1. Natural Logarithm Function: \( \ln(1)=0, \ln[\exp(x)]=x, \ln(\infty)=\infty \)

**Integral Definition:**
\[
\ln(x) = \int \frac{1}{x} \, dx
\]

**Derivative Definition:**
\[
\frac{d}{dx} \ln(x) = \frac{1}{x}
\]

**Evaluate:**
\[
I = \int \frac{1}{a} \, dx
\]

Use integral property:
\[
\int_a^b f(x) \, dx = \int_c^b f(x) \, dx - \int_c^a f(x) \, dx
\]

Assume: \( c = 1 \quad f(x) = 1/x \)

\[
\int_1^b f(x) \, dx - \int_1^a f(x) \, dx
\]

From definition:
\[
\ln(x) = \int_1^x \frac{1}{x} \, dx ; \quad \int_1^b 1/x \, dx = \ln(b) \quad \text{and} \quad \int_1^a 1/x \, dx = \ln(a)
\]

Therefore:
\[
\int_a^b 1/x \, dx = \ln(b) - \ln(a)
\]

Ans. \( I = \int \frac{1}{a} \, dx = \) \( \) (Show all work)

2. Exponential Function: \( \exp(0)=1, \exp(1)=1.7182818284 ..., \exp(\infty)=\infty, \exp(-\infty)=0 \)

**Integral Definition:**
\[
\int \exp(ax) \, dx = \frac{1}{a} \exp(ax) + C
\]

**Derivative Definition:**
\[
\frac{d}{dx} \exp(ax) = a \exp(ax)
\]

**Evaluate:**
\[
\int_0^x \exp(-ax) \, dx = ?
\]

\[
\int_0^x \exp(-ax) \, dx = -\frac{1}{a} \exp(-ax) \bigg|_0^x = -\frac{1}{a} [\exp(-ax) - \exp(-a(0))]
\]

\[
= \frac{1}{a} [1 - \exp(-ax)]
\]

Ans. \( \int_0^x \exp(-ax) \, dx = \) \( \) (Show all work)
3. Solve the following:

\[ \int_{1}^{e} \frac{\ln(x)}{x^2} dx = ? \]

**Hint:**

Integration by parts. Let \( u \) and \( v \) be differentiable functions. According to the product rule for differentials

\[ d(uv) = u dv + v du \]

Upon taking the antiderivative of both sides of the equation, we obtain

\[ uv = \int u dv + \int v du \]

This is the formula for integration by parts when written in the form

\[ \int u dv = uv - \int v du \quad \text{or} \quad \int f(x)g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx \]

where \( u = f(x) \) and \( v = g(x) \). The corresponding result for definite integrals over the interval \([a, b]\) is certainly valid if \( f(x) \) and \( g(x) \) are continuous and have continuous derivatives in \([a, b]\). See Problems 5.17 to 5.19.


Defining \( u \) and \( v \) parameters:

\[ u = \ln(x) \quad \rightarrow \quad du = dx / x \]

\[ dv = dx / x^2 \quad \rightarrow \quad v = -1 / x \]

Using the definition of integration by parts:

\[ \int_{1}^{e} \frac{\ln(x)}{x^2} dx = -\frac{\ln(x)}{x} \bigg|_{1}^{e} + \int_{1}^{e} \frac{1}{x^2} dx = -\left[\ln(e) / e - \ln(1) / 1\right] - \frac{1}{x} \bigg|_{1}^{e} \]

\[ = - \frac{1}{e} - \frac{1}{e} + 1 \]

\[ = 1 - \frac{2}{e} \]
4. Answer the following questions true or false (T/F).

   ____ F  The relationship between porosity and permeability can best be described by a power law model.
   ____ F  The estimates of permeability from core samples are usually higher than the estimates from pressure transient analysis.
   ____ T  Darcy's law indicates that flowrate and permeability have a linear relationship holding all other parameters constant.
   ____ T  Performing a pressure transient analysis requires the measurements of the (shut-in) pressures as well as the production history of the tested well prior to the test.
   ____ F  Averaging the high-frequency pressure and flowrate data is an efficient method for well test analysis/interpretation.
5. Data Models:
   a. Semilog (y-axis) Coordinates: \( k = b_{yl} \exp(m_{yl}\phi) \)

   ![East Tx Gas Well Core Data](image)

   Ans. Slope \((m_{yl})\): __________ 1/(fraction)  
   Intercept \((b_{yl})\): ____________________ (md)

   \[
   m_{yl} = \frac{\ln(k_2) - \ln(k_1)}{\phi_2 - \phi_1} \\
   = \frac{2.303 \log(k_2) - 2.303 \log(k_1)}{\phi_2 - \phi_1} \\
   m_{yl} = \frac{2.303 \log(0.605) - 2.303 \log(0.00055)}{0.10 - 0.00} = 70 \text{ (1/fraction)}
   \]

   \( b_{yl} = 0.00055 \text{ md} \)

   Note that \( \ln(x) = 2.303 \log(x) \)
b. Log-Log Coordinates: $\Delta p = \alpha t^\beta$

Ans. Slope ($\beta$): __________ (no units)  
Intercept ($\alpha$): _____________________ (psi)

$$\beta = \frac{\ln(\Delta p_2) - \ln(\Delta p_1)}{\ln(t_2) - \ln(t_1)}$$
$$\beta = \frac{2.303 \log(\Delta p_2) - \log(\Delta p_1)}{2.303 \log(t_2) - \log(t_1)}$$
$$\beta = \frac{\log(240) - \log(0.003)}{\log(10^2) - \log(10^{-3})} = 0.98$$

$\alpha = 2.65$ psi  
(note that $\Delta p_{1\text{hr}} = 2.65$ psi)