Example 1: Pressure Buildup Data Exhibiting Boundary Effects (Lee text Example 2.2)

**Given Data:** (Lee text (1st edition), Example 2.2)

These data are taken from Example 2.2 in the Lee text, *Well Testing*. These data are for a pressure "buildup" test run on an oil (liquid) well.

**Reservoir properties:**
\[ \phi = 0.039 \quad r_w = 0.198 \text{ ft} \quad c_i = 17 \times 10^{-6} \text{ psia}^{-1} \quad h = 69 \text{ ft} \]

**Oil properties:**
\[ B_o = 1.136 \text{ RB/STB} \quad \mu_o = 0.8 \text{ cp} \]

**Production parameters:**
\[ p_{wfi} = 3534 \text{ psia} \quad q_o = 250 \text{ STB/D} \quad t_p = 13,630 \text{ hr} \]

**Test Data and Data Functions:**

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<th>( \Delta p ), psi</th>
<th>( \Delta p' ), psi</th>
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**Required:**

Using the log-log type curve for a pressure buildup test performed on a well in a bounded reservoir (i.e., the "Ansah" type curve), estimate the reservoir drainage area, \( A \).
Example 1: Pressure Buildup Data Exhibiting Boundary Effects (Lee text Example 2.2)

Type Curve Analysis:

*Type Curve Match:* Ansah Type Curve

Matching Parameter, $t_{DF} = 1$ (symmetric reservoir)

\[
\begin{align*}
\frac{t_{DA}}{t_{DF}}_{\text{MP}} & = 1 & [t]_{\text{MP}} & = 0.048 \text{ hr} \\
\frac{P_{wD}}{P_{DF}}_{\text{MP}} & = 1 & [\Delta p']_{\text{MP}} & = 60.8 \text{ psi}
\end{align*}
\]

Formation Permeability: (as a check)

\[
k = 141.2 \frac{qB\mu}{h} \frac{[P_{wD}]_{\text{MP}}}{[\Delta p']_{\text{MP}}}
\]

\[
k = 141.2 \frac{(250 \text{ STB/D}) (1.136 \text{ RB/STB}) (0.8 \text{ cp})}{(69 \text{ ft}) (60.8 \text{ psi})} (1.0) = 7.65 \text{ md}
\]

Drainage Area:

\[
A = 0.0002637 \frac{k}{\phi \mu c_t} \frac{1}{t_{DF} \frac{t_{DA}}{t_{DF}}_{\text{MP}}}
\]

\[
A = 0.0002637 \left( \frac{7.65 \text{ md}}{(0.039) (0.80 \text{ cp}) (17.0 \times 10^{-6} \text{ psia}^{-1})} \right) \frac{1}{(1600 \text{ hr})} (1) = 139.7 \text{ acres}
\]
Type Curve Analysis of Well Test Data Including Boundary Effects

Example 1: Pressure Buildup Data Exhibiting Boundary Effects (Lee text Example 2.2)

Type Curve Plot: Ansah Type Curve (Reservoir Drainage Area/Pore Volume)

Type Curve Analysis—Boundary Effects Model (Pressure Buildup Case) (Lee Example 2.2)

Transient Radial Flow Regime, \( p_{D} \) and \( p_{D}' \) = 0.5

Region of Boundary-Dominated Reservoir Performance

Results for Lee Text Example 2.2:
\( (\Delta p/p_{D})_{MP} = 60.8 \text{ psi}, (\Delta t_{DAMF}) = 1600 \text{ hrs} \)
\( k = 7.649 \text{ md (forced)} \)
\( A = 139.7 \text{ acres (Well Centered in a Square)} \)

Base Data:
- \( q = 250 \text{ STB/D} \)
- \( B_{o} = 1.136 \text{ RB/STB} \)
- \( \mu_{w} = 0.8 \text{ cp} \)
- \( c = 17.0 \times 10^{-6} \text{ psia}^{-1} \)
- \( r_{e} = 0.196 \text{ ft} \)
- \( h = 69 \text{ ft} \)
- \( \phi = 0.039 \text{ (fraction)} \)

Line Type | Reservoir Type/Well Location
--- | ---
1x1 Square, Well at (0,0,0.5), \( t_{D}=1 \) | 1x1 Square, Well at (0.75,0.75), \( t_{D}=4 \)
1x1 Square, Well at (0.75,0.75), \( t_{D}=8 \) | 2x1 Rect., Well at (1.0,0.5, \( t_{D}=2 \)
2x1 Rect., Well at (1.5,0.5), \( t_{D}=8 \) | 8x1 Rect., Well at (4.0,0.5), \( t_{D}=8 \)
2x1 Rect., Well at (1.5,0.75), \( t_{D}=8 \) | 8x1 Rect., Well at (6.0,0.75), \( t_{D}=32 \)

Normalized Dimensionless Shut-in Time Based on Drainage Area, \( \Delta t_{DA}/t_{DF} \)
Log-Log Summary Plot: Simulation of Pressure Buildup Test Using Analysis Results

Legend: Data Functions
- Simulation
- \( \Delta \rho \)
- \( \Delta \rho^*; L=0.05 \)
- \( \Delta \rho^*; L=0.10 \)
- \( \Delta \rho^*; L=0.15 \)
- \( \Delta \rho_j \)
- \( \Delta \rho_j; L=0.10 \)

Data for Lee Text Example 2.2:
- \( q=250 \) STB/D
- \( B_o=1.136 \) RB/STB
- \( \mu_o=0.8 \) cp
- \( c_p=17.0 \times 10^{-6} \) psi a
- \( r_w=0.198 \) ft
- \( h=69 \) ft
- \( \phi=0.039 \) (fraction)

Results for Lee Text Example 2.2:
- \( k=7,649 \) md (forced)
- \( s=5.789 \)
- \( C_D=5626 \) (\( C_D e^{0.6x} \))
- \( A=139.7 \) acres (Well Centered in a Square)
Petroleum Engineering 324
Well Performance
Analysis of Pressure Buildup Tests

*Ants and savages put strangers to death.*
— Bertrand Russell (1950)

**Topic:** Analysis of Pressure Buildup Tests

**Objectives:** (things you should know and/or be able to do)

- Be able to derive the "Horner" relations for the analysis of a pressure "buildup" test sequence. Note that this approach explicitly includes the producing time, \( t_p \), where \( p_{ws} \) is plotted versus \( \log\left[\frac{t_p^2 + \Delta t}{\Delta t}\right] \).

The permeability and skin factor relations for Horner analysis are:

\[
k = \frac{162.6 \frac{qB\mu}{mh}}{s = 1.1513 \left[ \frac{(p_{ws,1hr} - p_{wf(\Delta t=0)})}{m} - \log\left[\frac{t_p}{t_p+1}\right] - \log\left[\frac{k}{\phi \mu c \rho_w^2}\right] + 3.2275 \right]}
\]

- Be able to derive the "MDH" relations for the analysis of a pressure "buildup" test sequence. Recall that this approach does not include the producing time, \( t_p \), where \( p_{ws} \) is plotted versus \( \log(\Delta t) \).

The permeability and skin factor relations for MDH analysis are:

\[
k = \frac{162.6 \frac{qB\mu}{mh}}{s = 1.1513 \left[ \frac{(p_{ws,1hr} - p_{wf(\Delta t=0)})}{m} - \log\left[\frac{k}{\phi \mu c \rho_w^2}\right] + 3.2275 \right]}
\]

- Be able to derive the "Modified Muskat" plotting functions for the determination of \( \bar{p} \) using late-time data from a pressure "buildup" test sequence.

  - "Modified Muskat" Pressure Equation: (valid for \( t_{pDA} > 0.1 \) and \( 0.05 < \Delta t_{DA} < 0.10 \))
    \[
    \bar{p} - p_{ws} = 118.6 \frac{qB\mu}{kh} \exp\left[-0.00388 \frac{k\Delta t}{\phi \mu c \rho_w^2}\right]
    \]
    (Lee text, Eq. 2.23, p. 40)
    
    or
    \[
    \bar{p} - p_{ws} = a \exp[-b\Delta t]
    \]
    (general form)

  - "Modified Muskat" Pressure Derivative Equation:
    \[
    \frac{d}{d\Delta t}[p_{ws}] = -ab \exp[-b\Delta t]
    \]

  - "Modified Muskat" Plotting Relation:
    \[
    p_{ws} = \bar{p} - \frac{1}{b} \frac{d}{d\Delta t}[p_{ws}]
    \]
    
    \( p_{ws} \) is plotted versus \( \frac{d}{d\Delta t}[p_{ws}] \) to determine \( \bar{p} \) as the intercept of the straight-line trend at \( \frac{d}{d\Delta t}[p_{ws}] = 0 \).
Lecture Outline:

- Example: Lee Text Example 2.2
  - Early-time Cartesian Analysis: Determination of $p_{wf}(\Delta t=0)$
  - Semilog Analysis: Determination of $k$, $s$, and $p^*$
    - MDH Method (Rate effects not accounted for).
    - Horner Method (Rate effects accounted for).
  - Log-log Analysis:
    - Early time validation of the unit slope line (wellbore storage).
    - Validate semilog straight-line ($\Delta p' = \text{constant}$, radial flow).
    - Late time validation of boundary effects.
- Late-time analysis: Determination of $\bar{p}$
  - Modified Muskat Method (plot of $p_{ws}$ versus $\frac{d[1]}{d\Delta t}[p_{ws}]$)
Reading Assignment:

Text Reading:
- Review Chapters 2 and 4 of the Lee Well Testing text, 1st edition.
  - Chapter 7—The Constant Terminal Rate Solution of the Diffusivity Equation and its Application to Oilwell Testing (Section 7.7).
  - Chapter 4—Oilwell Testing (Sections 4.11, 4.12, 4.12)

Derivations:
- Review attached notes.
  - Example Analysis: (Lee text (1st edition), Example 2.2).
  - Graphical Analysis (Lee text (1st edition), Example 2.2).
Derivation of the "Modified Muskat" Plotting Functions: (determination of $\bar{p}$)

- Base relations:
  - "Modified Muskat" Pressure Equation: (valid for $t_{pDA}>0.1$ and $0.05<\Delta t_{DA}<0.10$)
    \[
    \bar{p} - p_{ws} = 118.6 \frac{qB\mu}{kh} \exp\left[-0.00388 \frac{k\Delta t}{\phi\mu c_r r_e^2}\right]
    \]  
    (Lee text, Eq. 2.23, p. 40)
    or
    \[
    \bar{p} - p_{ws} = a \exp[-b\Delta t]
    \]
    (general form)
  - "Modified Muskat" Pressure Derivative Equation:
    \[
    \frac{d}{d\Delta t}[p_{ws}] = -ab \exp[-b\Delta t]
    \]

For the purposes of our analysis (i.e., the determination of $\bar{p}$), we simply assume that $a$ and $b$ are constants in the Modified Muskat equation and have no direct application (i.e., we will not use these parameters to estimate formation properties).

Combining the pressure and pressure derivative relations, we obtain

\[
p_{ws} = \bar{p} - \frac{1}{b} \frac{d}{d\Delta t}[p_{ws}]
\]

where $p_{ws}$ is plotted versus $\frac{d}{d\Delta t}[p_{ws}]$ to determine $\bar{p}$ as the intercept of the straight-line trend at $\frac{d}{d\Delta t}[p_{ws}] = 0$.

Derivation of the "Rectangular Hyperbola" Plotting Functions: (determination of $\bar{p}$)

Note: The "Rectangular Hyperbola" model is not as rigorous as the "Modified Muskat" model for late-time pressure buildup behavior. The "Rectangular Hyperbola" approach is provided for illustrative purposes — the "Modified Muskat" model should always be the preferred method.

- Base relations:
  - Rectangular Hyperbola "Pressure" Equation:
    \[
p_{ws} = \bar{p} - \frac{a}{b + \Delta t}
    \]
    (Approximation for late-time pressure behavior)
  - Rectangular Hyperbola "Pressure Derivative" Equation:
    \[
    \frac{d}{d\Delta t}[p_{ws}] = \frac{a}{(b + \Delta t)^2}
    \]

Combining the pressure and pressure derivative relations, we obtain

\[
p_{ws} + \Delta t\frac{d}{d\Delta t}[p_{ws}] = \bar{p} - b\frac{d}{d\Delta t}[p_{ws}]
\]

where $p_{ws} + \Delta t\frac{d}{d\Delta t}[p_{ws}]$ is plotted versus $\frac{d}{d\Delta t}[p_{ws}]$ to determine $\bar{p}$ as the intercept of the straight-line trend at $\frac{d}{d\Delta t}[p_{ws}] = 0$.

Alternatively, $p_{ws}$ can also be plotted versus $\left[\frac{dp_{ws}}{d\Delta t}\right]^{0.5}$ to determine $\bar{p}$ as the intercept of the straight-line trend at $\left[\frac{dp_{ws}}{d\Delta t}\right]^{0.5} = 0$. (This is probably the better approach)
Cartesian Plot: Early-Time Pressure Data

"Early Time" Cartesian Plot -- Lee Text Example 2.2
(Analysis of Wellbore Storage Dominated Data)

Regression Equations:
Wellbore Storage Equation:
\[ P_{w(t)} = 3534 + 973.33 \Delta t \]

Legend: Lee Text Example 2.2
● Pressure Data

Data for Lee Example 2.2
Reservoir Properties:
\( c_p = 17.0 \times 10^5 \) psia\(^{-1} \)
\( r_w = 0.180 \) ft
\( h_w = 68 \) ft
\( \phi = 0.039 \) (fraction)
\( k = 7.65 \) md
\( n = 5.79 \)

Oil Properties:
\( B_o = 1.136 \) RB/STB
\( 

Production Parameters:
\( q_o = 250 \) STBD
\( t_w = 13,830 \) hrs
\( P_{wf}(\Delta t = 0) = 3534 \) psia

Linear Portion Indicates Wellbore Storage Domination

\[ P_{wf}(\Delta t = 0) = 3534 \text{ psia} \]
MDH Semilog Plot: Pressure Data

Horner Semilog Plot: Pressure Data
**Log-Log Plot: Pressure and Pressure Derivative Data**

*Log-log Plot -- Simple Analysis (Lee Text Example 2.2)*

**Legend:**
- Lee Text Example 2.2
- $\Delta p$ (Symbol: $\Delta p$)
- $L_s=0.05$
- $L_s=0.10$
- $L_s=0.15$

**Wellbore Storage Transition Effects**

**Wellbore Storage Domination:**
- Pressure Drop Functions
- Unit Slope Behavior

**Boundary Effects**

**Horizontal Trend Indicates Radial Flow**

**Shut-In Time, $\Delta t$, hours ($t_f=13,630$ hrs)**

---

**Log-Log Plot: Pressure and Pressure Derivative Data (with Summary Analysis)**

*Log-log Summary Plot -- Lee Text Example 2.2 (Including Simulated Performance)*

**Legend:**
- Data Functions (Lee Text Example 2.2)
- $\Delta p$, $L_s=0.05$
- $\Delta p$, $L_s=0.10$
- $\Delta p$, $L_s=0.15$

**Wellbore Storage Distortion Region**

**Region of Boundary-Dominated Reservoir Performance**

**Pressure Drop Functions, psia**

**Shut-In Time, $\Delta t$, hours ($t_f=13,630$ hrs)**

---

**Data for Lee Example 2.2**
- Reservoir Properties:
  - $c_r=17.0 \times 10^{-6}$ psia$^{-1}$
  - $r_w=0.198$ ft
  - $h_w=60$ ft
  - $e=0.529$ (fraction)

- Oil Properties:
  - $B_o=1.138$ RB/STB
  - $\mu_o=0.8$ cp

- Production Parameters:
  - $q_p=250$ STB/D
  - $p_w(\Delta t=0)=3934$ psia

**Average Pressure Estimates:**
- $p_{avg}$ (Muskat 1 term) = 4408.5 psia
- $p_{avg}$ (Muskat 2 term) = 4408.9 psia
- $p_{avg}$ (RHM/reg) = 4422.8 psia
- $p_{avg}$ (MBH (Lee text)) = 4411.0 psia

**Results for Lee Text Example 2.2:**
- $k=0.65$ md (forced)
- $s=5.79$
- $C_s=562$ ($C_s g^2=9 \times 10^6$)
- $A=139.7$ acres (Well Centered in a Square)
Rectangular Hyperbola Plot (Method 1): Determination of Average Reservoir Pressure

Data for Lee Example 2.2
Reservoir Properties:
- \( r_w = 0.196 \text{ ft} \)
- \( h = 69 \text{ ft} \)
- \( f = 0.036 \text{ (fraction)} \)
- \( k = 7.86 \text{ md} \)
- \( s = 5.79 \)
- \( \mu_o = 0.8 \text{ cp} \)

Oil Properties:
- \( B_o = 1.136 \text{ RB/STB} \)

Production Parameters:
- \( q_p = 250 \text{ STB/D} \)
- \( t_p = 13,630 \text{ hrs} \)
- \( \rho_w = 35534 \text{ psi} \)

Average Pressure Estimates:
- \( p_{avg} \text{ Muskat (1 term)} = 4408.5 \text{ psi} \)
- \( p_{avg} \text{ Muskat (2 term)} = 4408.9 \text{ psi} \)
- \( p_{avg} \text{ RHM (reg)} = 4422.8 \text{ psi} \)
- \( p_{avg} \text{ MBH (Lee text)} = 4411.0 \text{ psi} \)

RHM Straight Line: Method 1
- \( p_{res} + \frac{1}{k} \frac{dp_{res}}{dt} = 4412 - 5.3 \frac{dp_{res}}{dt} \)

Rectangular Hyperbola Plot (Method 2): Determination of Average Reservoir Pressure

Data for Lee Example 2.2
Reservoir Properties:
- \( r_w = 0.196 \text{ ft} \)
- \( h = 69 \text{ ft} \)
- \( f = 0.036 \text{ (fraction)} \)
- \( k = 7.86 \text{ md} \)
- \( s = 5.79 \)
- \( \mu_o = 0.8 \text{ cp} \)

Oil Properties:
- \( B_o = 1.136 \text{ RB/STB} \)

Production Parameters:
- \( q_p = 250 \text{ STB/D} \)
- \( t_p = 13,630 \text{ hrs} \)
- \( \rho_w = 35534 \text{ psi} \)

Average Pressure Estimates:
- \( p_{avg} \text{ Muskat (1 term)} = 4408.5 \text{ psi} \)
- \( p_{avg} \text{ Muskat (2 term)} = 4408.9 \text{ psi} \)
- \( p_{avg} \text{ RHM (reg)} = 4422.8 \text{ psi} \)
- \( p_{avg} \text{ MBH (Lee text)} = 4411.0 \text{ psi} \)

RHM Straight Line: Method 2
- \( p_{res} = 4412 - 29 \frac{dp_{res}}{dt}^{0.5} \)

Correct RHM trend.
Modified Muskat Plot: Determination of Average Reservoir Pressure

Muskat Plotting Function Approach
(Lee Text Example 2.2)

Muskat Equations: (Time Format)
1-Term Muskat Equation:
\[ p_{avg} = 4408.5 - 83.798 \exp(-0.052809 \Delta t) \]
2-Term Muskat Equation:
\[ p_{avg} = 4408.9 - 79.331 \exp(-0.050128 \Delta t) - 58.845 \exp(-0.28359 \Delta t) \]

Average Pressure Estimates:
- \( p_{avg} \) Muskat (1 term) = 4408.5 psia
- \( p_{avg} \) Muskat (2 term) = 4408.9 psia
- \( p_{avg} \) RHM (reg) = 4422.8 psia
- \( p_{avg} \) MSH (Lee text) = 4411.0 psia

Data for Lee Example 2.2
Reservoir Properties:
- \( c_p = 17.0 \times 10^6 \) psia\(^{-1}\)
- \( r_w = 0.198 \) ft
- \( h = 88 \) ft
- \( \phi = 0.039 \) (fraction)
- \( k = 7.65 \) md
- \( n = 0.79 \)

Oil Properties:
- \( B_o = 1.138 \) RB/STB
- \( \mu_o = 8.8 \) cp

Production Parameters:
- \( q = 250 \) STB/D
- \( t_p = 13,820 \) hrs
- \( p_{wf}(t=0) = 3534 \) psia

Symbol Derivative
- \( L = 0.05 \)
- \( L = 0.10 \)
- \( L = 0.15 \)
- Moving Least Squares

Muskat Straight Line: \( (dp_{w}/dt) \) Format
\[ p_{avg} = 4408.5 - 20 \Delta p_{w}/\Delta t \]
Example Analysis: (Lee text (1st edition), Example 2.2)

**Given Data:** (Lee text (1st edition), Example 2.2)

These data are taken from Example 2.2 in the Lee text, *Well Testing*. These data are for a pressure "buildup" test run on an oil (liquid) well.

**Reservoir properties:**
- \( \phi = 0.039 \)
- \( r_w = 0.198 \text{ ft} \)
- \( c_f = 17 \times 10^{-6} \text{ psia}^{-1} \)
- \( h = 69 \text{ ft} \)

**Oil properties:**
- \( B_o = 1.136 \text{ RB/STB} \)
- \( \mu_o = 0.8 \text{ cp} \)

**Production parameters:**
- \( p_{wf}(\text{at } \Delta t = 0) = 3534 \text{ psia} \)
- \( q_o = 250 \text{ STB/D} \)
- \( t_p = 13,630 \text{ hr} \)

**Test Data and Data Functions:**

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<th>( p_{wsi}, \text{ psia} )</th>
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**Required:**

Estimate the following:

- Cartesian analysis of "early" time (wellbore storage distorted) data:
  a. The pressure at the start of the test, \( p_i \).
  b. The wellbore storage coefficient, \( C_s \).

- Semilog analysis of "middle" time (radial flow) data: MDH and Horner Analysis
  a. The formation permeability, \( k \).
  b. The near well skin factor, \( s \).
  c. Estimate the extrapolated pressure, \( p^* \).

- Estimate the average reservoir pressure, \( p \), using:
  a. The Rectangular Hyperbola Method (RHM) plot.
  b. The "Modified Muskat" Method plot.
  c. The Matthews-Brons-Hazebroek (MBH) method.
Log-Log Plot: Pressure and Pressure Derivative Data

Cartesian Plot: Early-Time Pressure Data
MDH Semilog Plot: Pressure Data

Semilog Plot – Lee Text Example 2.2
(Radial Flow Analysis and Average Pressure Determination)

Horner Semilog Plot: Pressure Data

Horner Plot – Lee Text Example 2.2
(Radial Flow Analysis and Average Pressure Determination)
Rectangular Hyperbola Plot (Method 1): Determination of Average Reservoir Pressure

Rectangular Hyperbola Method 1
(Lee Text Example 2.2)

Legend: Lee Text Example 2.2
Symbol Derivative
L=0.05
L=0.10
L=0.15
Moving Least Squares

Rectangular Hyperbola Plot (Method 2): Determination of Average Reservoir Pressure

Rectangular Hyperbola Method 2
(Lee Text Example 2.2)

Legend: Lee Text Example 2.2
Symbol Derivative
L=0.05
L=0.10
L=0.15
Moving Least Squares
Modified Muskat Plot: Determination of Average Reservoir Pressure

Legend: Lee Text Example 2.2
- ● L₀=0.05
- ■ L₀=0.10
- ○ L₀=0.15
- ▲ Moving Least Squares

Muskat Plotting Function Approach
(Lee Text Example 2.2)
Graphical Analysis  
(Lee text (1st edition), Example 2.2)

Preliminary Log-log Analysis:  
As with pressure drawdown analysis, our most important diagnostic plot is the log-log plot. This plot clearly shows the entire history of the test—the pressure drop and pressure drop derivative functions (i.e., the $\Delta p$ and $\Delta p'$ functions) can be used to establish specific flow regimes and to verify parameters estimated from other plots (e.g., $m_{wbs}$ (early-time Cartesian plot) and $m$ (semilog plot)).

From the log-log plot for this case, we can establish the following:

*First Early Time Region ($ETR_1$): Wellbore Storage Domination (Cartesian Analysis)*  

$ETR_1$: $\Delta t < 0.25$ hr  
$m_{ws} = 970$ psia/hr (any $\Delta p$-$\Delta t$ point on the "unit slope" line: $m_{w}=\left(\Delta p/\Delta t\right)_{usi}$)

*Second Early Time Region ($ETR_2$): Wellbore Storage Distortion (No Analysis)*  

$ETR_2$: $0.25 < \Delta t < 7$ hr

*Middle Time Region ($MTR$): Radial Flow Region (Semilog Analysis)*  

$MTR$: $7 < \Delta t < 25$ hr (approximate)  
$\Delta p_{radial\ flow} \approx 30$ psi (with $\Delta t_{start} = 7$ hours and $\Delta t_{end} = 25$ hours)  
$m = 2.303$ (30 psi) = 69.1 psia/cycle  
($m$ result from the log-log plot)

*Late Time Region ($ETR_3$): Boundary Effects ($\bar{p}$ analysis—various methods)*  

$LTR$: $\Delta t > 25$ hr

Early Time Cartesian Analysis:  

*Graphical Analysis: Early Time Cartesian Analysis*  

From the Cartesian plot of early-time pressure data, we extrapolate the early-time linear data trend to obtain:  
$p_{w}(\Delta t=0) = 3534$ psia  
$m_{ws} = 973.3$ psia/hr

*Results: Early Time Cartesian Analysis*  

Recalling the definition of the wellbore storage slope, $m_{wbs}$, we have  

$m_{wbs} = \frac{q_{sur}B}{24\ C_s}$

Solving for the wellbore storage coefficient, $C_s$, we obtain  

$C_s = \frac{q_{sur}B}{24\ m_{wbs}}$

Solving $C_s$ for this particular case, we have  

$C_s = \frac{(250\ STB/D)(1.136\ RB/STB)}{24\ (973.3\ psia/hr)}$

or  

$C_s = 0.0122\ RB/psia$
Early Time Cartesian Analysis: (continued)

Summary:

Early Time Cartesian Analysis

\( p_{wf}(\Delta t=0) = 3534 \text{ psia} \)

\( C_s = 0.0122 \text{ RB/psia} \)

Semilog Analysis: (Horner and MDH Semilog Plots)

Graphical Analysis: Log-log Plot

From the log-log plot we established that the semilog straight line should have the following properties:

Radial flow: \( \Delta t_{start}=7 \text{ hours and } \Delta t_{end}=25 \text{ hours} \)

\( m = 69.1 \text{ psia/cycle} \)

Using the definition of Horner time, we must convert the semilog straight-line criteria (in terms of \( \Delta t \)) to Horner time. Horner time is given by:

\[ HT = \frac{t_p + \Delta t}{\Delta t} \]

Given that \( t_p=13,630 \text{ hr} \), then

\( HT_{start}=1950 \text{ and } HT_{end}=550 \)

Graphical Analysis: Horner Plot

Using \( HT_{start}=1950 \text{ and } HT_{end}=550 \), we construct a straight-line through the pressure data on the Horner plot. The straight line has the following properties:

\( p^* = 4577 \text{ psia} \) (the pressure at \( (t_p + \Delta t)/\Delta t = 1 \))

\( p_{1hr} = 4287.6 \text{ psia} \) (i.e., the pressure at (13,630+1)/1= 13,631)

\( p_{HT=10^2} = 4437 \text{ psia} \) (arbitrary, but should be consistent)

\( p_{HT=10^3} = 4367 \text{ psia} \) (arbitrary, but should be consistent)

Computing the slope from \( p_{HT=10^2} \) and \( p_{HT=10^3} \), we have

\( m = (4437 \text{ psia}) - (4367 \text{ psia}) = 70 \text{ psia/cycle} \) (m result from the semilog plot)

Graphical Analysis: MDH Plot

Using \( \Delta t_{start}=7 \text{ hours and } \Delta t_{end}=25 \text{ hours} \), we construct a straight-line through the pressure data on the MDH plot \( (p_{ws} \text{ vs. } \log(\Delta t)) \). The straight line has the following properties:

\( p_{1hr} = 4287.6 \text{ psia} \)

\( p_{10hr} = 4357.6 \text{ psia} \) (arbitrary, but should be consistent)

\( p_{100hr} = 4427.6 \text{ psia} \) (arbitrary, but should be consistent)

Computing the slope from \( p_{10hr} \) and \( p_{100hr} \), we have

\( m = (4427.6 \text{ psia}) - (4357.6 \text{ psia}) = 70 \text{ psia/cycle} \) (m result from the semilog plot)

As with the previous pressure drawdown example, we compare the slope values \( m \)'s from the semilog and log-log analyses (70 and 69.1, psia/cycle) and conclude that we have good agreement. As we noted before, you are encouraged to "force" these results to be the same, by determining the most representative estimate of the horizontal \( \Delta p \) trend (log-log plot) and the best estimate of the slope of the pressure versus logarithm of time plot, then averaging or judiciously choosing the "forced" \( m \) value.
**Results:** Semilog Analysis

**Formation Permeability:**

Recalling the formula for estimating the formation permeability, we have

\[
k = 162.6 \frac{qB_i}{m h}
\]

Using our graphical analysis results (assuming \(m=70\) psia/cycle) we obtain

\[
k = \frac{162.6 \cdot (250\ \text{STB/D})(1.136\ \text{RB/STB})(0.8\ \text{cp})}{(70\ \text{psia/cycle})(69\ \text{ft})}
\]

or

\[k = 7.65\ \text{md}\]

**Skin Factor: Horner Analysis**

Recalling the formula for estimating the skin factor for the Horner method, we have

\[
s = 1.1513 \left[ \frac{(p_{ws,1hr} - p_{wf}(\Delta t=0))}{m} - \log \left( \frac{t_p}{t_p+1} \right) - \log \left( \frac{k}{\phi \mu c_r r_w^2} \right) + 3.2275 \right]
\]

Using our graphical analysis results (\(m=70\) psia/cycle and \(p_{1hr}=4287.6\) psia) we obtain

\[
s = 1.1513 \left[ \frac{(4287.6\ \text{psia} - 3534\ \text{psia})}{(70\ \text{psia/cycle})} - \log \left( \frac{13,630}{13,630+1} \right) \right.
\]

\[
+ 1.1513 \left[ - \log \left( \frac{(7.65\ \text{md})}{(0.039)(0.8\ \text{cp})(17\times10^{-6}\ \text{psia}^{-1})(0.198\ \text{ft})^2} \right) \right] + 3.2275
\]

or

\[s = 6.25\]

**Skin Factor: MDH Analysis**

Recalling the formula for estimating the skin factor for the MDH method, we have

\[
s = 1.1513 \left[ \frac{(p_{ws,1hr} - p_{wf}(\Delta t=0))}{m} - \log \left( \frac{k}{\phi \mu c_r r_w^2} \right) + 3.2275 \right]
\]

Using our graphical analysis results (\(m=70\) psia/cycle and \(p_{1hr}=4287.6\) psia) we obtain

\[
s = 1.1513 \left[ \frac{(4287.6\ \text{psia} - 3534\ \text{psia})}{(70\ \text{psia/cycle})} \right.
\]

\[
+ 1.1513 \left[ - \log \left( \frac{(7.65\ \text{md})}{(0.039)(0.8\ \text{cp})(17\times10^{-6}\ \text{psia}^{-1})(0.198\ \text{ft})^2} \right) \right] + 3.2275
\]

or

\[s = 6.25\]

**Summary:** Semilog Analysis

\[p^* = 4577\ \text{psia} \quad \text{(Horner Analysis)}\]

\[k = 7.65\ \text{md} \quad \text{(Horner and MDH Analysis)}\]

\[s = 6.25 \quad \text{(Horner and MDH Analysis)}\]
Late-Time Analysis: (Rectangular Hyperbola and Modified Muskat Plots, MBH Method)

**Graphical Analysis: Log-log Plot**

From the log-log plot we established that boundary effects occur as follows:

$$\Delta t_{\text{boundary}} \geq 25 \text{ hours}$$

**Graphical Analysis: Rectangular Hyperbola Plot**

$p_{ws} + \Delta t \frac{d}{d\Delta t}[p_{ws}]$ is plotted versus $\frac{d}{d\Delta t}[p_{ws}]$ to determine $\bar{p}$ as the intercept of the straight-line trend at $\frac{d}{d\Delta t}[p_{ws}] = 0$. Unfortunately, we have no definitive criteria as to the accuracy of the rectangular hyperbola equation, and as such, we are left to our intuition as to whether or not we have found the "proper" straight-line. Making the appropriate plot, we have the following analysis:

$$\bar{p}_{\text{RHM}} = 4412 \text{ psia} \text{ (the "incorrect" trend (both plots) give: } \bar{p}_{\text{RHM}} = 4422.8 \text{ psia)}$$

$$b_{\text{RHM}} = 8.93 \text{ hr}$$

**Graphical Analysis: Modified Muskat Plot**

$p_{ws}$ is plotted versus $\frac{d}{d\Delta t}[p_{ws}]$ to determine $\bar{p}$ as the intercept of the straight-line trend at $\frac{d}{d\Delta t}[p_{ws}] = 0$. We have the following criteria from Cobb and Ramey for analyzing pressure data using the Modified Muskat equation: $t_{pDA} > 0.1$ and $0.05 < \Delta t_{DA} < 0.10$.

As the producing time is very large ($t_p = 13,630 \text{ hr}$), we will assume that $t_{pDA} > 0.1$. However, we must check to see if the $0.05 < \Delta t_{DA} < 0.10$ criteria is met. Recalling the definition of $\Delta t_{DA}$, we have:

$$\Delta t_{DA} = \frac{k}{\phi \mu_c A} \frac{1}{\Delta t}$$

Solving for $\Delta t$, gives us

$$\Delta t = \frac{1}{0.0002637} \frac{\phi \mu_c A}{k} \Delta t_{DA}$$

Assuming that $A = 150 \text{ acres}$ (from Lee, Example 3.2), we have

$$\Delta t = \frac{1}{0.0002637} \frac{(0.039)(0.8 \text{ cp})(17 \times 10^{-6} \text{ psia}^{-1})[150 \text{ acres}](43,560 \text{ ft}^2/\text{acre})}{(7.65 \text{ md})} \Delta t_{DA}$$

or,

$$\Delta t = 1718 \Delta t_{DA}$$

Using this result for the $0.05 < \Delta t_{DA} < 0.10$ criteria, we have

$$85.9 < \Delta t < 171.8 \text{ hr}$$

Therefore, if we adhere rigorously to the Cobb and Ramey criteria, then this problem cannot be analyzed using the Modified Muskat equation (or its resulting plotting functions). However, we can "blindly" make the Modified Muskat plot and perform the appropriate straight-line analysis. This gives the following two cases:

- Line drawn through last 8 data points
  - $\bar{p}_{\text{MM}} = 4408.5 \text{ psia}$
  - $b_{\text{MM}} = 0.05 \text{ hr}$
Late-Time Analysis: (continued)

Graphical Analysis: Modified Muskat Plot

In addition to the single-term Muskat equation (used to develop the plotting function), we can also use a 2 (or more) term form of the Muskat equation—but we must use regression (curve fitting) to determine the trend. The 2-term Muskat equation gives essentially the same estimate as the single-term form—\( \bar{p}_{MM,2-term} = 4408.9 \) psia.

Analysis: Matthews-Brons-Hazebrook (MBH) Method

This approach was developed to "correct" the extrapolated pressure from Horner analysis, \( p^* \), to yield the average reservoir pressure, \( \bar{p} \). The MBH method is derived as a correlation of \( (p^*-\bar{p}) \) versus a dimensionless producing time function. While theoretically rigorous, the main disadvantage of the MBH method is that reservoir size and shape must be known—and the production history (i.e., \( t_p \)) must be well documented.

The MBH method consists of a series of \( y \) versus \( x \) correlations, made on different plots for various reservoir shapes and well configurations. The \( x \) and \( y \) functions are:

\[
x = t_{pDA} = 0.0002637 \frac{k}{\phi \mu c A} t_p
\]

\[
y = \frac{2.303}{m} (p^*-\bar{p})
\]

Calculating \( t_{pDA} \) for our case (again assuming that \( A = 150 \) acres), gives

\[
t_{pDA} = 0.0002637 \frac{(7.65 \text{ md})}{(0.039)(0.8 \text{ cp})(17 \times 10^6 \text{ psia}^{-1})[(150 \text{ acres})(43,560 \text{ ft}^2/\text{acre})]} (13,630 \text{ hr})
\]

or, \( t_{pDA} = 7.93 \) (or, assuming \( A = 160 \) acres, \( t_{pDA} = 7.43 \)).

Using Fig. 2.17 (Lee text, pg. 36) and evaluating the circular reservoir case at \( t_{pDA}=7.93 \), we have

\[
\frac{2.303}{m} (p^*-\bar{p}) = 5.58
\]

Solving for the average reservoir pressure, \( \bar{p} \)

\[
\bar{p} = p^* - \frac{m}{2.303} (5.58)
\]

Substituting our previous results, we have

\[
\bar{p} = (4577 \text{ psia}) - \frac{(70 \text{ psia/cycle})}{2.303} (5.58)
\]

or \( \bar{p} = 4407.4 \) psia.

It is important to note (again) that the MBH method assumes that the following properties are known accurately: reservoir size and shape, and the producing history. This may or may not be practical, and we note that the rectangular hyperbola and Modified Muskat relations make no such assumptions. Unfortunately, the uncertainty in this example may obscure the "best" technique—however, any or all of these approaches should provide reasonable estimates of the average reservoir pressure.

Summary: Late-Time Analysis

\[
\bar{p} = 4412 \text{ psia} \quad \text{(Rectangular Hyperbola Method, wrong trend \( \bar{p} = 4422 \) psia)}
\]

\[
\bar{p} = 4408.5 \text{ psia} \quad \text{(Case 1: Modified Muskat Method, 1-term form)}
\]

\[
\bar{p} = 4408.9 \text{ psia} \quad \text{(Case 2: Modified Muskat Method, 2-term form)}
\]

\[
\bar{p} = 4407.4 \text{ psia} \quad \text{(MBH Method)}
\]
Analysis of Pressure Buildup Tests

- Lee Text Example 2.2
  - Early-Time Cartesian Plot
  - "Horner" Semilog Plot (Includes Rate History)
  - "MDH" Semilog Plot (No Rate History)
  - Log-log Plot (For WBS & Radial Flow)
  - Rectangular Hyperbola Plot ($\bar{p}$)
  - Modified Muskat Plot ($\tilde{p}$)
Example 1 - Lee Text Example 2.2

Cartesian Plot: Early-Time Pressure Data

"Early Time" Cartesian Plot -- Lee Text Example 2.2
(Analysis of Wellbore Storage Dominated Data)

Legend: Lee Text Example 2.2
- Pressure Data

Shut-In Pressure, $p_{ws}$, psia

Shut-In Time, $\Delta t$, hours ($t_p=13,630$ hrs)
Example 1 - Lee Text Example 2.2

MDH Semilog Plot: Pressure Data

Semilog Plot -- Lee Text Example 2.2
(Radial Flow Analysis and Average Pressure Determination)

Legend: Lee Text Example 2.2
- Pressure Data

Data for Lee Example 2.2:
Reservoir Properties:
- \( c_f = 17.0 \times 10^6 \) psia\(^{-1} \)
- \( r_w = 0.198 \) ft
- \( h = 69 \) ft
- \( \phi = 0.039 \) (fraction)
- \( k = 7.65 \) md
- \( s = 5.79 \)

Oil Properties:
- \( B_o = 1.136 \) RB/STB
- \( \mu_o = 0.8 \) cp

Production Parameters:
- \( q_o = 250 \) STB/D
- \( t_p = 13,630 \) hrs
- \( p_{w(i)t=0} = 3534 \) psia
Horner Semilog Plot: Pressure Data

Horner Plot -- Lee Text Example 2.2
(Radial Flow Analysis and Average Pressure Determination)

Legend: Lee Text Example 2.2
○ Pressure Data

Data for Lee Example 2.2:
Reservoir Properties:
$c_f=17.0 \times 10^6$ psia$^{-1}$
$r_w=0.198$ ft
$h=69$ ft
$\phi=0.039$ (fraction)
$k=7.65$ md
$s=5.79$

Oil Properties:
$B_o=1.136$ RB/STB
$\mu_o=0.8$ cp

Production Parameters:
$q_o=250$ STB/D
$t_o=13,630$ hrs
$P_{wo}(t=0)=3534$ psia
Log-Log Plot: Pressure and Pressure Derivative Data

Legend: Lee Text Example 2.2

- $\Delta p$
- Symbol Derivative
  - $L=0.05$
  - $L=0.10$
  - $L=0.15$

Example 1 - Lee Text Example 2.2
Example 1 - Lee Text Example 2.2

**Modified Muskat Plot**: Determination of Average Reservoir Pressure

**Muskat Plotting Function Approach**
*(Lee Text Example 2.2)*

![Muskat Plot Diagram]

**Legend**:
- □ $L=0.05$
- ● $L=0.10$
- ○ $L=0.15$
- ▲ Moving Least Squares

**Variables**:
- $p_w$: psi
- $dp_{ws}/\Delta t$: psi/hr

**Axes**:
- X-axis: Pressure Derivative, $(dp_{ws}/\Delta t)$, psi/hr
- Y-axis: $p_w$, psi
Rectangular Hyperbola Plot (Method 1): Determination of Average Reservoir Pressure (RTH Method is not rigorous)
Rectangular Hyperbola Plot (Method 2): Determination of Average Reservoir Pressure (RTH Method is not rigorous)
Example 3: Oil Well in a Geologically Complex Reservoir System

Given Data:
These data are taken from an oil well in a geologically complex reservoir system—the well is very close to a salt plume (salt dome) in an offshore reservoir.

Reservoir properties:
\[ \phi = 0.30 \quad r_w = 0.28 \text{ ft} \quad c_l = 36.7 \times 10^{-6} \text{ psia}^{-1} \quad h = 59 \text{ ft} \]

Oil properties:
\[ B_o = 1.625 \text{ RB/STB} \quad \mu_o = 0.39 \text{ cp} \]

Production parameters:
\[ p_{wf} (at \Delta t = 0) = 9131 \text{ psia} \quad q_o = 1550 \text{ STB/D} \quad t_p = 120 \text{ hr} \]

Required:
Estimate the following:
- The distance to the fault (flow obstruction) using the "sealed" fault model (i.e., the "Stewart" type curve).
- The distance to the flow obstruction using the radial composite reservoir model (i.e., the "Raghavan/Tang-Brigham" type curve).

Type Curve Analysis: Stewart Type Curve

Type Curve Match:
Matching Parameter, fault configuration (single fault (\( \Delta t \)), parallel faults (\( \Delta t_e \))):
\[ \frac{[t_{DA}/t_{DP}]_{MP}}{[t_{MP}]_{MP}} = 1 \quad [t_{MP}]_{MP} = 6 \text{ hr} \quad \text{(single fault (\( \Delta t \)))} \]
\[ \frac{[t_{DA}/t_{DP}]_{MP}}{[t_{MP}]_{MP}} = 1 \quad [t_{MP}]_{MP} = 11 \text{ hr} \quad \text{(parallel faults (\( \Delta t_e \)))} \]
\[ [p_{MP}]_{MP} = 1 \quad [\Delta p]_{MP} = 10.5 \text{ psi} \quad \text{(fixed for both cases)} \]

Formation Permeability: (as a check)
\[ k = 141.2 \frac{qB\mu}{h} \left[ \frac{p_{wD}}{\Delta p} \right]_{MP} \quad \text{(or} \quad k = 141.2 \frac{qB\mu}{h} \left[ \frac{p_{wD}}{\Delta p} \right]_{MP} \quad \text{)} \]
\[ k = 141.2 \frac{1550 \text{ STB/D}}{(1.625 \text{ RB/D}) \times (0.39 \text{ cp})} \frac{(1.0)}{(59 \text{ ft})} = 223.9 \text{ md} \]

Distance to Flow Obstruction: (Stewart Type Curve—single fault (\( \Delta t \)))
\[ L = \sqrt{\frac{k}{\phi \mu c_l \left[ \frac{t_{MP}}{t_{MP}^2} \right] \left[ \frac{t_{DA}/t_{DP}}{t_{DA}/t_{DP}_{MP}} \right]}} \quad \text{or finally, we have} \]
\[ L = 287.2 \text{ ft} \]
Example 3: Oil Well in a Geologically Complex Reservoir System

Type Curve Analysis: Stewart Type Curve (Continued)

Distance to Flow Obstruction: (Stewart Type Curve—parallel faults (Δτ_e))

\[
L = \sqrt{\frac{k}{\phi \mu c_i} \left[ \frac{t_{\text{MP}}}{t_D/L_D} \right]_{\text{MP}}}
\]

\[
L = \sqrt{0.0002637 \frac{223.9 \text{ md}}{(223.9 \text{ md})} \frac{(11 \text{ hr})}{(0.30) (0.39 \text{ cp}) (36.7 \times 10^{-6} \text{ psia}^{-1})}}
\]

or finally, we have

\[L = 388.9 \text{ ft}\]

Type Curve Analysis: Raghavan/Tang-Brigham Type Curve

Type Curve Match:

• Analysis using: Δt

Matching Parameter, Mobility Ratio (λ)

\[
\eta_r = 1 \quad \lambda = 2 \quad \omega = 2
\]

\[
\left[ \frac{t_{DA}/t_D}{t_{\text{MP}}} \right]_{\text{MP}} = 1 \quad \left[ \frac{t_{\text{MP}}}{t_{\text{MP}}} \right]_{\text{MP}} = 11 \text{ hr} \quad \left[ \Delta P \right]_{\text{MP}} = 10.5 \text{ psi}
\]

• Analysis using: Δτ_e

Matching Parameter, Mobility Ratio (λ)

\[
\eta_r = 1 \times 10^{-3} \quad \lambda = 50 \quad \omega = 0.05
\]

\[
\left[ \frac{t_{DA}/t_D}{t_{\text{MP}}} \right]_{\text{MP}} = 1 \quad \left[ \frac{t_{\text{MP}}}{t_{\text{MP}}} \right]_{\text{MP}} = 11 \text{ hr} \quad \left[ \Delta P \right]_{\text{MP}} = 10.5 \text{ psi}
\]

Formation Permeability: (forced)

\[
k = 141.2 \frac{q B \mu}{h} \left[ \frac{p_w D_{\text{MP}}}{\Delta P} \right]_{\text{MP}} (\text{or } k = 141.2 \frac{q B H}{h} \left[ \frac{p_w D_{\text{MP}}}{\Delta P} \right]_{\text{MP}})
\]

\[
k = 141.2 \frac{(1550 \text{ STB/D}) (1.625 \text{ RB/STB}) (0.39 \text{ cp})}{(59 \text{ ft}) (10.5 \text{ psi})} = 223.9 \text{ md}
\]

Distance to Flow Obstruction: (Raghavan/Tang-Brigham Type Curve—Δt and Δτ_e Analysis)

\[
L = \sqrt{\frac{k}{\phi \mu c_i} \left[ \frac{t_{\text{MP}}}{t_D/L_D} \right]_{\text{MP}}}
\]

\[
L = \sqrt{0.0002637 \frac{223.9 \text{ md}}{(223.9 \text{ md})} \frac{(11 \text{ hr})}{(0.30) (0.39 \text{ cp}) (36.7 \times 10^{-6} \text{ psia}^{-1})}}
\]

or finally, we have

\[L = 388.9 \text{ ft}\]
Example 3: Oil Well in a Geologically Complex Reservoir System

Summary:
We again find that completely different reservoir models (i.e., the "sealed" fault and radial composite reservoir models) yield very similar, and some might say, equally plausible results. Because we know that the well is actually located adjacent to a salt plume (dome), we can defer to the "sealed" fault model. Further, there does not appear to be significant justification for using the radial composite reservoir model, other than the fact that it provides a reasonable match of the data.
Type Curve Analysis--Sealing Faults Model (Matched Using $\Delta t$)
(Well A11 Example Data)

Data Legend: Fault Type Curves
- Single Fault Case
- 2 Perpendicular Faults (2 at 90°)
- 2 Parallel Faults (2 at 180°)
- 3 Perpendicular Faults (3 at 90°)

Results for Well A11: (Matched using $\Delta t$)
$(\Delta p/p_D)_{RF}=10.5$ psi, $(\Delta t(t_D/L_D^2))_{RF}=6$ hrs
$k=223.9$ md, $L_{fault}=287.2$ ft (single fault)
Petroleum Engineering 324 — Well Performance
Type Curve Analysis of Well Test Data Including Boundary Effects
Example 3: Oil Well in a Geologically Complex Reservoir System

**Type Curve Plot:** Stewart Type Curve ("Sealed" Fault Model)—Data Plotted Using Effective Shut-In Time

**Type Curve Analysis--Sealing Faults Model (Matched Using $\Delta t_0$)
(Well A11 Example Data)**

Data Legend: Fault Type Curves
- Single Fault Case
- 2 Perpendicular Faults (2 at $90^\circ$)
- 2 Parallel Faults (2 at $180^\circ$)
- 3 Perpendicular Faults (3 at $90^\circ$)

Results for Well A11: (Matched using $\Delta t_0$)

$(\Delta p/p_0)_{\text{max}} = 10.5$ psi, $(\Delta t(t/L_D)^2)_{\text{max}} = 11$ hrs

$k = 223.9$ md, $L_{\text{fault}} = 388.9$ ft (2 parallel faults)
Petroleum Engineering 324 — Well Performance
Type Curve Analysis of Well Test Data Including Boundary Effects
Example 3: Oil Well in a Geologically Complex Reservoir System

Log-Log Plot: "Sealed" Fault Model—Simulation Results—Data Plotted Using Shut-In Time

Data for Well A-11:
(Buildup Test—Aug. 1995)
Oil Properties:
$B_o=1.625$ RB/STB
$\mu_o=0.39$ cp
Reservoir Properties:
$c_r=3.67 \times 10^{-5}$ psia$^{-1}$
$r_w=0.28$ ft
$h=59$ ft
$\phi=0.30$ (fraction)
Production Parameters:
$q=1550$ STB/D
$t_f=120$ hr
$p_w(\Delta t=0)=9131$ psia

Results for Well A-11:
(Buildup Analysis)
$k=346.4$ md
$S_w=4.02$
$C_g=2.15$ rb/psi
$(C_D=\text{not given})$
$L_{z_{1,\text{Fault}}}=310$ ft
(120 degree fault)

Data Legend:
- Δp Data Function
- Δp' Data Function
- Δp" Data Function
- Simulated Functions
Log-Log Plot: "Sealed" Fault Model—Simulation Results—Data Plotted Using Effective Shut-In Time

**Data for Well A-11:**
(Buildup Test--Aug. 1995)

**Oil Properties:**
- $B_o=1.625$ RB/STB
- $\mu_o=0.39$ cp

**Reservoir Properties:**
- $c_o=3.67 \times 10^{-5}$ psia$^{-1}$
- $r_w=0.28$ ft
- $h=59$ ft
- $\phi=0.30$ (fraction)

**Production Parameters:**
- $q=1550$ STB/D
- $t_p=120$ hr
- $p_w(\Delta t=0)=9131$ psia

**Results for Well A-11:**
(Buildup Analysis)
- $k=346.4$ md
- $S=4.02$
- $C_p=2.15$ rb/psi
- $C_D=37725$
- $L_{ef}=$Fault=280 ft
(parallel faults)

**Data Legend:**
- $\Delta p$ Data Function
- $\Delta p'$ Data Function
- $\Delta p_i$ Data Function
- $\Delta p'_i$ Data Function
--- Computed Functions

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Minor Endpoint Effects in Derivative Functions Caused by Effective Time Function.

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Effective Buildup Time, $\Delta t_e=\Delta t(1+\Delta t/t_p)$, hours ($t_p=120$ hr)
Log-Log Plot: Radial Composite Model—Simulation Results—Data Plotted Using Shut-In Time

Data for Well A-11:
(Buildup Test—Aug. 1995)

Oil Properties:
- $B_o = 1.625$ RB/STB
- $\mu_o = 0.39$ cp

Reservoir Properties:
- $c = 3.67 \times 10^{-5}$ psia$^{-1}$
- $r_o = 0.28$ ft
- $h = 59$ ft
- $\phi = 0.30$ (fraction)

Production Parameters:
- $q = 1550$ STB/D
- $t_p = 120$ hr
- $p_w(\Delta t = 0) = 9131$ psia

Results for Well A11:
(Radial Composite Solution)

- $k_r = 346.4$ md
- $C_o = 33,207$ ($C_o e^{2s} = 10.5$)
- $S = 4.03$
- $R_{1D} = r_1/r_w = 1052$
- or $r_1 = 295$ ft ($r_w = 0.28$ ft)
- $\eta_v = \omega / \lambda = 0.36$
- $\omega = (\phi c) / (\phi c)_2 = 1.14$
- $\lambda = (k_1 / \mu_1) / (k_2 / \mu_2) = 3.18$
- $k_2 = (k_1 / \mu_1) / (\lambda / \mu_2)$
- or $k_2 \approx 279$ md ($\mu_2 \approx 1$ cp)

Data Legend:
- $\Delta p$ Data Function
- $\Delta p'$ Data Function
- $\Delta p''$ Data Function
- $\Delta p'''$ Data Function
- Computed Functions

Radial Composite Reservoir Solution from:
Data for Well A-11:
(Buildup Test--Aug. 1995)
Oil Properties:
\[ B_o = 1.625 \text{ RB/STB} \]
\[ \mu_o = 0.39 \text{ cp} \]
Reservoir Properties:
\[ c_r = 3.67 \times 10^{-5} \text{ psia}^{-1} \]
\[ r_w = 0.28 \text{ ft} \]
\[ h = 59 \text{ ft} \]
\[ \phi = 0.30 \text{ (fraction)} \]
Production Parameters:
\[ q = 1550 \text{ STB/D} \]
\[ t_p = 120 \text{ hr} \]
\[ p_w(\Delta t = 0) = 9131 \text{ psia} \]

Results for Well A-11:
(Radial Composite Solution)
\[ k_r = 346.4 \text{ md} \]
\[ C_o = 33,207 \text{ (} C_o e^{25} = 10.5 \text{)} \]
\[ S_e = 4.03 \]
\[ R_{10} = r_1/r_w = 1020 \]
or \[ r_1 = 285.6 \text{ ft} \text{ (} r_w = 0.28 \text{ ft) } \]
\[ \eta_r = \omega \lambda = 9.55 \times 10^{-4} \]
\[ \omega = (k_r \phi_o)/(k_o \phi_o) = 0.042 \]
\[ \lambda = (k_r/\mu_r)/(k_o/\mu_o) = 44.0 \]
\[ k_2 = (k_r/\mu_r)/(\lambda/\mu_o) \]
or \[ k_2 \approx 20.2 \text{ md} \text{ (} \mu_o = 1 \text{ cp) } \]

Data Legend:
- \( \Delta p \) Data Function
- \( \Delta p' \) Data Function
- \( \Delta p'' \) Data Function
- Computed Functions

Minor Endpoint Effects in Derivative Functions Caused by Effective Time Function.

Radial Composite Reservoir Solution from: