 Unsolicited

Generalized Transient Pressure Solutions
With Wellbore Storage

By

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Abstract

This paper presents new methods for computing the wellbore pressure with wellbore storage and skin effects. These methods use existing analytical solutions for the constant rate case. Two types of wellbore storage (afterflow) are considered: the first is the constant wellbore storage case, and the second is the exponential wellbore flow rate case.

The accuracy of the methods were verified by comparing results with published ones. It is shown that the methods work for a variety of wellbore geometries. The methods were applied to fully-penetrated radial and vertically fractured wells.

Introduction

Pressure behavior of a well with wellbore storage (afterflow) have received a considerable attention during the last three decades. The study of pressure behavior with wellbore storage received its impetus from the classical paper by van Everdingen and Hurst. The early investigations on the pressure behavior with wellbore storage carried out by van Everdingen, Hurst, and Gladfelter et al., Ramey.

In the early part of 1970, interest in the pressure behavior with wellbore storage gained momentum again. Several contributions of lasting significance are due to Ramey, Agarwal et al, McKinley, and Wattenbarger and Ramey. In later years, Earlougher and Kersch and Gringarten et al introduced new
type-curves with different parametrization than the earlier ones. Many more type-curves with wellbore storage have appeared in the literature for different wellbore geometries and reservoir types since 1970.

At the most fundamental level, analytical solutions with wellbore storage (type-curves) have provided a better understanding of the basic behavior of wells at early times. These solutions (type-curves) have also been used for estimating reservoir parameters. Furthermore, the semilog analysis which is considered a basic tool for estimating permeability, skin, and reservoir pressure has also benefitted from type-curves.

Most of the above analytical solutions (type-curves) to well test problems are based on constant wellbore storage. In practice, the constant wellbore storage assumption is never entirely fulfilled and undetectable deficiencies of the solutions with the constant wellbore storage are very real for predicting the transient pressure behavior of a well. Refs. 12, 13, 14, and 15 reported wellbore storage variation for pumping as well as naturally flowing wells.

It is important to appreciate that the flow in the production string usually is two-phase for oil wells although it is usually single-phase at the wellbore. Such nonlinearities in fluid flow through the production string can make the wellbore storage vary considerably during the test. This problem has become apparent in recent years, largely as a result of capability of measuring downhole flow rate simultaneously with pressure. In principle, measured downhole flow rate can be used to eliminate the wellbore storage effect above the flowmeter. However, the wellbore storage effect on the formation pressure cannot be eliminated entirely as long as downhole rate is measured at the wellbore, not through perforations. The wellbore storage below the flowmeter
can be important for fractured wells. Furthermore, it may not be practical to measure the downhole rate for every well.

The objective of this work is to give a generalized method to compute solutions with wellbore storage and skin effects.

**BACKGROUND**

van Everdingen and Hurst\(^1\) presented the dimensionless wellbore pressure for a continuously varying flow rate as

\[
P_{\text{wbD}}(t_D) = \int_0^{t_D} q_D^*(\tau) p_{\text{wd}}(t_D - \tau) d\tau
\]

where

\[
P_{\text{wbD}} = \frac{kh}{141.2 \, qB \mu} [p_i - p_{\text{wf}}(t)]
\]

\[
t_D = \frac{0.0002537 \pi t}{\phi \mu \sigma r_w^2}
\]

\(P_{\text{wd}}(t_D)\) = the dimensionless wellbore pressure for the constant rate case which includes skin and storage effects if they exist.

\(q_D(t_D) = q_{\text{wb}}(t_D)/q_B\)

\(q_D^*(t_D) = dq_D(t_D)/dt_D\)

\(q\) = reference flow rate. If the stabilized sandface rate is available, then \(q_B\) should be replaced by \(q_{\text{res}}\).
\[ q_{WB}(t) \] = variable wellbore flow rate (flowmeter readings)

\[ q_D(t_D) \] = dimensionless sandface rate.

The nomenclature used above is slightly different than what was used in Ref. 16 because flow rate is not measured through perforations or the sandface. Therefore, the constant rate solution (the response function) can still be affected by wellbore storage. This point will be elaborated later. The name "sandface" flow rate is also changed to "downhole" or "wellbore" flow rate.

Eq. 1 is the general solution for the time-dependent internal boundary condition which can be used for different wellbore geometries and reservoir types as long as the fluid in the porous media is single phase. Eq. 1 also represents the solution of the forward problem. Thus, either measuring or knowing the time-dependent boundary condition, \( q_D(t_D) \), the wellbore pressure can be obtained using Eq. 1 and the constant rate solution (influence function or unit-response function) for the problem.

From an ideal point of view, one should measure the varying rate, \( q_D(t_D) \) as a function of time in order to realistically determine the wellbore pressure response. In that case, we carry out integration with known \( P_{WD} \). Unfortunately, the pressure is still the only observation which we monitor in many well tests. If flow rate data is not available, the wellbore flow rate behavior of a test well has to be approximated during the annulus unloading or afterflow for early time analyses.

Even if we measure the wellbore flow rate data, it is possible that there is enough compressible fluid below or beyond the measurement point which will affect the wellbore pressure behavior. For example, the wellbore volume below
the measurement point and/or the volume of an infinite-conductivity fracture may distort the wellbore pressure considerably. Thus, obtaining analytical solutions with wellbore storage is still important. Furthermore, it is also important to have readily available solutions with wellbore storage for automated (computerized) type-curve techniques which make use of a wide variety of solutions for different wellbore geometries and reservoir types.

For many reservoir engineering problems, the constant rate solutions \( p_{wD}'s \) are not available in simple forms; indeed, many of them are given in the Laplace image space (Laplace transform solutions). Thus, the Laplace transform of the wellbore pressure solution for a continuously varying rate can be written directly from Eq. 1 as

\[
\bar{P}_{wbD}(s) = s\bar{q}_D(s) \bar{P}_{wD}(s)
\]  

(2)

where

\( s = \text{Laplace transform variable.} \)

The principal difficulty with Eq. 2 is the inversion from the Laplace image space to the real time. The inversion of Eq. 2 or both \( \bar{q}_D \) and \( \bar{P}_{wD} \) usually do not appear in published tables for most transient problems, but at least in principle they can be inverted using the Mellin inversion theorem. However, the resulting integrals from the Mellin inversion may converge slowly and takes a considerable amount of computational effort and time. Furthermore, it is not always possible to find a suitable contour (path) for the complex integration.
Several numerical inversion methods are available for the inversion of Laplace transform solutions. Among these methods, the Stehfest algorithm has been used widely for the reservoir engineering problems. The Stehfest algorithm usually works well if the Laplace transform solution is well behaved and non-periodic. A study on the behavior of Stehfest algorithm and a survey on numerical Laplace transform inversion methods can be found in Ref. 18.

In this work, we will show that Eqs. 1 and 2 can be used simultaneously to compute the wellbore pressure with given $q_D$ or $\bar{q}_D$ and $p_{WD}$ or $p_{WD}$.

As we said earlier, the downhole flow rate, $q_D$, is the time-dependent inner boundary condition which can either be measured directly or inferred from other sources. If $q_D$ is available as measured data, the computation of the wellbore pressure is rather straightforward using Eqs. 1 or 2, since $p_{WD}$ is the constant rate solution of the problem. A detailed treatment of this problem can be found in Ref. 13, 14, 15, 16, 19, 20, 21, and 22. It is also possible that measuring $q_D$ may not eliminate afterflow or wellbore storage effects completely. Thus, two different formulations are available to approximate (infer) the downhole flow rate. These are: 1) constant wellbore storage case, and 2) exponential wellbore rate case.

**METHOD OF SOLUTIONS**

Here we will seek solutions for the wellbore pressure, $p_{WD}$, for the constant rate case with wellbore storage. For this case Eqs. 1 and 2 can be rewritten as, respectively,
\[ p_{WD}(t_D) = s q_D(t_D) + \int_0^{t_D} q_D(\tau) p_D'(t_D-\tau) d\tau \] (4)

and

\[ \bar{p}_{WD}(s) = \bar{q}_D(s) [s + s P_D(s)] \] (5)

where

\[ p_D(t_D) = \text{the dimensionless sandface pressure for constant rate case without wellbore storage and skin effects.} \]

\[ S = \text{steady state skin factor} \]

\[ p_D' = \frac{d p_D(t_D)}{d t_n} \]

\[ q_D(t_D) = \text{dimensionless wellbore rate and we assumed that it is approximated (inferred) rate.} \]

**Constant Wellbore Storage Case**

Sometimes it is possible that during annulus unloading or afterflow, the compressibility of the fluid in the production string may remain constant. Under this condition, the dimensionless wellbore flow rate can be approximated as

\[ q_D(t_D) = 1 - C_D \frac{d p_{WD}(t_D)}{d t_D} \] (6)

\[ C_D = \frac{5.615 C}{2 \pi \rho_r h r_w^2} \]
If the Laplace transform solution, \( p_D \), for the constant rate case is available, the Laplace transform of the dimensionless wellbore pressure solution for the constant wellbore storage and skin can be written directly from Eqs. 5 and 6 as

\[
\frac{-\bar{p}_{WD}(s)}{s+c_Ds^2[sp_D(s)+s]} = \frac{s\bar{p}_D(s) + s}{s+c_Ds^2[sp_D(s)+s]}
\]  

(7)

\( s \) = Laplace transform variable

For finite \( p_D \) and non-zero \( c_D \), Eq. 7 becomes as \( s \) approaches infinite \((t_D + 0)\) as

\[
\frac{-\bar{p}_{WD}(s)}{s+c_Ds^2} = \frac{1}{s^2c_D}
\]  

(7a)

Inversion of Eq. 7a yields

\[
\bar{p}_{WD}(t_D) = \frac{t_D}{c_D}
\]  

(7b)

As can be seen from Eq. 7b, the wellbore pressure behavior during the early time is independent of the reservoir. In other words, there exists a time limit \((t_D > 0)\), no matter how small, the wellbore pressure will change without affecting the reservoir (the formation side of the wellbore) pressure before this time limit if \( c_D \) is not zero.

As we said earlier, as in Eq. 2, in order to compute \( \bar{p}_{WD} \) from Eq. 7, \( p_D \) has to be available in Laplace image space, and it also requires a good numerical
Laplace transform inversion algorithm. An alternate method will be presented to compute \( p_{WD} \). The method presented here is similar to that which was presented by Cinco-Ley and Samaniego.\(^{23}\)

Substitution of Eq. 6 into Eq. 4 yields an integro-differential equation as\(^{1,7}\)

\[
p_{WD}(t_D) = \int_0^D [1 - C_D p_{WD}^\prime(\tau)] p_D(t_D - \tau) d\tau + S[1 - C_D p_{WD}(t_D)]
\]

\( \prime \) denotes derivatives with respect to time.

The integral in Eq. 8 can be approximated with an assumption that \( \frac{dp_{WD}}{dt_D} \) is constant between two successive data points, as

\[
p_{WD}(t_D) = \int_0^D p_D^\prime(t_D - \tau) d\tau - C_D \sum_{i=0}^{n} p_{WD}^\prime(t_{Di}) \int_{t_{Di}}^{t_{Di+1}} p_D(t_D - \tau) d\tau + S[1 - C_D p_{WD}(t_D)]
\]

Performing integrations in Eq. 9 yields

\[
p_{WD}(t_D) = p_D(t_D) + S + C_D \sum_{i=0}^{n} p_{WD}^\prime(t_{Di}) [p_D(t_D - t_{Di+1}) - p_D(t_D - t_{Di})] - C_D S p_{WD}^\prime(t_D)
\]
Approximation of \( p_{WD}(t_D) \) in Eq. 10 by the forward difference formula and solving for \( p_{WD}(t_{Dn+1}) \) yields

\[
p_{WD}(t_{Dn+1}) = [p_D(t_{Dn+1}) + S + C_D \sum + TTS \ p_{WD}(t_{Dn})]/(1 + TTS) \quad (11)
\]

where

\[
\sum = \sum_{i=0}^{n-1} \frac{[p_{WD}(t_{Di+1}) - p_{WD}(t_{Di})]}{t_{Di+1} - t_{Di}} \ [p_D(t_{Dn+1} - t_{Di+1}) - p_D(t_{Dn+1} - t_{Di})]
\]

\[
TTS = C_D \ [p_D(t_{Dn+1} - t_{Dn}) + S]/(t_{Dn+1} - t_{Dn})
\]

Notice that if \( C_D = 0 \) the solution reduces to the constant rate solution, \( p_D \).

All \( p_{WD} \)'s can be computed from Eq. 10 recursively knowing \( p_{WD}(t_{D1}) \) and the constant rate solution without wellbore storage. The constant rate solution, \( p_D \), can be computed either analytically or numerically. In fact, if we have numerical simulator without wellbore storage for complex wellbore as well as reservoir geometries, the wellbore pressure behavior with wellbore storage can be generated easily using Eq. 11.

As discussed earlier, \( p_{WD}(t_{D1}) \) can always be approximated by \( t_{D1}/C_D \) for almost any system if the wellbore storage is constant. However, the computation of \( p_{WD} \)'s from Eq. 11 becomes much easier if the short-time solution or approximation is available in either an analytical form or a Laplace transform form.

One should also be careful with the time discretization used in Eq. 11 since the derivatives approximated by the forward differences. We had good
experience with logarithmically-equitably spaced time discretization; i.e., log
\( t_{D_{i+1}} \) - log \( t_{D_{i}} \) = constant, for \( i = 0,1, \ldots, n \). One also can use a
different discretization depending on the problem.

We have applied Eq. 11 to compute \( P_{WD} \)'s for a radial and fully penetrated well
in an infinite reservoir. The solution of this problem can be found in Ref.
7. We calculated \( P_{WD} \)'s for \( S = 20 \) and \( C_D = 1000 \) and using \([\log (t_{D_{i+1}}) - \log \]
\( t_{D_{i}} \)]) = 1.0513, \( i = 0, 1, 2, \ldots \). The computation results are pre-
sented in Table 1.

The third column of Table 1 presents the calculated values of \( P_{WD} \) for selected
\( t_D \) values from Eq. 11 with \( P_{WD}(0.1) = 10^{-4} \), while the second column of the
same table presents the values of \( P_{WD} \) from Eq. 7 using the Stehfest\(^\text{17} \) algo-

rithm. The fourth column of Table 1 presents the relative percentage error
between two solutions. As seen from Table 1, differences of two solutions is
small, less than 0.6%, and the relative error decreases for large times. One
can further improve the solution by decreasing the time step size, however,
for most practical purposes, an accuracy less than 1% will be sufficient.

**Exponential Wellbore Rate Case**

van Everdingen\(^2 \) and Hurst\(^3 \) approximated the wellbore flow rate as

\[
q_D(t_D) = 1 - e^{-\beta t_D}
\]

(12)

where \( \beta \) is a positive constant. These authors stated that the constant, \( \beta \),
can be determined from the well and reservoir parameters.
The Laplace transform of the dimensionless wellbore pressure solution can also be written directly from Eqs. 4 and 12 as

\[ \frac{P_{WD}(s)}{s^2} = \frac{\beta [sP_D(s) + s]}{s(\beta + s)} \quad (13) \]

The wellbore pressure for the exponential wellbore rate case can be computed from Eq. 4 or Eq. 13 with Eq. 12. This case is much easier than the constant wellbore storage case because the wellbore flow rate is not a function of the wellbore pressure.

For a line source well, van Everdingen\(^2\), and Hurst\(^3\) presented the wellbore pressure solution for the exponential wellbore rate case using Eq. 4 and Eq. 12 as

\[ P_{WD}(t_D) = P_D(t_D) + S - \frac{1}{2} e^{\beta t_D} \left[ -\ln \beta - 2\gamma + \ln 4 + \text{Ei}(\beta t_D) + 2\beta \right] \quad (14) \]

This equation is only valid when \( t_D > 30 \). An approximate solution for \( t_D < 30 \) can be obtained from Eqs. 14 and 12 (the details of the derivation is given in Appendix A) as

\[ P_{WD}(t_D) = \left( 1 - e^{-\beta t_D} \right) \left[ 1 - \frac{1}{4} \left( \beta - \frac{\beta^2}{(21)^2} + \frac{\beta^3}{(31)^2} - \ldots \right) \right] P_D(t_D) \]

\[ - \frac{1}{8} e^{-\beta t_D} \left( e^{-1/4 t_D} \left( \beta t_D + \frac{\beta^2}{214} \left[ \frac{(4t_D)^2}{2} - \frac{4t_D}{2} \right] + \frac{\beta^3}{314} \right) \right) \]

\[ \left( \frac{(4t_D)^3}{3} - \frac{(4t_D)^2}{6} + \frac{4t_D}{6} + \ldots \right) + S \left( 1 - e^{-\beta t_D} \right) \quad (15) \]
Eq. 15 is very useful for two reasons: 1) the accuracy of the Stehfest numerical inversion algorithm can be check for small values of $t_D$, and 2) $p_{WD}$ can be computed for very small values of $t_D$.

Table 2 presents $p_{WD}$ values from Eq. 13 (using the Stehfest algorithm with the line source solution), Eq. 14, and Eq. 15 for $\beta=0.001$ and $S=10$. $p_{WD}$ values from Eq. 15 are exact for small times while $p_{WD}$ values from Eq. 14 are completely wrong during the same time. The performance of the Stehfest algorithm is remarkable even for very small times and also Eq. 15 works reasonably well for all times.

**APPLICATIONS**

**Vertically Fractured Wells**

Although one may think that all transient problems for vertically fractured wells have been solved over the past two decades. Recently, we needed a solution for an infinite - conductivity vertically fractured well producing at a constant rate with a wellbore storage, when we were developing a semi-automatic computerized type-curve algorithm using the non-linear least-squares method. We did not want to use published typecurves for two reasons; 1) there are only a few limited number of curves available, 2) digitizing these-type curves would have been a very tedious work.

It should be emphasized that measuring downhole flow rate is still important for fractured wells in order to eliminate the wellbore storage effect due to the fluid in the production string. As we said earlier, the fluid in the production string is usually two-phase and with a very complex flow regime,
while the fluid at the bottom hole and in the fracture volume remains slightly compressible.

In this section, we will apply the preceding developments to vertically fractured wells.

1. Uniform - Flux Fracture:

The dimensionless pressure at the wall for the uniform-flux fracture case is given as

\[ P_D = \sqrt{\pi} t_{Dx} \text{erf}\left(\frac{1}{2t_{Dx}}\right) - \frac{1}{2} Ei\left(-\frac{1}{4t_{Dx}}\right) \]  

(16)

where dimensionless time based on the half-fracture length, \( x_f \), is defined as

\[ t_{Dx} = t_D \left(\frac{x_f}{x_f}\right)^2 \]

When \( t_{Dx} > 10 \), Eq. 16 becomes

\[ P_D = \frac{1}{2} \left[ \text{In}_{Dx} + 2.80907 \right] \]  

(17)

with less than one percent error. For \( t_{Dx} < 0.1 \), Eq. 16 becomes

\[ P_D = \sqrt{\pi} t_{Dx} \]  

(18)

Constant Wellbore Storage Case

The wellbore pressure with wellbore storage and skin can be easily obtained from Eq. 7 using the Laplace transforms of Eq. 17 and 18 for small and large
t_{Dxf}. However, the Laplace transform of Eq. 16 can not be obtained easily for Eq. 7. Thus, for the middle time region 0.1 < t_{Dxf} < 10, P_{WD} cannot be computed from Eq. 7 for a given wellbore storage and skin.

The solution to this particular problem was presented by Raghavan26 for small t_{Dxf}. However, the evaluation of Raghavan's solution is not straightforward for non-zero skin values. Nevertheless, one can easily obtain the Laplace transform of P_{WD} from Eq. 7 using the Laplace transform of Eq. 16 for small t_{Dxf} as

$$\frac{P_{WD} (s)}{2s^2 + C_{Dxf} s^2} = \frac{s + 2s^2}{2s^2 + C_{Dxf} s^2} \left( \frac{s + 2s^2}{s^2 + C_{Dxf} s^2} \right)$$  \hspace{1cm} (19)

This equation is only valid when t_{Dxf} < 0.1. Thus, P_{WD}'s for t_{Dxf} < 0.1 can be computed from Eq. 19 using the Stahfens16 algorithm. In fact, an approximate analytical inversion of Eq. 19 was given by Raghavan26 (Eq. B-4 and B-5 with and without skin, respectively). In any case, P_{WD}'s for t_{Dxf} < 0.1 can be easily computed recursively from Eq. 11 using the early time P_{WD}'s from Eq. 19 and Eq. 16.

Table 3 presents P_{WD} values for S = 0.0, C_{Dxf} = 1.0 and using (log t_{Dxfi+1} - log (t_{Dxfi}) = 1.01, i = 0, 1, ..., n. The second column of Table 3 presents P_{WD} values [(P_{WD})_G] from Eq. 11 with P_{WD} (t_{Dxf}) = 10^{-4} and Eq. 16 while the third column presents P_{WD} values [(P_{WD})_L] from Eq. 19. Initially, the relative error is about half percent and becomes smaller for large t_{Dxf}. The error starts increasing when t_{Dxf} > 0.06 because the \sqrt{\pi t_{Dxf}} approximation becomes poor for p_D in Eq. 19 when t_{Dxf} > 0.06. Overall, the method works well for computing the wellbore pressure with storage and skin.
Exponential Wellbore Rate Case

The dimensionless wellbore pressure for the exponential wellbore rate case can be written directly from Eq. 4, Eq. 12 and Eq. 16 as

\[ P_{WD}(t_D) = P_D(t_{Dxf}) - \frac{\sqrt{\pi}}{2} e^{-\beta t_{Dxf}} \int_0^{t_{Dxf}} e^{\beta t} \frac{1}{\sqrt{t}} \text{err} \left( \frac{1}{2\sqrt{t}} \right) dt + S(1-e^{-\beta t_{Dxf}}) \tag{20} \]

where \( P_D(t_{Dxf}) \) is given by Eq. 16.

The integrand in Eq. 20 is infinite at its lower limit, \( t_{Dxf}=0 \). Thus, the short time approximation, will be given next, should be used for small \( t_{Dxf} \). Knowing the integral at small \( t_{Dxf} \), the \( P_{WD} \) can be computed numerically from Eq. 20 any required number of decimal places by several methods. The integral given in Eq. 20 is not a convolution type, thus, it saves a considerable amount of computation time.

For \( t_{Dxf} > 10 \), a simple expression can be derived for the Laplace transform of \( P_{WD} \) from Eq. 13 and the Laplace transform of Eq. 17 as

\[ \tilde{P}_{WD}(s) = \frac{1}{2 \beta} \frac{1}{s(s + \beta)} \left[ -\ln s + 2(1+\ln2) + 2s \right] \tag{21} \]

\( P_{WD} \) can be computed numerically from Eq. 21 for \( t_{Dxf} > 10 \) using Stehfest\textsuperscript{17} algorithm. The long time approximation given by Eq. 21 can be very useful since the integral given in Eq. 20 may become less accurate for large \( \beta t_{Dxf} \).

A short time \( P_{WD} \) solution for \( t_{Dxf} < 0.1 \) also can be written from Eq. 20 as

\[ P_{WD}(t_{Dxf}) = P_D(t_{Dxf}) - \sqrt{\pi} t_{Dxf} e^{-\beta t_{Dxf}} \sum_{n=0}^{\infty} \frac{(\beta t_{Dxf})^n}{(2n+1)n!} + S(1-e^{-\beta t_{Dxf}}) \tag{22} \]
A simpler expression than Eq. 22 can be obtained from Eqs. 4, 12 and 18 for short times as

\[ P_{WD}(t_{Dxf}) = \frac{2}{3} \beta \sqrt{\pi} t_{Dxf}^{3/2} + S \beta t_{Dxf} \]  

(23)

Figure 1 presents \( P_{WD} \) values as a function of \( t_{Dxf} \) from Eq. 23, and from the unit slope equation \( (t_{Dxf}/C_D) \) for \( C_{Dxf} = \beta = 1 \) and \( S = 0.0 \).

For zero or small skin, a log-log plot of \( P_{WD} \) versus \( t_{Dxf} \) should yield a straight line with a 3/2 slope. The relationship given in Eq. 23 can be used to calculate the fracture length if the other parameters and \( \beta \) are known.

The derivative of \( P_{WD} \) in Eq. 23 with respect to \( t_{Dxf} \) yields

\[ P_{WD}'(t_{Dxf}) = \beta (\sqrt{\pi} t_{Dxf} + S) \]  

(24)

\( P_{WD} \) is nothing more than \( \beta \) times the short time approximation. It should yield a half slope straight line on a log-log plot. This half slope identification-criteria is not restricted to zero or small skin as the 3/2 slope. Thus, it can be used for the identification of linear from regime with storage and skin effects.

2. Infinite Conductivity Fracture

The dimensions pressure for an infinite-conductivity vertically fractured well producing at a constant rate in an infinite reservoir is given by

\[ P_D = \frac{1}{2} \sqrt{\pi} t_{Dxf} \left[ \text{erf}\left(\frac{0.134}{\sqrt{t_{Dxf}}}\right) + \text{erf}\left(\frac{0.866}{\sqrt{t_{Dxf}}}\right) \right] \]
\[-0.067 \text{Ei}\left(-\frac{0.018}{t_{DF}}\right) - 0.433 \text{Ei}\left(-\frac{0.750}{t_{DF}}\right)\]  \hspace{1cm} (25)

This equation is similar to Eq. 16 and it has also the same short-time approximate solution, Eq. 18. It is also unfortunate that its Laplace transform can not be obtained readily to compute \(p_{WD}\) from Eq. 7 or Eq. 13.

The Laplace transform solution for the infinite-conductivity vertically fractured case presented by Kucuk and Brigham\textsuperscript{28} as

\[-\frac{1}{p_{D}(s)} = \frac{1}{8\lambda} \sum_{n=0}^{\infty} \left(\frac{A_{0}^{2}}{\lambda}\right)^{2} \text{Fek}_{2n}(0,\lambda)\] \hspace{1cm} (26)

where

\[\lambda = s/4\]

\[\text{Fek}_{2n}(0,\lambda) = \text{Mathieu functions}\]

\[\text{Fek'}_{2n}(0,\lambda) = \frac{\partial}{\partial \xi} [\text{Fek}_{2n}(\xi,\lambda)]_{\xi=0}\]

the differential of the Mathieu functions

\[A_{0}^{(2n)} = \text{Fourier coefficient and function of } \lambda.\]

There are some differences in \(p_{D}\) values from Eq. 25 and Eq. 26 due to the fact that Eq. 25 neglects the flow from the tips of the fracture. However, the differences are too small to be of practical importance. Because of time consuming computation of Mathieu functions, it takes a considerable amount of
computer time to compute $P_{D}$'s from Eq. 26 and used in Eq. 7 or Eq. 13 for $P_{WD}$ values. The method suggested here will use Eq. 25 and the algorithm given in Eq. 11, simpler than Eq. 26, to compute $P_{WD}$ values.

It can be seen from Table 4 that difference in $P_{WD}$'s from Eq. 11 with Eq. 25 and Eq. 7 with Eq. 26 are very small even though $P_{D}$ values from Eq. 25 and 26 are slightly different for $C_{Dxf} = 0.005$ and $S = 0.0$. These values also compare well with $P_{WD}$ values presented by Ramey and Gringarten.\textsuperscript{29} It is remarkable that the algorithm given in Eq. 11 to compute the wellbore pressure with storage and skin effects works very well for vertically fractured wells as well as for the radial flow.

For $t_{Dxf} > 10$, Eq. 26 can be approximated as\textsuperscript{28}

$$\tilde{P}_{D}(s) = -\left(\frac{y - 2\ln 2}{s} + \frac{1}{2s}\right) \ln s$$ \hspace{1cm} (27)

As in the uniform-flux fracture case, $\tilde{P}_{D}$ from Eq. 27 can be used in Eq. 7 or Eq. 13 to compute $P_{WD}$ values for $t_{Dxf} > 10$. Of course, inversion of Eq. 27 yields\textsuperscript{28}

$$P_{D}(t_{Dxf}) = \frac{1}{2}(\ln t_{Dxf} + 2.19537)$$ \hspace{1cm} (28)

The short time approximations for the infinite-conductivity fracture case is the same as the uniform-flux case. Thus, all short time approximations for both the constant wellbore storage and the exponential rate case for the uniform-flux fracture can also be used for the infinite-conductivity case.
CONCLUSIONS

1. A new generalized method is presented to compute the wellbore pressure with constant wellbore storage and skin using existing analytical solution for the constant rate case.

2. The method is extended to the exponential wellbore flow rate case.

3. The method is applied successfully to both, fully-penetrated radial and vertically fractured wells.

4. Solutions from the new method confirm the published solutions for fully penetrated radial and vertically fractured wells.

5. New short time solutions with wellbore storage and skin effects are presented.

REFERENCES


presented at the SPE-AIME 52nd Annual Fall Meeting, Denver, CO, Oct. 9-12, 1977.


**NOMENCLATURE**

B = oil formation-volume factor, res. bbls/STB  
C = wellbore storage coefficient, bbls/psi  
CD = dimensionless wellbore storage constant  
CDxf = dimensionless wellbore storage constant based on half fracture length
\( c_t \) = system total compressibility, psi\(^{-1} \)

\( h \) = formation thickness, ft

\( k \) = formation permeability, md

\( s \) = Laplace transform variable

\( P_D(t_D) \) = dimensionless formation pressure

\( p'_D \) = differential of \( P_D \)

\( P_i \) = initial pressure, psi

\( P_{WD} \) = dimensionless wellbore pressure

\( P_{wf} \) = bottom-hole flowing pressure, psi

\( q \) = stabilized or constant rate, STB/D

\( q_{wb} \) = downhole flow rate, bbls/day

\( q_D \) = dimensionless flow rate

\( r_w \) = wellbore radius, ft

\( S \) = skin factor

\( t \) = time, hours

\( t_D \) = dimensionless time

\( t_{Dxf} \) = dimensionless time based on half fracture length

\( \beta \) = a positive constant

\( \alpha = 0.00254k\beta/\phi \mu c \sigma r_w^2 \)

\( \phi \) = porosity

\( \mu \) = viscosity, cp

\( \tau \) = dummy integration variable

\( \gamma = 0.57722... = Euler's constant \)

\( Ei \) = exponential integral

\( - \) = Laplace transform of
APPENDIX A

The convolution integral for the exponential integral solution with the exponential wellbore rate can be written from Eq. 4 and Eq. 12 as

\[ p_{WD}(t_D) = \int_0^{t_D} \left[ 1 - e^{-\beta(t_D - \tau)} \right] \frac{e^{-\beta\tau}}{2\tau} d\tau + S(1 - e^{-\beta t_D}) \]  
(A-1)

A further simplification of Eq. A-1 yields

\[ p_{WD}(t_D) = p_D(t_D) - \frac{e^{-\beta t_D}}{2} \int_0^{t_D} e^{\beta\tau} \left( \frac{1}{2\tau} - \frac{1}{4\tau^2} \right) d\tau + S(1 - e^{-\beta t_D}) \]  
(A-2)

For small \( \beta\tau \), the series expansion can be substituted for \( e^{\beta\tau} \) in Eq. A-2 as

\[ p_{WD}(t_D) = p_D(t_D) \left( 1 - e^{-\beta t_D} \right) - \frac{e^{-\beta t_D}}{2} \int_0^{t_D} \left( \beta\tau + \frac{\beta^2\tau^2}{2!} + \frac{\beta^3\tau^3}{3!} + \ldots \right) \frac{1}{\tau} - \frac{1}{4\tau^2} d\tau \]

\[ + S(1 - e^{-\beta t_D}) \]  
(A-3)

The integral in Eq. A-3 can be written as

\[ I = \int_0^{t_D} \left( \beta + \frac{\beta^2\tau}{2!} + \frac{\beta^3\tau^2}{3!} + \ldots \right) e^{-\frac{1}{4\tau}} d\tau \]  
(A-4)

Equation A-4 can be further simplified by substituting \( \tau = -\frac{1}{4u} \).

\[ I = \int_{-\infty}^{-\frac{1}{4u}} \left[ \beta + \frac{\beta^2}{2!} \left( -\frac{1}{4u} \right) + \frac{\beta^3}{3!} \left( -\frac{1}{4u} \right)^2 + \ldots \right] e^{u} \frac{du}{4u^2} \]  
(A-5)

The series in Eq. A-5 can be written as
\[ I = \frac{1}{4} \sum_{n=0}^{\infty} (-1)^n \frac{\beta^{n+1}}{(n+1)!4^n} + \frac{1}{4t_D} \frac{e^u}{u^{n+1}} du \]  
\tag{A-6}

Evaluating the integral in Eq. A-6 yields

\[ I = \frac{1}{4} \sum_{n=0}^{\infty} (-1)^n \frac{\beta^{n+1}}{(n+1)!4^n} \]

\[ \left[ \left( -e^u \right)^{m-1} \sum_{k=1}^{\infty} \frac{1}{(m-1)(m-2)\ldots(m-k)4^k \Gamma(k)} + \frac{1}{(m-1)!} \text{Ei}(u) \right] \right]_{-\infty} \]
\tag{A-7}

where \( m = n+2 \)

Substituting \( u = \frac{1}{4t} \) and the integration limits in Eq. A-7 yields

\[ I = -\frac{e}{4t_D} \sum_{n=0}^{\infty} (-1)^n \frac{\beta^{n+1}}{(n+1)!4^n} \sum_{k=1}^{m-1} \frac{(-1)^{m-k}(4t_D)^{m-k}}{(m-1)(m-2)\ldots(m-k)} \]

\[ = \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \frac{\beta^{n+1}}{(n+1)!4^n} \frac{1}{4^n} \frac{1}{(n+1)!} \beta_{D}(t_D) \]
\tag{A-8}

Substituting Eq. A-8 in Eq. A-3 yields

\[ p_{wd}(t_D) = \left[ 1 - e^{\frac{-\beta t_D}{4t_D}} \right] \left[ 1 - \frac{1}{4} \sum_{n=0}^{\infty} (-1)^n \frac{\beta^{n+1}}{(n+1)!4^n} \right] \beta_{D}(t_D) \]

\[ = \frac{1}{8} e^{\frac{-\beta t_D}{4t_D}} - \frac{1}{8} e^{\frac{-\beta t_D}{4t_D}} \sum_{n=0}^{\infty} (-1)^n \frac{\beta^{n+1}}{(n+1)!4^n} \]
\[
\sum_{k=1}^{m-1} \frac{(-1)^{m-k} (4t_D)^{m-k}}{(m-1)(m-2)\ldots(m-k)} + S(1-e^{-\beta t_D})
\]  
(A-9)

where \( m = n+2 \)

The infinite series in Eq. A-9 converges uniformly for moderately small values of \( \beta \) and \( \beta t_D \). For \( n=3 \), Eq. A-9 becomes

\[
P_{wd}(t_D) = (1-e^{-\beta t_D}) \left( 1 - \frac{1}{4} \left( \beta - \frac{\beta^2}{(2!)^2} \frac{1}{4} + \frac{\beta^3}{(3!)^2} \frac{1}{4^2} - \frac{\beta^4}{(4!)^2} \frac{1}{4^3} + \ldots \right) \right) p_D(t_D)
\]

\[+ \frac{1}{8} e^{-\beta t_D} \left( e^{-\frac{\beta}{4t_D}} - \frac{\beta}{2!} \frac{(t_D)^2}{4} \left( 1 - \frac{\beta}{2!} \frac{(t_D)^2}{4} \right) - \frac{\beta^3}{(3!)^2} \frac{(t_D)^2}{4} \left( 1 - \frac{\beta}{2!} \frac{(t_D)^2}{4} \right) \right)\]

\[+ \frac{\beta^3}{(3!)^2} \left( 1 - \frac{(4t_D)^3}{3} + \frac{(4t_D)^2}{6} - \frac{4t_D}{6} \right) - \frac{\beta^4}{(4!)^2} \left( \frac{(4t_D)^4}{4} - \frac{(4t_D)^3}{12} \right)\]

\[+ \frac{(4t_D)^2}{24} - \frac{4t_D}{24} \right) + \ldots \ldots \right) + S(1-e^{-\beta t_D})
\]  
(A-10)

If \( \beta \ll 1 \) and \( \beta t_D \ll 1 \), Eq. A-10 can be further simplified to

\[
P_{wd}(t_D) = [-e^{-\beta t_D} (1 - \frac{1}{4})] p_D(t_D) - \frac{\beta t_D}{2} e^{-\beta t_D} e^{-\frac{t_D}{4t_D}} - e^{-\beta t_D} - \frac{1}{4t_D} + S(1-e^{-\beta t_D})
\]  
(A-11)
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<th>$t_D$</th>
<th>$P_{WD}$ (Analytical)</th>
<th>$P_{WD}$ (from Eq. 11)</th>
<th>Error, %</th>
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TABLE 3

COMPARISON OF COMPUTED WELLBORE PRESSURES
FOR A UNIFORM-FLUX FRACTURED WELL

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<th>$(P_{WD})_L$</th>
<th>Error, %</th>
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TABLE 4

COMPARISON OF WELLBORE Pressures
FOR AN INFINITE - CONDUCTIVITY VERTICALLY
FRACtURED WELL

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<th>$P_{WD}$ from Ref. 29</th>
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** Not available
Fig. 1 Dimensionless wellbore pressure for a vertically fractured well with constant wellbore storage and exponential wellbore flow rate.