Petroleum Engineering 620 — Fluid Flow in Petroleum Reservoirs
Fundamental Flow Lecture 3 — Material Balance Concepts

Art disturbs, science reassures...
— Georges Braque (1963)

**Topic:** Material Balance Concepts

**Objectives:**
- Be able to identify and apply the material balance relations for gas and compressible liquid systems:
  - **General Form:** (any material balance relation)
    \[ q \propto c \frac{d}{dt} [f(p)] \] \( f(p) = \) some function of pressure (e.g., \( \bar{p} \), \( \bar{p}/\bar{z} \), etc.)
  - **Gas Material Balance Equations:**
    - Dry Gas Case: \( \frac{\bar{p}}{\bar{z}} = \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right] \) (No Influx)
    - General Material Balance Formulation (from Dake): General Form
      \[
      \text{Withdrawal (rcf)} = \text{Gas Expansion (rcf)} + \text{Water Expansion \\ & Pore Compaction (rcf)} + \text{Influx (rcf)}
      \]
      \[
      G_p B_g + W_p B_w = G(B_g - B_{gi}) + GB_{gi} \frac{(c_w S_{wi} + c_f)}{1 - S_{wi}} (p_i - \bar{p}) + W_e B_w
      \]
    - General Material Balance Formulation (from Dake): \( \bar{p}/\bar{z} \) form
      \[
      \frac{\bar{p}}{\bar{z}} = \frac{p_i}{z_i} \left[ 1 - \frac{(c_w S_{wi} + c_f)}{(p_i - \bar{p})} \left( \frac{W_e - W_p}{GB_{gi}} \right) \right] \left[ 1 - \frac{G_p}{G} \right]
      \]
  - **Oil Material Balance Equations:**
    - Constant Compressibility Case: Oil above the Bubble-Point Pressure
      \( (p_i - \bar{p}) = \frac{1}{N c_i} \frac{B_o}{B_{oi}} N_p \)
    - Solution Gas Drive Case (from Dake):
      \[ N_p \left[ B_o + (R_p - R_s) B_g \right] + W_p B_w = \] (Withdrawal (RB))
      \[ N \left[ (B_o - B_{oi}) + (R_{si} - R_s) B_g \right] \] (Oil Expansion (RB))
      \[ + m N B_{oi} \left[ \frac{B_g}{B_{gi}} - 1 \right] \] (Gas Cap Expansion (RB))
      \[ + (1 + m) N B_{oi} \frac{(c_w S_{wi} + c_f)}{1 - S_{wi}} (p_i - \bar{p}) \] (Water Exp./Pore Comp.(RB))
      \[ + W_e B_w \] (Water Influx (RB))
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- Be familiar with and be able to apply the "Havlena-Odeh" formulations of the oil and gas material balance equations.
- Be able to compute the following using Cartesian coordinate plots (and the appropriate material balance relations for each case):
  - The wellbore storage coefficient, \( C_s \),
    \[
    p_{wf} = p_i - \frac{q_{sur}B}{24 C_s} t \quad \text{(} p_{wf} \text{ vs. } t, \text{ Drawdown tests)} \\
    p_{ws} = p_{wf}(\Delta t=0) + \frac{q_{sur}B}{24 C_s} \Delta t \quad \text{(} p_{ws} \text{ vs. } \Delta t, \text{ Buildup tests)}
    \]
  - The oil-in-place, \( N \) ("slightly compressible" liquid system)
    - **Average Pressure Formulation:**
      \[
      \bar{p} = p_i - 5.615 \frac{N_p B_o}{\phi h A c_i} \quad \text{(} \bar{p} \text{ vs. } N_p) \]
    - **Wellbore Pressure Formulation:**
      \[
      \frac{p_i - p_{wf}}{q} = b_{pss} + \frac{1}{N c_i B_{oi}} \frac{N_p}{q} \quad \text{(} \frac{p_i - p_{wf}}{q} \text{ vs. } \frac{N_p}{q})
      \]
      or, for a constant flowrate, \( q \), we have
      \[
      p_{wf} = p_i - q b_{pss} - 0.23395 \frac{q B_o}{\phi h A c_i} t \quad \text{(} p_{wf} \text{ vs. } t, t \text{ in hours)}
      \]
  - The gas-in-place, \( G \), for a dry gas system
    \[
    \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right] = \frac{p_i}{z_i} - \frac{p_i}{z_i G_p} G_p \quad \text{(} \frac{p}{z} \text{ vs. } G_p) \]

**Lecture Outline:**

- Material Balance Relations:
  - Derivation of material balance relations (see attached notes and Dake1)
  - **Oil material balance equation:** \( (\bar{p}=\text{psia}, \ p_i=\text{psia}, \ N_p=\text{STB}, \ N=\text{STB}, \ c_i=\text{psia}^{-1}, \ B_o=\text{RB/STB}, \ \phi=\text{fraction}, \ h=\text{ft}, \ A=\text{reservoir drainage area}, \ \text{ft}^2) \)
    \[
    \bar{p} = p_i - 5.615 \ \frac{N_p B_o}{\phi h A c_i} \quad \text{and} \quad N = \frac{\phi h A}{5.615 B_{oi}}, \text{ which gives } \bar{p} = p_i - \frac{N_p B_o}{N c_i B_{oi}}.
    \]
  - **Gas material balance equation:** \( (\bar{p}=\text{psia}, \ p_i=\text{psia}, \ \bar{z}, \ z_i=\text{dimensionless}, \ G_p=\text{MSCF}, \ G=\text{MSCF}, \ B_g=\text{RB/MSCF}, \ \phi=\text{fraction}, \ h=\text{ft}, \ A=\text{reservoir drainage area}, \ \text{ft}^2) \)
    \[
    \frac{p_i}{z_i} \left[ 1 - \frac{G_p}{G} \right] = \frac{p_i}{z_i} \left[ 1 - 5.615 \ \frac{G_p B_g}{\phi h A} \right] \quad \text{where } B_g = 5.02 \ \frac{\text{zT}}{p}, \ T \text{ in } ^\circ\text{R}
    \]
Lecture Outline: (Continued)

- **Wellbore Storage Material Balance Equation:**
  
  - The fundamental material balance equation for wellbore storage is given as:
    
    $$-q_{sf} = q_{sur} + \frac{24C_s}{B} \left[ \frac{dp_w}{dt} - \frac{dp_{bg}}{dt} \right]$$
    
    ($q_{sf}$ = sandface rate, $q_{sur}$ = surface rate)
  
  - Assuming $q_{sf} = 0$ and $p_{bg}$ = constant, then integrating, we obtain the material balance relation for the **Wellbore Storage Domination** flow regime. In this case all fluid is either produced from (drawdown) or into (buildup) the wellbore.
    
    - $p_{wf} = p_i - m_{wbs} t$  
      \[ (p_{wf} \text{ vs. } t, \text{ Drawdown tests}) \]
    
    - $p_{ws} = p_{wf}(\Delta t=0) + m_{wbs} \Delta t$  
      \[ (p_{ws} \text{ vs. } \Delta t, \text{ Buildup tests}) \]
  
  where
    
    $$m_{wbs} = \frac{q_{sur} B}{24 C_s}$$
    
    ($p_{wf}$ vs. $t$, Drawdown tests)

  and the wellbore storage coefficient terms ($C_s$ variables) are given by:

  - $C_s = V_{wb} c_{wb}$ for a wellbore filled with a compressible fluid
  
  - $C_s = \frac{144 A_{wb} g \phi}{5.615 p_c}$ for a wellbore with a rising or failing liquid level

- **Equation for Boundary-Dominated (or Pseudosteady-State) Flow**

  - The material balance equation for this case is given as:
    
    $$\bar{p} = p_i - 5.615 \frac{N_p B_o}{\phi h A C_t}$$
    
    \[ (\text{for } q \text{ constant, } N_p=qt) \]
  
  - The so-called "pseudosteady-state flow equation" is: (without derivation)
    
    $$\bar{p} = p_{wf} + q b_{pss}$$
    
    where
    
    $$b_{pss} = 141.2 \frac{\mu B_o}{kh} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} + s \right]$$
    
    \[ (\text{for a circular reservoir}) \]
  
  and
    
    $$b_{pss} = 141.2 \frac{\mu B_o}{kh} \left[ 1 - \frac{4}{e^2 C_t} \frac{A}{r_w^2} + s \right]$$
    
    \[ (\text{general reservoir } (\tau=0.577216...)) \]

  - Combining the material balance and pseudosteady-state flow equations gives
    
    $$p_i - p_{wf} = \frac{b_{pss}}{q} + \frac{1}{N_{C_t}} B_o \frac{N_p}{B_{oi} q}$$

  - For a constant flowrate, $q$, the above relation becomes
    
    $$p_{wf} = p_i - q b_{pss} - 0.23395 \frac{q B_o}{\phi h A C_t} t$$
    
    or writing more compactly, we have
    
    $$p_{wf} = p_{int} - m_{pss} t$$, where $m_{pss} = 0.23395 \frac{q B_o}{\phi h A C_t}$ and $p_{int} = p_i - q b_{pss}$.
References:


Reading Assignment:

• Review attached handout notes
  ■ Appendix A—Development of Material Balance Time Concept
  ■ Appendix A—Development and Use of the Material Balance Time for Boundary Dominated-Liquid Flow
  ■ Appendix B—Development, Proof and Use of the Material Balance Pseudotime for Boundary-Dominated Gas Flow

• Review attached handout notes—"Derivation of Material Balance Relations"
  ■ Reservoir Oils
    – Undersaturated oil reservoir case ($p>p_h$).
    – Saturated oil reservoir case ($p<p_h$).
  ■ Natural Gases
    – General case (includes water expansion/pore compaction and water influx).
    – Dry gas case.

• Review attached papers:
(Appendix A)

Derivation of "Material Balance" Time
(Liquid Flow Case)

From:
APPENDIX A

DEVELOPMENT OF MATERIAL BALANCE TIME CONCEPT

In this appendix, we present the pseudosteady-state flow equation for slightly compressible liquids derived from a material balance relation. This derivation rigorously illustrates the necessity for using the material balance time function, \( t_{mb} \), for pseudosteady-state flow.

In words, a material balance based on the amount of oil in a reservoir is given by

\[
\text{Amount of oil originally in reservoir} = \text{Amount of oil in reservoir at a later time} + \text{Change in hydrocarbon pore volume due to rock and water expansion.}
\]

This can be expressed mathematically as

\[
NB_{oi} = (N - N_p) B_o + \Delta V_r + \Delta V_w \tag{A-1}
\]

We can express the change in rock and water volume in terms of formation and water compressibility respectively. The general definition of isothermal compressibility is

\[
c = -\frac{1}{V} \left[ \frac{\partial V}{\partial P} \right]_T
\]

We can calculate the slope, \( \frac{\partial V}{\partial P} \), by approximating the tangent to the curve (volume versus pressure) by a chord slope, i.e.

\[
c = -\frac{1}{V_i} \left[ \frac{V - V_i}{P - P_i} \right]
\]

and rearranging this equation to obtain an expression for change in volume gives

\[
\Delta V \approx - c V_i (p - p_i) \tag{A-2}
\]

Since the initial volume for rock is the initial pore volume, \( V_{pi} \), Eq. (A-2) can be used to express change in rock volume as

\[
\Delta V_r = - c_f V_{pi} (p - p_i) \tag{A-3}
\]

and since the initial volume for water is given by \( V_{pi} S_{wi} \), the change in water volume is

\[
\Delta V_w = - c_w V_{wi} (p - p_i) = - c_w V_{pi} S_{wi} (p - p_i) \tag{A-4}
\]

Combining Eqs. (A-3) and (A-4) gives

\[
\Delta V_r + \Delta V_w = - c_f V_{pi} (p - p_i) - c_w V_{pi} S_{wi} (p - p_i)
\]
Grouping terms condenses this equation to

$$\Delta V_r + \Delta V_w = -(c_w S_{wi} + c_j) V_{pi} (p - p_i) \tag{A-5}$$

We can now substitute Eq. (A-5) into Eq. (A-1), giving

$$N B_{oi} = (N - N_p) B_o - (c_w S_{wi} + c_j) V_{pi} (p - p_i) \tag{A-6}$$

The original oil in place in reservoir volumes, $N B_{oi}$, can be expressed in terms of pore volume and initial water saturation

$$N B_{oi} = V_{pi} (1 - S_{wi})$$

or

$$V_{pi} = \frac{N B_{oi}}{(1 - S_{wi})} \tag{A-7}$$

We can substitute Eq. (A-7) for the pore volume term in Eq. (A-6), giving

$$N B_{oi} = (N - N_p) B_o - (c_w S_{wi} + c_j) \frac{N B_{oi}}{(1 - S_{wi})} (p - p_i) \tag{A-8}$$

We assume the pressure, $p$, represents the average pressure, $\bar{p}$, in our volumetric system. Rearranging Eq. (A-8) gives

$$N B_{oi} = (N - N_p) B_o + (c_w S_{wi} + c_j) \frac{N B_{oi}}{(1 - S_{wi})} (p_i - \bar{p})$$

or, grouping terms containing $N$ on the left-hand side of the equation, we have

$$N (B_{oi} - B_o) = - N_p B_o + (c_w S_{wi} + c_j) \frac{N B_{oi}}{(1 - S_{wi})} (p_i - \bar{p}) \tag{A-9}$$

We can express the change in oil volume, $B_{oi} - B_o$, in terms of the isothermal compressibility of oil, $c_o$. Recall that the definition of $c_o$ is

$$c_o = - \frac{1}{B_o} \left[ \frac{\partial B_o}{\partial p} \right]_T$$

As before, we can calculate the slope, $\frac{\partial B_o}{\partial p}$, by approximating the tangent to the curve (formation volume factor versus pressure) by a chord slope, i.e.

$$c_o \approx - \frac{1}{B_{oi}} \left[ \frac{B_o - B_{oi}}{p - p_i} \right]$$

Rearranging this equation gives the following expression for the change in oil volume, $B_{oi} - B_o$

$$B_o - B_{oi} = - c_o B_{oi} (p - p_i) = c_o B_{oi} (p_i - \bar{p}) \tag{A-10}$$
We can now substitute Eq. (A-10) into Eq. (A-9) for the change in oil volume, $B_{oi} - B_o$

$$- Nc_o B_{oi}(p_i - \bar{p}) = - N_p B_o + (c_w S_{wi} + c_f) \frac{N B_{oi}}{(1 - S_{wi})}(p_i - \bar{p})$$

which can be rearranged to give

$$N_p B_o = Nc_o B_{oi}(p_i - \bar{p}) + (c_w S_{wi} + c_f) \frac{N B_{oi}}{(1 - S_{wi})}(p_i - \bar{p})$$

$$N_p B_o = NB_{oi}(p_i - \bar{p}) \left[ c_o + \frac{(c_w S_{wi} + c_f)}{(1 - S_{wi})} \right]$$

or finally,

$$N_p B_o = \frac{NB_{oi}}{(1 - S_{wi})}(p_i - \bar{p}) \left[ c_o (1 - S_{wi}) + c_w S_{wi} + c_f \right] \quad \text{(A-11)}$$

If we use the expression derived by Perrine$^1$ and Martin$^2$ for total system compressibility, $c_t$,

$$c_t = c_o (1 - S_{wi}) + c_w S_{wi} + c_f \equiv c_o S_{oi} + c_w S_{wi} + c_f \quad \text{(A-12)}$$

we can simplify Eq. (A-11) and write

$$N_p B_o = \frac{NB_{oi}}{(1 - S_{wi})} c_t(p_i - \bar{p}) \quad \text{(A-13)}$$

Recall that

$$V_{pi} = \frac{NB_{oi}}{(1 - S_{wi})} \quad \text{(A-7)}$$

therefore,

$$N_p = \frac{V_{pi}}{B_o} c_t(p_i - \bar{p}), \text{ where } V_{pi} = \phi h A, \text{ so we can write}$$

$$N_p = \frac{\phi h A}{B_o} c_t(p_i - \bar{p})$$

Rearranging gives

$$(p_i - \bar{p}) \frac{h}{B_o} = \frac{N_p}{\phi c_r A}$$

Multiply through by $\frac{2\pi k}{q^\mu}$ to obtain

$$\frac{p_i - \bar{p}}{2\pi k h} = \frac{2\pi k}{q B_{oi}^\mu} \frac{N_p}{\phi \mu c_r A} \frac{1}{q} \quad \text{(A-14)}$$

The definitions of dimensionless pressure, $p_D$, and dimensionless time based on drainage area, $t_{AD}$ are

$$p_D = \frac{2\pi k h}{q B_{oi}^\mu} (p_i - p_{wf}) \quad \text{(A-15)}$$
and
\[ t_{AD} = \frac{k t}{\phi \mu c_p A} \]  \hspace{1cm} \text{(A-16)}

Therefore, if we define a material balance time as \( t_{mb} = \frac{N_p}{q} \), Eq. (A-16) becomes
\[ 2 \pi t_{AD,mb} = \frac{2 \pi k}{\phi \mu c_p A} \frac{N_p}{q} \]  \hspace{1cm} \text{(A-17)}

We can now substitute Eq (A-17) into Eq. (A-14) to obtain
\[ (p_i - \bar{p}) \frac{2 \pi k h}{q B_o \mu} = 2 \pi t_{AD,mb} \]  \hspace{1cm} \text{(A-18)}

Eq. A-18 is valid for all flow regimes (transient or pseudosteady-state). We can couple this relationship for \((p_i - \bar{p})\) with the general relationship for \((\bar{p} - p_{wf})\) derived by Camacho\textsuperscript{3}, which is valid for pseudosteady-state flow and which is given below
\[ (\bar{p} - p_{wf}) \frac{2 \pi k h}{q B_o \mu} = \frac{1}{2} \ln \left[ \frac{4}{e^\gamma} \frac{A}{C_A r_w^2} \right] + s \]  \hspace{1cm} \text{(A-19)}

Adding Eqs. A-18 and A-19 eliminates \( \bar{p} \), giving
\[ (p_i - p_{wf}) \frac{2 \pi k h}{q B_o \mu} = 2 \pi t_{AD,mb} + \frac{1}{2} \ln \left[ \frac{4}{e^\gamma} \frac{A}{C_A r_w^2} \right] + s \]  \hspace{1cm} \text{(A-20)}

Substituting the definition of dimensionless pressure, Eq. (A-15), into Eq. (A-20) gives the dimensionless form of the pseudosteady-state flow solution to the diffusivity equation,
\[ p_D = 2 \pi t_{AD,mb} + \frac{1}{2} \ln \left[ \frac{4}{e^\gamma} \frac{A}{C_A r_w^2} \right] + s \]  \hspace{1cm} \text{(A-21)}

This validates the use of the material balance time function, \( t_{mb} \), for pseudosteady-state flow.

REFERENCES used in APPENDIX A


Material Balance Notes
(from Department of Petroleum Engineering Course Notes — 1984)

- Black Oil Cases
  - Undersaturated Oil
  - Solution Gas Drive
- Dry Gas Case
VOLUMETRIC OIL RESERVOIRS

I. Undersaturated Reservoir

Objective: To derive a material balance equation for an undersaturated reservoir

A. Assumptions

1. $P > P_B$

2. No original or final gas cap

3. No water influx or production

B. Derivation of Material Balance Equation
BY VOLUMETRIC BALANCE

ORIGINAL VOLUME = FINAL VOLUME

ORIGINAL VOLUME = \(N_0\)

FINAL VOLUME = \((N-N_p)B_o + \) VOLUME OCCUPIED BY
WATER AND ROCK
EXPANSION AS
PRESSURE DECLINES

- ROCK AND WATER EXPANSION IMPORTANT IN
UNDERSATURATED RESERVOIRS

FROM DEFINITION OF COMPRESSIBILITY

\[
c_w = -\frac{1}{V_w} \left( \frac{dV_w}{dP} \right) = -\frac{1}{V_w} \frac{\Delta V_w}{\Delta P}
\]

THUS, CHANGE IN RESERVOIR WATER VOLUME DUE
TO PRESSURE CHANGE:

\[
\Delta V_w = -c_w V_w \Delta P
\]

AS PRESSURE DECREASES, MATRIX SUPPORTING
STRUCTURE COLLAPSES INTO PORE SPACE

\[
c_f = -\frac{1}{V_p} \left( \frac{dV_p}{dP} \right) = -\frac{1}{V_p} \frac{\Delta V_p}{\Delta P}
\]
THUS, CHANGE IN PORE VOLUME DUE TO PRESSURE CHANGE:

$$\Delta V_p = -c_f \Delta P \frac{V_{PI}}{\Delta P}$$

TOTAL CHANGE IN WATER VOLUME AND PORE VOLUME:

$$\Delta V_w + \Delta V_p = -c_w \frac{V_{WI}}{\Delta P} + c_f \frac{V_{PI}}{\Delta P} \Delta P$$

$$= \Delta V_{TOTAL}$$

NOTE THAT

$$V_w = S_w V_p$$

$$V_{WI} = S_{WI} V_{PI}$$

THUS

$$\Delta V_{TOTAL} = -c_w \frac{S_{WI}}{\Delta P} + c_f \frac{V_{PI}}{\Delta P} \Delta P$$

ALSO

$$V_{PI} = \frac{NB_{OI}}{1 - S_{WI}}$$

THUS

$$\Delta V_{TOTAL} = \frac{NB_{OI}}{1 - S_{WI}} \left[ c_w S_{WI} + c_f \right] \Delta P$$
THE VOLUMETRIC BALANCE BECOMES:

\[ N_{BOI} = (N-N_P)B_O - \frac{N_{BOI}}{1-S_{WI}} \left[ c_w S_{WI} + c_f \right] \Delta P \]

SOLVING FOR \( N \):

\[ N = \frac{N_P B_O}{B_O + B_{OI} \left( \frac{c_w S_{WI} + c_f}{1-S_{WI}} \right) (P_I-P) - B_{OI}} \quad (V-1) \]

TO SIMPLIFY, NOTE:

\[ c_O = -\frac{1}{V} \left( \frac{dV}{dP} \right) = -\frac{1}{V} \left( \frac{\Delta V}{\Delta P} \right) \]

IF \( V_{SC} \) IS VOLUME OF OIL IN STOCK TANK (STANDARD CONDITIONS)

\[ \frac{1}{V} \left( \frac{\Delta V}{\Delta P} \right) = -\frac{1}{V_{I/V_{SC}}} \frac{(V/V_{SC} - V_I/V_{SC})}{(P-P_I)} \]

\[ = -\frac{1}{B_{OI}} \frac{(B_O-B_{OI})}{(P-P_I)} \]

THEN

\[ B_O-B_{OI} = c_O B_{OI} (P_I-P) \]
SUBSTITUTING INTO EQUATION V-1

\[
N = \frac{N_P B_Q}{B_{OI} \left( c_0 + \frac{c_W S_{W1} + c_F}{1-S_{W1}} \right) (P_I - P)}
\]

DEFINE

\[
c_E = c_0 + \frac{c_W S_{W1} + c_F}{1-S_{W1}} = \frac{c_O S_{OI} + c_W S_{W1} + c_F}{(1-S_{W1})}
\]

THUS

\[
N = \frac{N_P B_Q}{B_{OI} c_E (P_I - P)} \quad \text{(V-2)}
\]

C. CONSIDERATIONS

1. Eqs. V-1 or V-2 should be used for estimating OOIP above bubble point where rock and water expansion not negligible

2. Difficulty in measuring \(c_F\) and \(c_W\) may limit accuracy
D. Example - Use of Material Balance to Determine Original Oil in Place in Undersaturated Reservoir

Problem

Determine the original oil in place for the undersaturated reservoir for which data are summarized below.

\[ N_p = 1.4 \times 10^6 \quad \text{STB} \]
\[ B_O = 1.46 \quad \text{RB/STB} \]
\[ B_{OI} = 1.39 \quad \text{RB/STB} \]
\[ c_w = 3.71 \times 10^{-6} \quad \text{PSI}^{-1} \]
\[ c_F = 3.52 \times 10^{-6} \quad \text{PSI}^{-1} \]
\[ S_{WI} = 32\% \]

The reservoir was discovered at an initial pressure of 4300 psi, pressure has declined to 2450 psi.

Solution

From equation V-1

\[ N = \frac{N_p B_O}{B_O + B_{OI} \left( \frac{c_w S_{WI} + c_F}{1 - S_{WI}} \right) (P_i - P) - B_{OI}} \]
\[
N = \frac{1.4 \times 10^6(1.46)}{1.46 + 1.39 \left\{ \frac{3.71 \times 10^{-6}(0.32) + 3.52 \times 10^{-6}}{1-0.32} \right\} (4300-2450)-1.39}
\]

\[= 2.33 \times 10^7 \text{ STB} \]

**NOTE:**

**IF** \( C_F \) **IS ASSUMED TO BE 0:**

\[
N = \frac{1.4 \times 10^6(1.46)}{1.46 + 1.39 \left\{ \frac{3.71 \times 10^{-6}(0.32) + 0}{1-0.32} \right\} (4300-2450)-1.39}
\]

\[= 2.74 \times 10^7 \text{ STB} \]

**CALCULATED VALUE OF N IS SIGNIFICANTLY INCREASED IF** \( C_F \) **NEGLIGENCE**
I. Saturated Reservoir - Solution Gas Drive

OBJECTIVE: TO DERIVE A MATERIAL BALANCE EQUATION FOR A SOLUTION GAS DRIVE RESERVOIR AND TO APPLY IT TO ESTIMATE ORIGINAL OIL IN PLACE (OOIP)

A. Assumptions

1. $P \leq P_B$

2. No original gas cap

3. No water influx or production

4. Negligible rock and water expansion

B. Derivation of Material Balance Equation

![Diagram of material balance equation]

Original conditions by volumetric balance

Original volume = Final volume
ORIGINAL OIL VOLUME = $N_B_{O1}$
ORIGINAL FREE GAS VOLUME = 0
FINAL OIL VOLUME = $(N-N_p)B_O$

- DETERMINE FINAL FREE GAS VOLUME BY PERFORMING A GAS BALANCE

ORIGINAL DISSOLVED GAS = $NR_{S1}$
FINAL DISSOLVED GAS = $(N-N_p)R_S$
GAS PRODUCED = $G_p$

THEREFORE,

FINAL FREE GAS = $NR_{S1} - (N-N_p)R_S - G_p$

- CONVERT TO RESERVOIR CONDITIONS

FINAL FREE GAS = $(NR_{S1} - (N-N_p)R_S - G_p)B_g/5.61$

THE VOLUMETRIC BALANCE BECOMES:

$NB_{O1} = (N-N_p)B_O + (NR_{S1} - (N-N_p)R_S - G_p)B_g/5.61$

SOLVING FOR N

$$N = \frac{N_pB_O - (N_pR_S - G_p)B_g/5.61}{B_O + (R_{S1} - R_S)B_g/5.61 - B_{O1}} \quad (V-3)$$
TO SIMPLIFY, NOTE THAT

\[ B_T = B_O + (R_{SI} - R_S)B_G/5.61 \]

\[ R_P = G_P/N_P \]

ALSO,

\[ B_{OI} = B_{TI} \quad \text{(NO GAS EVOLVED AT P_B)} \]

SUBSTITUTING INTO EQUATION V-3

\[ N = \frac{N_P \{B_T + (R_P - R_{SI})B_G/5.61\}}{B_T - B_{TI}} \quad (V-4) \]

SUBSTITUTING FOR B_O IN THE NUMERATOR ONLY, AN ALTERNATIVE FORM IS

\[ N = \frac{N_P \{B_O + (R_P - R_S)B_G/5.61\}}{B_T - B_{TI}} \quad (V-5) \]
III. Derivation of Material Balance Equation

- From Volumetric Balance

**Initial Volume = Final Volume**

\[ G(B_{G1})/5.61 = (G-G_p)(B_G/5.61) + (W_E-W_p)(B_W) \]

Where \( B_G \) = Gas Formation Volume Factor, RCF/SCF

\[ G(B_{G1})/5.61 = (G-G_p)(B_G/5.61) - W_p(B_W) + W_E(B_W) \]

This can be rearranged to be

\[ G(B_G-B_{G1})/5.61 = (G_p(B_G)/5.61) + W_p(B_W) - W_E(B_W) \]
THE GENERALIZED FORM OF THE MATERIAL BALANCE EQUATION

\[
\frac{G_p(B_G)/5.61 + W_p(B_w)}{(B_G - B_{G1})/5.61} = G + \frac{W_e(B_w)}{(B_G - B_{G1})/5.61}
\]

(X-6)

A. PRESSURE DEPLETION CASE - NO WATER INFLUX

\[W_e = W_p = 0\]

Therefore:

\[G(B_G - B_{G1})/5.61 = G_p(B_G)/5.61\]

or

\[G_p = G(1 - B_{G1}/B_G)\]  

(X-7)
\[ B_G = \frac{V_{R.C.}}{V_{S.C.}} = \frac{P_{s.c.}}{P_{R.C.}} \cdot \frac{T_{R.C.}}{s.c.} \cdot Z_{R.C.} \]

\[ \frac{B_G}{B_{GI}} = \left( \frac{P_1}{Z_1} \right) \left( \frac{z}{p} \right) = \text{constant} \]

Then \[ G_p = G(1 - (P/z)(z_1/P_1)) \]

This can be rearranged to be

\[ (P_1/Z_1)(1-G_p/G) = P/z \]  \hspace{1cm} (X-8)

This suggests a plot of \((P/z)\) vs. \(G_p\)

When \((P/z) = 0\), note that

\[ P_1/Z_1 \cdot (1-G_p/G) = 0 \]

or

\[ G_p = G \]

Also, the plot should be linear and thus readily extrapolated to \(P/z = 0\).
B. Example Problem - Determination of OGIP and Drive Mechanism Using a P/z Plot

Problem

An isopach map of the "Zapata Sand" in the Woodford Field in Atascosa County, Texas indicated an original gas in place of 44 mmscf. Production from the field has resulted in the following:

<table>
<thead>
<tr>
<th>Reservoir Pressure (psia)</th>
<th>Gp (mmscf)</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>0.00</td>
<td>0.80</td>
</tr>
<tr>
<td>3500</td>
<td>2.46</td>
<td>0.73</td>
</tr>
<tr>
<td>3000</td>
<td>4.92</td>
<td>0.66</td>
</tr>
<tr>
<td>2500</td>
<td>7.88</td>
<td>0.60</td>
</tr>
<tr>
<td>2000</td>
<td>11.20</td>
<td>0.55</td>
</tr>
<tr>
<td>200</td>
<td>---</td>
<td>0.94</td>
</tr>
</tbody>
</table>

The Z factors were derived from fluid analysis data. A volumetric type depletion is suspected.

Perform a P/z plot to confirm original gas in place estimates and the suspected drive mechanisms.
**Solution**

<table>
<thead>
<tr>
<th>$G_p$ (MMSCF)</th>
<th>$P/z$ (PSIA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>5000</td>
</tr>
<tr>
<td>2.46</td>
<td>4795</td>
</tr>
<tr>
<td>4.92</td>
<td>4545</td>
</tr>
<tr>
<td>7.88</td>
<td>4167</td>
</tr>
<tr>
<td>11.20</td>
<td>3636</td>
</tr>
</tbody>
</table>

(See Graph)

The characteristics of the best straight line fit of the data set are:

A) $G = 43.6$ MMSCF (by extrapolation)

B) The curve gives some indication of nonlinearity but still fits the expected gas-in-place estimate

C) The straight line and good agreement with original gas-in-place estimate confirm the volumetric depletion characteristics