Problem:

(20 pts) 1. You are to PLOT the steady-state linear flow relation in cartesian coordinates, IDENTIFY the slope, intercept and the direction of the flow. Use the equation given below.

\[ p_x = p_w + \frac{1}{c_x} \frac{q_{sc}B_{\mu}}{kA} x \]  

(Darcy's law (linear flow)) ............................................................. (1)

(30 pts) 2. You are to PLOT the steady-state radial flow relation in cartesian AND semi-log coordinates, IDENTIFY the slope, intercept (for the semi-log plot) and the direction of the flow. Use the equation given below.

\[ p_r = p_w + \frac{1}{c_r} \frac{q_{sc}B_{\mu}}{kh} \ln\left(\frac{r}{r_w}\right) \]  

(Darcy's law (radial flow)) ............................................................. (2)
(50 pts) 3. You are given Darcy’s Law in differential form for a finite (steady-state) radial reservoir system: (Darcy units)
\[ q = \frac{2\pi h}{B\mu} k(r) r \frac{dp}{dr} \]
(liquid case — \( B \) and \( \mu \) are constants)

\( k(r) \) indicates a radial distribution of permeability.
Specific to this case, we have:
for \( r_w < r < r_s \): \( k(r) = k_s \) (altered permeability)
for \( r_s < r < r_e \): \( k(r) = k \) (unaltered permeability)

**Required:**

a. Derive an expression for reservoir pressure (\( p_r \)) — for \( r_s < r < r_e \) (outside of the "altered" or "skin" zone).

b. Explain the behavior of the reservoir pressure (\( p_r \)) result for:
   - \( k_s = k \): No damage or stimulation.
   - \( k_s < k \): Damage case (what is the maximum damage case?)
   - \( k_s > k \): Stimulation case (what is the maximum stimulation case?)

Starting:
\[ q = \frac{2\pi h}{B\mu} k(r) r \frac{dp}{dr} \]
Rearranging:
\[ \frac{1}{k(r)} \frac{1}{r} \frac{dp}{dr} = \frac{2\pi h}{qB\mu} \]
Recalling:
\( k(r) = k_s \); for \( r_w < r < r_s \)
\( k(r) = k \); for \( r_s < r < r_e \)

Using these criteria, and expanding for integration:
\[ \frac{1}{k_s} \int_{r_w}^{r_s} \frac{1}{r} \frac{dr}{r} + \frac{1}{k} \int_{r_s}^{r_e} \frac{1}{r} \frac{dr}{r} = \frac{2\pi h}{qB\mu} \int p_r \frac{dp}{p_w} \]

Completing the integration, we have:
\[ \frac{1}{k_s} \ln \left( \frac{r_s}{r_w} \right) + \frac{1}{k} \ln \left( \frac{r}{r_s} \right) = \frac{2\pi h}{qB\mu} (p_r - p_w) \]

Multiplying through by the permeability, \( k \), we obtain the following result:
\[ \frac{k}{k_s} \ln \left( \frac{r_s}{r_w} \right) + \ln \left( \frac{r}{r_s} \right) = \frac{2\pi kh}{qB\mu} (p_r - p_w) \]

Solving for the pressure, \( p_r \), yields
\[ p_r = p_w + \frac{qB\mu}{2\pi kh} \left( \frac{k}{k_s} \ln \left( \frac{r_s}{r_w} \right) + \ln \left( \frac{r}{r_s} \right) \right) \]

Comment:
   - \( k_s = k \): No damage or stimulation.
   - \( k_s < k \): Maximum damage \( \rightarrow k_s = 0 \).
   - \( k_s > k \): Maximum stimulation \( \rightarrow k_s = \infty \).