Table IV. Smoothed Values of Relative Conductivity

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<td>18.70</td>
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</table>

Standard deviations

- \( \eta = \) absolute viscosity, pound second/square foot
- \( \kappa = \) thermometric conductivity, square feet/second
- \( \nu = \) kinematic viscosity, square feet/second
- \( \sigma = \) specific weight, pounds/cubic foot
- \( \tau = \) shear, pounds/square foot
- \( \nu = \) shear at wall, pounds/square foot
- \( \eta = \) absolute viscosity, pound second/square foot
- \( \kappa = \) thermometric conductivity, square feet/second
- \( \nu = \) kinematic viscosity, square feet/second
- \( \tau = \) shear, pounds/square foot

Dimensionless parameters

- \( Pr = \) molecular Prandtl number \( \nu / \kappa \)
- \( Pr = \) eddy Prandtl number \( \nu / \sigma \)
- \( Re = \) Reynolds number \( 2\eta u / \nu \)
- \( \nu = \) velocity parameter (Equation 9)
- \( \tau = \) distance parameter (Equation 8)

Subscript

- \( w = \) wall

Flow of Gases through Consolidated Porous Media

**DAVID CORNEL** AND **DONALD L. KATZ**

*University of Michigan, Ann Arbor, Mich.*

The flow of gases through consolidated porous media is of importance in widely differing applications such as aerodynamic problems, friction processes, and the analysis of behavior of natural gas reservoirs. The purpose of this paper is to correlate and predict the flow of gases through the interstices of consolidated porous media at rates leading to deviations from viscous flow that occur in the inertial, quasiturbulent, or turbulent flow region.

The measurement of viscous flow of gases through consolidated porous media has been widely studied (20). Deviations from viscous flow of gases at low flow rates, including molecular streaming and other effects, have received considerable attention (6, 14, 20). The measurement of the porosity (20) and the electrical resistivity factor (1, 28, 29) of consolidated porous media has received thorough study. The pore size distribution of such materials has been given wide consideration (20).

Correspondingly little attention has been given to the deviations from viscous flow at high flow rates through consolidated porous media. Green and Duwez (18) presented an article on the flow of gases through porous metals during this research. Fancher and Lewis (12) have given data on the flow of air, oil, and water through sandstones and ceramic samples in which the...
effects of turbulent flow were noted. Carlson and Eastman (7) investigated turbulence as a factor influencing permeability measurements. In unconsolidated porous media both the viscous and turbulent regions have been correlated by Brownell and coworkers (5, 6).

The present work involved a further investigation into viscous and turbulent flow of gases through consolidated porous media. It consisted of four phases:

1. Writing a flow equation for viscous and turbulent flow in terms of the fundamental variables
2. Transforming this flow equation into one based only on measurable quantities
3. Measurement of flow in the viscous and turbulent regions and the physical characteristics of representative consolidated samples
4. Correlation of data on consolidated porous samples including sandstones, limestones, and dolomites

It is believed that the theory and results are of a general nature and are applicable to unconsolidated materials as well as consolidated ones, although only the latter have been considered.

Turbulence may be defined as a state of fluid motion in which random fluctuation in velocity takes place with time at a given point, although such a definition excludes many of the flow processes in chemical engineering now referred to as turbulence. Green and Duwez (18) state that the explanation of deviation from Darcy's law due to the onset of turbulence within the pores of the medium appears unsatisfactory, and this view is held by many. The authors readily admit that the detailed mechanism of flow in the interstices of a porous medium is not well understood. However, it is believed that turbulence as defined takes place within the porous medium as deviation from Darcy's law appears. This turbulence may only be present in the large voids, whereas laminar flow still takes place in the bulk of the moving stream. The turbulence, however, is responsible for the extra pressure drop required for the flow in that the stream lines of flow lose energy to fluid in the recirculating pockets, which vary in size and shape with increasing velocities. It is possible that the auxiliary fluid movement within the pockets becomes steady state so that the definition given here no longer applies. Turbulence as used in this paper does not mean that fluctuations in velocity with time take place at all points in the flowing streams as it does at all points in the bulk of a stream flowing at high velocities in a pipe.

Equation for Viscous and Turbulent Flow Is Derived from Kozeny's Equation

Most flow problems involving the flow of gases in continuous systems may be broken down into three major classes which have been widely studied experimentally and theoretically. These are molecular streaming, viscous flow, and turbulent flow. The systems to be considered here consist of solids having a regular, continuous internal pore structure that permits the flow of gases under a pressure gradient. Under certain conditions each of these mechanisms may be expected to be important. The following analysis is limited to viscous and turbulent flow. Several excellent bibliographies of material on flow through porous media exist (6, 9, 10, 22) making it unnecessary to repeat such information here. Reference will be made, however, to the work that forms the foundation of the present development. The Kozeny equation for viscous flow through porous media is discussed and extended, and then a general equation for viscous and turbulent flow is derived.

The term viscous flow is used here to indicate the flow mechanism in porous media that depends on the fluid viscosity but not its density. Turbulent flow designates the flow region in which the pressure drop is proportional to the velocity raised to a power between 1.0 and 2.0. Bakhmeteff and Fedoroff (8), Green and Duwez (18), and others have sought to exclude fine-grained porous media from the systems in which turbulent flow may occur. However, turbulence actually was observed in porous media by Fancher and Lewis (12), leaving little doubt as to the existence of turbulence in porous media. Undoubtedly there are differences in the scale and spectrum of turbulence in consolidated porous media as compared to other systems, but since present experimental evidence and theoretical background are inadequate for satisfactory classification, the observed effects will be designated by the general term of turbulence. Further work will be necessary to determine whether or not random fluctuations in the fluid velocity at any point are actually occurring.

The Kozeny equation for viscous flow through porous media (17) as described by Carman (8) is a modification of Poiseuille's law:

\[-\frac{\Delta P_f}{L_B} = \frac{(32)(\mu)}{(\nu) (\rho_g) (D_B)}\]  

(1)

The hydraulic radius \(m\) for circular pipes is given by

\[m = \frac{D_B}{4}\]  

(2)

Poiseuille's law in terms of the hydraulic radius then becomes, for pipes

\[-\frac{\Delta P_f}{L_B} = \frac{(2.0)(\mu)}{(\nu) (m^2)}\]  

(3)

In general, the hydraulic radius of porous beds will be given by

\[m = \frac{D_B}{\delta}\]  

(4)

in which \(\delta\) is characteristic of the system's geometry. The constant of Equation 3 will no longer be 2.0 but will have some other value, \(k_o\), which is called the Kozeny constant and is believed to be approximately constant at a value of about 2.5.

| Table I. Values of \(k_o\) for Certain Geometric Configurations |
|-------------------|---|---|
| Shape             | \(k_o\) | \(k\) |
| Circle            | 2.0 | 1.0 |
| Square            | 1.78| 0.96–1.12 |
| Opening between four tangent circles located on square centers | ... | 0.35–0.60 |

In addition to the substitution of \(k_o\) for the 2.0 term in Equation 3, Kozeny gave the relationship between the effective velocity \(v_k\) and the superficial velocity \(v_s\) in terms of the porosity \(X\) and the length of the bed \(L_B\) as in Equation 5.

\[v_k = \left(\frac{v_s}{L_B}\right) \left(\frac{L_A}{X}\right)\]  

(5)

Eliminating \(v_k\) from Equation 3, Equation 6 (Kozeny's equation) is obtained.

\[-\frac{\Delta P_f}{L_B} = \left(\frac{k_o}{(\mu)} (\rho_g) (\mu) \left(\frac{(X)}{(L_A)}\right)\right)\]  

(6)

Previously, all the terms in Equation 6 were directly measurable except the ratio, \(L_A/L_B\). Lacking other evidence, various values of \(L_A/L_B\) have been assumed including \(\pi/2\), \(\sqrt{2}\), 1.40, and 1.304.

A method has been derived for the evaluation of \(L_A/L_B\) from measurements of porosity and the electrical resistance of a sample saturated with a salt solution of known conductivity which is applicable to consolidated porous media that are nonconductors of electricity. The resistivity factor \(F\) was defined by Archie (1) in connection with electrical logging of gas and oil wells as the ratio of the electrical resistance of a sample saturated with a conducting brine to the resistance of a volume of the brine of the same size and shape as the sample. The value of \(F\) depends on the available cross-sectional void area of the sample and on the increased distance that the electrical current must flow because of the irregular path. The available cross-sectional void area of the sample is the total cross-sectional area multiplied by \(X\) \((L_A/L_B)\). The length of the path is the length of the sample multiplied by \(L_A/L_B\). Since \(F\) is directly proportional to the
length and inversely proportional to the area, the correct expression for \( F \) is shown in Equation 7.

\[
F = \frac{(L_0)^3}{(L_3)(L_5)} \quad (7)
\]

Equations having a slightly different form have appeared in the literature (23, 24). These, however, are based on a cross-sectional area proportional to the porosity rather than \( \phi (L_0/L_3) \).

In Modified Equation, All Quantities May Be Measured Directly except \( k_5 \)

Substituting Equation 7 in Equation 6, a new form of Kozeny's equation is obtained in which all the quantities except \( k_5 \) are directly measurable.

\[
-\frac{\Delta P_1}{L_5} = \frac{(k_0) (v_0) (\mu) (F)}{(g_0) (m^2)} \quad (8)
\]

Equation 8 should prove to be very valuable in establishing the nature of Kozeny's constant more accurately than has been possible previously, since all the terms except \( k_0 \) can be established experimentally.

It was preferred, however, to work with the effective pore diameter, rather than hydraulic radius, because the pore size distribution of the materials used was more easily measured than was the surface area needed in computing the hydraulic radius. Furthermore, there is some doubt concerning the validity of the hydraulic radius when there is a wide distribution of pore sizes because the small pores increase the area greatly, whereas the large pores contribute the porosity important to the flow. Substituting Equation 4 in Equation 8 and defining \( k_1 \)

\[
k_1 = 32/(k_0(\delta)^4) \quad (9)
\]

Equation 10, used in this work, becomes

\[
-\frac{\Delta P_1}{L_5} = \frac{(32)(\mu)(\phi)(F)}{(k_1)(g_0)(D^4)} \quad (10)
\]

For circular pipes, \( k_1 = 1.0, F = 1.0, v_0 = v_B \), \( L_5 = L_B \), and Equation 10 becomes Poiseuille's law, Equation 1.

For porous media with a value of \( k_1 \) of about 2.5 and \( \delta \) of about 5.0, \( k_1 \) will have a value of about 0.5. Since the emphasis of this work was on turbulent rather than viscous flow, \( k_1 \) was taken to be 0.5, and no attempt was made to refine this value further. The values of \( D_B \) obtained by the use of \( k_1 \) equal to 0.5 in Equation 10 fell within the range of pore diameters determined by the pore size distribution measurements. Table 1 includes the estimated values of \( k_1 \) for certain geometric configurations.

Since the Kozeny equation applies only to laminar flow, it is necessary to go to some such method as the use of a friction factor–Reynolds number plot in order to handle both laminar and turbulent flow. In flow through rough pipes it is necessary to multiply the friction factor and to divide the Reynolds number by some factor, \( k_3 \), in order to bring all the data in the turbulent region together. The factor, \( k_3 \), may then be correlated with the roughness ratio, \( \epsilon/D \), in the case of rough pipes. It is assumed that such a factor exists for consolidated porous media and may be correlated similarly with the same \( \epsilon/D \) ratio. Lacking specific information as to the nature of \( \epsilon \), the correlation actually made was one of \( k_3 \) versus \( D_B \) with the type of consolidated porous media as a parameter. This type of correlation is not completely satisfactory in that it is based on a derived quantity, \( D_B \). It was used, however, because it provides an interpretation of \( k_3 \) that is consistent with the knowledge of the behavior of rough pipes.

In addition to the use of \( k_3 \) for consolidated porous media it is also necessary to multiply the Reynolds number by \( k_3 \) to satisfy the laminar flow theory. The friction factor and Reynolds number for porous media are given by Equations 11 and 12.

\[
f = \frac{(2)(g_2)(D_3)(-\Delta P)(k_3)}{(L_3)(v_0^2)(a)} \quad (11)
\]

By use of Equations 5 and 7 these forms of the friction factor and Reynolds number may be transformed into expressions in terms of measurable quantities as

\[
f = \frac{(2)(g_2)(D_3)(-\Delta P)(k_3)(X/\epsilon)}{(L_3)(g_3)(F^1/3)(X/\epsilon)} \quad (13)
\]

\[
Re = \frac{(D_3)(g_2)(F^{1/3})(k_3)}{(\mu)(X/\epsilon)} \quad (14)
\]

As pointed out by many previous investigators the curve on a friction factor–Reynolds number plot may be represented by a quadratic equation. For consolidated porous media this quadratic equation is

\[
-\frac{dP_1}{dL_5} = \frac{(32)(\mu)(\phi)}{(k_1)(g_0)(D^4)} + \frac{(32)(\mu)(\phi\phi^2)}{(k_1)(g_0)(D^4)} \quad (15)
\]

For gases, neglecting kinetic energy changes, this equation may be integrated and put in terms of measurable quantities

\[
\frac{(P_1^2 - P_3^2)(L_5)}{L_5} = \frac{(32)(\mu)(\phi)}{(k_1)(g_0)(D^4)} \left( \frac{2(\mu)(R)(T)(G)}{(M)(g_3)} \right) \quad (16)
\]

Green and Dwuez (15) have presented an equation similar to Equation 18, given here as equation 17.

\[
\frac{(P_1^2 - P_3^2)(L_5)}{L_5} = \frac{(\alpha)(2)(\alpha)(R)(T)(G)}{(M)(g_3)} + \frac{\beta(\gamma)(R)(T)(G)}{(M)(g_3)} \quad (17)
\]

The friction factor and Reynolds number then become equations 18 and 19 in terms of \( \alpha \) and \( \beta \).

\[
f = \frac{(64)(g_2)(1/\beta)(\phi)(-\Delta P)}{(L_3)(g_3)} \quad (18)
\]

\[
Re = \frac{(\beta/\alpha)(G_3)}{(\mu)} \quad (19)
\]

The constants, \( \alpha \) and \( \beta \), can be broken down as

\[
\alpha = \frac{(32)(F)}{(k_3)(D^4)} \quad (20)
\]

\[
\beta = \frac{(32)(F^1/\epsilon)}{(k_3)(D^4)(X/\epsilon)} \quad (21)
\]

Equations 20 and 21 provide a means for isolating geometrical factors for correlation.

Equation 17 may be rearranged for convenience in analyzing experimental data

\[
\frac{(P_1^2 - P_3^2)(M)(G_3)}{(L_3)(g_2)(R)(T)(\mu)(G_3)} = \frac{(\beta)(G_3)}{(\mu)} + \alpha \quad (22)
\]

It is evident that in this form a plot of \( (P_1^2 - P_3^2)(M)(G_3)/L_3(g_2)(R)(T)(\mu)(G_3) \) versus \( (G_3)/(\mu) \) on coordinate paper yields a straight line having a slope equal to \( \beta \) and an intercept equal to \( \alpha \). Thus, \( \alpha \) and \( \beta \) can be obtained readily from all the data taken with different gases. This type of plot has been used by several previous authors, among whom are Martin (18) and Ergun (11).

Data Were Obtained for Four Gases Flowing through 24 Samples of Sandstones, Limestones, and Dolomites

Cylindrical samples of sandstone, limestone, and dolomite rock were cut by use of a 4-inch diamond core drill from bulk samples. They were dried, extracted with benzene, dried, and extracted with water to remove the benzene-soluble and water-soluble materials. The samples were then dried in an oven at
Their pore volume was measured by evacuating weighed samples, saturating them with water, and reweighing. The bulk volume was determined by displacement in a pycnometer, and the porosity was computed from these measurements. The pore size distribution was obtained by the displacement of distilled water from the sample by air. Saturated samples were placed on cellulose tissue saturated with water and resting on a porous porcelain barrier having a bubble pressure for air over water of 35 pounds per square inch gage. The air pressure on the sample in the apparatus was maintained constant by a manual-loading pressure regulator, and the apparatus was kept in a constant temperature room to avoid temperature variations. The pore sizes were calculated as those for the displacement of liquid from a circular capillary. Figure 1 shows the pore size distribution curves for several of the samples studied.

The electrical resistivity factor was determined by saturating the samples with 0.100 N potassium chloride. The samples were placed between two platinum-platinum electrodes backed by felt saturated with the 0.100 N potassium chloride and clamped in a frame constructed of Lucite. The resistivity of the samples was determined by means of a Wheatstone bridge using a 1000 cycle/second audio-frequency voltage and headphones as a detector. The dimensions of the sample were determined with a micrometer and the resistivity factor \( F \) of each sample was calculated from the known conductivity of the potassium chloride solution (15) and the sample size.

The samples were re-extracted with water, dried, and mounted in Lucite in a standard metallurgical mounting press; care was taken not to crush the sample during application of pressure on the plastic. The exposed faces of the samples were all produced by fracturing the sample. Cuts were made through the plastic and the ends of the samples were fractured off by means of a blow on a thin chisel.

The Lucite-mounted samples were sealed into the sample holder of the flow apparatus using an O-ring gasket as in Figure 2. Dried filtered air, water-pumped nitrogen, pure grade methane, and helium were passed through the samples and the upstream, downstream, and, under some conditions, the differential pressures were measured. The flow rate was controlled manually with needle valves. The gas leaving the sample was saturated with water and was metered with a calibrated wet-test meter or by a soap film moving in a buret and timed with a stop watch. The temperature of the gas stream just before and after the sample was measured by means of calibrated copper-constantan thermocouples mounted in a sample holder by means of thermocouple glands. Nearly all the samples were removed from the sample holder and reversed. No change in the flow characteristics was observed with the change in the flow direction. Typical flow data appear in Table II. The flow rate and pressure measurements were accurate within 5% except for the data taken on very low permeability samples for which the flow rates were extremely small. In addition, since considerable variations in the properties of different samples taken from the same bulk sample were found, the results should be interpreted with due regard to the variable nature of the materials being considered.

Table II. Typical Data for Flow of Gases through Consolidated Porous Media—Wilcox Sandstone, No. 1

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<td>2.50 &lt;sup&gt;10&lt;/sup&gt;</td>
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<td>17</td>
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<td>1.20 &lt;sup&gt;10&lt;/sup&gt;</td>
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<td>2.50 &lt;sup&gt;10&lt;/sup&gt;</td>
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Twenty-four sandstone, limestone, and dolomite samples were analyzed. Their characteristics are given in Table III. The photomicrographs of the exposed faces of six of these samples are given as Figure 3. Each of the sandstones, with the exception of the Spraberry sandstone, had a well-defined quartz grain structure. The Spraberry sandstone had very fine grains with a great deal of cementing material and had a more dense appearance than did the others. The fractured faces of the brown dolomite samples revealed an irregular crystalline structure as did the lime-
stone. The limestone sample, however, was ground flat for the photomicrograph to show the nature of the internal porosity more clearly.

Samples from 0.65 to 1.31 inches long and about 0.72 inch in diameter were used for the flow measurements. Somewhat longer samples were used for the other measurements of the properties of the samples, since the ends of the samples were fractured off after mounting in plastic prior to the flow measurements. The permeability of the samples varied from 0.01 to 500 millidarcies. Porosities of from 2 to 25% were found. Electrical resistivity factors of 12 to 140 were observed. The pore size distribution of four types of samples was obtained for the larger pores and ranged from 6 $\times$ 10$^{-4}$ to 2 $\times$ 10$^{-4}$ foot. The flow measurements were made with gases having molecular weights of 4.0 (He), 16.04 (CH$_4$), 28.2 (N$_2$), and 29.0 (air). This selection of gases provided viscosities ranging between about 0.01 and 0.02 cp. (4, 10, 18, 21).

### Table III. Summarized Characteristics of the Consolidated Samples Studied

<table>
<thead>
<tr>
<th>Sample Type</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$K$ (Permeability)</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$B_1$</th>
<th>$B_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wilcox*</td>
<td>15.5</td>
<td>0.175</td>
<td>4.90</td>
<td>10$^{-1}$</td>
<td>9.23 $\times$ 10$^{-1}$</td>
<td>192</td>
<td>4.59 $\times$ 10$^{-1}$</td>
<td>1.64</td>
<td>1.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bromide</td>
<td>17.4</td>
<td>0.158</td>
<td>3.77</td>
<td>10$^{-1}$</td>
<td>3.92 $\times$ 10$^{-1}$</td>
<td>145</td>
<td>4.08 $\times$ 10$^{-1}$</td>
<td>1.63</td>
<td>1.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spraberry*</td>
<td>16.6</td>
<td>0.160</td>
<td>6.35</td>
<td>10$^{-1}$</td>
<td>9.95 $\times$ 10$^{-1}$</td>
<td>220</td>
<td>4.12 $\times$ 10$^{-1}$</td>
<td>1.66</td>
<td>1.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Burbank</td>
<td>36.2</td>
<td>0.110</td>
<td>4.22</td>
<td>10$^{-1}$</td>
<td>2.38 $\times$ 10$^{-1}$</td>
<td>2.0</td>
<td>1.73 $\times$ 10$^{-3}$</td>
<td>1.99</td>
<td>1.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spraberry</td>
<td>62.5</td>
<td>0.080</td>
<td>3.91</td>
<td>10$^{-1}$</td>
<td>9.09 $\times$ 10$^{-1}$</td>
<td>3.74</td>
<td>1.73 $\times$ 10$^{-3}$</td>
<td>1.99</td>
<td>1.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oterton</td>
<td>90.3</td>
<td>0.084</td>
<td>6.53</td>
<td>10$^{-1}$</td>
<td>4.49 $\times$ 10$^{-1}$</td>
<td>15.0</td>
<td>1.75 $\times$ 10$^{-3}$</td>
<td>1.92</td>
<td>1.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Torpedo*</td>
<td>36.0</td>
<td>0.133</td>
<td>4.60</td>
<td>10$^{-1}$</td>
<td>1.31 $\times$ 10$^{-1}$</td>
<td>11.0</td>
<td>1.73 $\times$ 10$^{-3}$</td>
<td>2.41</td>
<td>1.60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brown</td>
<td>25.1</td>
<td>0.150</td>
<td>8.63</td>
<td>10$^{-1}$</td>
<td>3.41 $\times$ 10$^{-1}$</td>
<td>15.0</td>
<td>1.73 $\times$ 10$^{-3}$</td>
<td>1.92</td>
<td>1.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Limestone</td>
<td>10.6</td>
<td>0.238</td>
<td>5.80</td>
<td>10$^{-1}$</td>
<td>2.20 $\times$ 10$^{-1}$</td>
<td>220</td>
<td>4.12 $\times$ 10$^{-3}$</td>
<td>1.99</td>
<td>1.63</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Sandstone samples.

Flow Data Correlated on Friction Factor Plot

Figure 4 shows typical flow data for three Wilcox sandstones plotted in the form described by Equation 22 to fall on a straight line. The quantities $\alpha$ and $\beta$ can be obtained directly from this plot as its intercept and slope. Summarized values of $\alpha$ and $\beta$ are given in Table III. The low permeability samples all showed deviations on this type of straight-line data plot at low mean pressures because of molecular streaming. Figure 5 shows typical flow data on low permeability bromide sandstones for which molecular streaming was a factor. Molecular streaming, however,
was not under study in this work, and its effects were avoided by extrapolating the data taken at higher mean pressures. Similar flow data were taken on a total of 24 consolidated porous samples.

The values of $k_2$ given in Table III were computed from the experimentally determined values of $\alpha$, $\beta$, $F$, and $X$ and plotted versus $D_x$ with the type of material as a parameter as in Figures 6 and 7. These correlations of $k_2$ show some scattering of the data. This is undoubtedly due to the sampling and experimental errors and to the arbitrary assumption of $k_1 = 0.5$. The average deviation of the values of $k_2$ for all of the samples excluding the bromide sandstones was 12.4%. The various bromide sandstones differed considerably in their characteristics as is apparent from the photomicrographs in Figure 3 and, consequently, a good correlation would not be expected for them.

The conventional means of presenting fluid flow data is a dimensionless friction factor–Reynolds number plot. Such a plot is given as Figure 8 for the flow data of this investigation and that of Green and Duwez (18) for porous metals. Three expressions are given for the friction factor and Reynolds number. The first forms are due to Green and Duwez and are based on $\alpha$ and $\beta$, the empirical flow constants. These would be used in correlating flow data. The second set of forms of $f$ and Re may be used if fluid flow data are not available and flow rates must be predicted. Knowledge of the permeability, porosity, and electrical resistivity factor with Figures 6 and 7 giving $k_2$ values is sufficient to predict the entire range of viscous and turbulent flow. The final forms of $f$ and Re are the basic theoretical friction factor and Reynolds number from which the other two forms

**Figure 4. Flow Data Plot to Obtain $\alpha$ and $\beta$—Wilcox Sandstones**

$X = \text{Air} \quad = \text{N}_2 \quad \Delta = \text{He}$

**Figure 5. Flow Data Plot to Obtain $\alpha$ and $\beta$—Bromide Sandstones**

$X = \text{Air} \quad = \text{N}_2 \quad \Delta = \text{He}$

**Figure 6. $k_2$ Correlation for Sandstones**

- $\bullet$ Wilcox Nos. 1–3
- $\bullet$ Bromide Nos. 4–8
- $\bullet$ Burbank Nos. 9–10
- $\bullet$ Spraberry Nos. 11–12
- $\bullet$ Outcrop Nos. 13–14
- $\bullet$ Torpedo Nos. 15–16

Data taken at low mean pressures affected by molecular streaming have been omitted from this plot.

**Relationship to Other Methods Is Shown**

Previous applications of the Kozeny equation to flow problems have required the estimation of the increased length of the path through an irregular bed. The use of Equation 7 enables one to measure this property of the bed directly. Table III lists the

**Figure 7. $k_2$ Correlation for Limestones and Dolomites**

- $\bullet$ Brown dolomites Nos. 17, 20–22
- $\bullet$ Canyon Reef limestone Nos. 23–24
measured values of $L_2/L_3$ for the samples studied. The $L_2/L_3$ ratios vary from 1.36 to 2.26. Since this ratio is squared in the laminar flow equation, it is evident that it is of considerable importance in any fluid flow correlation.

Heretofore, the evaluation of the Kozeny constant has depended on an assumed value of $L_2/L_3$. With a method of measuring $L_2/L_3$ directly now available it is possible to calculate the Kozeny constant accurately for any porous media whose permeability, porosity, surface area, and electrical resistivity factor are known.

The theory developed here permits an analysis of the $F_f$ and $F_{Re}$ terms presented by Brownell, Dombrowski, and Dickey (7).

$$F_f = \frac{(D_p)(\beta)(\sigma)}{(c)(h_2)(h_3)(X^{1/2})} = \frac{(L_3)}{(L_2) (h_2)(h_3)(X^{1/2})}$$  (23)

$$F_{Re} = \frac{(c)(\beta/\alpha)}{D_p} = \frac{c}{(h_2)(X^{1/2})} = \frac{(c)(L_3)(h_2)(h_3)}{(L_2)(h_2)(X)}$$  (24)

The constant ($c$) relates the particle diameter to the effective diameter of the flow path.

$$D_p = (c)(D_e)$$  (25)

These values of $F_f$ and $F_{Re}$ should not be interpreted as varying with $X^{-1}$ and $X^{-1}$ when the porosity of a porous medium is altered, because actual changes in the porosity will also change the values of $L_2/L_3$ and $L_3$ in the usual case, thus complicating the nature of the porosity function. The presence of the constant ($c$) is due to the fact that two different reference points on the friction factor plot have been used.

**Summary**

Procedures for predicting and correlating turbulent flow through consolidated porous media are shown using the permeability, porosity, and electrical resistivity factor of the solid and the properties of the fluid. A method of evaluating the actual length of the path through a porous medium by means of an electrical resistivity measurement is shown. A procedure for evaluating the Kozeny constant more accurately than previously possible is shown. The relationship of this work to that of Brownell, Kozeny, Green and Duwez, and others is given.

**Acknowledgment**

This work was carried out under the Socony-Vacuum fellowship in chemical engineering at the University of Michigan. The samples studied and the pure methane were provided by the Phillips Petroleum Co.

**Nomenclature**

- $c, c_1$: dimensionless proportionality factors
- $D_p$: effective diameter of pore structure, feet
- $D_e$: particle diameter of unconsolidated particles, feet
- $F$: electrical resistivity factor: electrical resistance of

![Graph](image-url)

**Figure 8. Friction Factor–Reynolds No. Plot**
Radiation-Conduction Correction for Temperature Measurements in Hot Gases

W. E. WEST, JR., AND J. W. WESTWATER

University of Illinois, Urbana, Ill.

A C C U R A T E measurement of the temperature of a hot gas is a problem beset with difficulties. Consider the common case of a hot gas flowing through a duct. A number of possible temperature measuring devices could be used, but objections can be cited for each.

If some sort of radiation meter is sighted through a peep-hole, it "sees" the duct walls as well as the gas. The duct walls cannot be at the gas temperature unless the outside of the duct is provided with perfect insulation. Usually the duct is losing heat to the surroundings, and the duct temperature and gas temperature are different. Thus the pyrometer may give readings that are in error, possibly by several hundred degrees (2). One radiation scheme does give good accuracy; this is the line-reversal method which consists essentially of measuring the radiation emitted by a salt suspended in the hot gas. This method is suitable for flames only and is so inconvenient to use that it is rarely seen in industry.

If a thermometer or similar probe device is inserted into the gas stream, it will have a radiation error, because the probe can see the cool duct walls. There will be a conduction error also (the so-called fin effect) caused by the flow of heat along the probe to the duct wall.

Two methods of attacking the problem are possible. One consists of trying different designs of temperature measuring devices until one is found that produces a direct reading of the true gas temperature. A large body of literature deals with this approach. Some of the methods use shielded thermocouples (3, 4, 14), heated shields (3, 15, 20), heated resistance elements of resistance thermometers (10), heated thermocouples (12), thermocouples of zero diameter (5, 9, 10), high velocity suction tubes (4, 14, 15), a clever arrangement of two orifices (3, 13, 17), and other arrangements as well.

The second method consists of using some simple measuring device which produces an "inaccurate" temperature, the error of which can be calculated accurately. This is the subject of the present paper; the discussion is restricted to probe devices.

The conduction error inherent in all probes has been well under-