Petroleum Engineering 620 — Fluid Flow in Petroleum Reservoirs
Fundamental Flow Lecture 2 — Non-Laminar Flow in Porous Media

*He has one of those terribly weak natures that are not susceptible to influence.*

— Oscar Wilde (1895)

**Topic:** Non-Laminar Flow in Porous Media

**Objectives:** (things you should know and/or be able to do)

- Be familiar with and be able to develop flow relations for gases and compressible liquids, in terms of pressure, pressure-squared, and pseudopressure using the **non-laminar** Forchheimer flow relation, which is quadratic in terms of velocity.

**Lecture Outline:**

- Developments using the Forchheimer, non-laminar flow relation.
  - Liquid flow relations
  - Gas flow relations
  - Plotting functions

- Various applications of the non-laminar flow equations (discussion)
  - Transient flow (isochronal tests)
  - Pseudosteady-state flow (flow-after-flow tests)

**Reading Assignment:**

- Review attached notes
Exercises: For your own practice/skills building—do **NOT** turn in!

- Derive the horizontal linear Forchheimer relations for gas flow.
- You are to provide a critical and detailed review (at least 1 page) for the following papers:

For each paper you are to address the following questions: (Type or write neatly)

- **Problem:**
  - What is/are the problem(s) solved?
  - What are the underlying physical principles used in the solution(s)?
- **Assumptions and Limitations:**
  - What are the assumptions and limitations of the solutions/results?
  - How serious are these assumptions and limitations?
- **Practical Applications:**
  - What are the practical applications of the solutions/results?
  - If there are no obvious "practical" applications, then how *could* the solutions/results be used in practice?
- **Discussion:**
  - Discuss the author(s)'s view of the solutions/results.
  - Discuss your own view of the solutions/results.
- **Recommendations/Extensions:**
  - How could the solutions/results be extended or improved?
  - Are there applications other than those given by the author(s) where the solution(s) or the concepts used in the solution(s) could be applied?
Forchheimer Equation for Non-Laminar Flow in Porous Media

(from Petroleum Engineering 620 Course Notes — 1997)
Forchheimer Equation for Non-laminar Flow in Porous Media

The Forchheimer equation for non-laminar flow in porous media is given by

\[ -\frac{dp}{dx} = \frac{\mu}{k} \frac{\nu}{k} + c \beta \gamma \nu^2 \]  \hspace{1cm} (1)

where,

\[ c = \frac{1}{(1.01325 \times 10^6 \text{ dyne cm}^{-2})(12 \text{ in/ft}) (2.54 \text{ cm/1 in})} \]

or

\[ c = 5.23794 \times 10^{-8} \text{ dyne cm}^{-2} \frac{cm}{cm^2 \text{ ft}} \]

\[ \beta = \text{"inertial flow coefficient," ft}^{-1} \]

By inspection we note that the first term on the right-hand-side (RHS) of Eq. 1 is simply Darcy's law (for laminar flow). The \( c \beta \gamma \nu^2 \) term is the "add-on" term used to account for non-laminar flow.

The average velocity, \( \nu \), is given as

\[ \nu = \frac{1}{A} q_{res} = \frac{1}{A} q_{sc} \frac{B}{1} \]  \hspace{1cm} (2)

where,

\( \nu = \text{average velocity} \)
\( A = \text{cross-sectional area} \)
\( q_{res} = \text{volumetric flowrate (reservoir volume)} \)
\( q_{sc} = \text{volumetric flowrate ("standard" volume)} \)
\( B = \text{formation volume factor, res vol/std vol} \)

Substituting Eq. 2 into Eq. 1, we obtain

\[ -\frac{dp}{dx} = \frac{\mu B}{kA} q_{sc} + \frac{c \beta \gamma B^2}{A^2} q_{sc}^2 \]  \hspace{1cm} (3)
For a dry gas, we have

\[ \frac{S_g}{\rho_g} = \frac{1}{B_g} \frac{S_g}{S_{sc}} \]  

(4)

Substituting Eq. 4 into Eq. 3, we obtain

\[-\frac{dp}{dx} = \frac{mg B_g}{k A} q_{sc} + \frac{C G}{m g A^2} S_{sc} q_{sc} q_{sc} \]

Dividing through by \( mg B_g \)

\[-\frac{1}{mg B_g} \frac{dp}{dx} = \frac{1}{k A} q_{sc} + \frac{C G}{m g A^2} S_{sc} q_{sc} q_{sc} \]  

(5)

Multiplying through by \( m_n B_n \) \((m_n = \frac{m}{\rho_n}; B_n = B_g \rho_n)\), we obtain

\[-\frac{m_n B_n}{mg B_g} \frac{dp}{dx} = \frac{m_n B_n}{k A} q_{sc} + \frac{C G}{m g A^2} S_{sc} m_n B_n q_{sc} q_{sc} \]

Separating

\[-\frac{m_n B_n}{mg B_g} \frac{dp}{dx} = \left[ \frac{m_n B_n}{k A} q_{sc} + \frac{C G}{m g A^2} S_{sc} m_n B_n q_{sc} q_{sc} \right] dx \]

Integrating

\[-m_n B_n \int_{p_1}^{p_2} \frac{1}{mg B_g} \frac{dp}{dx} = \left[ \frac{m_n B_n}{k A} q_{sc} + \frac{C G}{m g A^2} S_{sc} m_n B_n q_{sc} q_{sc} \right] \int_{0}^{l} dx \]

The gas formation volume factor, \( B_g \), is defined as

\[ B_g = \frac{P_{sc}}{\rho} \frac{T}{T_{sc}} \frac{z}{z_{sc}} \]  

(7)
Substituting Eq. 7 into the integral on the left-hand-side (LHS) of Eq. 6, we have

\[ I = -\mu_1 \bar{B}_1 \int_{\bar{p}_1}^{\bar{p}_2} \frac{1}{\mu_1 \bar{E}_1} \frac{p}{\bar{M}_1 \bar{E}_1} \, dp = -\mu_1 \bar{B}_1 \int_{\bar{p}_1}^{\bar{p}_2} \frac{p}{\mu_2} \, dp \] (assume \(T_n = T\))

Using the initial reservoir pressure, \(\bar{p}_1\), as the "normalizing" pressure, \(\bar{p}_n\), we have

\[ I = \frac{\mu_1 \bar{B}_1}{\bar{p}_1} \int_{\bar{p}_1}^{\bar{p}_2} \frac{p}{\mu_2} \, dp \] (reversing limits)

Expanding the integral

\[ I = \frac{\mu_1 \bar{B}_1}{\bar{p}_1} \int_{\bar{p}_1}^{\bar{p}_2} \frac{p}{\mu_2} \, dp - \frac{\mu_1 \bar{B}_1}{\bar{p}_1} \int_{\bar{p}_1}^{\bar{p}_2} \frac{p}{\mu_2} \, dp \] (8)

Or

\[ I = \int_{\bar{p}_1}^{\bar{p}_2} \rho_p(p) \, dp - \int_{\bar{p}_1}^{\bar{p}_2} \rho_p(p) \, dp \] (9)

where

\[ \rho_p(p) = \frac{\mu_1 \bar{B}_1}{\bar{p}_1} \int_{\bar{p}_1}^{p} \frac{p}{\mu_2} \, dp \] (10)

Substituting Eq. 9 into Eq. 6, we obtain

\[ \int_{\bar{p}_1}^{\bar{p}_2} \rho_p(p) \, dp - \int_{\bar{p}_1}^{\bar{p}_2} \rho_p(p) \, dp = \left[ \frac{\mu_1 \bar{B}_1}{kA} q_{SC} + \frac{cE}{MqA^2} L q_{SC} \frac{\mu_1 \bar{B}_1}{\bar{M} q_{SC}} q_{SC}^2 \right] \int_0^L dx \]

Completing the integration and rearranging

\[ \frac{\int_{\bar{p}_1}^{\bar{p}_2} \rho_p(p) \, dp - \int_{\bar{p}_1}^{\bar{p}_2} \rho_p(p) \, dp}{L} = \frac{\mu_1 \bar{B}_1}{kA} q_{SC} + \frac{cE}{MqA^2} L q_{SC} \frac{\mu_1 \bar{B}_1}{\bar{M} q_{SC}} q_{SC}^2 \]

Dividing through by \(\frac{\mu_1 \bar{B}_1}{A} q_{SC}\)

\[ \frac{A}{\mu_1 \bar{B}_1} \frac{1}{L} \left( \int_{\bar{p}_1}^{\bar{p}_2} \rho_p(p) \, dp - \int_{\bar{p}_1}^{\bar{p}_2} \rho_p(p) \, dp \right) = \frac{1}{k} + \frac{cE L q_{SC}}{MqA} q_{SC} \] (11)
From Darcy's law (i.e., steady-state laminar flow) we have,

\[ q_{sc} = \frac{k_{DL} A \left( P_0(r_1) - P_0(r_2) \right)}{\mu' B' L} \]

or

\[ \frac{1}{k_{DL}} = A \frac{1}{\mu' B'} q_{sc} L \]  \hspace{1cm} (12)

Substituting Eq. 12 into Eq. 11

\[ \frac{1}{k_{DL}} = \frac{1}{k} + \frac{c B q_{sc}}{\mu q A} q_{sc} \]  \hspace{1cm} (13)

where Eq. 13 is of the form,

\[ y = b + mx \]

where

\[ y = \frac{1}{k_{DL}} \quad ; \quad x = q_{sc} \quad ; \quad m = \frac{c B q_{sc}}{\mu q A} \quad \text{and} \quad b = \frac{1}{k} \]

and

\[ \mu q = \] is taken at \( \bar{p} = \frac{1}{2} (p_1 + p_2) \)

A plot of Eq. 13 gives

![Graph](image-url)