Quote du Jour:  
Nothing blocks happiness like happiness remembered.
— Andre Gide (1902)

Topic: Relative Permeability

Objectives: (things you should know and/or be able to do)

- Be familiar with the concept of "relative permeability" and the factors which should and should not affect this function. You should also be familiar with the laboratory techniques for measuring relative permeability.
- Be familiar with and be able to derive the Purcell-Burdine relative permeability equations, which are given as:

$$k_{rw} = (S_w^*)^2 \int_0^{S_w^*} \frac{1}{p_c^2} dS_w^*$$  and  $$k_{rm} = (1-S_w^*)^2 \int_0^{1} \frac{1}{p_c^2} dS_w^*$$

- Be familiar with and be able to derive the Brooks-Corey-Burdine equations for relative permeability based on the combination of the Burdine relative permeability equations (shown above) and the Brooks and Corey capillary pressure model. These results are given by:

$$k_{rw} = k_{rw}^o S_w^*(3+2/\lambda)$$ and $$k_{rm} = k_{rm}^o (1-S_w^*)^2 \left[1-S_w^* (1+2/\lambda)\right],$$

where the Brooks and Corey capillary pressure model is given by:

$$p_c = p_d S_w^{-\lambda}.$$

and $$k_{rw}^o$$ and $$k_{rm}^o$$ are the "endpoint" relative permeability values.

Lecture Outline:

- Factors which affect relative permeability:
  - "Bundle of tubes" concept for the $$p_c$$ and $$k_c$$ functions
  - Influence of pressure drop
  - Influence of fluid viscosity ratios
  - Influence of capillary pressure "end effects"

- Derivation of the Brooks-Corey-Burdine equation for relative permeability.
  - Burdine equation for effective permeability:

$$kk_{rw} = 10.66 \phi^3 g_w^2 \beta n \int_0^{S_w^*} \frac{1}{p_c^2} dS_w^*$$
Lecture Outline: Continued

- Brooks-Corey capillary pressure model:

\[ p_c = p_d S^*_w \lambda \] where \( S^*_w = \frac{S_w - S_{wi}}{1 - S_{wi}} \).

- Integration: Using the Purcell-Burdine identity, and substituting the Brooks-Corey capillary pressure model, we have

\[ I = \int_a^b \frac{1}{P_c} \frac{dS^*_w}{p^*_d} = \frac{1}{p^*_d} \left[ \frac{\lambda}{\lambda + 2} \right] b^{(1 + 2/\lambda)} - a^{(1 + 2/\lambda)} \]

which yields

\[ k_{rw} = k_{rw}^o S^*_w^{(3 + 2/\lambda)} \quad \text{and} \quad k_{rn} = k_{rn}^o (1 - S^*_w)^{1/2} \left[ 1 - S^*_w^{(1 + 2/\lambda)} \right], \]

where \( k_{rw}^o \) and \( k_{rn}^o \) are the "endpoint" relative permeability values and are included for completeness.

- Development of a type curve matching approach for relative permeability data based on the Brooks-Corey model.

  - \( k_{rwD} \) and \( k_{rnD} \) versus \( S_{wD} \) Type Curve Approach:
    - Plot \( k_{rwD} = k_{rw}/k_{rw}^o \) and \( k_{rnD} = k_{rn}/k_{rn}^o \) versus \( S_{wD} \) and overlay these trends with a transparent plot of \( k_{rw} \) and \( k_{rn} \) versus \( 1 - S_w \).
    - Once the \( k_{rw} \) and \( k_{rn} \) versus \( 1 - S_w \) trends are "matched" on top of the \( k_{rwD} \) and \( k_{rnD} \) versus \( S_{wD} \) curves, read the \( \lambda \) parameter, then read the \( k_{rw}/k_{rwD} \), \( k_{rn}/k_{rnD} \), \( 1 - S_w \), and \( S_{wD} \) ratios from the data grids. Using these results we can estimate the following:
      - Wetting phase endpoint relative permeability:
        \[ k_{rw}^o = \frac{(k_{rw})_{MP}}{(k_{rwD})_{MP}} \]
      - Non-wetting phase endpoint relative permeability:
        \[ k_{rn}^o = \frac{(k_{rn})_{MP}}{(k_{rnD})_{MP}} \]
      - Irreducible wetting phase saturation:
        \[ S_{wi} = 1 - \frac{(1 - S_w)_{MP}}{(S_{wD})_{MP}} \]

      Where "MP" is the "match point" value of a function. We note that the simplicity of this approach is that the ratios of the functions can be read at any convenient location on the overlay plots, hence the name "match point."

  - \( (k_{rnD}/k_{rwD}) \) versus \( S_{wD} \) Type Curve Approach:
    - Plot \( (k_{rnD}/k_{rwD}) \) versus \( S_{wD} \) and overlay these trends with a transparent plot of \( (k_{rn}/k_{rw}) \) versus \( 1 - S_w \).
    - Once the \( (k_{rn}/k_{rw}) \) versus \( 1 - S_w \) trends are "matched" on top of the \( (k_{rnD}/k_{rwD}) \) versus \( S_{wD} \) curves, read the \( \lambda \) parameter, then read the \( [(k_{rn}/k_{rw})/(k_{rnD}/k_{rwD})] \) and \( (1 - S_w) \) ratios from the data grids. Using these results we can estimate the following:
      - Endpoint relative permeability ratio:
        \[ \frac{k_{rn}^o}{k_{rw}^o} = \frac{(k_{rn}/k_{rw})_{MP}}{(k_{rnD}/k_{rwD})_{MP}} \]
Reading Assignment:

- Review attached notes.
  - Various Factors Affecting Relative Permeability.
  - Type Curve Matching of Relative Permeability Data.
Exercises: For your own practice/skills building—do NOT turn in!

• In each of these derivations/problems you are to work in complete detail and you must show all work.
  ■ Derive the Brooks-Corey-Burdine equations for relative permeability.
• You are to provide a critical and detailed review (at least 1 page) for the following paper(s):

For each paper you are to address the following questions: (Type or write neatly)

• **Problem:**
  – What is/are the problem(s) solved?
  – What are the underlying physical principles used in the solution(s)?
• **Assumptions and Limitations:**
  – What are the assumptions and limitations of the solutions/results?
  – How serious are these assumptions and limitations?
• **Practical Applications:**
  – What are the practical applications of the solutions/results?
  – If there are no obvious "practical" applications, then how could the solutions/results be used in practice?
• **Discussion:**
  – Discuss the author(s)'s view of the solutions/results.
  – Discuss your own view of the solutions/results.
• **Recommendations/Extensions:**
  – How could the solutions/results be extended or improved?
  – Are there applications other than those given by the author(s) where the solution(s) or the concepts used in the solution(s) could be applied?
Various Factors Affecting Relative Permeability
(from Petroleum Engineering 620 Course Notes — 1997)

Petroleum Engineering 620
Fluid Flow in Reservoirs
Fig. 31 – Hypothetical Capillary-pressure Curve for Bundle of Capillaries with Uniform Lengths and Uniform Diameters

Fig. 32 – Hypothetical Capillary-pressure Curve for Bundle of Capillaries with Uniform Lengths and Wide Distribution of Diameters

Steady State Water-Oil Relative Permeability (Imbibition Displacement)

Unsteady State Water-Oil Relative Permeability (Imbibition)

Relative Permeability to Oil & Water for Drainage and Imbibition (Water Wet)

FIG. 4 — COMPARISON OF EXPERIMENTAL AND THEORETICAL SATELLITE GRADIENTS DUE TO BOUNDARY EFFECT.

Note

$q_g = 0.15 \text{ cc/sec.}
q_0 = 0.000336 \text{ cc/sec.}$

FIG. 5 — COMPARISON OF EXPERIMENTAL AND THEORETICAL SATELLITE GRADIENTS DUE TO BOUNDARY EFFECT.

Note

$q_g = 0.264 \text{ cc/sec.}
q_0 = 0.0022 \text{ cc/sec.}$

FIG. 6 — COMPARISON OF EXPERIMENTAL AND THEORETICAL SATELLITE GRADIENTS DUE TO BOUNDARY EFFECT.

Note

$q_g = 0.80 \text{ cc/sec.}
q_0 = 0.00288 \text{ cc/sec.}$

FIG. 12 — EFFECT OF FLOW RATE ON THE RELATIVE PERMEABILITY-SATURATION RELATION (DRAINAGE CONDITIONS).

Note
Berea outcrop sample
L = 30.7 cm

FIG. 20 — EFFECT OF CORE LENGTH ON SINGLE CORE DYNAMIC RELATIVE PERMEABILITY (DRAINAGE CONDITIONS).

FIG. 1 - EFFECT OF PRESSURE GRADIENT ON RELATIVE PERMEABILITY
(LACK OF CAPILLARY CONTACT BETWEEN CORES). PENN STATE.

Effect of Oil-Water Viscosity Ratio on $K_{rw}$ & $K_{ro}$ from Unsteady State Displacement Tests

**FIG. 5 — EFFECT OF OIL VISCOSITY ON RELATIVE PERMEABILITY.
SINGLE CORE DYNAMIC.**

FIG. 12 — RELATIVE PERMEABILITY — THREE METHODS.

Brooks/Corey/Burdine
Relative Permeability Relation

and

Type Curve Matching
of Relative Permeability Data

(from Petroleum Engineering 620 Course Notes — 1995)
Brooks/ Corey-Burdine Relative Permeability Relation and Type Curve Analysis of Relative Permeability Data

The relative permeability (that is, the ratio of the effective permeability at a particular saturation to the absolute permeability) of a porous medium can be derived using a "bundle of capillary tubes" model, as shown in detail by Nakornthap and Evans (SPEE, May 1986). The "wetting" and "non-wetting" phase relative permeabilities are given by:

Wetting Phase Relative Permeability:

\[ k_w = (s_w^*)^2 \frac{\int_0^{s_w^*} \frac{1}{\phi_e^2} \, ds_w^*}{\int_0^{1} \frac{1}{\phi_e^2} \, ds_w^*} \]  \hspace{1cm} (1)

Non-Wetting Phase Relative Permeability:

\[ k_m = (1 - s_w^*)^2 \frac{\int_0^{s_w^*} \frac{1}{\phi_e^2} \, ds_w^*}{\int_0^{1} \frac{1}{\phi_e^2} \, ds_w^*} \]  \hspace{1cm} (2)

Eqs. 1 and 2 were originally credited to Burdine (ASNE Trans, 1958) but were also given by Wylie and Gardner (World Oil, March-April 1958). Regardless, this concept is quite old and has seen considerable application in reservoir engineering.

In Eqs. 1 and 2 the effective saturation, \( s_w^* \), is given by:

\[ s_w^* = \frac{s_w - s_w^e}{1 - s_w^e} \]  \hspace{1cm} (3)

Obviously, we need to relate capillary pressure and saturation. This approach will use the Brooks and Corey power law model for relating capillary pressure and saturation:

\[ P_c = P_{c1} s_w^{1/\lambda} = P_{c1} \left[ \frac{s_w - s_w^e}{1 - s_w^e} \right]^{1/\lambda} \]  \hspace{1cm} (4)
Isolating the capillary pressure integral we have

\[ I = \int_{a}^{b} \frac{1}{\frac{\lambda}{z} - \frac{1}{n}} \, ds_w \]  

(5)

Substituting Eq. 4 into Eq. 5, we have

\[ I = \int_{a}^{b} \frac{1}{\frac{\lambda}{z} \cdot \frac{1}{n} - \frac{1}{n}} \, ds_w \]

or

\[ I = \frac{1}{\frac{\lambda}{z}} \int_{a}^{b} \frac{1}{\frac{1}{n}} \, ds_w \]

reducing to

\[ I = \frac{1}{\frac{\lambda}{z}} \int_{a}^{b} \left(1 + \frac{z}{\lambda}\right) \, ds_w \]

or

\[ I = \frac{1}{\frac{\lambda}{z}} \left[ \frac{\lambda}{\lambda + z} \right] \left(1 + \frac{z}{\lambda}\right) \int_{a}^{b} \, ds_w \]

(6)

Summarizing the results of the various integrals

<table>
<thead>
<tr>
<th>Term</th>
<th>(a)</th>
<th>(b)</th>
<th>[ I ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{nw}) Numerator</td>
<td>0</td>
<td>(s_w^*)</td>
<td>(\frac{1}{\frac{\lambda}{z}} \left[ \frac{\lambda}{\lambda + z} \right] \left(1 + \frac{z}{\lambda}\right) \int_{a}^{b} , ds_w )</td>
</tr>
<tr>
<td>(k_{nm}) Numerator</td>
<td>(s_w^*)</td>
<td>1</td>
<td>(\frac{1}{\frac{\lambda}{z}} \left[ \frac{\lambda}{\lambda + z} \right] \left(1 + \frac{z}{\lambda}\right) \int_{a}^{b} , ds_w )</td>
</tr>
<tr>
<td>General Denominator</td>
<td>0</td>
<td>1</td>
<td>(\frac{1}{\frac{\lambda}{z}} \left[ \frac{\lambda}{\lambda + z} \right] \left(1 + \frac{z}{\lambda}\right) \int_{a}^{b} , ds_w )</td>
</tr>
</tbody>
</table>

Substituting these results into Eqs. 1 and 2, we have

\[ k_{nw} = (s_w^*)^2 \, s_w^* \left(1 + \frac{z}{\lambda}\right) = s_w^* \left(1 + \frac{z}{\lambda}\right) \]

(7)

\[ k_{nm} = (1 - s_w^*)^2 \left(1 - s_w^* \left(1 + \frac{z}{\lambda}\right) \right) \]

(8)

or

\[ k_{nw} = s_w^* \left(\frac{\lambda z - z}{\lambda}\right) \]

(9)

\[ k_{nm} = (1 - s_w^*)^2 \left(1 - s_w^* \left(\frac{\lambda + z}{\lambda}\right) \right) \]

(10)
Unfortunately, the Burdine approach neglects the fact that relative permeability data do not necessarily extrapolate to unity. Graphically, we have

\[ ki = k_{rW} - k_m \]

Adjusting the relative permeabilities for non-unity intercepts, we have

\[
k_{rW} = k_{rW}^{*} s_w^{*} \left( 1 + z/\lambda \right) \quad (11)
\]

\[
k_m = k_m^{*} \left( 1 - s_w^{*} \right)^2 \left[ 1 - s_w^{*} \left( 1 + z/\lambda \right) \right] \quad (12)
\]

Substituting the effective saturation, \( s_w^{*} \) (Eq. 3) into Eqs. 11 and 12 gives

\[
k_{rW} = k_{rW}^{*} \frac{s_w - s_w^{*}}{1 - s_w^{*}} \left( 1 + z/\lambda \right) \quad (13)
\]

\[
k_m = k_m^{*} \left[ \frac{1 - s_w^{*}}{1 - s_w^{*}} \right]^2 \left[ 1 - \left[ \frac{s_w - s_w^{*}}{1 - s_w^{*}} \right] \left( 1 + z/\lambda \right) \right] \quad (14)
\]

where Eqs. 13 and 14 could be fitted to relative permeability data. An alternate approach would be to develop a "type curve" and match relative permeability onto these type curves.

Defining a dimensionless saturation, \( s_{wp} \), we have

\[
s_{wp} = 1 - s_w^{*} = \frac{1 - s_w - s_w^{*}}{1 - s_w} - \frac{s_w - s_w^{*}}{1 - s_w^{*}}
\]
or
\[ S_{wp} = \frac{1 - S_w}{1 - S_{wi}} \]  
(15)

or finally
\[ S_{wp} = 1 - S_{wd} \]  
(16)

And the "dimensionless" relative permeability functions are given by
\[ k_{rwd} = \frac{k_{rw}}{k_{rw}^0} \]  
(17)

\[ k_{rmd} = \frac{k_{rm}}{k_{rm}^0} \]  
(18)

Substituting Eqs. 16 and 17 into Eq. 11
\[ k_{rwd} = (1 - S_{wd}) \]  
(19)

Substituting Eqs. 16 and 18 into Eq. 12
\[ k_{rmd} = \frac{S_{wp}^2}{1 - (1 - S_{wd})} \]  
(20)

Using the type curve approach we will overlay a scaled plot of \( k_{rw} \) and \( k_{rm} \) versus \( 1 - S_w \) over a plot of \( k_{rwd} \) and \( k_{rmd} \) versus \( S_{wd} \). Once the data are "matched" onto the type curves we simply read the "match point" values (ratios of \( k_{rw}/k_{rwd} \), \( k_{rm}/k_{rmd} \), and \( (1 - S_w)/S_{wd} \)). These relations are given by
\[ k_{rw}^0 = \frac{(k_{rw})_{MP}}{(k_{rwd})_{MP}} \]  
(21)

\[ k_{rm}^0 = \frac{(k_{rm})_{MP}}{(k_{rmd})_{MP}} \]  
(22)

\[ S_{wi} = 1 - \frac{(1 - S_w)_{MP}}{(S_{wd})_{MP}} \]  
(23)
Another approach is to use the relative permeability ratio function given by $k_{rMD}/k_{rWD}$. Using Eqs. 19 and 20 we have

$$
rac{k_{rMD}}{k_{rWD}} = \frac{s_{wD}^2}{1 - (1-s_{wD})} \left[ 1 - (1-s_{wD}) \frac{1 + 2/\lambda}{(s_{wD} + 2/\lambda)} \right]
$$

or

$$
rac{k_{rMD}}{k_{rWD}} = \frac{s_{wD}^2}{(1-s_{wD})^2} \frac{1 - (1-s_{wD})}{(1-s_{wD}) \frac{1 + 2/\lambda}{(s_{wD} + 2/\lambda)}}
$$

or

$$
rac{k_{rMD}}{k_{rWD}} = \frac{s_{wD}^2}{(1-s_{wD})^2} \left[ \frac{-1}{(1-s_{wD}) \left( s_{wD} + \frac{2}{\lambda} \right)} - 1 \right]
$$

Matching on the $k_{rMD}/k_{rWD}$ vs. $s_{wD}$ type curve gives

$$
\frac{s_w}{s_{wD}} = 1 - \frac{(1-s_{wD})}{(s_{wD})_{MP}}
$$

and

$$
\frac{k_{rM}}{k_{rW}} = \frac{(k_{rMD}/k_{rWD})_{MP}}{(k_{rMD}/k_{rWD})_{MP}}
$$

The $k_{rMD}/k_{rWD}$ type curve approach tends to have less difficulty than trying to match $k_{rM}$ and $k_{rW}$ simultaneously. Unfortunately, the effect of matching on $k_{rMD}/k_{rWD}$ may simply "smooth" the character of the $k_{rM}$ and $k_{rW}$ trends—but this approach (using $k_{rMD}/k_{rWD}$) appears to be better.

The question remains as to how to resolve $k_{rM}/k_{rW}$ from the match point. Standing (or notes) provides

$$
k_{rM} = 1.81 - 2.62 s_{wL} + 1.15 s_{wL}^2 \quad 0.12 < s_{wL} < 0.5
$$

and

$$
k_{rM} = 1 \quad s_{wL} \leq 0.12
$$

This correlation can be used to resolve $k_{rM}/k_{rW}$. 
Type Curve for Brooks and Corey/Burdine $k_r/S_{wb}$ Method

$k_{md}(S_{wd}) = k_{md}/k_m^2 = S_{wd}^2/[1-(1-S_{wb})(1-S_{md})]$}

$S_{wd} = 1 - S_w = (1-S_w)/(1-S_w)$ (note that scale is reversed for familiarity)
Data Grid for Brooks and Corey/Burdine $k_r/S_wD$ Method
Type Curve for Brooks and Corey/Burdine $k_r/S_{wD}$ Method (Semilog Approach)

$k_{rd}(S_{wD}) = k_{rd}/k_{rn} = S_{wD}^{2}(1-S_{wD})^{(3+2\lambda)}$

$k_{rw}(S_{wD}) = k_{rw}/k_{rw}^{o} = (1-S_{wD})^{(3+2\lambda)}$

$S_{wD} = 1 - S_{w}^{*} = (1 - S_{w})/(1 - S_{w})$ (note that scale is reversed for familiarity)
Type Curve for Brooks and Corey/Burdine $k_r/S_{wD}$ Method

$$k_{rd}(S_w)/k_{wD}(S_w) = (k_r/k_w)(k_{wD}/k_{wD})^*$$
$$= S_{wD}^2(1-(1-S_{wD})^{1+2A})(1-S_{wD})^{3+2A}$$

$S_{wD} = 1 - S_w = (1-S_w)/(1-S_{wi})$
Data Grid for Brooks and Corey/Burdine $k_f/S_{wD}$ or $k_f/S_w^*$ Method

$1 - S_w$ (fraction) or $S_w^* = (S_w - S_{wi})/(1 - S_{wi})$
IP/P571 (Gas-Oil System)--Type Curve for Brooks and Corey/Burdine $k_r/S_wD$ Method
IP/P571 (Gas-Oil System)--Type Curve for Brooks and Corey/Burdine $k_r/S_{wd}$ Method (Semilog Format)
Table 4

GAS-OIL RELATIVE PERMEABILITY

IP Petroleum Company, Inc.
Pfluger A 16-3 Well
Sugg Ranch Field
Sterling County, Texas
SRS 1512/RSN 2552

<table>
<thead>
<tr>
<th>Sample:</th>
<th>P 571</th>
<th>Porosity, (% BV):</th>
<th>11.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (ft):</td>
<td>7776.1</td>
<td>Initial Water Sat. (% PV):</td>
<td>36.4</td>
</tr>
<tr>
<td>Perm.to Gas (md):</td>
<td>12.1</td>
<td>Perm.to Oil @ Swi (md):</td>
<td>4.34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gas Saturation (% PV)</th>
<th>Relative Permeability (Fraction)</th>
<th>Gas-Oil Ratio (Ko/Kr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gas (Kr)</td>
<td>Oil (Kr)</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>4.1</td>
<td>0.0092</td>
<td>0.6277</td>
</tr>
<tr>
<td>6.3</td>
<td>0.0145</td>
<td>0.4956</td>
</tr>
<tr>
<td>8.6</td>
<td>0.0211</td>
<td>0.3332</td>
</tr>
<tr>
<td>10.6</td>
<td>0.0312</td>
<td>0.2460</td>
</tr>
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<td>12.7</td>
<td>0.0464</td>
<td>0.1970</td>
</tr>
<tr>
<td>16.8</td>
<td>0.0741</td>
<td>0.1240</td>
</tr>
<tr>
<td>20.6</td>
<td>0.0960</td>
<td>0.0757</td>
</tr>
<tr>
<td>24.2</td>
<td>0.1186</td>
<td>0.0433</td>
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<tr>
<td>27.7</td>
<td>0.1446</td>
<td>0.0245</td>
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<tr>
<td>30.8</td>
<td>0.1763</td>
<td>0.0139</td>
</tr>
<tr>
<td>32.9</td>
<td>0.2043</td>
<td>0.0084</td>
</tr>
<tr>
<td>34.9</td>
<td>0.2381</td>
<td>0.0053</td>
</tr>
</tbody>
</table>
Figure 9
GAS-OIL RELATIVE PERMEABILITY

IP Petroleum Company, Inc.
Pfluger A 16-3 Well
Sugg Ranch Field
Sterling County, Texas
SRS 1512/RSH 2552

Sample: P 571  Porosity, %BV: 11.6
Depth, ft.: 7776.1  Initial Water Sat. (%PV): 36.4
Perm. to Gas (md): 12.1  Perm. to Oil @ Swi (md): 4.34

RELATIVE PERMEABILITY (FRACTION)

Krg

Kro

GAS SATURATION (% P. V.)

[Graph and data points]
Figure 10

GAS-OIL RELATIVE PERMEABILITY RATIO

IP Petroleum Company, Inc.
Pflugerville 16-3 Well
Sugg Ranch Field
Sterling County, Texas
SRS 1512/RSE 2552

Sample: P 571  Porosity, % BV: 11.6
Depth, ft.: 7776.1  Initial Water Sat. (% PV): 36.4
Perm. to Gas (md): 12.1  Perm. to Oil @ Swi (md): 4.34

GAS SATURATION (X P. V.)