Orientation:
This is a very straightforward examination. You have 1 problem, an application of the gas material balance equation to a field data set. However, you must realize that this problem represents the majority of your course grade — therefore, you must provide your absolute best effort.

I would also like for you to prepare an extended outline of your course notes (notes you have taken in class) — probably something on the order of 5-6 pages, handwritten. Imagine that you are only permitted 5-6 pages of study notes for an exam on the entire course. This is what I would like for you to submit. This will also be a component of your grade.

Because of the nature of the examination, you are strictly forbidden from working together in any capacity — sharing notes, sharing work, etc. are expressly forbidden. I understand that conversations occur as to clarifications or possible directions a particular problem may take — however, you are not permitted any collaboration beyond such general conversations. I ask that you respect this condition, failure to do so will result in extreme penalties.

Finally, grading will be assigned based on a rank ordering of best to worst work. I can not overemphasize the need to submit your best possible work. An inadequate effort will lead to a poor grade in the course.

Problem:
You are given the following reference articles:


You are to perform the following tasks:

1. Rederive the model given by Hales to represent the material balance behavior of a water influx system (Eq. 1 in the JPT version of the paper). You must show all details.
2. Rederive the models given by Moran and Samaniego to represent the material balance behavior of a water influx system (Case 3: Eq. 18 and Case 6: Eqs. 24-30). You must show all details.
3. Using the data given by Hales (JPT version and preprint), you are to analyze these data in complete detail using:
   a. Hales' method.
   b. Moran and Samaniego's methods given by "Case 3" and "Case 6."

Format of Materials to be Submitted: (follow this format exactly)

You are to use the following format when submitting your exam:

- Coversheet.
- Problem
  — Derivation of Hales' method.
  — Derivation of Moran and Samaniego's methods ("Case 3" and "Case 6").
  — Analysis of Hales' data using Hales' method and Moran and Samaniego's methods ("Case 3" and "Case 6").
  — Summary table of results and 1-2 paragraphs of discussion.
- 5-6 page extended outline of materials covered this semester.
A Production Mechanism Diagnosis Approach to the Gas Material Balance
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Abstract
This paper presents the derivation of a more general gas material balance equation than that currently available in the literature, that considers a variable effective compressibility, water influx, an associated shale pore volume, that the reservoir could be naturally fractured, and abnormal pressure conditions. The diagnosis of the reservoir production mechanisms problem is thoroughly addressed. With presently available conventional material balance methods and limited diagnosis techniques, under conditions of a partial reservoir characterization, it is not possible to positively differentiate between an abnormal volumetric reservoir and one connected to a shale pore volume. This work discusses new diagnosis methods such as \( \frac{\partial(p/z)}{\partial G_p} \) vs \( G_p \) or \( (p_i - p) \) and \( -\frac{\partial(p/z)}{\partial p} \) vs \( \partial G_p/\partial p \).

The new methods are illustrated through its application to the most discussed gas field behavior examples published in the literature, among them the McEwen’s water influx case, the volumetric behavior, Begg’s case, and the Anderson L.

Derivation of a General Material Balance Equation.
We follow basically the derivation of Fetkovich et al. \(^5\). The main difference is that in the present study the possibility of a naturally fractured formation is considered. The details of this general gas material balance equation (GGMBE) are shown in reference 6. The general form of this GGMBE is

\[
G(B_u - B_g) + GB_w \tilde{c}_c(p) \Delta p = (G_p - W_p R_w - G_w) B_u + (W_p - W_{in} - W_w) B_w,
\]

where the cumulative effective compressibility term \( \tilde{c}_e(p) \) is pressure-dependent, consisting of a cumulative pore volume \( (PV) \) compressibility \( c_e(p) \) for a double porosity system\(^1\), cumulative total water compressibility \( c_w(p) \), and the total pore and water volumes associates (i.e., in pressure communication) with the gas reservoir,

\[
c_e(p) = \phi_m \epsilon_m + \phi_w \epsilon_w + \phi_m S_m \epsilon_m + M (\phi_m \epsilon_m + \phi_p \epsilon_p + \phi_w \epsilon_w)
\]

\[
\phi_m (1 - S_m) + \phi_p
\]

and \( \Delta p \) is the average reservoir pressure drop (\( = p_i - p \)).
The first two terms of the numerator of Eq. 2 represent the effective total cumulative compressibility for a double porosity reservoir

$$
\bar{c}_f(p) = \phi_{ma} \bar{c}_{ma}(p) + \phi_{ri} \bar{c}_{ri}(p).
$$

(3)

Eq. 1 can be written in terms of pressure; first dividing through by GB_{pa} and expressing $B_g = (p_{sc} / T_{sc}) (zT/p)$ gives an alternate GGMBE:

$$
\frac{p}{z} (1 - \bar{c}_e(p) \Delta p) = \frac{p_i}{z_i} \left[ 1 - \frac{1}{G_p} [G_p - W_p R_{sw} - G_{sw} + (W_p - W_{sw} - \frac{W_{sw}}{B_w}) B_g] \right].
$$

(4)

For the conditions of a single porosity (conventional) reservoir, Eq. 2 can be simplified, and the result will be Eq. 3 of Fetkovich et al.,

$$
\bar{c}_e(p) = \frac{S_{wi} \bar{c}_{sw} + \bar{c}_f(p) + M \left( \bar{c}_{sw}(p) + \bar{c}_f(p) \right)}{1 - S_{wi}}.
$$

(5)

The interbedded nonpay volume ($V_{pNNP}$) and limited aquifer contributions ($V_{pAQ}$) support are quantified in terms of the M ratio,

$$
M = \frac{V_{pNNP} + V_{pAQ}}{V_{pr}}.
$$

(6)

The cumulative effective compressibility $\bar{c}_e(p)$ accounts for cumulative changes in volume from initial pressure to the current pressure. Fetkovich et al. clearly explain (their Fig. 2) the meaning of cumulative effective compressibility. For instance, for the case of a single porosity reservoir (Eqs. 4 and 5), the cumulative PV compressibility $\bar{c}_f(p)$ has been defined as

$$
\bar{c}_f(p) = \frac{1}{V_{pr}} \left[ V_{pr} - V_{p}(p) \right] / \frac{p_i - p}{p_i - p}.
$$

(7)

The term in brackets is the slope of the chord from the initial condition $(p_i, V_{pr})$ to any lower pressure $(p, V_p)$; this implies that $\bar{c}_f(p)$ is a function of both pressure and the initial conditions. This cumulative compressibility function $\bar{c}_f(p)$ should be used with material balance equations that consider the cumulative pressure drop $(p_i - p)$, i.e., $p$ vs. $G_p$ plots. This parameter can be best estimated by special core analysis under in-situ reservoir conditions.

**Cumulative Total Water Compressibility.** $\bar{c}_{sw}(p)$. The pressure support provided by water is made up of two components. The reservoir pressure decrease causes the water expansion and the release of gas and its expansion. The cumulative total compressibility is expressed as

$$
\bar{c}_{sw} = \frac{1}{B_{sw}(p)} \left[ B_{sw}(p) - B_{sw}(p_i) \right] / \frac{p_i - p}{p_i - p},
$$

(8)

where the total water formation volume factor $B_{sw}$ is defined as

$$
B_{sw}(p) = B_{sw}(p) + [R_{swt} - R_{sw}(p)] B_g(p).
$$

(9)

To calculate $B_{sw}(p)$, values of $B_g(p)$, $R_{sw}(p)$ and $z(p)$ can be obtained from correlations when necessary, but laboratory data measured on representative samples taken from the reservoir are always superior to general correlations, and should be used whenever possible.

**Associated Water Volume Ratio M.** The total compressibility effect on the gas material balance depends for a naturally fractured system, on the magnitudes of matrix, fractures and total water compressibilities and on the total pore and water volumes in pressure communication with the gas reservoir (this includes connate water and the $PV$ within the net pay).

Associated water ($AW$) and $PV$s external to the net pay include nonnet pay (NNP), such as interbedded shales and dirty sands, plus external water volume found in limited aquifers. Following the developments of Fetkovich et al., the associated volume (water and $PV$s) is expressed (Eq. 6) as a ratio relative to the $PV$ of the net pay reservoir ($V_{pr}$),

$$
M = M_{NNP} + M_{AQ},
$$

(10)

where

$$
M_{NNP} = \frac{V_{pNNP}}{V_{pr}},
$$

and

$$
M_{AQ} = \frac{V_{pAQ}}{V_{pr}}.
$$

(11)

(12)

For the simplest case when $M = 0$ there will be pressure support only from connate water and the net $PV$.

**M_{NNP}.** Let's start assuming that properties and thickness of the net pay and nonnet pay are readily available from well log analysis. The nonnet pay water volume ratio $M_{NNP}$.
comprises interbedded reservoir $PV$, including shales and poor quality rock, assumed to be fully water saturated. Based on this definition, $M_{NP}$ can be expressed in terms of the net to gross ratio $R_{NG}$ defined as

$$R_{NG} = \frac{h_r}{h_r + h_{NNP}} = \frac{h_r}{h_i}$$

(13)

Considering different porosities in the net pay and nonnet pay, $M_{NNP}$ is given by

$$M_{NNP} = \frac{(\phi h_A)_{NNP}}{(\phi h_A)_r} = \frac{\phi_{NNP}}{\phi_r} \left( \frac{1 - R_{NG}}{R_{NG}} \right)$$

(14)

$$M_{AQ}$$: Associated aquifers of sufficient permeability (greater than 100 md) and limited areal extent can be treated as part of the total cumulative compressibility$^{15}$. $M_{AQ}$ is defined as

$$M_{AQ} = \frac{(\phi h_A)_{AQ}}{(\phi h_A)_r}$$

(15)

As expected, for large aquifers the required treatment will be the superposition methods originally discussed by van Everdingen and Hurst$^9$. For these cases the $\bar{c_e}(p)$ term can still be used, but it should only contain the effect of net pay and nonnet pay volumes; i.e., $M = M_{NNP}$ (from Eq. 10).

Diagnosis of the Production mechanism.

The diagnosis of the production mechanism of a gas reservoir is an important information regarding the implementation of the optimum exploitation conditions of these systems. A review of the literature indicates that the energy plot discussed by Beggs$^1$, where the reservoir behavior is plotted in a log-log graph in terms of $(1 - z_i/p)/z_i G_p$, is the most used for these purposes. This graph can be regarded as the $\Delta p$ vs. $t$ Ramey's first log-log method for the flow diagnosis in well test analysis$^{10}$. This problem is emphasized under limited reservoir characterization conditions.

The following discussion will present methods for the diagnosis of the production mechanism based on cartesian graphs.

Methods Based on the Derivative $-\partial(p/z)/\partial G_p$.

The following discussion will present methods for the diagnosis of the production mechanism based on cartesian graphs.

Case 1: Volumetric reservoir.

Deriving the $GGMBE$ given by Eq. 4 with respect to pressure$^{11}$,

$$-\frac{\partial(p/z)}{\partial G_p} = \frac{p_i}{z_i G}$$

(16)

This expression indicates that for the producing conditions of a volumetric (closed) reservoir, a graph of $-\partial(p/z)/\partial G_p$ vs. $G_p$ should result in a constant value, equal to $p_i/z_i G$, which can be used to estimate the original gas in place, G. Fig. 1 shows this type of graph. Continuing with the discussion between the well test analysis flow diagnosis problem and the production mechanism diagnosis for gas reservoirs, this graph is similar to the derivative methods of Tiab and Kumar$^{12}$ and Bourdet et al.$^{13}$ A graph as previously described can be very useful as complement for analyzing the behavior of gas reservoirs.

Case 2: Over pressured Naturally Fractured Volumetric Reservoir (ONFR).

As previously stated regarding Eq. 2, the $c_e(p)$ total effective compressibility of the system is considered as variable, or pressure dependent. Deriving Eq. 4 with respect to the cumulative gas production $G_p$,

$$-\frac{\partial(p/z)}{\partial G_p} = \frac{p_i}{z_i G} \frac{\partial(c_e(p)\Delta p)}{\partial G_p} \frac{1}{z_i} (1 - c_e(p)\Delta p)$$

(17)

Based on an analogy analysis between Eqs. 16 and 17, due to the second negative term in the latter equation, a graph of $-\partial(p/z)/\partial p$ vs. $G_p$ will result at early production times in smaller ordinate values than those corresponding to a volumetric reservoir (case 1), Fig. 2; later after the inflection point (●), either a straight line or concave behaviors can be shown. The shape of the late reservoir behavior, is strictly related with the values of the $c_e(p)$ parameter as the reservoir pressure decreases. For the case of a constant $c_e(p)$, Eq. 17 indicates that a straight-line behavior would result (curve b, Fig.2), and with the information about the intercept to the origin an estimate of the original gas in place G can be obtained. For conditions of variable $c_e(p)$ an upward concave behavior will be shown, curve c (Fig. 2).

Case 3: Conventional (Single Porosity) reservoir-aquifer system, considering solution gas in water.
Starting from the GGMBE (Eq. 4), for a reservoir with an associated aquifer, considering a constant \( \bar{c}_e \), deriving with respect to the cumulative gas production \( G_p \),

\[
\frac{\partial (p/z)}{\partial G_p} = \frac{P_i}{z \bar{c}_e} \left[ 1 - \frac{\partial}{\partial G_p} \left( \frac{W_e - W_p B_w}{B_g} \frac{1}{z \bar{c}_e} + W_p R_{sw} \right) \right] \tag{18}
\]

Fig. 3 shows a graph of the reservoir behavior for the conditions already stated; it can be observed from Eq. 18 that this straight line equation has a slope and an intercept of equal value, \( p_i/z \bar{c}_e \), which can be used to estimate the original gas in place \( G \). The direction of production time increase is also indicated in this schematic figure.

Based on a comparison of the behavior of a volumetric reservoir (Eq. 16, Fig. 1) and the reservoir behavior presented on Fig. 3, and through Eq. 18 that includes a negative derivative of the net water influx and the solution gas released from water, in can be concluded that a graph of \( -\partial (p/z)/\partial G_p \) vs. \( G_p \) or \( (p_i - p) \) should follow a straight line with negative slope, as shown in Fig. 4. The value of the slope of this reservoir behavior graph is strictly related to the two physical factors previously stated, the net water influx and the solution gas released from the produced water. For instant, as the activity of the aquifer (dimension) increases, the slope will also increase.

**Methods Based on the Derivative** \( -\partial (p/z)/\partial p \) vs. \( \partial G_p/\partial p \).

### Case 4: Volumetric Reservoir

Considering the physical conditions of these reservoirs, the derivative of the GGMBE given by Eq. 4 gives

\[
\frac{\partial}{\partial p} \left( \frac{p}{z \bar{c}_e} \right) = -\frac{P_i}{z \bar{c}_e} \frac{\partial G_p}{\partial p} \tag{19}
\]

This is the basic starting equation for the methods to be discussed in this section.

For volumetric gas reservoirs the change (derivative) of the cumulative production with respect to pressure decreases with the increase in production time. This is due to a higher rate decline at early production times. In addition, the change of \( p/z \) with respect to pressure decreases as pressure decreases, because the \( z \) compressibility factor increases at low pressure levels. This discussion can be better understood as follows. Deriving the left term of Eq. 19:

\[
\frac{\partial}{\partial p} \left( \frac{p}{z \bar{c}_e} \right) = \frac{1}{z} - \frac{p}{z^2 \partial p} \bar{c}_e \tag{20}
\]

Thus, considering the physical behavior of the \( z \) parameter at low pressures, \( \partial (p/z)/\partial p \) will show a decreasing behavior. It also can be concluded that for the (theoretical) limiting or minimum atmospheric reservoir pressure \( (p = 0 \text{ psi}) \), the value of \( \partial (p/z)/\partial p \big|_{p=0} \) will be 1 \( (z=1 \text{ at this pressure}) \), as shown in Fig. 5. In conclusion, Eq. 19 that describes the derivative behavior of a volumetric gas reservoir, is a straight line with zero interception and positive slope \( p_i/(z \bar{c}_e) \). Based on this slope, the original gas in place \( G \) can be estimated. A further check for the estimation of \( G \) can be as follows. For conditions of the minimum value of \( \partial (p/z)/\partial p \big|_{p=0} \), the value for the abscissa \( -\partial G_p/\partial p \big|_{p=0} \) could be applied in Eq. 19 to estimate the original gas in place \( G \):

\[
G = P_i \left[ \bar{c}_e \frac{\partial G_p}{\partial p} \right]_{p=0} \tag{21}
\]

### Case 5: Overpressured Naturally Fractured Reservoir (ONFR)

Deriving the GGMBE with respect to the reservoir pressure, assuming a constant cumulative effective compressibility \( \bar{c}_e(p) \), the derivative \( \partial (p/z)/\partial p \) is given by

\[
\frac{\partial}{\partial p} \left( \frac{p}{z \bar{c}_e} \right) = -\frac{P_i}{z \bar{c}_e} \frac{\partial G_p}{\partial p} \tag{22}
\]

Considering a variable cumulative effective compressibility \( \bar{c}_e(p) \), the derivative can be expressed as

\[
\frac{\partial}{\partial p} \left( \frac{p}{z \bar{c}_e} \right) = -\frac{P_i}{z \bar{c}_e} \frac{\partial G_p}{\partial p} \frac{1}{z \bar{c}_e} \tag{23}
\]

An analysis of the parameters of Eq. 23 will allow the conclusion that for an over pressured naturally fractured reservoir, the derivative \( \partial (p/z)/\partial G_p \) presents increasing positive values with respect to the exploitation time. The easiest way to analyze this problem is for the simplest case of a constant effective cumulative compressibility \( \bar{c}_e \) considered in Eq. 22, where as the exploitation time increases its numerator also increases due to the decrease in pressure, and its denominator decreases because the reservoir pressure drop \( (p_i - p) \) increases. This behavior can be observed schematically in Fig. 6, part (a).

It can be stated that the behavior of an ONFR will show several straight lines in a graph of \( \partial (p/z)/\partial p \) vs. \( \partial G_p/\partial p \), which are strictly related to the variability of the effective
cumulative compressibility $c_e(p)$, as shown in Fig. 6. As was previously stated in the last paragraph, while the effective cumulative compressibility $c_e(p)$ has a high value or keeps increasing during the transient reservoir behavior period, the derivative $\partial(p/z)/\partial p$ will also increase up to a maximum value at the end of part (b) of Fig. 6, then it will decrease, as indicated in part (c) of this figure. In addition, it has been discussed that the $z$ gas compressibility factor decreases in a pressure range starting from the initial pressure and ending in an intermediate pressure (see for instance the Standing and Katz correlation, Fig. 1.3, page 21 of Craft and Hawkins).

For the case of a closed (limited) reservoir ($M=0$, Eqs. 2 and 6), the cumulative effective compressibility $c_e(p)$ will only be a function of the cumulative PV compressibility and of the cumulative total water compressibility $c_w(p)$, showing as the reservoir pressure declines a faster decrease with exploitation time than that for the general case, where the interbedded nonpay volume $V_{NP}$ and aquifer contribution are considered. Thus, in accordance to Eq. 23, the relation between the derivatives $\partial(p/z)/\partial p$ vs $-\partial G_p/\partial p$ will decrease as the exploitation time increases, as shown in Fig. 6 part (c).

A further explanation of the reservoir behavior shown in Fig. 6, part (b) will be as follows. When the reservoir fluids are produced, the reservoir pressure decreases, causing an increase of the effective stress (equal to the overburden stress minus the reservoir pressure), resulting in a deformation of the formation. This compressive effect provides an additional energy source for the gas production. This effect will show in this graph through decreasing values of the derivative $-\partial G_p/\partial p$, due to a decrease in the cumulative PV effective compressibility caused by the formation deformation (pore collapse) already discussed.

Summarizing, based on Eqs. 22 and 23 and the previously presented discussion for this Case 5, the possible general behavior of an ONFR is as shown in Fig. 6.

Case 6: Conventional (single porosity) reservoir-aquifer system, neglecting solution gas in water ($R_{sw} = 0$) and injection of fluids ($G_{inj} = W_{inj} = 0$).

For these conditions already discussed in case 3, Eq. 4 can be expressed as

$$\frac{p}{z} = \frac{p_i}{z} - \frac{1}{G_p} \left[ G_p + (W_p B_w - W_e) \frac{1}{B_g} \right]. \hspace{1cm} (24)$$

Deriving this equation with respect to pressure

$$\frac{\partial}{\partial p} \left( \frac{p}{z} \right) = \frac{p_i}{z} \left[ \frac{\partial G_p}{\partial p} - \frac{\partial}{\partial p} \left( \frac{W_e - W_p B_w}{B_g} \right) \right] \hspace{1cm} (25)$$

It can be observed from this Eq. 24 that for these conditions the derivative $\partial G_p/\partial p$ is not proportional to the derivative $\partial(p/z)/\partial p$, as it was for case 3 previously discussed, Eq. 18; the difference in this last Eq. 25 is the negative net water influx derivative term, which results in a decreasing $\partial(p/z)/\partial p$ derivative with the exploitation time.

Writing Eq. 25 in terms of the net water influx

$$\left( W_{en} = W_e - W_p B_w \right)$$

$$\frac{\partial}{\partial p} \left( \frac{p}{z} \right) = \frac{p_i}{z} \left[ \frac{\partial W_{en}}{\partial p} - \frac{\partial G_p}{\partial p} \right]. \hspace{1cm} (26)$$

If we consider an increasing aquifer activity, the change of the cumulative gas production with respect to pressure will be bigger than those of the net water influx, resulting in decreasing values for the $\partial(p/z)/\partial p$ derivative.

This previous discussion for the present case can be continued considering the first version for the GGMBE written in terms of the gas volume factor $B_g$; for the conditions of these reservoirs, Eq. 1 can be simplified as

$$G_p B_g = G(B_g - B_{gl}) + (W_e - W_p B_w). \hspace{1cm} (27)$$

Deriving this equation

$$\frac{\partial G_p}{\partial p} = G \left( \frac{B_g - B_{gl}}{B_g} \right) + \frac{\partial}{\partial p} \left( \frac{W_e - W_p B_w}{B_g} \right). \hspace{1cm} (28)$$

Writing this equation in terms of the net water influx $W_{en}$,

$$\frac{\partial W_{en}}{\partial p} = \frac{\partial G_p}{\partial p} = -G \left( \frac{B_{gl}}{B_g} \frac{\partial B_g}{\partial p} \right) \hspace{1cm} (29)$$

Inserting this equation in Eq. 25

$$\frac{\partial}{\partial p} \left( \frac{p}{z} \right) = \frac{p_i}{z} \left[ \frac{B_{gl}}{B_g^2} \frac{\partial B_g}{\partial p} \right]. \hspace{1cm} (30)$$

The derivative of the gas formation volume factor $\partial B_g/\partial p$ presents negative values, that increase in absolute value as the reservoir pressure decreases. The absolute value of this derivative decreases with the activity (size) of the associated aquifer. In the limit for conditions of a very strong (large) aquifer where the reservoir pressure drop is small, the derivative $\partial B_g/\partial p$ presents a small negative value, resulting in value of $\partial(p/z)/\partial p$ close or equal to zero.
In summary, from the previous discussion based on Eq. 30, the behavior of a gas reservoir with an associated aquifer, presented in a graph of $\frac{\partial (p/z)}{\partial p}$ vs $-\frac{\partial G_p}{\partial p}$, Fig. 7, will show a negative slope straight line, that tends to reach a zero value at long exploitation times. For a very strong aquifer, the derivative $\frac{\partial (p/z)}{\partial p}$ reaches a constant positive, close to zero, value, following a horizontal straight line, and the derivative $-\frac{\partial G_p}{\partial p}$ continues to increase, as shown in Fig. 7.

Field Examples
McEwen’s Gas Reservoir with an Associated Aquifer\(^1\).
A Gulf Coast gas reservoir was analyzed by the author. A conventional $\frac{p}{z} vs. G_p$ graph shows the typical concave upward curve. Fig. 8 shows a diagnostic graph of the reservoir behavior in terms of $\frac{\partial (p/z)}{\partial p}$ vs $G_p$. From the results shown in this graph three different reservoir behaviors corresponding to different water influx effects can be observed; the last part starting in point e and ending in f indicates a more intensive influx than the first two affecting this reservoir.

Estimates from other diagnostic mechanism methods, for instance one based on the derivative of the GGMBE with respect to the exploitation time $t$, resulted in values for the original gas in place $G$ of 195.35 Bscf\(^6\), which compares well with the 180 Bscf calculated by McEwen.

Begg’s Volumetric Gas Reservoir\(^1\).
Fig. 9 shows a typical behavior for a volumetric reservoir. A Comparison is presented between two methods for the estimation of this derivative, mainly through the derivation of an interpolation polynomial of second degree and using a centered difference approximation\(^17\).

The estimation of the original gas in place $G$ by means of the extrapolation to the minimum point, results in a value of 46.4 MMMscf, which can be compared with the value calculated by Begg of 48.348 MMMscf.

The Anderson L Gas Condensate Reservoir.
This reservoir has been studied by several authors and it is usually recognized as the best example of a high-pressure gas condensate reservoir, with the characteristic concave downward $\frac{p}{z} vs. G_p$ behavior. The reservoir was abandoned after producing 55 Bscf, but pressure tests of public record were discontinued after 40 Bscf had been produced.

The behavior of this reservoir is shown in Fig. 10 in terms of $\frac{\partial (p/z)}{\partial p}$ vs $-\frac{\partial G_p}{\partial p}$. The first finding is that the behavior of this reservoir is highly complex. The abnormal reservoir pressure causes an increase of the cumulative effective compressibility due to the inelastic compression of the formation, shown in Fig. 10 by the positive slope straight lines ab, de and fg, and also results in a decrease of the compressibility generated by the pore volume collapse, indicated by the negative slope straight line behavior gh. Starting in point b, a decrease on the y axis values of $\frac{d (p/z)}{dp}$ occurs, ending at point c, which indicates a possible water influx coming, form shales in contact with the reservoir, which ceases after a certain time. On a different basis, this was also stated by Duggan\(^19\).

Conclusions.
The main aim of this paper has been to present a more general gas material balance equation (GGMBE) than that currently available in the literature, that considers as an improvement that the reservoir could be naturally fractured. The diagnosis of the production mechanism is thoroughly addressed, presenting new methods of analysis.

From the results of this study the following conclusions are pertinent.

1. Applying the GGMBE derived in this work, new and more robust methods for the diagnosis of the production mechanisms of gas reservoirs have been derived.
2. Through the implementation of the new diagnostic methods, new alternate procedures were derived for the estimation of the original gas in place $G$, the cumulative effective compressibility $\tilde{c}_e (p)$, and the volume ratio $M$.
3. The new diagnostic methods result in a better reservoir characterization, because having accurate information regarding the production mechanisms and the original gas in place, will allow a better forecasting of the reservoir behavior.

Nomenclature

- $B_g$ = gas FVF.
- $B_w$ = water FVF, RB/STB.
- $\tilde{c}_e$ = effective cumulative compressibility, psia\(^{-1}\).
- $\tilde{c}_f$ = in-situ formation compressibility, psia\(^{-1}\).
- $\tilde{c}_{fr}$ = in-situ formation compressibility of fractures, caverns and vugs, psia\(^{-1}\).
- $\tilde{c}_{rw}$ = water compressibility, psia\(^{-1}\).
- $G$ = original gas in place, Mscf.
- $G_{ij}$ = cumulative gas injection, Mscf.
- $G_p$ = cumulative gas production, Mscf.
- $M$ = volume ratio, dimensionless.
- $p$ = reservoir pressure, psia.
- $p_i$ = initial reservoir pressure, psia.
- $p_{cw}$ = pressure at standard conditions, psia.
- $R_{sw}$ = Solution gas water ratio, scf/res cf.
- $S_w$ = connate water saturation, fraction.
- $t$ = time, days.
- $V_{p/A}$ = associated (non-net pay and aquifer) pore volume.
occupied by gas, ft³.

\[ V_{gr} = \text{net reservoir pore volume occupied by gas, ft}^3. \]

\[ V_{pa} = \text{pore volume of the associated rock, ft}^3. \]

\[ V_{ph} = \text{net reservoir pore volume occupied by gas and water, ft}^3. \]

\[ V_{wa} = \text{water volume in the associated pore volume, ft}^3. \]

\[ V_{wr} = \text{net reservoir pore volume occupied by water, ft}^3. \]

\[ W_e = \text{cumulative water influx, res bbl.} \]

\[ W_{es} = \text{net water influx, } W_e - W_p B_w, \text{ Eq. } 28, \text{ res bbl.} \]

\[ W_{inv} = \text{cumulative water injection, STB.} \]

\[ W_p = \text{cumulative water production, STB.} \]

\[ z = \text{gas compressibility factor, dimensionless.} \]

\[ \phi_{fr} = \text{porosity of fractures, caverns and vugs, fraction.} \]

\[ \phi_{ma} = \text{matrix porosity.} \]

References


8. Earlougher, R.C., Jr.: Advances in Well Test Analysis, SPE Monograph Series No. 5, Richardson, Texas (1967).


Fig. 1. Diagnosis of the production mechanism of a volumetric reservoir based on the derivative \( \frac{\partial(p/z)}{\partial G_p} \).
Fig. 2. Diagnosis of the production mechanism of an abnormally pressured naturally fractured gas reservoir, based on the derivative $-\partial (p/z) / \partial G_p$.
(a) volumetric;
(b) constant effective compressibility;
(c) variable effective compressibility $c_e(p)$.

Fig. 3. Diagnosis of the production mechanism for a water drive gas reservoir, based on the derivative $-\partial (p/z) / \partial G_p$ vs. $\partial ((W_e - W_p B_w) B_g + W_p R_{sw}) / \partial G_p$.

$\frac{b}{t} = \frac{p_i}{(z_i G)}$

$m = \frac{p_i}{(z_i G)}$

Fig. 4. Diagnosis of the production mechanism for a water drive gas reservoir, based on the derivative $-\partial (p/z) / \partial G_p$ vs. $\partial G_p$.

Fig. 5. Diagnosis of the production mechanism for a volumetric gas reservoir, based on a graph of $\partial (p/z) / \partial p$ vs. $\partial G_p / \partial p$. 
Fig. 6. Diagnosis of the production mechanism for over-pressured or naturally fractured gas reservoirs, based on a graph of $\frac{\partial (p/z)}{\partial p}$ vs. $-\frac{\partial G_p}{\partial p}$, for various effective compressibility conditions: (a) high, (b) increasing and (c) small and decreasing.

Fig. 7. Diagnosis of the production mechanism for a water drive gas reservoir, based on a graph of $\frac{\partial (p/z)}{\partial p}$ vs. $-\frac{\partial G_p}{\partial p}$.

Fig. 8. Diagnosis of the production mechanism for the water drive gas reservoir of McEwen$^{12}$, based on a graph of $\frac{\partial (p/z)}{\partial G_p}$ vs. $-\frac{\partial G_p}{\partial p}$.

Fig. 9. Diagnosis of the production mechanism based on a graph of $\frac{\partial (p/z)}{\partial p}$ vs. $-\frac{\partial G_p}{\partial p}$ for the gas reservoir data presented by Beggs.
Fig. 10. Diagnosis of the production mechanism for the Anderson L gas reservoir, based on a graph of $\partial (p/z)/\partial p$ vs. $-\partial G_p/\partial p$. 
A METHOD FOR PREDICTING TOTAL GAS-IN-PLACE FROM RESERVOIR PERFORMANCE DATA

by Hugh B. Hales, Mobil Exploration and Producing Services, Inc.

ABSTRACT

This paper describes a new method for combining pressure decline data and production rates to determine original gas volumes for gas reservoirs. Use of the method is illustrated by application to actual field data.

The new method is similar to the classical analysis which extrapolates a plot of p/z versus cumulative production to the original gas-in-place. However, it allows for the influx of water into the original reservoir volume in a more rigorous manner than previous methods. It also provides an estimate of the reliability of the predicted original gas-in-place values.

An illustration of the analysis of pressure decline data from an actual reservoir shows one of many applications which have confirmed the utility of the model despite substantial scatter in the data.

INTRODUCTION

Pressure decline analyses have become a basic reservoir engineering tool, particularly for gas reservoirs. It is readily apparent from the gas law of physical chemistry that the reservoir pressure-gas deviation factor ratio, p/z, should decline linearly with cumulative reservoir production, provided the reservoir is highly permeable and has a constant volume. Unfortunately, many reservoirs do not have a constant volume as a result of water influx which occurs with declining reservoir pressure. Most methods of accounting for this complication are based on the early work of Van Everdingen and Hurst. Van Everdingen and Hurst used the Laplace transform to solve the partial differential equation governing single phase, radial flow in porous media and were able to relate production at an internal reservoir boundary to the pressure at that boundary. Many investigators have used this solution to account for water influx at the edge of a reservoir. These include the well-known techniques of Van Everdingen, Timmerman, and McMahon, Hurst, Carter and Tracy, and Odeh and Havelena. The literature abounds in applications and variations of these methods.

These methods can be visualized as modeling reservoirs as large tanks which gradually fill with water as the tank pressures decrease. Many workers have questioned the validity of such a model. In particular, Chierici and Pizzi, and McEwan have investigated the errors in the predicted hydrocarbon volumes which result from the unavoidable errors in the pressure measurements. Furthermore, Bruns et al, have shown that the effects of water influx on the nature of the pressure decline curve are diverse. Water influx can cause a concave upward pressure history, a concave downward pressure history, or an "S" shaped curve. Pressure decline with strong water influx can also closely resemble the straight line pressure decline of a depletion type reservoir.

This work is an attempt to overcome the shortcomings of previous methods. The improvements are two-fold. First, the model includes a moving water/hydrocarbon interface. In analogy with previous models, this allows a non-constant tank volume. Second, the method provides not only a best estimate of the hydrocarbons in place, but also an estimate of the accuracy of these results.

THEORY

This work involves the development and use of a model which predicts reservoir pres-
A method for predicting total gas-in-place as a function of cumulative production and time. A schematic drawing of the model is shown below.

<table>
<thead>
<tr>
<th>Gas</th>
<th>Wall or Wells</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>Water</td>
</tr>
</tbody>
</table>

The principal assumptions of the model are, first, that the gas has a very small viscosity compared with that of the encroaching water, and second, that the water encroaches linearly along a single boundary of the reservoir. The first assumption infers that the pressure drop throughout the gas-containing region of the reservoir is small. The bottom hole pressures at each of the wells are assumed equal to each other and to the pressure at the gas/aquifer interface. Since the pressure is uniform throughout the gas-containing region, the actual shape of this region, which is shown rectangular above, is of no significance, nor are the well locations of importance. The assumed linear flow from the infinite aquifer implies that the streamlines within the aquifer are everywhere straight and parallel. This assumption is somewhat arbitrary. Radial flow could just as easily have been assumed. However, prospective applications frequently illustrated water influx along only the edge of the reservoir rather than on all sides. Linear flow is thought to be the best approximation in most cases. A third assumption is that flow within the aquifer follows Darcy's law.

The model differs from previous models in that the constant pressure area, i.e. the water-free zone, is allowed to shrink as the aquifer encroaches. In the invaded region, the pressure drop is assumed to be linear. This corresponds to a constant residual gas saturation throughout. However, the relative permeability of the water phase in this region is reduced as a result of this residual gas.

With these assumptions, the reservoir pressure, \( p \), can be determined as a function of the cumulative production, \( G_p \), and of time, \( t \), as follows:

\[
p = p_i + \frac{\alpha^2 \rho G}{Km} \frac{\partial}{\partial t} \left[ \frac{z}{p}(G_p - G) \right] \frac{z}{p}(G - G_p) - \frac{z}{p} G
\]

\[
= a \left[ \frac{\partial}{\partial \alpha} \frac{z}{p}(G_p - G) \right] ds \left( t - \frac{s}{t} \right) \quad \cdots (1)
\]

A derivation of this equation is included in the Appendix.

Equation (1) must be solved for \( G \), the initial gas-in-place. There is some arbitrariness as to how this is accomplished. In this work \( a \) and \( p_i \) were also considered unknowns. Each set of data points, pressure and production as a function of time, provide a separate equation of form (1). Generally there are many more than three sets of data points, and hence the unknowns, \( G \), \( a \), and \( p_i \), are adjusted to optimize the agreement of each calculated \( p \) with the measured reservoir pressure.

A computer program was written to perform this optimization. The program incorporates the calculation of \( z \)-factors as a function of temperature, pressure, and gas density, and the conversion of well head pressures to reservoir values. Finite difference methods are used to evaluate the derivative and the integral in the equation.

The program provides values of three constants in equation (1), \( a \), \( p_i \), and \( G \), which provide the closest match to the pressure decline data. \( a \) can be thought of as a shape factor or as a measure of the water influx; \( p_i \) is the initial reservoir pressure; and \( G \) is the initial gas-in-place volume. Errors in the pressure measurements are often apparent and hence, the value of extrapolating this much information from the data is questionable. In order to assess the reliability of the gas-in-place estimate obtained, a standard deviation for this value can be calculated. This is done by assuming a linearized form of equation (1):

\[
\Delta p = \frac{\partial p}{\partial G} \Delta G + \frac{\partial p}{\partial p_i} \Delta p_i + \frac{\partial p}{\partial a} \Delta a \quad \cdots (2)
\]

where \( \Delta p \), \( \Delta G \), \( \Delta p_i \), and \( \Delta a \) are deviations from their optimum values and the derivatives evaluated at the optimum values. If the functional relationship is linear as assumed in equation (2), the equation for calculating the error in the parameters is developed in many statistical texts, e.g. Carnahan, Luther, and Wilkes. The equation for the standard deviation in the estimated gas-in-place value is:

\[
\sigma_G^2 = \sum \left( \frac{\partial p}{\partial p_i} \right)^2 \sum \left( \frac{\partial p}{\partial a} \right)^2 - \left( \sum \frac{\partial p}{\partial p_i} \frac{\partial p}{\partial a} \right)^2 \frac{\sigma^2}{V} \quad \cdots (3)
\]
where $\sigma_p$ is the standard deviation in the pressure data fit, and $\nu$ is given by:

$$\nu = \sqrt{\sum \frac{(p_{\text{fit}} - p_{\text{data}})^2}{\sigma_p^2} + \sum \frac{(p_{\text{fit}} - p_{\text{data}})^2}{\sigma_p^2} + \sum \frac{(p_{\text{fit}} - p_{\text{data}})^2}{\sigma_p^2}}$$

In both equations, the sums are taken for all data points to be matched.

The value of the initial gas-in-place, $G$, obtained from the best fit to the pressure data, can be thought of as the most probable gas-in-place, i.e. there is a 50% probability that the gas-in-place is higher and a 50% probability that the gas-in-place is lower. The standard deviation in the gas-in-place obtained from equation (3) can be used to estimate the probability that other initial gas-in-place values are valid. Table 1 shows the probabilities of having the amount of gas indicated by those values in the reservoir; 1% chance of 123.3 BCF, etc.

The linearized pressure assumption of equation (2) is valid only for small variations in the pressure, i.e. small errors. Pressure data errors may often exceed the validity of the formulation. Nevertheless, this analysis can serve as a useful qualitative tool in evaluating the reliability of the results.

In addition to the assumed linearity of the pressure function, this statistical analysis also assumes 1) that the mathematical representation of the process shown in equation (1) is an accurate representation of the physical phenomenon, 2) that the measured times, production volumes, and various input constants are exact, so that data inaccuracies occur only in the pressure measurements, and 3) that the error in pressure measurements are randomly distributed.

RESULTS

Table 2 shows production and pressure decline data for six wells assumed from geological data to flow from the same gas reservoir. However, it is immediately clear from a plot of $p_x$ against cumulative production, shown in Figure 1, that Well 6 is declining much more rapidly than the other wells. Therefore there must be a communication barrier between this segment of the reservoir and the rest. Figure 2 shows the pressure decline data of Well 6 as a function of its own production. The solid line in the figure shows the pressure decline curve predicted by optimization of equation (1). This procedure predicted an original gas-in-place of 11 bscf (standard deviation: 1 bscf) and no water influx. This result also suggests total isolation of Well 6 from the other wells since gas leakage into the Well 6 reservoir from the other would have provided pressure maintenance similar to a water drive.

Figure 3 illustrates the pressure decline of the other five wells as a function of their cumulative production. The solid line in the figure shows again the pressure decline curves predicted by the optimization of equation (1). This procedure predicted an original gas-in-place of 138 bscf with a moderate water influx. The standard deviation in this estimated volume is 38 bscf. Sufficient scatter exists in the data that to the eye, a straight line fit is just about as good as the optimum fit. However, extrapolation of the straight line fit to zero pressure, suggests that the gas-in-place is 243 bscf. This is 2.76 standard deviations greater than the optimum value. Therefore one would estimate a less than 1% probability that this much gas actually existed in the reservoir. Even though this is a gross approximation, it does confirm that reasonable estimates of gas-in-place can be predicted from performance data even though substantial errors exist in the data.

CONCLUSIONS

This study presents a new method for analyzing pressure decline data which can result in meaningful estimates of initial gas volumes despite substantial scatter in the decline data.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tr>
<td>$A$</td>
<td>area of reservoir face through which water is encroaching</td>
</tr>
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</tr>
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</tr>
<tr>
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<td>time</td>
</tr>
<tr>
<td>$T$</td>
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</tr>
<tr>
<td>$u$</td>
<td>fluid flux through reservoir</td>
</tr>
<tr>
<td>$V_m$</td>
<td>molar volume</td>
</tr>
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A METHOD FOR PREDICTING TOTAL GAS-IN-PLACE FROM RESERVOIR PERFORMANCE DATA

\( \mu_w \) viscosity of water at reservoir conditions.

\( \Phi \) porosity.

\( \sigma_x \) standard deviation in \( x \), \( \frac{\sigma_x^2}{\mu} \frac{\partial^2 P}{\partial x^2} \)

Subscripts:

\( i \) pertains to initial condition of reservoir.

\( p \) based on performance data.

\( j \) pertains to the \( j \)-th data point.

\( x_i \) at position \( x_i \).

ACKNOWLEDGEMENTS

This work was expedited by valuable discussions with Mr. R. J. Redmore. The author also wishes to express his appreciation to Mobil Oil Exploration and Production Southeast Inc., which supported the work and allowed its publication.

REFERENCES


APPENDIX: MATHEMATICAL DERIVATION OF MODEL

Consider the gas reservoir shown conceptually in Figure A-1. Initially the gas existed exclusively in Zones A and B. An infinite aquifer extended to the right of this area in Zone C. However, as gas is produced from Zone A and the pressure reduced, water encroaches into this area forming a two phase region; Zone B. In order to mathematically relate the pressure to the production, the fluid flow in each of these zones must be considered separately.

Zone A contains only gas. If we assume that this gas has a very large mobility, then it is clear from Darcy's law (see Nomenclature)

\[ u = \frac{-k}{\mu} \frac{\partial P}{\partial x} \]  

that the pressure gradient is small. That is to say, if the permeability is large and/or the gas viscosity is small, the pressure throughout the gas Zone A, is uniform. In Zone C, if we assume a one-dimensional Darcy flow of the water, a material balance provides the following partial differential equation:

\[ c\Phi \frac{\partial P}{\partial t} = \frac{\partial}{\partial x} \left( \frac{k}{\mu_w} \frac{\partial P}{\partial x} \right) \]  

The appropriate boundary conditions for this partial differential equation are:

\[ P = P_i \quad \text{at} \quad t = 0 \]  

\[ P = P_i \quad \text{as} \quad x \to \infty \]  

\[ u = \frac{-k}{\mu_w} \frac{\partial P}{\partial x} \quad \text{at} \quad x = L_1 \]

The solution to equation (A-2) with boundary conditions (A-3) is given by Carslaw and Jaeger. This solution takes the form:

\[ P_{x_1} = P_i - \frac{\mu_w}{k \Phi} \left( \frac{1}{2} \right) \int_0^t \frac{u(t) \partial P}{\sqrt{\pi} \sigma_x} \right) \]  

In Zone B, it is assumed that there is a uniform mass flux throughout. Therefore, this flux can be related to the pressures at
the two boundaries of Zone B through the equation:

\[ u(t) = \frac{k_{rw} \mu_w}{L - L_1} \cdot \frac{P - P_1}{L_1} \quad \cdots \cdots \cdots (A-5) \]

A mass balance of the form:

Cumulative Production = Initial Gas-in-Place - Remaining Gas-in-Place

can be written in the form:

\[ Q_p = \frac{A \phi}{RT} \left( \frac{L_1 P_1 - L P}{z} \right) \quad \cdots \cdots \cdots (A-6) \]

Furthermore, the movement of the gas-water interface can be related to the water flux with equation:

\[ u(t) = -\phi \frac{dL}{dt} \quad \cdots \cdots \cdots (A-7) \]

Equations (A-4) through (A-7) provide four equations in the unknowns \( p, P_x, u, L \). A function of the form \( p(z) \) is desired. Hence, \( P_x, u, \) and \( L \) must be eliminated by combination of the above four equations to provide the single relationship. Elimination of \( P_x \) from equation (A-4) by substituting equation (A-5), results in the following equation:

\[ p = p_1 - \frac{\mu_w}{k_{rw}} u(t)(L_1 - L) \]

\[ -\left( \frac{\mu_w}{k \phi \pi} \right)^{1/2} \int_0^t u(t - \tau) \frac{d\tau}{\tau^{1/2}} \quad \cdots (A-8) \]

Substituting \( L \) from equation (A-6) into (A-7) and further substituting the resulting expression for \( u \) with the expression for \( L \) into equation (A-8) results in the following expression:

\[ p = p_1 + \frac{2 \pi c}{k_{rw}} \int [ \frac{z}{G_p - G} ] \left[ \frac{z (G - G_p)}{p_1} - \frac{z L}{L_1} \right] \]

\[ - \left[ \frac{\partial}{\partial s} \left[ \frac{z}{p(G_p - G)} \right] \right] \frac{ds}{(t-s)^{1/2}} \quad \cdots (A-9) \]

In this equation, \( \alpha \) is defined as:

\[ \alpha = \left( \frac{\mu_w}{k \phi \pi} \right)^{1/2} \frac{RT}{A} \quad \cdots \cdots (A-10) \]

and \( L_1 \) has been eliminated from the expression by substitution of the special case of equation (A-6) in which \( L = 0 \) when \( G_p = G \). Equation (A-9) forms the basis for the model and appears as equation (1) in the text.
TABLE 1

<table>
<thead>
<tr>
<th>Probability that Variation Exceeds Value at Right</th>
<th>Variation from G (Number of Standard Deviation)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>.001</td>
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<tr>
<td>.01</td>
<td>2.33</td>
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<tr>
<td>.1</td>
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<tr>
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<td>.84</td>
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<tr>
<td>.3</td>
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<tr>
<td>.7</td>
<td>-.52</td>
</tr>
<tr>
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<td>-.84</td>
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<td>-1.28</td>
</tr>
<tr>
<td>.99</td>
<td>-2.33</td>
</tr>
<tr>
<td>.999</td>
<td>-3.09</td>
</tr>
</tbody>
</table>

* "Variation from G" is defined as [(actual G)-(most probable G)]/σG

This formula should be used in conjunction with above table to calculate the probability of various (actual G)'s.
## TABLE 2
PRESSURE DECLINE DATA

<table>
<thead>
<tr>
<th>TIME* (DAYS)</th>
<th>CUMULATIVE RESERVOIR REDUCTION (bcf)</th>
<th>BOTTOM HOLE PRESSURE (psi)</th>
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</thead>
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<tr>
<td>WELL 1</td>
<td>294</td>
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<tr>
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<td></td>
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*Time since start of production.
Fig. 1 - Pressure decline data for all wells in reservoir.
Fig. 2 - Pressure decline data for Well 6.
Fig. 3 - Pressure decline data for Wells 1 thru 5.
Fig. A1 - Schematic illustration of model.
Pressure Decline Analysis for Prediction of Gas Reservoir Volumes

H.B. Hales, SPE, Mobil Exploration and Producing Services Inc.

Summary
This paper describes a new method for combining pressure decline data and production rates to determine original gas volumes for gas reservoirs. The method is illustrated by application to actual field data. The new method is similar to the classical analysis which extrapolates a plot of \( p/z \) vs. cumulative production to the original gas in place. However, it allows for the influx of water into the original reservoir volume in a more rigorous manner than previous methods. It also provides an estimate of the reliability of the predicted original-gas-in-place values.

Introduction
Pressure decline analysis has become a basic reservoir engineering tool, particularly for gas reservoirs. It is readily apparent from the gas law of physical chemistry that the reservoir-pressure/gas-deviation-factor ratio \( p/z \) should decline linearly with cumulative reservoir production, provided the reservoir is highly permeable and has a constant volume. Unfortunately, many reservoirs do not have a constant volume as a result of water influx that occurs with declining reservoir pressure. Most methods of accounting for this complication are based on the early work of van Everdingen and Hurst.\(^1\) Van Everdingen and Hurst used the Laplace transform to solve the partial differential equation governing single-phase radial flow in porous media and were able to relate production at an internal reservoir boundary to the pressure at that boundary. Many investigators have used this solution to account for water influx at the edge of a reservoir. These include the well-known techniques of van Everdingen \textit{et al.},\(^2\) Hurst,\(^3\) Carter and Tracy,\(^4\) and Odeh and Havlena.\(^5,6\) The literature abounds in applications and variations of these methods.

These methods can be visualized as modeling reservoirs as large tanks which gradually fill with water as the tank pressures decrease. Many workers have questioned the validity of such a model. In particular, Chierici \textit{et al.}\(^7\) and McEwen\(^8\) have investigated the errors in the predicted hydrocarbon volumes that result from the unavoidable errors in the pressure measurements. Furthermore, Bruns \textit{et al.}\(^9\) have shown that the effects of water influx on the nature of the pressure decline curve are diverse. Water influx can cause a concave upward pressure history, a concave downward pressure history, or an S-shaped curve. Pressure decline with strong water influx also can resemble closely the straight-line pressure decline of a depletion-type reservoir.

This work is an attempt to overcome the shortcomings of previous methods. The improvements are twofold. First, the model includes a moving water/hydrocarbon interface. In analogy with previous models, this allows a nonconstant tank volume. Second, the method provides not only a best estimate of the hydrocarbons in place but also an estimate of the accuracy of these results.

Theory
This work involves the development and use of a model that predicts reservoir pressure as a function of cumulative production and time. A schematic of the model is shown here.
The principal assumptions of the model are (1) that the gas has a very small viscosity compared with that of the encroaching water and (2) that the water encroaches linearly along a single boundary of the reservoir. The first assumption infers that the pressure drop throughout the gas-containing region of the reservoir is small. The bottomhole pressures at each of the wells are assumed equal to each other and to the pressure at the gas/aquifer interface. Since the pressure is uniform throughout the gas-containing region, the actual shape of this region, which is shown rectangular, is of no significance, nor are the well locations of importance. The assumed linear flow from the infinite aquifer implies that the streamlines within the aquifer are everywhere straight and parallel. This assumption is somewhat arbitrary. Radial flow could have been assumed just as easily. However, prospective applications frequently illustrated water influx along only the edge of the reservoir rather than on all sides. Linear flow is thought to be the best approximation in most cases. A third assumption is that flow within the aquifer follows Darcy's law.

The model differs from previous models in that the constant-pressure area (i.e., the water-free zone) is allowed to shrink as the aquifer encroaches. In the invaded region, the pressure drop is assumed linear. This corresponds to a constant residual gas saturation throughout. However, the relative permeability of the water phase in this region is reduced as a result of this residual gas.

With these assumptions, the reservoir pressure \( p \) can be determined as a function of the cumulative production \( G_p \) and of time \( t \) as follows.

\[
p = p_i + \frac{\alpha^2 \pi c}{k_{rw}} \frac{\partial}{\partial t} \left[ \frac{z}{p} (G_p - G) \right] - \frac{z_i}{p_i} G - \alpha \int_0^t \frac{\partial}{\partial S} \left[ \frac{z}{p} (G_p - G) \right] dS \frac{dS}{(t-S)^{1/2}}.
\]

A derivation of this equation is included in the Appendix.

Eq. 1 must be solved for \( G \), the initial gas in place – a somewhat arbitrary process. In this work, \( \alpha \) and \( p_j \) also were considered unknowns. Each set of data points (pressure and production as a function of time) provides a separate equation of the form of Eq. 1. Generally, there are many more than three sets of data points; hence, the unknowns \( G \), \( \alpha \), and \( p_j \) are adjusted to optimize the agreement of each calculated \( p \) with the measured reservoir pressure.

A computer program was written to perform this optimization. The program incorporates the calculation of \( z \) factors as a function of temperature, pressure, and gas density and the conversion of wellhead pressures to reservoir values when single-phase flow predominates in the well. Finite difference methods are used to evaluate the derivative and the integral in the equation. The derivative with respect to time was taken as the backward difference approximation, and the integral was evaluated as

\[
2 \sum_{n=1}^{i} \left[ \frac{z_n (G_{pn} - G)}{p_n} - \frac{z_{n-1} (G_{pn-1} - G)}{p_{n-1}} \right] \frac{\sqrt{t_j - t_{n-1}} - \sqrt{t_j - t_n}}{t_n - t_{n-1}}.
\]

The program provides values of three constants in Eq. 1: \( \alpha \), \( p_j \), and \( G \), which provide the closest match to the pressure decline data. \( \alpha \) can be thought of as a shape factor or as a measure of the water influx. \( p_j \) is the initial reservoir pressure. \( G \) is the initial gas-in-place volume. Errors in the pressure measurements are often apparent; hence, the value of extrapolating this much information from the data is questionable.

To assess the reliability of the gas-in-place estimate obtained, a standard deviation for this value can be calculated by assuming a linearized form of Eq. 1:

\[
\Delta p = \frac{\partial p}{\partial G} \Delta G + \frac{\partial p}{\partial p_j} \Delta p_j + \frac{\partial p}{\partial \alpha} \Delta \alpha,
\]

where \( \Delta p \), \( \Delta G \), \( \Delta p_j \), and \( \Delta \alpha \) are deviations from their optimal values and the derivatives are evaluated at the optimal values. If the functional relationship is linear as assumed in Eq. 2, the equation for calculating the error in the parameters is developed in many statistical analysis texts – e.g., Carnahan et al.\textsuperscript{10} The equation for the standard deviation in the estimated gas-in-place value is

\[
\sigma_G^2 = \left[ \sum_j \left( \frac{\partial p_j}{\partial G} \right)^2 \sum_j \left( \frac{\partial p_j}{\partial \alpha} \right)^2 \right] \left( \sum_j \left( \frac{\partial p_j}{\partial p_j} \right)^2 \right) \sigma_p^2 \nu,
\]

where \( \sigma_p \) is the standard deviation in the pressure data fit and \( \nu \) is given by

\[
\nu = \left[ \sum_j \left( \frac{\partial p_j}{\partial G} \right)^2 \sum_j \frac{\partial p_j}{\partial p_j} \sum_j \frac{\partial p_j}{\partial \alpha} \frac{\partial p_j}{\partial \alpha} \right]^{-1} \left[ \sum_j \left( \frac{\partial p_j}{\partial G} \right)^2 \left( \frac{\partial p_j}{\partial p_j} \right)^2 \left( \frac{\partial p_j}{\partial \alpha} \right)^2 \right]^{-1} \left[ \sum_j \frac{\partial p_j}{\partial G} \sum_j \frac{\partial p_j}{\partial p_j} \sum_j \frac{\partial p_j}{\partial \alpha} \right]^{-1} \left( \sum_j \left( \frac{\partial p_j}{\partial G} \right)^2 \right) \sigma_p^2.
\]

In both equations, the sums are taken for all \( n \) data points to be matched.

The value of the initial gas in place, \( G \), obtained from the best fit to the pressure data, can be thought...
of as the most probable gas in place, i.e., there is a 50% probability that the gas in place is higher and a 50% probability that the gas in place is lower. The standard deviation in the gas in place obtained from Eq. 3 can be used to estimate the probability that other initial-gas-in-place values are valid. In Table 1, the left column shows the probabilities of having the amount of gas indicated by those values in the right column. For example, if the optimal estimate for the gas in place were 100 Bcf with a standard deviation of 10 Bcf, there is a 99% probability that at least 87.2 Bcf of gas exists in the reservoir or a 1% chance of 123.3 Bcf, etc.

The linearized pressure assumption of Eq. 2 is valid only for small variations in the pressure—i.e., small errors. Pressure data errors often may exceed the validity of the formulation. Nevertheless, this analysis can serve as a useful qualitative tool in evaluating the reliability of the results.

In addition to the assumed linearity of the pressure function, this statistical analysis assumes that (1) the mathematical representation of the process shown in Eq. 1 is an accurate representation of the physical phenomenon, (2) the measured times, production volumes, and various input constants are exact so that data inaccuracies occur only in the pressure measurements, and (3) the error in pressure measurements are distributed randomly.

Results
Table 2 shows production and pressure decline data for six wells that, from geological data, seem to be in the same gas reservoir. However, it is immediately clear from a plot of p/z against cumulative production (Fig. 1) that Well 6 is declining much more rapidly than the other wells. Therefore, there must be a communication barrier between this segment of the reservoir and the rest. Fig. 2 shows the pressure decline data of Well 6 as a function of its own production. The solid line in the figure shows the pressure decline curve predicted by optimization of Eq. 1. This procedure predicted an original gas in place of 11 Bscf (standard deviation: 1 Bscf) and no water influx. This result also suggests total isolation of Well 6 from the other wells since gas leakage into the Well 6 reservoir from the others would have provided pressure maintenance similar to a water drive.

Fig. 3 illustrates the pressure decline of the other five wells as a function of their cumulative production. The solid line in the figure shows again the pressure decline curves predicted by the optimization of Eq. 1. This procedure predicted an original gas in place of 138 Bscf with a moderate water influx. The standard deviation in this estimated volume is 38 Bscf. Sufficient scatter exists in the data that, to the eye, a straight-line fit is just about as good as the optimal fit. However, extrapolation of the straight-line fit to zero pressure suggests that the gas in place is 243 Bscf. This is 2.76 standard deviations greater than the optimal value. Therefore, one would estimate a less than 1% probability that this much gas actually existed in the reservoir. Even

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*Time since start of production.
though this is a gross approximation, it does confirm that reasonable estimates of gas in place can be predicted from performance data even though there are substantial errors in the data.

Conclusions

This study presents a new method for analyzing pressure decline data that can result in meaningful estimates of initial gas volumes despite substantial scatter in the decline data.

Nomenclature

\[ A = \text{area of reservoir face through which water is encroaching} \]
\[ c = \text{total compressibility (rock and water compressibilities)} \]
\[ G = \text{initial gas in place in reservoir} \]
\[ G_p = \text{cumulative production} \]
\[ L = \text{length of gas reservoir} \]
\[ k = \text{reservoir permeability} \]
\[ k_{rw} = \text{relative permeability of water in invaded zone} \]
\[ n = \text{number of pressure data points} \]
\[ P = \text{reservoir pressure} \]
\[ R = \text{perfect gas law constant, } pV_m = RT \]
\[ t = \text{time} \]
\[ T = \text{reservoir temperature} \]
\[ u = \text{fluid flux through reservoir} \]
\[ V_m = \text{molar volume} \]
\[ x = \text{distance} \]
\[ z = \text{perfect gas law deviation factor, } pV_m = zRT \]
\[ \alpha = \left( \frac{\mu_w}{Kc\phi \pi} \right)^{\frac{1}{2}} \frac{RT}{A} \]
\[ \mu_w = \text{viscosity of water at reservoir conditions} \]
\[ \sigma_x = \text{standard deviation in } x \]
\[ \sigma_p^2 = \sum_{j=1}^{n} \frac{(p_{ji} - P_j)^2}{(n-3)} \]
\[ \phi = \text{porosity} \]

Subscripts

\[ i = \text{initial condition of reservoir} \]
\[ j = \text{ith data point} \]
\[ p = \text{based on performance data} \]
\[ x_i = \text{at position } x_i \]

Acknowledgments

This work was expedited by valuable discussions with R.J. Redmore, and its final publication was made possible through the help of K.D. Stidham. I also express my appreciation to Mobil Oil Exploration and Production Southeast Inc., which supported the work and allowed its publication.

References


APPENDIX

Mathematical Derivation of Model

Consider the gas reservoir shown conceptually in Fig. 4. Initially, the gas existed exclusively in Zones A and B. An infinite aquifer extended to the right boundary of this area in Zone A. However, as gas is produced from Zone A and the pressure reduced, water encroaches into this area forming a two-phase region, Zone B. To relate the pressure to the production mathematically, the fluid flow in each of these zones must be considered separately.

Zone A contains only gas. If we assume that this gas has a very large mobility, then it is clear from Darcy's law (see Nomenclature),

$$u = -\frac{k}{\mu} \frac{\partial p}{\partial x}, \quad \ldots \quad (A-1)$$

that the pressure gradient is small. That is, if the permeability is large and/or the gas viscosity is small, the pressure is uniform throughout the gas Zone A. In Zone C, if we assume a one-dimensional Darcy flow of the water, a material balance provides the following partial differential equation.

$$c\phi \frac{\partial p}{\partial t} = \frac{\partial}{\partial x} \left( \frac{k}{\mu_w} \frac{\partial p}{\partial x} \right). \quad \ldots \quad (A-2)$$

The appropriate boundary conditions for this partial differential equation are

$$\begin{align*}
  p &= p_i, & \text{at } t = 0 \\
  p &= p_i, & \text{as } x \to \infty \\
  u &= \frac{k}{\mu_w} \frac{\partial p}{\partial x}, & \text{at } x = L_i
\end{align*} \quad \ldots \quad (A-3)$$

The solution to Eq. A-2 with boundary conditions (Eq. A-3) is given by Carslaw and Jaeger. This solution takes the form

$$P_{x_i} = p_i - \left( \frac{\mu_w}{k \phi \mu} \right)^{\frac{1}{2}} \int_0^t u(t-\tau) \frac{dt}{\tau^{\frac{1}{2}}} \quad \ldots \quad (A-4)$$

In Zone B, it is assumed that there is a uniform mass flux throughout. Therefore, this flux can be related to the pressures at the two boundaries of Zone B through

$$u(t) = \frac{k}{\mu_w} k_{rw} \left( \frac{P - P_{x_i}}{L - L_i} \right) \quad \ldots \quad (A-5)$$

A mass balance of the form

cumulative production = initial gas in place − remaining gas in place

can be written in the form

$$G_p = \frac{A \phi}{RT} \left( \frac{L_i P_i}{z_i} - \frac{L P}{z} \right) \quad \ldots \quad (A-6)$$

Furthermore, the movement of the gas/water interface can be related to the water flux with

$$u(t) = -\frac{\phi}{\mu} \frac{dL}{dt} \quad \ldots \quad (A-7)$$

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Eqs. A-4 through A-7 provide four equations in the unknowns \( p, p_{x_i}, u, L, \) and \( G_p \). A function of the form \( P(G_p) \) is desired. Hence, \( p_{x_i}, u, \) and \( L \) must be eliminated by combination of the preceding equations to provide the single relationship. Elimination of \( p_{x_i} \) from Eq. A-4 by substituting Eq. A-5 results in

\[
p = p_i - \frac{\mu_w}{k} \frac{u(t)}{k_w} (L_i - L)
\]

\[-\left(\frac{\mu_w}{k_c \phi \pi}\right)^{\frac{1}{2}} \int_0^t u(t-\tau) \frac{d\tau}{\tau^{\frac{1}{2}}} \quad \ldots \quad (A-8)
\]

Substituting \( L \) from Eq. A-6 into Eq. A-7 and further substituting the resulting expression for \( u \) with the expression for \( L \) into Eq. A-8 results in

\[
p = p_i + \frac{\alpha^2 \pi c}{k_w} \frac{\partial}{\partial t} \left[ \frac{z}{p} (G_p - G) \right]
\]

\[-\left(\frac{z}{p} (G - G_p) - \frac{z_i}{p_i} G \right) - \alpha \int_0^t \frac{\partial}{\partial S} \left[ \frac{z}{p} (G_p \right)
\]

\[-G) \left( \frac{dS}{(t-S)^{\frac{1}{2}}} \right) \right] \quad \ldots \quad (A-9)
\]

In this equation, \( \alpha \) is defined as

\[
\alpha = \left( \frac{\mu_w}{k_c \phi \pi} \right)^{\frac{1}{2}} \frac{RT}{A} \quad \ldots \quad (A-10)
\]

and \( L_i \) has been eliminated from the expression by substitution of the special case of Eq. A-6 in which \( L = 0 \) when \( G_p = G \). Eq. A-9 forms the basis for the model and appears as Eq. 1 in the text.

**SI Metric Conversion Factors**

- cu ft \( \times \) 2.831 685 \( \quad \) E \( - \) 02 = m \( ^3 \)
- °F \( \times \) (°F \( - \) 32)/1.8 = °C
- psi \( \times \) 6.894 757 \( \quad \) E \( + \) 00 = kPa
- scf \( \times \) 2.863 64 \( \quad \) E \( - \) 02 = std m \( ^3 \)

JPT