Petroleum Engineering 324
Well Performance
Concepts and Applications in Wellbore Storage Distortion

We pardon to the extent that we love.
— La Rochefoucauld (1665)

Topic: Concepts and Applications in Wellbore Storage Distortion

Objectives: (things you should know and/or be able to do)

- Be familiar with and, based on physical principles, be able to derive the relations to model the phenomena of "wellbore storage." In particular, you should be able to derive the following:

  - **General Rate Relation:**
    
    \[ q_D = \frac{q_{sf}}{q} = 1 - C_D \left[ \frac{dp_{wd}}{dt_D} - \frac{dp_{id}}{dt_D} \right] \] (dimensionless form)

    \[ (q_{sf} - q)B = 24C_s \left[ \frac{dp_{wf}}{dt} - \frac{dp_{ff}}{dt} \right] \] (field units form)

  - **Definition of \( C_s \) for a fluid filled wellbore:**
    \[ C_s = c_{wb} V_{wb} \]

  - **Definition of \( C_s \) for a well with a rising or falling liquid level:**
    \[ C_s = \frac{144g}{5.615} \frac{A_{wb}}{\rho g g_c} \]

  - **Pressure Relations:** (for small times/wellbore storage domination)

    For wellbore storage domination conditions (which only occur at small times) we assume that \( q_{sf} = 0 \) and \( p_{ff} = \text{constant} \). These assumptions yield the following:

    - **Dimensionless Pressure Identity:** (for small times)
      \[ p_{wd} = \frac{t_D}{C_D} \]

    - **Pressure Drawdown and Buildup Relations:** (for small times)
      
      **Drawdown Case**
      \[ p_{wf} = p_i - \frac{qB}{24C_s} t \]

      **Buildup Case**
      \[ p_{ws} = p_{wf}(\Delta t = 0) + \frac{qB}{24C_s} \Delta t \]

  - **Laplace Domain Identity:** (valid for all times)
    \[ \bar{p}_{wd}(u) = \frac{1}{\bar{p}_{id}(u)} + u^2 C_D \]
    \[ \text{where,} \]
    \[ u = \text{Laplace transform parameter. Recall that } \bar{f}(u) = \int_0^\infty e^{-ut} f(t) \, dt. \]
    \[ \bar{p}_{wd}(u) = \text{Laplace transform of the dimensionless wellbore pressure, including wellbore storage and skin effects.} \]
    \[ \bar{p}_{id}(u) = \text{Laplace transform of the dimensionless wellbore pressure, including skin effects, but no wellbore storage effects.} \]
    \[ C_D = \text{Dimensionless wellbore storage coefficient.} \]
Lecture Outline:
- Discussion of wellbore storage models
  - Wellbore filled with a compressible fluid
  - Rising or falling liquid level
  - Dimensionless rate relation
- General expressions for wellbore storage
  - Real space convolution integral form
  - Laplace space form
- Development of approximate solutions using the log approximation for the $p_{sD}$ function (i.e., for radial flow).
  - Explicit form in the Laplace domain
  - Possible approaches to solving this relation (SPE 21826)

**Approximate Solution:** (Constant Approximation for $p_{sD}(t_D) — SPE 21826$)

**Dimensionless Pressure Relation:**

$$p_{wCD}(t_D) = p_{sD}(t_D) \left[ 1 - \exp \left[ -\frac{t_D}{p_{sD}(t_D)C_D} \right] \right]$$

**Dimensionless Pressure Derivative Relation:**

$$p'_{wCD}(t_D) = t_D \frac{d}{dt_D} \left[ p_{wCD}(t_D) \right]$$

or, using the $p_{wCD}$ result, we have

$$p'_{wCD}(t_D) = \frac{p_{wCD}(t_D)}{p_{sD}(t_D)} t_D \frac{d}{dt_D} \left[ p_{sD}(t_D) \right]$$

$$+ \frac{t_D}{C_D} \left[ 1 - \frac{1}{p_{sD}(t_D)} t_D \frac{d}{dt_D} \left[ p_{sD}(t_D) \right] \right] \left[ 1 - \frac{p_{wCD}(t_D)}{p_{sD}(t_D)} \right]$$

Reading Assignment:

**Text Reading:**
- Review Chapters 1 and 4 of the Lee Well Testing text, 1st edition.
  - Chapter 7—The Constant Terminal Rate Solution of the Diffusivity Equation and its Application to Oilwell Testing (Section 7.11).
  - Chapter 4—Oilwell Testing (Sections 4.14, 4.19, 4.21)

**Derivations:**
- Review attached notes.
  - Development and application of concepts in wellbore storage distortion.

**Reference Articles:**
Dimensionless Pressure Plot: Radial Flow with Wellbore Storage and Skin Effects

Dimensionless Pressure Derivative Plot: Radial Flow with Wellbore Storage/Skin Effects
Dimensionless Wellbore Flowrate Plot: Radial Flow with Wellbore Storage/Skin Effects
Dimensionless Pressure Plot: Constant Approximation for the \( p_D(t_D) \) Function

Dimensionless Pressure Derivative Plot: Constant Approximation for the \( p_D(t_D) \) Function
Dimensionless Pressure Plot: Linear Approximation for the $p_{D}(t_{D})$ Function

Dimensionless Pressure Derivative Plot: Linear Approximation for the $p_{D}(t_{D})$ Function
Dimensionless Pressure Plot: Quadratic Approximation for the $p_{sd}(t_D)$ Function

Dimensionless Pressure Derivative Plot: Quadratic Approximation for the $p_{sd}(t_D)$ Function
"Bourdet-Gringarten" Format Type Curve Plot: $p_D$ and $p_D'$ Functions
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"Bourdet-Gringarten" Format Type Curves for Radial Flow with Wellbore Storage and Skin Effects

"Bourdet-Gringarten" Format Type Curve Plot: $p_D$, $p_D'$, and $p_{Dr1}$ Functions

Legend: Radial Flow Type Curves
- $p_D$ Type Curve
- $p_D'$ Type Curve
- $p_{Dr1} = p_D/2p_D'$ Type Curve

Wellbore Storage Domination Region
$p_D$, $p_D' = Unit Slope Line$
and $p_{Dr1} = 1/2$

Wellbore Storage Distortion Region

Radial Flow Region
$p_D' = 1/2$, $p_{Dr1} = p_D$
"Bourdet-Gringarten" Format Type Curve Plot: $p_D$ and $p_D^\prime$ Functions

Type Curve for an Unfractured Well in an Infinite-Acting Homogeneous Reservoir with Wellbore Storage and Skin Effects

Legend: Radial Flow Type Curves
- $p_D$ Type Curve
- $p_D^\prime$ Type Curve

Wellbore Storage Domination Region $p_D = \text{Unit Slope Line}$

Radial Flow Region $p_D^\prime = 1/2$
"Bourdet-Gringarten" Format Type Curve Plot: $p_{Di}$, $p_{Di}'$, and $p_{Di1}$ Functions

Type Curve for an Unfractured Well in an Infinite-Acting Homogeneous Reservoir with Wellbore Storage and Skin Effects

Legend: Radial Flow Type Curves
- $p_{Di}$ Type Curve
- $p_{Di}'$ Type Curve
- $p_{Di1} = p_{Di}/2p_{Di}'$ Type Curve

Wellbore Storage Domination Region
$p_{Di}$, $p_{Di}'$ = Unit Slope Line
and $p_{Di1} = 1/2$

Radial Flow Region
$p_{Di}' = 1/2$, $p_{Di1} = p_{Di}$

Wellbore Storage Distortion Region

$t_{Di}/C_{Di}$ vs. $t_{Di}/C_{Di}$
"Bourdet-Gringarten" Format Type Curve Plot: $p_{Di}$ and $p_{Di}’$ Functions

Legend: Radial Flow Type Curves
- $p_{Di}$ Type Curve
- $p_{Di}’$ Type Curve

Type Curve for an Unfractured Well in an
Infinite-Acting Homogeneous Reservoir with
Wellbore Storage and Skin Effects

Wellbore Storage
Domination Region
$p_{Di}$, $p_{Di}’$ = Unit Slope Line

Wellbore Storage
Distortion Region

Radial Flow Region
$p_{D} = 1/2$
"Bourdet-Gringarten" Format Type Curve Plot: $p_D$ and $p_{Dir2}$ Functions

Legend: Radial Flow Type Curves
- $p_D$ Type Curve
- $p_{Dir2} = p_D^{1/2}$ Type Curve

Wellbore Storage Domination Region
$p_D = \text{Unit Slope Line}$
and $p_{Dir2} = 1/2$

Radial Flow Region
$p_{Dir2} = 1$

Wellbore Storage Distortion Region
Development and Application of Concepts in Wellbore Storage Distortion

(from Petroleum Engineering 620 Course Notes -- 1994)

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Well Performance
Development and Application of Concepts in Wellbore Storage Distortion

The issue of wellbore storage distortion virtually dictates the analysis and interpretation of well test data—particularly at early times, when wellbore storage effects can dominate the wellbore pressure response. In order to resolve issues related to wellbore storage, we must develop a mathematical model and apply this model to observe the various aspects of wellbore storage distorted pressure behavior.

The mass balance in the wellbore is given as

$$m_{in} - m_{out} = \frac{dm}{dt}$$  \hspace{1cm} (1)

where

$$m_{in} = \left(5.615 \, \frac{ft^3}{bbl} \right) \left( \frac{lbm}{ft^3} \right) \left( q_{in} \right) \left( 528 \, \frac{lbm}{bbl} \right)$$

$$m_{out} = \left(5.615 \, \frac{ft^3}{bbl} \right) \left( \frac{lbm}{ft^3} \right) \left( q_{out} \right) \left( 528 \, \frac{lbm}{bbl} \right)$$

$$m = \left( V_{wb} \, 528 \, \frac{lbm}{bbl} \right) \left( 528 \, \frac{lbm}{bbl} \right) \left( 5.615 \, \frac{ft^3}{bbl} \right)$$

Substituting these relations into the mass balance relation (Eq. 1)

$$\left[ 5.615 \, \frac{ft^3}{bbl} \right] \left[ \frac{lbm}{ft^3} \right] \left[ \frac{lbm}{bbl} \right] = \left[ 5.615 \, \frac{ft^3}{bbl} \right] \left[ \frac{lbm}{ft^3} \right] \left[ \frac{lbm}{bbl} \right] \left( q_{in} - q_{out} \right) \left( \frac{lbm}{bbl} \right) \left( \frac{lbm}{bbl} \right) \left( 528 \, \frac{lbm}{bbl} \right)$$

Cancelling like terms gives

$$\left( q_{in} - q_{out} \right) \left( 528 \, \frac{lbm}{bbl} \right) \left( \frac{lbm}{bbl} \right) = 24 \, \frac{ft}{hr} \left( V_{wb} \right) \left( \frac{lbm}{bbl} \right) \left( \frac{lbm}{bbl} \right) \left( 528 \, \frac{lbm}{bbl} \right)$$  \hspace{1cm} (2)

where $V_{wb}$ is given by the shaded area in the figure shown above.
There are two cases of interest to us. The first case is that of a fluid-filled wellbore as illustrated on the previous page. The second case is that of pumping well with a rising or falling liquid level, as shown below.

The two cases both require detailed knowledge of the \( \frac{dV_{wb}}{dt} \) behavior—in other words, we need mathematical expressions for the \( \frac{dV_{wb}}{dt} \) terms in order to develop the mass balance relations for each case.

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**Case 1: Fluid-Filled Wellbore**

\[
\Delta V_{wb} = (p_i - p) c_{wb} V_{wb} \tag{3}
\]

where Eq. 3 is the "material balance" equation for a slightly compressible liquid.

Taking the limit of Eq. 3 with respect to a time differential gives

\[
\lim_{\Delta t \to 0} \frac{\Delta V_{wb}}{\Delta t} = c_{wb} V_{wb} \lim_{\Delta t \to 0} \frac{(p_i - p)}{\Delta t}
\]

or

\[
\frac{dV_{wb}}{dt} = -c_{wb} V_{wb} \frac{dp}{dt} \tag{4}
\]

From pseudo-steady-state, we have \( p = p_{sf} + c_3 \); yields \( \frac{dp}{dt} = \frac{dp_{sf}}{dt} \)

Combining Eqs. 4 and 5 gives

\[
\frac{dV_{wb}}{dt} = c_{wb} V_{wb} \left[ \frac{dp_{sf}}{dt} - \frac{dp_{wf}}{dt} \right] \tag{6}
\]

Substituting Eq. 6 into Eq. 2 and using \( q_{in} = q_{sf} \) and \( q_{out} = q \) gives

\[
(q_{sf} - q) B = 24 c_{wb} V_{wb} \left[ \frac{dp_{sf}}{dt} - \frac{dp_{wf}}{dt} \right] \tag{7}
\]
Defining the wellbore storage constant for this case
\[ C_s = Q_{wb} V_{wb} \]
Substituting Eq. 8 into Eq. 7 gives
\[ (Q_{sf} - Q) \frac{dP_{sf}}{dt} = \frac{dP_{t}}{dt} \]
\[ 24C_s \frac{dV_{wb}}{dt} = \frac{dP_{t}}{dt} \]

Case 2: Rising or Falling Liquid Level
\[ \Delta V_{wb} = \frac{1}{5.615 \text{ ft}^3 \text{ bbl}^{-1}} \cdot \text{Awb} \frac{1}{\text{ft}^2} \Delta z \frac{1}{\text{ft}} \]

where Eq. 10 represents the volumetric change in liquid for a given a change in the fluid level, \( \Delta z \). Dividing by \( \Delta t \) and taking the limit as \( \Delta t \to 0 \) gives
\[ \lim_{\Delta t \to 0} \frac{\Delta V_{wb}}{\Delta t} = \frac{1}{5.615} \frac{\text{Awb}}{\text{ft}^2} \lim_{\Delta t \to 0} \frac{\Delta z}{\Delta t} \]
or
\[ \frac{dV_{wb}}{dt} = \frac{1}{5.615} \frac{\text{Awb}}{\text{ft}^2} \frac{d\Delta z}{dt} \]

for a rising/falling liquid column the pressure profiles are given by
\[ P_{wf} = P_{sf} + \frac{1}{144 \text{ ft}^{-2}} \frac{1}{\text{in}^2} \frac{\text{lbm}}{\text{ft}^2} \frac{9 \text{ ft}^2}{\text{sec}^2} \Delta z \frac{1}{\text{ft}} \]

Solving for \( \Delta z \)
\[ \Delta z = \frac{144}{9} \frac{g}{q} \frac{\text{lbm-ft}}{\text{sec}^2} \frac{P_{wf} - P_{sf}}{q} \]

Dividing through Eq. 12 by \( \Delta t \) and taking the limit as \( \Delta t \to 0 \), we have
\[ \lim_{\Delta t \to 0} \frac{\Delta z}{\Delta t} = \frac{144}{9} \frac{g}{q} \frac{\text{lbm-ft}}{\text{sec}^2} \lim_{\Delta t \to 0} \frac{P_{wf} - P_{sf}}{\Delta t} \]
or
\[ \frac{dz}{dt} = \frac{144}{9} \frac{g}{q} \frac{\text{lbm-ft}}{\text{sec}^2} \left( \frac{dP_{wf} - dP_{t}}{dt} \right) \]

Substituting Eq. 13 into Eq. 11
\[ \frac{dV_{wb}}{dt} = \frac{144}{5.615} \frac{\text{Awb}}{\text{ft}^2} \frac{9 \text{ ft}^2}{\text{sec}^2} \left( \frac{dP_{wf} - dP_{t}}{dt} \right) \]
Substituting Eq. 14 into Eq. 2 and using \( Q_{in} = Q_{sf} \) and \( Q_{out} = Q \) gives
\[
(Q_{sf} - Q)B = 2Q \left( \frac{144}{5.615} \frac{A_w b}{g \rho_{w c}} \right) \left[ \frac{dP_w}{dt} - \frac{dP_{nt}}{dt} \right]
\]
(15)

Defining the wellbore storage constant for this case we have
\[
C_s = \frac{144}{5.615} \frac{A_w b}{g \rho_{w c}}
\]
(16)

Substituting Eq. 16 into Eq. 15 gives
\[
(Q_{sf} - Q)B = 24 C_s \left[ \frac{dP_w}{dt} - \frac{dP_{nt}}{dt} \right]
\]
(17)

where Eqs. 9 and 17 are identical. Using this relation as a general form we can manipulate this result into an inner boundary (rate) condition. Rearranging gives
\[
Q_{sf} = Q + 24 C_s \left[ \frac{dP_w}{dt} - \frac{dP_{nt}}{dt} \right]
\]

or dividing through by the surface flowrate, \( Q \), we obtain
\[
\frac{Q_{sf}}{Q} = 1 + 24 C_s \left[ \frac{dP_w}{dt} - \frac{dP_{nt}}{dt} \right]
\]
(18)

where
\[
\frac{Q_{sf}}{Q} = \frac{Q_{sf}}{Q} \quad \text{(a rate ratio)}
\]
(19)

Recalling the definitions of dimensionless time and dimensionless pressure we have
\[
t_D = t_{dc} \frac{k}{\mu Q_w} t \quad (20) \quad \beta_D = \beta_{dc} \frac{k h (p_i - p)}{Q_w u}
\]
(21)

where \( t_{dc} \) and \( \beta_{dc} \) are given by

<table>
<thead>
<tr>
<th>Darcy Units</th>
<th>Field Units</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{dc} )</td>
<td>1</td>
<td>2.637\times10^{-4}</td>
</tr>
<tr>
<td>( \beta_{dc} )</td>
<td>2\pi</td>
<td>7.081\times10^{-8}</td>
</tr>
<tr>
<td>( 1/\beta_{dc} )</td>
<td>1/2\pi</td>
<td>141.2</td>
</tr>
</tbody>
</table>

Solving Eq. 20 for \( t \) and Eq. 21 for \( p \) gives
\[
t = \frac{1}{t_{dc}} \frac{dP_w}{dt} \quad (22)
\]

\[
p = p_i - \frac{1}{\beta_{dc} \frac{k h}{Q_w u}} \beta_D
\]
(23)
considering a generic $dp/dt$ term and substituting Eqs. 22 and 23
\[
\frac{dp}{dt} = d \left[ \frac{1}{\frac{1}{T_{dc}} + \frac{q_{8}}{\phi h c r_w^2 \frac{dP}{dP}} \frac{dp}{dt}} \right] = -\frac{1}{T_{dc}} \frac{q_{8}}{\phi h c r_w^2 \frac{dP}{dP}} \frac{dp}{dt}
\]
which reduces to
\[
\frac{dp}{dt} = -\frac{t_{dc}}{T_{dc}} \frac{q_{8}}{\phi h c r_w^2} \frac{dp}{dt}
\]  
(24)

Substituting Eq. 24 into Eq. 18
\[
\frac{q_{d}}{t_{dc}} = 1 + \left( \frac{24 C_5}{q_{8}} \right) \left( -\frac{t_{dc}}{T_{dc}} \frac{q_{8}}{\phi h c r_w^2} \frac{dp}{dt} \right) \left[ \frac{dp_{w}}{dt} - \frac{dp_{D}}{dt} \right]
\]
or
\[
\frac{q_{d}}{t_{dc}} = 1 - \frac{t_{dc}}{T_{dc}} \frac{24 C_5}{\phi h c r_w^2} \left[ \frac{dp_{w}}{dt} - \frac{dp_{D}}{dt} \right]
\]

Defining the dimensionless wellbore storage coefficient
\[
C_{D} = \frac{t_{dc}}{T_{dc}} \frac{24 C_5}{\phi h c r_w^2}
\]
which gives
\[
\frac{q_{d}}{t_{dc}} = 1 - C_{D} \left[ \frac{dp_{w}}{dt} - \frac{dp_{D}}{dt} \right]
\]  
(25)

Eq. 26 is the general dimensionless rate relation for the so-called "wellbore phase redistribution" case as defined by Fair, W.B.: "Pressure Buildup Analysis with Wellbore Phase Redistribution," SPEJ (April 1981) p. 269-270. In the Fair article the tubing pressure term was dubbed the "phase redistribution pressure" and carried a semi-empirical definition.

Assuming a constant tubing pressure we have
\[
\frac{q_{d}}{t_{dc}} = 1 + \frac{24 C_5}{q_{8}} \frac{dp_{w}}{dt}
\]  
(27)
or
\[
\frac{q_{d}}{t_{dc}} = 1 - C_{D} \frac{dp_{w}}{dt}
\]  
(28)
Note that at early times \( q_f \equiv 0 \) and \( q_d \equiv 0 \). Combining this observation with Eq. 27 gives

\[
0 = l + 24 c_s \frac{dpwf}{dt} \quad (t \rightarrow 0)
\]

separating and isolating the \( dpwf \) term

\[
dpwf = -\frac{q_b}{24c_s} \quad (t \rightarrow 0) \tag{29}
\]

Integrating

\[
p_i - pwf = \frac{q_b}{24c_s} \int_0^t dt \quad \text{or}
\]

\[
p_i - pwf = \frac{q_b}{24c_s} t \quad (t \rightarrow 0) \tag{30a}
\]

or

\[
pwf = p_i - \frac{q_b}{24c_s} t \quad (t \rightarrow 0) \tag{30b}
\]

Eq. 30b shows that at small times \((t \rightarrow 0)\), for wellbore storage distorted data, a plot of \( pwf \) versus time will yield a straight line of intercept, \( p_i \). This plot allows us to verify the initial pressure for a given drawdown or buildup tests. This concept is illustrated below.

### Drawdown

![Drawdown Diagram]

Using

\[
\Delta p = \frac{1}{Pec} \frac{q_{sw}}{kh} P_i, \quad t = \frac{1}{t_{dc}} \frac{\mu c r_i^2}{k} t_d, \quad \text{and} \quad C_s = \frac{Pec}{t_{dc}} \frac{\mu c r_i^2}{24} c_i \text{in Eq. 30a}
\]

\[
\frac{1}{Pec} \frac{q_{sw}}{kh} P_i = \frac{q_b}{24} \left( \frac{t_{dc}}{Pec} \frac{24}{\mu c r_i^2} \frac{1}{k} \right) \left( \frac{1}{t_{dc}} \frac{\mu c r_i^2}{k} \right)
\]

which gives

\[
B = t_d / c_d \quad (t_d \rightarrow 0, \text{Unit Slope Line}) \tag{30c}
\]

### Buildup

![Buildup Diagram]
Summary of Results

General Rate Relation

\[
\frac{Q_{wf} - Q_{w}}{t_{wf} - t} = 24 C_5 \left[ \frac{dP_{w0}}{dt} - \frac{dP_0}{dt} \right]
\]
(9) or (17)

Definition of Wellbore Storage Constants

\[ C_5 = C_w V_{wb} \quad \text{Fluid-Filled Wellbore Case} \quad (8) \]

\[ C_5 = \frac{144}{5.615} \frac{A_{wb}}{y(9/86)} \quad \text{Rising/Falling Liquid Level Case} \quad (16) \]

Dimensionless Rate Relation

\[ \Phi_D = 1 - C_D \left[ \frac{dP_{w0}}{dD} - \frac{dP_D}{dD} \right] \quad (26) \]

Early Time Pressure Relation

\[ P_{wf} = P_i - Q_0 \frac{t - t_0}{24 C_5} \quad (30b) \quad \text{or} \quad \frac{\bar{P}_D}{C_0} = \frac{t}{t_{d-0}} \quad (30c) \]

Application of Wellbore Storage Concepts in Convolution

Recalling the convolution integral for a continuously changing flowrate

\[ P_{w0}(t_0) = \int_0^{t_0} q'(t) P_{d0}(t_0 - t) dt \quad (31) \]

where

\[ P_{d0} = P_0 + 1' \quad (32) \]

where 1' is the dimensionless skin factor. Taking the Laplace transform of Eq. 31 gives

\[ \bar{P}_{w0}(m) = \left[ m \bar{P}_D(m) - P_0(t_0=0) \right] \bar{P}_{d0}(m) \]

which reduces to

\[ \bar{P}_{w0}(m) = m \bar{P}_D(m) \bar{P}_{d0}(m) \quad (83) \]

Taking the Laplace transform of Eq. 26 we have

\[ \bar{q}_D(m) = \frac{1}{m} - C_D \left[ m \bar{P}_{w0}(m) - P_0(t_0=0) \right] \]

which reduces to

\[ \bar{q}_D(m) = \frac{1}{m} - C_D m \bar{P}_{w0}(m) \quad (34) \]
Substituting Eq. 34 into Eq. 33 we have
\[ \bar{P}_{wD}(u) = \mu \left[ \frac{1}{\bar{P}_{wD}(u)} - c_D \bar{P}_{wD}(u) \right] \bar{P}_{sd}(u) \]

Multiplying through by \( \frac{1}{\bar{P}_{wD}(u)} \) and \( \frac{1}{\bar{P}_{sd}(u)} \) gives
\[ \frac{1}{\bar{P}_{sd}(u)} = \frac{1}{\bar{P}_{wD}(u)} - c_D \mu^2 \]

Or solving for \( \bar{P}_{wD}(u) \) we obtain
\[ \bar{P}_{wD}(u) = \frac{1}{1 + c_D \mu^2} \]

Recalling our Laplace domain solutions for the infinite-acting radial flow case (line source solution) we have at the wellbore (i.e., \( \nu_0 = 1 \))
\[ \bar{P}_{sd}(u) = \frac{1}{\mu} \ln \left( \frac{\sqrt{\mu}}{\nu} \right) + \frac{\nu'}{\mu} \]

And
\[ \bar{P}_{sd}(u) = \frac{1}{2\mu} \ln \left( \frac{4 - 1 \nu}{\nu} \right) + \frac{\nu'}{\mu} \]

Substituting Eq. 36 into Eq. 35 gives us
\[ \bar{P}_{wD}(u) = \frac{1}{1 + c_D \mu^2} \]

Where it is very unlikely that we can analytically invert Eq. 38 into any reasonably usable form. Agrawal, Al-Hussainy, and Ramzy, "Investigation of Wellbore Storage and Skin Effect in Unsteady Liquid Flow: I. Analytical Treatment," SPEJ (Sept. 1970), 278-290, give the following solution for Eq. 38, via the use of the residue theorem
\[ \bar{P}_{wD}(\sigma', \tau, \theta) = \int_0^\infty \frac{[1 - \exp(-\mu^2 \tau)] J_0(\mu)}{\bar{P}_{wD}(\sigma, \tau, \theta)} \left\{ [1 - \mu^2 \tau \sigma' + \frac{1}{2} \pi \mu^2 \tau \sigma' J_0(\mu)]^2 + \left[ \frac{1}{2} \pi \sigma' \mu^2 J_0(\mu) \right]^2 \right\} d\mu \]

Where obviously, the form given by Eq. 39 is of little use for practical applications.
Substituting Eq. 37 (the log approximation) into Eq. 35 holds some promise -- but no unique result as we will see

\[ \phi_w(\mu) = \frac{1}{\frac{1}{2} \ln\left(\frac{\phi}{\phi_0} \frac{1}{\mu^2} \right) + \delta} + c_\phi \mu^2 \]  

or collecting we have

\[ \phi_w(\mu) = \frac{1}{\frac{1}{2} \ln\left(\frac{C}{\mu} \right) + c_\phi \mu^2} \]  

where

\[ C = \frac{A}{e^{2\gamma} e^{-2\beta}} \]  

Unfortunately, the form illustrated by Eq. 40 does not enlighten us as to an explicit inverse form. Considerable effort has been expended to resolve Eq. 40, but to no avail. The paper by Blasingame, "Advances in the use of Convolution Methods in Well Test Analysis," paper SPE 21826, presented at the 1991 Denver SPE meeting provides several approximations for modeling wellbore storage distorted pressure data.

Recalling the Schapery approach for approximate inversion we have

\[ f(t) = \mu \tilde{f}(\mu) \quad \text{where } \mu = \frac{t}{t} \quad \text{where } \mu = \frac{t}{t} \text{ for linear } \tau \text{ case} \]

\[ \mu = \frac{t}{t} \text{ for log } (t) \text{ case} \]

So the dilemma is how to choose \( \mu \) ? For purposes of argument let's just use \( \mu = 1 \) for \( c_0 \) term and \( \mu = t \) for \( \ln(\mu) \) term. Using this in Eq. 39 gives

\[ \phi_w(\mu, c_0, t_0) = \frac{1}{\frac{1}{2} \ln\left(\frac{e^2}{e^2} \frac{t_0}{c_0} \right) + \delta} + c_0 \frac{t_0}{c_0} \]  

where \( c_0(\delta, c_0, t_0) \) is the remainder or error term.
where we note that the generalized form of Eq. 43 is given by

\[ P_{w0}(s', t_d) = \frac{1}{P_{o}(t_d) + \frac{C_P}{t_d}} \quad (44) \]

We do not propose Eq. 44 as a general solution, but rather as a possible analysis and interpretation mechanism.