Final Exam

for

Petroleum Engineering 620
Fluid Flow in Reservoirs

Department of Petroleum Engineering
Texas A&M University — Summer 2001
Instructor: Tom Blasingame

This package includes:
— Exam Guidelines
— Exam Problem 1
— Exam Problem 2
— Exam Problem 3

This document (as well as other files pertaining to PETE 620) are located at:
http://pumpjack.tamu.edu/~t-blasingame/P620_01B/
Deadline/Due Date
The final exam is due by 5:00 p.m. Monday 20 August 2001 at my office in Richardson Rm. 401T. A penalty of 1 letter grade will be assigned for work submitted after this deadline.

General Rules:
1. You are to work independently—collaborative efforts are not permitted (i.e., joint work). You are permitted to discuss problems—but sharing of solutions, plots, programs, etc. will result in penalties of 1 to 2 letter grades. In extreme cases, work sharing may result in a failure grade (F).
2. You must show all work for credit. Unsupported work will not be given credit.
3. You are given specific instructions for each problem, follow these guidelines exactly—failure to do so will result in penalties.
4. Be as neat and organized as possible.
5. You must give an OUTLINE for each problem solution that also includes references or other citations when appropriate.

Format for Individual Problems:
1. For derivation problems, YOU MUST USE THE FOLLOWING FORMAT:
   — Type or write neatly, use lined paper (if handwritten) and ONLY write (or print) on the front of the page.
   — Number your pages as follows in the upper right corner of each page: Problem #/page #/total pages for this problem.
2. For computer-aided solutions, YOU MUST PROVIDE THE FOLLOWING:
   — Write a complete development of the equations and algorithms used and give a written summary description of the problem.
   — Include a copy of the computer code, the input file, and an abbreviated output file for each case.
   — For plots, you are to give a summary table (abbreviated to 1 or 2 pages) of data for each plot, then present an appropriately scaled plot that clearly identifies all regions of interest.

Report Format:
1. YOUR PROJECTS WILL NOT BE RETURNED FOR QUITE SOME TIME, so if you want a copy then you need to make one before turning in your project.
2. The report is to be bound with the following information given on a coverpage:
   Name: (printed)
   Course: Petroleum Engineering 620
   Date: Day-Month-Year
   Assignment: Final Exam
3. Contents of the final exam report: Separate each section with a cover page that includes a section title. Include the following materials in this order:
   — The exam assignment sheets (this material)
   — Project Summary (at least 1-2 pages, discuss results, observations, etc.)
   — Problem 1
   — Problem 2
   — Problem 3
1. Derivation of a Rate Relations for Pseudosteady-State Flow (liquid)

Given:

You are given the following "liquid" relations for material balance and pseudosteady-state radial flow. Assume Darcy units (no need for conversion factors).

**Material Balance Equation**: (valid for all times)

\[ \bar{p} = p_i - \frac{B}{\Phi c_i} N_p \]

**Pseudosteady-State Flow Relation**: (only valid for pseudosteady-state flow)

\[ \bar{p} = p_{wf} + q b_{pss} \]

Recall that the definition of the cumulative production, \( N_p \), is given by:

\[ N_p = \int_0^t q(t) dt \] or, a more useful result is given by, \( q = \frac{d}{dt} (N_p) \).

**Required**: 

1.1. You are to derive the flowrate relation for each of the following cases:

1.1.a. \( p_{wf} = \text{constant} \)
1.1.b. \( dp_{wf}/dt = \text{constant} \)
1.1.c. \( p_{wf} = f(t) \) (arbitrary)

**Hints**:

a. To start, take the time derivative of both the material balance and pseudosteady-state flow relations — equate and establish the base differential equation.

b. At \( t=0 \), \( c = q_i = \frac{1}{b_{pss}} (p_i - p_{wf}) \).
2. Derivation of Pseudosteady-State Gas Flowrate Relation ($p/z$-Form)

**Given:**
This exercise involves the derivation of the $p/z$ form of the pseudosteady-state gas flowrate relation. The lead reference for this work is:


The appropriate portion of this thesis (Appendix B) is attached for your use. You are free to "follow" this work, but any and all derivations should include your own efforts.

**Required:**

2.1. Derivation of the $p/z$ Form of the Pseudosteady-State Gas Flowrate Relations:

**Gas Material Balance Relation:**

$$\frac{p}{z} = \frac{p_i}{z_i} \left[ 1 - \frac{G}{G_p} \right]$$ .................................................. (1)

**Pseudosteady-State Gas Flow Equation: ($p/z$ form)**

$$q_g = J_g \left[ \frac{\mu_p}{\mu} \right] \int p^{\frac{1}{z}} \frac{dp}{p^{\frac{1}{z}} \frac{d(p/z)}{d(p/z)}}$$ ........................................ (2)

You are to combine Eqs. 1 and 2 to yield: ($p_{wf}$=constant, but $p_{wf} \neq 0$)

**Rate Equation: ($p/z$ form)**

$$q_g(t) = q_{gi} \left[ \frac{p_{wf}^{\frac{1}{z}z_{wf}}}{p_{i}^{\frac{1}{z}z_{i}}} \right] \left[ 1 + \frac{1 - \frac{p_{wf}^{\frac{1}{z}z_{wf}}}{p_{i}^{\frac{1}{z}z_{i}}}}{1 + \frac{p_{wf}^{\frac{1}{z}z_{wf}}}{p_{i}^{\frac{1}{z}z_{i}}}} \exp \left( \frac{-2q_{gi}t}{G_p} \frac{p_{wD}}{p_{i}^{\frac{1}{z}z_{i}}} \right) \left[ 1 - \frac{1}{\frac{p_{wf}^{\frac{1}{z}z_{wf}}}{p_{i}^{\frac{1}{z}z_{i}}}} \right] \right]^2 \left[ 1 - \frac{1}{\frac{p_{wf}^{\frac{1}{z}z_{wf}}}{p_{i}^{\frac{1}{z}z_{i}}}} \exp \left( \frac{-2q_{gi}t}{G_p} \frac{p_{wD}}{p_{i}^{\frac{1}{z}z_{i}}} \right) \left[ 1 - \frac{1}{\frac{p_{wf}^{\frac{1}{z}z_{wf}}}{p_{i}^{\frac{1}{z}z_{i}}}} \right] \right]^{-1}$$ .................................................. (3)

**Rate Equation: ($q_{Dd}$ form)**

$$q_{Dd} = \frac{p_{wD}^2}{(1 - p_{wD}^2)} \left[ 1 + \frac{(1 - p_{wD})}{(1 + p_{wD})} \exp (- p_{wD} t_{Dd}) \right]^2 \left[ 1 - \frac{(1 - p_{wD})}{(1 + p_{wD})} \exp (- p_{wD} t_{Dd}) \right]^{-1}$$ ........................................ (4)
2. (Continued)

Cumulative Production Equation: \( (G_{pDd} \text{ form}) \)

\[
G_{pDd} = 1 - p_{wD} \left[ \frac{1 + \frac{(1 - p_{wD})}{(1 + p_{wD})} \exp \left( - p_{wD} t_{Dd} \right)}{1 - \frac{(1 - p_{wD})}{(1 + p_{wD})} \exp \left( - p_{wD} t_{Dd} \right)} \right]
\]  
............................................ (5)

Tasks:
2.1.a. Derive the \( p/z \) form of the pseudosteady-state gas flow relation for the case of \( p_{wf} \) constant, but \( p_{wf} \neq 0 \). Provide the \( q_{g} \), \( q_{Dd} \), and \( G_{pDd} \) results. You can use the attached Appendix B text from Knowles, but you must SHOW ALL DETAILS in your work.

2.1.b. Comment on applications that could be made using these semi-analytical models derived for the \( q_{g} \), \( q_{Dd} \), and \( G_{pDd} \) functions (recall that \( p_{wf} \) is constant, but \( p_{wf} \neq 0 \)).

Hint:

2.2 Derivation of the \( q_{Dd} G_{pDd} \) Identity for the \( p/z \)-Form of the Pseudo-steady-State Gas Flow Relation.

You are to combine Eqs. 4 and 5 to yield:

Rate-Cumulative Production Equation: \( (q_{Dd} G_{pDd} \text{ form}) \)

\[
q_{Dd} = 1 - \frac{2}{(1 - p_{wD})^2} G_{pDd} + \frac{1}{(1 - p_{wD})^2} G_{pDd}^2
\]  
............................................ (6)

Tasks:
2.2.a. Derive the \( q_{Dd} G_{pDd} \) identity for this case (Eq. 6). You can use the attached Appendix B text from Knowles, but you must SHOW ALL DETAILS in your work.

2.2.b. Comment on applications that could be made using this semi-analytical rate-cumulative production performance model.

Hint:
3. Solutions for a Well Produced at a Constant Bottomhole Flowing Pressure

Given:

This problem has been given to previous classes in different forms—be sure to solve the problem as it is specifically posed in this examination (there are differences).

— The governing relations for this case are:

‡ Dimensionless Diffusivity Equation

\[
\frac{1}{r_D^2} \frac{\partial}{\partial r_D} \left( r_D \frac{\partial p_D}{\partial r_D} \right) = \frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_D}{\partial t_D} \quad .............................................(1)
\]

‡ Dimensionless Variables

\[
r_D = \frac{r}{r_w} \quad .................................................................(2)
\]

\[
p_D = \frac{(p_i - p(r,t))}{(p_i - p_{wf})} \quad \text{(Darcy Units)}\quad .............................................(3)
\]

\[
t_D = \frac{kt}{\phi \mu c r_w^2} \quad \text{(Darcy Units)}\quad .............................................(4)
\]

‡ Initial Condition (Uniform Pressure Distribution)

\[
p_D(r_D,t_D<0) = 0 \quad .................................................................(5)
\]

‡ Inner Boundary Condition (Constant Pressure at the Well)

\[
p_D(r_D=1,t_D) = 1 \quad .................................................................(6)
\]

‡ Outer Boundary Conditions

a. "Infinite-Acting" Reservoir

\[
p_D(r_D \to \infty,t_D) = 0 \quad \text{(No reservoir boundary)} \quad .............................................(7)
\]

b. "No Flow" at the Outer Boundary

\[
\left[ r_D \frac{\partial p_D}{\partial r_D} \right]_{r_D=r_eD} = 0 \quad \text{(No flux across the reservoir boundary)} \quad ............(8)
\]

c. Constant Pressure Boundary

\[
p_D(r_eD,t_D) = 0 \quad \text{(Constant pressure at the reservoir boundary)} .......(9)
\]

You are given the following Laplace domain identity for the dimensionless rate profile, for the case of a well produced at a constant wellbore pressure:

\[
\bar{q}_D(r_D=1,u) = -\left[ r_D \frac{d p_D}{d t_D} \right]_{r_D=1} \quad .............................................(10)
\]
3. (Continued)

Required:

3.1. You are to derive the following $q_D(r_D=1,u)$ solutions: (Laplace domain solutions)

3.1.a. "Infinite-Acting" Reservoir Case

**Cylindrical Source Form**

\[
q_D(r_D=1,u) = \frac{1}{u} \frac{K_1(\sqrt{u})}{K_0(\sqrt{u})} \tag{11}
\]

**Line Source Form**

\[
q_D(r_D=1,u) = \frac{1}{u} \frac{1}{K_0(\sqrt{u})} \tag{12}
\]

3.1.b. "No Flow" at the Outer Boundary Case

**Cylindrical Source Form**

\[
q_D(r_D=1,u) = \frac{1}{u} \frac{\pi K_1(\sqrt{u}) I_1(\sqrt{\pi r_{eD}}) - \pi I_1(\sqrt{\pi r_{eD}}) K_1(\sqrt{\pi r_{eD}})}{I_0(\sqrt{\pi}) K_1(\sqrt{\pi r_{eD}}) + K_0(\sqrt{\pi}) I_1(\sqrt{\pi r_{eD}})} \tag{13}
\]

**Line Source Form**

\[
q_D(r_D=1,u) = \frac{1}{u} \frac{I_1(\sqrt{\pi r_{eD}})}{I_0(\sqrt{\pi}) K_1(\sqrt{\pi r_{eD}}) + I_1(\sqrt{\pi r_{eD}}) K_0(\sqrt{\pi})} \tag{14}
\]

or, rearranging slightly, we have:

\[
q_D(r_D=1,u) = \frac{1}{u} \frac{1}{K_0(\sqrt{u}) + I_0(\sqrt{u}) \frac{K_1(\sqrt{\pi r_{eD}})}{I_1(\sqrt{\pi r_{eD}})}} \tag{15}
\]

3.2. Using the solutions given in Part 2.1, you are to perform the following tasks:

3.2.a. "Infinite-Acting" Reservoir Case:

- Derive the $q_D(r_D=1,t_D)$ function using Eq. 12.

3.2.b. "No Flow" at the Outer Boundary Case:

- Derive the $q_D(r_D=1,t_D)$ function using Eq. 15.

Note: The tasks assigned in Parts 2.2.a and 2.2.b are potentially VERY DIFFICULT, and there is no guarantee of an explicit solution—your effort should be significant. For some guidance, you can consult:

3. (Continued)

3.3. Presuming the following solutions:

"Infinite-Acting" Reservoir Case: (transient flow only)

\[ q_{Dcp}(r_D=1, t_D) \approx \frac{1}{2} \ln \frac{4}{e^{t_D/r_D} r_D^2} \] ...........................................(16)

"No Flow" at the Outer Boundary Case: (pseudosteady-state flow only)

\[ q_{Dcp}(r_D=1, t_D) \approx \frac{1}{\ln(r_{eD}) - 3/4} \exp \left( -\frac{2}{r_{eD}^2} \frac{1}{\ln(r_{eD}) - 3/4} t_D \right) \] ............ (17)

3.3.a. You are to "combine" Eqs. 16 and 17 to yield a "total performance" solutions — i.e., a relation for all \( t_D \). You may have to resort to semi-analytical expansions and/or "switch" conditions to turn one solution on and the other solution off.

3.3.b. You are to compare the result obtained from 3.3.a to the numerical inverse Laplace transform of Eq. 15. You are to use the Stehfest algorithm and the following parameters:

- \( 1 \times 10^1 < t_D < 1 \times 10^8 \)
- \( r_{eD} = 1 \times 10^3 \)

You are to plot the \( q_D(r_D=1, t_D) \) solutions as follows:

- \( \log(q_D(r_D=1, t_D)) \) versus \( t_D \)
- \( \log(q_D(r_D=1, t_D)) \) versus \( \log(t_D) \)

The Gaver-Stehfest formula for numerical Laplace transform inversion is given by:

\[ f_{\text{Gaver-Stehfest}}(T) = \frac{\ln(2)}{T} \sum_{i=1}^{N} V_i \cdot f \left( \frac{\ln(2)}{T} i \right) \]

And the Stehfest extrapolation coefficients are given as:

\[ V_i = (-1)^{\left\lfloor \frac{i}{2} \right\rfloor} \sum_{k=\left\lfloor \frac{i+1}{2} \right\rfloor}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{n!}{2^k k! (2k)!} \left( \frac{n-k}{2} \right)! k! (k-1)! (i-k)! (2k-i)! \]

You are free to use any programming device you wish—most students will probably prefer to use MS Excel to compute the numerical inversion, and the easiest mechanism for Excel is to have the coefficients already computed. The Gaver-Stehfest coefficients are given in a table attached to this assignment.

This part of the exam (3.3.b) is likely to be time consuming (setup, debugging, etc.) be sure to factor this into your efforts.
3. (Continued)

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