{wf} = \text{constant} \) — but probably valid for any production/pressure scenario.

Variation of average compressibility/mobility ratio with time.
Why not use liquid pseudopressure?

\[
\frac{Q_o}{2\pi hk} \log \frac{r_o}{r} = \int_p^{p_o} \frac{k_o/k}{\mu_o \beta} \, dp
\]

Evinger and Muskat (1942) note that:

The indefinite integral may be evaluated, as was done for the two-phase system, and the pressure distribution may be determined. However, it will be sufficient for the calculation of the productivity factor to consider only the limiting form ... (i.e., the constant property liquid relation).
Diffusivity Equations
Dry Gas Relations (Base Relations)

- **Diffusivity Equations for a "Dry Gas:"
  - **General Form for Gas:**
    \[
    \nabla \cdot \left[ \frac{p}{\mu_g z} \nabla p \right] = \frac{\phi c_t}{k} \frac{p \partial p}{z \partial t}
    \]
  - **Diffusivity Relations:**
    **Pseudopressure/Time:**
    \[
    \nabla^2 p_p = \frac{\phi \mu_g c_t}{k} \frac{\partial p_p}{\partial t}
    \]
    **Pseudopressure/Pseudotime:**
    \[
    \nabla^2 \left( \frac{p}{\mu_g} \right) = \frac{\phi}{k} \left( \mu_g c_t \right) p_n \frac{\partial p_p}{\partial t_a}
    \]
  - **Definitions:**
    **Pseudopressure:**
    \[
    p_{pg} = \left[ \frac{\mu_g z}{p} \right]_{p_n} \int_{p_{base}}^{p} \frac{p}{\mu_g z} dp
    \]
    **Pseudotime:**
    \[
    t_a = \left[ \mu_g c_t \right] \int_0^t \frac{1}{\mu_g(p)c_t(p)} dt
    \]
Diffusivity Equations

Dry Gas Pseudotime Condition \((\mu_g c_g \text{ vs. } p, \ T=200 \text{ Deg F})\)

\[ t_a = \left[ \mu_g c_t \right]_n \int_0^t \frac{1}{\mu_g(p)c_t(p)} \, dt \]

"Dry Gas" Pseudotime Condition: \((\mu_g c_g \text{ vs. } p)\)

- Concept: IF \(\mu_g c_g \cong \text{constant}\), THEN pseudotime NOT required.
- \(\mu_g c_g\) is NEVER constant — pseudotime is always required (for liquid eq.).
- However, can generate numerical solution for gas cases (no pseudotime).

Orientation — Diffusivity Equations (oil, gas, multiphase flow)
Diffusivity Equations
Dry Gas — $p^2$ Relations

- **Diffusivity Equations for a "Dry Gas:"** $p^2$ Relations
  - $p^2$ Form — Full Formulation:
    \[
    \nabla^2(p^2) - \frac{\partial}{\partial p^2}[\ln(\mu_g z)]\nabla(p^2)^2 = \frac{\phi \mu_g c_t}{k} \frac{\partial}{\partial t}(p^2)
    \]
  - $p^2$ Form — Approximation:
    \[
    \nabla^2(p^2) = \frac{\phi \mu_g c_t}{k} \frac{\partial}{\partial t}(p^2)
    \]
Diffusivity Equations

Dry Gas $p^2$ Condition ($\mu_g z$ vs. $p$, $T=200$ Deg F)

\[
p_{pg} = \left[ \frac{\mu_g z}{p} \right] p_n \int_{p_{base}}^{p} \frac{p}{\mu_g z} \, dp
\]

"Dry Gas" PVT Properties: ($\mu_g z$ vs. $p$)
- Concept: IF ($\mu_g z$) = constant, THEN $p^2$-variable valid.
- ($\mu_g z$) $\approx$ constant for $p<2000$ psia.
- Even with numerical solutions, $p^2$ formulation would not be appropriate.
Diffusivity Equations
Dry Gas — $p$ Relations

Diffusivity Equations for a "Dry Gas:" $p$ Relations

- $p$ Form — Full Formulation:

$$\nabla^2 p - \frac{\partial}{\partial p} \left[ \ln \left( \frac{\mu_g z}{p} \right) \right] (\nabla p)^2 = \frac{\phi \mu_g c_t}{k} \frac{\partial p}{\partial t}$$

- $p$ Form — Approximation:

$$\nabla^2 p = \frac{\phi \mu_g c_t}{k} \frac{\partial p}{\partial t}$$
Diffusivity Equations

Dry Gas $p$ Condition ($p/(\mu_g z)$ vs. $p$, $T=200$ Deg F)

\[
p_{pg} = \left[ \frac{\mu_g z}{p} \right]_{p_n}^{p_{base}} \int_{p_{base}}^{p} \frac{p}{\mu_g z} \, dp
\]

● "Dry Gas" PVT Properties: ($p/(\mu_g z)$ vs. $p$)
  - Concept: IF $p/(\mu_g z)$ = constant, THEN $p$-variable is valid.
  - $p/(\mu_g z)$ is NEVER constant — pseudopressure required (for liquid eq.).
  - $p$ formulation is never appropriate (even if generated numerically).
Diffusivity Equations
Multiphase Case — $p$-Form Relations (Perrine-Martin)

**Gas Equation:**

\[
\nabla \cdot \left[ \left( \frac{k_g}{\mu_g B_g} + R_{so} \frac{k_o}{\mu_o B_o} + R_{sw} \frac{k_w}{\mu_w B_w} \right) \nabla p \right] = \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_g}{B_g} + R_{so} \frac{S_o}{B_o} + R_{sw} \frac{S_w}{B_w} \right) \right]
\]

**Oil Equation:**

\[
\nabla \cdot \left[ \frac{k_o}{\mu_o B_o} \nabla p \right] = \frac{\partial}{\partial t} \left[ \phi \frac{S_o}{B_o} \right]
\]

**Water Equation:**

\[
\nabla \cdot \left[ \frac{k_w}{\mu_w B_w} \nabla p \right] = \frac{\partial}{\partial t} \left[ \phi \frac{S_w}{B_w} \right]
\]

**Multiphase Equation:**

\[
\nabla^2 p = \phi \frac{c_t}{\lambda_t} \frac{\partial p}{\partial t}
\]

\[
\lambda_t = \frac{k_o}{\mu_o} + \frac{k_g}{\mu_g} + \frac{k_w}{\mu_w}
\]

**Compressibility Terms:**

\[
c_o = -\frac{1}{B_o} \frac{dB_o}{dp} + \frac{B_g}{B_o} \frac{dR_{so}}{dp}
\]

\[
c_w = -\frac{1}{B_w} \frac{dB_w}{dp} + \frac{B_g}{B_w} \frac{dR_{sw}}{dp}
\]

\[
c_g = -\frac{1}{B_g} \frac{dB_g}{dp}
\]

\[
c_t = c_o S_o + c_w S_w + c_g S_g + c_f
\]
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End of Module

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