Simplified Equations of Flow in Gas Drive Reservoirs and the Theoretical Foundation of Multiphase Pressure Buildup Analyses

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ABSTRACT

Simplified equations are developed for the flow of fluids in gas drive reservoirs in which the effects of gravity can be neglected. The results show that the pressure distribution is governed by a nonlinear heat flow type of equation, and the saturation distributions are related to the pressure by the same equations as those developed by Muskat for the average pressure and saturations in gas drive reservoirs.

Under certain conditions the equation for the pressure can be approximated by the linear form of the heat flow equation. This equation is analogous to the equation for the pressure in single-phase compressible flow. The analysis reveals that the equation for the pressure in multiphase flows can be obtained directly from the equation for the pressure in single-phase flows by simply replacing the single-phase compressibility by the total compressibility and the single-phase mobility by the sum of the mobilities of all the fluids present. Perrine's method of pressure buildup analysis for multiphase flows is based on these same substitutions. Thus, the development presented in this paper constitutes a theoretical justification for Perrine's method of pressure buildup analysis.

INTRODUCTION

One of the most difficult problems associated with the production of oil is the determination of the flow of the fluids within the reservoirs. One purpose of this paper is to present the development of simplified equations for the flow in gas drive reservoirs. The other purpose is to present some of the needed theoretical foundation for the methods currently being used in the analysis of multiphase pressure buildup data. The analysis presented in this paper makes use of a partial linearization of the equations governing multiphase flows. It is felt that the process of linearizing or partially linearizing the nonlinear equations of multiphase flows can lead to useful results since this process has proved extremely valuable in the study of other fields of science and engineering where nonlinear equations are encountered.

Pressure buildup data of shut-in wells have been used for many years to estimate the static reservoir pressure. These data have also been used in studying the average permeability both far away from and close to the wellbores. The majority of the buildup data are obtained from wells producing two or three fluids. As yet the theoretical foundations for the methods of buildup analysis used in connection with multiphase buildup data have not been established.

A number of methods of analysis of pressure buildup data have been developed.* Most of these methods apply only to single-phase flows, and almost all of the theoretical analysis is confined to single-phase flows. Perrine1 developed a method of analysis for multiphase flows from the method for single-phase flows of Miller, Dyes, and Hutchinson.2 He did this by simply replacing the single-phase compressibility by the multiphase compressibility and replacing the single-phase mobility by the sum of the mobilities of the fluids in the multiphase flows. Obviously, such a procedure lacks a certain amount of theoretical justification, and such practices can lead to erroneous results. Fortunately, Perrine's results have a theoretical justification, and this justification is presented in the following discussion.

DISCUSSION

It is assumed that gravitational effects and the compressibility of the rock can be neglected. Under these assumptions the equations governing the simultaneous flow of gas, oil and water in porous media are:

\[ \nabla \cdot \left( \frac{R}{\mu \beta_r} + \frac{R_s \phi P}{\mu \beta_r} + \frac{S}{\mu \beta_r} \right) \nabla P \]

\[ = \phi \frac{\partial}{\partial t} \left( \frac{R}{\mu \beta_r} S + \frac{R_s \phi P}{\mu \beta_r} + S \right) \]  \( \ldots \) \( \text{(1)} \)

\[ \nabla \cdot \left( \frac{k_w}{\mu \beta_w} \nabla P \right) = \phi \frac{\partial}{\partial t} \left( 
\frac{S}{\mu \beta r} \right) \]  \( \ldots \) \( \text{(2)} \)

and

\[ \nabla \cdot \left( \frac{k_w}{\mu \beta_w} \nabla P \right) = \phi \frac{\partial}{\partial t} \left( 
\frac{S}{\mu \beta r} \right) \]  \( \ldots \) \( \text{(3)} \)

where \( \nabla \) is a vector operator.

The preceding equations can be expanded into the following expressions.

1References given at end of paper.
*See AIME Symbols list in Text. AIME (1955) 207, 265, for definitions of symbols not defined in the text.

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\[ \left( \frac{R, k_s^* + R, k_p^*}{\mu_s^*} + \frac{k_v}{\mu_w^*} \right) \nabla^4 P + \left( \frac{R, \alpha}{\mu_s^*} + \frac{R, \alpha}{\mu_w^*} \right) \nabla^2 P + \left( \frac{R, \alpha}{\mu_s^*} + \frac{R, \alpha}{\mu_w^*} \right) \nabla^2 P = \frac{\partial}{\partial t} \left( \frac{S_o}{\beta_o} \right) \]

\[ + \frac{1}{\mu_s^*} \frac{\partial k_v}{\partial S_o} \nabla^2 P \cdot \nabla S_o + \left[ \frac{k_v}{\mu_w^*} + \frac{k_v}{\mu_w^*} \frac{\partial}{\partial S_w} \left( \frac{R, \alpha}{\mu_w^*} \right) \nabla^2 P \cdot \nabla P \right. \]

\[ \left. + \frac{1}{\mu_w^*} \frac{\partial k_v}{\partial S_w} \nabla^2 P \cdot \nabla S_w + \frac{1}{\mu_s^*} \frac{\partial k_v}{\partial S_w} \nabla P \cdot \nabla S_w \right] \]

\[ + \frac{1}{\mu_w^*} \frac{\partial k_v}{\partial S_w} \nabla P \cdot \nabla P = \frac{\partial}{\partial t} \left( \frac{S_o}{\beta_o} + \frac{R, \alpha}{\beta_o} \right) \]

(4)

\[ \frac{k_v}{\mu_s^*} \nabla^4 P + \frac{1}{\mu_s^*} \frac{\partial k_v}{\partial S_o} \nabla P \cdot \nabla S_o + \frac{1}{\mu_s^*} \frac{\partial k_v}{\partial S_w} \nabla P \cdot \nabla S_w + \frac{1}{\mu_w^*} \frac{\partial k_v}{\partial S_w} \nabla P \cdot \nabla P = \frac{\partial}{\partial t} \left( \frac{S_o}{\beta_o} \right) \]

(5)

and

\[ \frac{k_v}{\mu_s^*} \nabla^4 P + \frac{1}{\mu_s^*} \frac{\partial k_v}{\partial S_o} \nabla P \cdot \nabla S_o + \frac{1}{\mu_w^*} \frac{\partial k_v}{\partial S_w} \nabla P \cdot \nabla S_w + \frac{1}{\mu_w^*} \frac{\partial k_v}{\partial S_w} \nabla P \cdot \nabla P = \frac{\partial}{\partial t} \left( \frac{S_o}{\beta_o} \right) \]

(6)

where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \).

In those cases where the pressure and saturation gradients are small the vector products, \( \nabla P \cdot \nabla P, \nabla S_o \cdot \nabla S_o \), and \( \nabla S_o \cdot \nabla S_w \) are small compared to the magnitudes of \( \nabla P, \nabla S_o \), and \( \nabla S_w \). For these cases the terms containing the products of the pressure and saturation gradients can be neglected for a first approximation, and Eqs. 4, 5 and 6 reduce to

\[ \nabla^2 P = \frac{\partial}{\partial t} \left( \frac{S_o}{\beta_o} \right) \]

(7)

\[ \nabla^2 P = \frac{\phi \beta_o}{k_v} S_o \frac{\partial}{\partial t} \left( \frac{S_o}{\beta_o} \right) \]

(8)

and

\[ \nabla^2 P = \frac{\phi \beta_o}{k_v} S_o \frac{\partial}{\partial t} \left( \frac{S_o}{\beta_o} \right) \]

(9)

It should be remembered that only the variations of \( P \), \( S_o \), and \( S_w \) with time must be small in order for the three preceding equations to be valid. The variations of \( P \), \( S_o \), and \( S_w \) with time are not assumed to be small.

The elimination of \( \nabla^2 P \) from Eqs. 7, 8 and 9 leads to the following system of ordinary differential equations:

\[ \frac{dS_o}{dP} = S_o \beta_o + \frac{S_o}{\beta_o} C_i \]

(10)

and

\[ \frac{dS_w}{dP} = S_w \beta_o + \frac{S_w}{\beta_o} C_i \]

(11)

where the primes denote derivatives with respect to \( P \), \( C_i \) is the total compressibility and \( \lambda \) is the sum of the mobilities of the fluids present. Thus,

\[ C_i = \frac{S_o \beta_o'}{\beta_o} + \frac{S_w \beta_o'}{\beta_o} - \frac{S_o \beta_o' R_{o'}'}{\beta_o'} + \frac{S_w \beta_o' R_{o'}'}{\beta_o'} - \frac{S_o \beta_o'}{\beta_o} \]

and \( \lambda = \lambda_1 + \lambda_2 + \lambda_4 \).

Eq. 10 states that the rate of change of the oil saturation with pressure is equal to the oil saturation, multiplied by the negative of the compressibility of the oil, plus the ratio of the mobility of the oil phase to the total mobility multiplied by the total compressibility. Eq. 11 constitutes a similar statement for the water phase. Eqs. 10 and 11 can be readily integrated by numerical or graphical methods.

Eqs. 10 and 11 are the same relations as those given by Muskat for solution gas drive reservoirs but are in a different form. The development presented herein indicates that the integral of these equations yields relations between the saturations and pressure which are valid at all points in the reservoir where the saturation and pressure gradients are small. In Muskat's application of these equations, the saturations and the pressure represent average values for the reservoir.

The initial values of saturations and pressure generally used in a Muskat analysis of a solution gas drive reservoir correspond to the initial reservoir conditions before any oil has been produced. In reservoirs in which large amounts of fluid migrate from one portion of the reservoir to another by gravity segregation, the saturations and pressure are not related by Eqs. 10 and 11. Since in most cases relatively long times are required for appreciable amounts of fluid migration due to gravity to take place, Eqs. 10 and 11 may be expected to apply to those phenomena which occur in relatively short time periods. The pressure buildup, after the well has been closed in, is an example of such a phenomenon. In applying Eqs. 10 and 11 to short period phenomena, the initial pressure and saturations should correspond to the reservoir conditions at the beginning of the phenomena and not to the initial reservoir conditions.

Combining Eqs. 10 and 11 with any one of the Eqs. 7, 8 and 9 yields

\[ \nabla^2 P = \frac{\phi C_i}{\lambda} \frac{\partial}{\partial t} \]

(12)

Eq. 12 is the well known heat flow or diffusion equation with the coefficient a function of \( P \). In the derivation of Eq. 12 the pressure gradient was assumed to be small. The partial derivative of \( P \) with respect to time was not limited to small values. Thus, the nonlinear character of Eq. 12 should have a physical meaning. Unfortunately, the difficulties involved in solving Eq. 12 in its nonlinear form are much greater than where it is approximated by linear equations.

In the analysis of certain phenomena which occur over relatively short time periods, the coefficient, \( \frac{\phi C_i}{\lambda} \), in Eq. 12 can be approximated by a constant. In essence, Perrine obtained the linear approximation to Eq. 12 intuitively from the equation for the flow of a single-phase compressible fluid. The development of Eq. 12 presented herein supplies a theoretical justification for the use of Perrine's method of multiphase buildup analysis.

It appears that Eq. 12 can be used as a basis, on which a number of methods of single-phase buildup analysis can be extended to multiphase flows; however, the author has not explored these possibilities in any detail. Also, it appears that the equations developed herein can be applied to problems other than the analysis of pressure buildup data. In particular, the solution of Eqs. 10, 11 and 12 which satisfies the proper
boundary conditions should yield a first approximation to the distribution of the pressure and saturations in gas drive reservoirs except, possibly, near the wells. Near the wellbores the flow can usually be approximated by steady-state relations.

The equations developed herein contain two serious limitations. The assumption that the sands are homogeneous is one, and the assumption that the effects of gravity can be neglected is the other. Actually, the derivation can be modified to allow small variations in the permeability without changing the resulting equations. The reason for this is that the variations in the permeability only lead to terms which are small and can be neglected. The effects of gravity may result in large variations in the saturations in the vertical direction. In such cases the derivation presented in this paper breaks down. It is interesting to note that the results obtained by Cook indicate the gas saturation is not critical in determining the reservoir pressure.

**Conclusions**

1. The pressure distribution in a homogeneous gas drive reservoir in which the effects of gravity can be neglected is determined by the heat flow equation.

2. For homogeneous gas drive reservoirs in which the effects of gravity can be neglected, the distribution of the saturation is related to the pressure by the same differential equations as those given by Muskat for the average pressure and saturations in a gas drive reservoir.

3. The Muskat solution gas drive analysis is more general than is implied by the derivation given by Muskat which is based on a tank-type model.

4. Perrine's method of multiphase pressure buildup analysis has a firm theoretical foundation.

**References**