PRESSURE BUILD-UP IN WELLS

BY

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Synopsis

The report presents a method of analysis of the pressure build-up curve obtained from a closed-in well by plotting the bottom hole pressure against the logarithm of \( t_0 + t \), where \( t_0 \) is the closed-in time and \( t \) is the past producing life of the well.

Methods are also given for extrapolating the recorded pressures to infinite closed-in time for the cases of

I— a new well far from any reservoir boundary
II— a new well close to a fault, but far from any other boundary
III— a well in a finite reservoir.

Several illustrative examples are discussed.

Acknowledgements

The author wishes to state that he was not concerned with the development of the theories herein presented. They were evolved over a considerable period by the reservoir engineering section of the production department of the head office of the Bataafsche Petroleum Maatschappij in The Hague.

Among others who worked on the matters herein presented should be mentioned Messrs B. P. Boots, F. Brons, D. N. Dietz, M. Jakobs and W. R. van Wijk**.

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PART I—THEORY

1—Basic Equations and Assumptions

The mathematical study of the sub-surface flow of reservoir fluid requires that certain simplifying assumptions be made as to the nature of the porous medium and the fluids which it contains. In effect, the only practicable method at present available requires such sweeping simplifications in order to obtain a solution at all, that the solution so obtained requires considerable testing in practice in order to determine its usefulness and its limitations.

Thus it is general practice to develop the flow equations assuming the reservoir to be homogeneous, horizontal and of uniform thickness throughout. The fluid is assumed to obey Darcy’s law and to be present in one phase only. Furthermore, it is assumed that the compressibility and the absolute viscosity of the fluid remain sensibly constant over the range of temperature and pressure variation encountered in the formation and that the density of the fluid obeys an exponential type law

\[ p = p_0 e^{c(p-p)} \]  

where \( p \) is the density at some pressure \( p \), \( p_0 \) is the density at some standard pressure (conveniently taken as the original reservoir pressure \( p_0 \)), and \( c \) is the compressibility (assumed constant).

Then if we consider one well drilled into such a reservoir and assume furthermore that the flow to the well is radial (which implies either an infinite reservoir or a finite circular reservoir with the well at its centre) it may be shown that the equation of flow (1) is

\[ \frac{\partial p}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) = \frac{f \mu}{k} \frac{\partial p}{\partial t} \]  

where \( r \) is the distance from the centre line of the well in centimetres, \( t \) is the time in seconds, \( p \) is the reservoir pressure in atmospheres at distance \( r \) and time \( t \), \( f \) is the formation porosity expressed as a fraction of the bulk volume, \( k \) is the formation permeability in darcies, \( \mu \) is the fluid viscosity in centipoises and \( c \) is the fluid compressibility measured in volumes per volume per atmosphere.

Of these basic assumptions it would seem that by far the most critical is the one which requires the presence of only a single phase of reservoir fluid, in that both the compressibility and the permeability are very sensitive to changes in pressure below the bubble point. However, although the theory is developed for the case where pressures are everywhere above the bubble point, the equations often seem to fit when this condition does not hold.

2—Basic Solution to the Equation of Flow

There are a number of exact solutions of equation II (for various boundary conditions) given in the literature, but these suffer from the disadvantage that they involve complicated integrals and Bessel functions which make them very unwieldy for calculation purposes. We therefore make use of the so-called “point-source” solution

\[ p = p_0 + \frac{q_p}{4 \pi k \mu} \text{Ei} \left( -\frac{r^2 \mu c}{4k} \right) \]  

where \( p_0 \) is the initial reservoir pressure in atmospheres, \( h \) is the formation thickness in centimetres and \( q \) is a constant rate of production of the well expressed as cubic centimetres of subsurface volume (under original conditions) per second.

This equation III is an exact solution of equation II for the following boundary conditions:

(i) external boundary at infinity at constant pressure \( p_0 \)

(ii) internal boundary (i.e. the well radius) vanishing and with a constant flow rate \( q \) across it (i.e. for a mathematical sink in an infinite reservoir).

Thus the only error introduced by using equation III as the basic solution for the case of an infinite reservoir is in considering the well radius as infinitely small. This error is considered to be negligible for the applications of this report.

3—Pressure Build-up in a Single Well in an Infinite Reservoir

a) Constant Production Rate before Closing in

Consider a single well in an infinite reservoir

\[ \text{Ei} (-x) = - \int_{x}^{\infty} e^u \frac{du}{u} \]  

are available from references (2) and (3).

This function is \( -\infty \) for \( x \) zero and increases monotonically to zero as \( x \) goes from zero to \( +\infty \). As \( x \) approaches zero, \( \text{Ei} (-x) = - \ln x - 0.5772 \ldots \), and so for small values of \( x \), say for \( x \) smaller than about \( 0.1 \), we may write with close approximation

\[ \text{Ei} (-x) \approx \ln x + 0.5772 \ldots \]
which was completed and first brought into production at time zero, and which subsequently produced at a constant rate \( q \) until time \( t_0 \), when it was closed in. Then, ignoring the effects of the after-production*, the well pressure \( p_w \) at time \( t_0 + \delta \) (i.e. \( \delta \) after closing in) may be obtained by superimposing two solutions of the form of equation III and then writing \( r_w \) for \( r \) thus

\[
p_w = p_0 + \frac{q \mu}{4\pi kh} \left( \frac{1}{\ln \left( \frac{t_0 + \delta}{\delta} \right)} \right) \quad \text{..... (IV)}
\]

where \( p_w \) = pressure at well bore in atmospheres and \( r_w \) = well radius in centimetres.

Now for small values of its argument the \( Ei \)-function may be accurately approximated by a logarithmic function. (See footnote to page 2.) If this approximation be in fact made in equation IV above we derive the basic build-up equation for a single well in an infinite reservoir as

\[
p_w = p_0 - \frac{q \mu}{4\pi kh} \ln \left( \frac{t_0 + \delta}{\delta} \right) \quad \text{..... (V)}
\]

The error involved in this approximation will be very small after a comparatively short time. It will, indeed, have dropped to \( \frac{1}{2} \% \) as soon as \( \delta > \frac{25\pi^2\mu c}{k} \), a condition which will usually be satisfied within a matter of seconds after closing in the well. As we have introduced errors into the calculations (by completely ignoring the after-production) which may affect the validity of our equations for a period of an hour or more after closing the well in, it is clear that both the approximation to the exact solution by the \( Ei \)-function and the approximation to the \( Ei \)-function by the \( \ln \)-function are entirely justifiable.

Thus in the case of a well which has produced uniformly since completion at a rate \( q \) from an infinite reservoir we may expect the bottom hole pressure to build up in accordance with equation V.

\[ b) \text{ Variable Production Rate before Closing in} \]

In the previous paragraph we derived a build-up equation V for a well which produces uniformly at rate \( q \) from time zero to time \( t_0 \) and is then closed

\* When a well is closed in at the surface, production from the formation does not cease immediately. Instead, there is some as yet undetermined period of time during which the formation produces fluid into the well bore (at a decreasing rate) thereby building up the pressure within the well. It is this quantity of fluid—the volume which is produced by the formation into the well after closing in at the surface—which is herein termed the "after-production".

However, such conditions do not normally obtain, and so some correction must be applied to take account of the varying rates at which a well will have produced during its history. Two methods of correction may be used, one of which may be said to enable a theoretically precise solution to be obtained (at least in principle) while the other is nothing but a good working approximation.

To illustrate the precise method we suppose that the production history of the well was as shown by the broken line in figure 1. To this we approximate by a series of steps (as shown) and then modify the equation V to read

\[
p_w = p_0 - \frac{q \mu}{4\pi kh} \left( \ln \left( \frac{t_0 + \delta}{\delta} \right) \right) \quad \text{..... (VI)}
\]

where the \( t \)'s and the \( q \)'s are as indicated in fig. 1, and are so chosen that they represent the same total production as the well actually made.

However, this equation VI is rather laborious in application, and will normally be more precise than is warranted by the other inaccuracies which are unavoidably present—in fact only rarely has it ever proved of value to apply this elaborate method. Instead, the equation V is usually modified by simply introducing a so-called corrected time \( t_c \), and writing

\[
p_w = p_0 - \frac{q \mu}{4\pi kh} \ln \left( \frac{t_c + \delta}{\delta} \right) \quad \text{..... (VII)}
\]

where \( q \) is calculated from the last established production rate before closing in the well; \( t_c \) is obtained by dividing the total cumulative production of the well by the last established production rate.
c) **Interpretation of Build-up Equations V, VI, VII**

It is immediately evident that if we plot $p_w$ against $\ln \frac{t_0 + \theta}{\delta}$ (using $t_0$ for $t_0$ as necessary) or against $q_0 \ln \frac{t_0 + \delta}{t_0 + \delta - t_1}$ etc. if we are using the accurate correction method, we may expect the points to fall on a straight line, at least after the effects of the after-production have disappeared. If indeed it does prove possible to draw such a straight line, two deductions can be made immediately. They are:

(i) by extrapolating the line to $\ln \frac{t_0 + \delta}{\delta} = 0$

(or $q_0 \ln \frac{t_0 + \delta}{t_0 + \delta - t_1} + q_1 \ln \frac{t_0 + \delta - t_1}{t_0 + \delta - t_2} + \ldots$ etc. $= 0$ in the accurate case), which is equivalent to extrapolating to $\delta$ infinite, we may read $p_w = p_i$, which is of course equal to $p_w$ the fully built up pressure of the well. This value is, of course, identical with the initial pressure, as we are at present only considering an infinite reservoir.

(ii) The gradient of this line is equal to $-\frac{q \mu}{4\pi k \delta}$ (or to $\mu$ in the accurate case) and so knowing values for $q$, $\mu$ and $h$ it is possible to determine a value for the permeability $k$, measured in situ. This value of $k$ has the advantage that it is a mean value for the whole well drainage area.

\[ d) \text{ Conditions of Applicability} \]

The theory detailed above is, however, only applicable strictly to an infinite reservoir, which is a theoretical conception which does not exist in fact. Thus the above method can only be expected to be applicable in the case of a well which has not yet produced sufficient fluid material to have diminished the overall static reservoir pressure, i.e. a new well in which the effects of the reservoir boundary have not yet become apparent.

4—**Influence of a Fault in an Otherwise Infinite Reservoir**

The problem of a well producing from a point distant “a” cm from a linear barrier fault may be simply solved by the method of images. This means that instead of considering one well $Q$ producing from a semi-infinite reservoir bounded by the linear fault (fig. 2a) we may consider two similar wells $Q$ and $Q'$ producing from an infinite reservoir (fig. 2b), where the fault has now been removed and the well $Q'$ is inserted in such a manner as to have an effect equivalent to that of the fault. This requirement is simply that $Q'$ be identical to $Q$ and at the mirror image of $Q$ in the fault plane, i.e. at a distance $2a$ from $Q$. Thus the pressure in $Q$ at time $t_0 + \delta$ (after closing in) is

\[ P_w = P_0 + \frac{q \mu}{4\pi k h} \ln \frac{t_0 + \delta}{\delta} - \frac{r_w^2 f_{\mu c}}{4k(t_0 + \delta)} - \frac{r_w^2 f_{\mu c}}{4k \delta} + \frac{a f_{\mu c}}{k(t_0 + \delta)} - \frac{a f_{\mu c}}{k \delta} \ldots \ldots \ldots (VII) \]

where the first two $E_1$-functions, unchanged from equation IV, represent the effect of the well $Q$, and the last two $E_1$-functions are the contribution of the image well $Q'$. As before we may substitute the $\ln$-function for the $E_1$-function in the first two terms to give the build-up equation

\[ P_w = P_0 - \frac{q \mu}{4\pi k h} \ln \frac{t_0 + \delta}{\delta} - \frac{a f_{\mu c}}{k(t_0 + \delta)} + \frac{a f_{\mu c}}{k \delta} \ldots \ldots \ldots (IX) \]

\[ \text{Fig. 2. Application of the method of images to the case of the linear barrier fault.} \]
Extrapolation of this line, however, would give a final pressure below the true value \( (p_0) \). However, as \( \delta \) becomes very large the arguments of both the Ei-functions in equation IX become small and they may then be approximated to by \( \ln \)-functions. Thus for very large \( \delta \) equation IX becomes

\[
P_w = p_0 - \frac{q\mu}{2\pi kh} \ln \frac{t_0 + \delta}{\delta} \quad \text{(XI)}
\]

that is the last part of the build-up curve is a straight line of slope \( \frac{q\mu}{2\pi kh} \) i.e. twice the normal value) when plotted against \( \ln \frac{t_0 + \delta}{\delta} \); this part of the curve may be correctly extrapolated to \( p_w = p_0 \) at \( \ln \frac{t_0 + \delta}{\delta} = 0 \).

Clearly there will be a range of transition values of \( \ln \frac{t_0 + \delta}{\delta} \) where the line of the form of equation X merges into the line of the form of equation XI. It is also theoretically possible to determine the quantity \( \frac{a + f \mu c}{k} \) (and hence \( a \)) by fitting the build-up curve to the exact equation IX and thus to determine the distance of a fault from a well (although not, of course, its orientation).

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**Fig. 3. Illustration of the theoretical case of a linear barrier fault.**
A calculated plot of equation IX is included (fig. 3) to illustrate the form of the build-up which can be expected.

S-Well at the Centre of a Finite Circular Reservoir

Exact solutions to the flow equation II are available in the literature for the case of a single well in the centre of a finite circular reservoir. A solution (4) which only errs insofar as it treats the well as a mathematical sink in a finite reservoir (i.e. as is accurate in this case as is the Ei-solution for an infinite reservoir) is

\[ p = p_e + \frac{q \mu}{4 \pi k h} \left\{ Ei \left( -\frac{r^2 f \mu c}{4 k t} \right) - \frac{r^2 f \mu c}{4 k t} \right\} \]

where for simplicity we define the function \( y \) by

\[ y(u) = Ei(-u) + \frac{u}{e^u} \]

It is of importance to be able to consider this function \( y \) as a known function. Accordingly a plot of \( y \) for a large range in values of \( u \) is appended (fig. 10).

Now this equation XIII is not even a mathematical solution of the basic flow equation II. However, the closeness of the approximation of this equation to equation XII may be easily shown to be usually satisfactory, the error is certainly never more than a few percent. As an example fig. 4 shows the accuracy of equation XIII by comparison with equation XII in the particular case where \( r_e/r_s = 1000 \).

Just as was done in paragraph I, 3a, we superimpose two solutions of the form of equation XIII and write \( r_s \) for \( r \) to give our approximation to the build-up equation in a finite reservoir as

\[ p_e(t) = p_e + \frac{q \mu}{4 \pi k h} \left\{ \ln \left( \frac{t}{t_s} + \frac{1}{\theta} \right) + y \left( \frac{r^2 f \mu c}{4 k (t_s + \theta)} \right) \right\} \]

where \( \theta \) is very large the second \( y \)-function will be almost zero and the first will be nearly constant, and the equation XIV reduces to

\[ p_e(t) = p_e + \frac{q \mu}{4 \pi k h} \left\{ \ln \left( \frac{t_s + \frac{1}{\theta}}{\theta} \right) + y(u) \right\} \]

where for convenience we write \( u_1 \) for \( r^2 f \mu c/4 k t_s \).

Thus we still have that, over the range of values of \( \theta \) which will normally be measured, a plot of \( p_e(t) \) against \( \ln (t_s + \frac{1}{\theta}) \) will be linear and its slope will still be \( \frac{q \mu}{4 \pi k h} \). However, linear extrapolation to \( \ln (t_s + \frac{1}{\theta}) = 0 \) will indicate a false value for the final build up (which false value we may indicate by \( p^* \)) defined by
SECTION II, PREPRINT 7

\[ p^* = p_s - \frac{q \mu}{4\pi k h} y(u_1) \quad \text{...(XVI)} \]

Thus, knowing \( p_s \) and having determined \( \frac{q \mu}{4\pi k h} \) and \( p^* \) from the build-up curve, we may solve equation XVI for \( y(u_1) \) and thus we may derive a value of \( u_1 = \frac{r^2 f \mu c}{4k t} \).

Now if we return to the former equation XIV and let \( \phi \) become infinite, the equation becomes

\[ \frac{p - p_w}{r^2} = \frac{q}{2\pi k h} \]

Curves:
- Curve I: Theoretical case of equation XII
- Curve II: Approximation of this report, equation XIII

Both curves plotted for the special case \( r_2 = 1000 \).

Fig. 4. Comparison between precise theory and approximation of this report for the case of a well in a finite circular reservoir.
p = \frac{q t_*}{\pi^{\frac{5}{2}} \theta c} = p_0 \left( \frac{q \mu}{4 \pi k \theta h} \right) \\
\div \left( \frac{r_*^5 \mu c}{4 k t_*} \right) \ldots (XVII)

where \( p \) is the final static (i.e. fully built up) closed in pressure, and then, substituting the known values of \( p_0 \), \( \frac{q \mu}{4 \pi k \theta h} \), and \( \frac{r_*^5 \mu c}{4 k t_*} \) in XVII we may obtain a value of \( p \), thereby correctly extrapolating the build-up curve.

PART II—APPLICATIONS

1—Introduction

Part II of this report is intended to be largely complete in itself. The object of this part is, by the use of several examples, to illustrate the methods of applying the more important equations developed in Part I. These equations are collected for easy reference in the Summary of Equations Chart, Appendix A; each equation is repeated in two forms namely exactly as derived in Part I and also as modified for use with practical oil-field units. All the symbols used in the report are collected in a table (Appendix B) to which reference should be made for the units which are to be used for the various quantities involved, and for the values of the numerical constants of conversion (A, B, D, F and G) to be used with the particular units employed.

The examples chosen to illustrate the use of the methods here presented are all from wells in the Casabe field, Colombia, and have been selected from a batch of 66 pressure surveys in this field made between August, 1948, and April, 1950. Those which have been selected for inclusion in this report have been so chosen because they clearly emphasize certain points of particular interest; it must be emphasized that the accuracy and compatibility of the experimental data obtained from the five wells here considered is in no way superior to that of the other 61 which are not included.

It is perhaps of interest to state that the oil-bearing formations of the Casabe field consist of three series of lenticular multiple sands 'A', 'B' and 'C' of which the 'A' and 'B' sands are of Oligocene and the 'C' sand is of Eocene age. They are fine grained to silty, rather argillaceous, and fairly well consolidated. The oil is heavy (20°-21° API) and viscous (40 cp ±). Some doubt exists as to the bubble point. It was first thought that the oil in all sands was undersaturated, but it now appears that pressures in the 'A' sands at least are below the bubble point. This, however, does not appear to affect the build-up curves.

2—Examples of the Pressure Build-up in a New Well

The earlier theory developed in Part I (I, 3) is designed to apply to a single well in an infinite field. Such a condition never obtains, of course, but it is further pointed out (I, 3d) that the case of a new well in a finite field is similar so long as the total withdrawal from the well has been kept small. It is difficult to lay down a criterion for the order of this smallness—practical applications seem to indicate that a normal well may be allowed to produce for a period of weeks or even of several months and it will still obey the "infinite reservoir" theory. Thus we may apply the infinite reservoir theory to the following case.

CB-161 was completed to the 'A' sands on 7th February 1950, and it was closed in from 16th February to 8th March while continual bottom hole pressure measurements were being taken. As will be apparent from the analysis of the results, it was not really necessary to leave the well closed in for such a long period in order to obtain adequate data, although it is interesting to observe how closely the pressure in the well followed the predicted course over such a long time.

This being a new well, we apply the theory for a single well in an infinite reservoir, as has been already explained. Reference to the Summary of Equations Chart, Appendix A, shows that the equation for the build-up in this case is number V, which in practical units is

\[ p = p_0 - \frac{G q u}{C \theta h} \log_{10} \left( \frac{t + \delta}{\theta} \right) \ldots (Va) \]

Reference to the table of Appendix B gives a definition of these symbols and of the units in which they are to be measured. The U.S. system of units being in use in Casabe, we must use the value of \( G \) given in the tabulation as pertaining to this system, namely 162.6. Thus equation Va is modified to

\[ p = p_0 - \frac{162.6 q u}{C \theta h} \log_{10} \left( \frac{t + \delta}{\theta} \right) \ldots (Vb) \]

which is thus the equation which we must attempt...
to fit to the experimental data. As the method of analysis consists of plotting \( \log_{10} \left( \frac{t + \delta}{\delta} \right) \) against \( p_* \), we firstly require a value for \( t_0 \). As has been previously explained (1, 3b) the equation \( V \) (or its modified form, \( V_b \)) assumes that \( q \), the rate of production, has remained constant for the whole life of the well \( (t_0) \). This not being in fact true, the method of correction is to take the last available production rate—in this particular case, 641 bbl/day—as the value for \( q \), and to use a corrected time \( t_* \) instead of the theoretical \( t_0 \), where \( t_0 \) is obtained by dividing the total cumulative production of the well (in this case, 5847 bbl) by this last production rate.

Hence for \( t_* \) in \( V_b \) above we substitute:

\[
t_0 = \frac{\text{Total cum. prod.}}{\text{Last prod. rate}} = \frac{5847 \text{ bbl}}{641 \text{ bbl/day}} = 219 \text{ hours}
\]

Thus the build-up equation \( V_b \) may be modified by the insertion of this value of \( t_* \) for \( t_0 \). This gives:

\[
p_* = p_0 - 162.6 \frac{k}{\sqrt{c}} \log_{10} \left( \frac{219 + \delta}{\delta} \right) \quad (Vc)
\]

The value of \( \delta = 641 \) bbl/day could now be inserted, but it is perhaps more convenient to leave this until a later stage.

The method of treating the experimentally obtained pressure data may be most easily explained by reproducing the measured values exactly as they were reported from well CB-161:

<table>
<thead>
<tr>
<th>Pressure ( p_* ) (psig)</th>
<th>C.I. time ( \delta ) (hours)</th>
<th>( \frac{219 + \delta}{\delta} )</th>
<th>( \log_{10} \frac{219 + \delta}{\delta} )</th>
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<td>1.599</td>
<td>0.4947</td>
</tr>
</tbody>
</table>

Columns 1 and 2 are derived directly from the bottom hole pressure readings, columns 3 and 4 are calculated from column 2, and finally the values of \( p_* \) from column 1 are plotted against the corresponding values of \( \log_{10} \frac{219 + \delta}{\delta} \) from column 4 as in figure 5.

It is convenient to plot \( p_* \) vertically and in a conventional manner, but to plot \( \log_{10} \frac{219 + \delta}{\delta} \) horizontally from right to left, i.e. with the zero on the right hand side as has been done in fig. 5, as this gives a more vivid impression of rising pressure.

If agreement is to be obtained with the theory, and thus with the derived equation \( V_c \), the points when so plotted should fall on a straight line, excepting only possibly the points corresponding to small \( \delta \) when the effects of the after-production (see footnote on page 3) may be still felt.

As can be seen from fig. 5, the accuracy with which these experimental points do in fact plot on a straight line in this case—and particularly the very last point which represents nearly 20 days closed in—is really quite remarkable.

The interpretation of this figure 5 is simple. Firstly we may deduce a value for \( p_0 \), which is done by extending the straight line plot to the point corresponding to \( \log_{10} \frac{219 + \delta}{\delta} = 0 \) and reading the corresponding pressure which gives in this case a value for \( p_0 \) of 1280 psig. Although we have defined \( p_0 \) to be the initial reservoir pressure, this must be interpreted somewhat widely. In this case, for example, the well considered was drilled as an infilling well into an already heavily drilled field, and so the pressure \( p_0 = 1280 \) psig must be interpreted as the reservoir pressure at the well at the moment of its completion.

In addition to this pressure determination we may determine the permeability \( k \) from the slope of the straight line. This can perhaps best be done by selecting two arbitrary and fairly widely separated points A and B on the line (fig. 5). The pressure difference between A and B is \( 1272 - 1182 = 90 \) psi, and the corresponding difference in \( \log_{10} \frac{219 + \delta}{\delta} \) is \( 1.2 - 1.1 \approx 1.1 \). The slope of the line is then obtained by division thus
Fig. 5 and 6. Observed pressure build-up curves in new wells.
SECTION II, PREPRINT 7

Examples of the Pressure Build-up in a Well near a Linear Barrier Fault

The theory which has been developed for a linear barrier fault is strictly only applicable to a well in an otherwise infinite reservoir. However, we may approximate to this condition by a new well close to a fault and considerably farther from any other barrier. Such a well is CB-123, completed to the C-sands at the beginning of September 1949; it was closed in for test from 4th November 1949 to 5th January 1950. Reference to Appendix A shows that the relevant equations are numbers IX, X and XI, which, substituting the values of A, D, F and G appropriate to U.S. units from Appendix B, become

\[ p_\infty = p_0 - \frac{q \mu}{C_s kh} \left( \frac{162.6 \log_{10} t + \theta}{10} \right) \]

\[ -70.60 E_i \left( -\frac{3793 \alpha^2 f \mu c}{k (t_s + \theta)} \right) \]

\[ + 70.60 E_i \left( -\frac{3793 \alpha^2 f \mu c}{k \theta} \right) \]

which is valid for all ranges of \( \theta \), and may be approximated to by

\[ p_\infty = p_0 - \frac{q \mu}{C_s kh} \left( \frac{162.6 \log_{10} t + \theta}{10} \right) \]

\[ -70.60 E_i \left( -\frac{3793 \alpha^2 f \mu c}{k t_s} \right) \]

for all very large values of \( \theta \), and by

\[ p_\infty = p_0 - \frac{q \mu}{C_s kh} \log_{10} \frac{t + \theta}{\theta} \]

for very large \( \theta \).

The exact equation IXa is not of great interest as far as the applications are concerned. Instead, the two approximations Xa and XIa are used.

Firstly we require a value of \( t_0 \), which is derived as for the previous example. We use \( q = 275 \) bbl/day and \( t_0 = 1353 \) hours. The value of \( q \), as before, we do not yet substitute. However, we insert the value of \( t_0 \) in the two approximate equations which become

\[ p_\infty = p_0 - \frac{q \mu}{C_s kh} \left( \frac{3353 + \theta}{\theta} \right) \]

\[ -70.60 E_i \left( -\frac{3793 \alpha^2 f \mu c}{1353 k} \right) \]

for all but very large \( \theta \), and

\[ p_\infty = p_0 - \frac{q \mu}{C_s kh} \log_{10} \frac{3353 + \theta}{\theta} \]

The points corresponding to very large closed-in times do not, of course, plot with the same accuracy on the straight line. This is because the earlier group of 17 points were obtained from one (or perhaps two) runs of the pressure gauge, while the last 8 points are eight spot readings taken at widely spaced values of \( \theta \).
Fig. 7. Observed pressure build-up curve in well affected by a linear barrier fault.
SECTION II, PREPRINT 7

whence $a^2 = 8.83 \times 10^4$
or $a = 297$ feet

However, this figure is not in agreement with the present (rather obscure) subsurface picture which places the fault at about 1100 feet from the well.

Instead of proceeding in the manner just detailed, however, we may modify our approach thus.

Firstly, we accept the possibility that the stopes of the lines I and II (figure 7) may not be exactly in the ratio of $2:1$. If we suppose this ratio to be $b:1$, a first approximation to the build-up equation may be obtained by modifying equation IX to read

$$q!L_1t+.& = P._C.kh G \log_{10}.& .(D_a!LC) + (b-1) AEl (D_a!PC) .(rXb)$$

$c = 5.1 \times 10^{-4}$ volts/foot

$k = 270$ md

the above equation XIX becomes

$$-Ei \left( -\frac{3793 \times a^2 \times .25 \times 40 \times 5.1 \times 10^{-4}}{1353 \times 270} \right) = 2.53$$
i.e. $-Ei (-.530 \times 10^{-6} a^2) = 2.53$

and reference to a table of Ei-functions (2, 3) shows that, for this value of $-Ei (-.530 \times 10^{-6} a^2)$ the quantity

$.530 \times 10^{-6} a^2 = .0468$

whence $a^2 = 8.83 \times 10^4$
or $a = 297$ feet

However, this figure is not in agreement with the present (rather obscure) subsurface picture which places the fault at about 1100 feet from the well.

Instead of proceeding in the manner just detailed, however, we may modify our approach thus.

Firstly, we accept the possibility that the stopes of the lines I and II (figure 7) may not be exactly in the ratio of $2:1$. If we suppose this ratio to be $b:1$, a first approximation to the build-up equation may be obtained by modifying equation IX to read

$$(in \ practical \ units)$$

$$p_0 = p_a - \frac{q!L_1}{C_a.kh} G \log_{10} t + \frac{9}{b}$$

$$-(b-1) AEl \left( -\frac{D_a!PC}{k(t_0 + \frac{8}{b})} \right)$$

$$+(b-1) AEl \left( -\frac{D_a!PC}{k \frac{8}{b}} \right) \ldots (IXb)$$

The problem of fitting this equation to the observed points can then be performed thus.

An approximate value of "b" is derived from the ratio of the two stopes, a value of $p_0$ is obtained by the extrapolation of the last part of the build-up curve and a value of $a$ is obtained just as previously described i.e. by equating the $-Ei$ function to $2.3 \times \log_{10} t + \frac{9}{b}$ at the point of intersection of the two straight lines, it is convenient that, given this point of intersection, the value of "a" thereby determined is independent of the slopes of the lines.

Due to some uncertainty in the value of "b" (for in practice $b$ must usually be very large indeed before line I is thoroughly established, and so it is possible for some of the later points in the transition zone to be mistaken for points on this line I) it may not now be taken as definite that the best possible values of $a$, $b$ and $p_0$ have now been chosen, although it may be expected that they will not differ too widely from the final values.

Thus the equation IXb is now plotted to see how nearly the calculated curve fits the observed points.

1.62.6 $\frac{q!L_1}{C_a.kh} = \text{slope of line II} = 125$

and substituting the known values

$q = 275 \text{ bbl/day (Ref. page 11)}$

$\mu = 40 \text{ cp}$ \{from PVT data\}

$h = 57 \text{ ft (Electric log)}$

we have

$$162.6 \frac{q!L_1}{C_a.kh} = \text{slope of line II} = 125$$
i.e. $33.700 = 125$

whence $k = 270$ md.

We may also estimate the distance of the fault from the well by the following method.

We equate the function $-Ei \left( -\frac{3793 a^2 \mu C}{1353 k} \right)$ to the value of $2.3 \log_{10} \frac{1353 + \frac{9}{b}}{t + \frac{8}{b}}$ at the point of intersection of the two straight lines I and II (fig. 7).

These lines intersect at $\log_{10} \frac{1353 + \frac{9}{b}}{t + \frac{8}{b}} = 1.1$, and so we have the equation

$$-Ei \left( -\frac{3793 a^2 \mu C}{1353 k} \right) = 2.3 \times 1.1 = 2.53 \ldots (XIX)$$

Now if we substitute the known values

$I = 25$

$\mu = 40 \text{ cp}$

$\mu = 40 \text{ cp}$

$\mu = 40 \text{ cp}$

† Note that the factor 2.3 is a universally applicable constant for this method (actually it is in 10).
in the event of an unsatisfactory fit, the three quantities \( a \), \( b \) and \( p_0 \) may be somewhat modified, and a process of trial and error will finally lead to the best values. Fortunately this seems to be fairly quick in application.

The end result of such a process may be seen in figure 7, in which the broken line has been calculated to fit the observed points using the method outlined above.

The final "best" values of the quantities \( a \), \( b \) and \( p_0 \) were not widely divergent from the values already obtained, as may be seen in the following tabulation.
There is a further possible use of this linear barrier fault theory.

Figure 8 shows the usual $p_*$ vs. $\log_{10}\frac{t_0+\delta}{\delta}$ plot of well CB-117. The unexpectedly high pressure reading at 1134 psig may be easily explained by the presence of a fault. Thus a straight line II may first be drawn through the other (earlier) points, and its slope measured—in this example it has a value of 96 (whence $k$ may be obtained as before, formula Xb).

Then through the exceptionally high pressure point at 1134 psig we draw a line I at twice the slope of the line II, i.e. at a slope of 192 and extrapolate this to give a value of $p_0 = 1312$ psig.

This method must, of course, be used with considerable discretion.

4—Example of the Pressure Build-up in a Well in a Finite Reservoir

The method again is to plot $p_*$ against $\log_{10}\frac{t_0+\delta}{\delta}$ and from the straight line plot so obtained to calculate $k$. However, linear extrapolation of the straight line region of this build-up curve does not give the value of the reservoir pressure.

As an example of the method we include the results of a long survey made on well CB-110 ($p_*$ plotted against $\log_{10}\frac{t_0+\delta}{\delta}$) figure 9. The relevant equations are XV, XVI and XVII. We proceed thus:

The slope of the initial straight part is measured to be 121, and then from equation XV we equate this value to the coefficient of $\log_{10}\frac{t_0+\delta}{\delta}$ to give an equation $121 = G \frac{q \mu}{C_w k h}$ and taking $G = 162.6$ (U.S. units, Appendix B) we get

$$\frac{q \mu}{C_w k h} = \frac{121}{162.6} = .744$$

which may be solved for $k$, knowing $q$, $\mu$, $C_w$ and $h$, as before.

If we extrapolate the straight line, however, we get a false pressure reading $p_* = 1313$ psig. To correct this we also need to know the value of $p_0$. We take in this case 1343 psig (as measured in a

---

**Figure 9.** Observed pressure build-up curve in well in a finite reservoir.
previous pressure survey) and use equation XVI, which is (Ref. Appendix A)

\[ p^* = p_e - A \frac{q \mu}{C \cdot k h} \left( \frac{B_n \mu \rho c}{k t_o} \right) \]  

(XVIa)

Substituting then the values

\[ p^* = 1343 \text{ psig} \]
\[ p_0 = 1343 \text{ psig} \]
\[ A = 70.60 \] (ref. Appendix B)

and \( \frac{q \mu}{C \cdot k h} = .744 \) (from slope of linear part of curve, as calculated above)

in equation XVIa, we have

\[ 1313 = 1343 - 70.60 \times .744 \frac{(B_n \mu \rho c)}{k t_o} \]

which may be solved for \( y \) to give

\[ y \frac{(B_n \mu \rho c)}{k t_o} = 1343 - 1313 = .571 \]

This \( y \)-function is plotted in fig. 10. Reference to this figure shows that if \( y(u) = .571 \), then \( u = .537 \). Thus we have for this case

\[ \frac{B_n \mu \rho c}{k t_o} = .537 \]

Now we go to equation XVII, which we write

\[ p_e = p_e - A \frac{q \mu}{C \cdot k h} \left( \frac{B_n \mu \rho c}{k t_o} \right) \]  

...... (XVIIa)

and we substitute the values

\[ p_e = 1343 \text{ psig} \]
\[ A = 70.60 \]
\[ \frac{q \mu}{C \cdot k h} = .744 \] as obtained above
\[ \frac{B_n \mu \rho c}{k t_o} = .537 \]

\[ p_e = 1343 - 70.60 \times .744 \times .537 \]

As the calculated static pressure at the well after infinite time closed in.

Agreement is here not too good—the apparent error in \( p_e \) is about 55 psi as may be seen from figure 9. This is, as yet, the only well which we have had closed in for long enough to be able to make such a check. Accordingly this method must still be treated as unproven.

5—Comments

In the application of these methods there are certain definite dangers. Firstly it is not always clear which part of the build-up curve is to be used to determine \( k \). It is not uncommon for many of the early pressure readings to fall on a straight line, when plotted against \( \log t \), although they have been taken during the period of the after-production. This report does not attempt to cover this period of after-production, and this early straight line, having a slope often many times greater than the true value of \( \frac{Gq \rho c}{C \cdot k h} \), will thus give incorrect values of \( k \) and \( p_0 \).

Secondly, when the after-production has ceased, all wells show an initial slope of \( \frac{Gq \rho c}{C \cdot k h} \) and yet for extrapolation purposes it is necessary to know which case must be catered for; infinite, semi-infinite or finite reservoir. Thus a good deal of care must be exercised in analysing the results of build-up curves if reliable data are to be obtained.

After analysing a large number of wells by this method we have obtained the impression that acceptable values for the permeability are more frequently obtained than for the extrapolated pressure. This is probably due to the fact that whereas one is content with only an approximate value of \( k \), the limits of error within which the reservoir pressure is required are very much smaller.

Bibliography

(2) Jahnke und Emde, Funktionentafeln Section 1, pp. 6-8.
APPENDIX A—SUMMARY OF EQUATIONS CHART

<table>
<thead>
<tr>
<th>EQUATION No.</th>
<th>REF. PAGE No.</th>
<th>BOUNDARY CONDITIONS</th>
<th>QUANTITY EVALUATED</th>
<th>RANGE OF VALUES OF TIME $t$ FOR WHICH EQU. IS VALID</th>
<th>THE MORE IMPORTANT EQUATIONS</th>
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<tr>
<td>III 3</td>
<td></td>
<td>Pressure at shut-in</td>
<td>All values</td>
<td>$p = p_e + \left(\frac{q^2}{2gh_0}\right) e^{-\frac{t}{\tau}}$</td>
<td>$p = p_e + \frac{Ag_p}{4kh_0} e^{-\frac{t}{\tau}}$</td>
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<tr>
<td>V 3</td>
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<td>Pressure</td>
<td>All values</td>
<td>$p_0 = p_e - \frac{q^2}{2gh_0} \ln \left(\frac{t}{\tau}\right)$</td>
<td>$p_0 = p_e - \frac{Ag_p}{4kh_0} \ln \left(\frac{t}{\tau}\right)$</td>
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<td>Pressure in buildup</td>
<td>All values</td>
<td>$p_w = p_e - \frac{q^2}{4kh_0} \ln \left(\frac{t}{\tau}\right)$</td>
<td>$p_w = p_e - \frac{Ag_p}{4kh_0} \ln \left(\frac{t}{\tau}\right)$</td>
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<td>X 4</td>
<td></td>
<td>All except very large values</td>
<td>$p_w = p_e - \frac{q^2}{4kh_0} \ln \left(\frac{t}{\tau}\right)$</td>
<td>$p_w = p_e - \frac{Ag_p}{4kh_0} \ln \left(\frac{t}{\tau}\right)$</td>
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<td>X 4</td>
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<td>Very large values</td>
<td>$p_w = p_e - \frac{q^2}{4kh_0} \ln \left(\frac{t}{\tau}\right)$</td>
<td>$p_w = p_e - \frac{Ag_p}{4kh_0} \ln \left(\frac{t}{\tau}\right)$</td>
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<td>Pressure at any point</td>
<td>All values</td>
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<td>$p = p_e + \frac{Ag_p}{4kh_0} \left(1 - e^{-\frac{t}{\tau}}\right)$</td>
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<td>XIV 9</td>
<td></td>
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<td>All values</td>
<td>$p_w = p_e - \frac{q^2}{4kh_0} \ln \left(\frac{t}{\tau}\right)$</td>
<td>$p_w = p_e - \frac{Ag_p}{4kh_0} \ln \left(\frac{t}{\tau}\right)$</td>
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<tr>
<td>XV 8</td>
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<td>$p_e = p_e$</td>
<td>$p_e = p_e$</td>
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<td>Closed-in pressure</td>
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<td>$p_e = p_e$</td>
<td>$p_e = p_e$</td>
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Note: Some of the equations which apply to pressure buildup will be strictly applicable while the effect of the after-production is still significant.
## APPENDIX B — LIST OF SYMBOLS

<table>
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<th>SYMBOL</th>
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<th>UNITS</th>
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<tr>
<td>A</td>
<td>A numerical constant</td>
<td>—</td>
</tr>
<tr>
<td>a</td>
<td>Distance from well to fault</td>
<td>cm</td>
</tr>
<tr>
<td>B</td>
<td>A numerical constant</td>
<td>—</td>
</tr>
<tr>
<td>C</td>
<td>Shrinkage factor, tank volume divided by subsurface volume</td>
<td>—</td>
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<tr>
<td>c</td>
<td>Compressibility</td>
<td>vols/vol/psi</td>
</tr>
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<td>D</td>
<td>The exponential integral function (cf. footnote to page 2)</td>
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<tr>
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<td>The exponential constant</td>
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<td>a fraction</td>
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<td>Pay thickness</td>
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<td>Bessel function of the first kind of order zero</td>
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</tr>
<tr>
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<td>Bessel function of the first kind of order unity</td>
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<td>darcy</td>
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<td>Logarithm to base e</td>
<td>—</td>
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<tr>
<td>log 10</td>
<td>Logarithm to base 10</td>
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<td>p</td>
<td>Viscosity</td>
<td>cp</td>
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<tr>
<td>P</td>
<td>Pressure at distance r and time t</td>
<td>atm</td>
</tr>
<tr>
<td>P₀</td>
<td>Initial reservoir pressure, or reservoir pressure at moment of completion of well</td>
<td>atm</td>
</tr>
<tr>
<td>Pₚ</td>
<td>Final closed-in static pressure in the well</td>
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<tr>
<td>Pₑ</td>
<td>Pressure in the well during build-up</td>
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<td>Pₑ</td>
<td>False extrapolated pressure</td>
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<tr>
<td>Qₑ</td>
<td>Total inflow over circle r = rₑ</td>
<td>cc of subsurface volume</td>
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<tr>
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<td>Rate of production of well, assumed constant †</td>
<td>cc of subsurface volume per sec.</td>
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<td>q₀, q₁, q₂, q₃</td>
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<td>cc of subsurface volume per sec.</td>
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<td>r</td>
<td>Distance from well centre-line</td>
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</tr>
<tr>
<td>rₑ</td>
<td>Radius of well</td>
<td>cm</td>
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<td>ρ</td>
<td>Density of reservoir fluid at pressure</td>
<td>gm/cc</td>
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<tr>
<td>ρₛ</td>
<td>Density of reservoir fluid at pressure ρₛ</td>
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<tr>
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<td>Value of tₑ corrected for non-constancy of q †</td>
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<td>x</td>
<td>An independent variable</td>
<td>—</td>
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<td>xₑ</td>
<td>The nᵗʰ root (in order of increasing magnitude) of f₁ (x) = 0</td>
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</tr>
<tr>
<td>y</td>
<td>The y-function, defined by y(α) = Ei(-α) + ½α²</td>
<td>—</td>
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† For methods of correction when q has not been kept constant, see paragraph I, 3b.
Fig. 10. Graph of the function $y(u)$. 

$$y(u) = \frac{1}{1} e^{iu} + Ei(-u)$$